Barcelona '10

# **Stabilized** estimation of primaries by sparse inversion

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# EPSI

**Estimation of Primaries by Sparse Inversion** (van Groenestijn and Verschuur, 2009)

-based on Amundsen inversion, division of up/down going wavefields

recorded data  predicted data from primary IR

$$\mathbf{P}^- = \mathbf{X_o}(\mathbf{Q}^+ + \mathbf{R}\mathbf{P}^-)$$

$\mathbf{P}$ total up-going wavefield

$\mathbf{Q}$ down-going source signature

$\mathbf{R}$ reflectivity of free surface (assume -1)

$\mathbf{X_o}$ primary impulse response

(all single-frequency data volume, implicit $\omega$)

# EPSI

**Estimation of Primaries by Sparse Inversion** (van Groenestijn and Verschuur, 2009)

-based on Amundsen inversion, division of up/down going wavefields

recorded data  predicted data from primary IR

$$\mathbf{P^-} = \mathbf{X_o}(\mathbf{Q^+} + \mathbf{RP^-})$$

$$f(\mathbf{X_o}, \mathbf{Q}) = ||\mathbf{P^-} - \mathbf{X_o}(\mathbf{Q^+} + \mathbf{RP^-})||_2^2$$

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**Estimation of Primaries by Sparse Inversion** (van Groenestijn and Verschuur, 2009)

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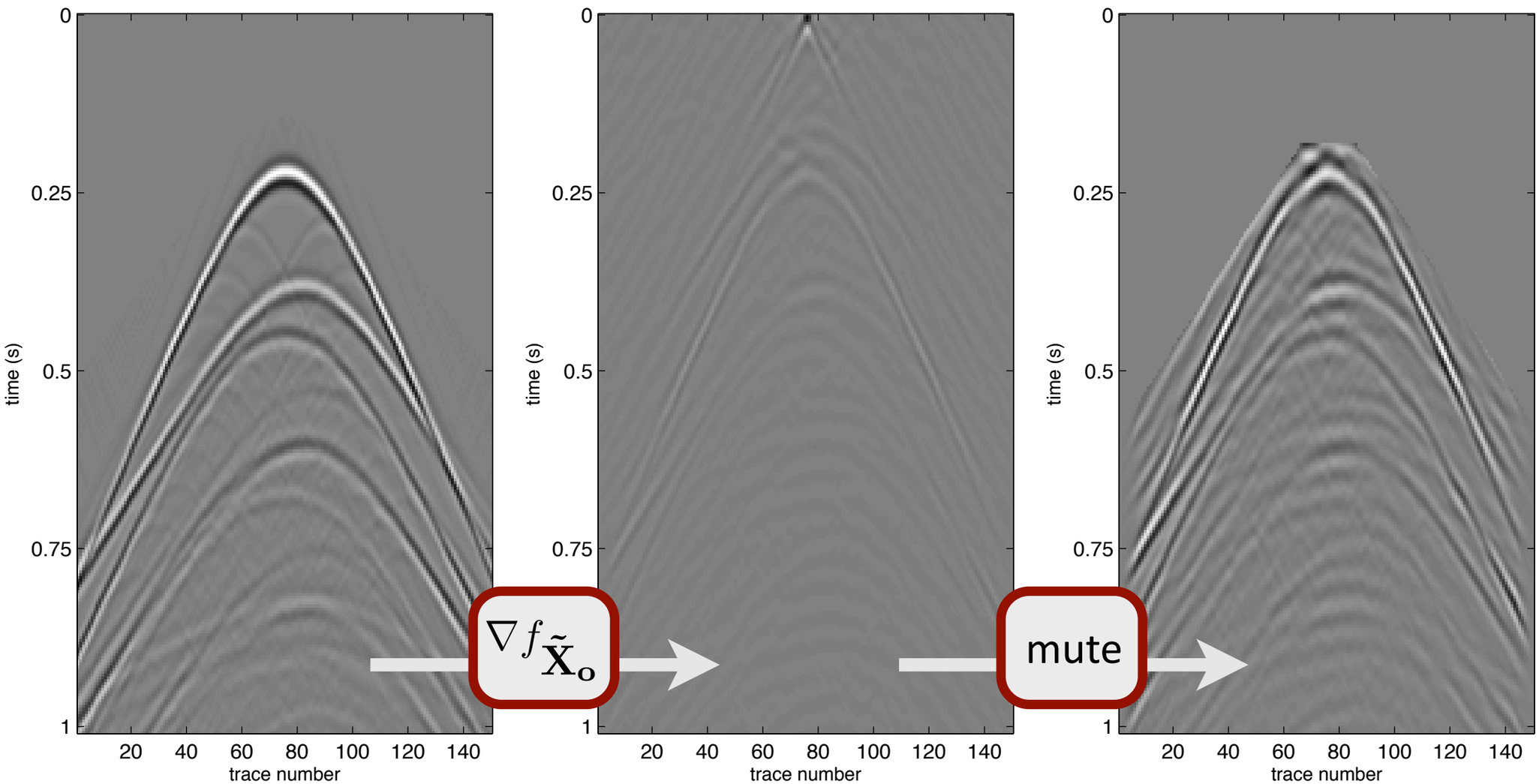
$$\mathbf{P}^- = \mathbf{X_o}(\mathbf{Q}^+ + \mathbf{R}\mathbf{P}^-)$$

$$f(\mathbf{X_o}, \mathbf{Q}) = ||\mathbf{P}^- - \mathbf{X_o}(\mathbf{Q}^+ + \mathbf{R}\mathbf{P}^-)||_2^2$$

$$\nabla f_{\tilde{\mathbf{X}}_\mathbf{o}} = \left(\mathbf{P}^- - \tilde{\mathbf{X}}_\mathbf{o}(\mathbf{Q}^+ + \mathbf{R}\mathbf{P}^-)\right)\left(\mathbf{Q}^+ + \mathbf{R}\mathbf{P}^-\right)^H$$

# EPSI

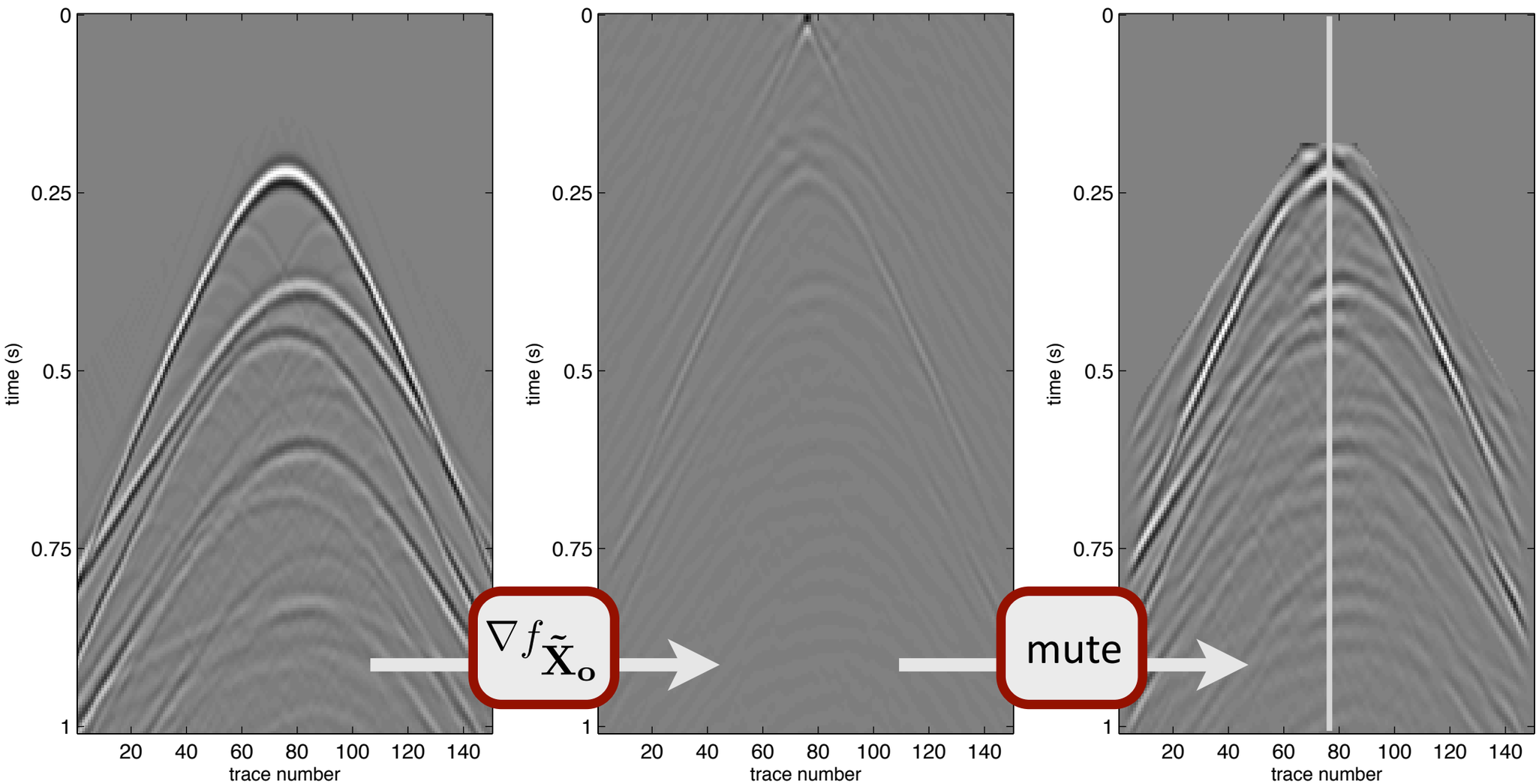
## Primary event estimation step




input data | gradient | muted gradient

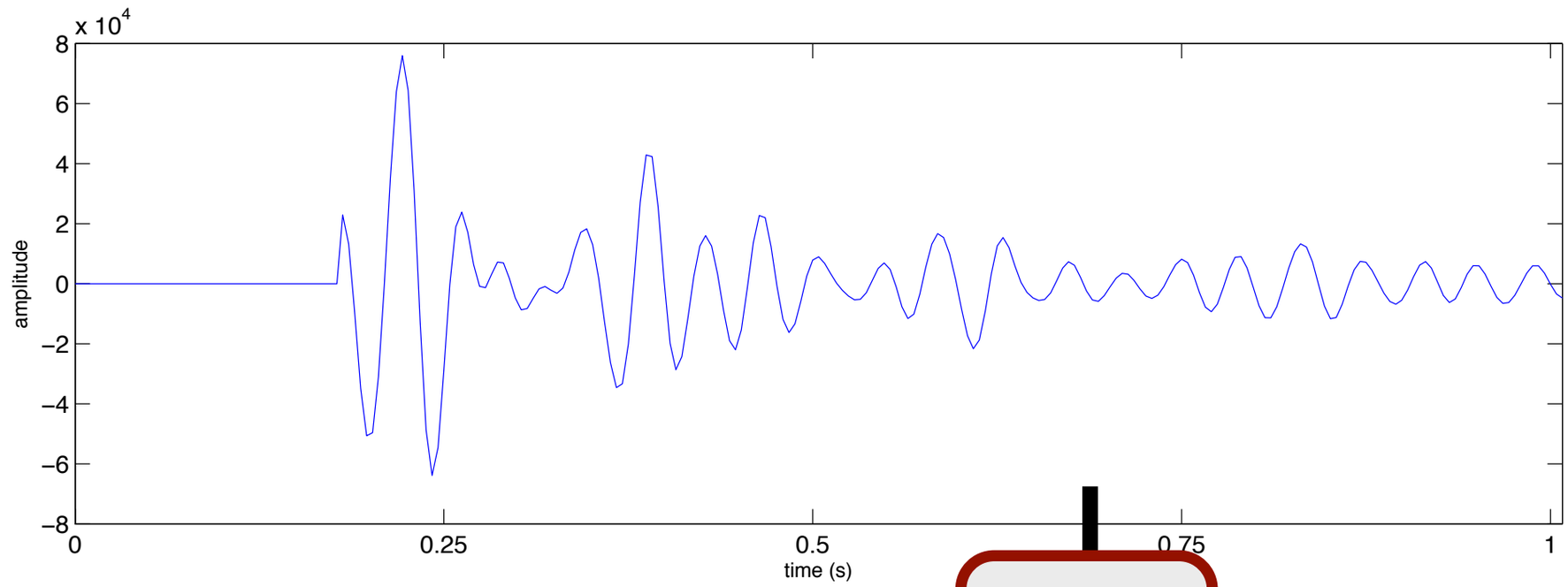
# EPSI

## Primary event estimation step



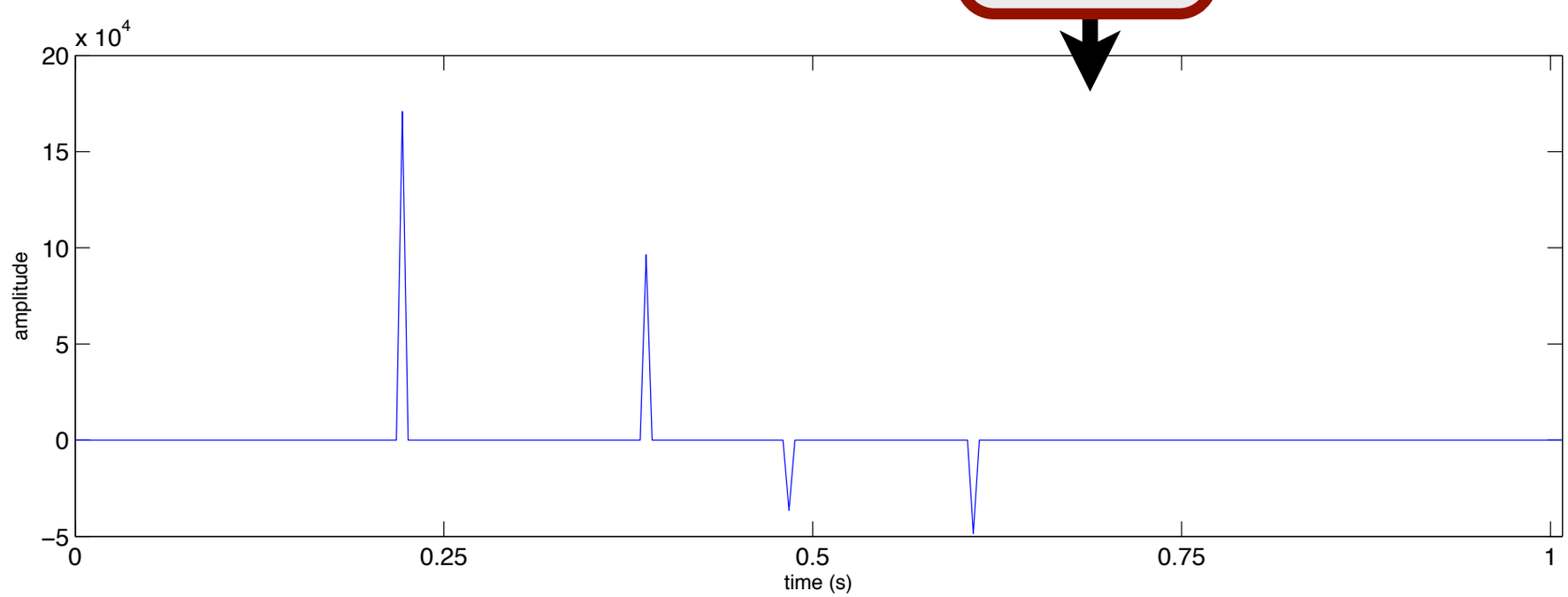

input data | gradient | muted gradient

# EPSI Primary event estimation step




muted gradient

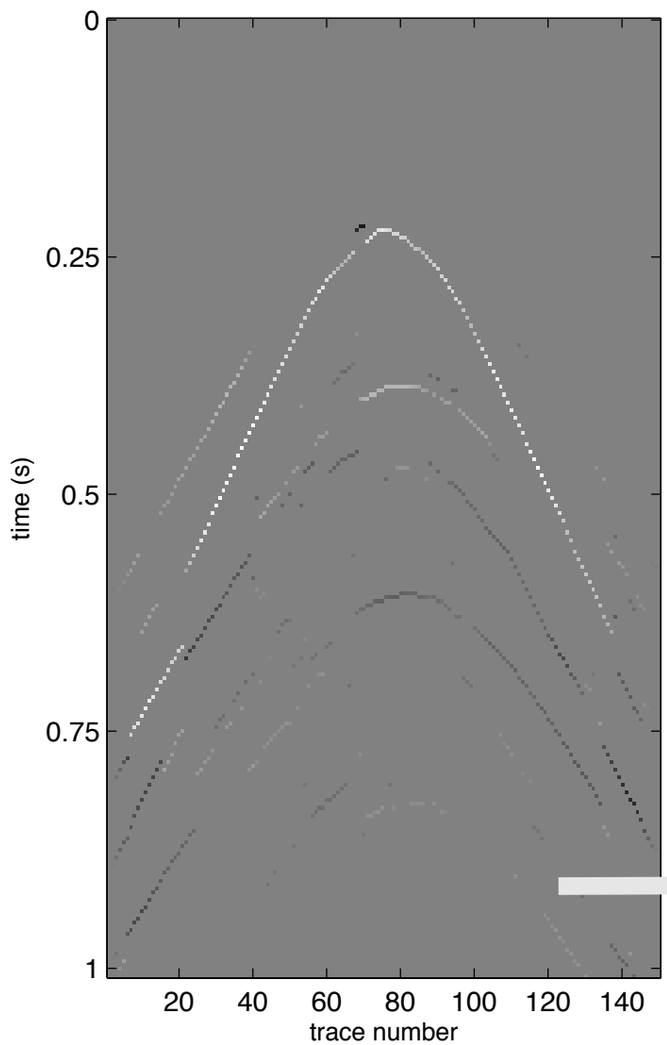
sparsity




4 events picked (per trace)

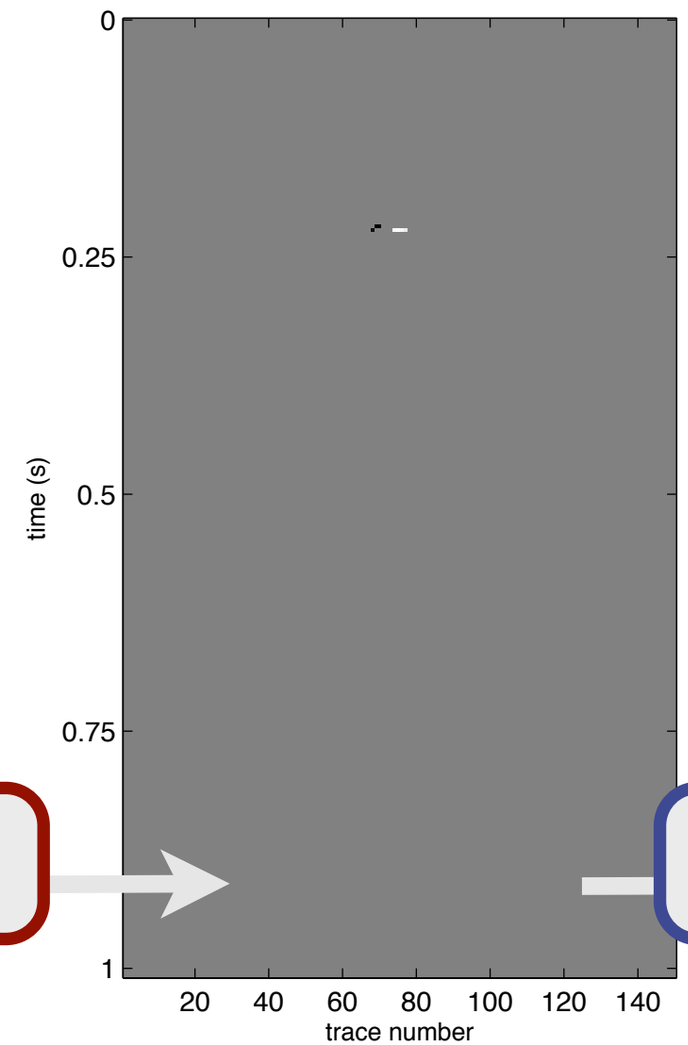
# EPSI

## Primary event estimation step




4 events picked (per trace)

window




windowed picked events

line srch + update

# EPSI

## Wavelet matching step

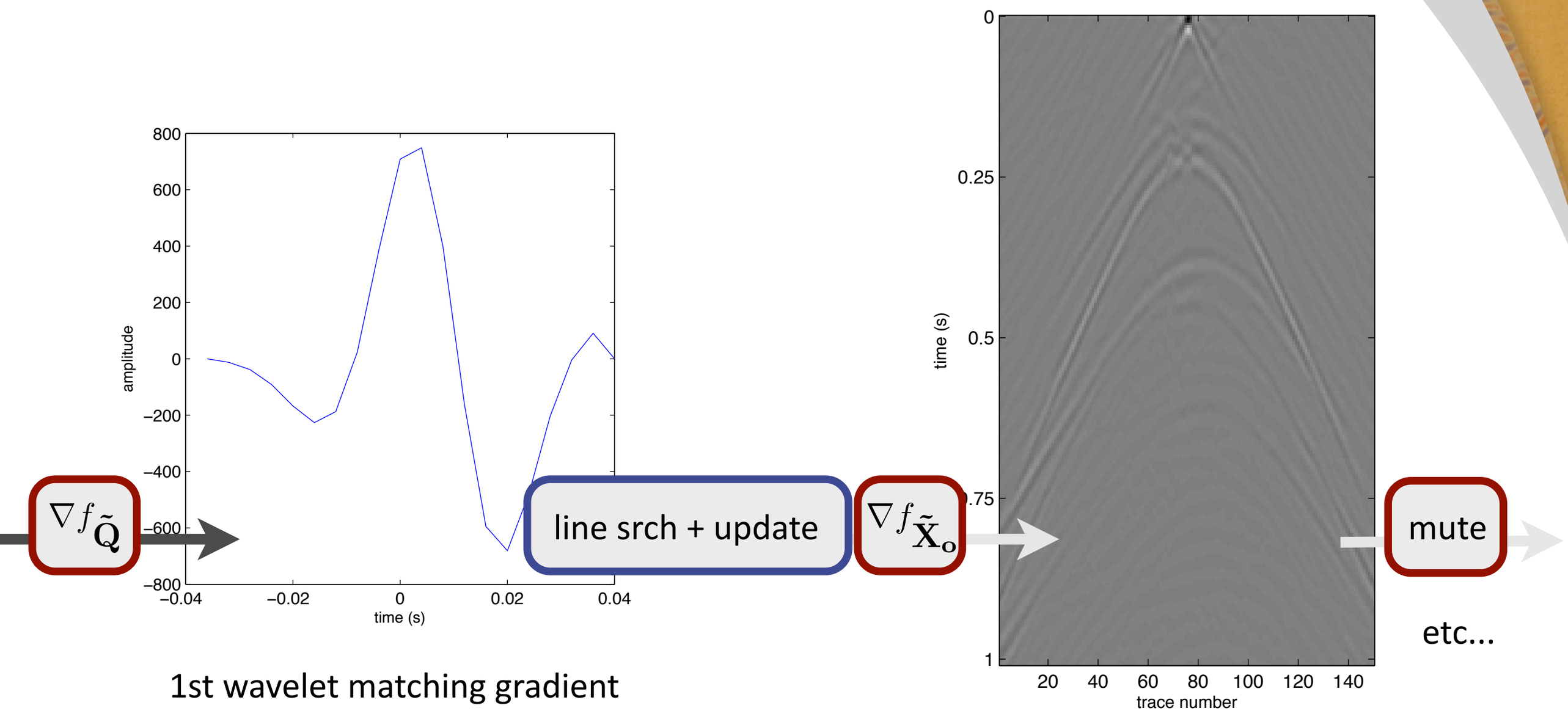
$$\mathbf{P}^- = \mathbf{X_o}(\mathbf{Q}^+ + \mathbf{R}\mathbf{P}^-)$$

$$f(\mathbf{X_o}, \mathbf{Q}) = ||\mathbf{P}^- - \mathbf{X_o}(\mathbf{Q}^+ + \mathbf{R}\mathbf{P}^-)||_2^2$$

$$\nabla f_{\tilde{\mathbf{Q}}} = \mathbf{X_o}^H \left(\mathbf{P}^- - \mathbf{X_o}(\tilde{\mathbf{Q}}^+ + \mathbf{R}\mathbf{P}^-)\right)$$

# EPSI

## Wavelet matching step

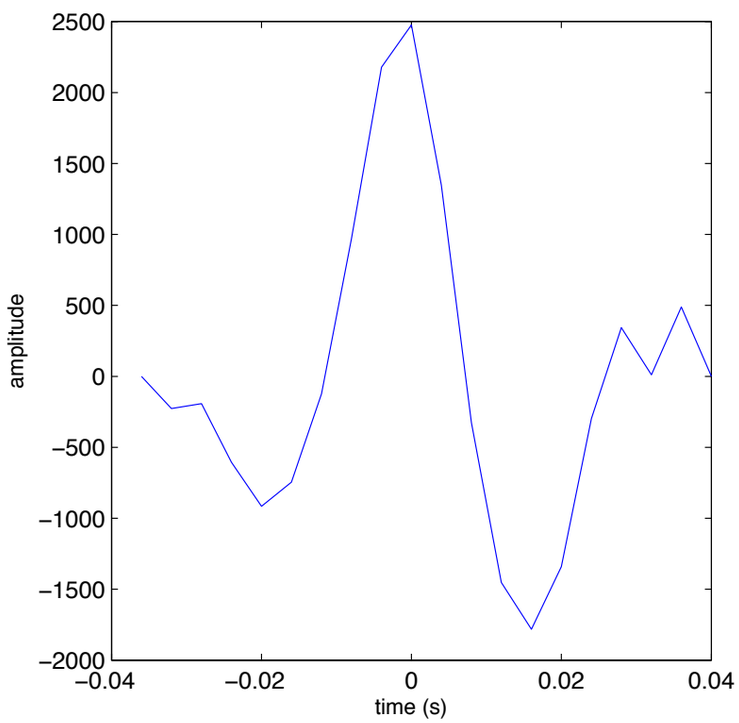



1st wavelet matching gradient

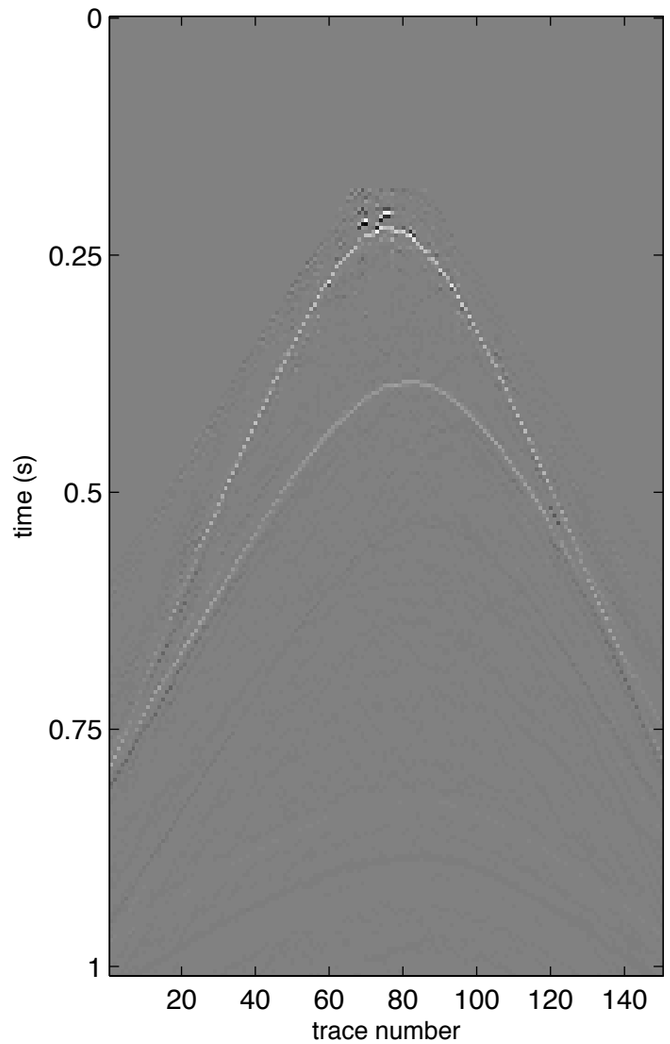
2nd Xo Gradient

# EPSI

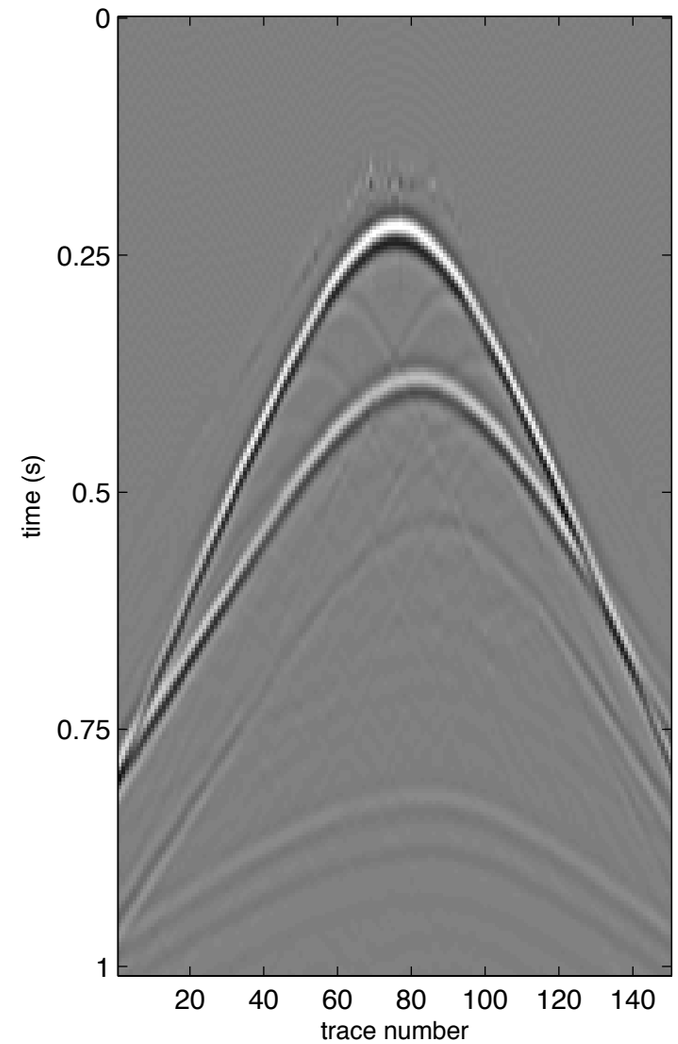
## Final results (60 iterations)




Final wavelet



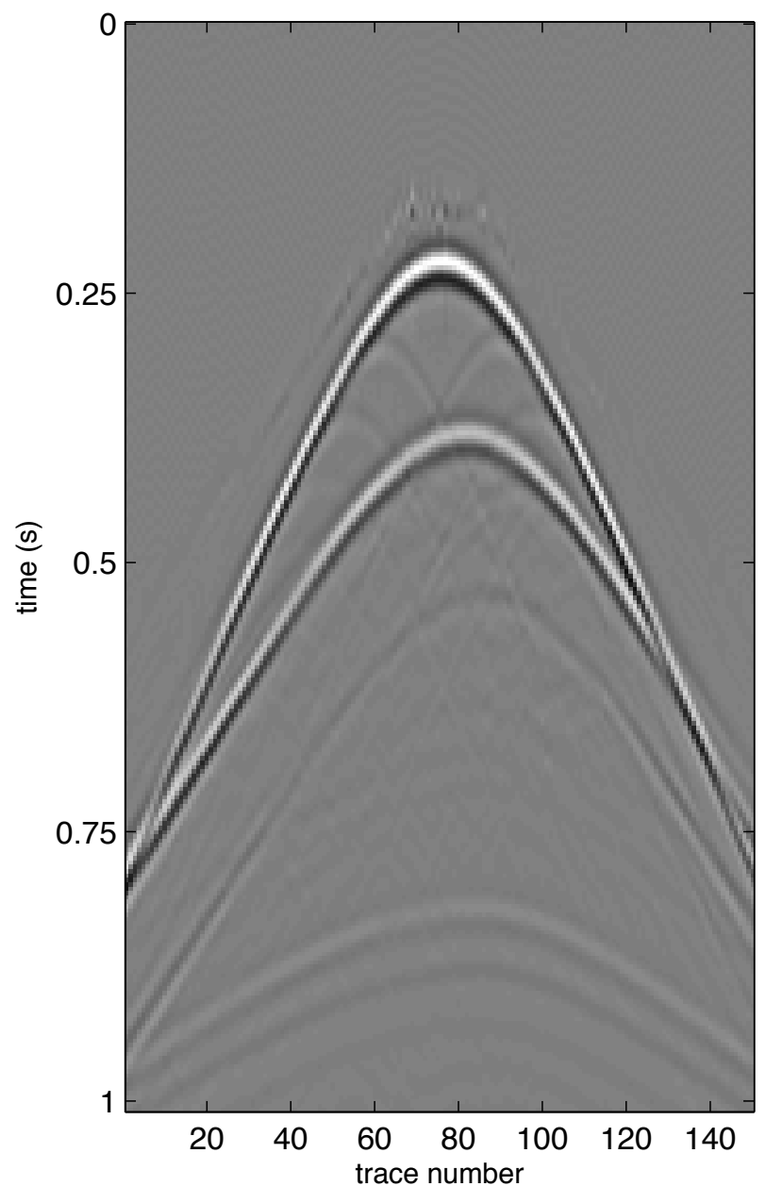

Final Green's function



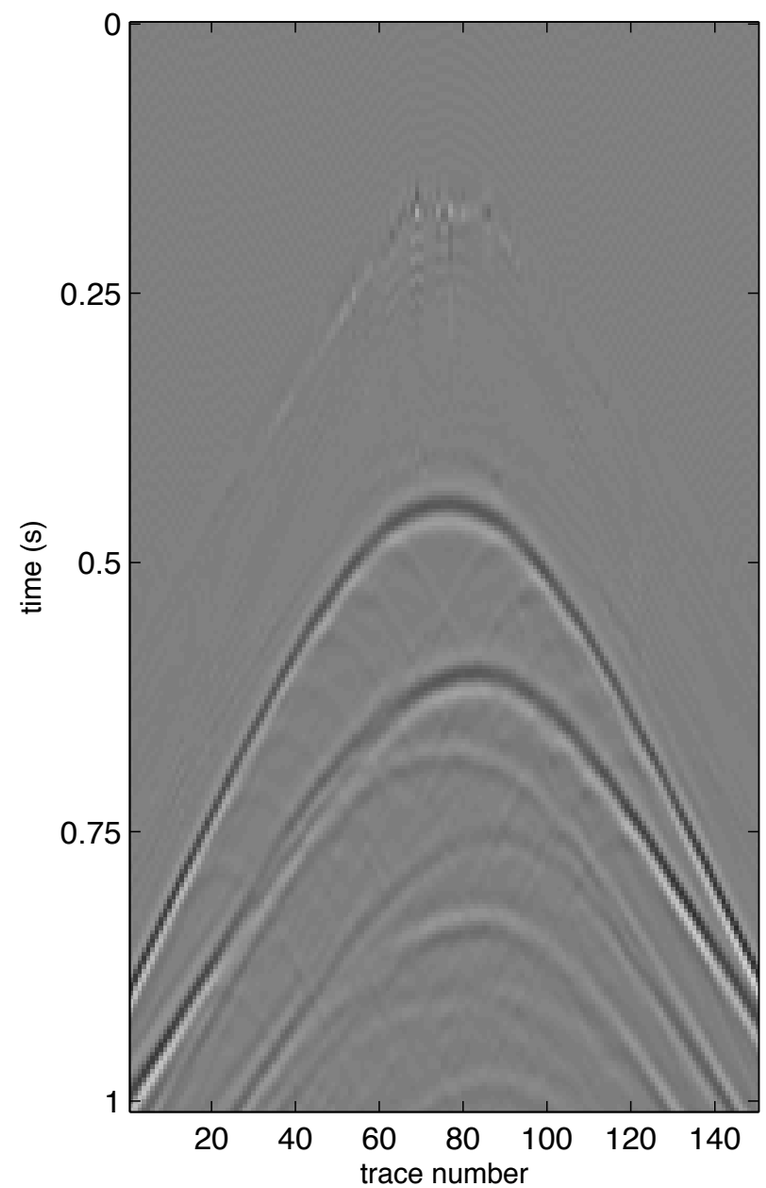

Final estimated primary

# EPSI

## Final results (60 iterations)



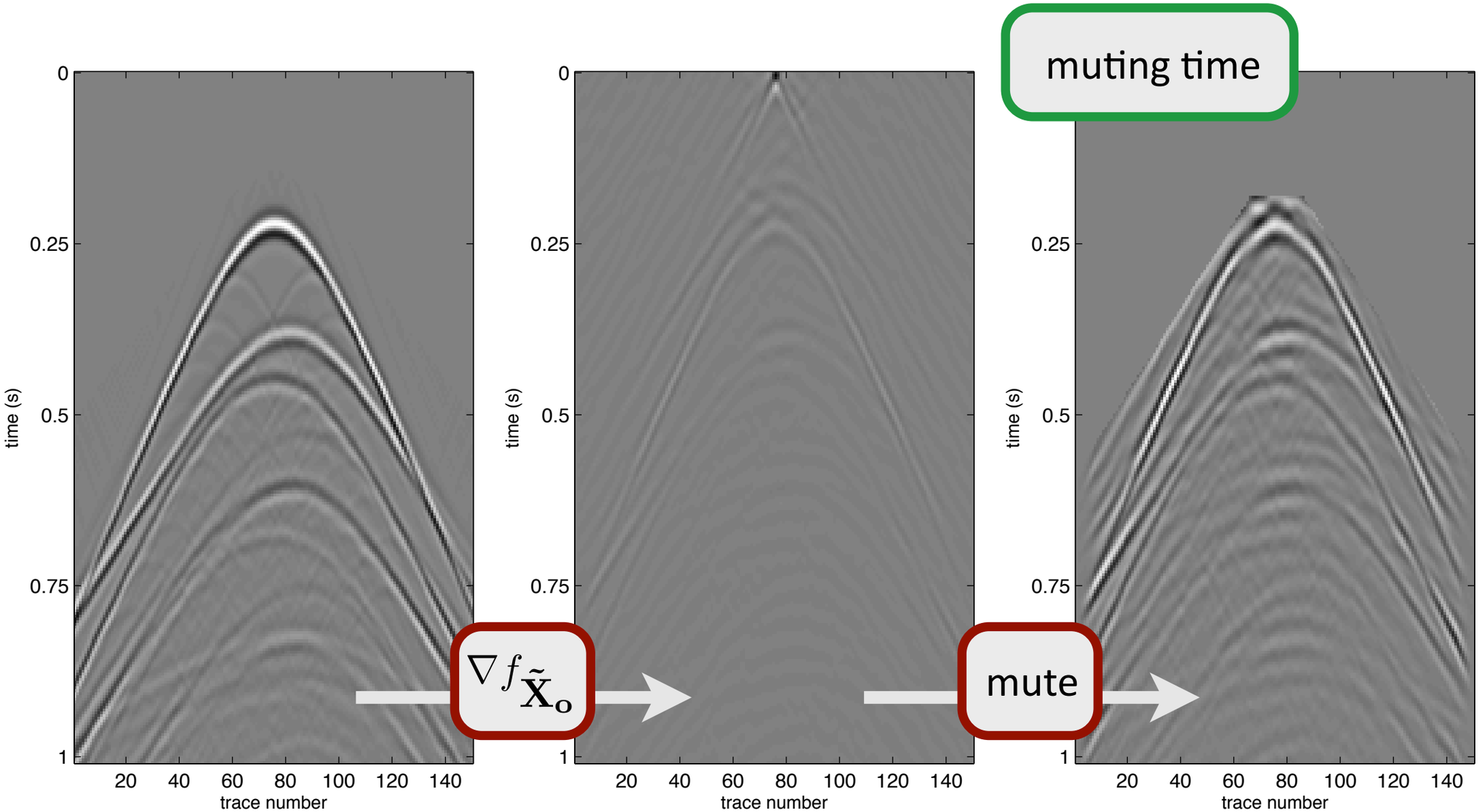

Final estimated primary




Data minus estimated primary

# EPSI

## Primary event estimation step

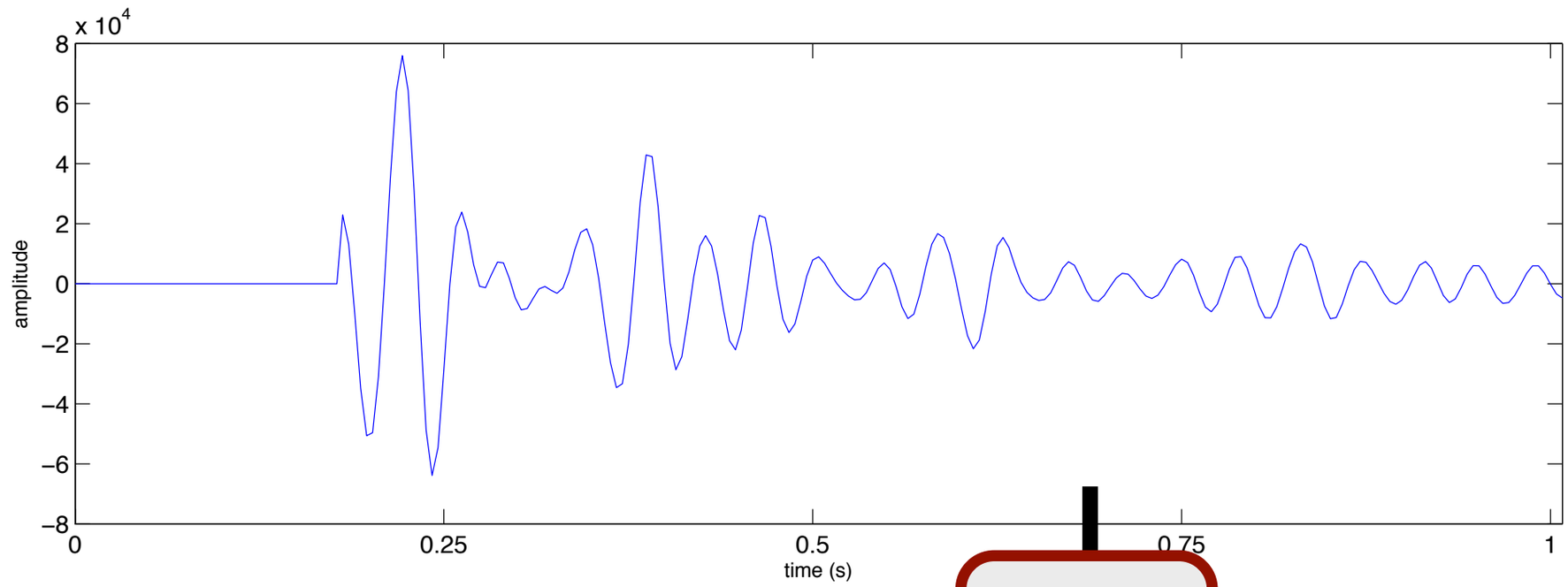



input data

gradient

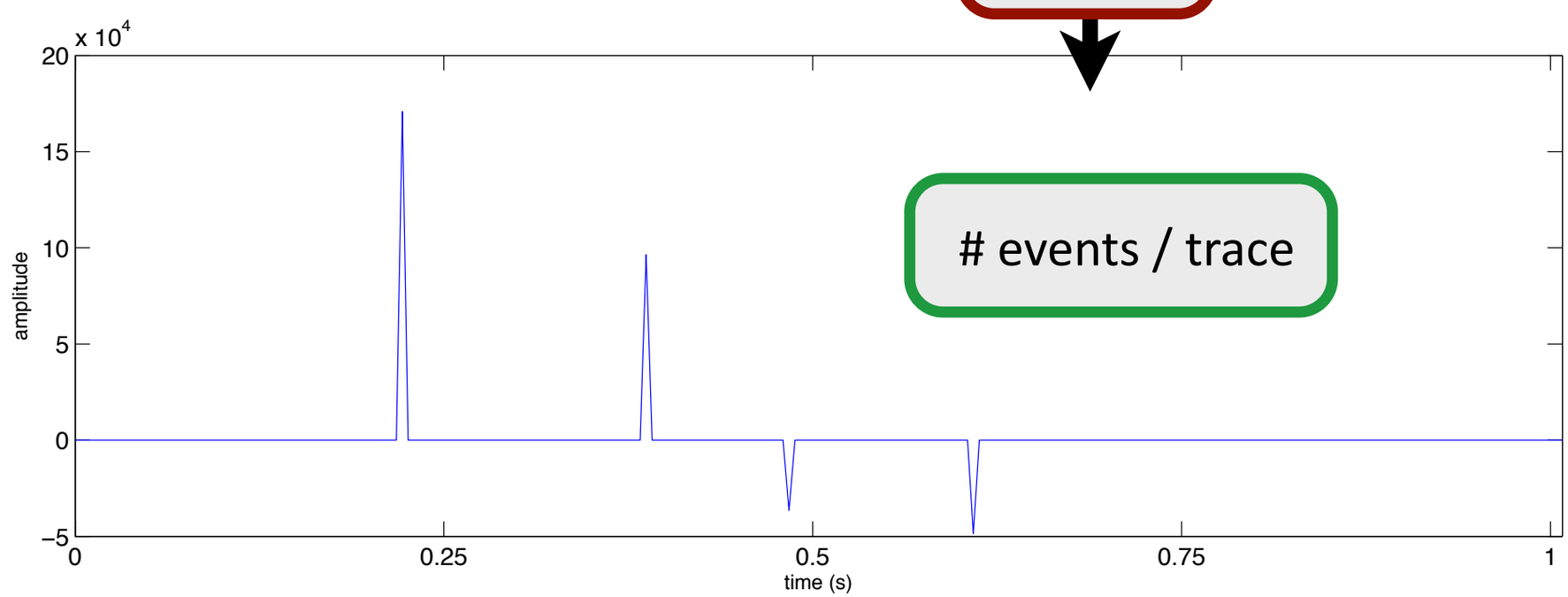
muted gradient

# EPSI Primary event estimation step




muted gradient

sparsity

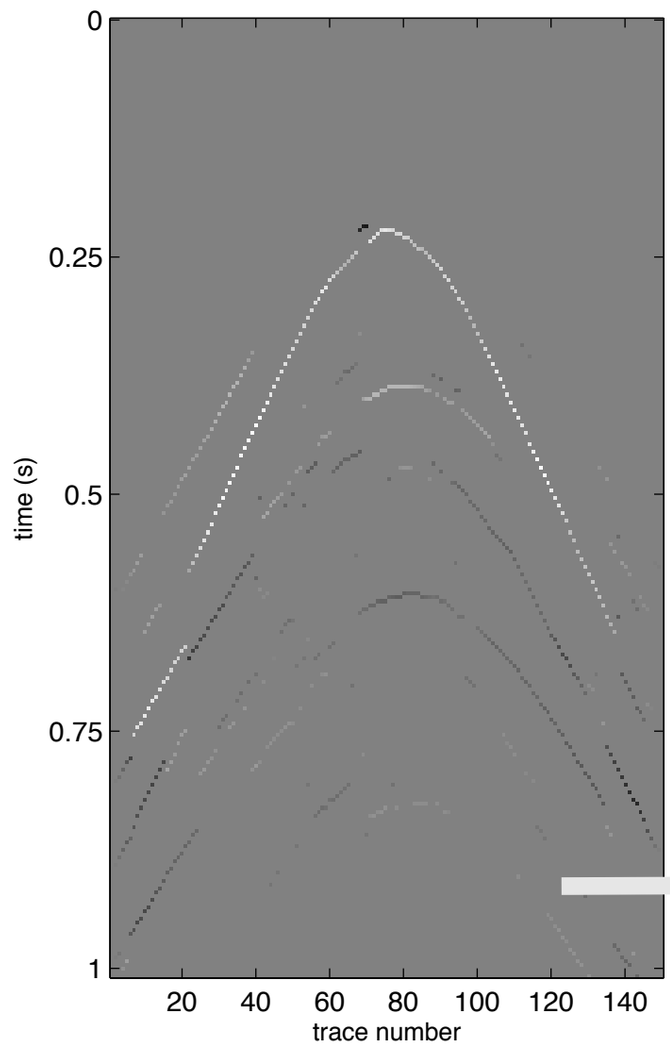



# events / trace

4 events picked (per trace)

# EPSI

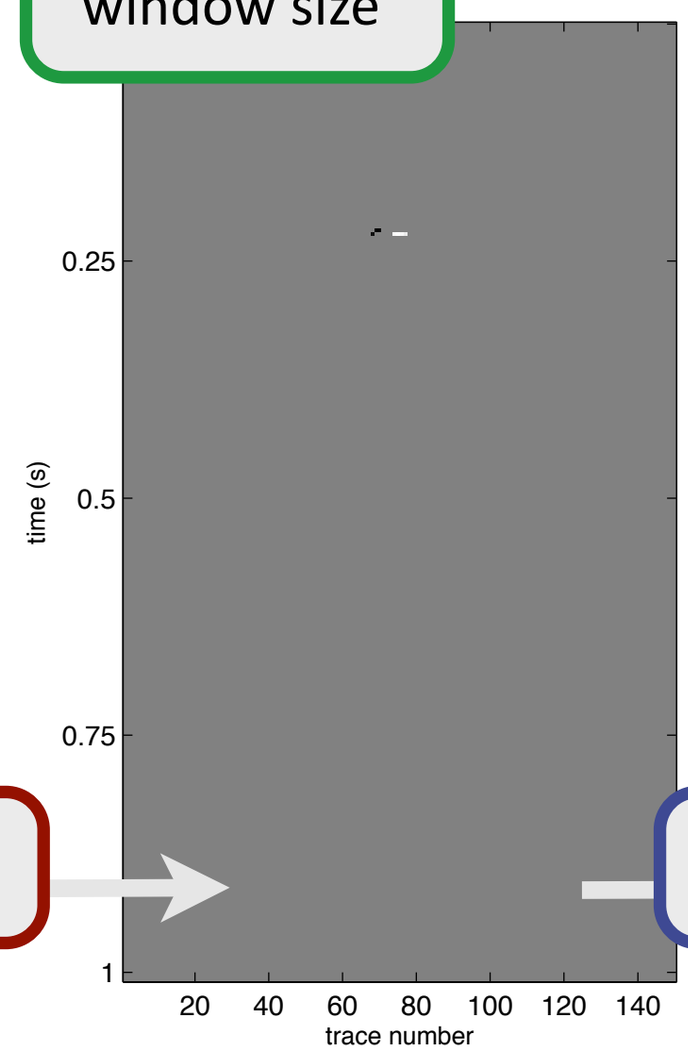
## Primary event estimation step




4 events picked (per trace)

window

window size



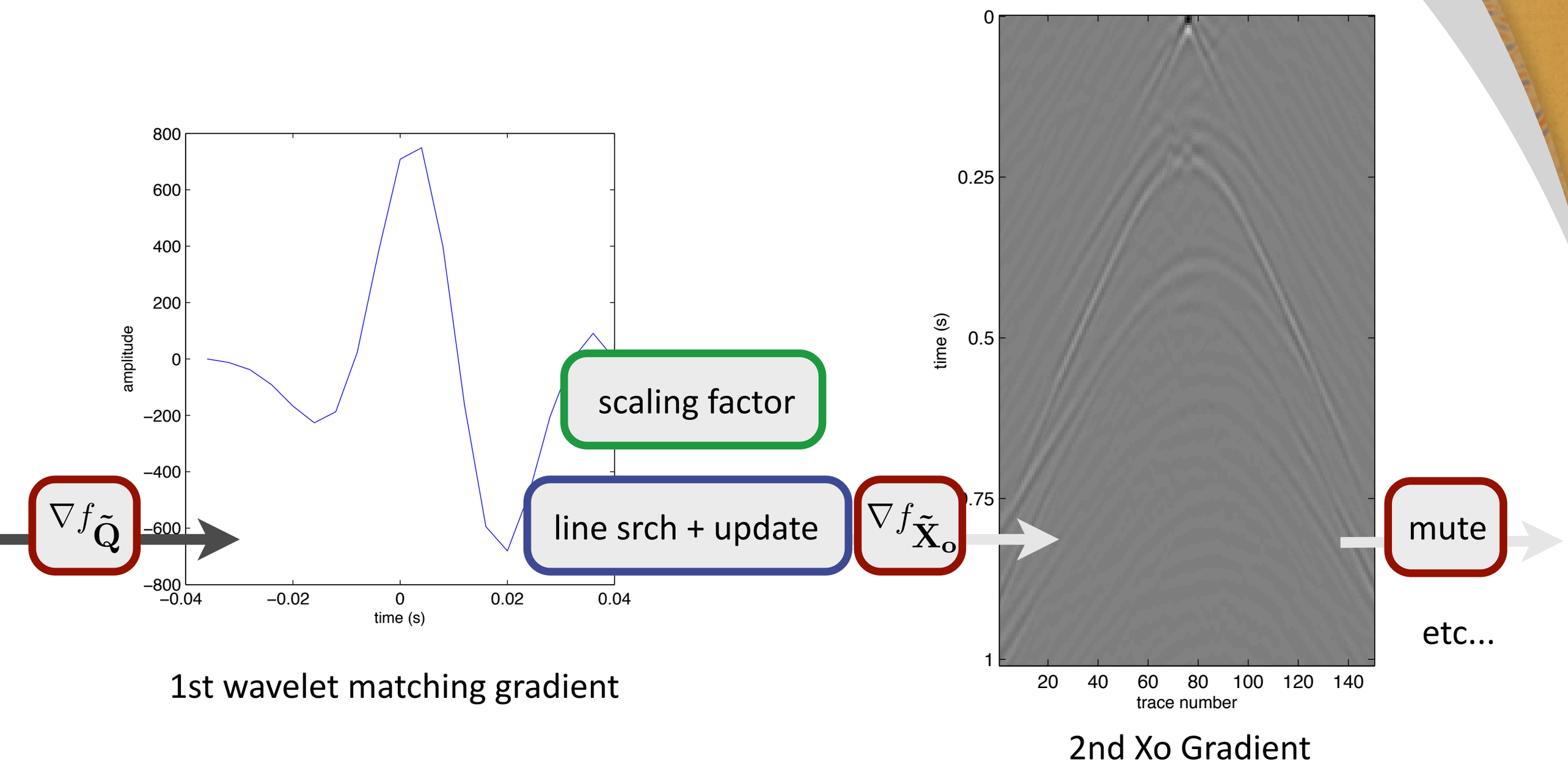

windowed picked events

scaling factor

line srch + update

# EPSI

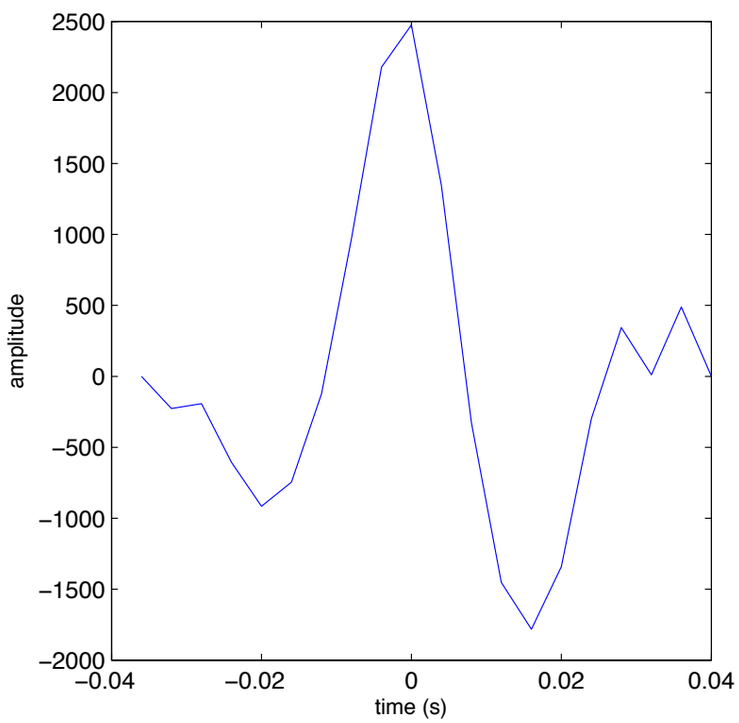
## Wavelet matching step



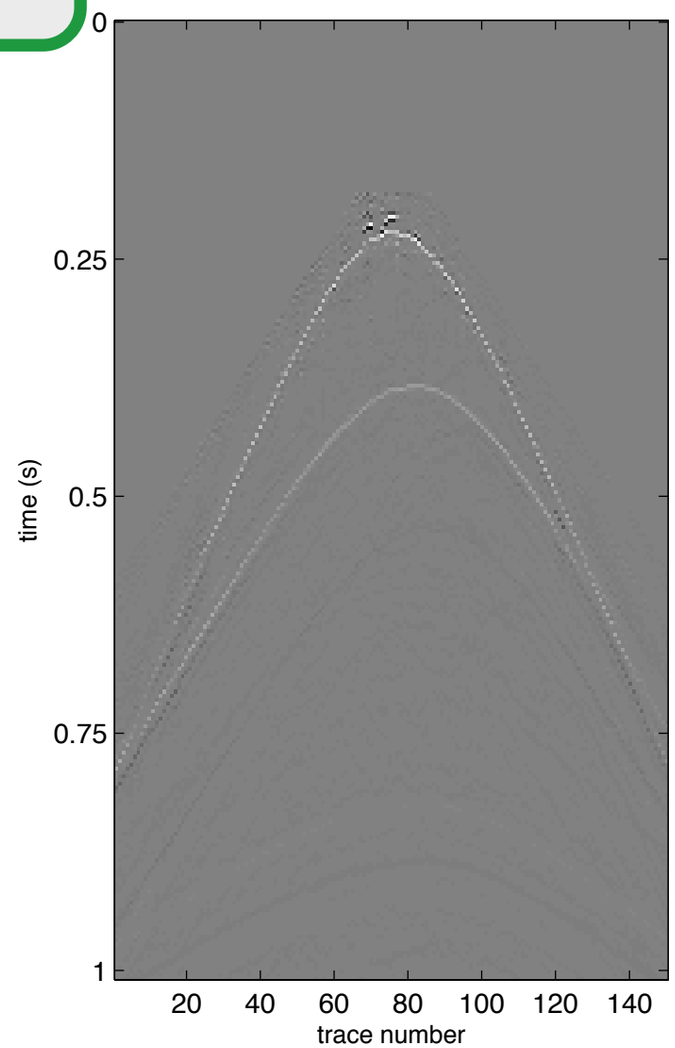
# EPSI

## Final results (60 iterations)

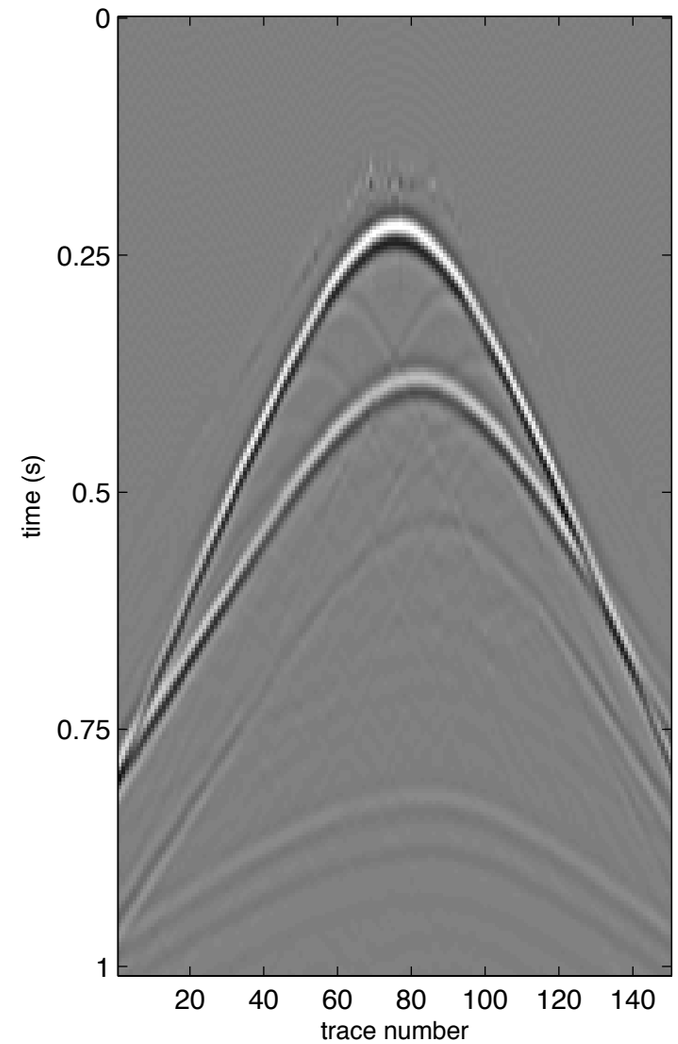
stopping criterion




Final wavelet




Final Green's function




Final estimated primary

# EPSI

Uses sparsity assumption on $\mathbf{X_o}$

$$\underset{\mathbf{X_o},\,\mathbf{Q}^+ \in \mathbf{Q}_\Lambda}{\textbf{minimize}} \quad \mathrm{nnz}(\mathbf{X_o}) \quad \textbf{s.t.} \; ||\mathbf{P}^- - \mathbf{X_o}(\mathbf{Q}^+ + \mathbf{R}\mathbf{P}^-)||_2^2 \leq \sigma$$

But approximates the solution with k iterations of projected gradient

$$\underset{\mathbf{X_o},\,\mathbf{Q}^+ \in \mathbf{Q}_\Lambda}{\textbf{minimize}} \quad ||\mathbf{P}^- - \mathbf{X_o}(\mathbf{Q}^+ + \mathbf{R}\mathbf{P}^-)||_2^2 \quad \textbf{s.t.} \; \mathrm{nnz}(\mathbf{X_o}) \leq \tau$$

This is an NP-hard problem:

- existence of local minima
- no convergence guarantees

$\mathcal{T}$ number. spike per iteration

$\mathbf{Q}_\Lambda$ short time-windowed wavelet (implies smooth spectrum)

# Convex relaxation

Use L1-norm relaxation for the sparsity objective

$$\underset{\mathbf{X_o},\,\mathbf{Q}^+ \in \mathbf{Q}_\Lambda}{\textbf{minimize}} \quad ||\mathbf{X_o}||_1 \quad \textbf{s.t.} \quad ||\mathbf{P}^- - \mathbf{X_o}(\mathbf{Q}^+ + \mathbf{R}\mathbf{P}^-)||_2^2 \leq \sigma$$

Bi-convex problem, but turns into two convex problems we know how to solve via alternating optimization

-Standard approach in blind image deconvolution

-no need for windowing primary events at each iteration

# Alternating optimization

Use L1-norm relaxation for the sparsity objective

$$\underset{\mathbf{X_o}}{\textbf{minimize}} \quad ||\mathbf{X_o}||_1 \quad \textbf{s.t.} \quad ||\mathbf{P}^- - \mathbf{X_o}(\mathbf{Q_k^+} + \mathbf{RP}^-)||_2^2 \leq \sigma$$

Fix source signature, turns into $\ell_1$- minimization

# Operator form

$$\mathbf{P}^- = \mathbf{X_o}(\mathbf{S}^+ + \mathbf{R}\mathbf{P}^-)$$

Define linear operator A that maps Green's func to up-going wavefield

$$\mathbf{A}\mathbf{x_o} := \mathcal{F}_{\mathrm{t}}^* \mathrm{BlockDiag}_\omega[(\mathbf{Q}^+ - \mathbf{P}^-)^* \otimes \mathbf{I}]\mathcal{F}_{\mathrm{t}}\mathbf{x_o} = \mathbf{p}^-$$

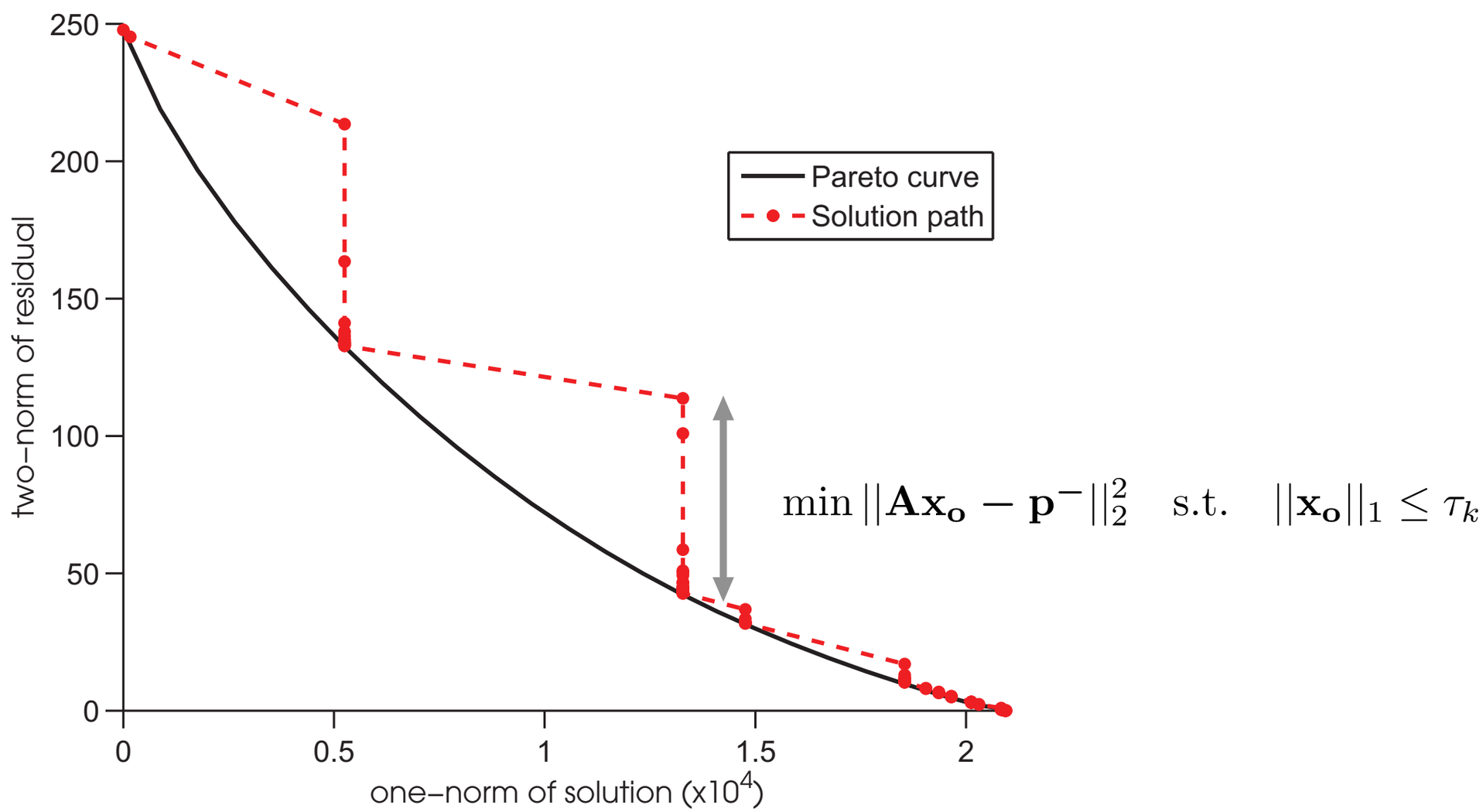
$$\mathbf{p}^- := \mathrm{vec}(\mathbf{P}^-)$$
$$\mathbf{x_o} := \mathrm{vec}(\mathbf{X_o})$$

# L1 minimization

$$\min ||\mathbf{x_o}||_1 \quad \text{s.t.} \quad ||\mathbf{p}^- - \mathbf{A}\mathbf{x_o}||_2^2 \leq \sigma$$

Use SPGl1 (van den Berg, Friedlander, 2008)

- a projected gradient based method (seismic data-volumes are huge)
- uses root-finding to find the final one-norm



# L1 minimization

$$\min ||\mathbf{x_o}||_1 \quad \text{s.t.} \quad ||\mathbf{p}^- - \mathbf{A}\mathbf{x_o}||_2^2 \leq \sigma$$

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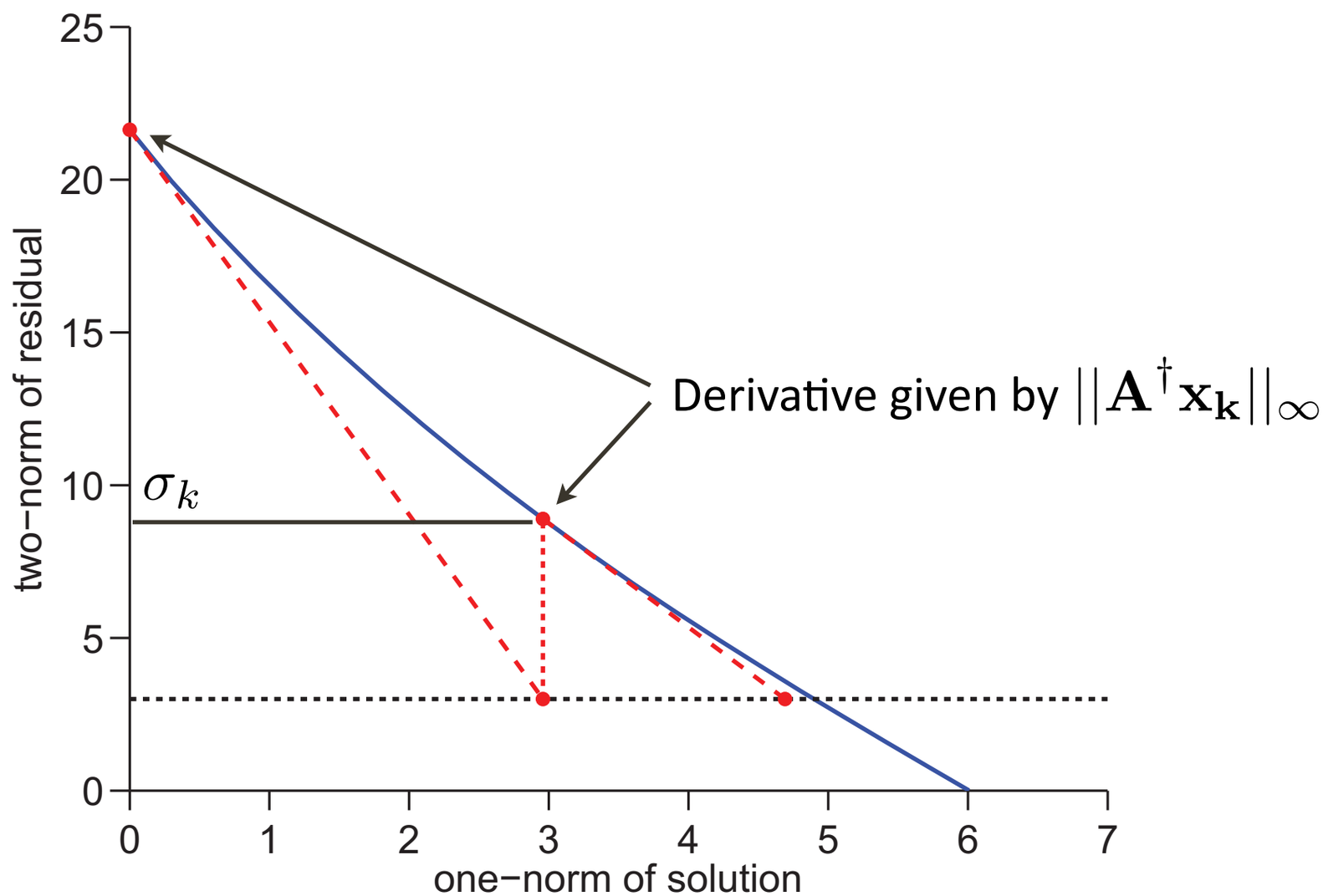


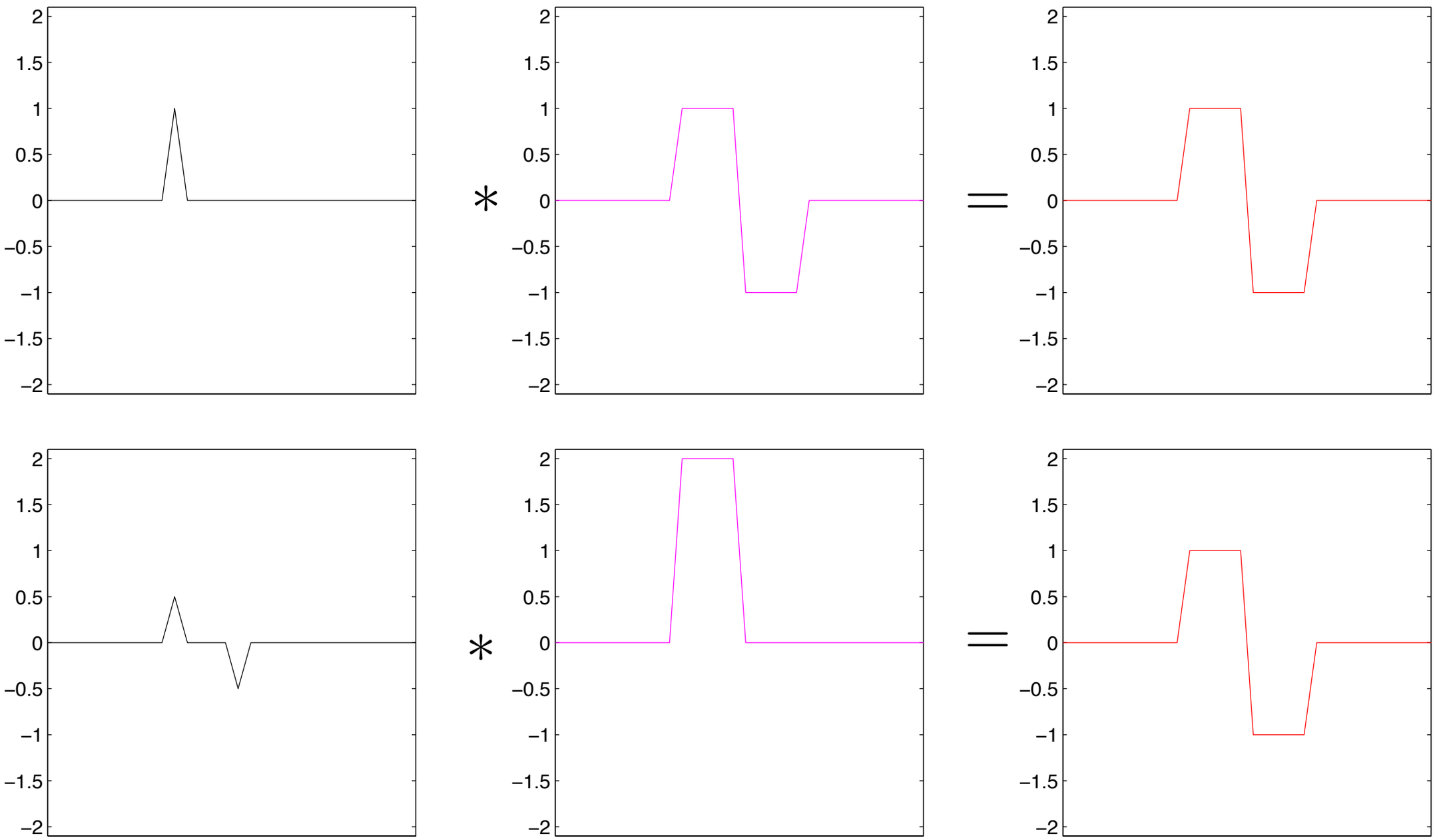
# Alternating optimization

Wavelet matching at Pareto curve

$$\underset{\mathbf{Q}^+ \in \mathbf{Q}_\Lambda}{\textbf{minimize}} \quad ||\mathbf{X}_{\mathbf{O}k}||_1 \quad \textbf{s.t.} \quad ||\mathbf{P}^- - \mathbf{X}_{\mathbf{O}k}(\mathbf{Q}^+ + \mathbf{R}\mathbf{P}^-)||_2^2 < \sigma_k$$

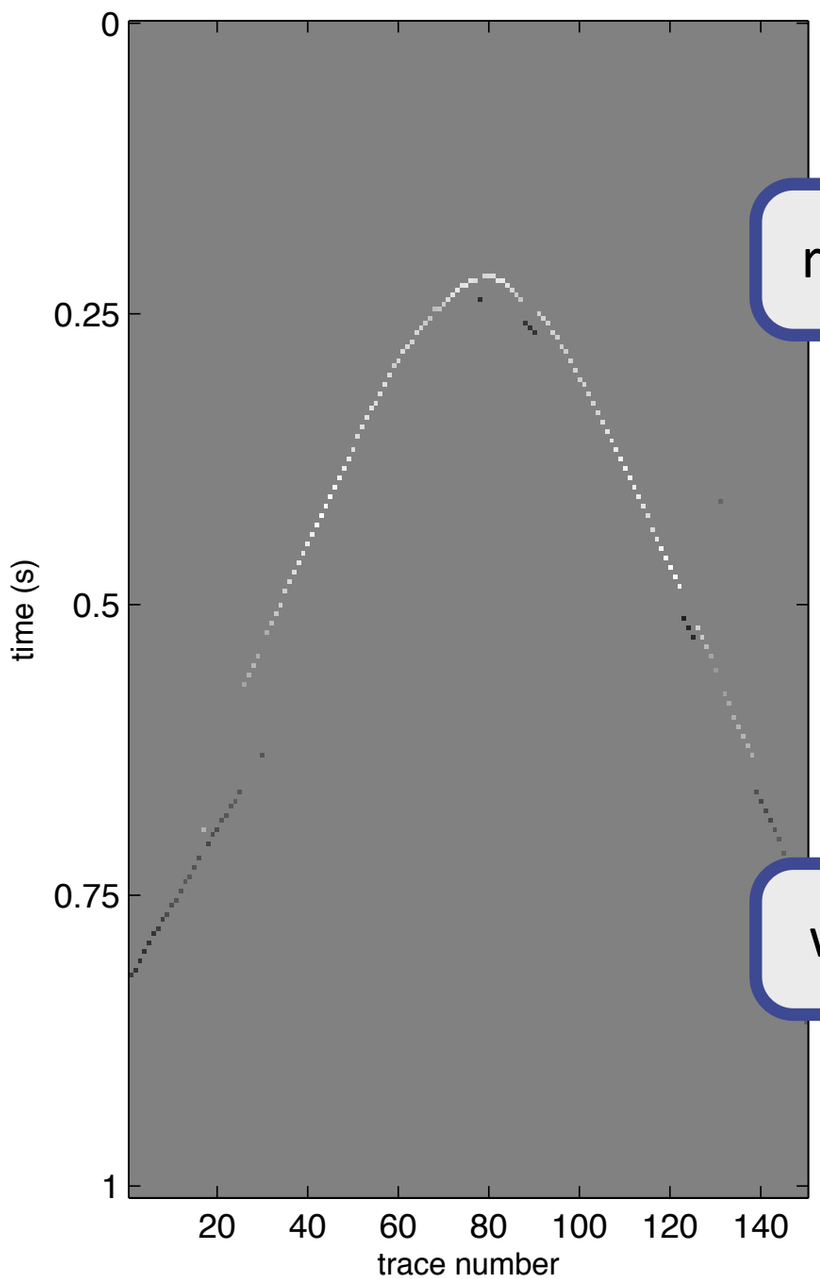
Fix primary impulse response, get least-squares matching for $\mathbf{Q}^+$ past $\ell_2$ mismatch tolerance $\sigma_k$

# Wavelet ambiguity




same $\ell_1$ norm

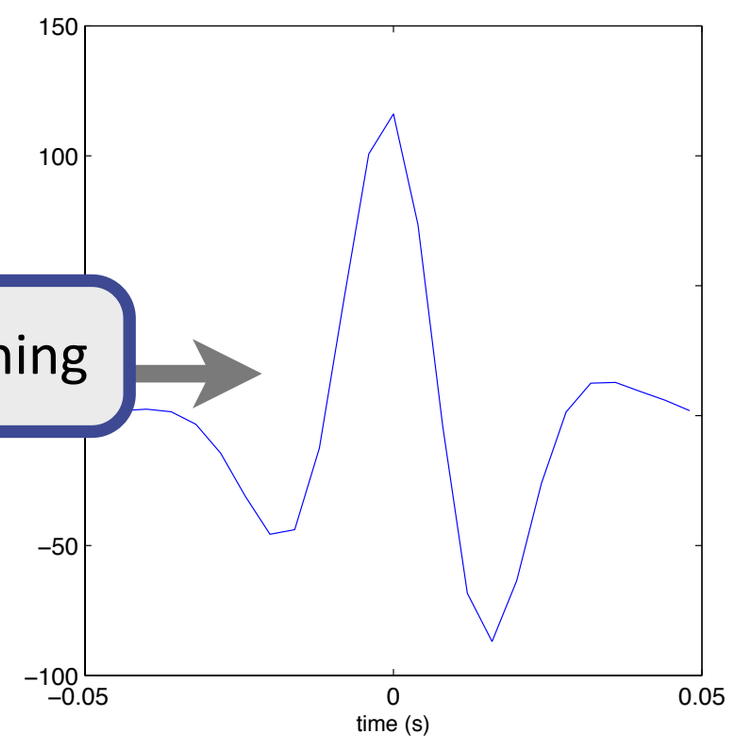
# Wavelet ambiguity




reset Green's Func

$$\mathbf{x}_{k=1} = \text{zero vector}$$

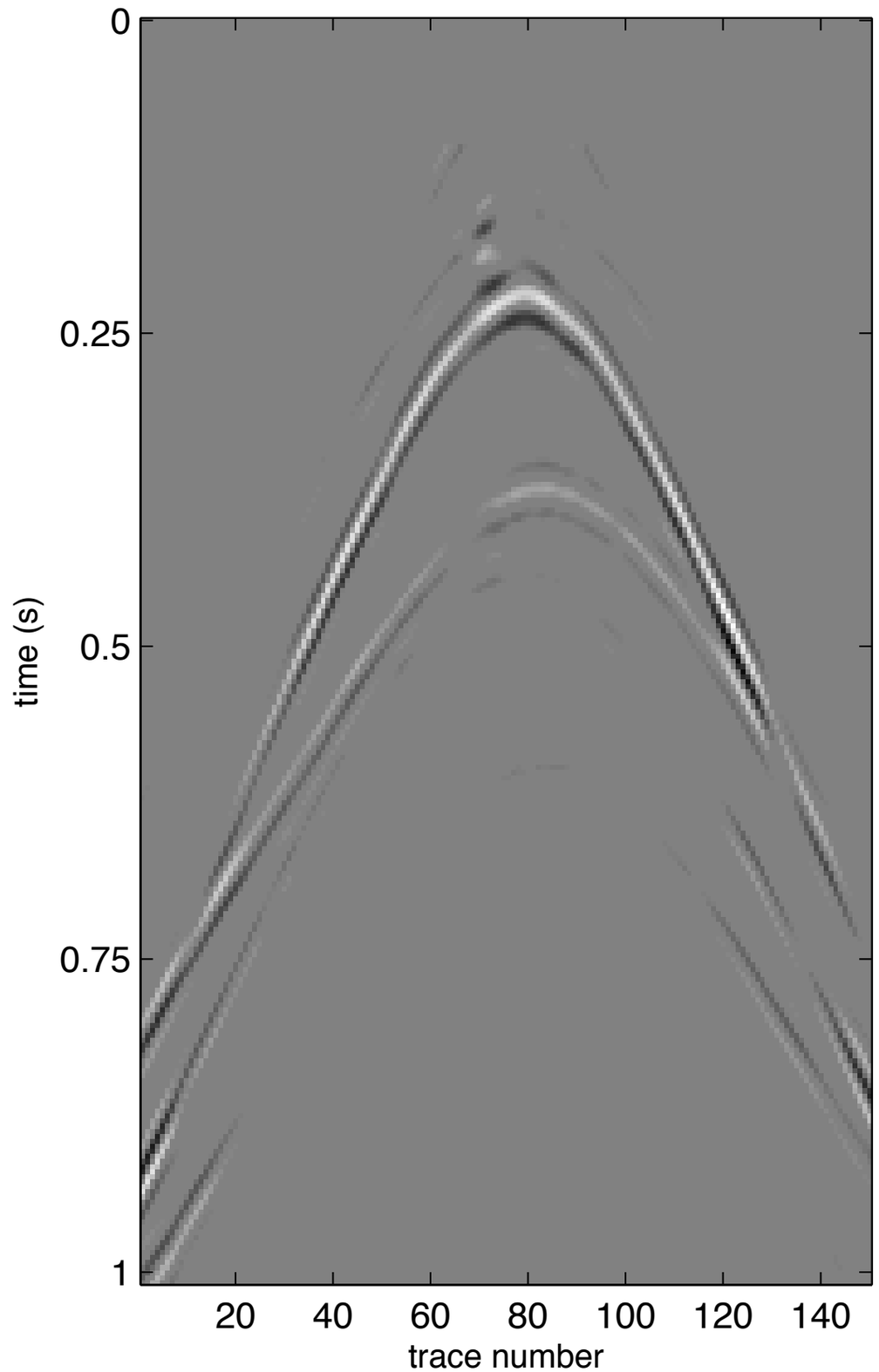
wavelet matching



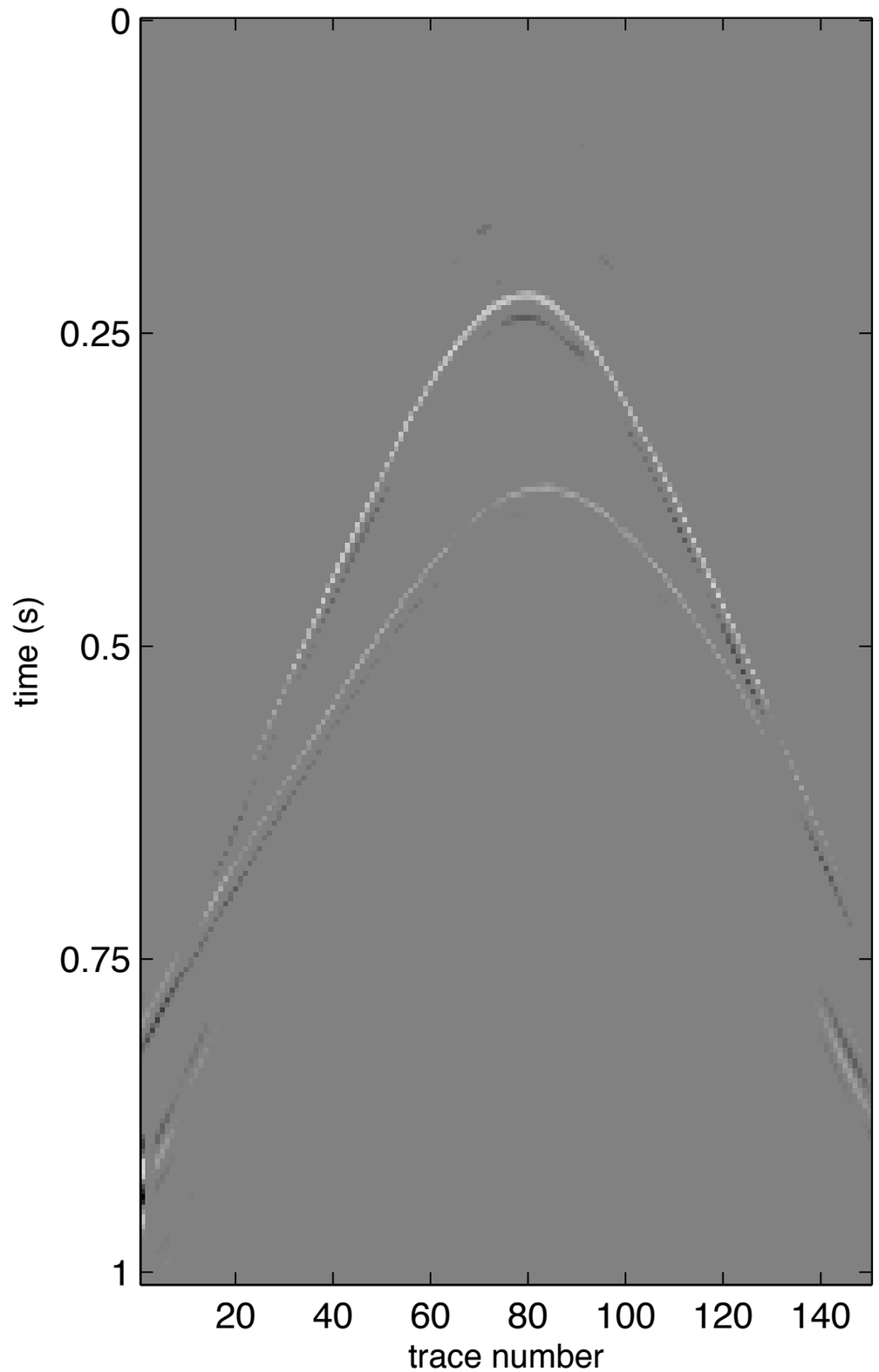

$$= \mathbf{Q}^{+}_{k=1}$$

pick only max event

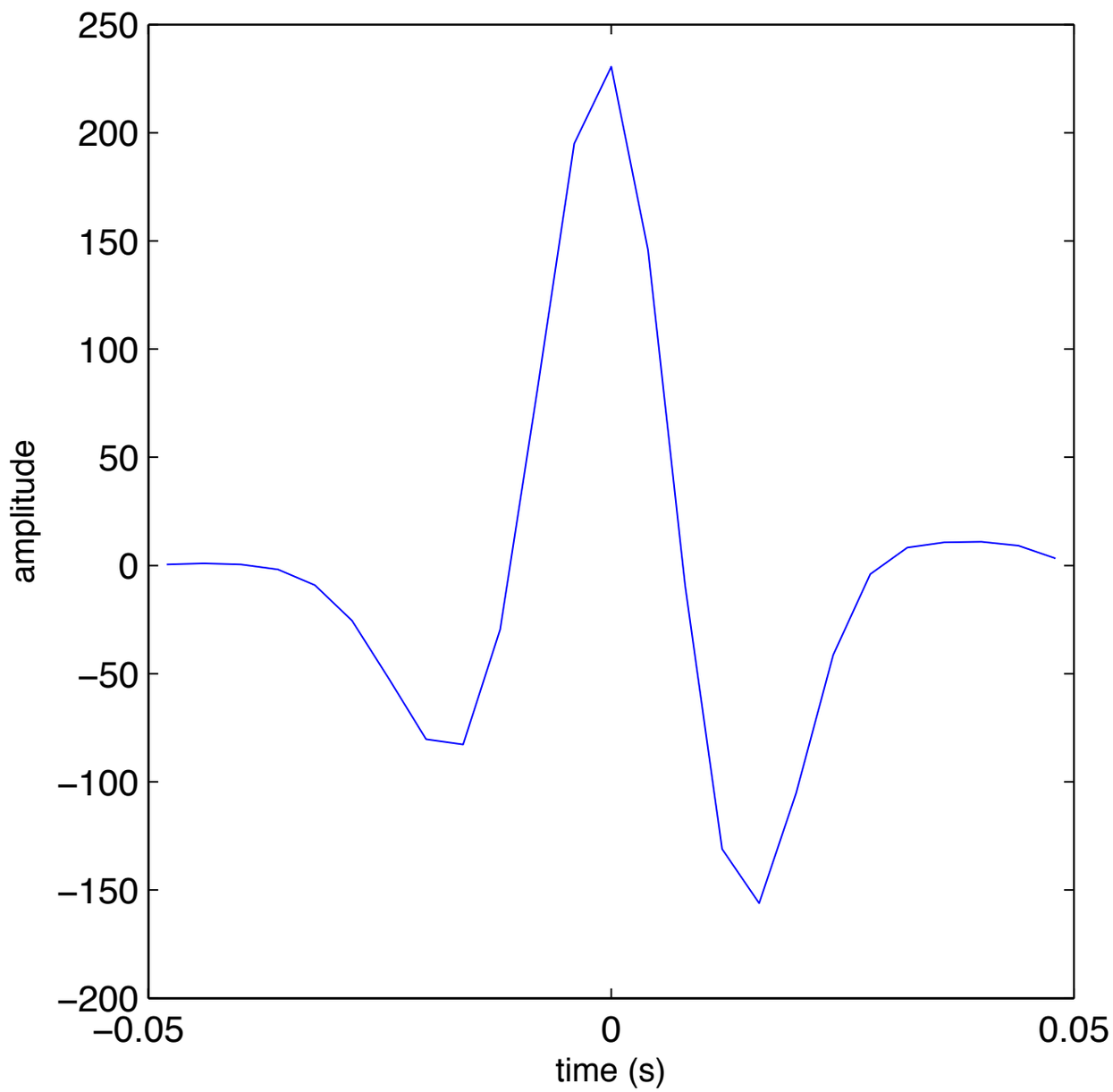
# IR estimation 1 - start



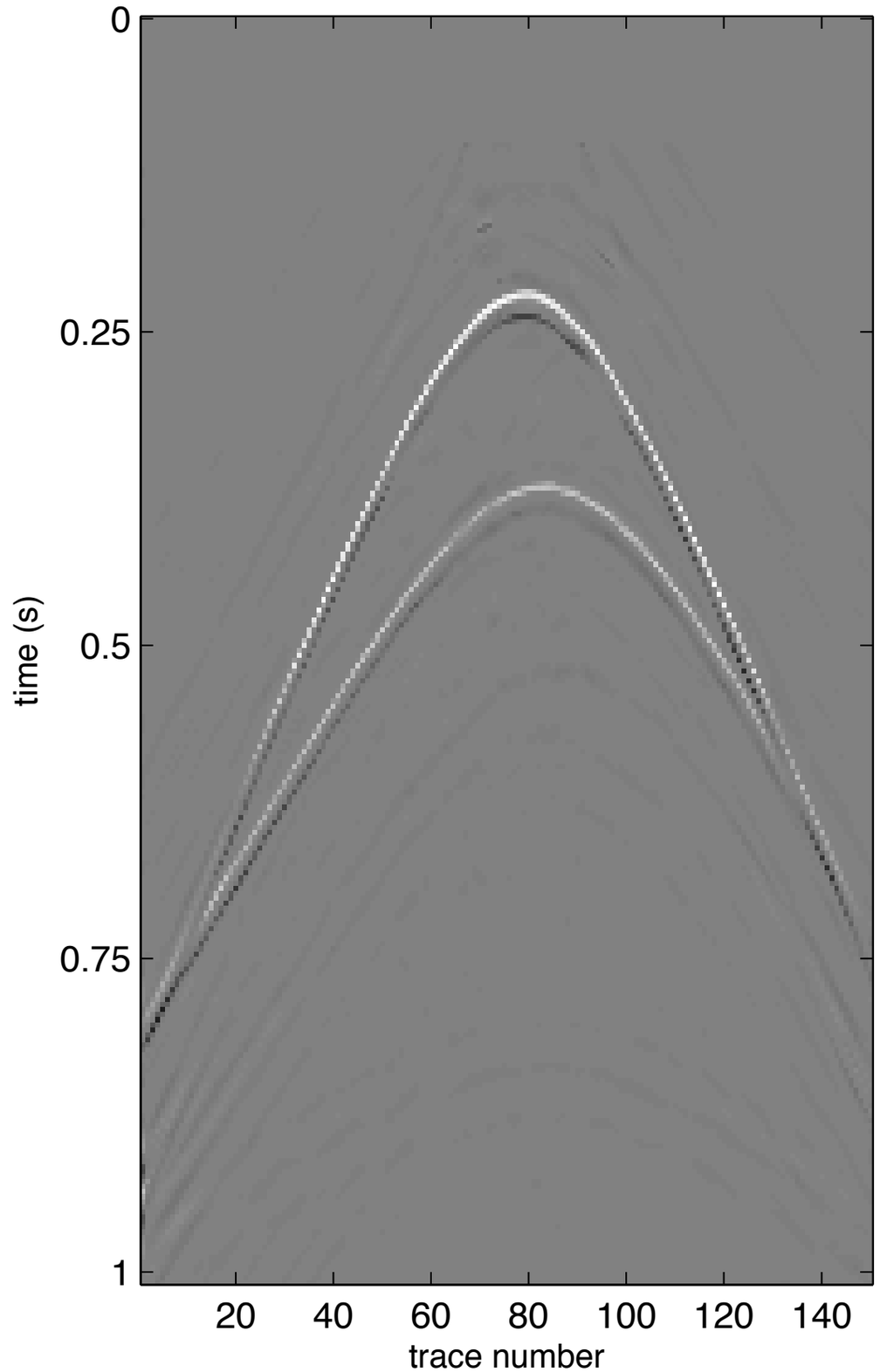
# IR estimation 1 - Pareto



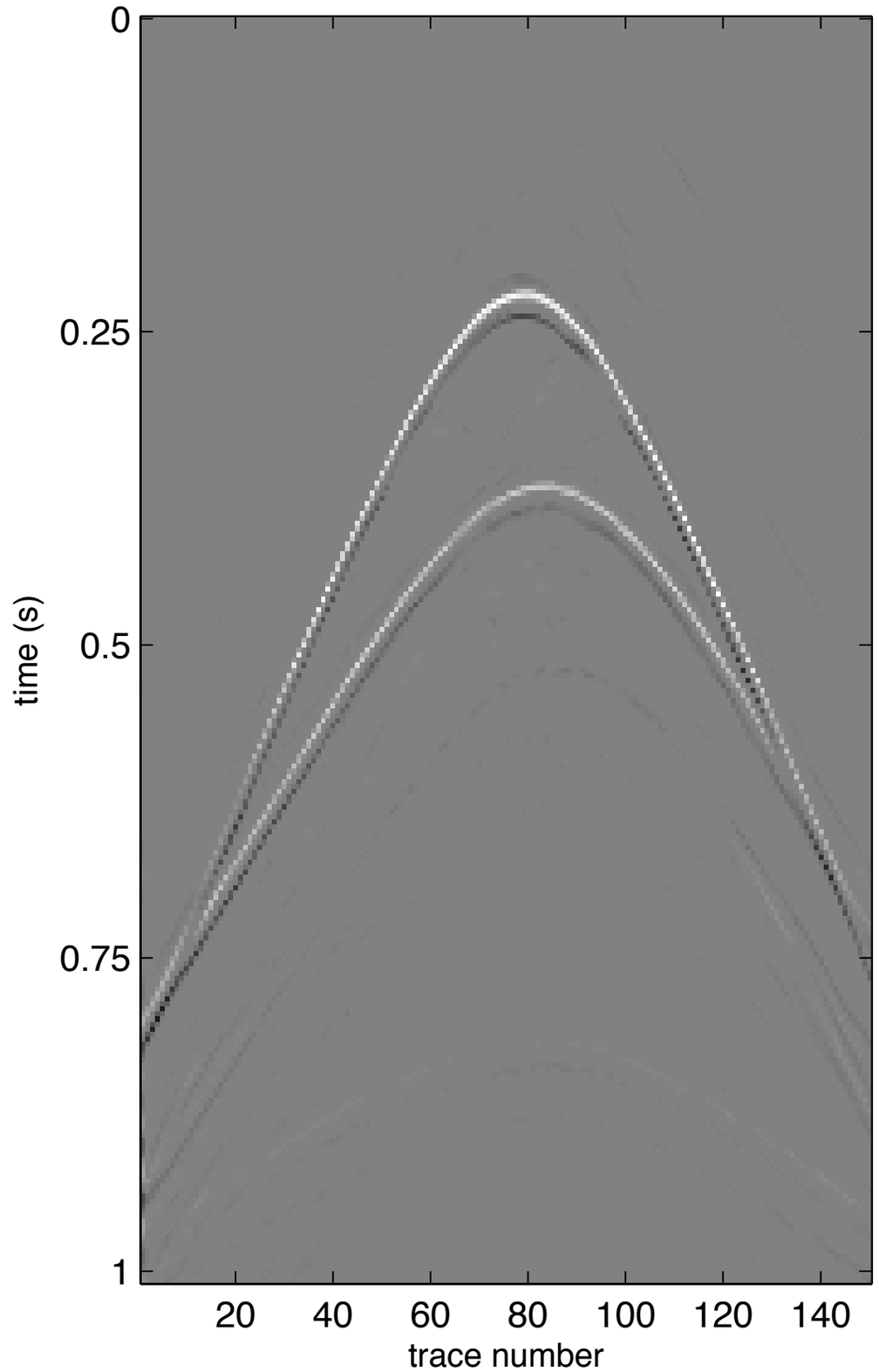
# Wavelet matching 2



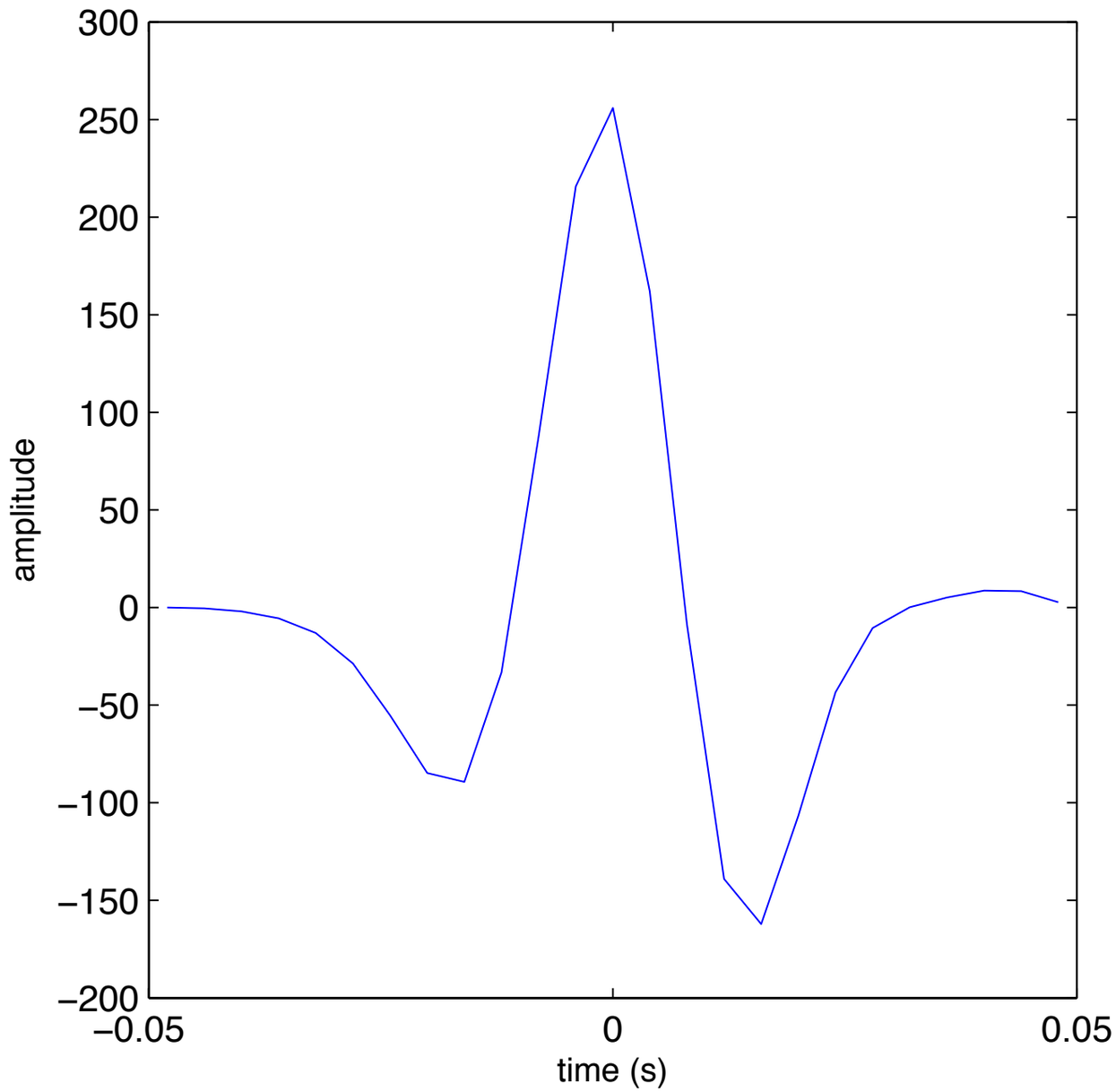
# IR estimation 2 - start



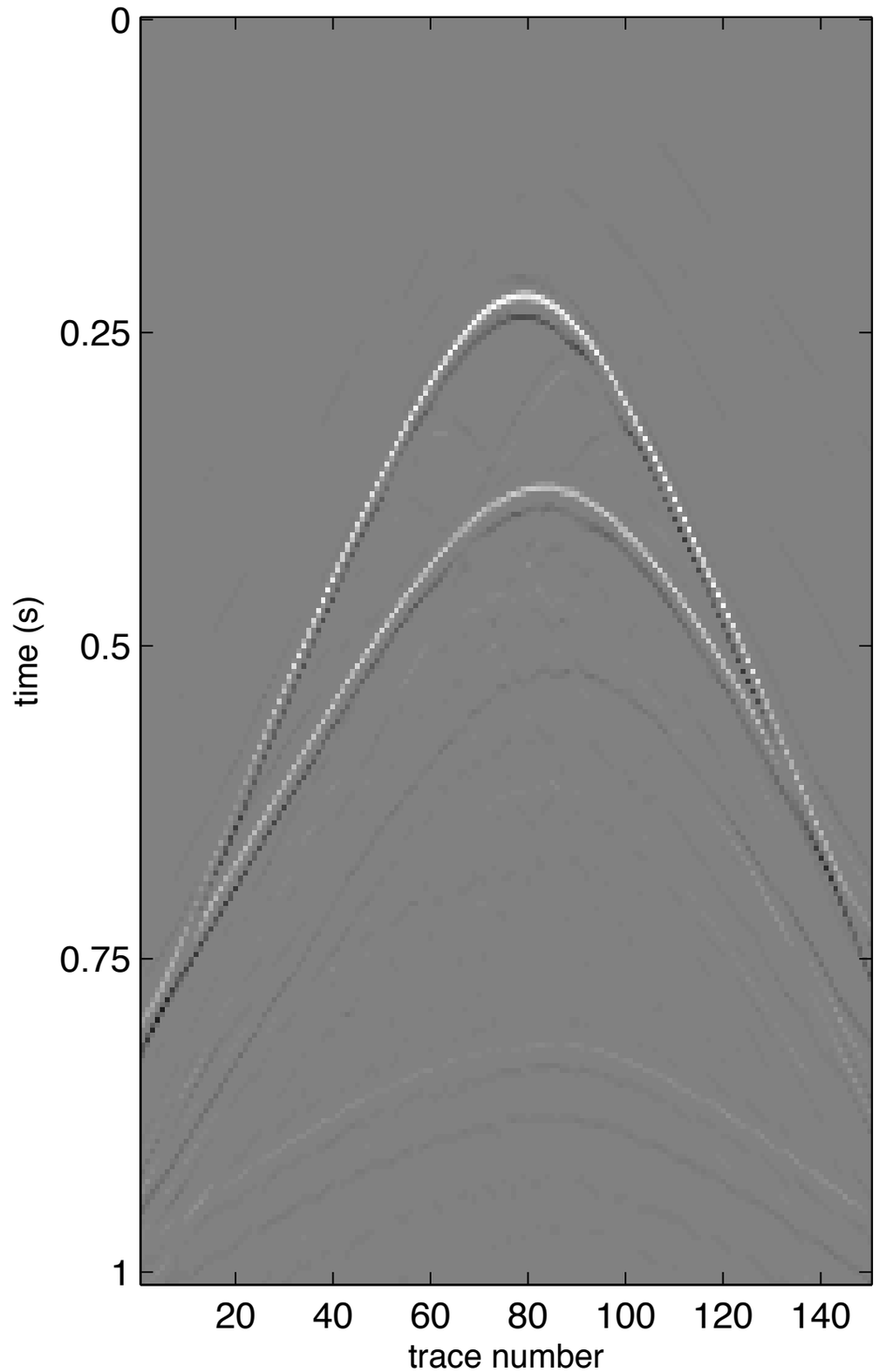
# IR estimation 2 - Pareto



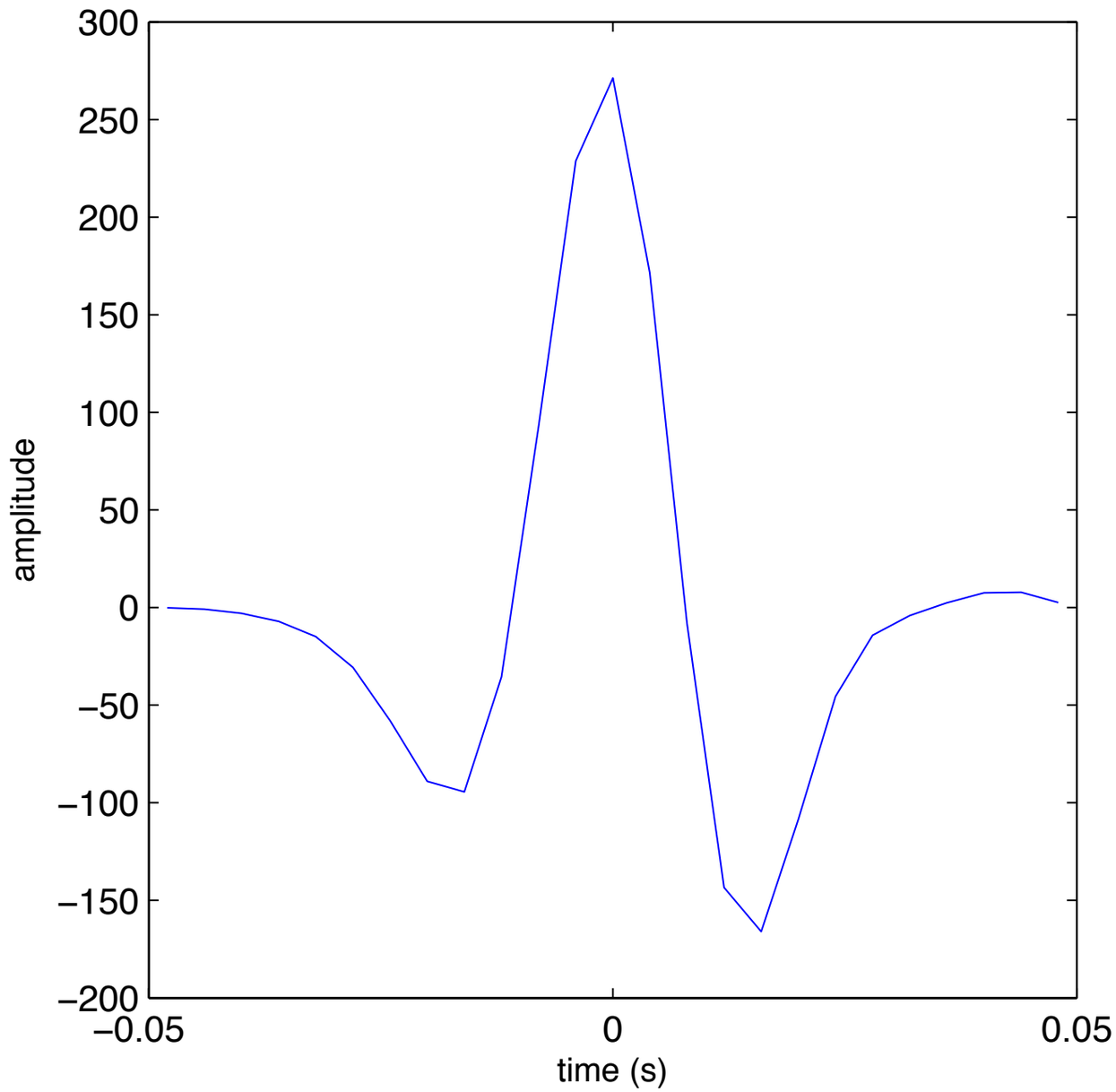
# Wavelet matching 3



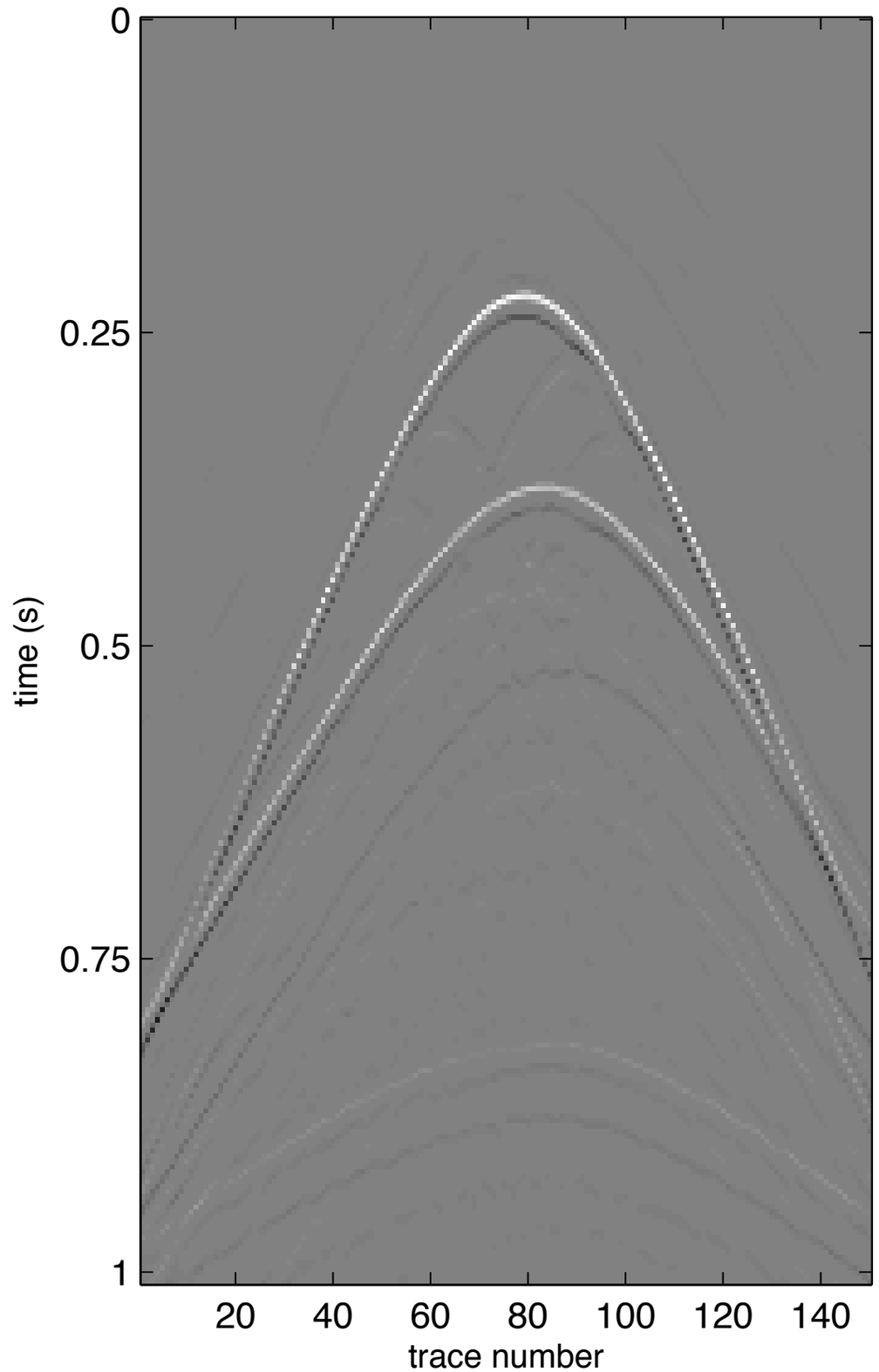
# IR estimation 3 - Pareto



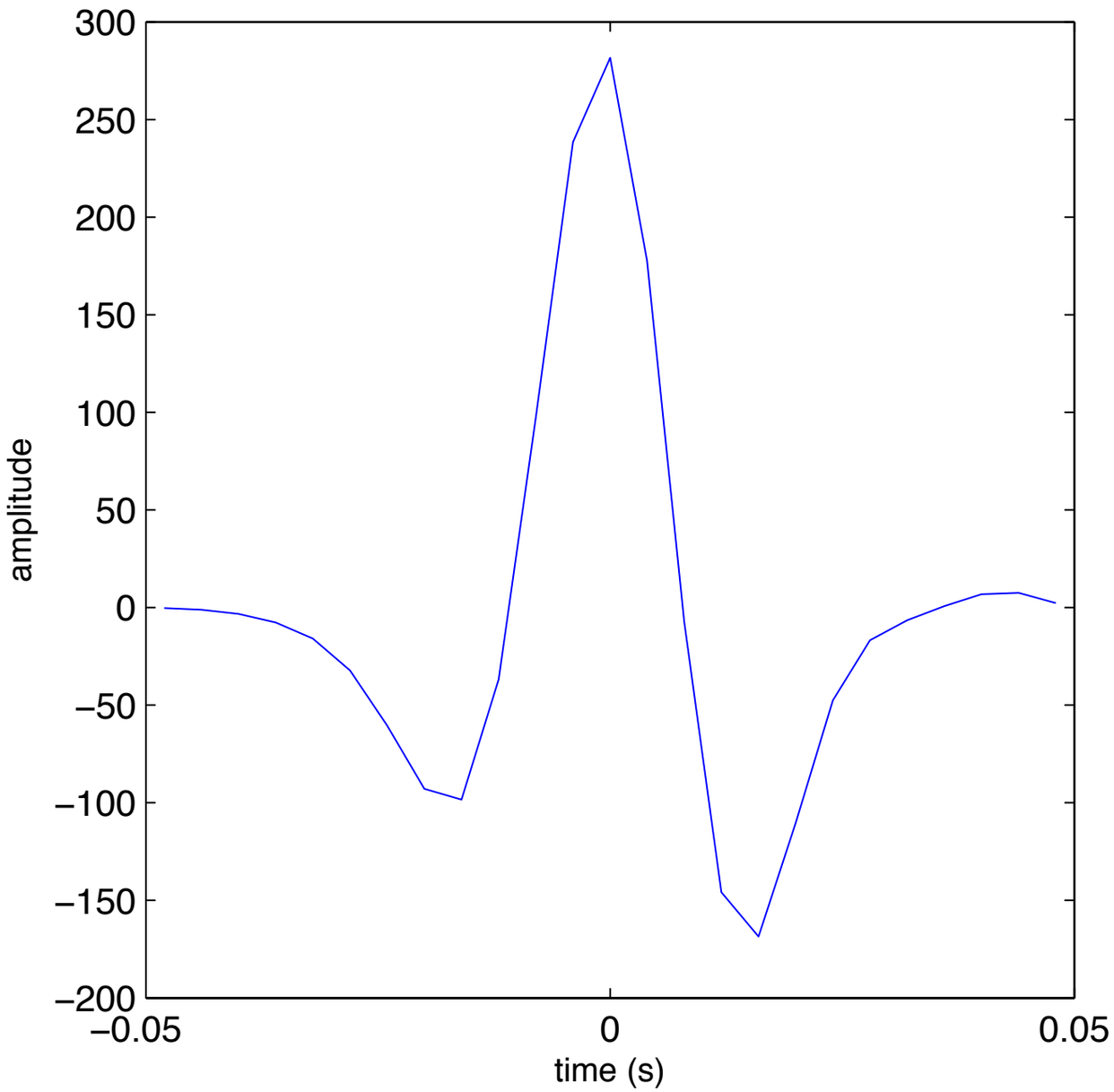
# Wavelet matching 4

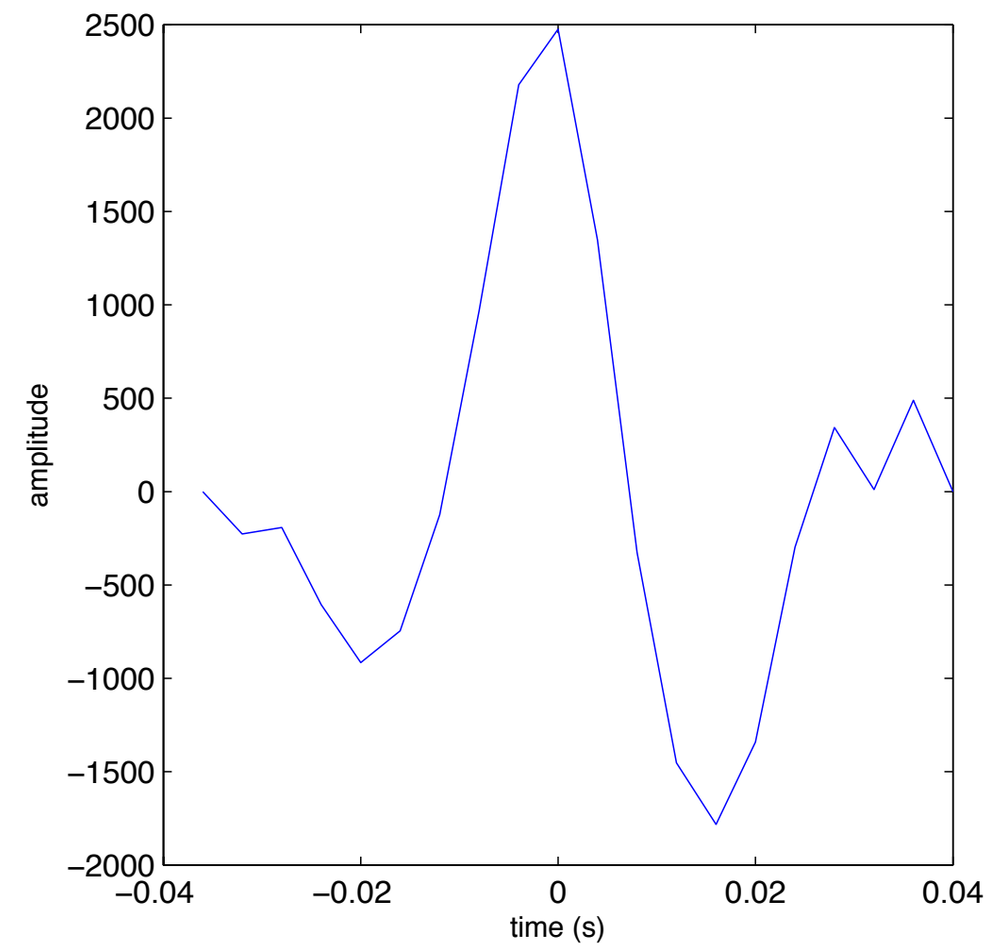


# IR estimation 4 - Pareto



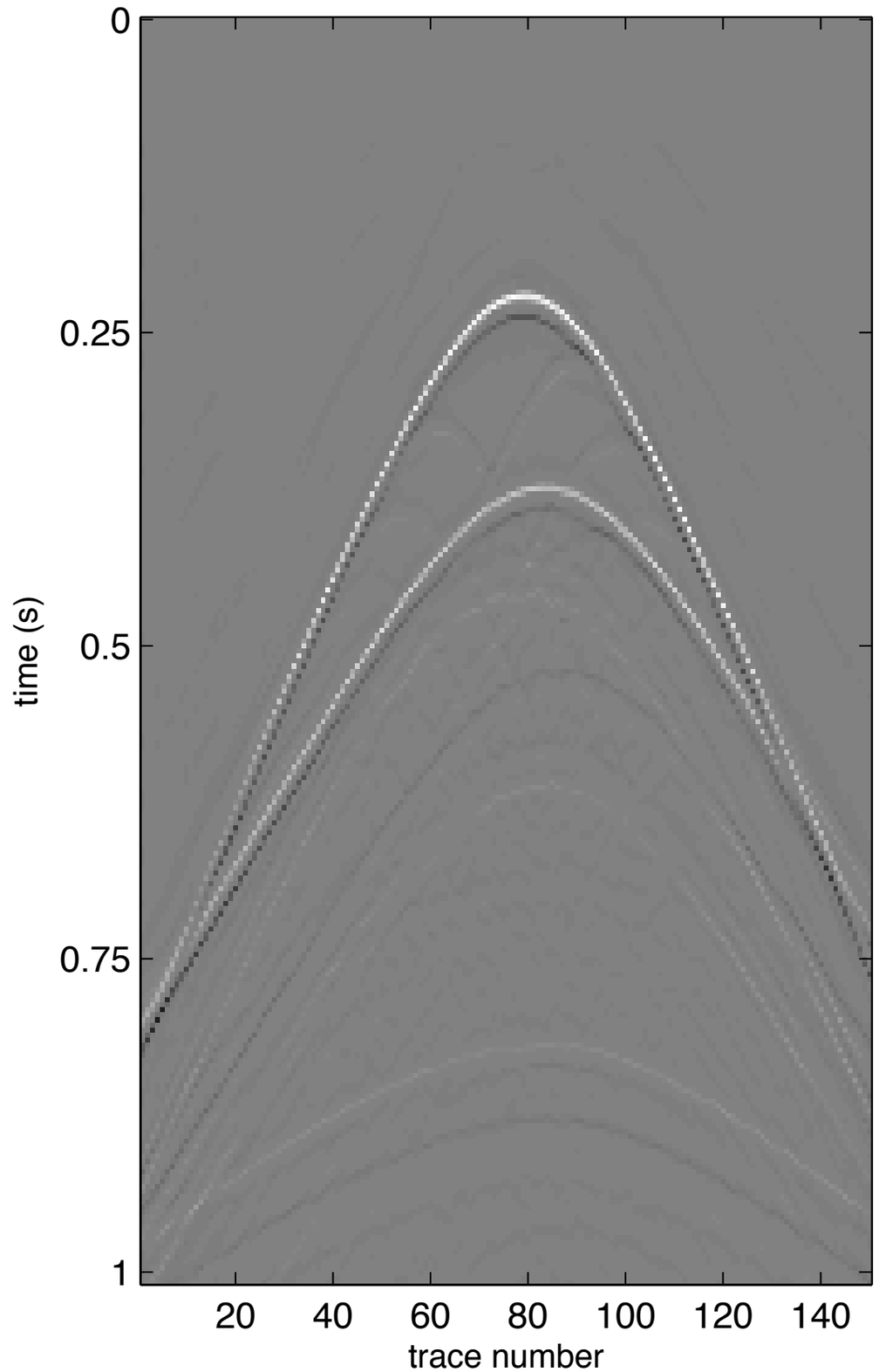
# Wavelet matching 5




EPSI result

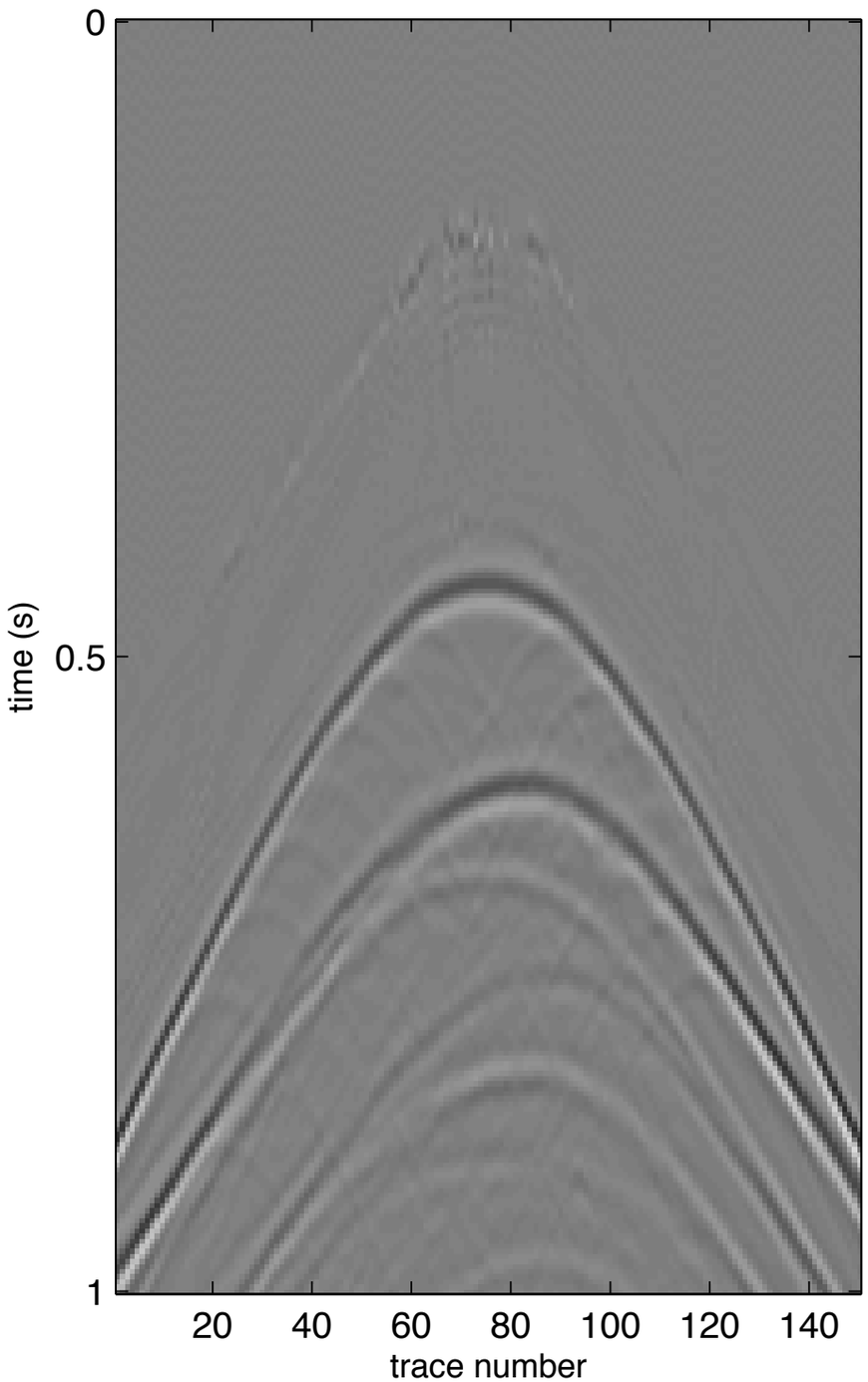
# IR estimation 5 - Pareto



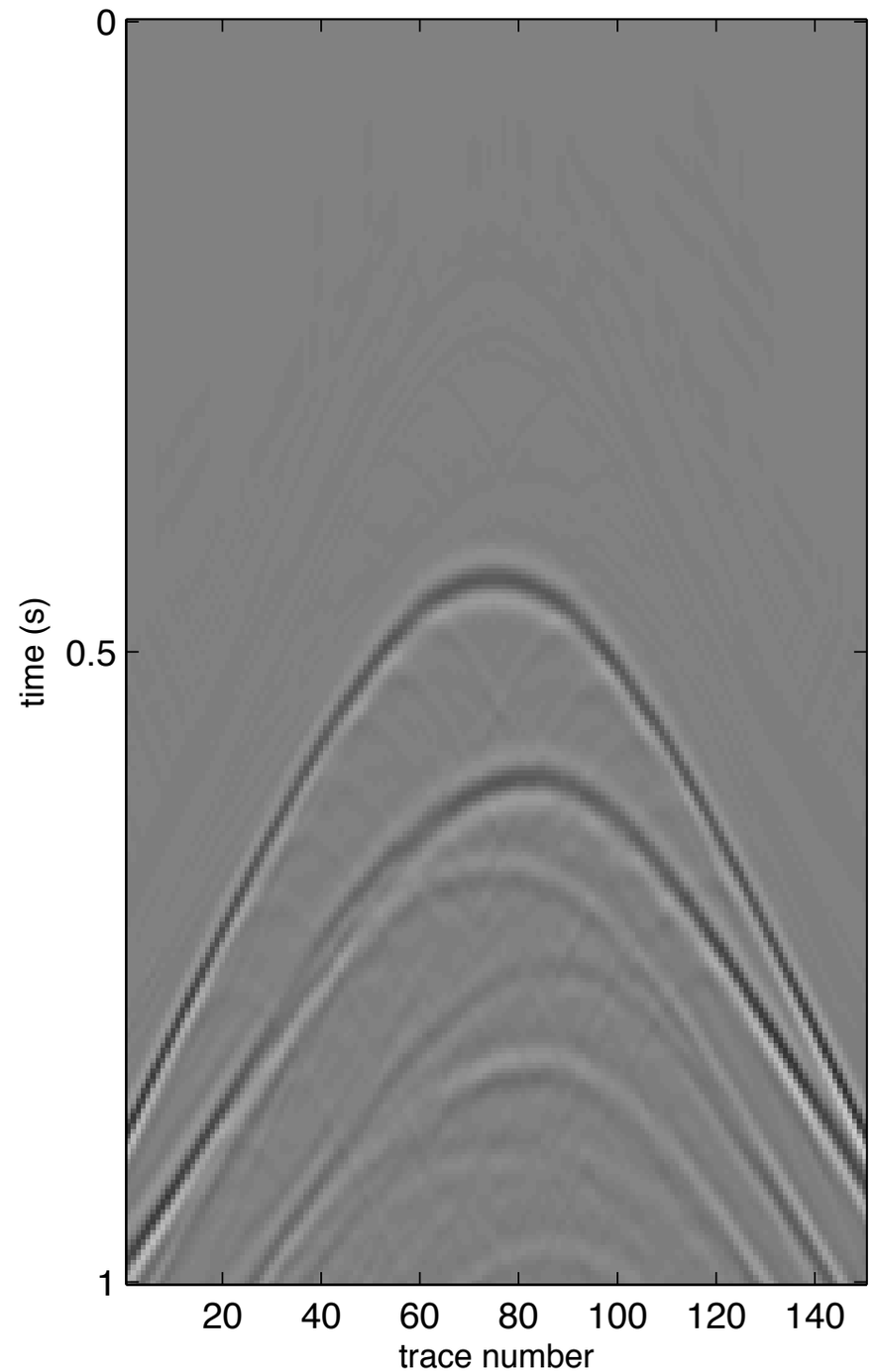

total ~60 gradient updates

# Sparsity vs L1

Data minus estimated primary
(one shot)



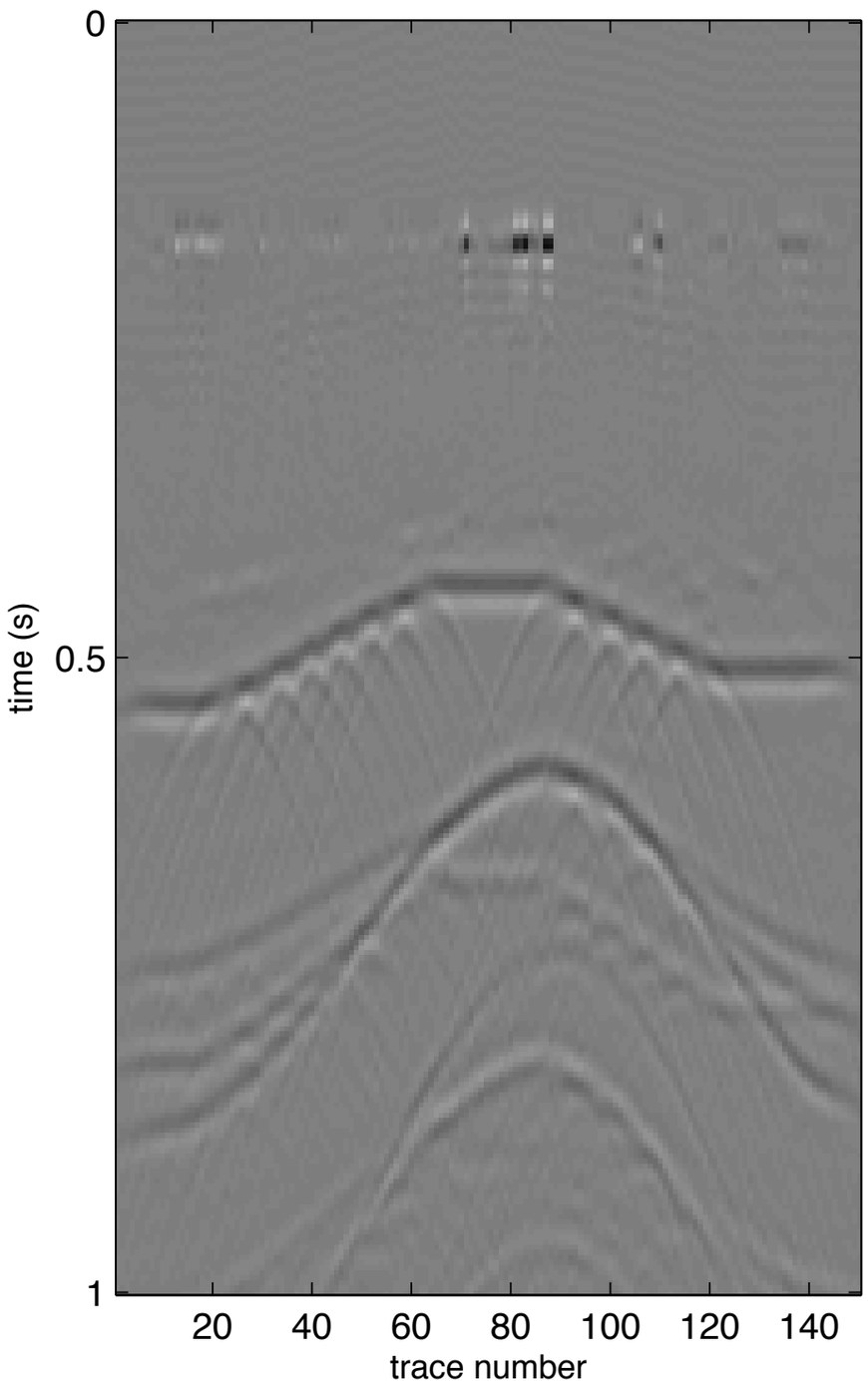

Sparse EPSI



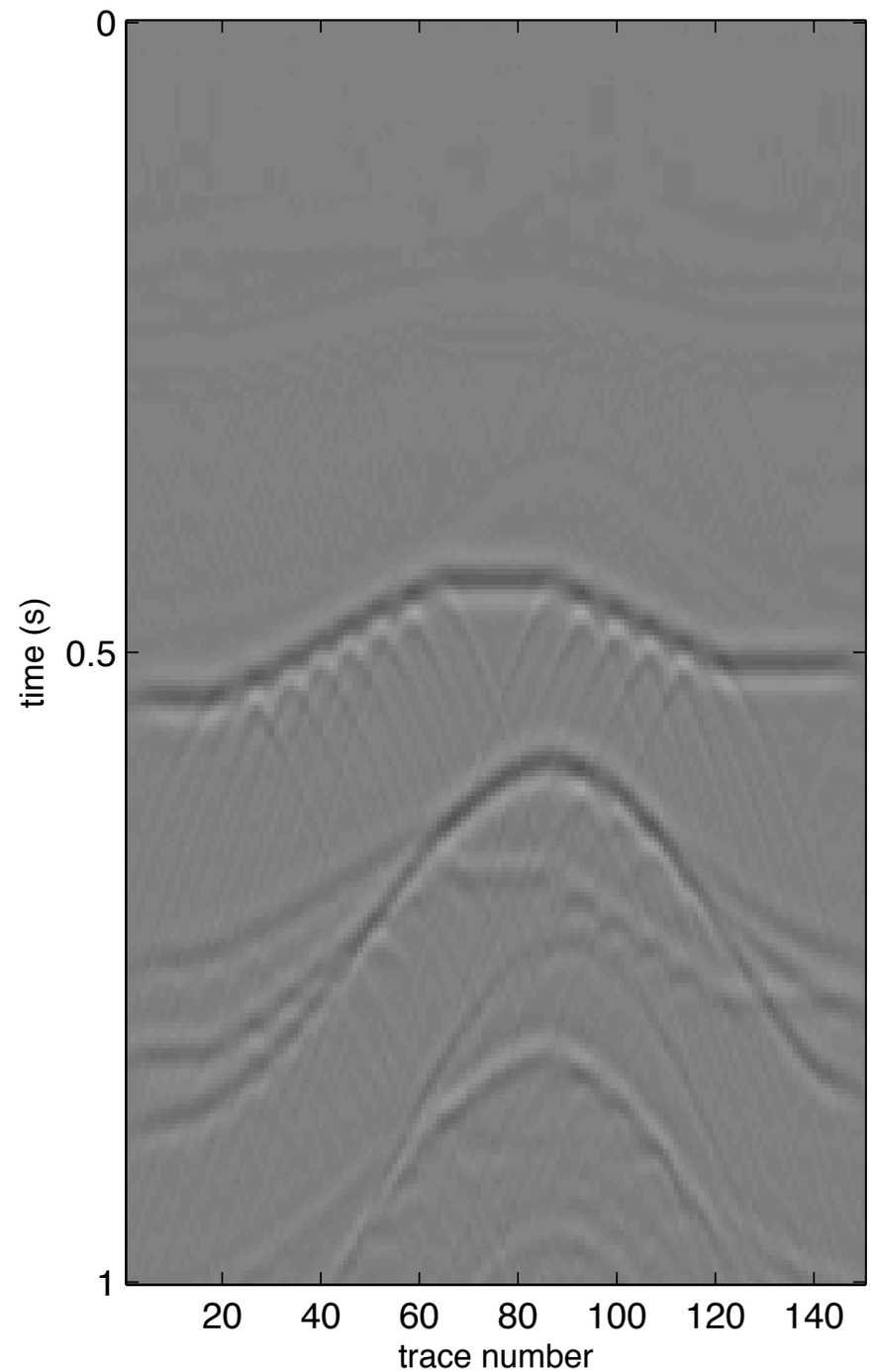

$\ell_1$ EPSI

# Sparsity vs L1

Data minus estimated primary
(zero-offset)



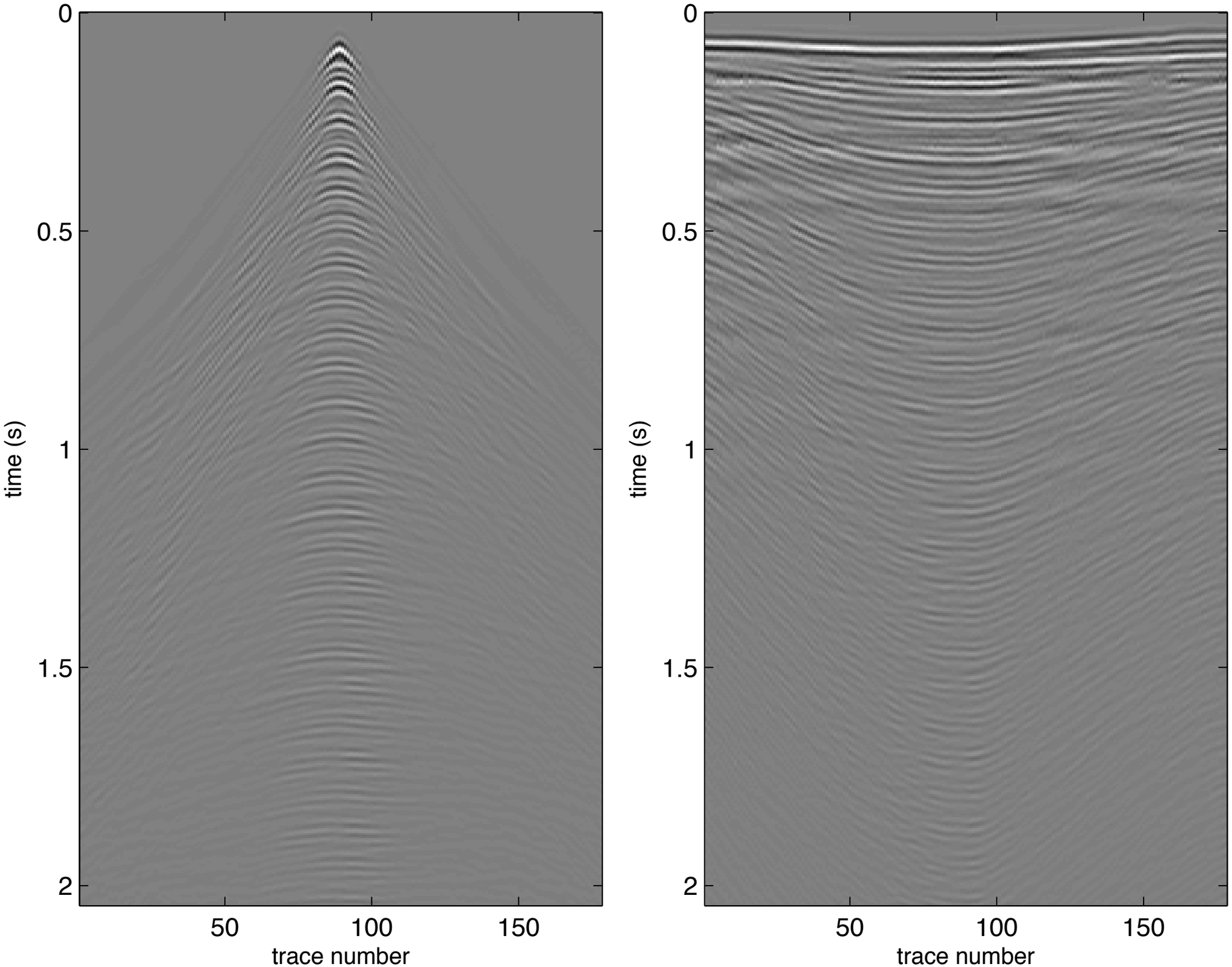

Sparse EPSI




$\ell_1$ EPSI

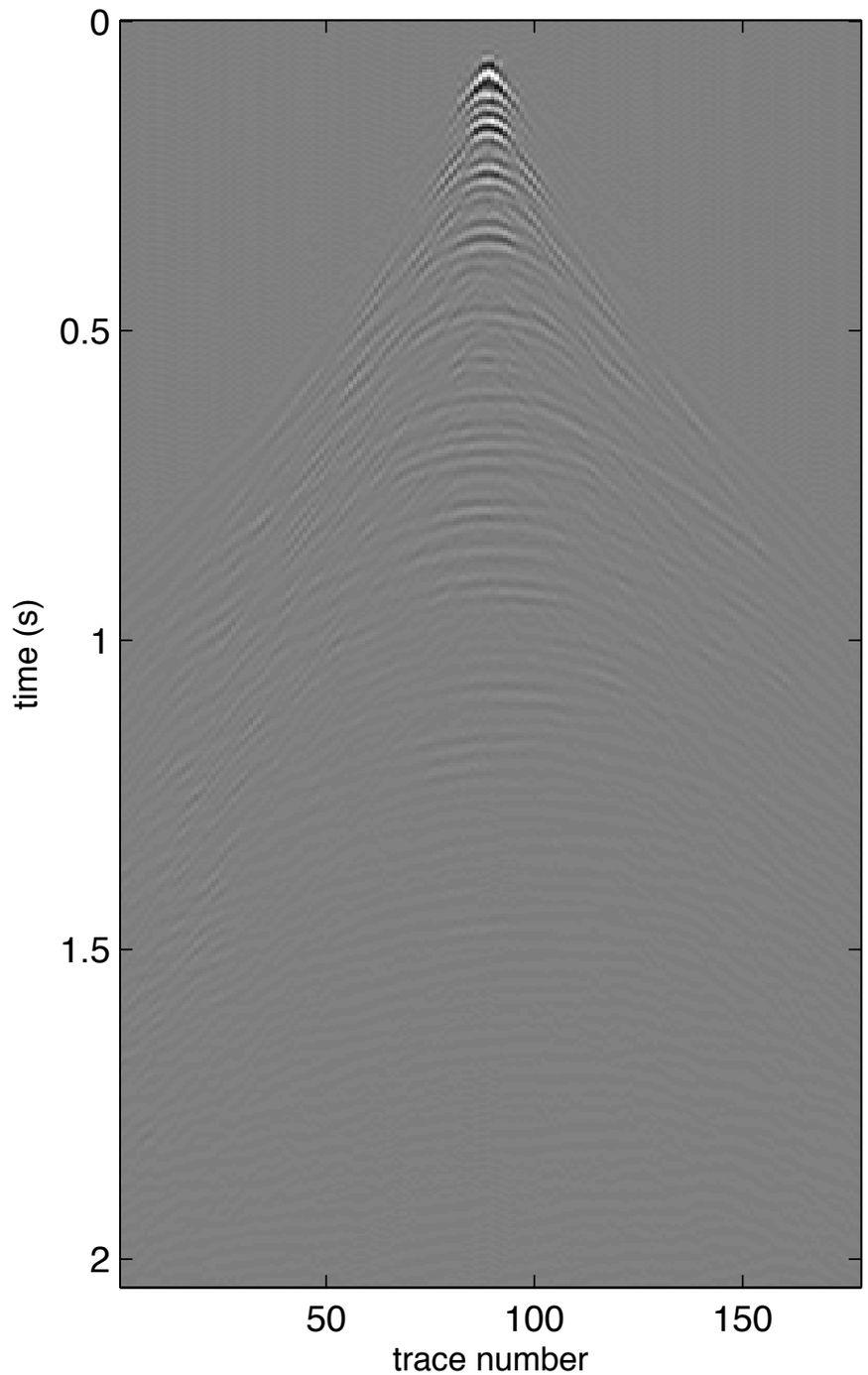
# Sparsity vs L1

Gulf of Suez

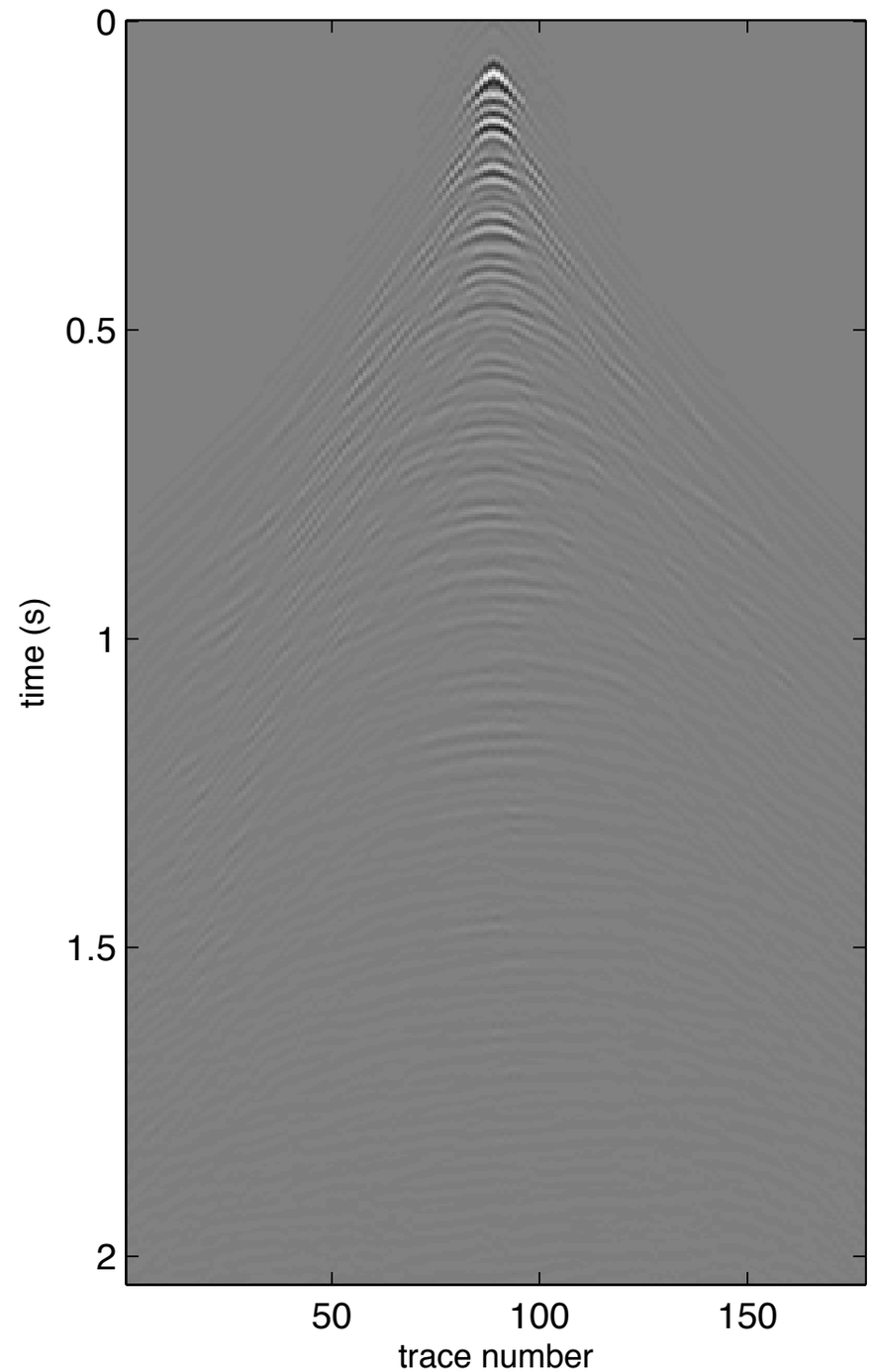


# Sparsity vs L1

Gulf of Suez
(one shot)



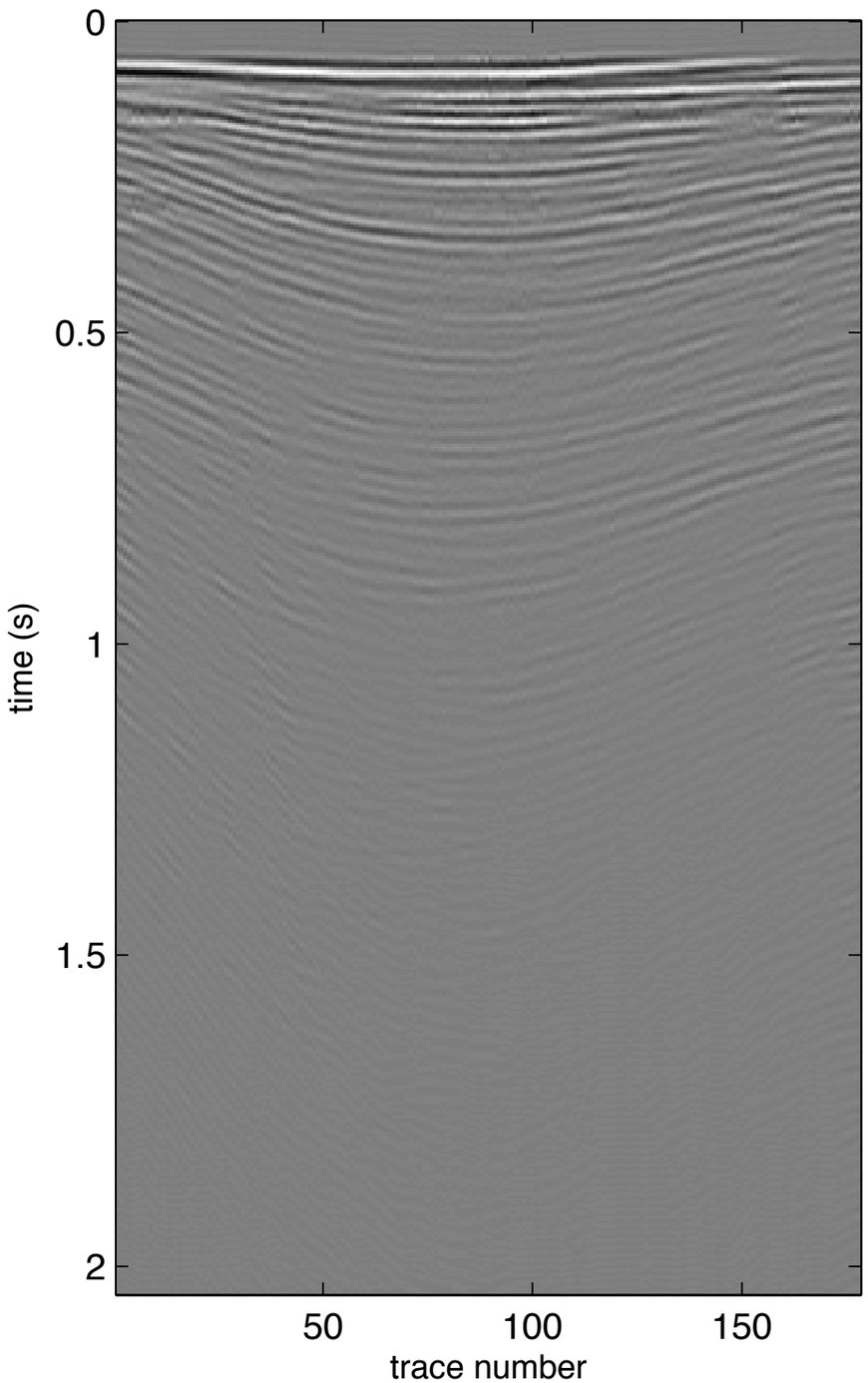

Sparse EPSI



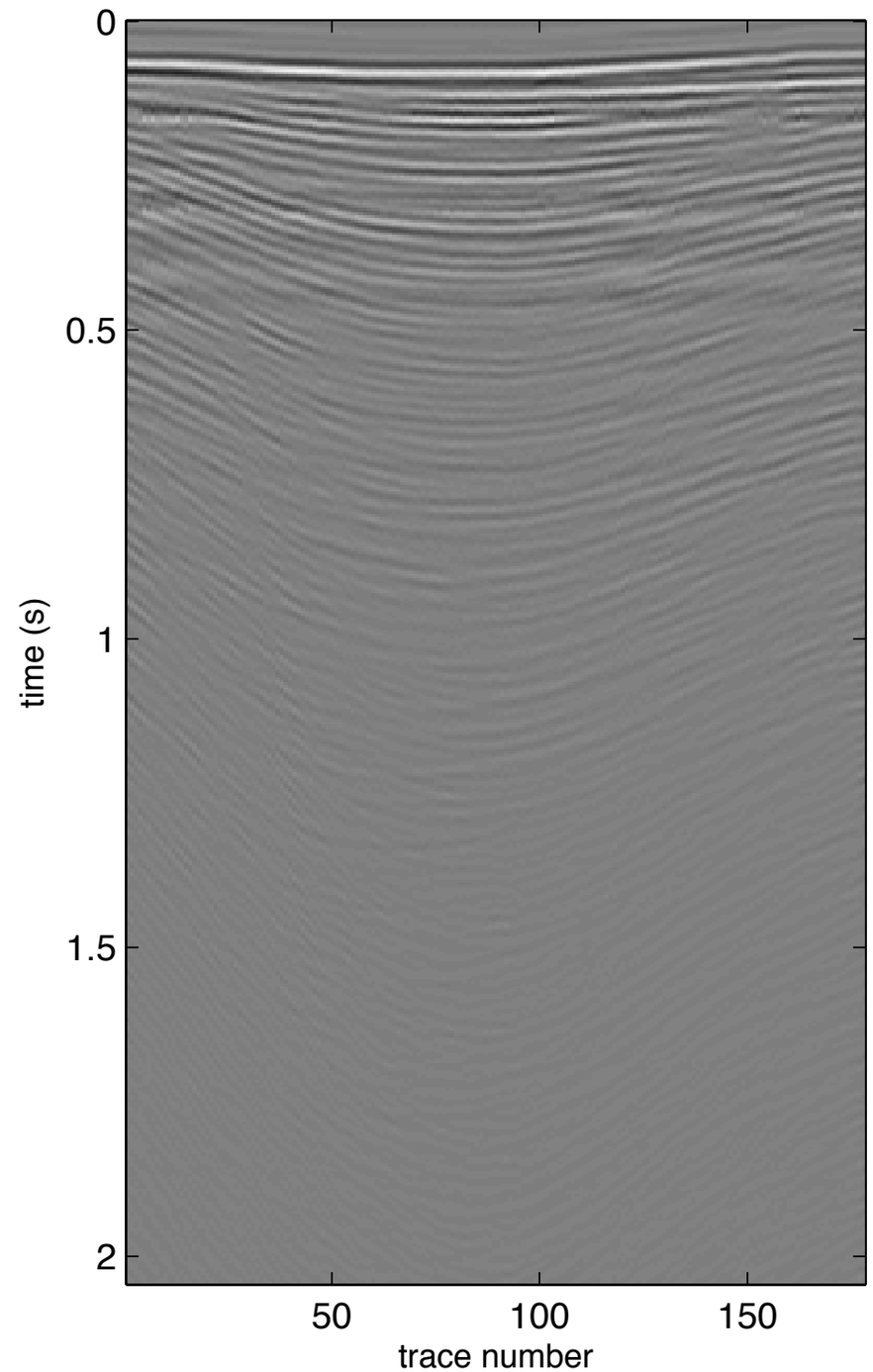

$\ell_1$ EPSI

# Sparsity vs L1

Gulf of Suez
(zero-offset)



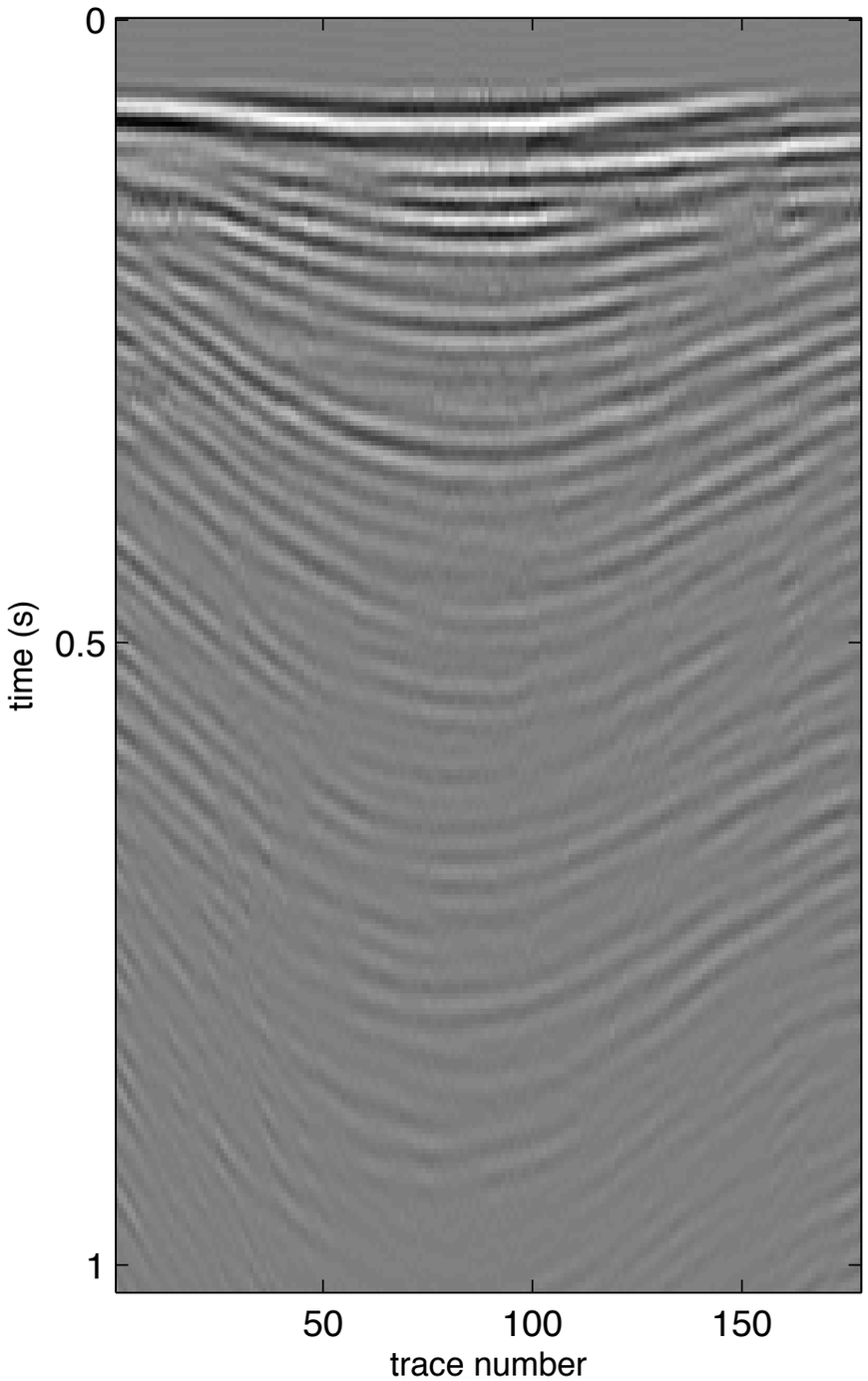

Sparse EPSI



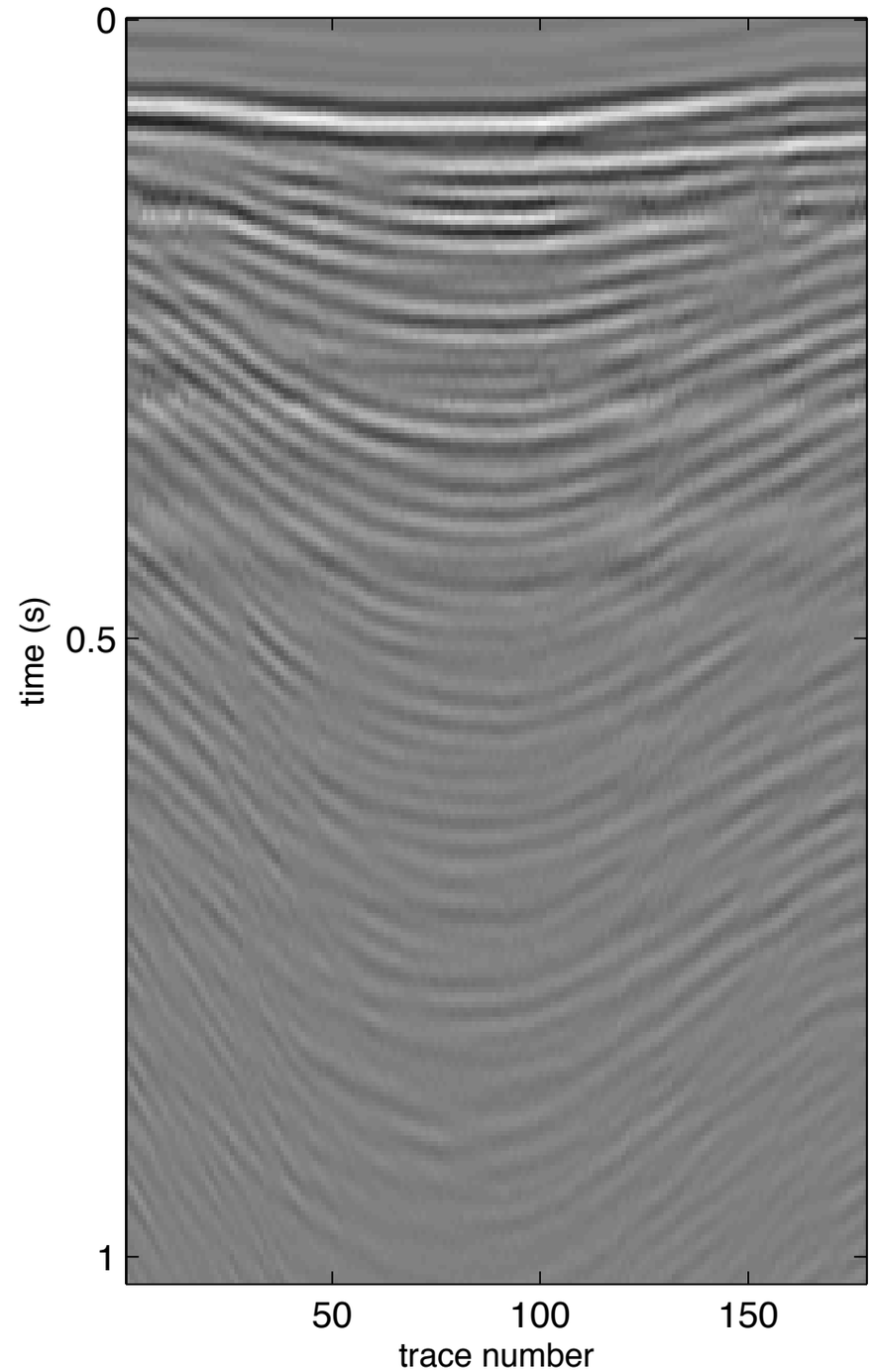

$\ell_1$ EPSI

# Sparsity vs L1

Gulf of Suez
(zero-offset zoomed)



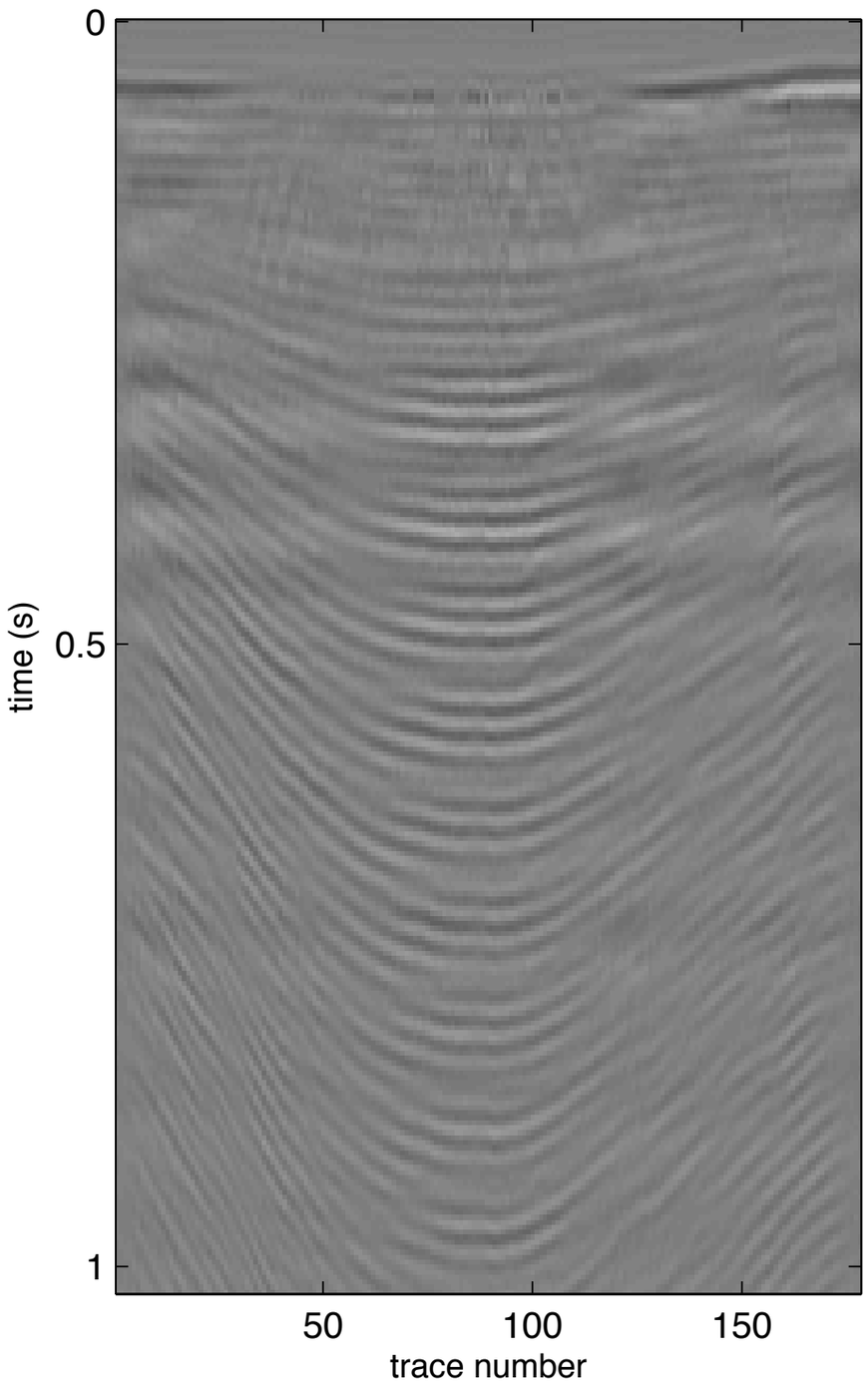

Sparse EPSI



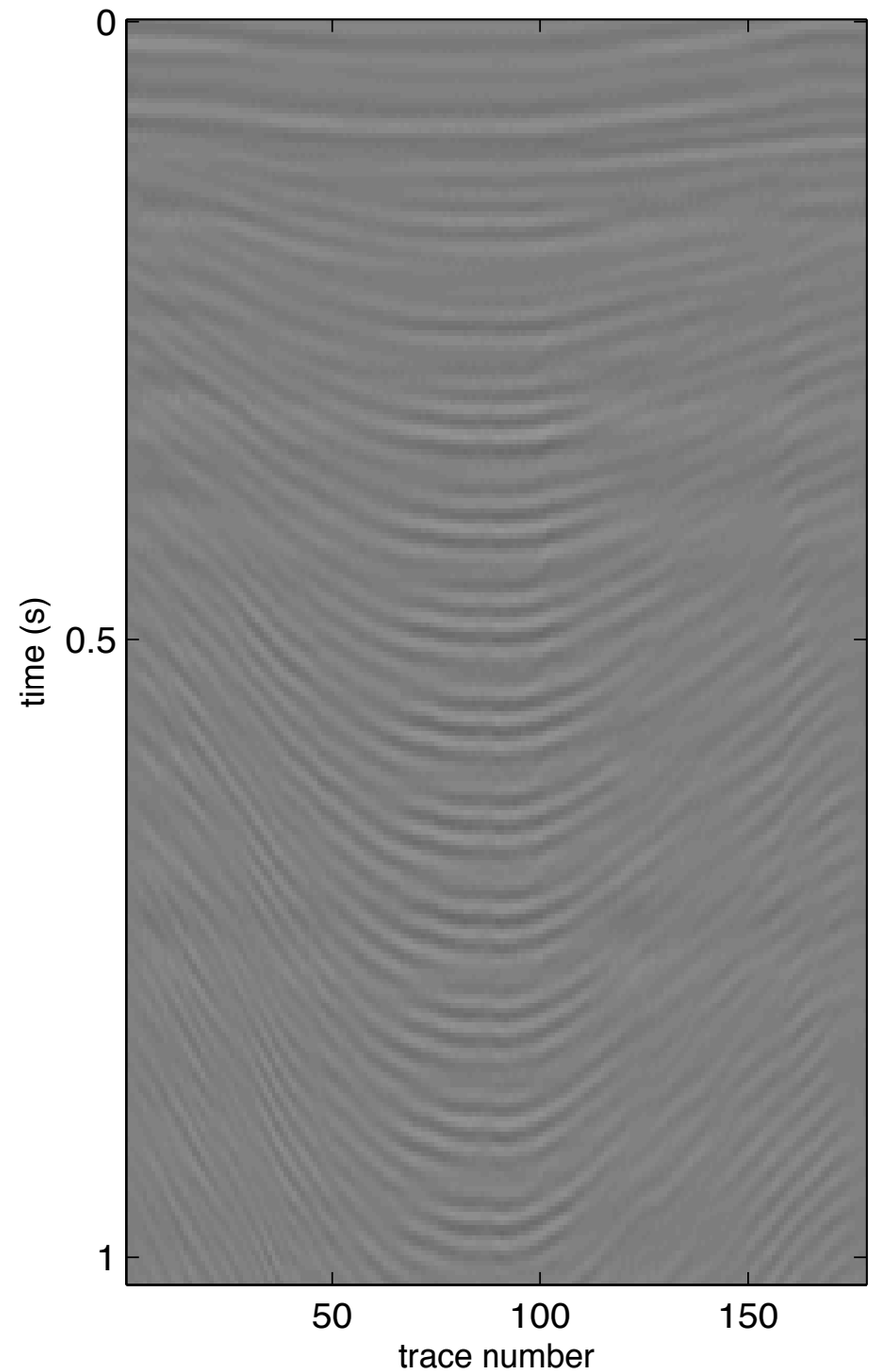

$\ell_1$ EPSI

# Sparsity vs L1

Data minus estimated primary
(zero-offset)




Sparse EPSI




$\ell_1$ EPSI

# EPSI requires

- tight muting window
- physical locations of the primaries (window at each gradient update)
- number of reflection events inside this window
- source wavelet length

# L1 reformulation

- ~~tight muting window~~
- ~~physical locations of the primaries (window at each gradient update)~~
- ~~number of reflection events inside this window~~
- noise level in input data (use GCV in the future)
- source wavelet length

# conclusions

- **less** parameters to tweak
- **improved convergence** properties of EPSI by convex relaxation
- **improved quality** of first arrivals
- cast EPSI into blind deconvolution **framework** using alternating optimization
- **removed gradient scaling issues** btw wavelet matching and IR estimation
- future work on intelligently setting $\sigma$ (GCV)

# acknowledgements

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