Robust FWI With Source Estimation

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Abstract: A growing body of work underscores the importance of robust modeling for data with large outliers or artifacts that are not captured by the forward model. Effectively, the least-squares penalty on the residual is replaced by a robust penalty, such as Huber, Hybrid l1-l2 or Student's t.

We show that it is essential to also use a robust penalty for source estimation, and present a general approach to robust waveform inversion with robust source estimation. There is no closed form solution for the optimal source weights (as in the classic case), but we need only solve a set of independent scalar optimization problems, which we do using a few iterations of a Newton-like method at a negligible cost. We show numerical examples illustrating robust source estimation and robust waveform inversion on synthetic data with outliers.

Review of NLLS FWI formulation:

$$\min_{\mathbf{m},\boldsymbol{\alpha}} \quad \phi(\mathbf{m},\boldsymbol{\alpha}) := \frac{1}{2}\sum_{i,\omega} \|\alpha_{i,\omega}\boldsymbol{\mathcal{F}}_{i,\omega}(\mathbf{m}) - \mathbf{d}_{i,\omega}\|_2^2$$

In the standard (NNLS) formulation, **m** is the velocity model, **d** are data, and $\boldsymbol{\mathcal{F}}$ is the forward model.

Sources are computed on the fly, during every iteration:

$$\hat{\alpha}^k_{i,\omega} = (\boldsymbol{\mathcal{F}}^k_{i,\omega})^T d_{i,\omega}/\|\boldsymbol{\mathcal{F}}^k_{i,\omega}\|_2^2$$

Review of Robust FWI:

- A lot of effort in conventional FWI is spent *pre-processing* and doing *complex modeling*.
- Robust formulations can recover good models despite *errors in data* and *errors in modeling*.
- Examples of robust models include Huber, Hybrid, Cauchy, and Student's t.

Statistical Perspective on Robust FWI:

- We design robust penalties by *modeling* the error in FWI using some particular parametric distribution (e.g. Gaussian, Student's t):

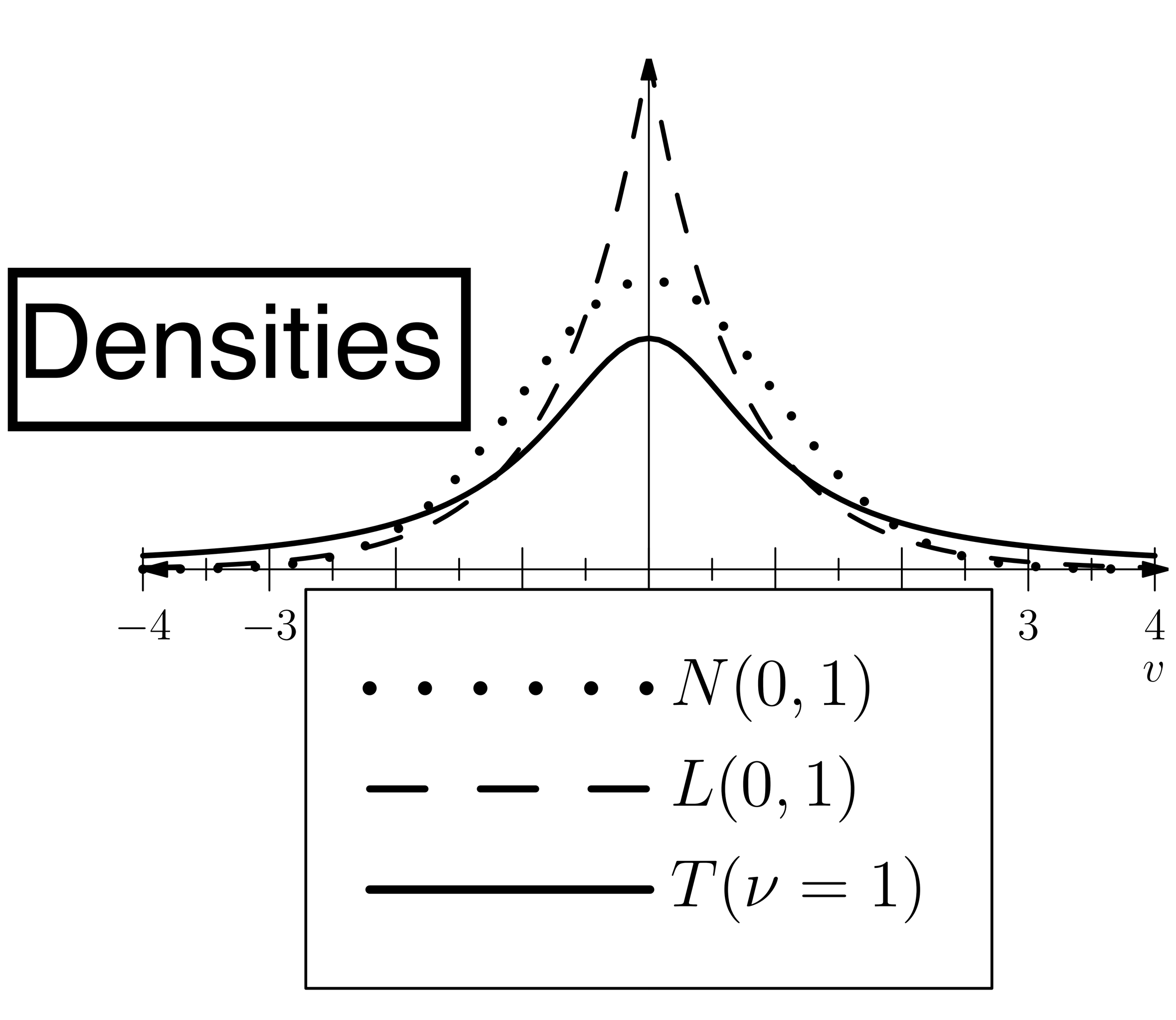
$$\mathbf{d}_{i,\omega} = \alpha_{i,\omega}\boldsymbol{\mathcal{F}}_{i,\omega}(\mathbf{m}) + \boldsymbol{\epsilon}_{i,\omega}$$
$$\boldsymbol{\epsilon}_{i,\omega} = \exp(-p(\mathbf{x}))$$

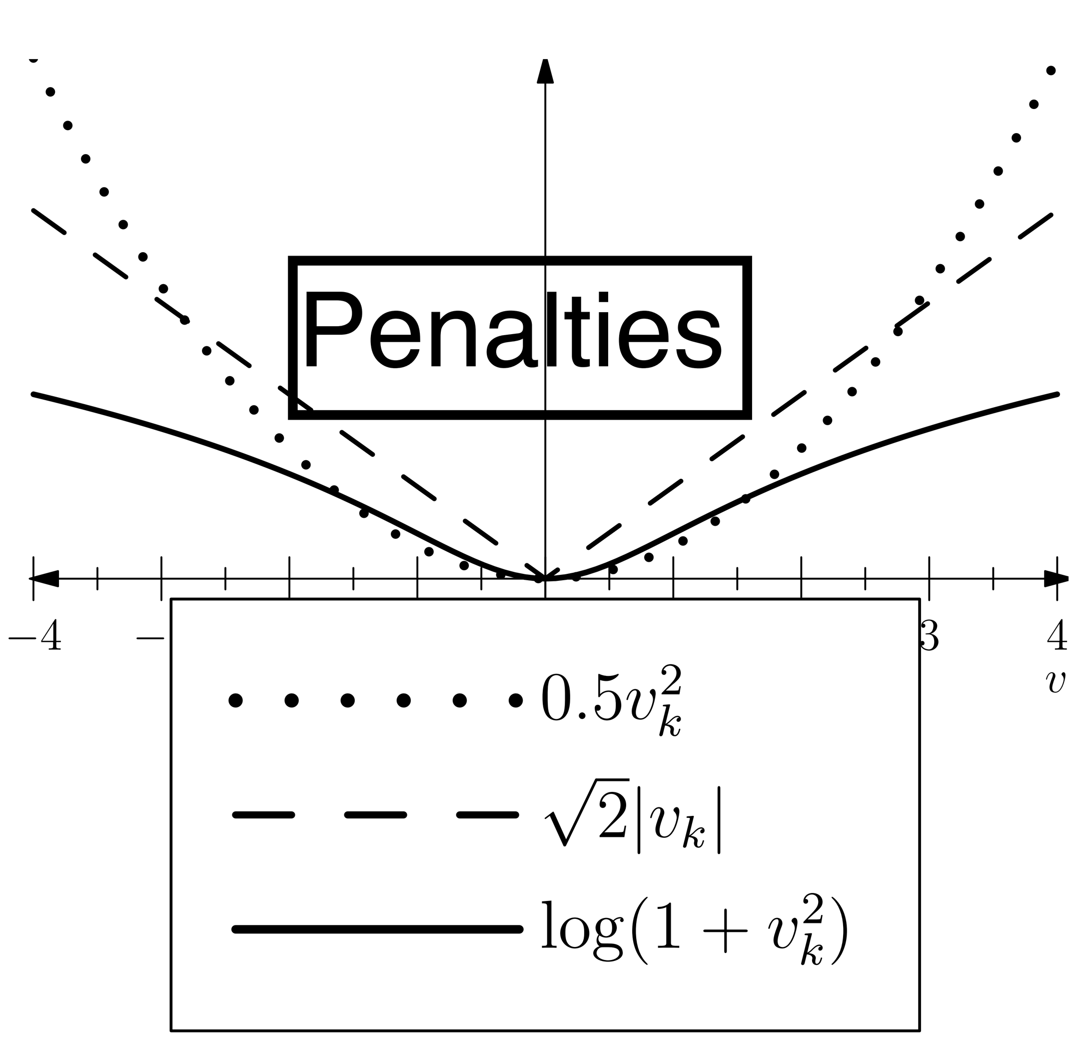
- We then solve for the maximum likelihood estimate:

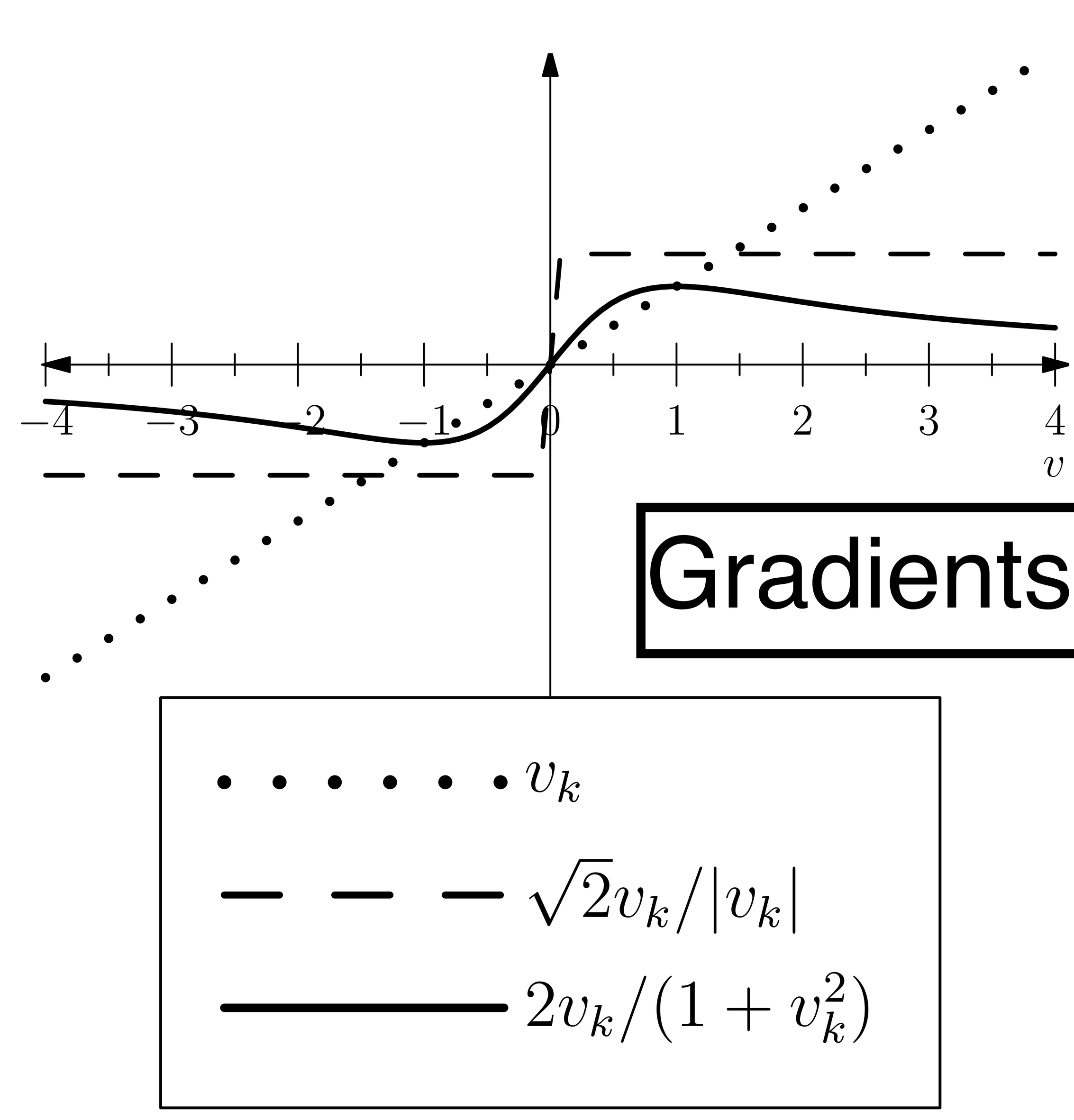
$$\min_{\mathbf{m},\,\boldsymbol{\alpha}} \sum_{i,\omega} p(\alpha_{i,\omega}\boldsymbol{\mathcal{F}}_{i,\omega}(\mathbf{m}) - \mathbf{d}_{i,\omega})$$

- For Student's t formulation, the problem becomes

$$\min_{\mathbf{m},\,\boldsymbol{\alpha}} \sum_{i,\omega,j} \log(\nu + (\alpha_{i,\omega}\boldsymbol{\mathcal{F}}_{i,\omega,j}(\mathbf{m}) - \mathbf{d}_{i,\omega,j})^2$$







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General Formulation for Source Estimation:

- At each iteration, re-estimate sources on the fly by solving the problem

$$\hat{\alpha}^k_{i,\omega} = \arg\min_{\alpha} p(\alpha \boldsymbol{\mathcal{F}}^k_{i,\omega} - \mathbf{d}_{i,\omega})$$

- For the case of least squares, the loss *p* is the quadratic, and we get the usual closed form solution.

- For the Student's t objective, the source estimation subproblem is given by

$$\hat{\alpha}^k_{i,\omega} = \arg\min_{\alpha} \sum_j \log(\nu + (\alpha \boldsymbol{\mathcal{F}}^k_{i,\omega,j} - \mathbf{d}_{i,\omega,j})^2)$$

- This problem can be solved with a scalar modified Newton's method.

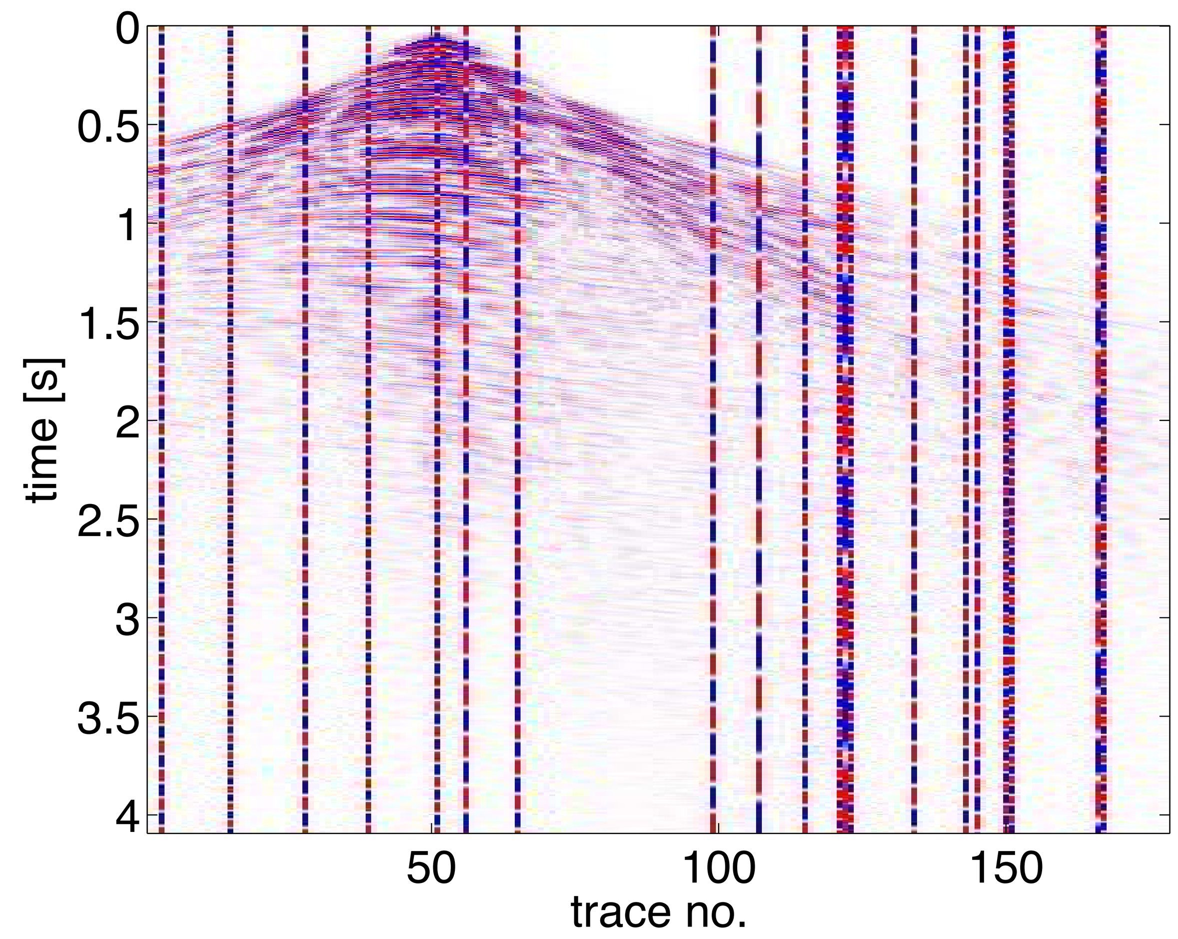


Figure 1 *Data with outliers in the form of bad traces.*

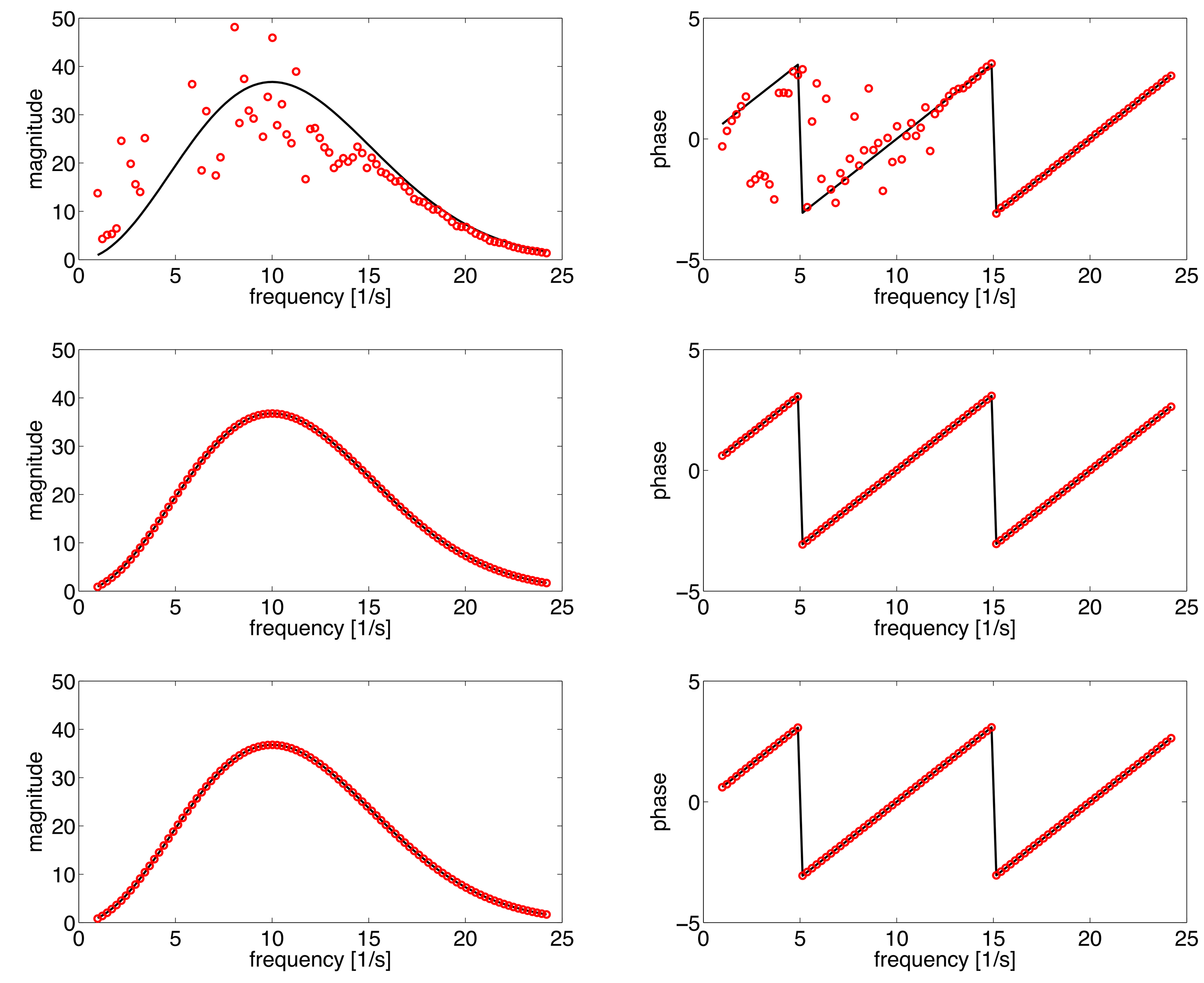
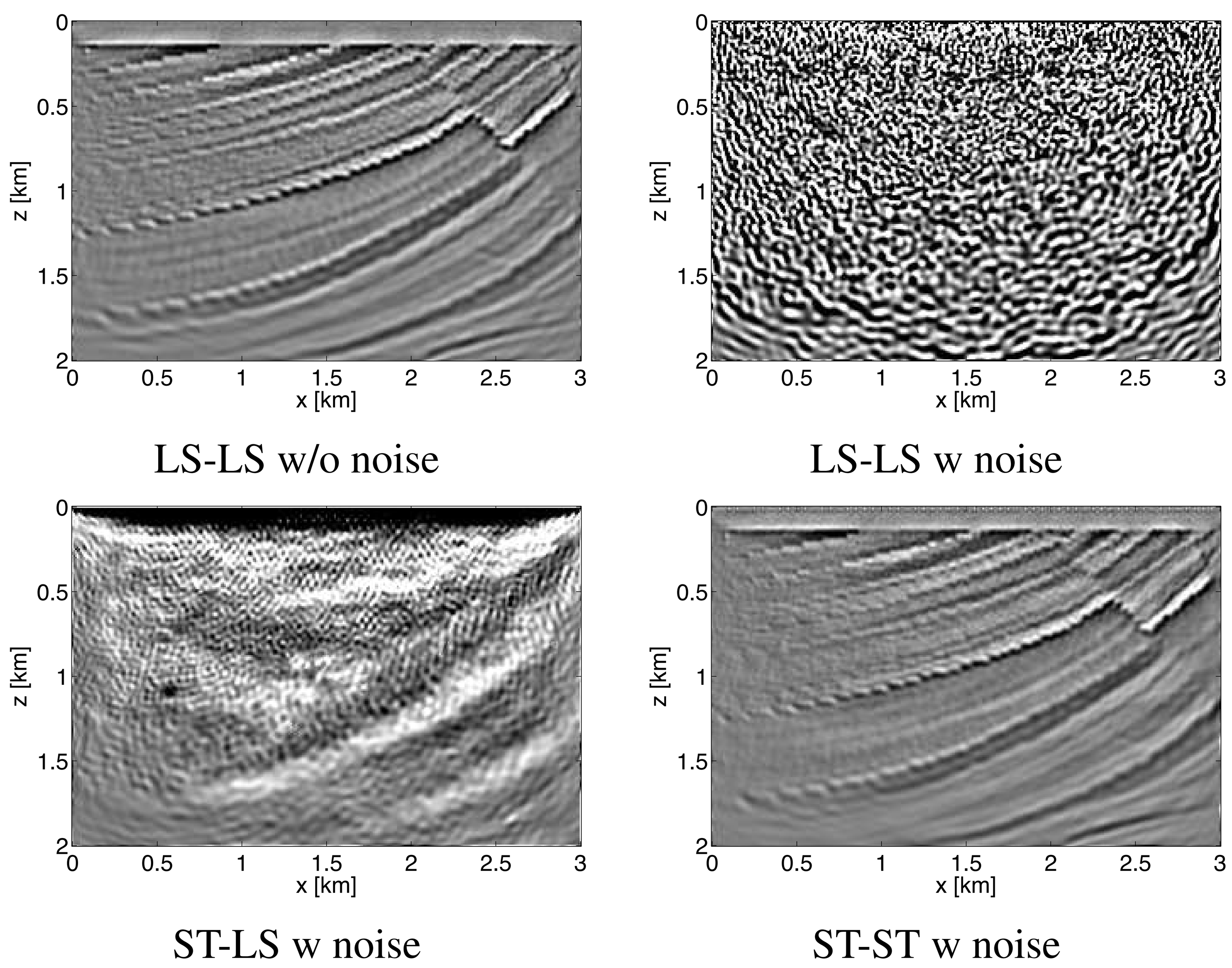


Figure 2 *Estimated source wavelet using Least-Squares (top), Hybrid (middle) and Students t (bottom) approaches.*



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