Vienna '11

Estimating Primaries by Sparse Inversion in a Curvelet-like Representation Domain

Tim T.Y. Lin and Felix J. Herrmann

SLIM
University of British Columbia

EPSI Problem

Estimation of Primaries by Sparse Inversion (van Groenestijn and Verschuur, 2009)

recorded data   predicted data from primary IR

$$\mathbf{P} = \mathbf{G}(\mathbf{Q} + \mathbf{R}\mathbf{P})$$

$\mathbf{P}$	**total up-going wavefield**
$\mathbf{Q}$	**down-going source signature**
$\mathbf{R}$	**reflectivity of free surface (assume -1)**
$\mathbf{G}$	**primary impulse response**

(all monochromatic data matrix, implicit ω)

EPSI Problem

Estimation of Primaries by Sparse Inversion (van Groenestijn and Verschuur, 2009)

recorded data   predicted data from primary IR

$$\mathbf{P} = \mathbf{G}(\mathbf{Q} + \mathbf{R}\mathbf{P})$$

Inversion objective:

$$f(\mathbf{G}, \mathbf{Q}) = \frac{1}{2}\|\mathbf{P} - \mathbf{G}(\mathbf{Q} + \mathbf{R}\mathbf{P})\|_2^2$$

EPSI Problem

In time domain **(lower-case: whole dataset in time domain)**

recorded data predicted data from primary IR

$$\mathbf{p} = \mathcal{M}(\mathbf{g}, \mathbf{q})$$

$$\mathcal{M}(\mathbf{g}, \mathbf{q}) := \mathcal{F}_{\mathrm{t}}^{\dagger} \mathrm{BlockDiag}_{\omega_1 \cdots \omega_{nf}} [(q(\omega)\mathbf{I} - \mathbf{P})^{\dagger} \otimes \mathbf{I}] \mathcal{F}_{\mathrm{t}} \mathbf{g}$$

Inversion objective:

$$f(\mathbf{g}, \mathbf{q}) = \frac{1}{2} \|\mathbf{p} - \mathcal{M}(\mathbf{g}, \mathbf{q})\|_2^2$$

EPSI Problem

Linearizations

$$\mathbf{p} = \mathcal{M}(\mathbf{g}, \mathbf{q})$$

$$\mathbf{M}_{\tilde{q}} = \left(\frac{\partial \mathcal{M}}{\partial \mathbf{g}} \right)_{\tilde{q}}$$

$$\mathbf{M}_{\tilde{g}} = \left(\frac{\partial \mathcal{M}}{\partial \mathbf{q}} \right)_{\tilde{g}}$$

In fact it is bilinear:

$$\mathbf{M}_{\tilde{q}}\mathbf{g} = \mathcal{M}(\mathbf{g}, \tilde{\mathbf{q}}) \qquad \mathbf{M}_{\tilde{g}}\mathbf{q} = \mathcal{M}(\mathbf{q}, \tilde{\mathbf{g}})$$

EPSI Problem

Linearizations

$$\mathbf{p} = \mathcal{M}(\mathbf{g}, \mathbf{q})$$

$$\mathbf{M}_{\tilde{q}} = \left(\frac{\partial \mathcal{M}}{\partial \mathbf{g}}\right)_{\tilde{q}}$$

$$\mathbf{M}_{\tilde{g}} = \left(\frac{\partial \mathcal{M}}{\partial \mathbf{q}}\right)_{\tilde{g}}$$

Associated objectives:

$$f_{\tilde{q}}(\mathbf{g}) = \frac{1}{2}\|\mathbf{p} - \mathbf{M}_{\tilde{q}}\mathbf{g}\|_2^2 \qquad f_{\tilde{g}}(\mathbf{q}) = \frac{1}{2}\|\mathbf{p} - \mathbf{M}_{\tilde{g}}\mathbf{q}\|_2^2$$

EPSI Procedure

Do:

$$\mathbf{g}_{k+1} = \mathbf{g}_k + \alpha \nabla f_{q_k}(\mathbf{g}_k)$$
$$\mathbf{q}_{k+1} = \mathbf{q}_k + \beta \nabla f_{g_{k+1}}(\mathbf{q}_k)$$

Alternating updates (Gauss-Sidel) to the linearized problem

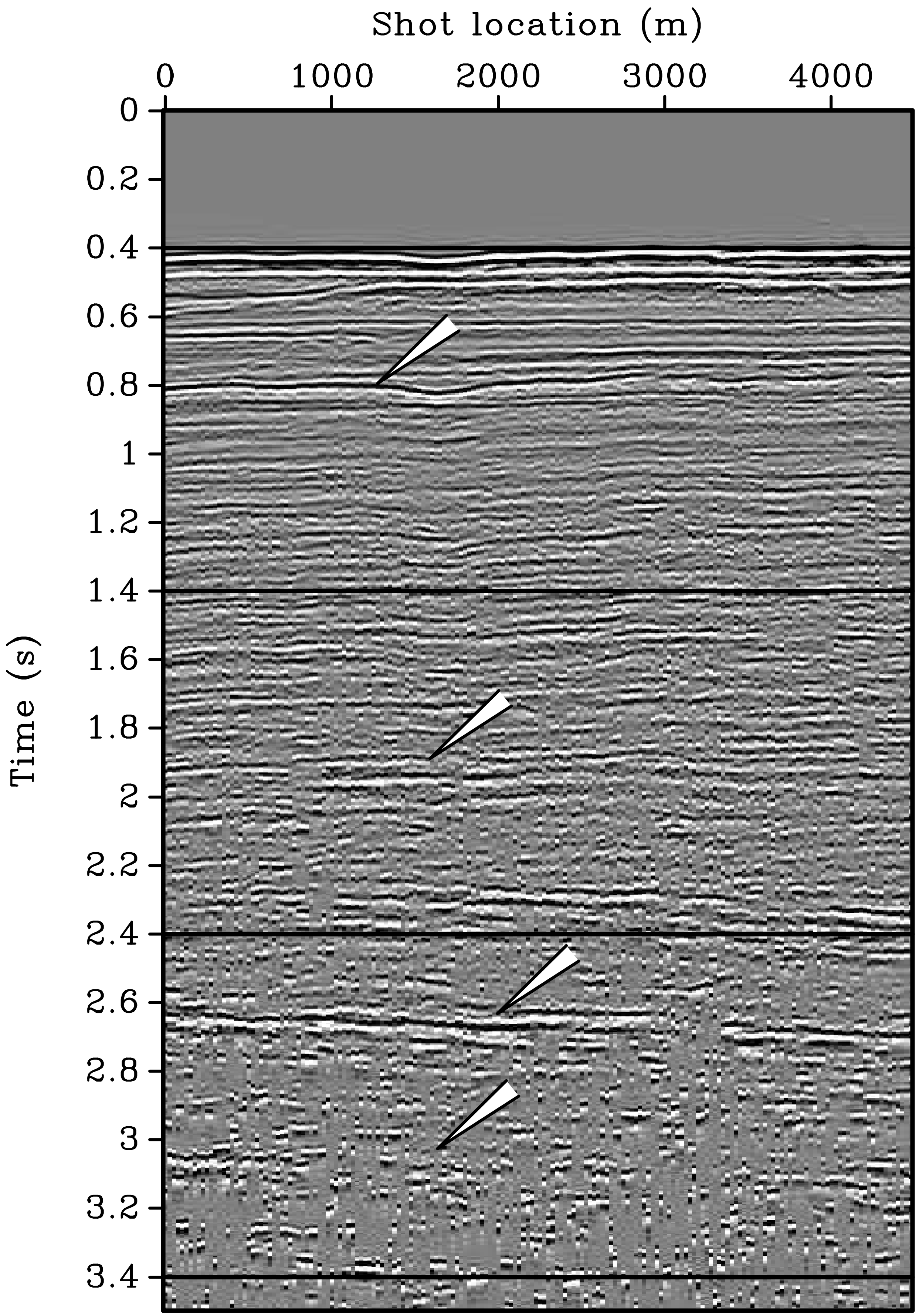
EPSI Procedure

Do:

$$\mathbf{g}_{k+1} = \mathbf{g}_k + \alpha \mathcal{S}(\nabla f_{q_k}(\mathbf{g}_k))$$
$$\mathbf{q}_{k+1} = \mathbf{q}_k + \beta \nabla f_{g_{k+1}}(\mathbf{q}_k)$$

Gradient sparsity

$\mathcal{S}$: pick largest ρ elements per trace


Shot location (m)
0
1000
2000
3000
4000
Time (s)
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
2.2
2.4
2.6
2.8
3
3.2
3.4

EPSI Procedure

Related to two underlying sub-problems:

$$\min_{\mathbf{g}} \|\mathbf{p} - \mathbf{M}_{\tilde{q}}\mathbf{g}\|_2 \quad \text{s.t.} \quad \mathrm{nnz}(\mathbf{g}) \leq \rho$$

$$\min_{\mathbf{q}} \|\mathbf{p} - \mathbf{M}_{\tilde{g}}\mathbf{q}\|_2$$

Which approximates:

$$\min_{\mathbf{g}} \mathrm{nnz}(\mathbf{g}) \quad \text{s.t.} \quad \|\mathbf{p} - \mathbf{M}_{\tilde{q}}\mathbf{g}\|_2 \leq \sigma$$

(notion of sparsest solution)

$$\min_{\mathbf{q}} \|\mathbf{p} - \mathbf{M}_{\tilde{g}}\mathbf{q}\|_2$$

EPSI Procedure

Can be made non-combinatorial (convex) by:

$$\min_{\mathbf{g}} \|\mathbf{g}\|_1 \quad \text{s.t.} \quad \|\mathbf{p} - \mathbf{M}_{\tilde{q}}\mathbf{g}\|_2 \leq \sigma$$

(minimum L1 solution usually the sparsest solution)

$$\min_{\mathbf{q}} \|\mathbf{p} - \mathbf{M}_{\tilde{g}}\mathbf{q}\|_2$$

Convex EPSI

Do:

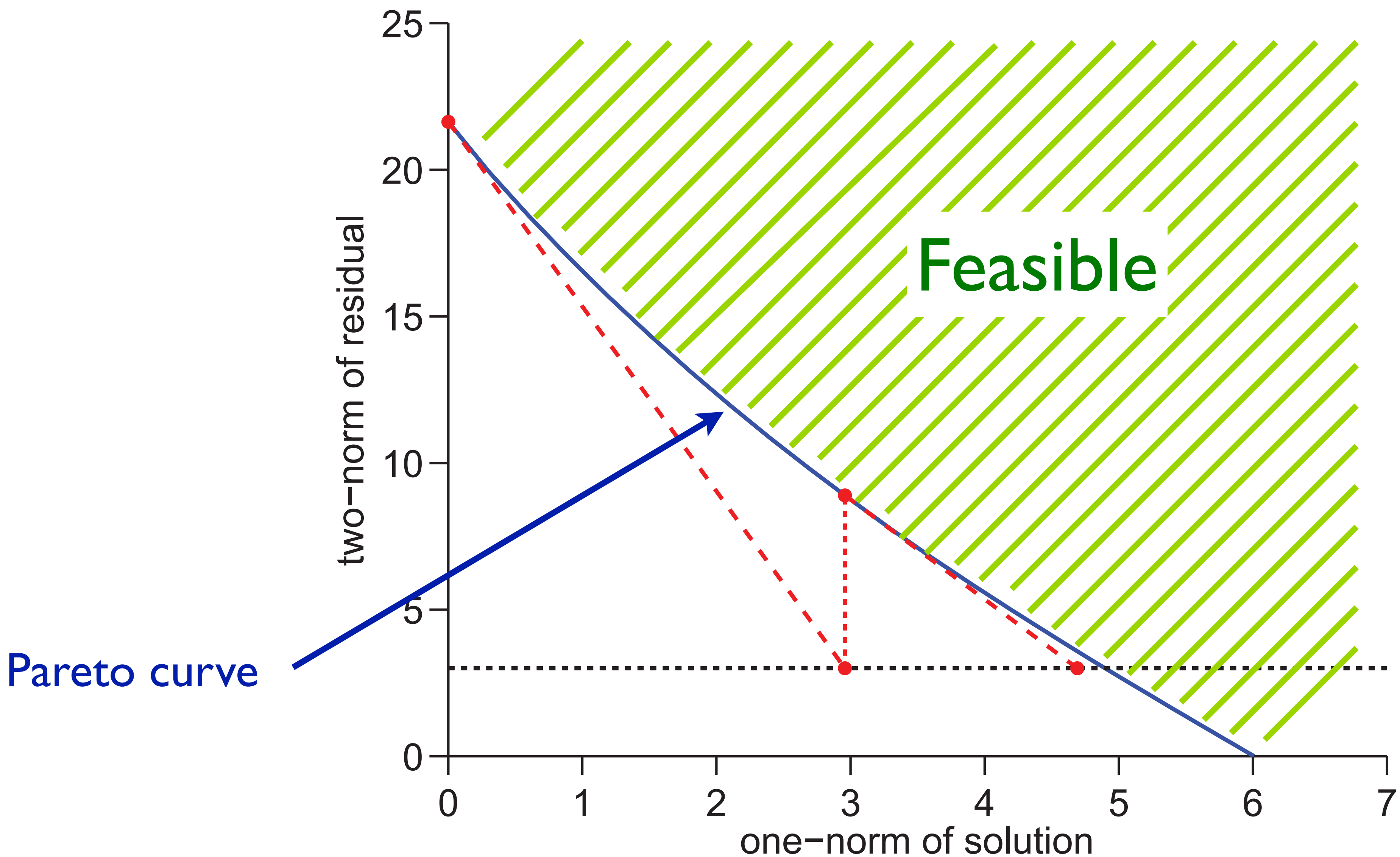
$$\mathbf{g}_{k+1} = \mathbf{g}_k + \alpha \, \mathrm{SoftTh}_{\phi}(\nabla f_{q_k}(\mathbf{g}_k))$$
$$\mathbf{q}_{k+1} = \mathbf{q}_k + \beta \nabla f_{g_{k+1}}(\mathbf{q}_k)$$

Soft-thresholding solves an L1 minimization problem, but how is ϕ **determined?**

Pareto curve

$$\begin{array}{ll} \text{minimize} & \|x\|_1 \\ \text{subject to} & \|Ax - b\|_2 \leq \sigma \end{array}$$

Look at the solution space and the line of optimal solutions (Pareto curve)

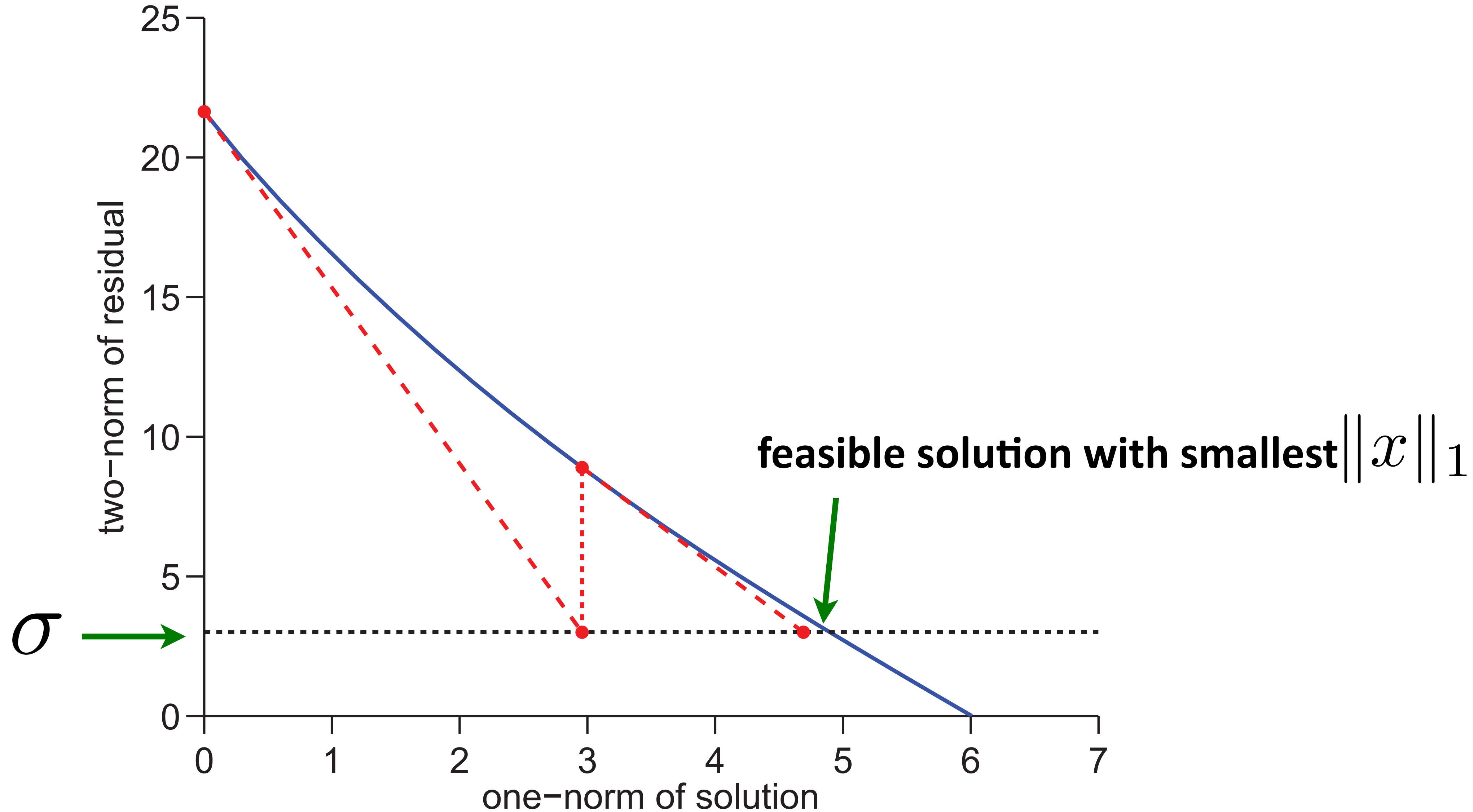


(van den Berg, Friedlander, 2008)

Pareto curve

$$\begin{array}{ll} \text{minimize} & \|x\|_1 \\ \text{subject to} & \|Ax-b\|_2 \le \sigma \end{array}$$

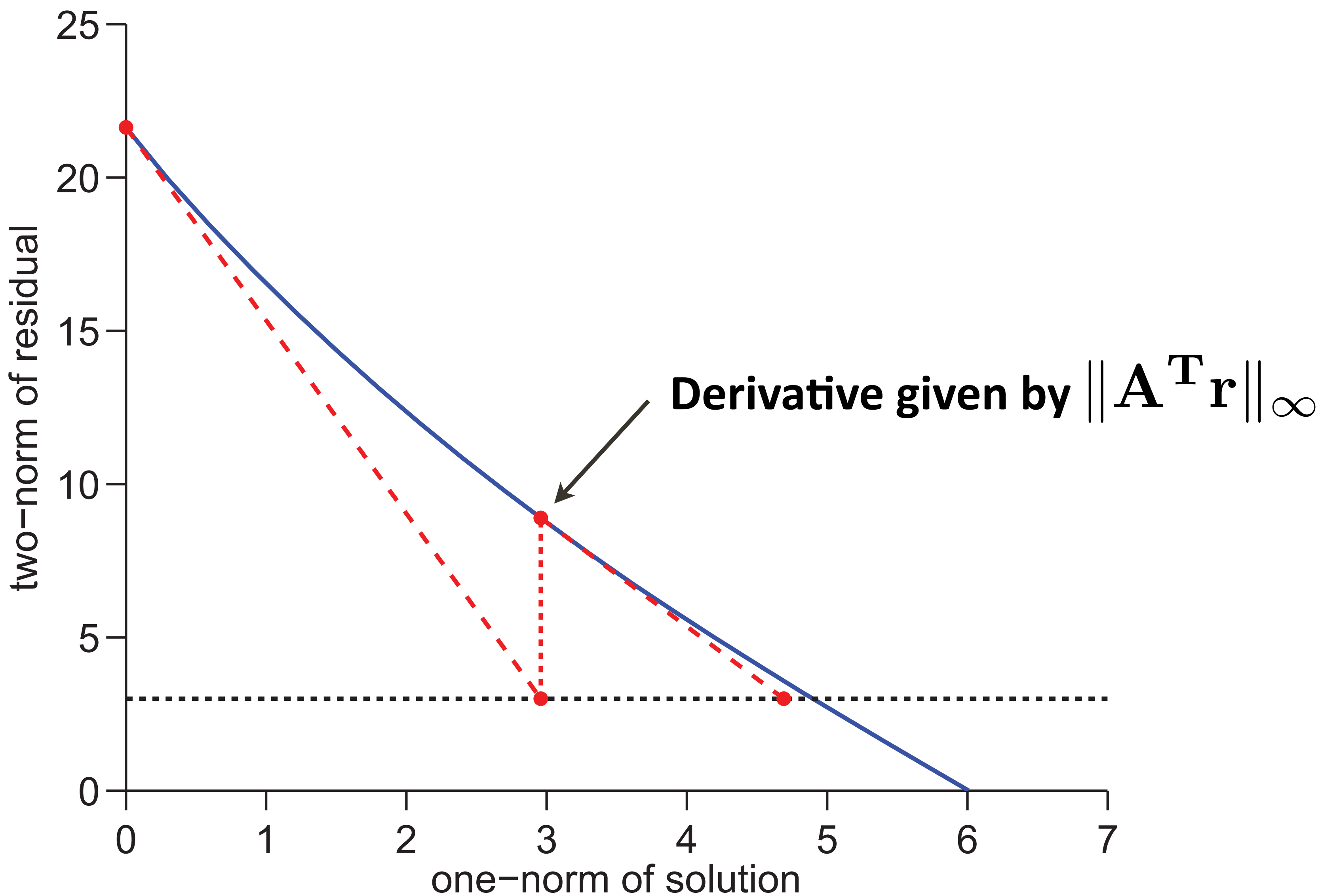
Look at the solution space and the line of optimal solutions (Pareto curve)



Pareto curve

$$\begin{array}{ll} \text{minimize} & \|x\|_1 \\ \text{subject to} & \|Ax - b\|_2 \leq \sigma \end{array}$$

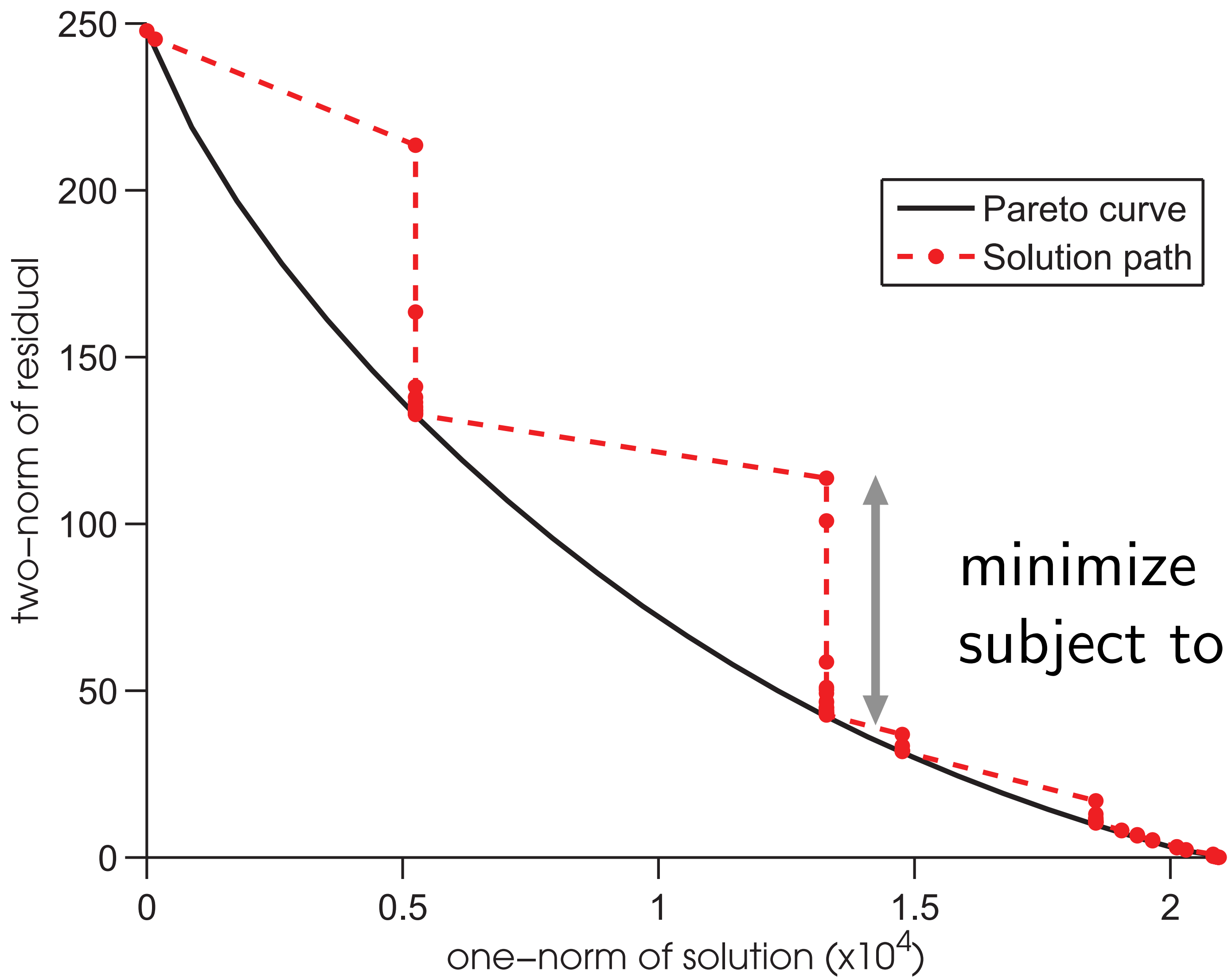
Look at the solution space and the line of optimal solutions (Pareto curve)



Pareto curve

$$\begin{array}{ll}\text{minimize} & \|x\|_1 \\ \text{subject to} & \|Ax - b\|_2 \leq \sigma\end{array}$$

Look at the solution space and the line of optimal solutions (Pareto curve)

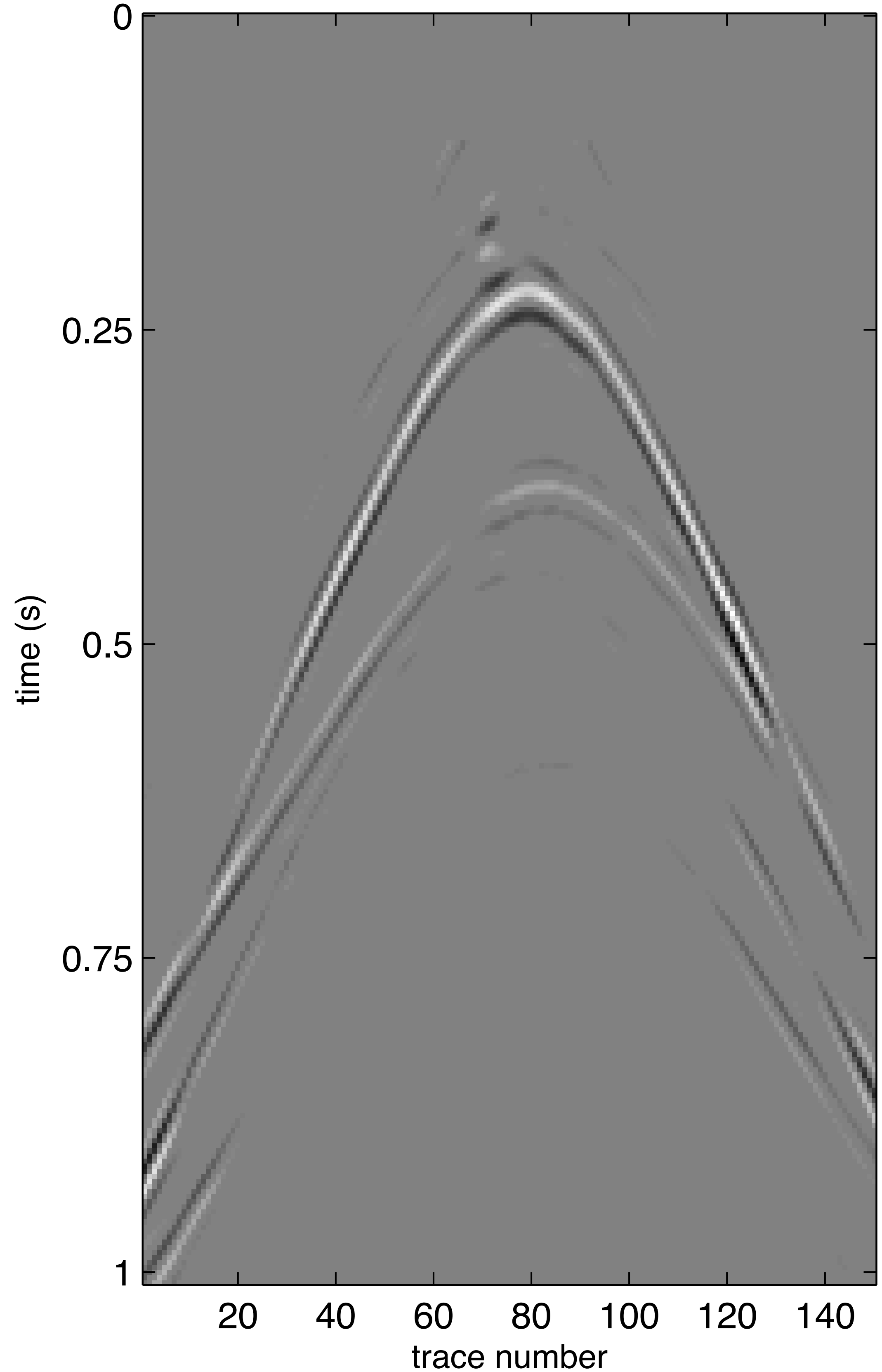


$$\begin{array}{ll}\text{minimize} & \|Ax - b\|_2 \\ \text{subject to} & \|x\|_1 \leq \tau\end{array}$$

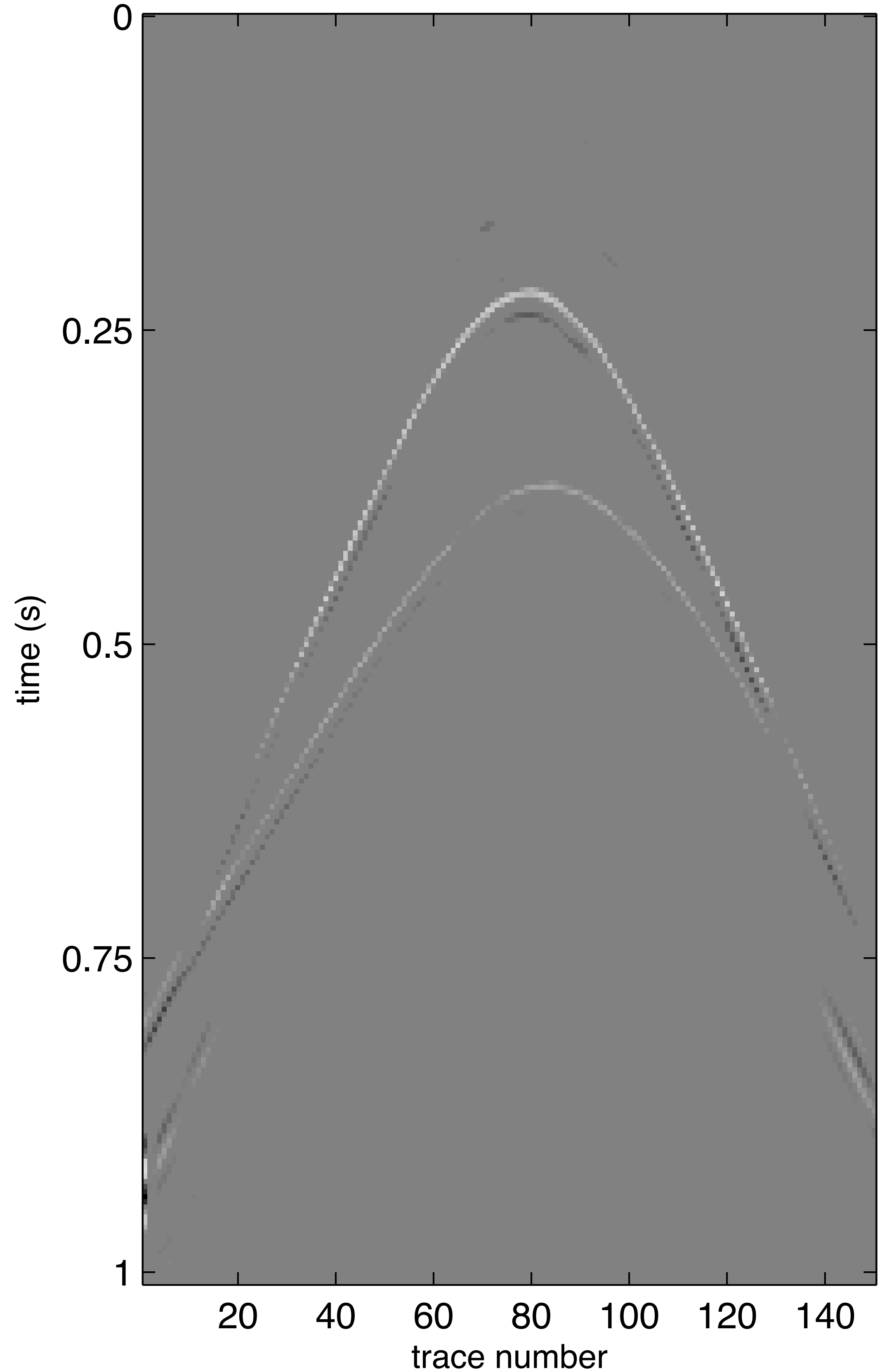
solve with SPG

(spectral projected gradients)

SPG start



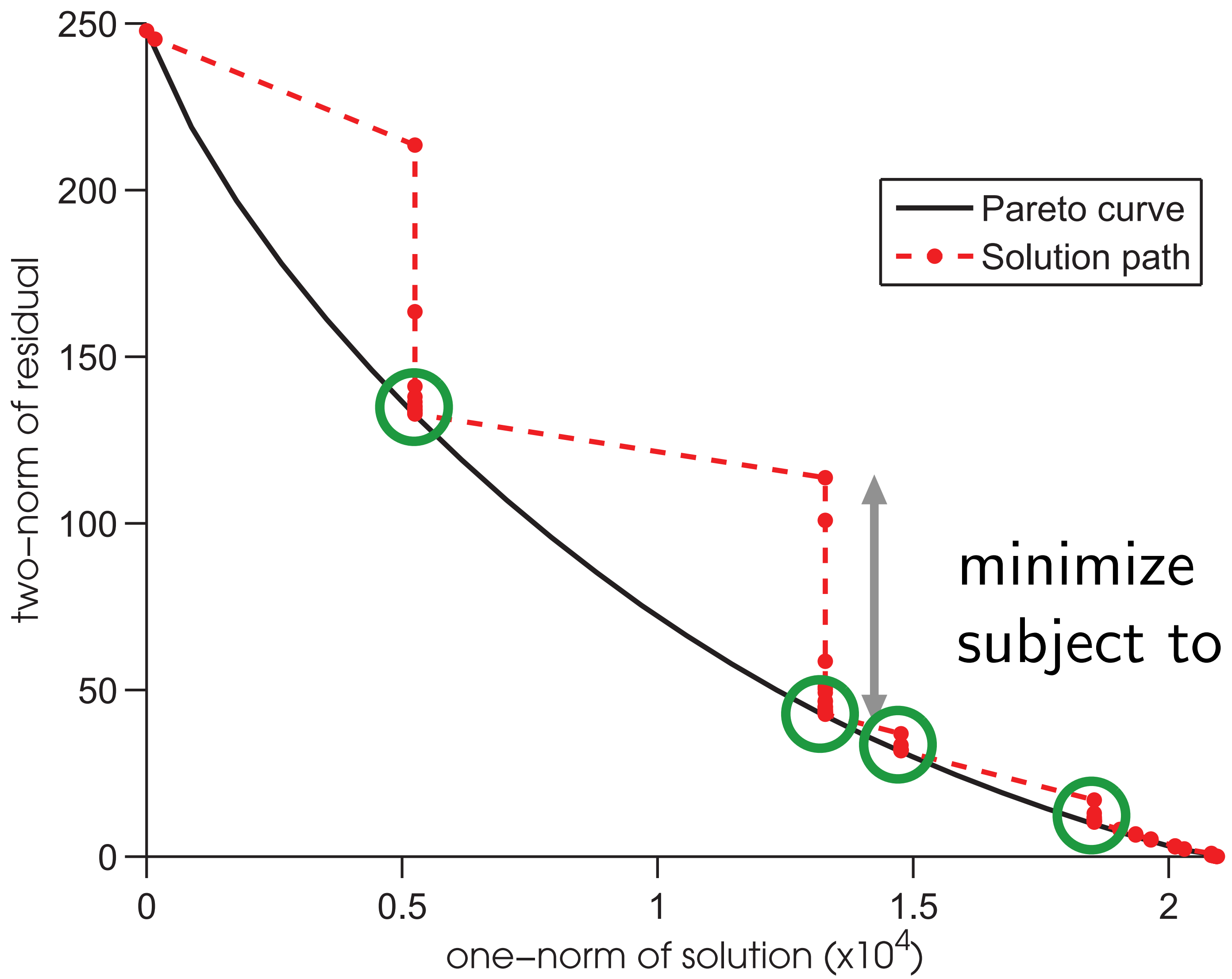
SPG at Pareto curve



Pareto curve

$$\begin{array}{ll} \text{minimize} & \|x\|_1 \\ \text{subject to} & \|Ax - b\|_2 \leq \sigma \end{array}$$

Only solve least-squares matching for q when solution reaches Pareto curve



$$\begin{array}{ll} \text{minimize} & \|Ax - b\|_2 \\ \text{subject to} & \|x\|_1 \leq \tau \end{array}$$

SLIM

Robust EPSI procedure

While $\|\mathbf{p} - \mathcal{M}(\mathbf{g}_k, \mathbf{q}_k)\|_2 > \sigma$

determine new τ_k from the Pareto curve

$$\mathbf{g}_{k+1} = \arg\min_{\mathbf{g}} \|\mathbf{p} - \mathbf{M}_{q_k}\mathbf{g}\|_2 \text{ s.t. } \|\mathbf{g}\|_1 \leq \tau_k$$

(Solve with SPGL1 until Pareto curve reached)

$$\mathbf{q}_{k+1} = \arg\min_{\mathbf{q}} \|\mathbf{p} - \mathbf{M}_{g_{k+1}}\mathbf{q}\|_2$$

(Solve with LSQR)

REPSI in transform domain

Modify just the problem for g:

$$\min_{\mathbf{g}} \|\mathbf{g}\|_1 \quad \text{s.t.} \quad \|\mathbf{p} - \mathbf{M}_{\tilde{q}}\mathbf{g}\|_2 \leq \sigma$$

$$\min_{\mathbf{q}} \|\mathbf{p} - \mathbf{M}_{\tilde{g}}\mathbf{q}\|_2$$

REPSI in transform domain

Modify just the problem for g:

$$\min_{\mathbf{x}} \|\mathbf{x}\|_1 \quad \text{s.t.} \quad \|\mathbf{p} - \mathbf{M}_{\tilde{q}}\mathbf{S}^\dagger\mathbf{x}\|_2 \leq \sigma, \quad \mathbf{g} = \mathbf{S}^\dagger\mathbf{x}$$

(basis pursuit)

$$\min_{\mathbf{q}} \|\mathbf{p} - \mathbf{M}_{\tilde{g}}\mathbf{q}\|_2$$

$\mathbf{S}$: sparsifying representation for seismic signals

- **Should have spatially localized support**
- **ex: nd-Wavelets, Curvelets, etc...**

$\mathbf{S}^\dagger$: synthesis operator for $\mathbf{S}$

REPSI in transform domain

While $\|\mathbf{p} - \mathcal{M}(\mathbf{g}_k, \mathbf{q}_k)\|_2 > \sigma$

determine new τ_k from the Pareto curve

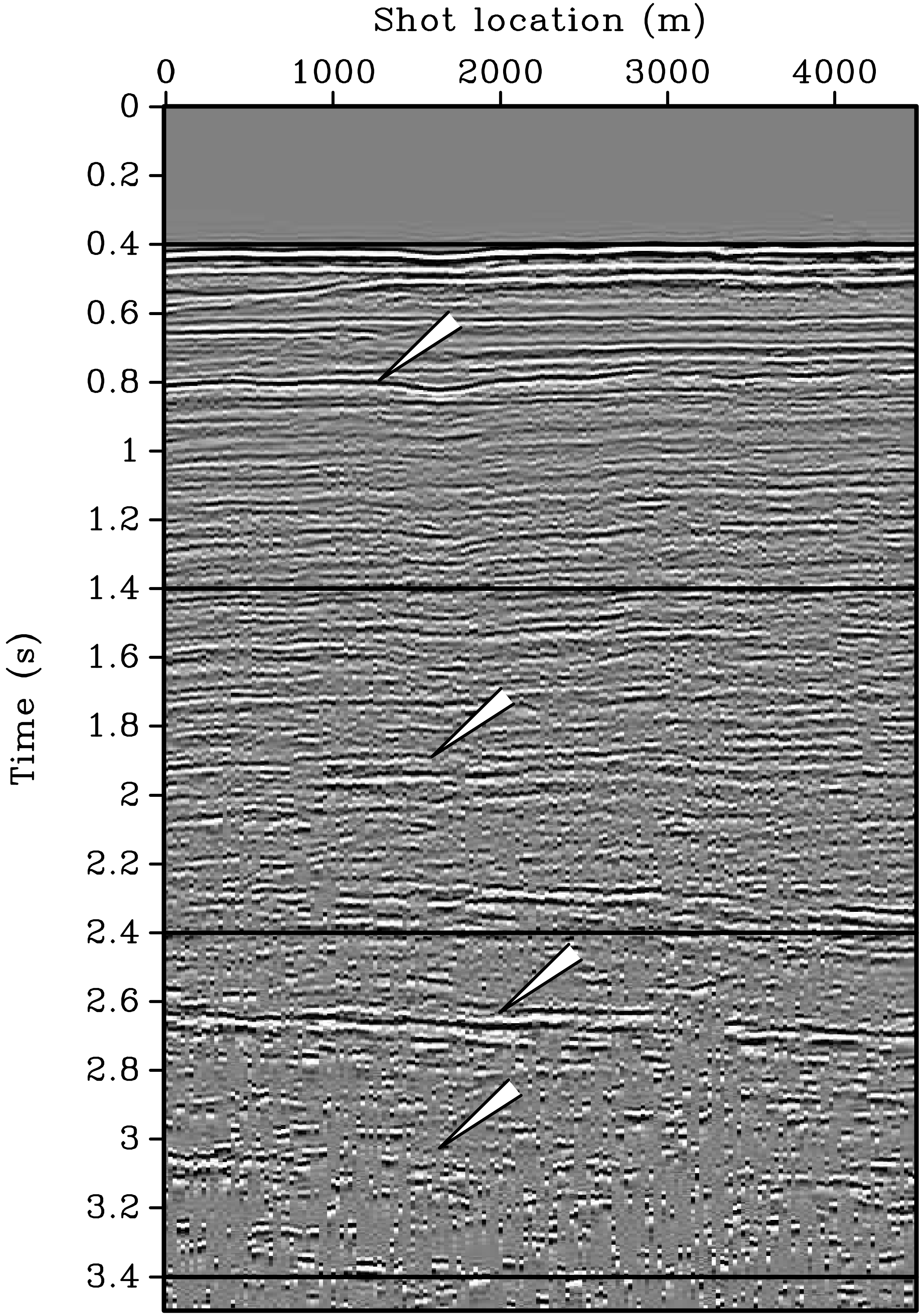
$$\mathbf{x}_{k+1} = \arg\min_{\mathbf{x}} \|\mathbf{p} - \mathbf{M}_{q_k}\mathbf{S}^\dagger\mathbf{x}\|_2 \text{ s.t. } \|\mathbf{x}\|_1 \leq \tau_k$$

(Solve with SPGL1 until Pareto curve reached)

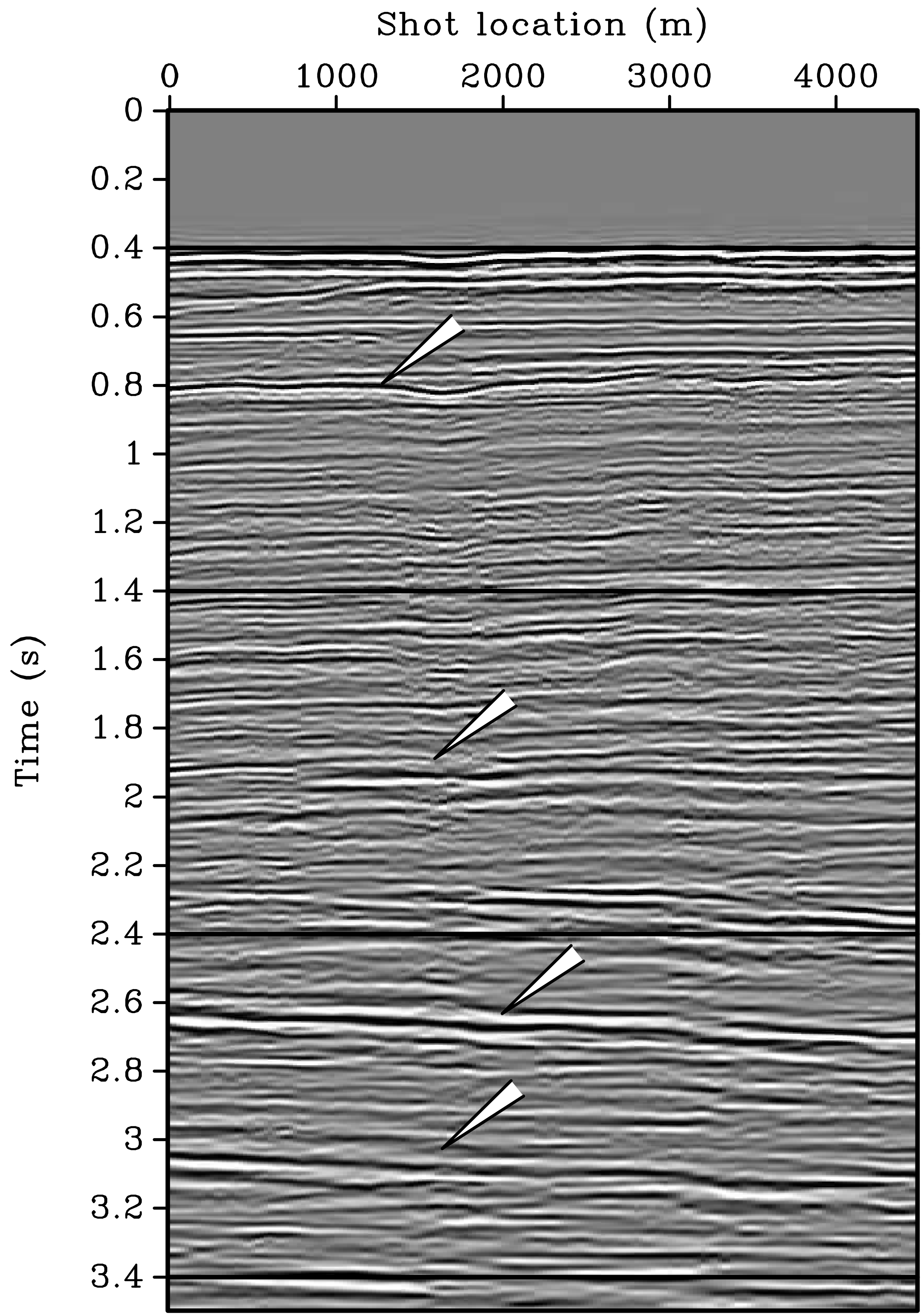
$$\mathbf{g}_{k+1} = \mathbf{S}^\dagger\mathbf{x}_{k+1}$$

$$\mathbf{q}_{k+1} = \arg\min_{\mathbf{q}} \|\mathbf{p} - \mathbf{M}_{g_{k+1}}\mathbf{q}\|_2$$

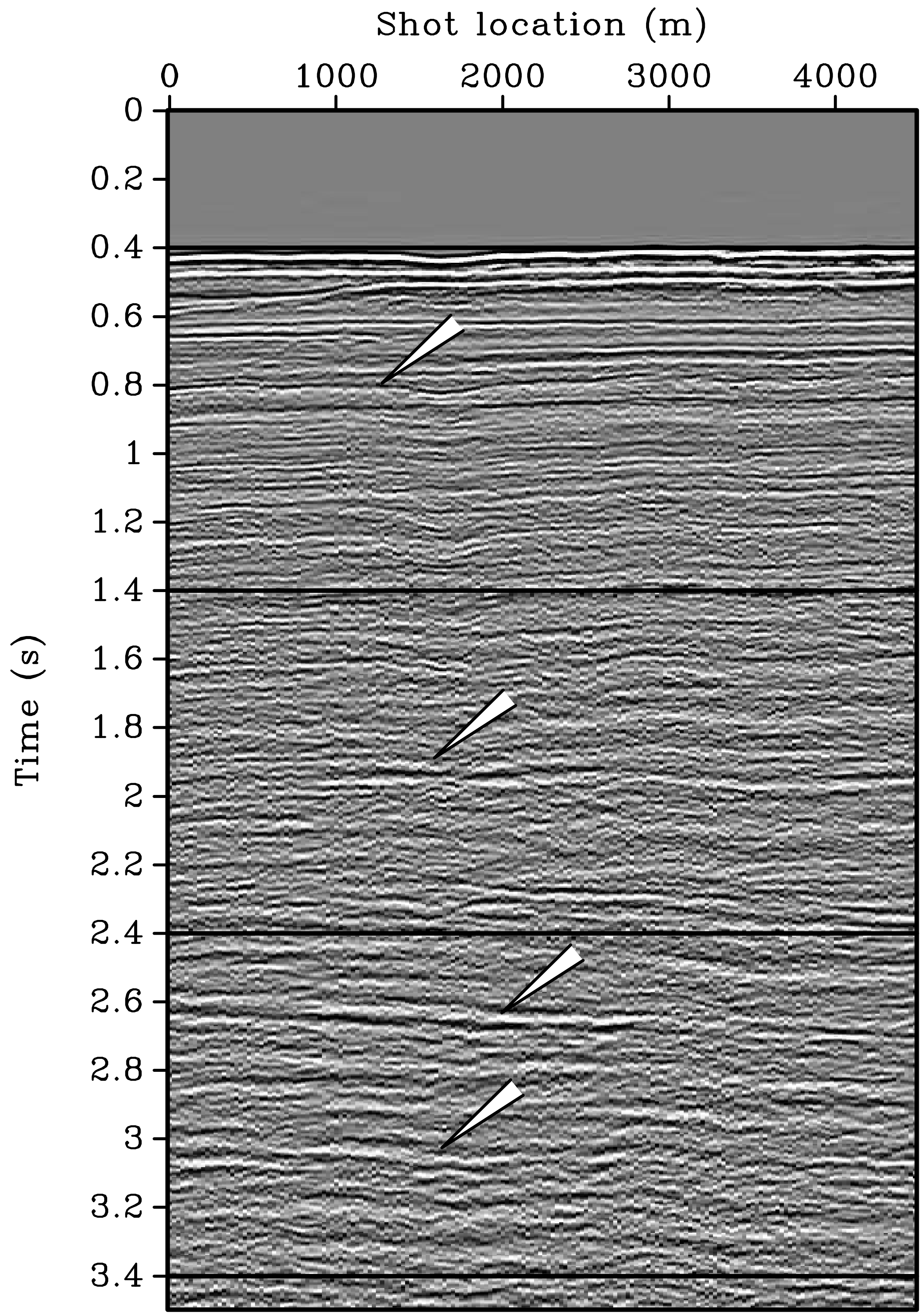
(Solve with LSQR)



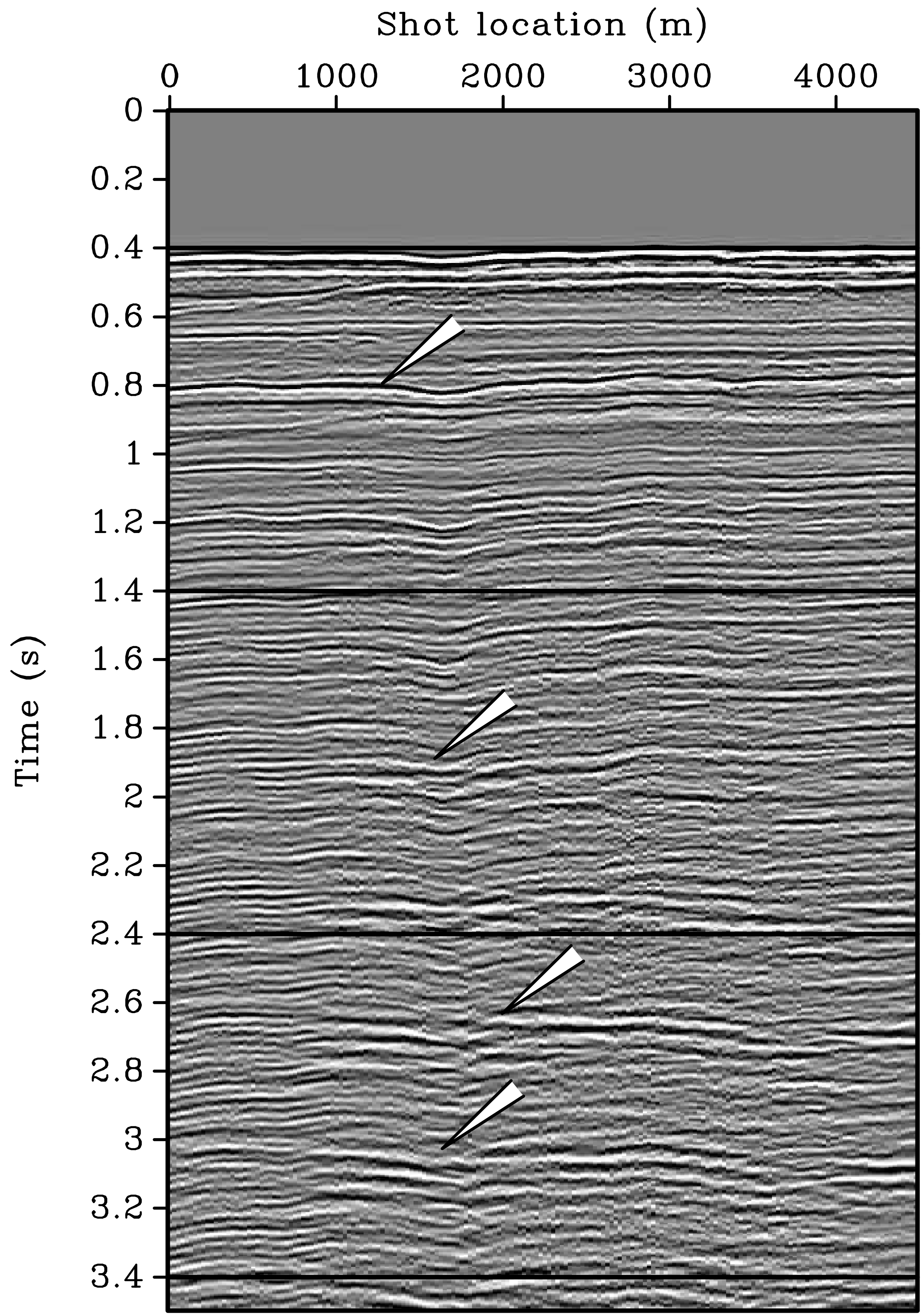
North sea EPSI



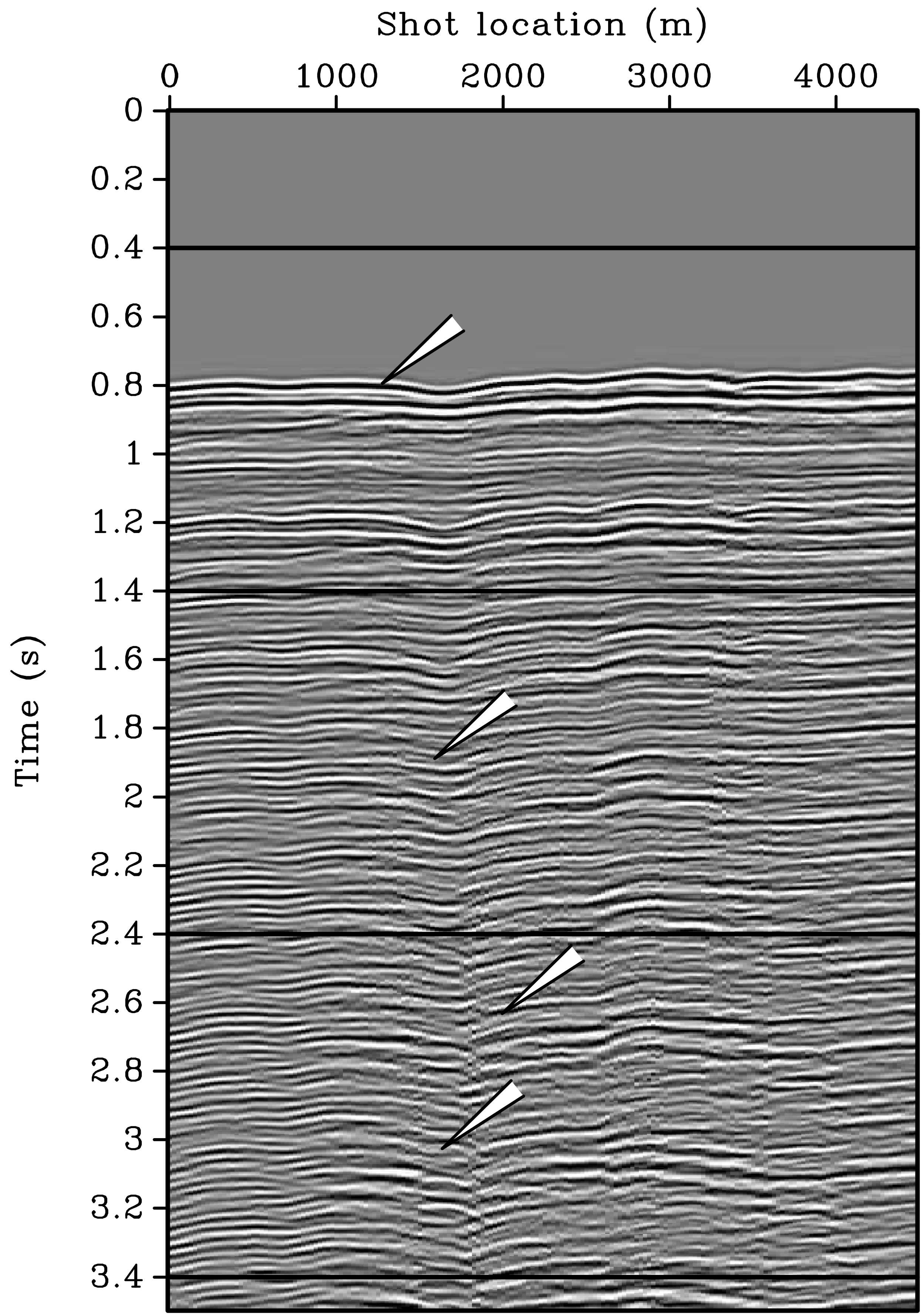
North sea EPSI + Sp



North sea SRME



North sea data



North sea pred. mul

(show Gulf of Suez results here)

summary

- L1-convexification behaves nicely and has few free parameters
- Follows the Pareto curve into a series of projected gradient problems
- Easily incorporates seeking the solution in a transform domain that promotes continuity

Acknowledgements

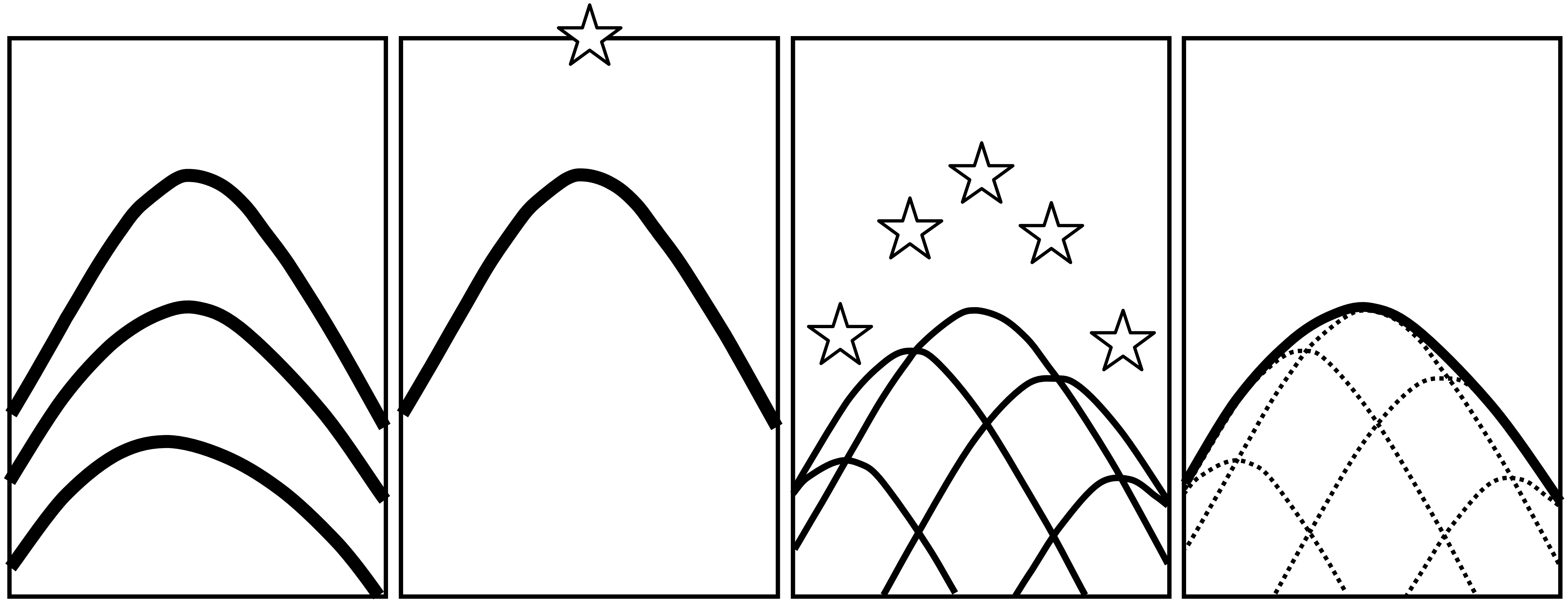
Special thanks to G.J. van Groenestijn, Eric Verschuur, and the rest of the members of DELPHI

M. Friedlander and E. van den Berg

SINBAD

NSERC CRSNG

This work was in part financially supported by the Natural Sciences and Engineering Research Council of Canada Discovery Grant (22R81254) and the Collaborative Research and Development Grant DNOISE II (375142-08). This research was carried out as part of the SINBAD II project with support from the following organizations: BG Group, BP, Chevron, ConocoPhillips, Petrobras, Total SA, and WesternGeco.



(van Groenestijn and Verschuur 08)