Sparsity-promoting migration with surface-related multiples

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Abstract

Multiples, especially the surface-related multiples, form a significant part of the total up-going wavefield. If not properly dealt with, they can lead to false reflectors in the final image. So conventionally practitioners remove them prior to migration. Recently research has revealed that multiples can actually provide extra illumination so different methods are proposed to address the issue that how to use multiples in seismic imaging, but with various kinds of limitations. In this abstract, we combine primary estimation and sparsity-promoting migration into one convex-optimization process to include information from multiples. Synthetic examples show that multiples do make active contributions to seismic migration. Also by this combination, we can benefit from better recoveries of the Green's function by using sparsity-promoting algorithms since reflectivity is sparser than the Green's function.

Introduction

In seismic imaging, we want to use as much information from seismic data as possible. Due to the fact that few imaging methods take multiples into account, multiple reflections, which form a significant component of the total up-going wavefield, will result in false reflectors in the final image (Verschuur, 2006). This explains why practitioners remove (at least surface-related) multiples from data prior to migration. Recently people have gradually realized that multiples can provide extra information. Because multiples travel more than once between reflectors, they contain higher spatial wavenumber components especially at far offsets, provide wider illumination angles, and are more sensitive to velocity changes (Verschuur, 2006). Therefore, finding a way to incorporate these additional information potentially leads to better images.

However, how to use multiples in imaging is still an open problem and methods addressing this issue have emerged in the literature. Reiter et al. (1991) introduced a free surface to the background-velocity model so the Green's function can explain multiple reflections to some extent. Berkhout et al. (Berkhout and Vershuur, 1994; Berkhout et al., 2009) proposed a double-illumination method that regards the surface-related multiples as the response when the primary reflections at the surface are regarded as the areal source, as well as another method by using focal transform (Berkhout and Verschuur, 2006) to map first-order multiples to primaries. Youn and Zhou (2001) and Sandberg et al. (2010) both used methods based on wavefield extrapolation and correlation using two-way wave-equation. He et al. (2007) used 3D wave-equation interferometric migration for VSP data containing free-surface multiples. To summarize, multiples were used to different extents with various kinds of limitations, e.g., violating the Born-scattering assumptions (Reiter et al., 1991), or being confined to certain migration method (Berkhout and Vershuur, 1994; Youn and Zhou, 2001; Sandberg et al., 2010) or certain type of data (He et al., 2007).

Here we propose combining EPSI (Estimation of Primaries via Sparse Inversion) (van Groenestijn and Verschuur, 2009a) with migration as a way to benefit from surface-related multiples. EPSI is able to invert from the whole up-going wavefield (ignoring internal multiples) for the Green's function from which we can perform migration. By incorporating EPSI, our method also offers the flexibility of choosing the right migration method for each individual case. By combining EPSI and migration, we can also obtain better recoveries of the Green's function with sparsity-promoting algorithms since the reflectivity is sparser than the Green's function.

Mapping wavefields back to the Green's function by EPSI

Verschuur et al. (1992) established the following monochromatic relationship between the up-going wavefield and the Green's function:

$$\hat{\mathbf{P}} = \hat{\mathbf{G}}(\hat{\mathbf{Q}} - \hat{\mathbf{P}}), \tag{1}$$

where $\hat{\mathbf{G}}$ represents the Green's function, $\hat{\mathbf{P}}$ the total up-going wavefield, and $\hat{\mathbf{Q}}$ the source signature. In this expression, we assume that the surface reflection operator can be given by $\mathbf{R} = -\mathbf{I}$ and that the source is stationary, i.e., $\hat{\mathbf{Q}} = \mathrm{q}\mathbf{I}$. Hatted quantities represent monochromatic variables. Each upper-case variable is a matrix comprised of a single frequency slice of the wavefield. Based on this relationship, EPSI inverts for $\mathbf{G}$ and $\mathbf{Q}$ simultaneously from the total data $\hat{\mathbf{P}}$ (van Groenestijn and Verschuur, 2009a). But for the scope of this abstract, we temporarily assume that $\mathbf{Q}$ is known.

To facilitate further discussion, the EPSI operator $\mathbf{E}$ is mathematically formulated so that it is in the canonical form of a linear operator acting on a vector that is conducive to writing down an optimization problem that find a solution with sparsity promoting (Lin and Herrmann, 2010):

$$\underbrace{\mathscr{F}_t^{*}\mathrm{BlockDiag}_{\mathrm{f}}[(\hat{\mathbf{Q}} - \hat{\mathbf{P}})^{*} \otimes \mathbf{I}]\mathscr{F}_{\mathbf{t}}}_{\mathbf{E}}\,\mathbf{g} = \mathbf{p} \tag{2}$$

where lower case quantities, e.g., $\mathbf{g}$ and $\mathbf{p}$, represent vectorized wavefield; $\mathscr{F}_t$ is the Fourier transform that operates along the time axis of the vectorized wavefield $\mathbf{g}$, and its adjoint operator $\mathscr{F}_t^{*}$ brings the wavefield back to the time domain. The block diagonal term varies over frequencies. The symbol $\otimes$ refers to the Kronecker product which turns matrix multiplication into matrix-vector multiplications.

With this expression, the convex version of EPSI (Lin and Herrmann, 2010) can be expressed as

$$\tilde{\mathbf{g}} = \underset{\mathbf{g}}{\operatorname{argmin}} ||\mathbf{g}||_1 \quad \text{subject to } ||\mathbf{p} - \mathbf{E}\mathbf{g}||_2 \leq \sigma \tag{3}$$

where σ is the residual between the recorded data and the up-going wavefield modeled by applying the EPSI operator $\mathbf{E}$ to the prediction of Green's function $\mathbf{g}$. This σ is highly linked to the noise level of the recorded data.

Despite the fact that EPSI is a relative new technique, there are already some successful applications (van Groenestijn and Verschuur, 2009b; Lin and Herrmann, 2010; Baardman et al., 2010) to primary estimation and near-offset data reconstruction.

Combining EPSI with sparsity-promoting migration

By combining EPSI with migration, one benefit is that it offers the flexibility to choose from a wide range of migration methods. Least-squares migration (Nemeth et al., 1999) is used here for its potential further extension to FWI. Since perturbations in the velocity $\delta\mathbf{m}$ is our prime objective, we include the linearized Born-scattering operator $\mathbf{K}$, which, when applied to $\delta\mathbf{m}$, produces a linearized scattering multiple-free wavefield $\delta\mathbf{b}$. Now assuming that the Green's function $\mathbf{g}$ is a good approximation of $\delta\mathbf{b}$, we can invert for $\delta\mathbf{m}$ by solving the following sparity-promoting problem (Herrmann and Li, 2010) in the curvelet domain (Candès et al., 2006):

$$\delta\tilde{\mathbf{m}} = \mathbf{S}^* \underset{\delta\mathbf{x}}{\operatorname{argmin}} ||\delta\mathbf{x}||_1 \quad \text{subject to } ||\mathbf{g} - \mathbf{K}\mathbf{S}^*\delta\mathbf{x}||_2 \leq \sigma \tag{4}$$

where $\mathbf{S}^*$ denotes the curvelet synthesis operator.

While sparsity-promoting migration only admits multiple-free data input, we overcome this restriction by including the EPSI operator, which is able to model the surface-related multiples (but not internal multiples). Since our background-velocity model is smooth, we ignore the internal multiples and identify the whole up-going wavefield with the medium perturbations. As a result, we can estimate this perturbation from $\mathbf{p}$ by solving the following problem when σ is adjusted to allow for misfit from internal multiples:

$$\delta\tilde{\mathbf{m}} = \mathbf{S}^* \underset{\delta\mathbf{x}}{\operatorname{argmin}} ||\delta\mathbf{x}||_1 \quad \text{subject to } ||\mathbf{p} - \mathbf{E}\mathbf{K}\mathbf{S}^*\delta\mathbf{x}||_2 \leq \sigma. \tag{5}$$

The benefit is at least three-fold. Firstly, we do not need to remove either the surface-related multiples or the source signature since they are all handled by EPSI. Secondly, since inverting $\mathbf{K}$ and $\mathbf{E}$ are both represented in ℓ_1-norm minimization form for inversion stability and well-posedness, we expect the combined inversion process to be stable as well. Thirdly, since information from surface-related multiples is used, equation (5) should recover more structural information than equation (4).

Examples with a synthetic 2D seismic line

So far we have been working with linearized data. The salt dome model (van Groenestijn and Verschuur, 2009a) is used to make a synthetic 2D seismic line that contains 128 receivers and shots with a 20-meter receiver/shot spacing. We approximate the Green's function $\mathbf{g}$ by applying the frequency-domain Born-scattering operator $\mathbf{K}$ to the reflectivity $\delta\mathbf{m}$ and the whole up-going wavefield $\mathbf{p}$ by applying the combined operator $\mathbf{EK}$ to $\delta\mathbf{m}$. An acoustic FD (Finite Difference) modeled wavefield and a pre-calculated wavelet are used to build the EPSI operator $\mathbf{E}$.

We first work on the whole seismic line and compare three possible scenarios when inverting for $\delta\mathbf{m}$: from multiple-free $\mathbf{g}$ by inverting the Born-scattering operator $\mathbf{K}$, from $\mathbf{p}$ that contains multiples, again by inverting $\mathbf{K}$, and from $\mathbf{p}$ but by inverting the combined operator $\mathbf{EK}$.

SPGℓ_1 (van den Berg and Friedlander, 2008) is used as the solver. The results after 50 **SPGℓ_1** iterations are shown in Figure (1). Scenario (1) intuitively gives a good result, shown in the top panel, thanks to the absence of multiples. In scenario (2), artifacts are clearly seen in the middle panel, mostly in forms of spatially displaced copies of true reflectors, showing the harm of multiples if they are not properly dealt with. By including the EPSI operator $\mathbf{E}$, we again get clear image in scenario (3), shown in the bottom panel, indicating that surface-related multiples are now well handled. This also verifies the previous conjecture that the combined inversion process with $\mathbf{EK}$ is stable.

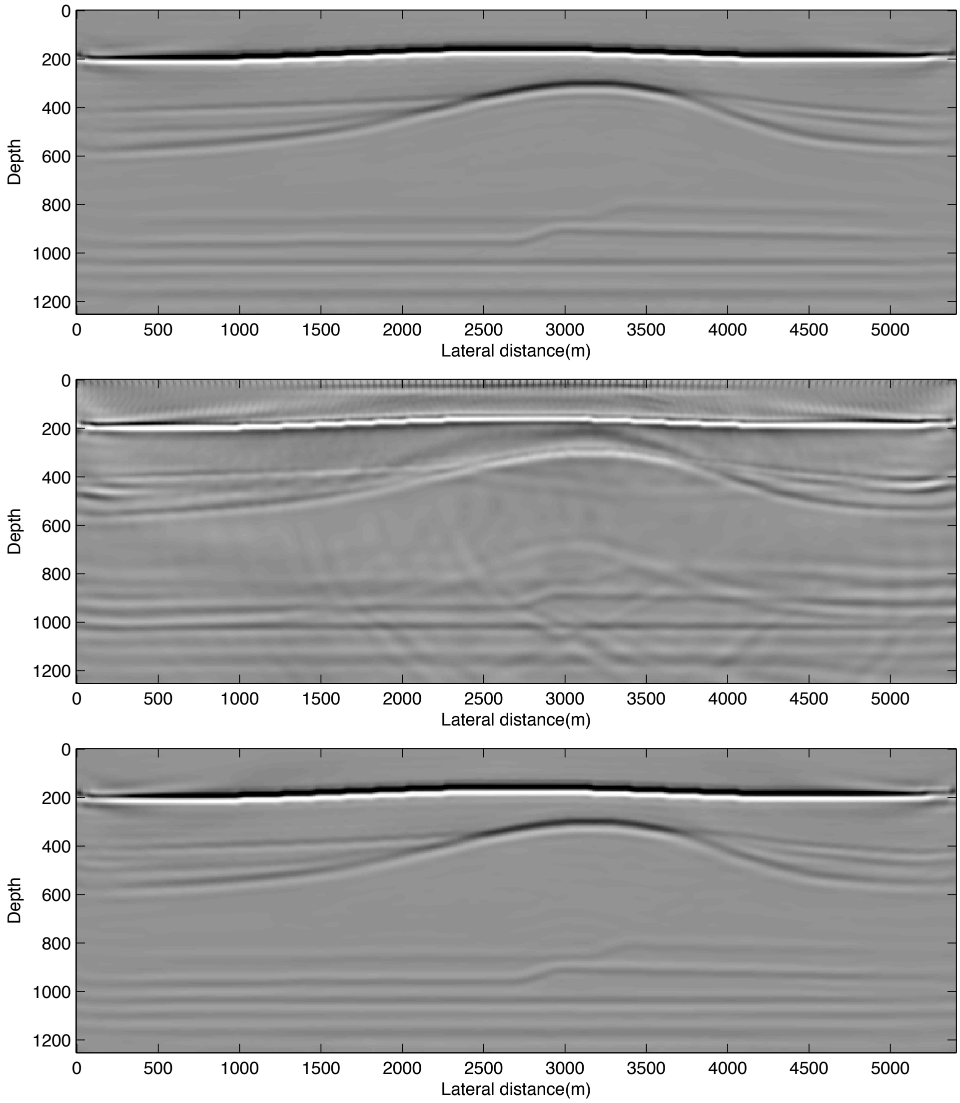

Figure 1: Inversion results from the seismic line

Examples with incomplete data

Next we work on subsampled data to see whether migration really benefits from using multiples. 16 out of all 128 shots are selected randomly from the seismic line with a 300-meter missing near-offset in each shot record, and 7 out of a total number of 98 frequencies are randomly chosen for migration. **RM** is the restriction and measurement operator corresponding to our subsampling scheme here.

We compare two scenarios to invert for $\delta\mathbf{m}$: from the subsampled multiple-free data **RMg** with operator **RMK**, and from the subsampled total data **RMp** that contains multiples with operator **RMEK**. If multiples really help imaging, scenario (2) should lead to better results.

The results of scenario (1) and (2) after 100 $\mathbf{SPG}\ell_1$ iterations are shown in the top and bottom panels of Figure (2) respectively. At least at those positions with insufficient illumination, e.g., at the corners, better image is obtained by including information from multiples. The $\delta\mathbf{m}$s we inverted in scenario (1) and (2) have SNRs of 3.08dB and 3.72dB respectively compared with the true reflectivity, indicating that multiples do help somewhat to improve image quality.

To summarize, by combining EPSI and sparsity-promoting migration into one convex optimization problem, surface-related multiples are now making active contributions to the final image. We will next work on field data and extend this work to FWI in the future.

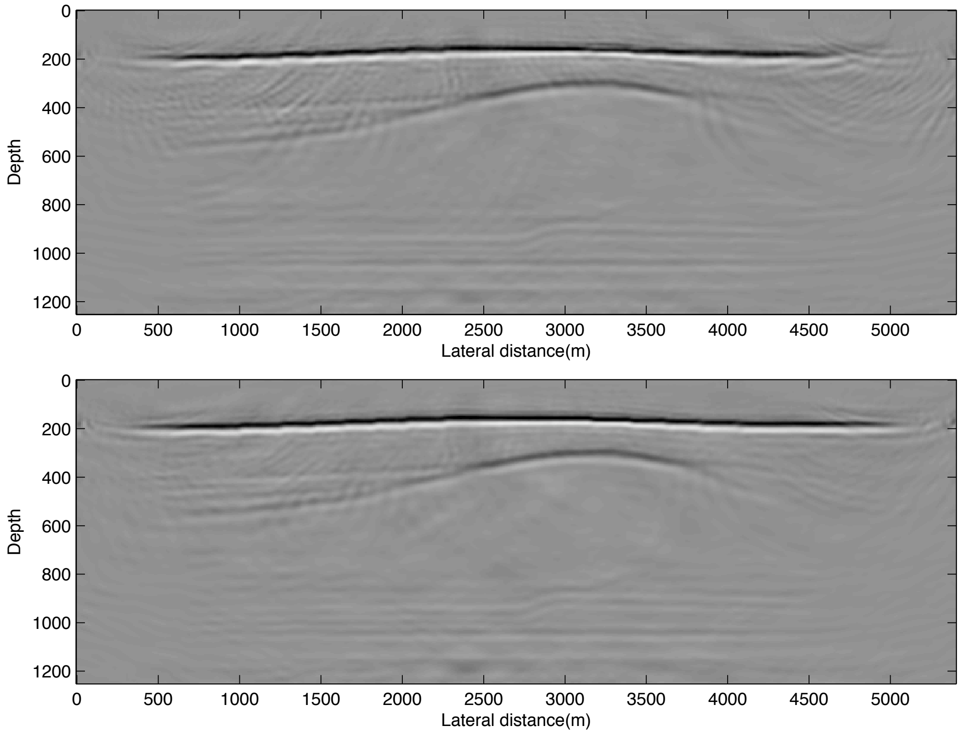

Figure 2: Inversion results from incomplete data

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