Sparsity-promoting migration with surface-related multiples

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Motivation

Migration from data with surface multiples
Motivation

Migration from data without surface multiples
Motivation

So...

\[ \begin{array}{c}
\text{Time (s)} \\
\text{Lateral Distance (m)}
\end{array} \quad \begin{array}{c}
0 \\
2000 \\
4000
\end{array} \quad \begin{array}{c}
0 \\
2000 \\
4000
\end{array} \quad \begin{array}{c}
0 \\
2000 \\
4000
\end{array} \]
Motivation

So...
Rethink multiples

amplitude spectrum: primaries @15Hz
Rethink multiples

amplitude spectrum: multiples @15Hz
Rethink multiples

**Surface-free Green’s function**

Receiver range

primaries: one shot–gather
Rethink multiples

Surface-free Green’s function
Exploit extra illumination

Lin, Tu, and Herrmann, 2010
Verschuur and Berkhout, 2011
Exploit extra illumination

From the formulation of SRME

\[ \hat{G} \hat{Q} + \hat{G}(-\hat{P}) = \hat{P} \]
Exploit extra illumination

- Invert the Green’s function directly from the total up-going wavefield.
- EPSI (Estimation of Primary via Sparse inversion) exploits the sparsity of the Green’s function in data space.
Motivation

• How to exploit this extra illumination in seismic imaging?
• How to exploit the sparsity in the image space to facilitate the inversion of the Green’s function?
Relate data space and model space

SEISMIC IMAGE

How?

TOTAL UP-GOING WAVEFIELD

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EPSI operator relates...

EPSI (modeling) operator

Surface-free Green’s function

Invert the EPSI operator

Total up-going wavefield

Groenestijn and Verschuur, 2009
Lin and Herrmann, 2010
EPSI Formulation

EPSI follows the formulation of SRME:

\[ \hat{P} = \hat{G}(\hat{Q} - \hat{P}) \]

Reformulating the EPSI operator:

\[ (\mathcal{F}_t^* \text{BlockDiag}_{1...nf} [(\hat{Q} - \hat{P})^* \otimes I] \mathcal{F}_t g = p) \]
Robust EPSI

Robust EPSI:

\[ \tilde{g} = \min_{g} \| g \|_1 \quad \text{subject to} \quad \| p - Eg \|_2 \leq \sigma \]

- sparsity promoting part
- data fitting part

Lin and Herrmann, 2010
Migration operator relates...

Model Perturbation

Linearized Born-scattering operator

Invert the Born-scattering operator

Linearized Surface-free Green's function
Sparsity promoting migration

By leveraging curvelet domain sparsity in the image space:

\[
\delta \tilde{m} = S^* \min_{\delta x} ||\delta x||_1 \text{ subject to } ||\delta g - KS^* \delta x||_2 \leq \sigma
\]
What about combine...

- **Model Perturbation**
- **EPSI & Migration**
- **Combined Inversion**
- **Total Up-going Wavefield**

Lin, Tu, and Herrmann, 2010

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Approximate EPSI

EPSI follows the formulation of SRME:

$$\hat{P} \approx \Delta \hat{G}(\hat{Q} - \hat{P})$$

Reformulating the EPSI operator:

$$\mathcal{F}_t^* \text{BlockDiag}_{1\ldots nf}[(\hat{Q} - \hat{P})^* \otimes I] \mathcal{F}_t \delta g \approx p$$
Approx. Robust EPSI

**Robust EPSI:**

\[
\delta \tilde{g} = \min_{\delta g} \| \delta g \|_1 \quad \text{subject to} \quad \| p - E \delta g \|_2 \leq \sigma
\]

sparsity promoting part

data fitting part
Combine EPSI with migration

We identify the total up-going wavefield with model perturbations:

\[ \delta \tilde{m} = S^* \min_{\delta x} ||\delta x||_1 \text{ subject to } ||p - E \underbrace{KS^* \delta x}_{\delta g}||_2 \leq \sigma \]
Numerical experiments

Linearized data:

• surface-free data
  \[ p_1 = K \delta m \]

• total data
  \[ p_2 = EK \delta m \]
Data preview: surface free data

total shots: 128, shot number: 65
Data preview:
total data

total shots: 128, shot number: 65

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Case study 1

- scenario 1: inversion from surface-free data
- scenario 2: inversion from total data
- scenario 3: combined inversion from total data
Sparse inversion of surface-free data
Sparse inversion of total data
Sparse inversion of data with multiples with EPSI
Sparse inversion of surface-free data
Do multiples really help?

Both scenario 1 and scenario 3 give good results...Will this still be true when we invert from incomplete data?
Case study 2

From incomplete data

• scenario 1: inversion from surface-free data
• scenario 2: combined inversion from total data
Incomplete data

• 16 sequential shots selected randomly from a total number of 128 shots
• 300m missing near-offset
Sparse inversion from surface-free data

SNR: 3.08dB (compared to true dm)
Combined inversion of total data

SNR: 3.72dB (compared to true dm)
Recovered Green’s function

![Graph showing a recovered Green's function with axes labeled Time(s) and Offset(m).]

- Total shots: 128, shot number: 65, SNR: 15.4dB

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In EPSI’s point of view

- Model perturbation is sparser than the Green’s function
- Primary estimation in image space
- One joint inversion outperforms two separate inversions
Case study 3

From 12 simultaneous shots

- scenario 1: EPSI & migration
- scenario 2: EPSI -> migration
Data preview

Receiver number

Time (s)

20 40 60 80 100 120

total shots: 12

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Combined inversion

SNR: 5.30dB (compared to true dm)
Two separate inversions

SNR: 4.52dB (compared to true dm)
Recovered Green’s function in combined inversion

total shots: 128, shot number: 65, SNR: 22.4dB
Recovered Green’s function in separate inversions

total shots: 128, shot number: 65, SNR: 5.4dB
Conclusions

By combing EPSI with migration:

• we reap benefits in seismic imaging by exploiting the extra illumination from surface multiples

• better primary estimation results by exploiting sparsity in image space
Future plans

- How to adapt EPSI for incomplete data
- Speed-up this joint inversion by introducing simultaneous sources
- extend this work to FWI
EPSI for incomplete data

EPSI contains term $G^*P$, matrix multiplication breaks when $P$ is incomplete along both columns and rows.
Compressively simulate $G$

EPSI needs full $G$ too for $G*P$ term.

- $P=USV^*$, use $U$ as a simultaneous shot term for $G$
- $G*P=P*G$ by reciprocity, apply the simultaneous shot term on the right.
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Thanks for your attention