

# Sparsity-promoting migration with surface-related multiples

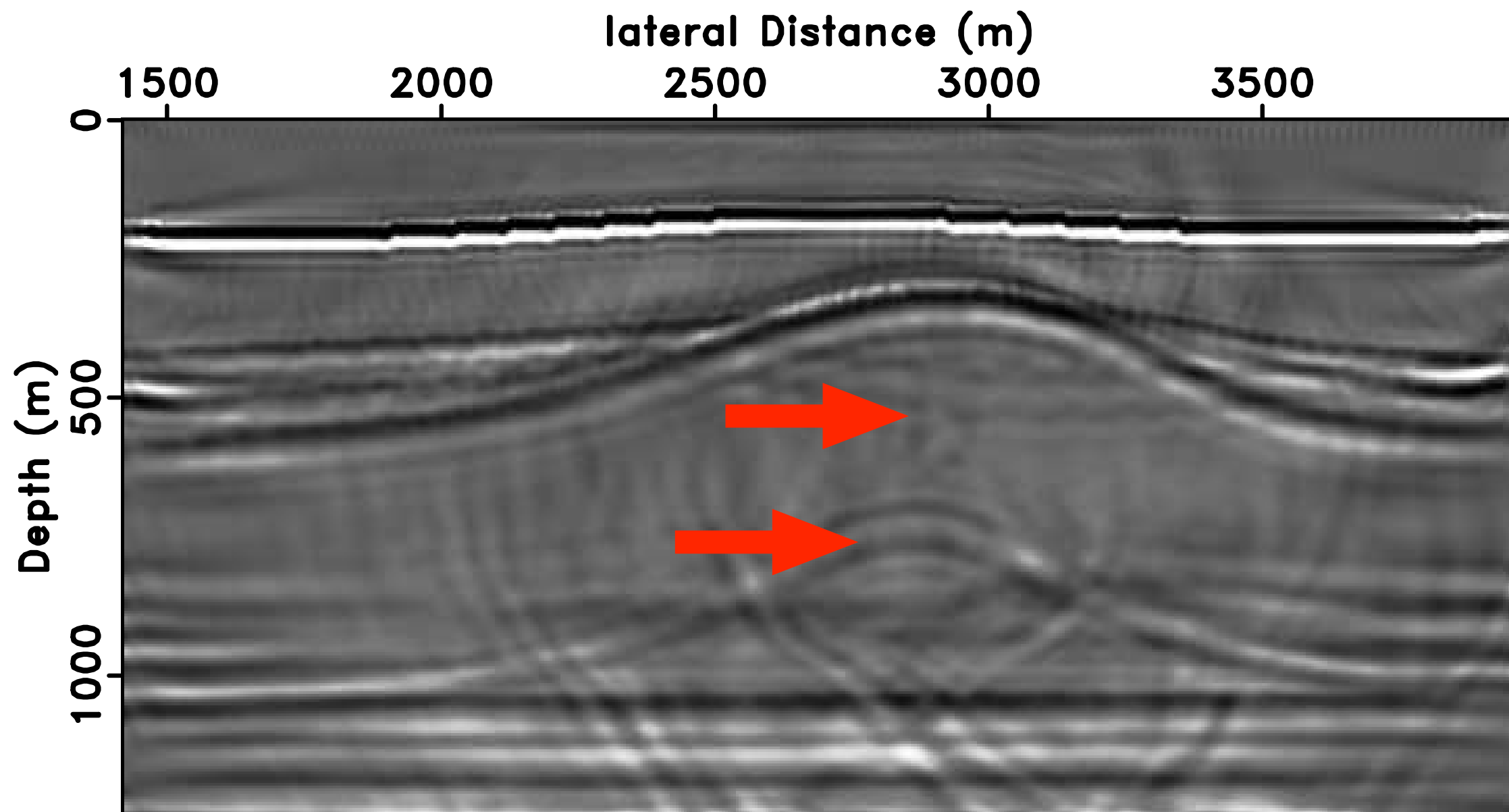
Ning Tu, Tim Lin and Felix Herrmann

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**SLIM**   
University of British Columbia

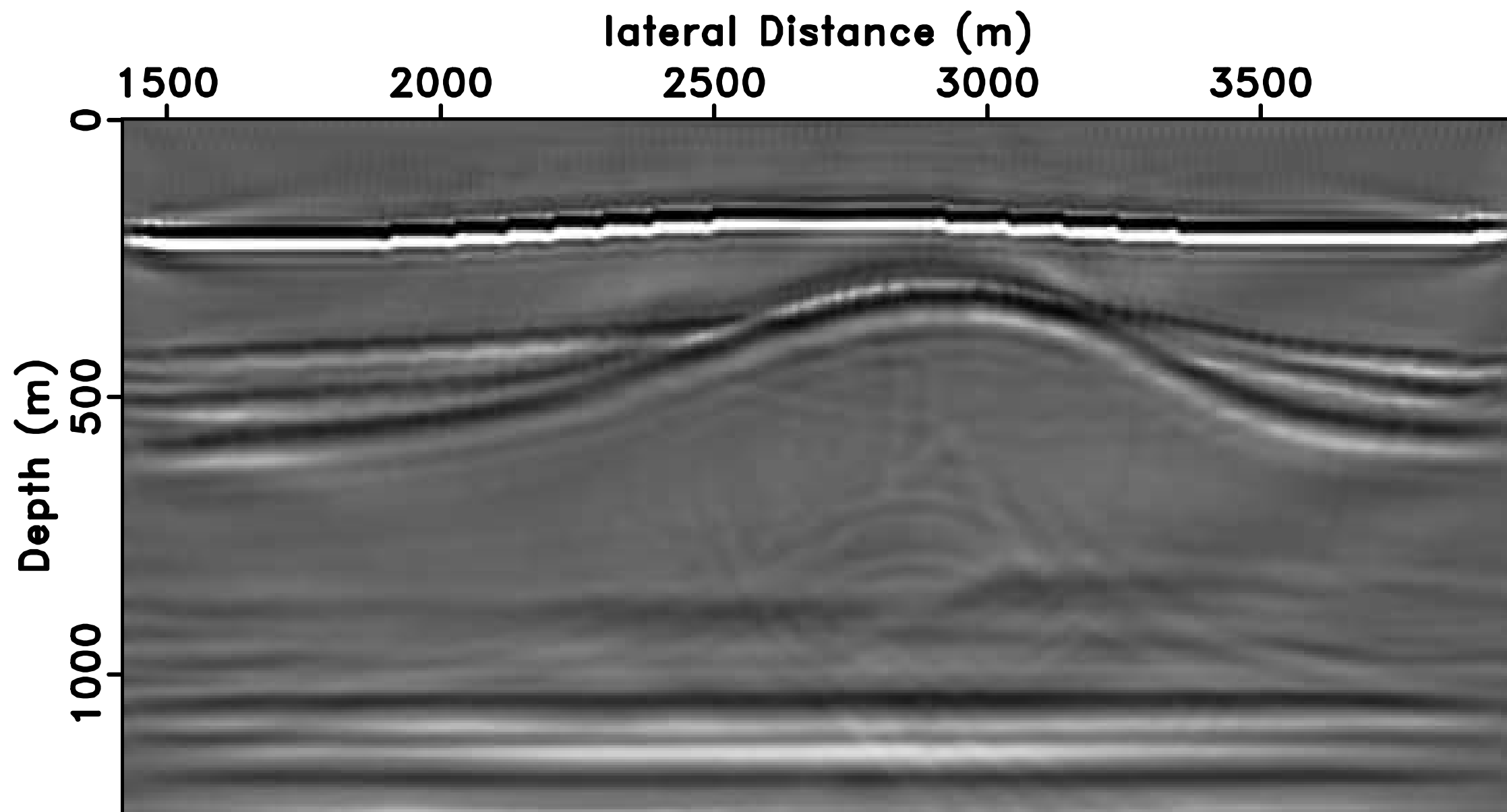


# Motivation



Migration from data with surface multiples

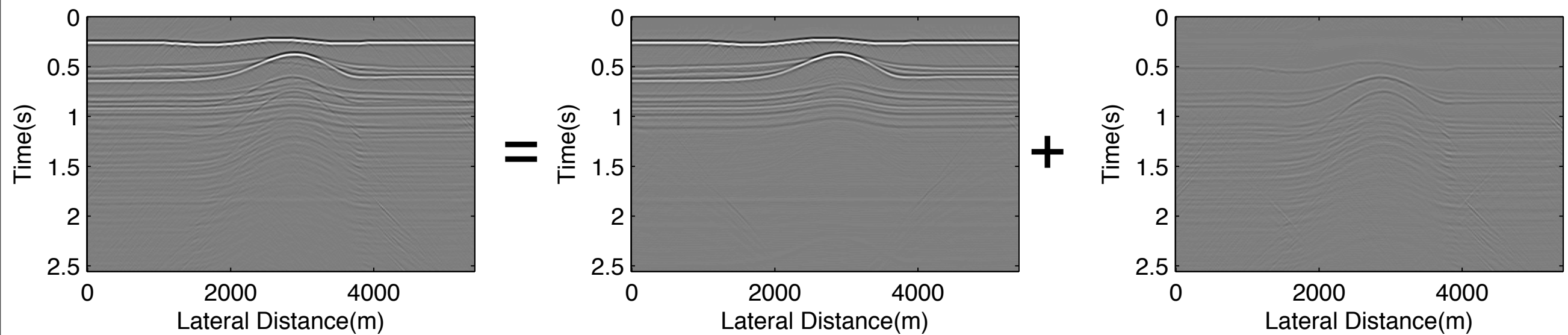
# Motivation



Migration from data without surface multiples

# Motivation

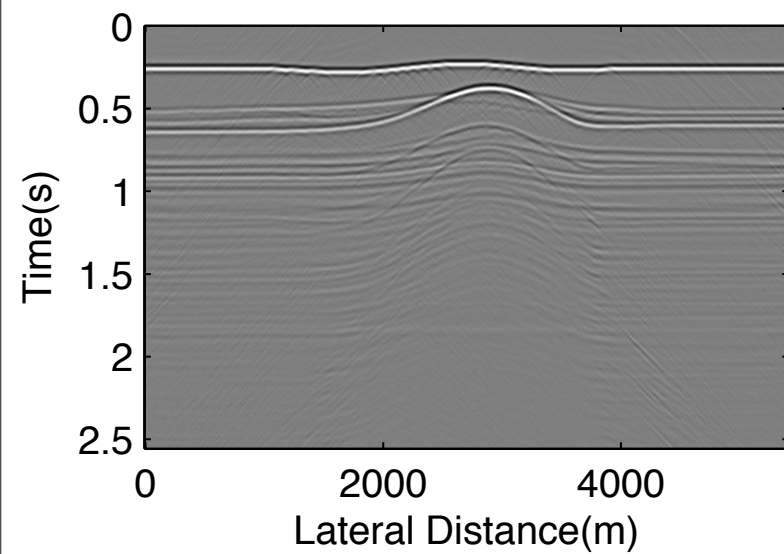
So...



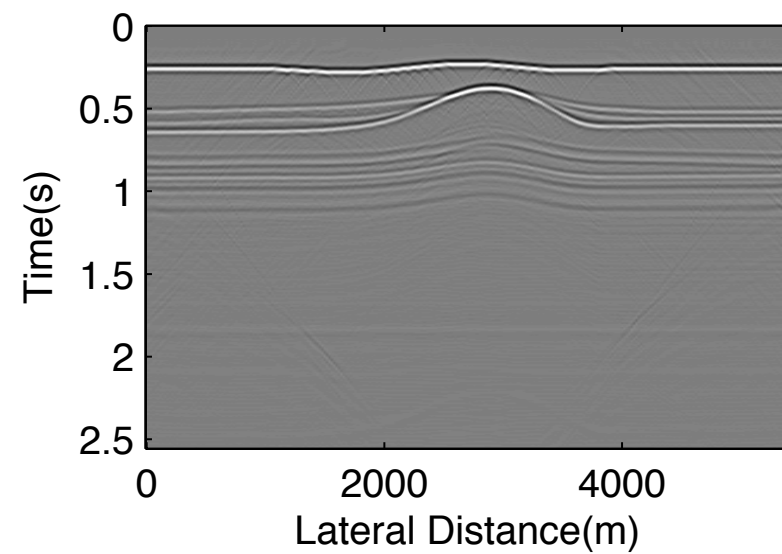


# Motivation

So...



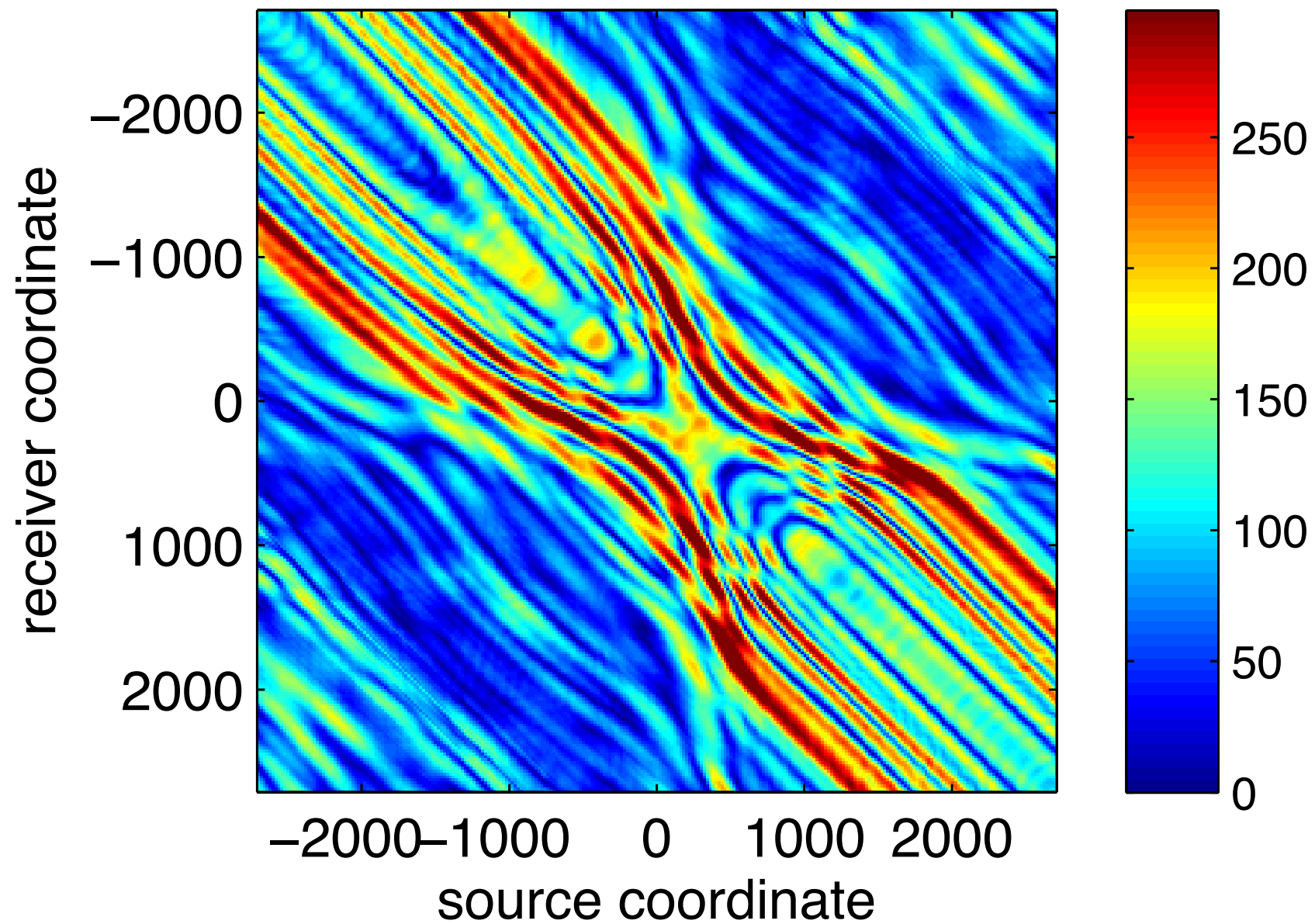
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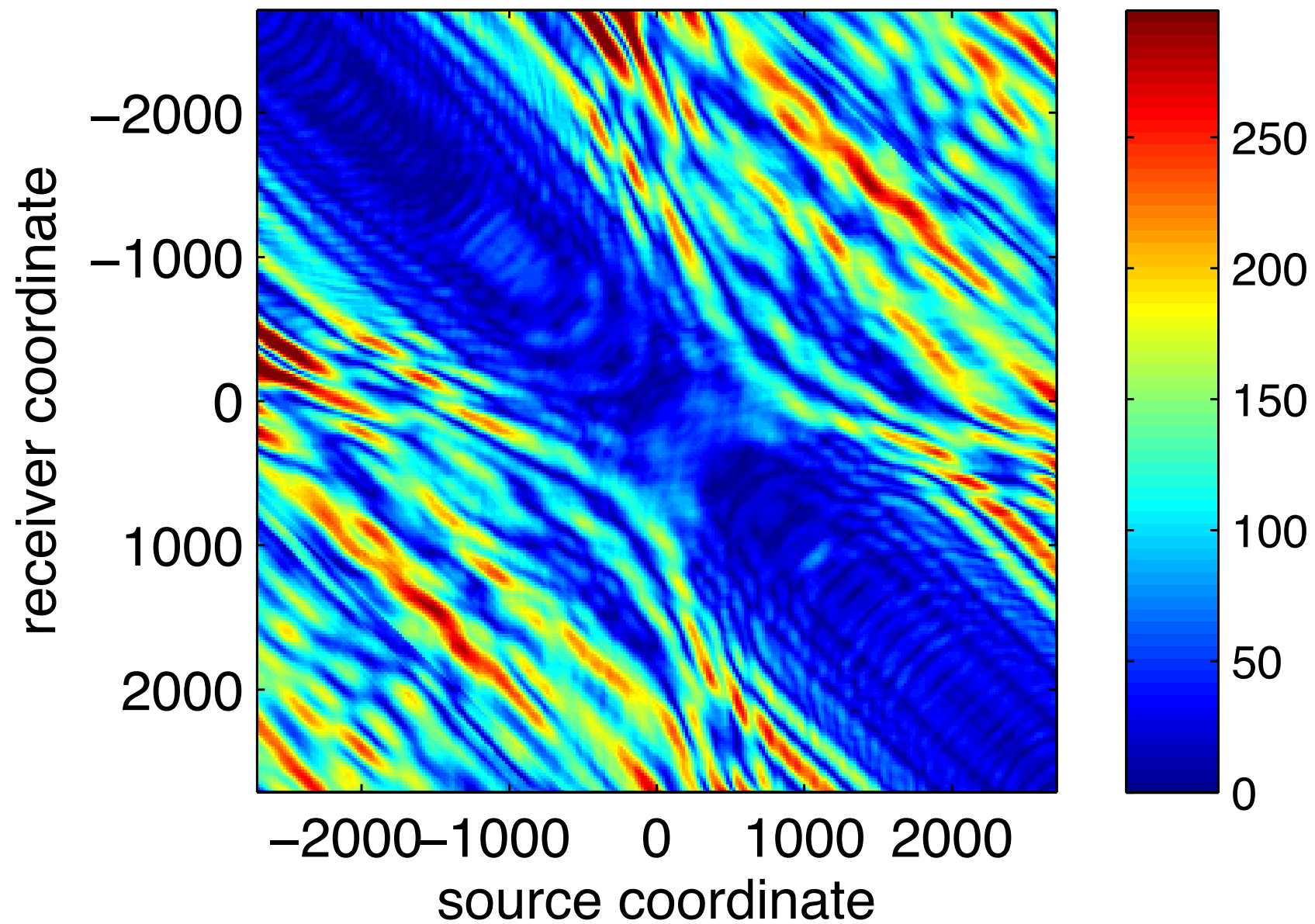


# Rethink multiples



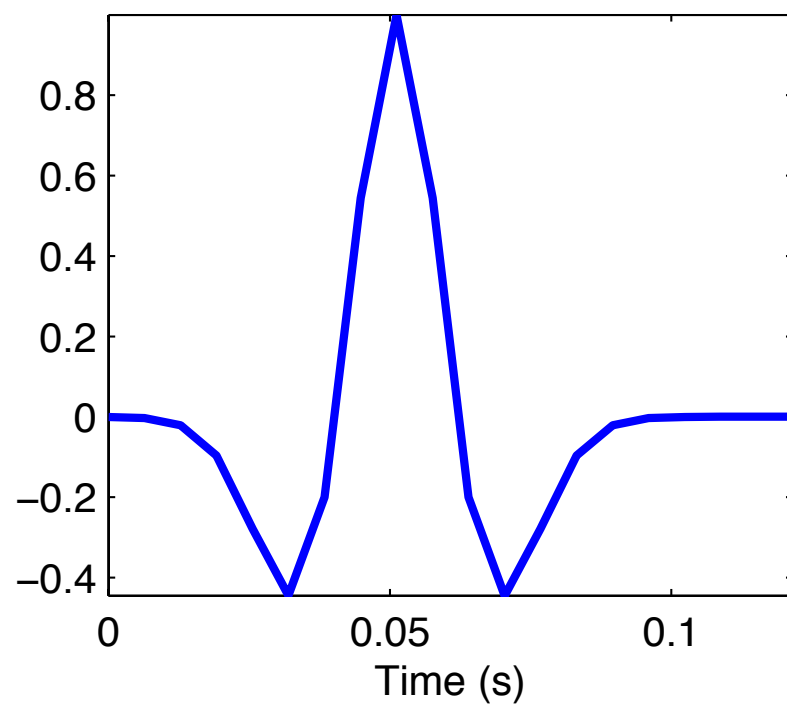


# Rethink multiples

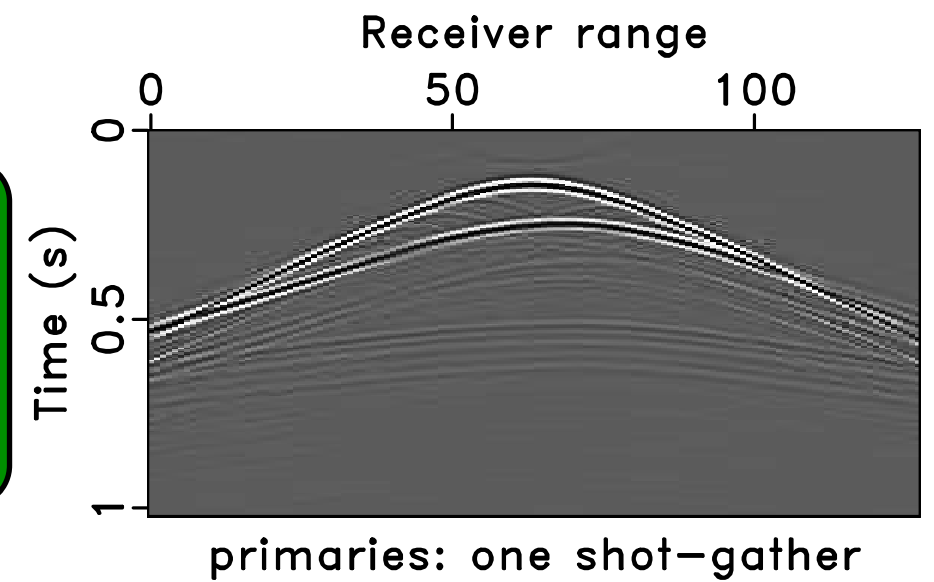


amplitude spectrum: multiples @15Hz

# Rethink multiples



**SURFACE-FREE  
GREEN'S  
FUNCTION**





# Rethink multiples



# Exploit extra illumination





# Exploit extra illumination

## From the formulation of SRME

$$\begin{array}{ccccc} \text{primaries} & & \text{surface multiples} & & \\ \underbrace{\hat{\mathbf{G}}\hat{\mathbf{Q}}} & + & \underbrace{\hat{\mathbf{G}}(-\hat{\mathbf{P}})} & = & \hat{\mathbf{P}} \end{array}$$

Groenestijn and Verschuur, 2009

Lin and Herrmann, 2010

## Exploit extra illumination

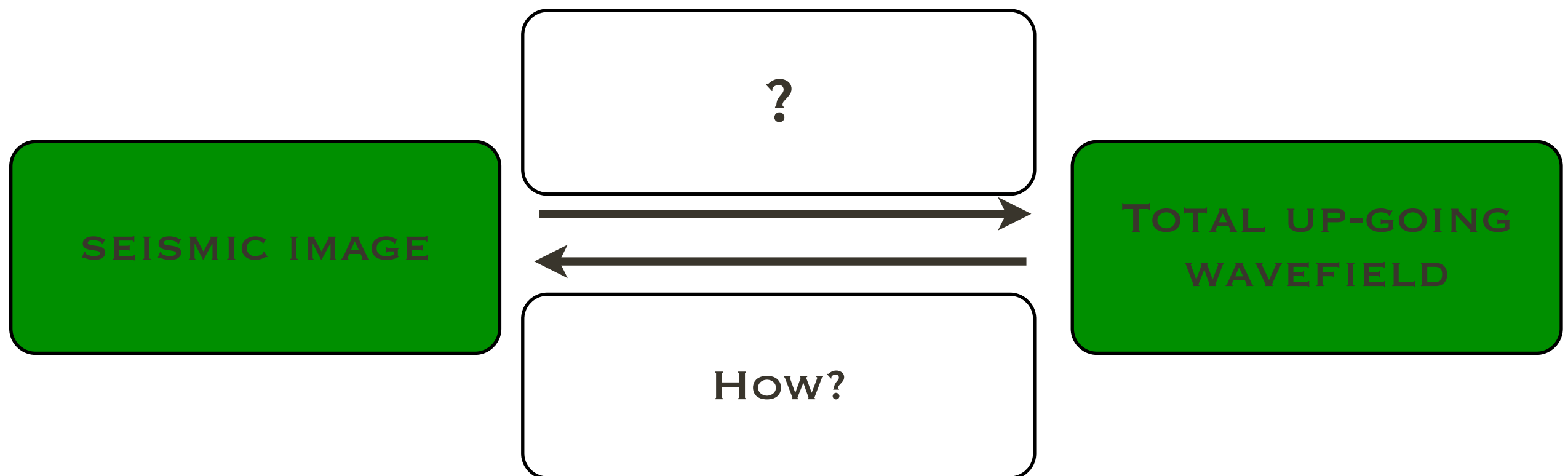
- **Invert the Green's function directly from the total up-going wavefield.**
- **EPSI (Estimation of Primary via Sparse inversion) exploits the sparsity of the Green's function in data space.**



# Motivation

- **How to exploit this extra illumination in seismic imaging?**
- **How to exploit the sparsity in the image space to facilitate the inversion of the Green's function?**

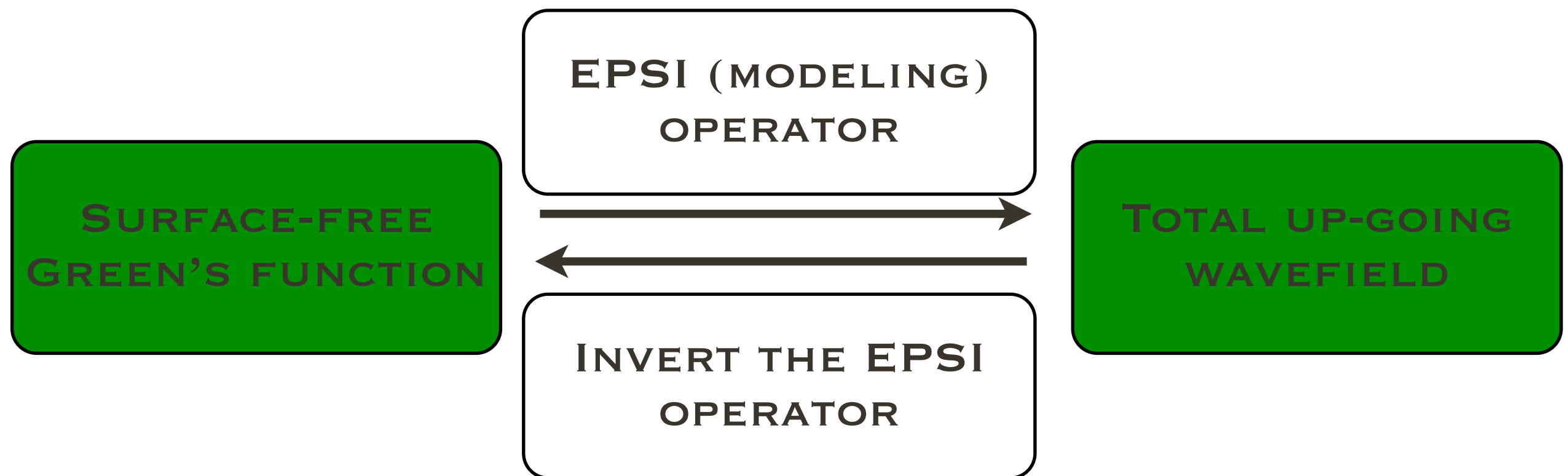
# Relate data space and model space



Groenestijn and Verschuur, 2009

Lin and Herrmann, 2010

# EPSI operator relates...





# EPSI Formulation

**EPSI follows the formulation of SRME:**

$$\hat{\mathbf{P}} = \hat{\mathbf{G}}(\hat{\mathbf{Q}} - \hat{\mathbf{P}})$$

**Reformulating the EPSI operator:**

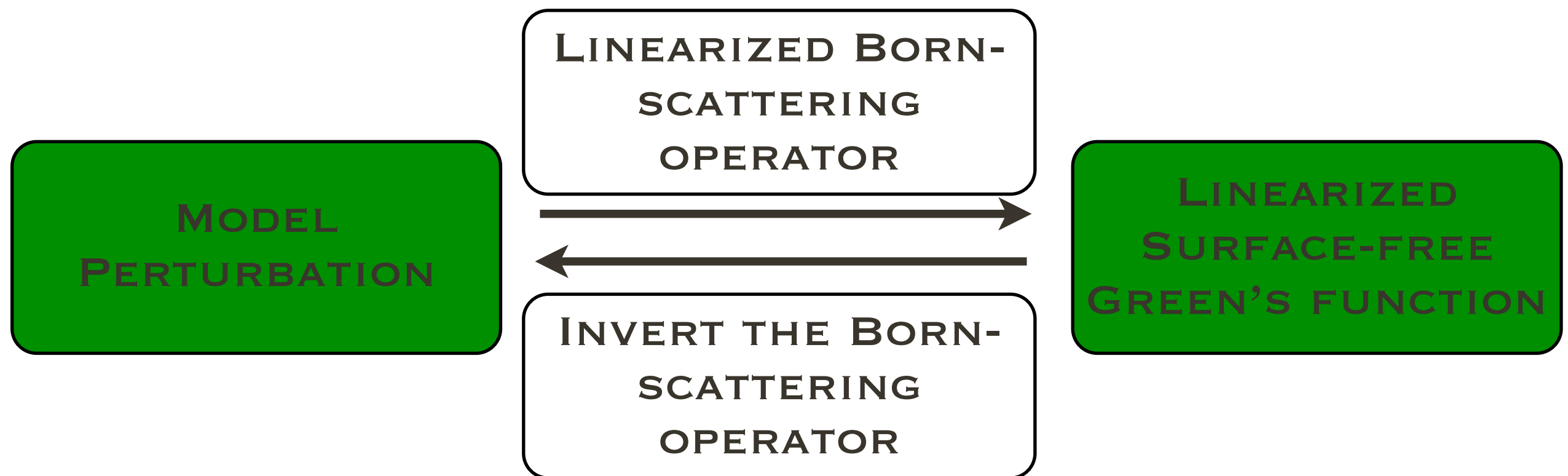
$$\underbrace{\mathcal{F}_t^* \text{BlockDiag}_{1\dots nf}[(\hat{\mathbf{Q}} - \hat{\mathbf{P}})^* \otimes \mathbf{I}] \mathcal{F}_t}_{\mathbf{E}} \mathbf{g} = \mathbf{p}$$

# Robust EPSI

## Robust EPSI:

$$\underbrace{\tilde{\mathbf{g}} = \min_{\mathbf{g}} \|\mathbf{g}\|_1}_{\text{sparsity promoting part}} \quad \text{subject to} \quad \underbrace{\|\mathbf{p} - \mathbf{E}\mathbf{g}\|_2 \leq \sigma}_{\text{data fitting part}}$$

# Migration operator relates...



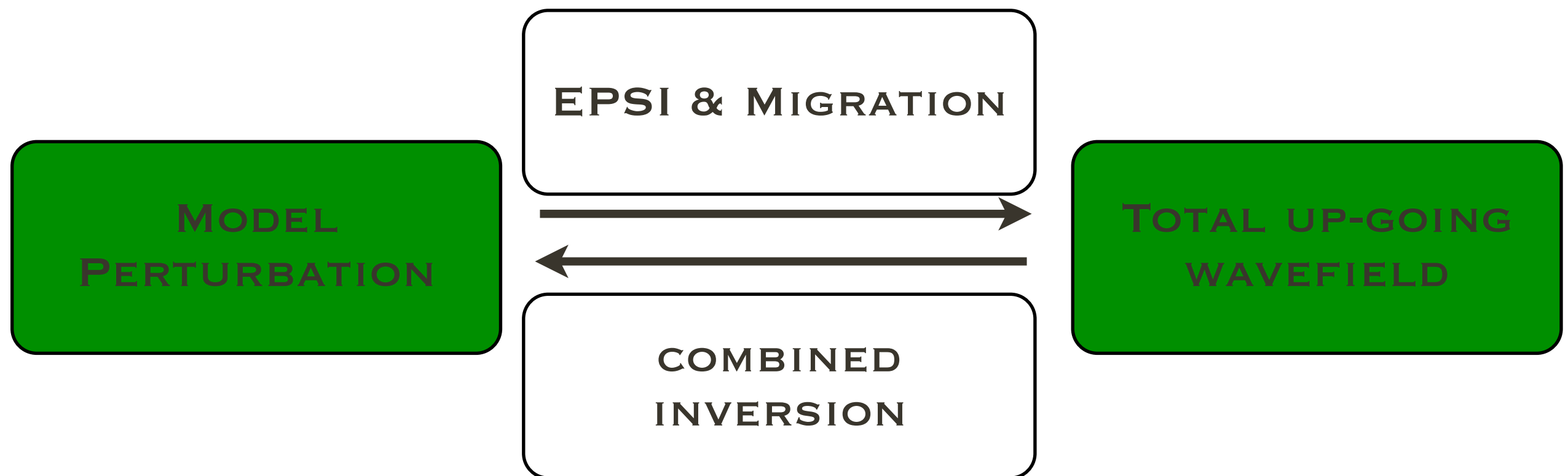
# Sparsity promoting migration

**By leveraging curvelet domain sparsity in the image space:**

$$\delta \tilde{\mathbf{m}} = \mathbf{S}^* \min_{\delta \mathbf{x}} \|\delta \mathbf{x}\|_1 \text{ subject to } \|\delta \mathbf{g} - \mathbf{KS}^* \delta \mathbf{x}\|_2 \leq \sigma$$



# What about combine...



# Approximate EPSI

**EPSI follows the formulation of SRME:**

$$\hat{\mathbf{P}} \approx \Delta \hat{\mathbf{G}}(\hat{\mathbf{Q}} - \hat{\mathbf{P}})$$

**Reformulating the EPSI operator:**

$$\underbrace{\mathcal{F}_t^* \text{BlockDiag}_{1 \dots n_f} [(\hat{\mathbf{Q}} - \hat{\mathbf{P}})^* \otimes \mathbf{I}] \mathcal{F}_t}_{\mathbf{E}} \delta \mathbf{g} \approx \mathbf{p}$$

# Approx. Robust EPSI

## Robust EPSI:

$$\underbrace{\delta \tilde{\mathbf{g}} = \min_{\delta \mathbf{g}} \|\delta \mathbf{g}\|_1}_{\text{sparsity promoting part}} \quad \text{subject to} \quad \underbrace{\|\mathbf{p} - \mathbf{E}\delta \mathbf{g}\|_2}_{\text{data fitting part}} \leq \sigma$$

# Combine EPSI with migration

**We identify the total up-going wavefield with model perturbations:**

$$\delta \tilde{\mathbf{m}} = \mathbf{S}^* \min_{\delta \mathbf{x}} ||\delta \mathbf{x}||_1 \text{ subject to } ||\mathbf{p} - \mathbf{E} \underbrace{\mathbf{K} \mathbf{S}^* \delta \mathbf{x}}_{\delta \mathbf{g}}||_2 \leq \sigma$$



# Numerical experiments

## Linearized data:

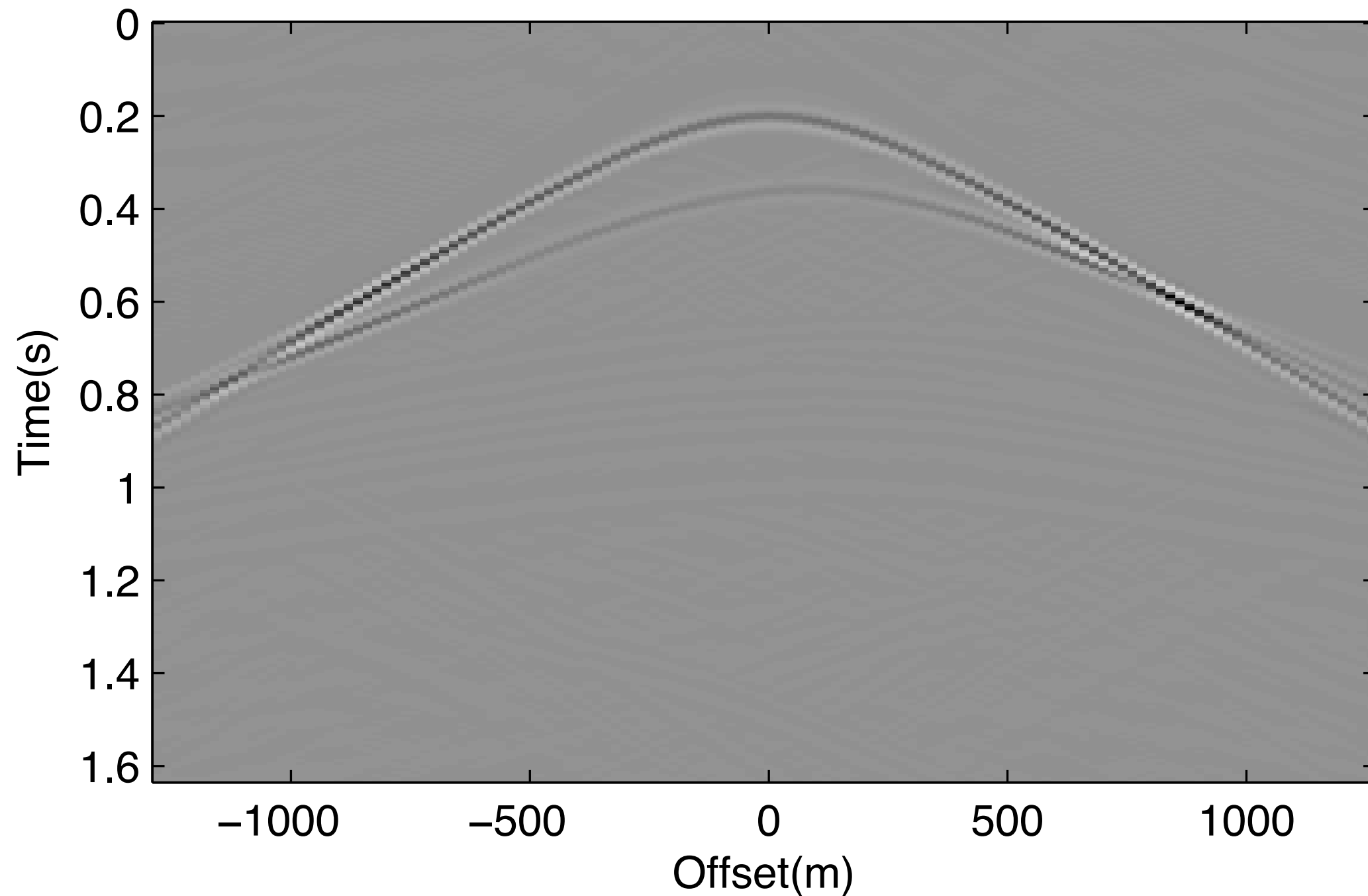
- **surface-free data**

$$p_1 = \mathbf{K} \delta \mathbf{m}$$

- **total data**

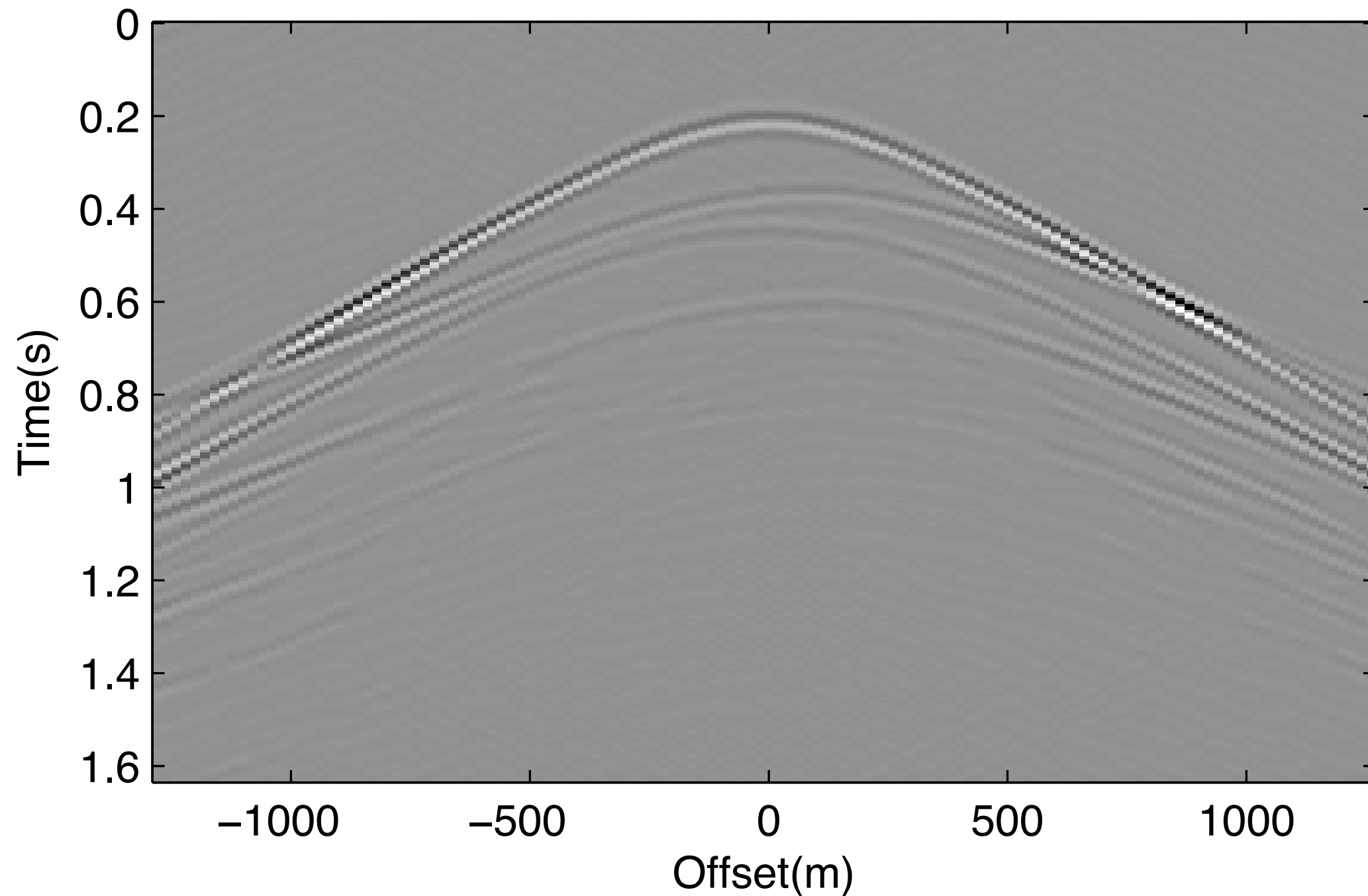
$$p_2 = \mathbf{E} \mathbf{K} \delta \mathbf{m}$$

# Data preview: surface free data



total shots: 128, shot number: 65

# Data preview: total data

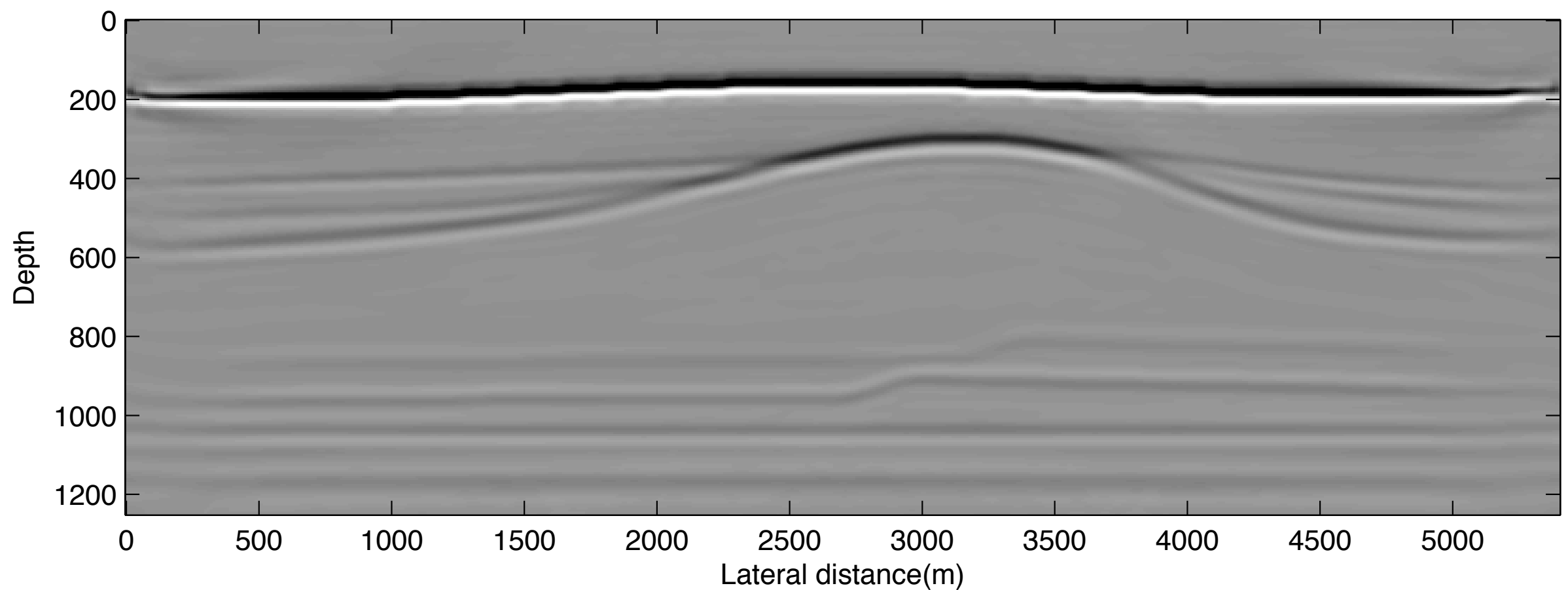


total shots: 128, shot number: 65

# Case study 1

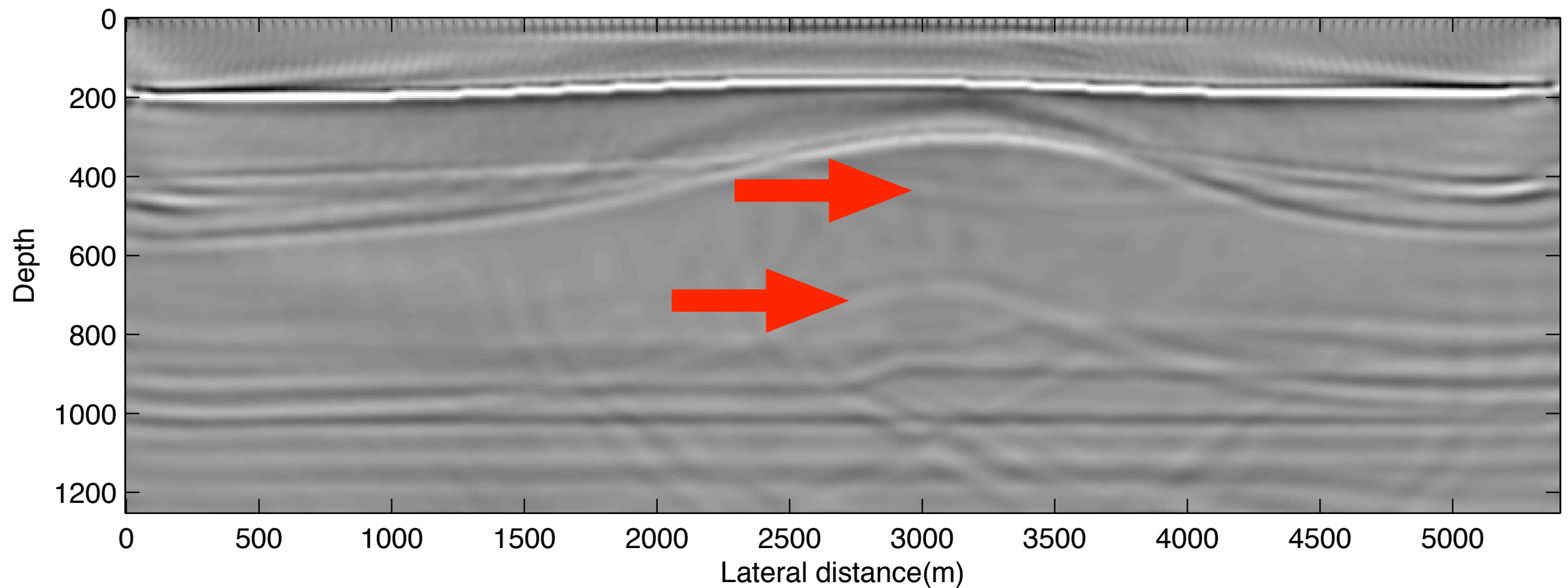
- **scenario 1: inversion from surface-free data**
- **scenario 2: inversion from total data**
- **scenario 3: combined inversion from total data**

# Sparse inversion of surface-free data

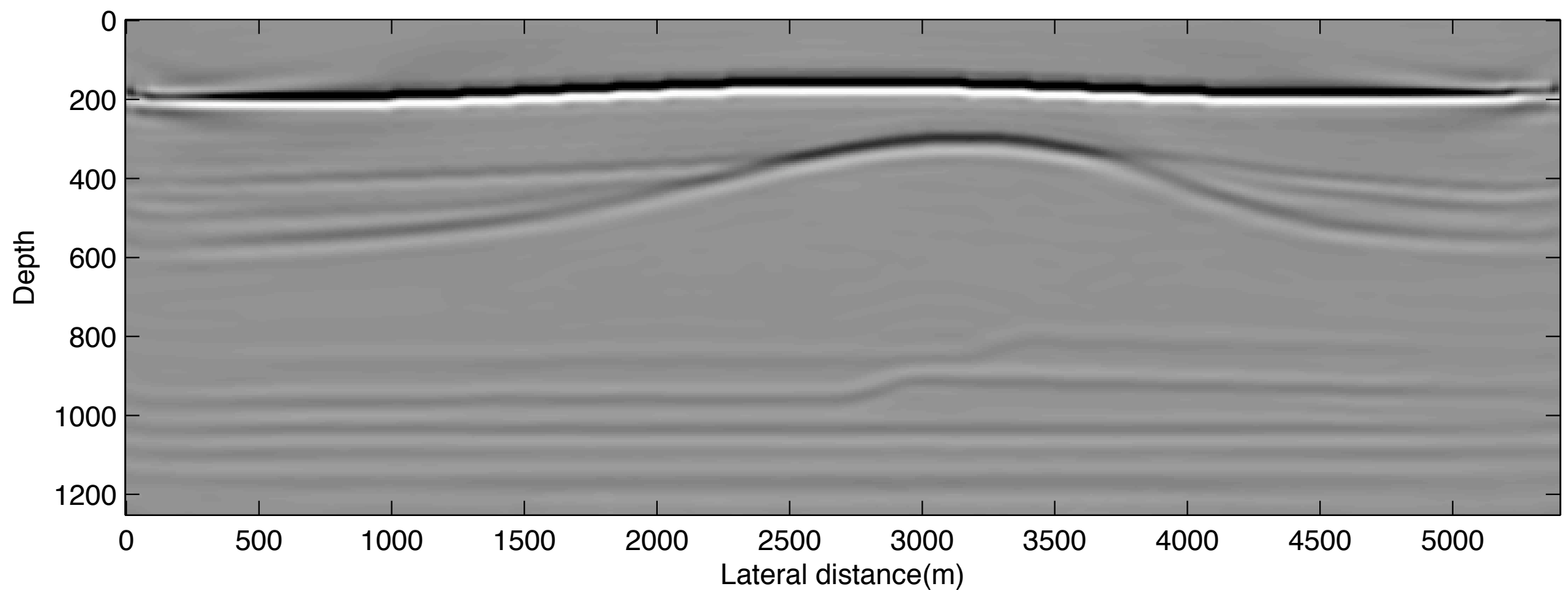




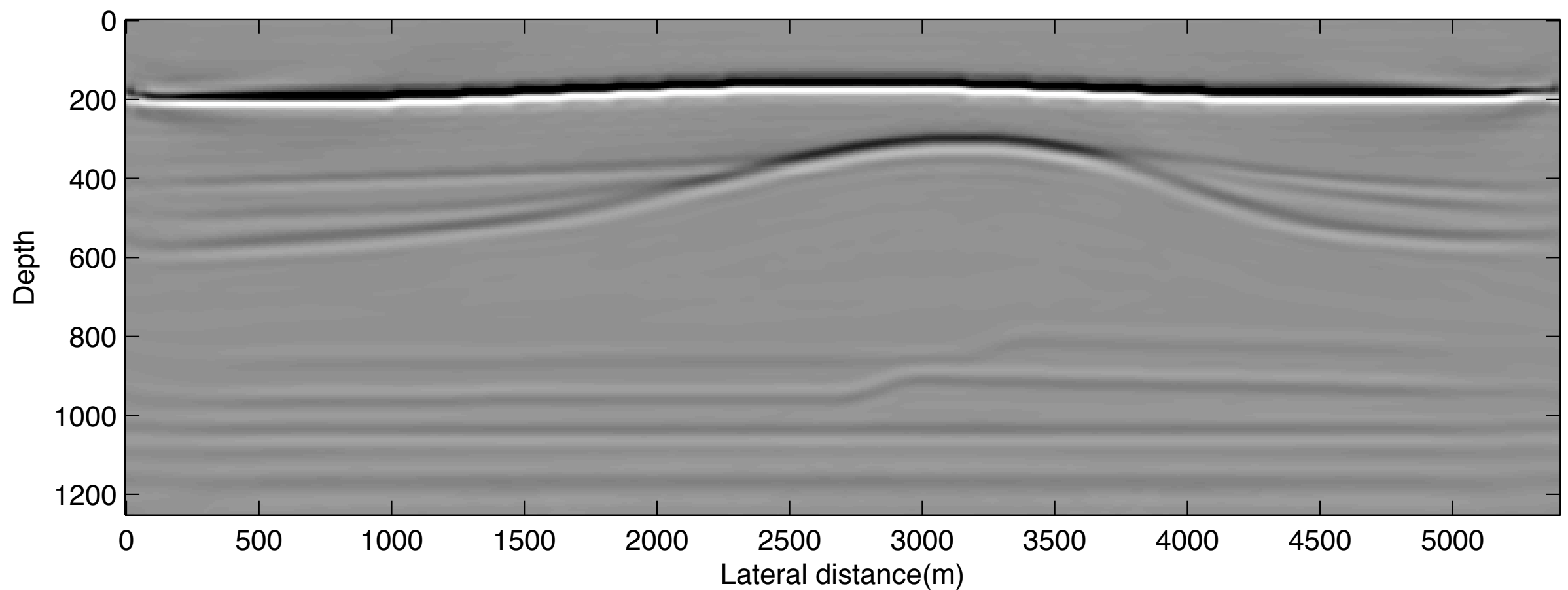
# Sparse inversion of total data



# Sparse inversion of data with multiples with EPSI



# Sparse inversion of surface-free data



# Do multiples really help ?

**Both scenario 1 and scenario 3 give good results...Will this still be true when we invert from incomplete data?**

# Case study 2

## From incomplete data

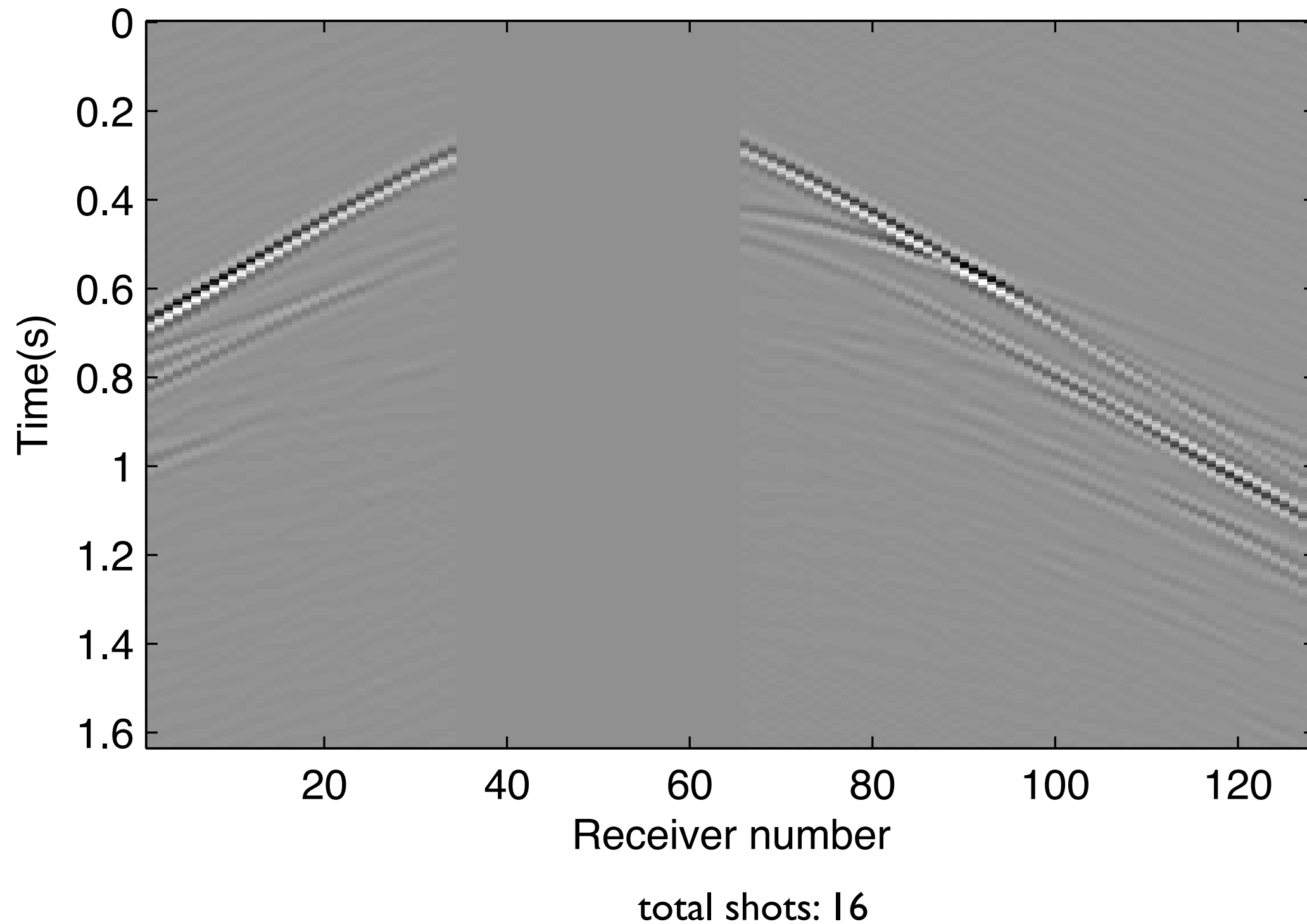
- **scenario 1: inversion from surface-free data**
- **scenario 2: combined inversion from total data**



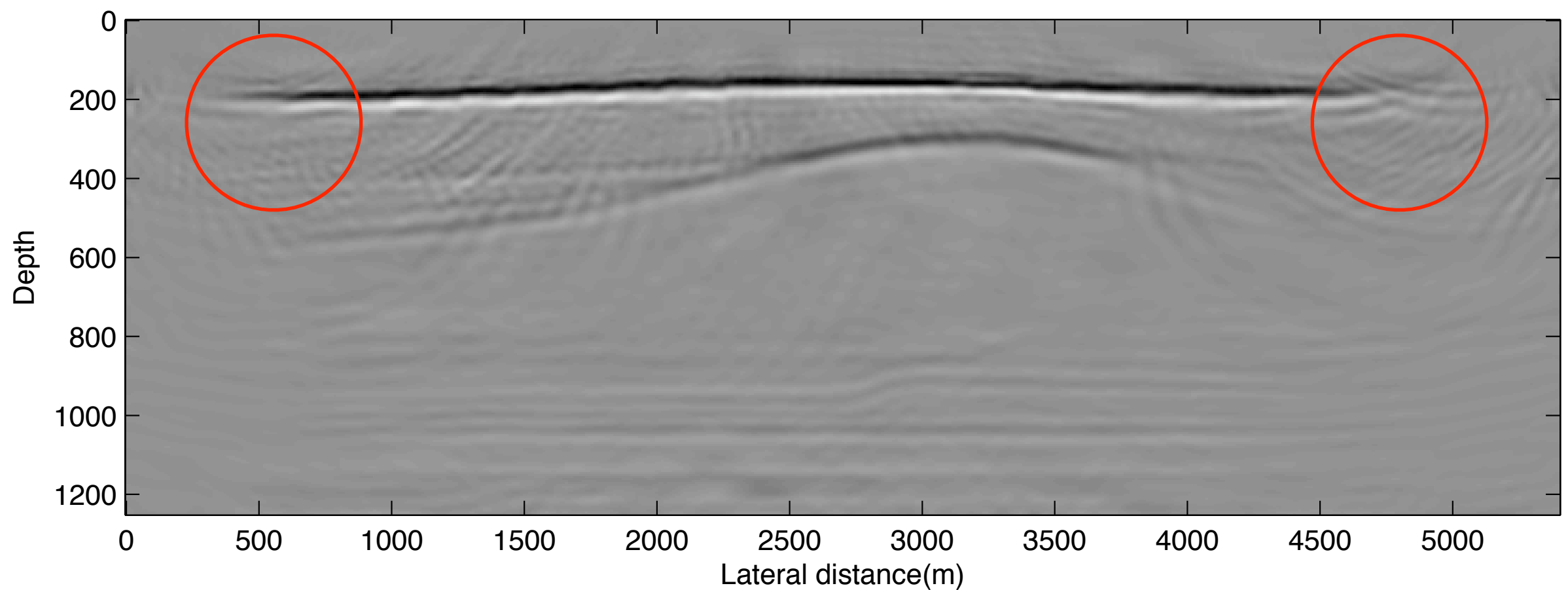
# Incomplete data

- **16 sequential shots selected randomly from a total number of 128 shots**
- **300m missing near-offset**

# Data preview

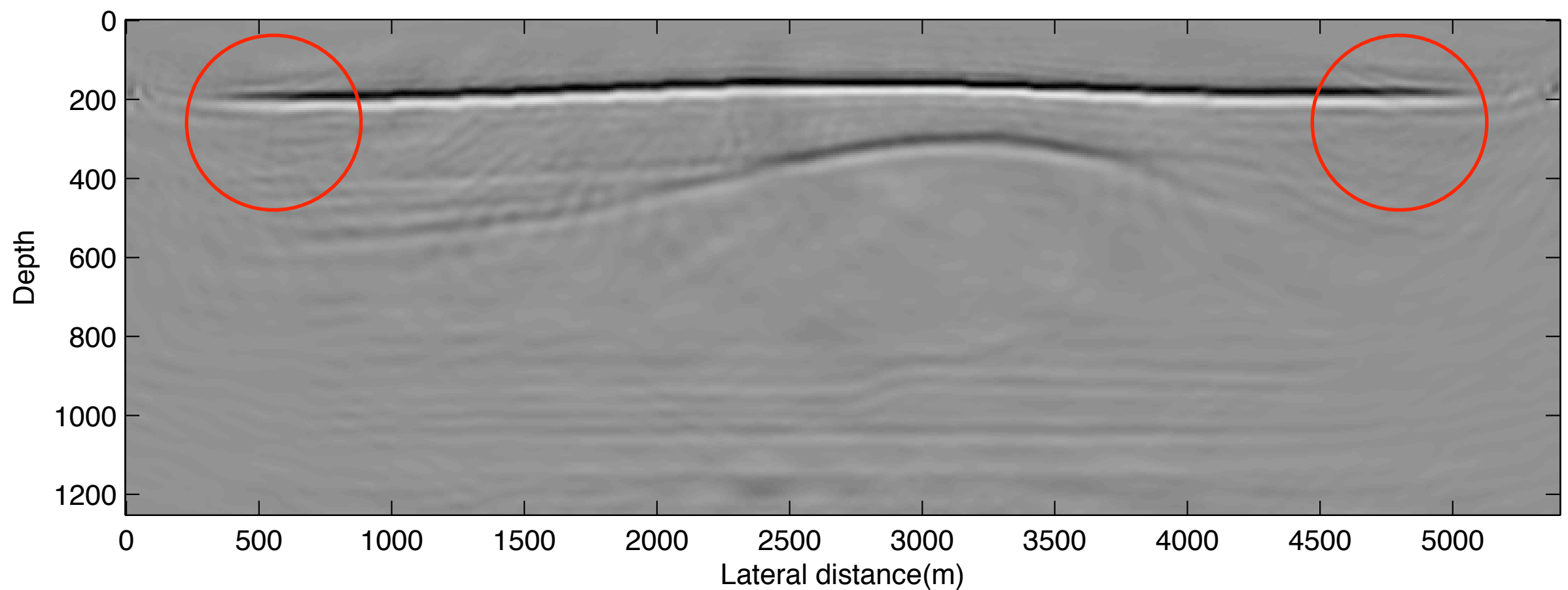


# Sparse inversion from surface-free data



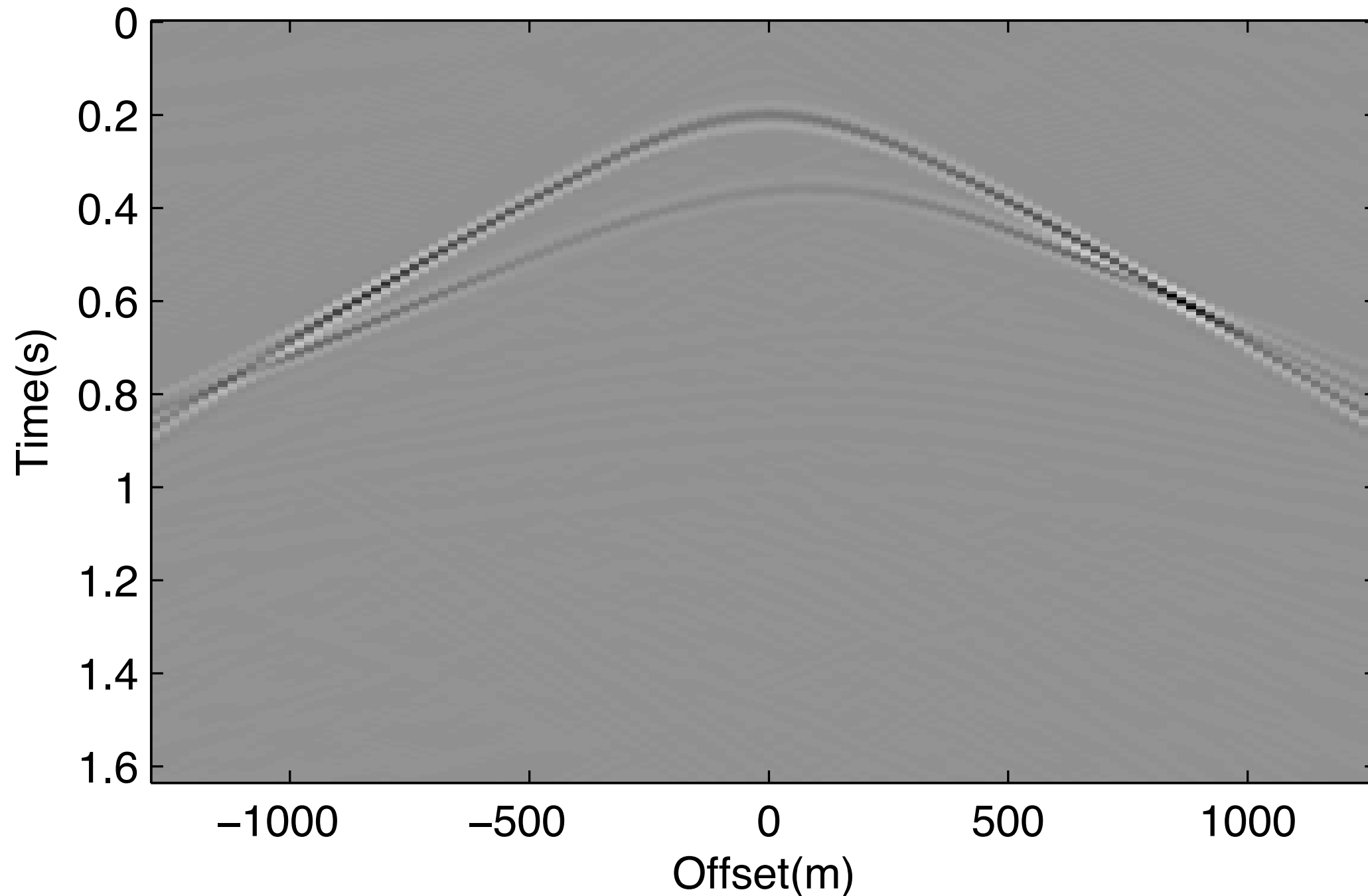
SNR: 3.08dB (compared to true dm)

# Combined inversion of total data



SNR: 3.72dB (compared to true dm)

# Recovered Green's function



total shots: 128, shot number: 65, SNR: 15.4dB

## In EPSI's point of view

- **Model perturbation is sparser than the Green's function**
- **Primary estimation in image space**
- **One joint inversion outperforms two separate inversions**

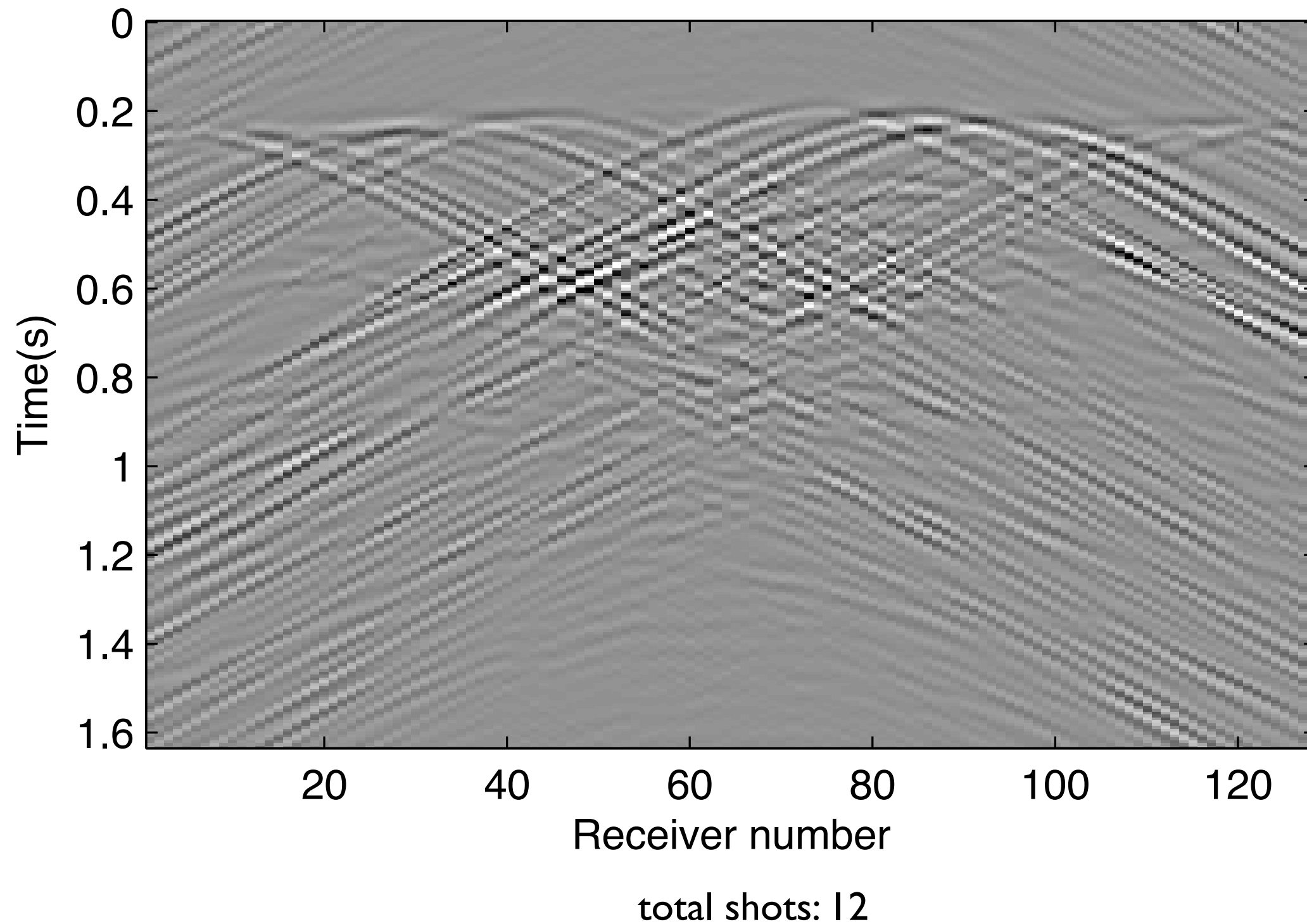


# Case study 3

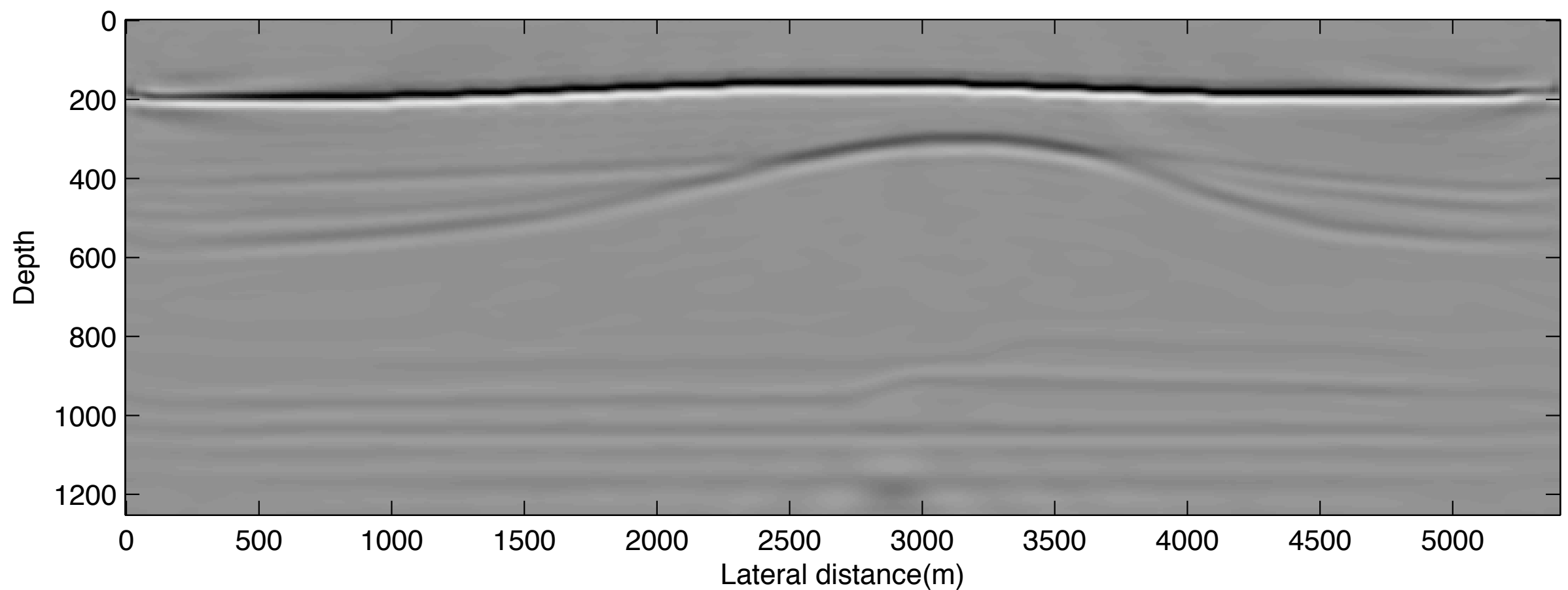
**From 12 simultaneous shots**

- **scenario 1: EPSI & migration**
- **scenario 2: EPSI -> migration**

# Data preview

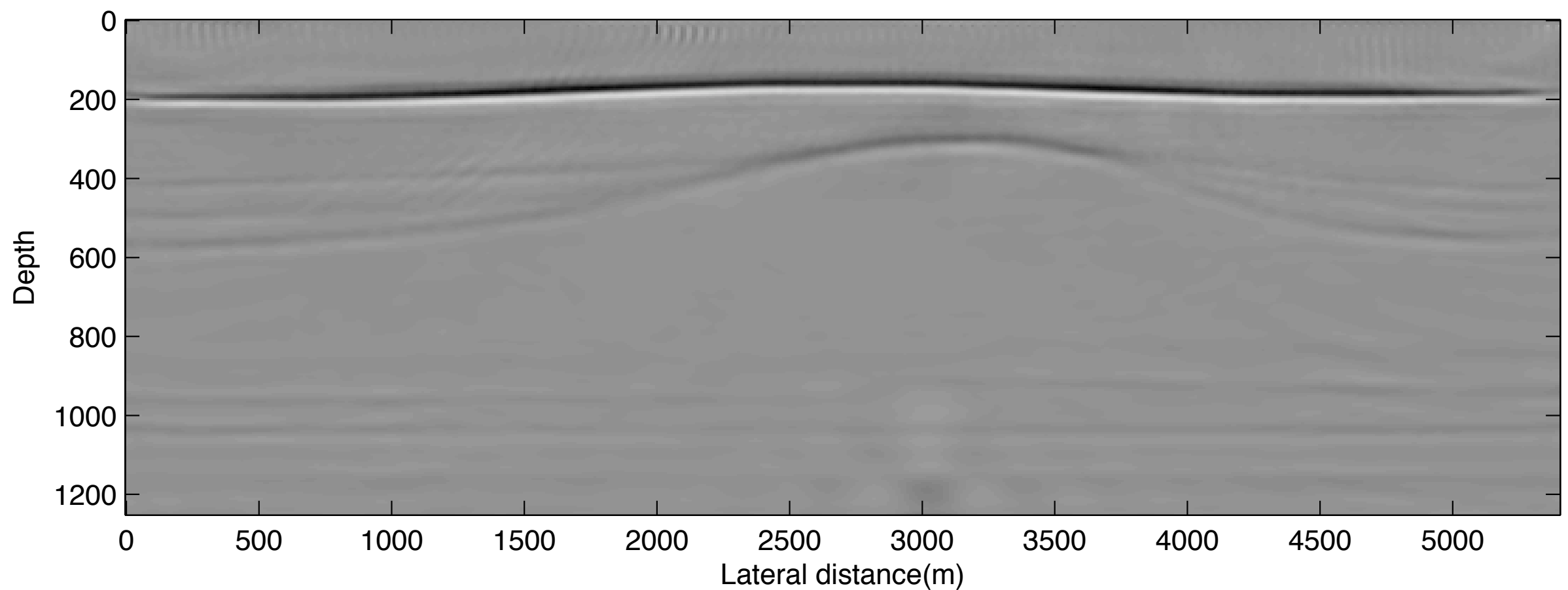


# Combined inversion



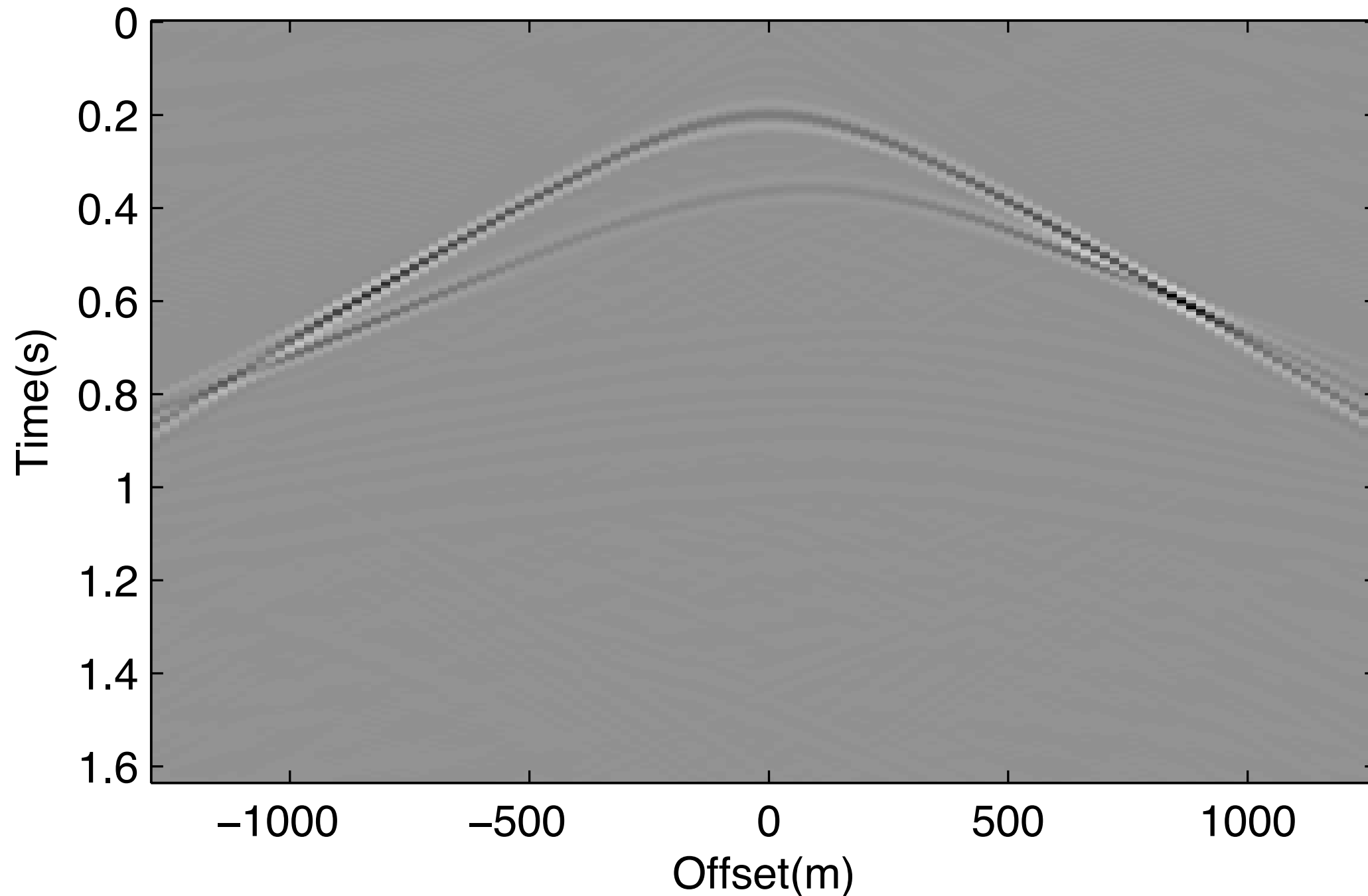
SNR: 5.30dB (compared to true dm)

# Two separate inversions



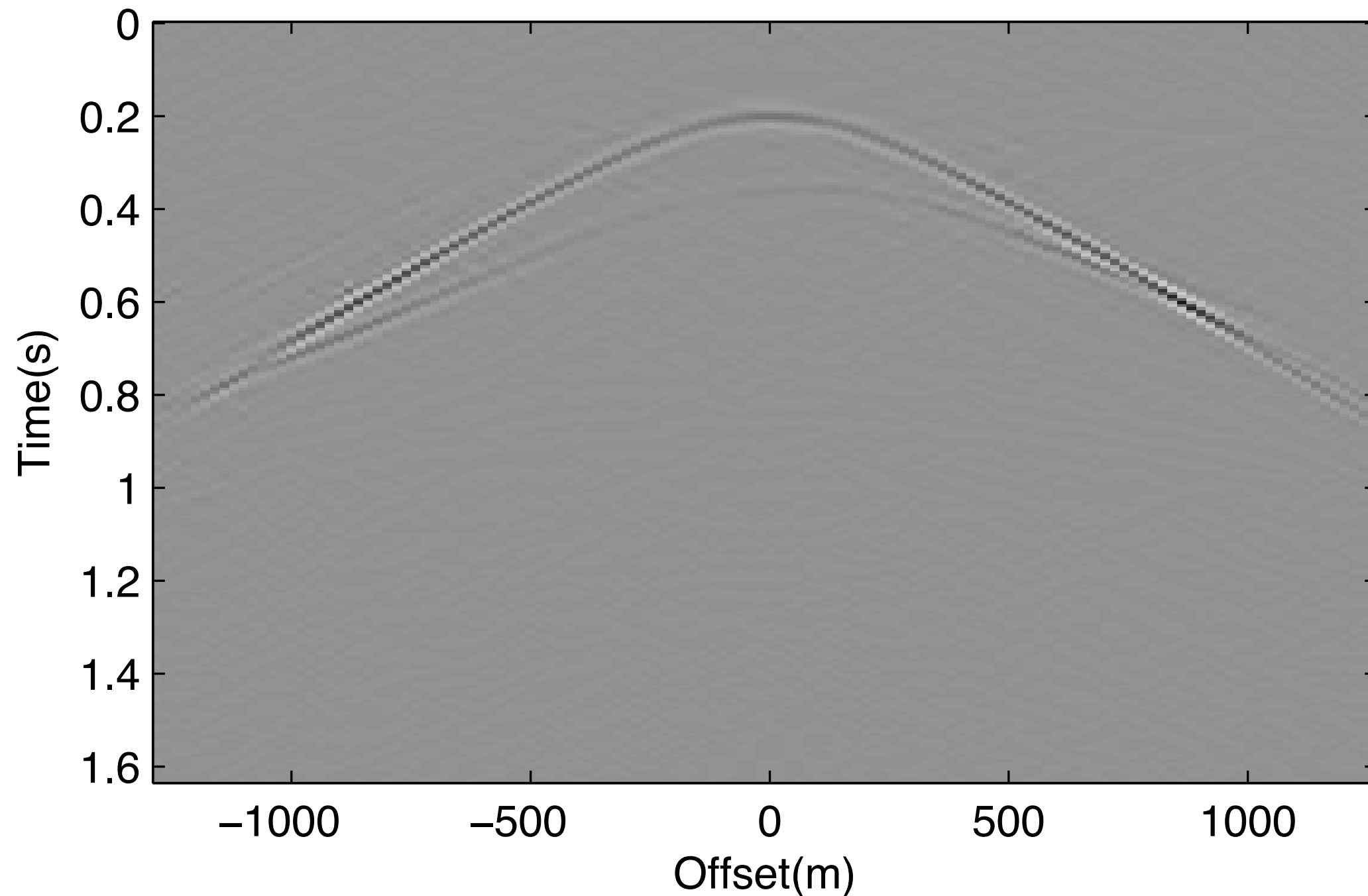
SNR: 4.52dB (compared to true dm)

# Recovered Green's function in combined inversion



total shots: 128, shot number: 65, SNR: 22.4dB

# Recovered Green's function in separate inversions



total shots: 128, shot number: 65, SNR: 5.4dB



# Conclusions

**By combining EPSI with migration:**

- **we reap benefits in seismic imaging by exploiting the extra illumination from surface multiples**
- **better primary estimation results by exploiting sparsity in image space**

## Future plans

- **How to adapt EPSI for incomplete data**
- **Speed-up this joint inversion by introducing simultaneous sources**
- **extend this work to FWI**

# EPSI for incomplete data

**EPSI contains term  $G^*P$ , matrix multiplication breaks when  $P$  is incomplete along both columns and rows.**

# Compressively simulate $G$

**EPSI needs full  $G$  too for  $G^*P$  term.**

- **$P=USV^*$ , use  $U$  as a simultaneous shot term for  $G$**
- **$G^*P=P^*G$  by reciprocity, apply the simultaneous shot term on the right.**

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- Thank Eric Verschuur for his advice on working with incomplete data



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# Thanks for your attention