Randomized sampling strategies

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Drivers & impediments

"Acquisition costs"

- Full-waveform inversion requires ***hifi*** data

"Data deluge"

- *"Curse of dimensionality"* compounded by over restrictive "*Nyquist sampling criterion*"

"*Limits on computational resources*"

- End of "*Moore's law*"

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Wish list

Acquisition & inversion costs determined by *structure of data & complexity* of the *subsurface*

- sampling *criteria* that are *dominated by* transform-domain *sparsity* and *not* by the *size* of the *discretization*

Controllable error that depends on

- degree of *subsampling / dimensionality* reduction
- available computational resources

Strategy

Adapt recent ***compressive sensing*** (CS)

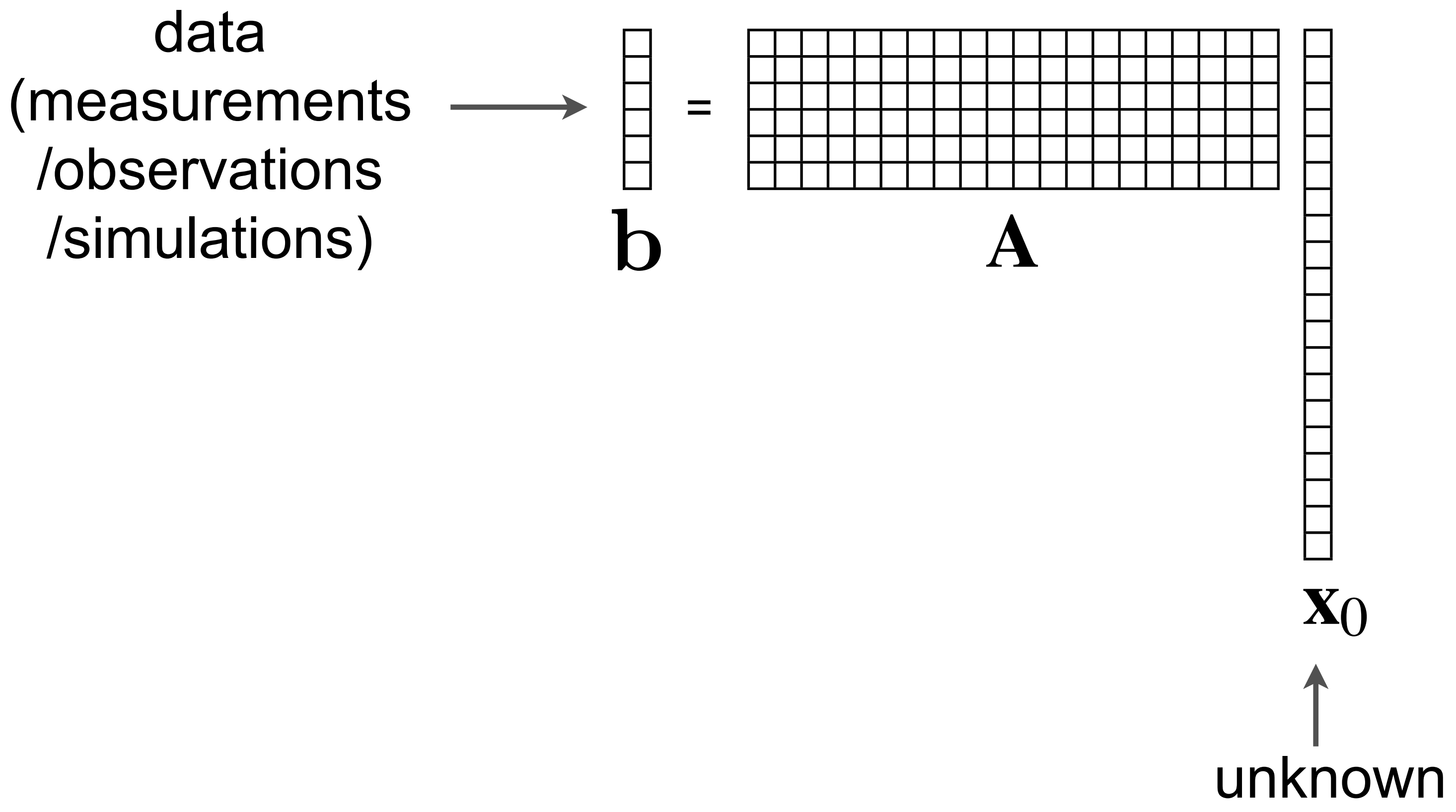
- *randomized* subsampling - turn *aliases/interferences* into *noise*
- *sparsity* promotion - removes subsampling *noise* by exploiting signal *structure*

This is really an *"acquisition" design problem*

Let's have a look at a stylized *recovery* problem first...

Problem statement

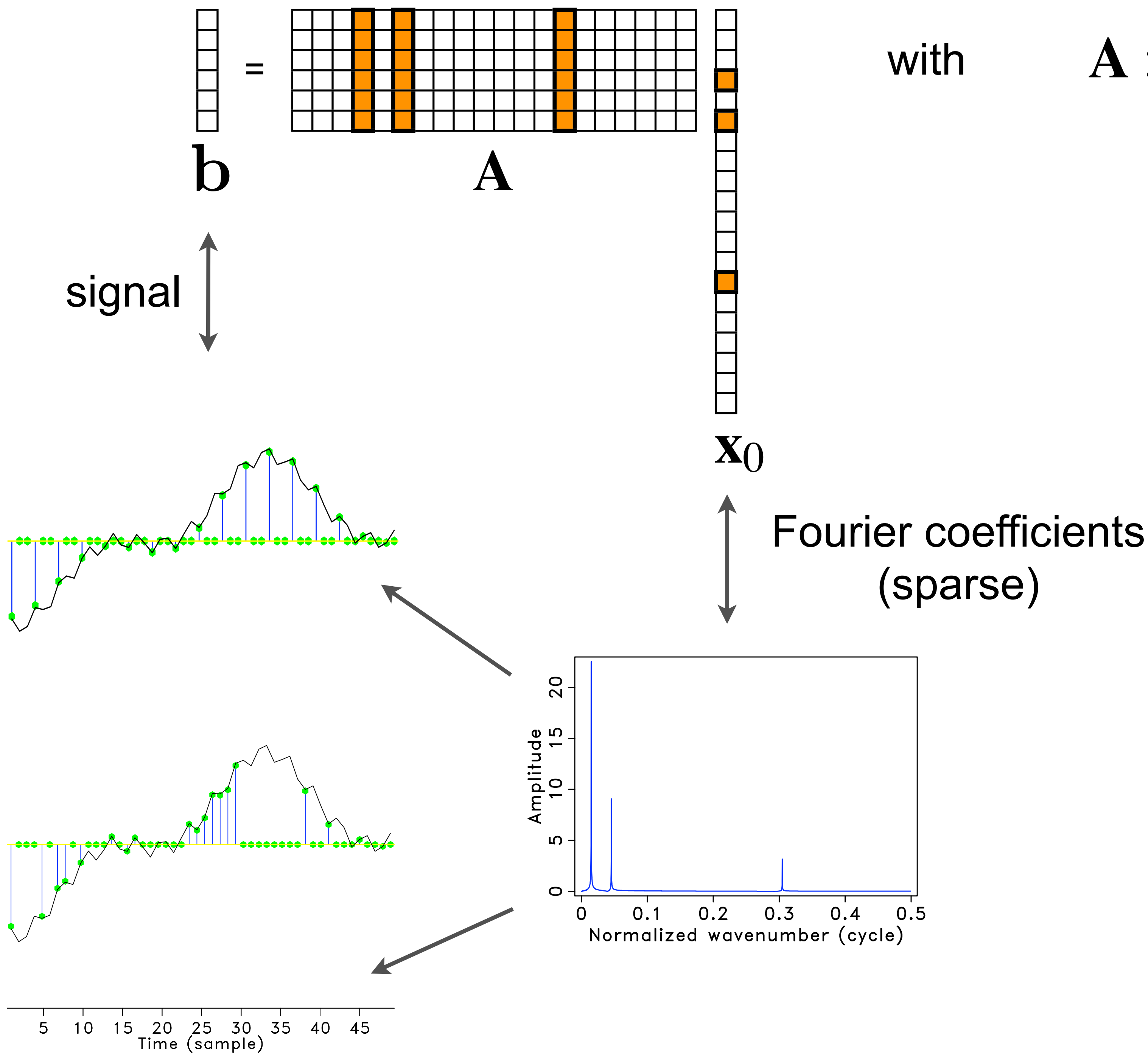
Consider the following (severely) *underdetermined* system of *linear* equations:



Is it possible to recover $\mathbf{x}_0$ accurately from $\mathbf{b}$?

The new field of *Compressive Sensing* attempts to answer this.

Sparse recovery

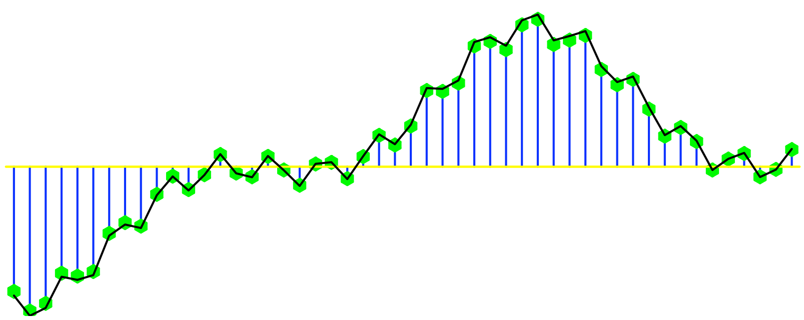


with $\mathbf{A} := \mathbf{R}\mathbf{F}^H$

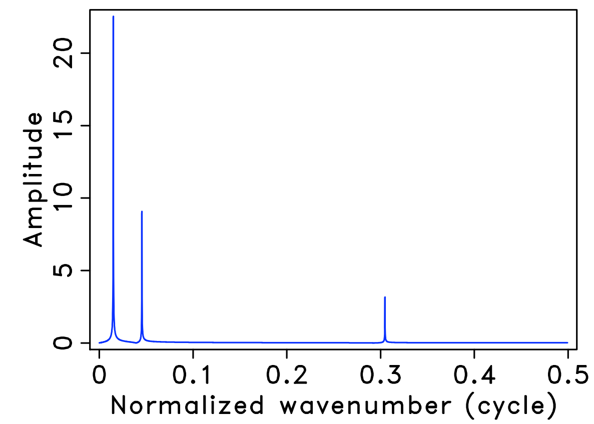
restriction operator

Fourier transform

Coarse sampling schemes

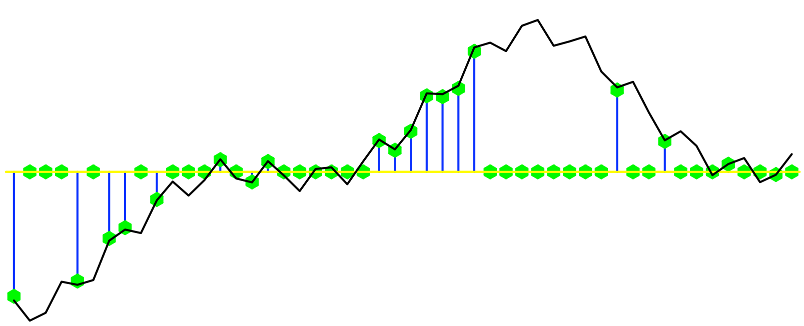


Fourier → transform

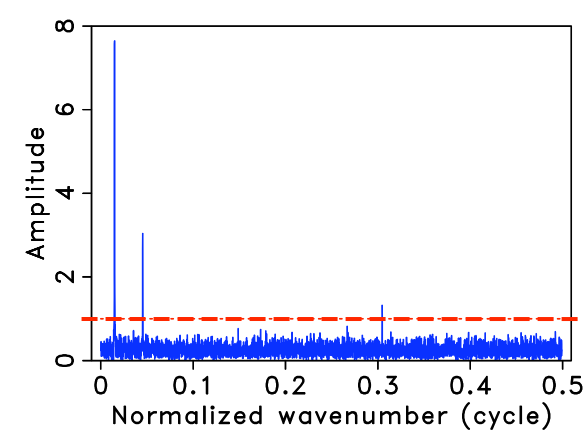


few significant coefficients

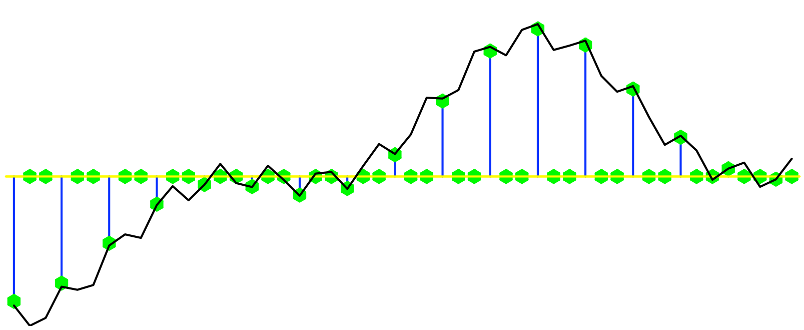
3-fold under-sampling



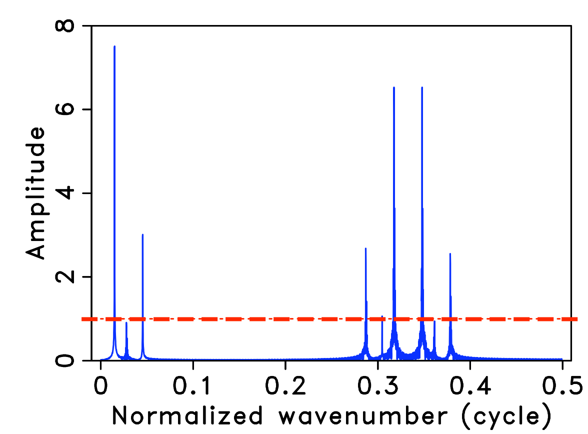
Fourier → transform



✓ significant coefficients detected



Fourier → transform



✗ ambiguity

[Hennenfent & Herrmann, '08]

Sparse one-norm recovery

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Signal model

$$\mathbf{b} = \mathbf{A}\mathbf{x}_0 \quad \text{where} \quad \mathbf{b} \in \mathbb{R}^n$$

and $\mathbf{x}_0$ *k* sparse

Sparse one-norm recovery

$$\tilde{\mathbf{x}} = \arg\min_{\mathbf{x}} ||\mathbf{x}||_1 \stackrel{\text{def}}{=} \sum_{i=1}^{N} |x[i]| \quad \text{subject to} \quad \mathbf{b} = \mathbf{A}\mathbf{x}$$

with $\mathrm{n} \ll N$

Study recovery as a function of

- the subsampling ratio n/N
- “over sampling” ratio k/n

[Sacchi ’98]
[Candès et.al, Donoho, ’06]

The math of Compressive Sensing [Candès et.al, '06]

Recovery is *possible* & *stable* as long as each *subset S* of *k* columns of $\mathbf{A} \in \mathbb{R}^{n \times N}$ with $k \leq N$ the # of nonzeros *approximately* behaves as an orthogonal basis.

In that case, we have

$$(1 - \hat{\delta}_k)\|\mathbf{x}_S\|_{\ell_2}^2 \leq \|\mathbf{A}_S \mathbf{x}_S\|_{\ell_2}^2 \leq (1 + \hat{\delta}_k)\|\mathbf{x}_S\|_{\ell_2}^2,$$

where *S* runs over all sets with cardinality $\leq k$

- the smaller the *restricted isometry constant* (RIP) $\hat{\delta}_k$ the more *energy* is captured and the more *stable* the *inversion* of **A**
- determined by the *mutual coherence* of the cols in **A**

Let's adapt this theory to seismic acquisition and processing

Key elements

- *sparsifying transform*
  - typically **localized** in the time-space domain to handle the complexity of seismic data
- *advantageous coarse randomized sampling*
  - generates incoherent random undersampling "noise" in the sparsifying domain
- *sparsity-promoting solver*
  - requires few matrix-vector multiplications

Extension

Extend CS framework:

$$\mathbf{A} := \mathbf{R}\mathbf{M}\mathbf{S}^H$$

restriction matrix   measurement matrix   sparsity matrix

Expected to perform well when

$$\mu = \max_{1 \le i \ne j \le N} |\left(\mathbf{R}\mathbf{M}\mathbf{s}^i\right)^H \mathbf{R}\mathbf{M}\mathbf{s}^j|$$

Generalizes to *redundant* transforms for cases where

- max of RIP constants for **M**, **S** are small [Rauhut et.al, '06]
- or $\mathrm{S}\mathrm{S}^H\mathrm{x}$ remains sparse for **x** sparse [Candès et.al, '10]

Open research topic...

Empirical performance analysis

Selection of the appropriate sparsifying transform

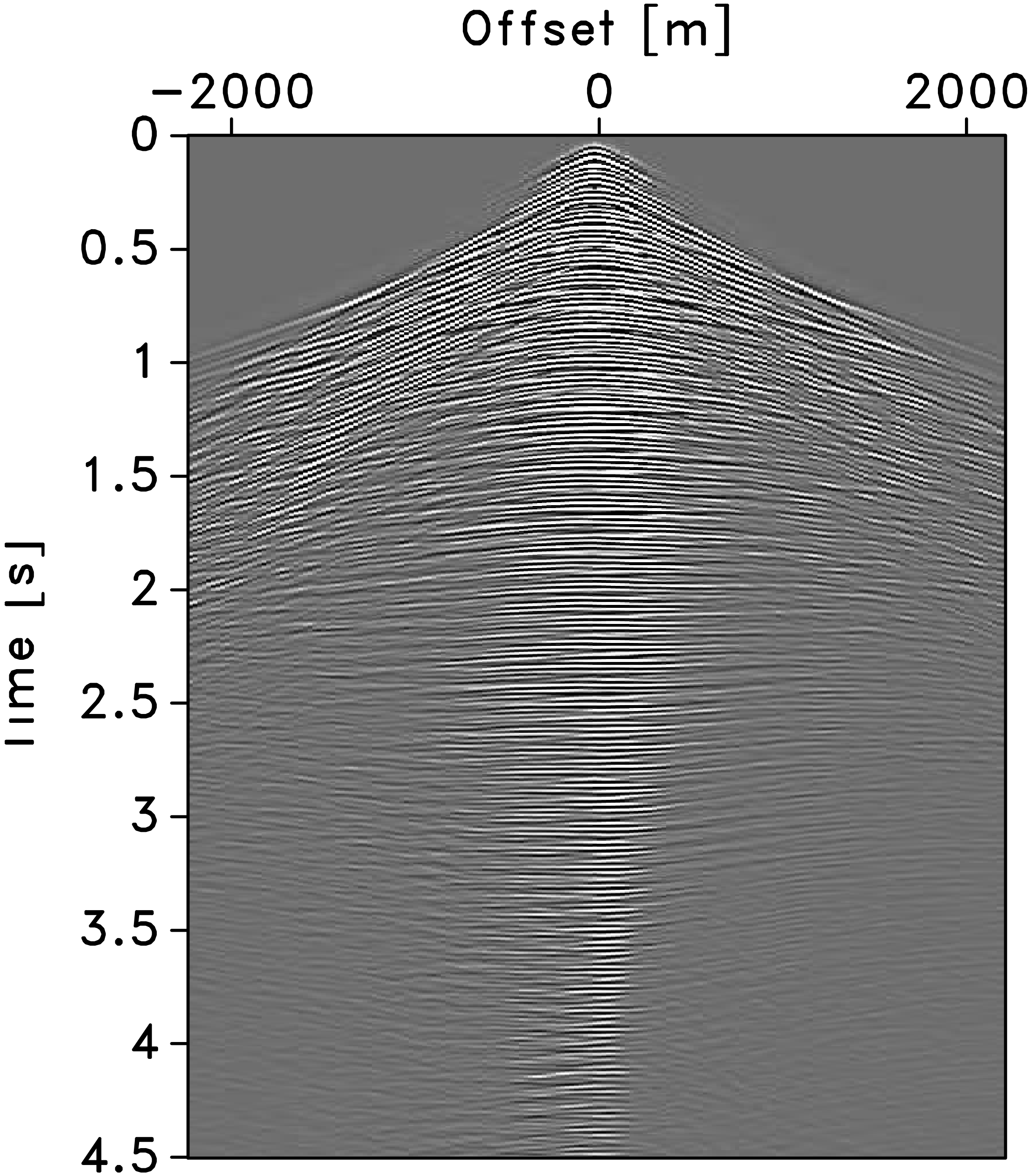
- nonlinear approximation error

$$\mathrm{SNR}(\rho) = -20\log\frac{\|\mathbf{f}-\mathbf{f}_\rho\|}{\|\mathbf{f}\|} \quad \text{with} \quad \rho = k/P$$

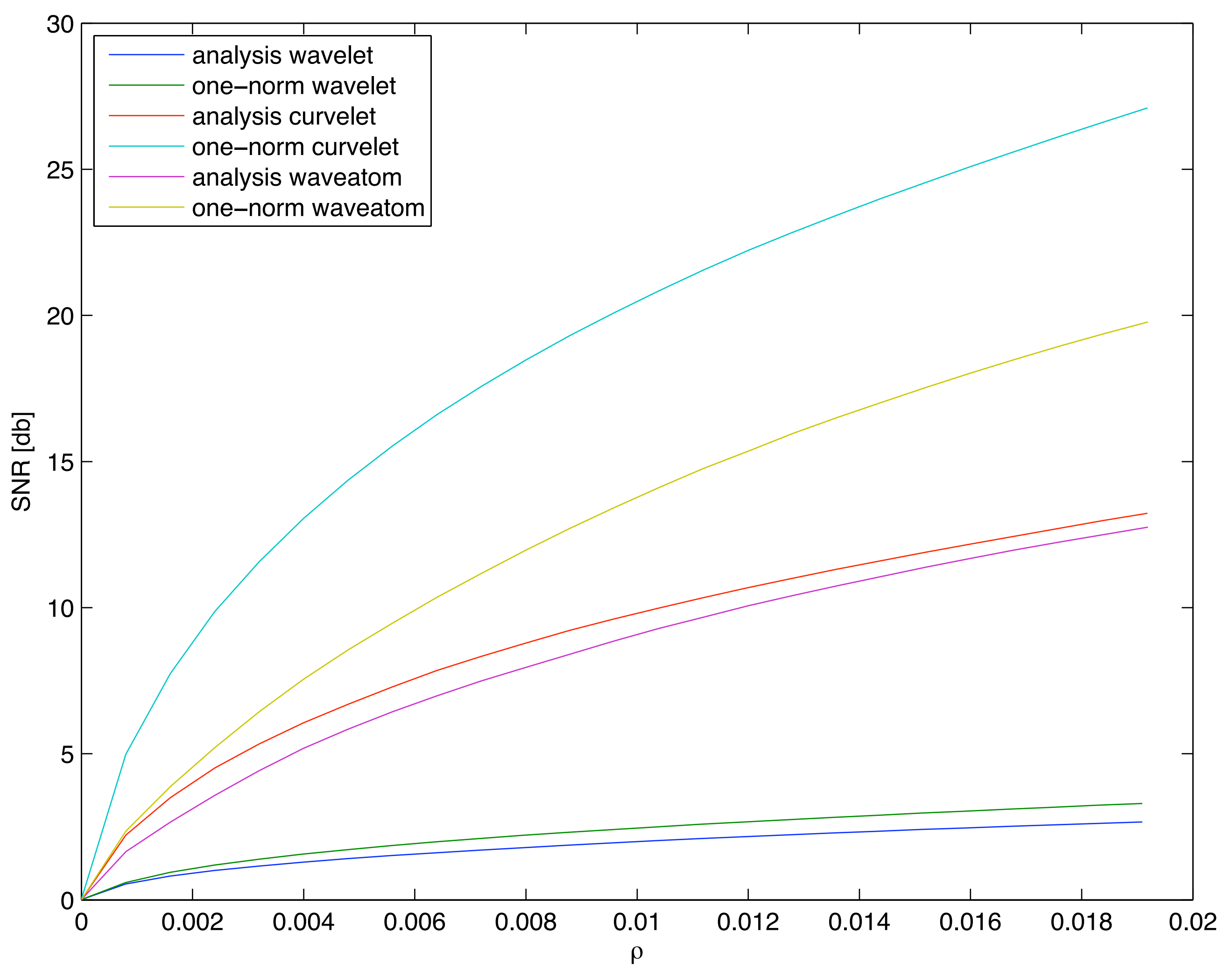
[FJH, '10]

Nonlinear approximation error

common receiver gather

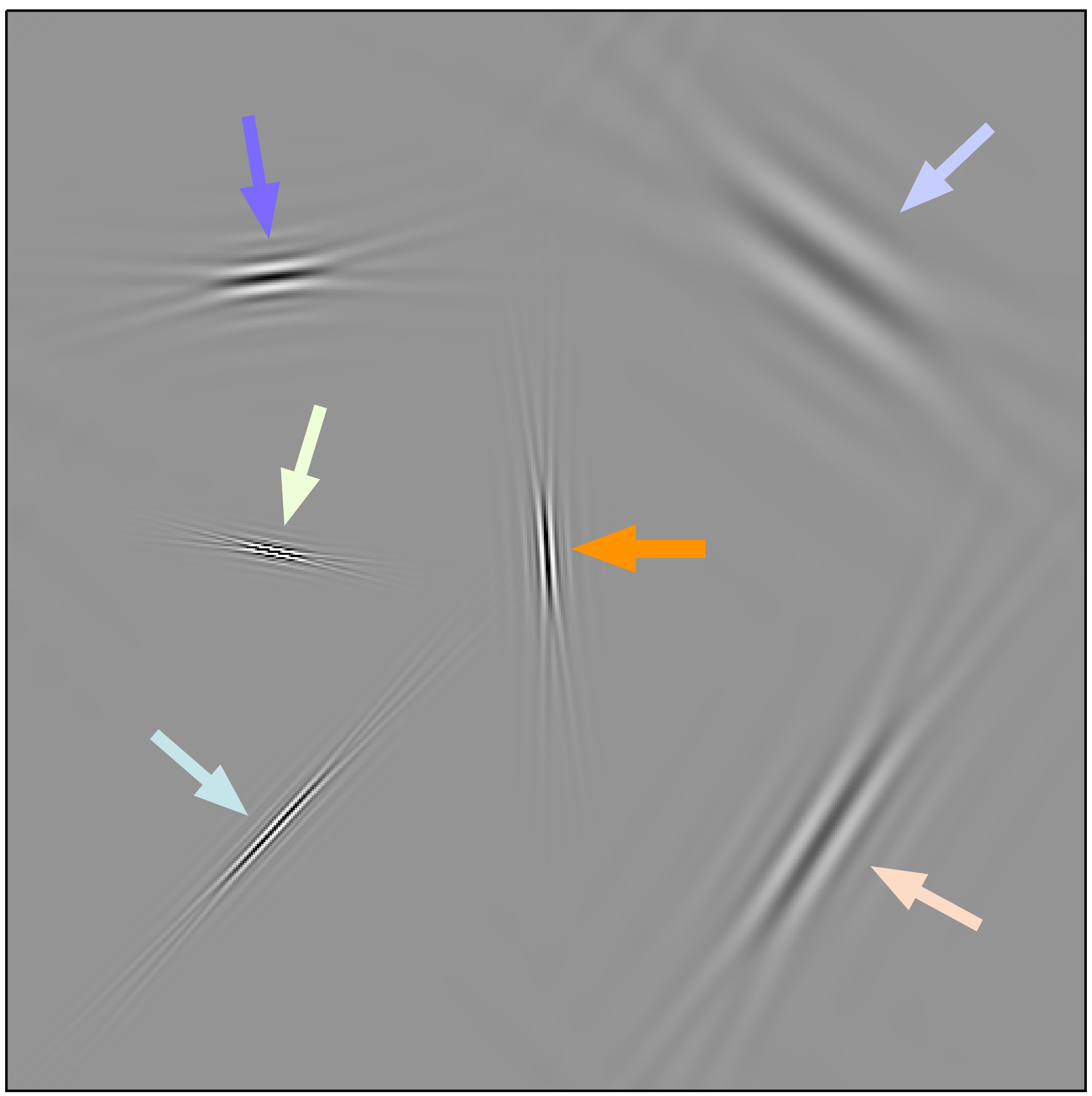


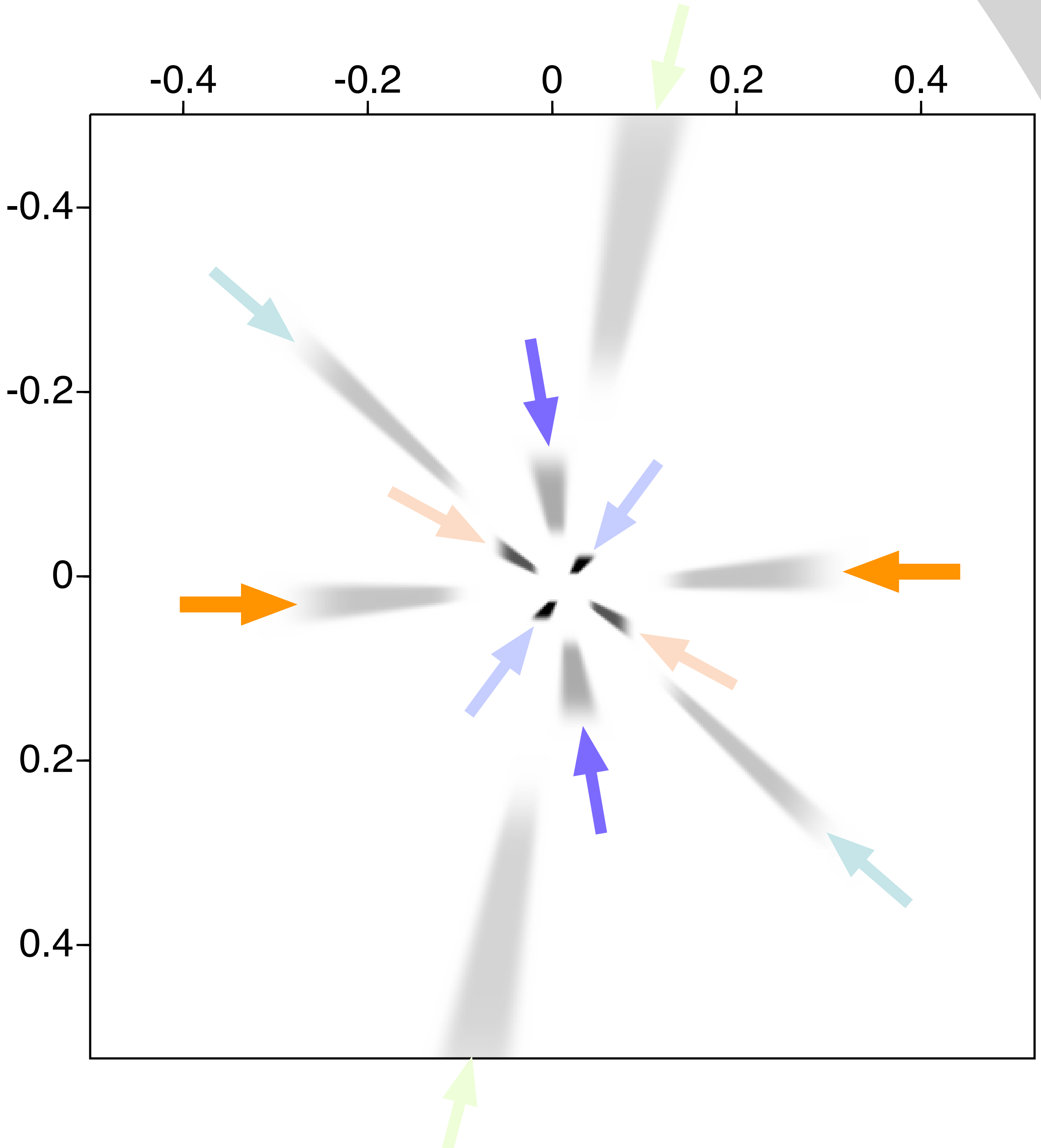
recovery error



[Demanet et. al., '06]

Curvelets





Key elements

☑ *sparsifying transform*

- typically **localized** in the time-space domain to handle the complexity of seismic data
- **curvelets**

☐ *advantageous coarse sampling*

- generates incoherent random undersampling "noise" in the sparsifying domain

☐ *sparsity-promoting solver*

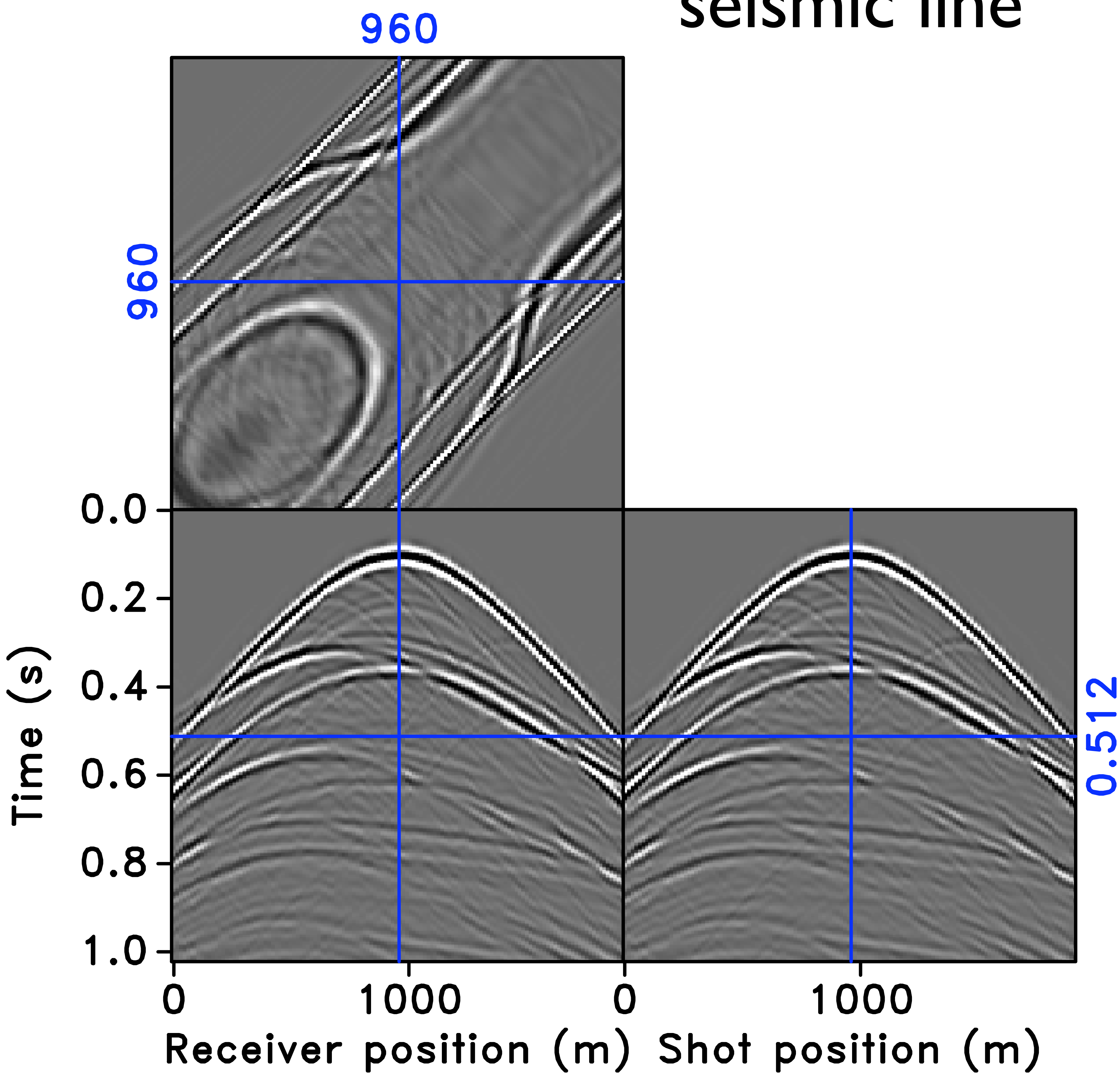
- requires few matrix-vector multiplications

Case study I

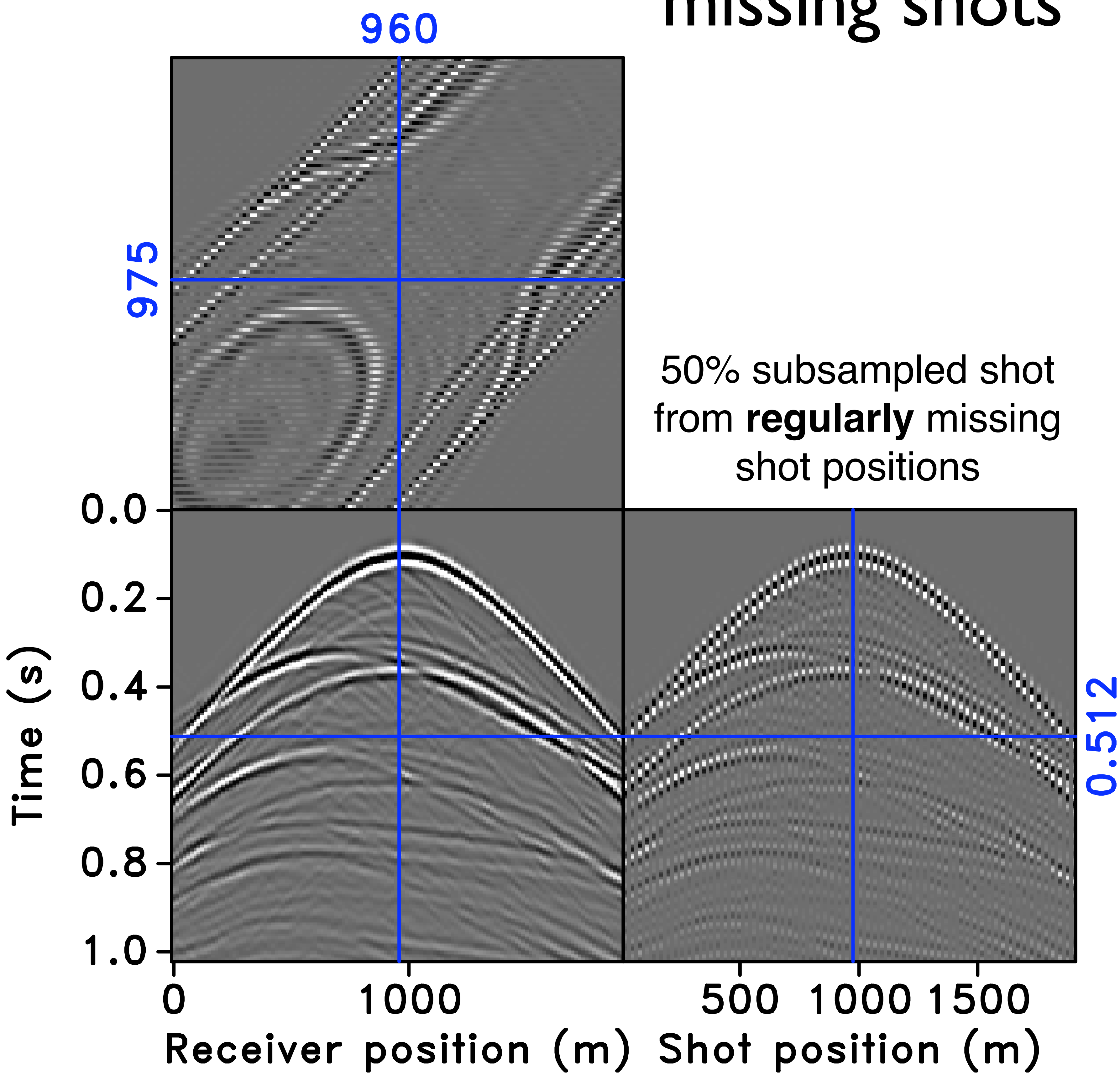
Acquisition design according to Compressive Sensing

- ***Periodic*** *subsampling* vs ***randomized jittered*** sampling of ***sequential*** sources in 2- and 3-D

seismic line

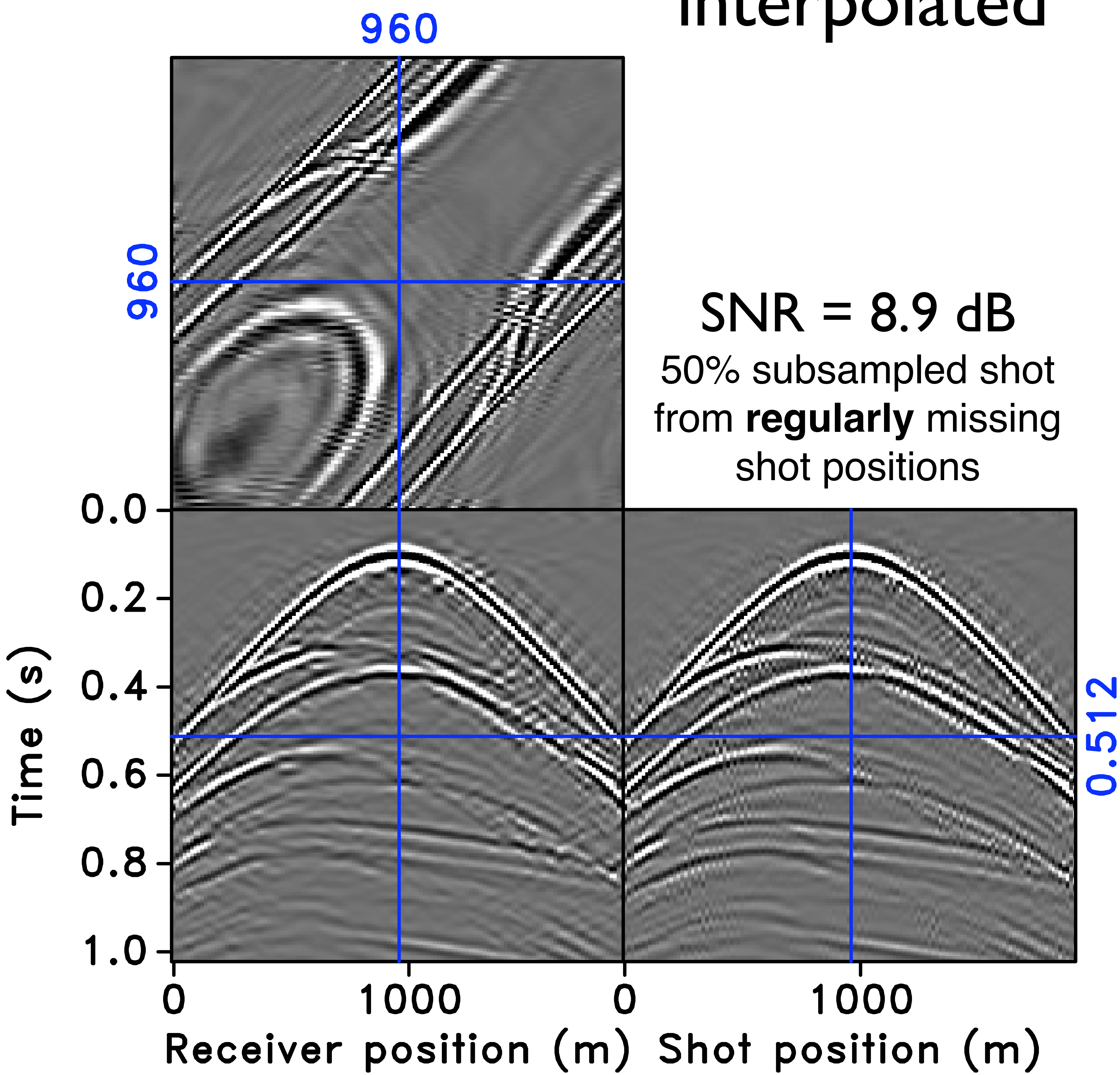


missing shots



50% subsampled shot from **regularly** missing shot positions

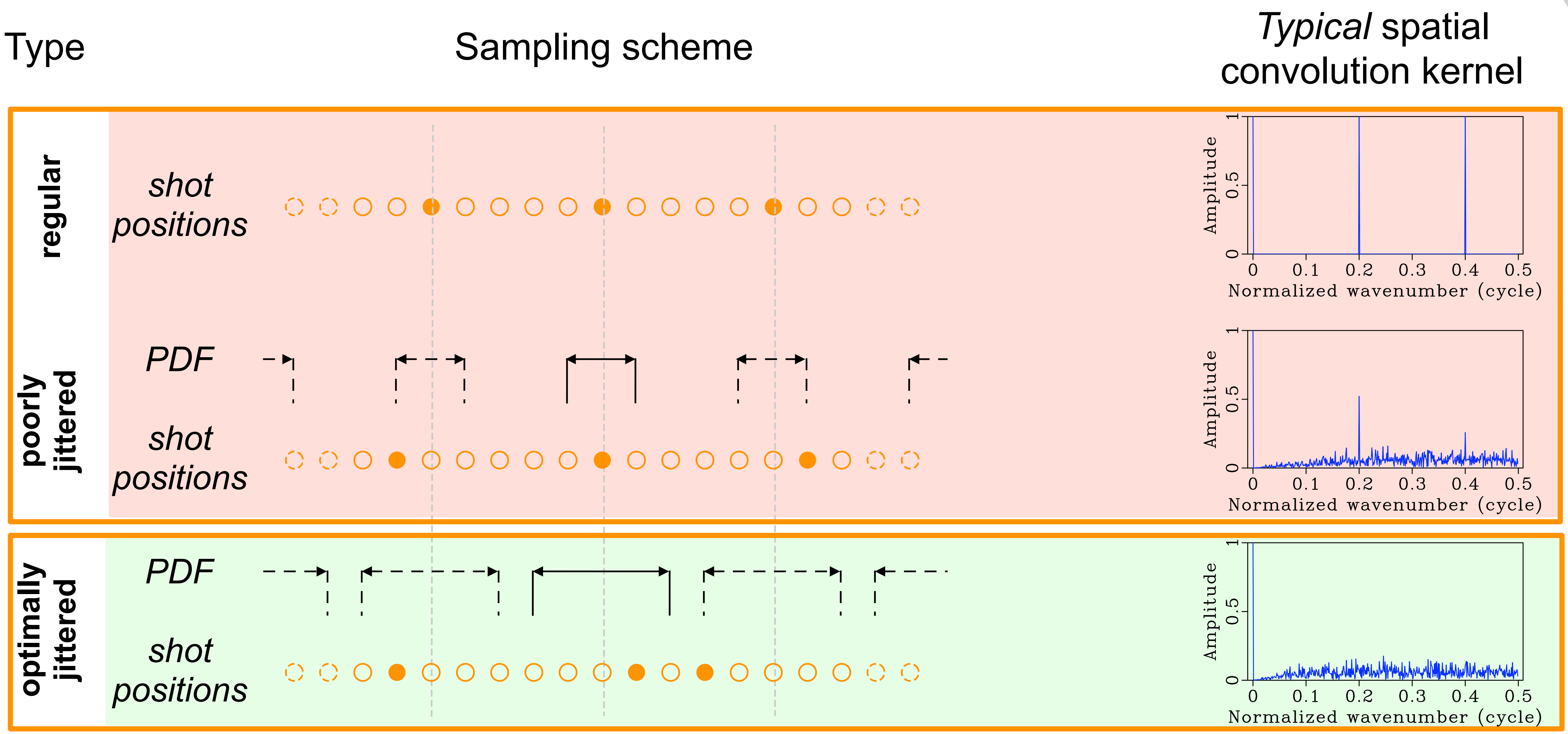
interpolated



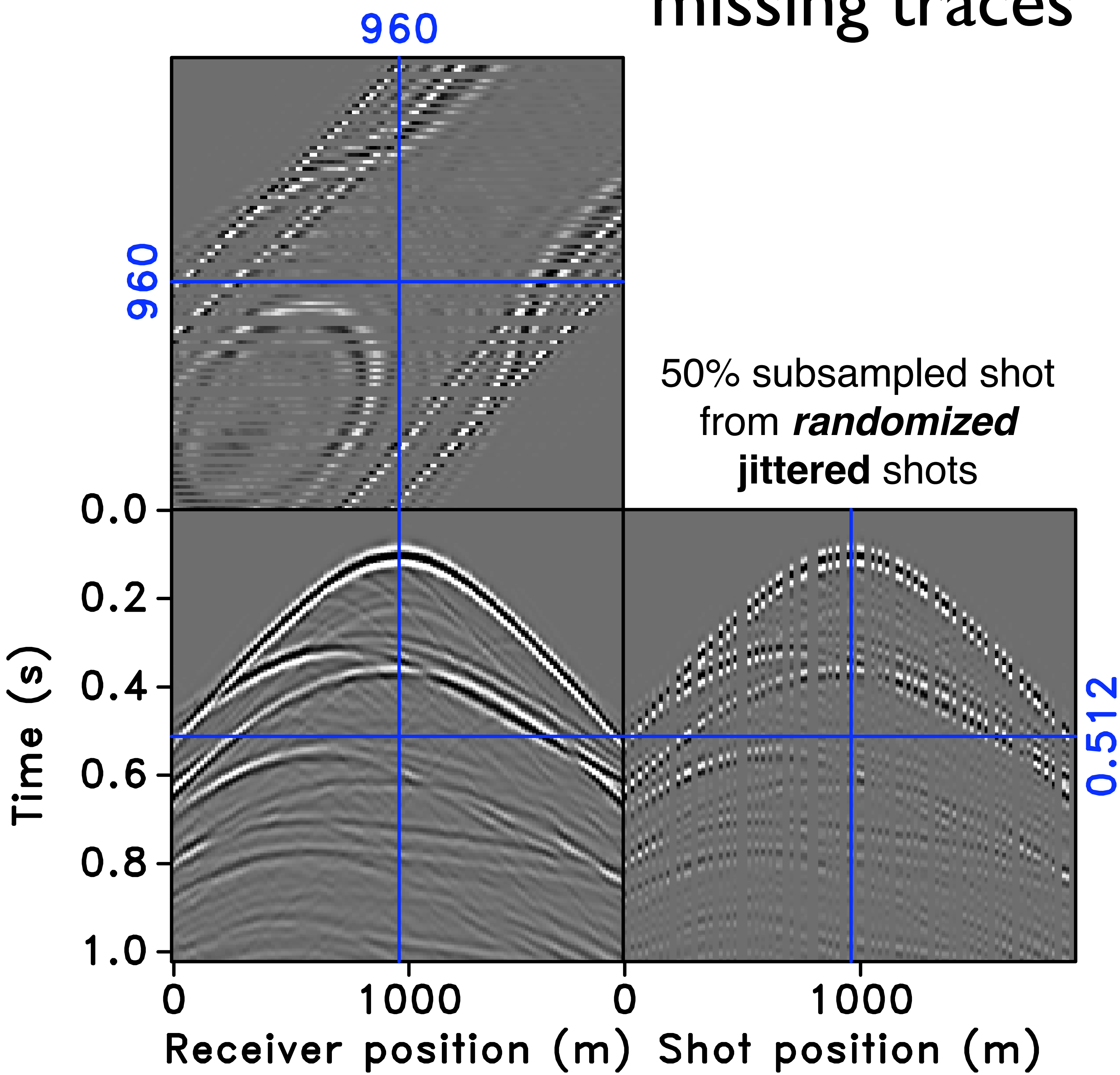
SNR = 8.9 dB

50% subsampled shot from **regularly** missing shot positions

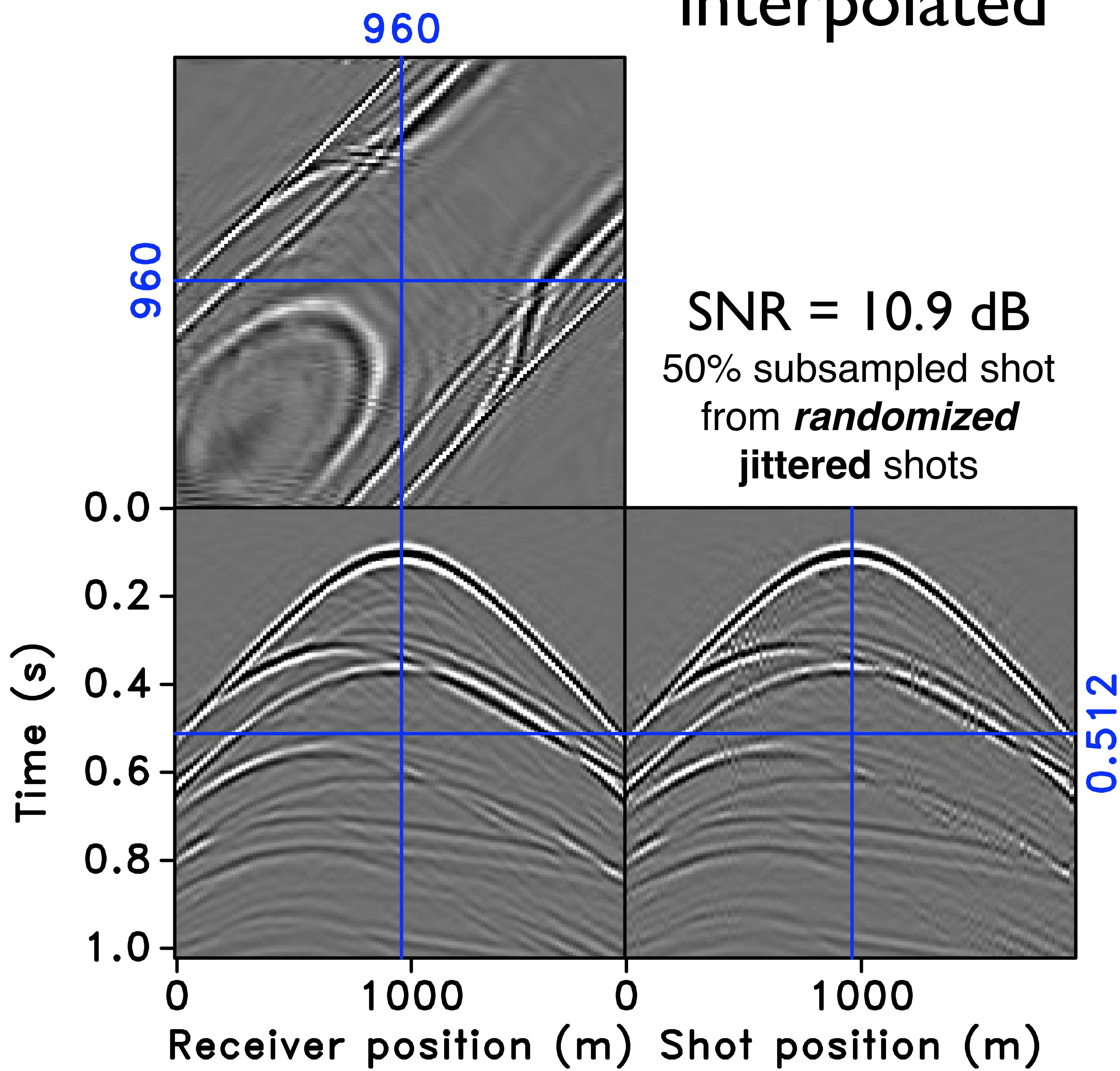
Jittered sampling



missing traces



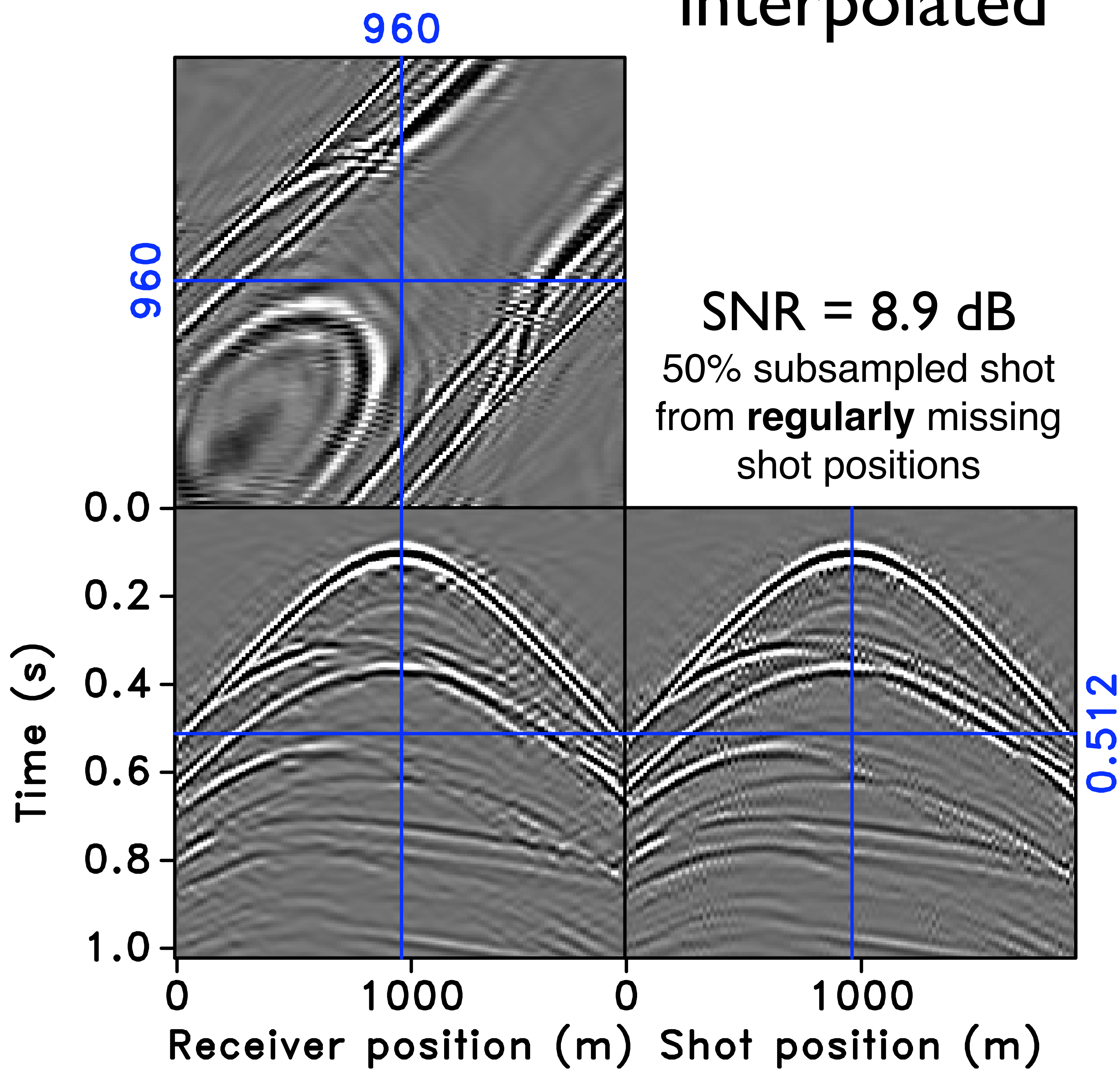
interpolated



SNR = 10.9 dB

50% subsampled shot
from ***randomized***
jittered shots

interpolated



SNR = 8.9 dB

50% subsampled shot from **regularly** missing shot positions

1 & 2-D jittered samplings

[Tang et. al., '09-'10]

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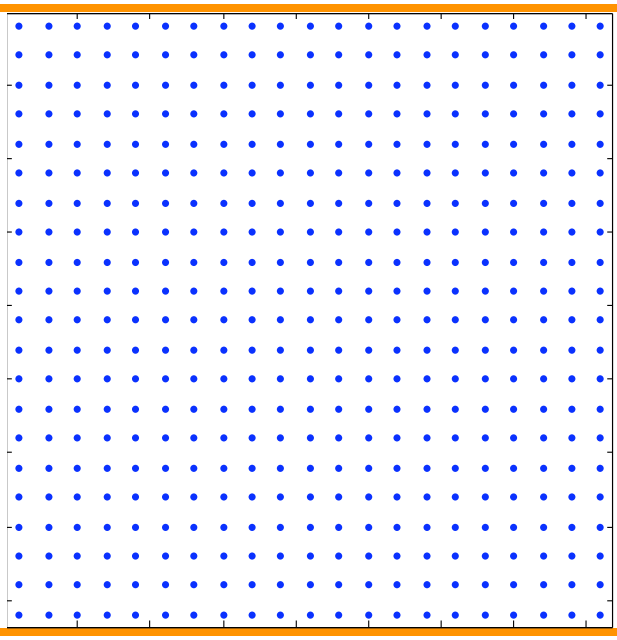
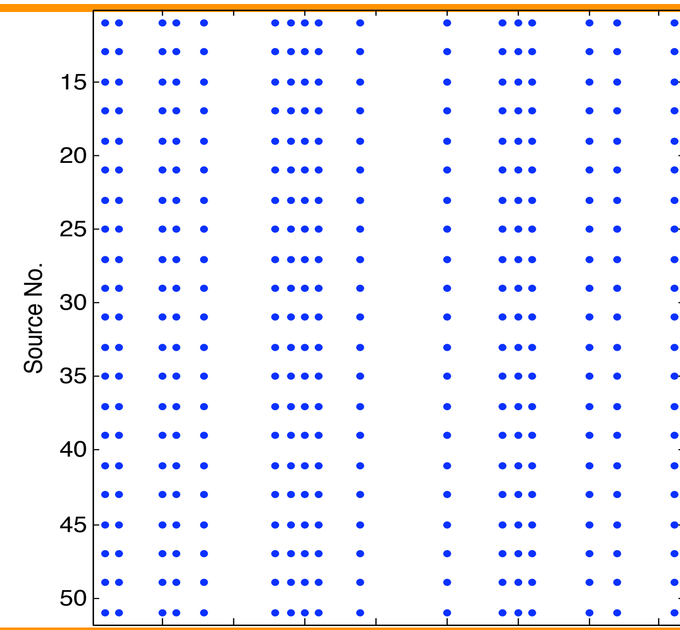
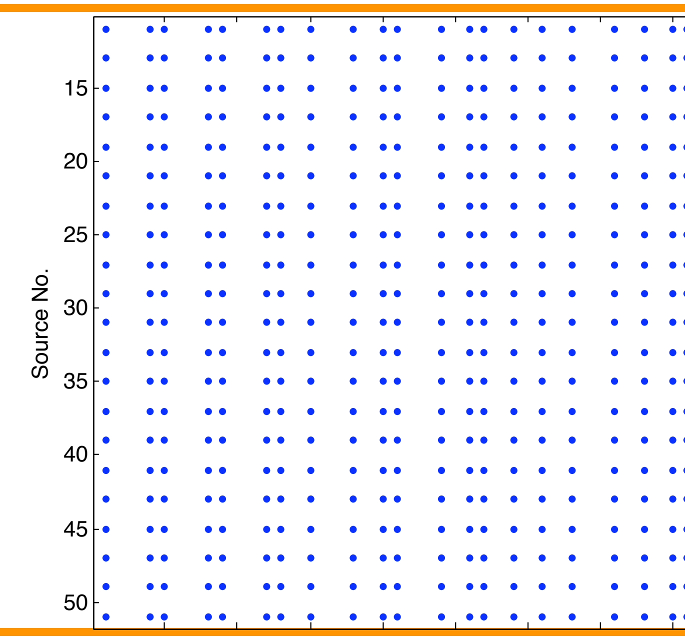
regular



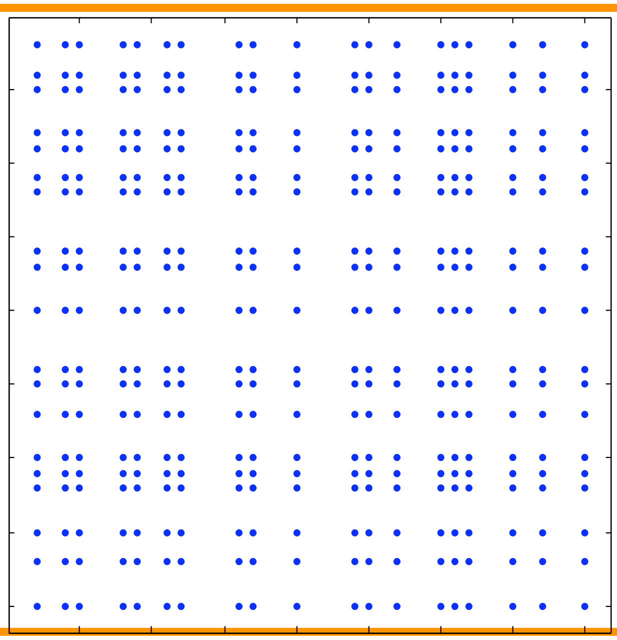
uniform



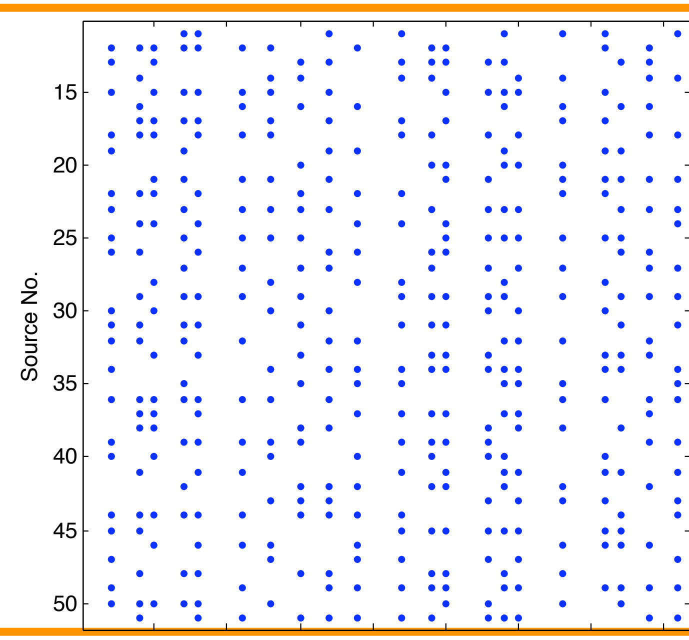
jittered



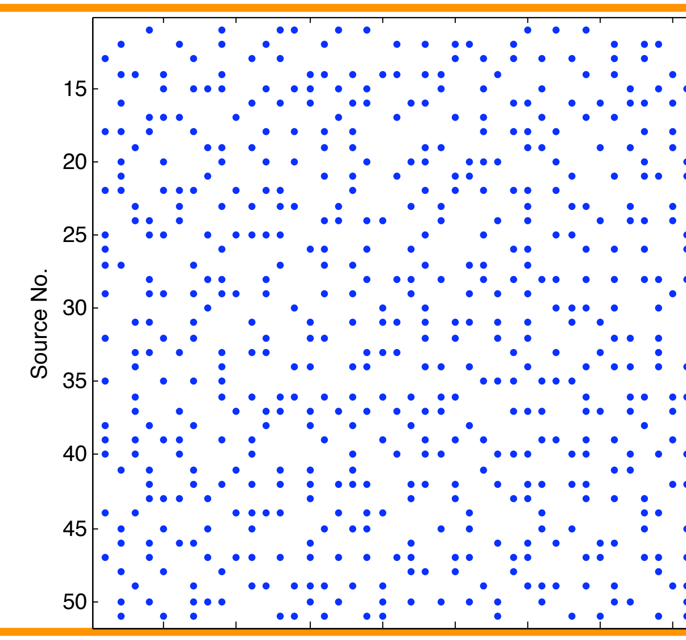
separable 2d jittered



non-seperable 2d jittered



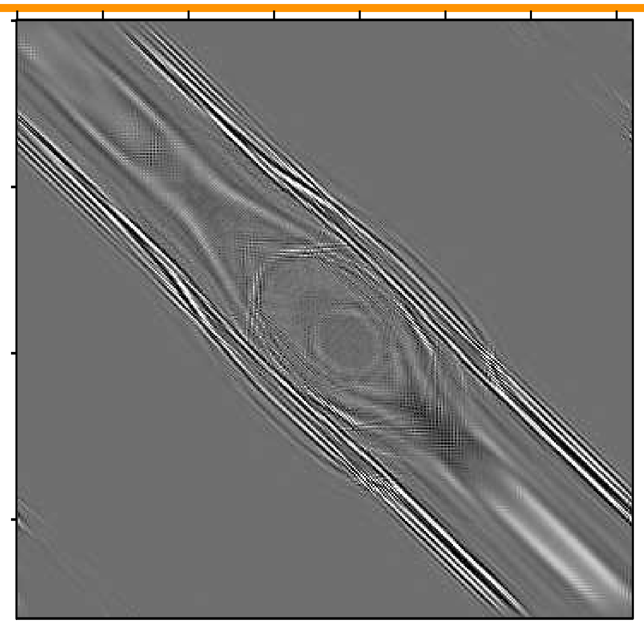
fully 2d jittered



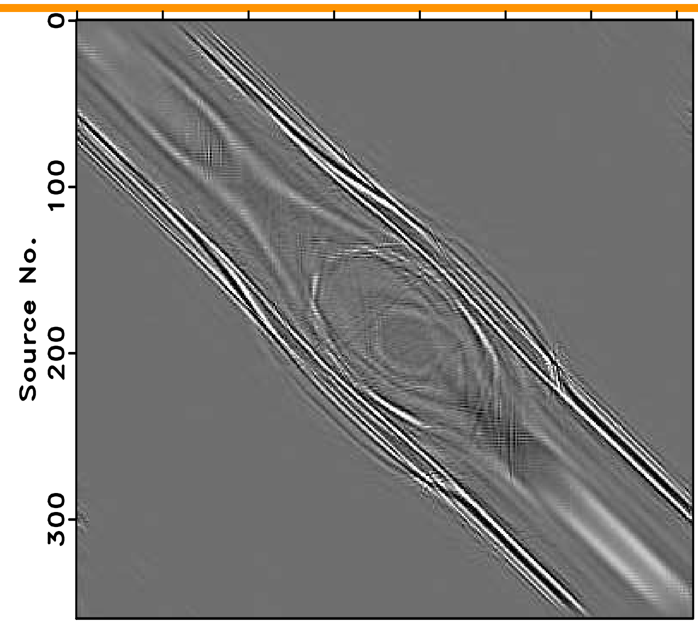
Spectra become increasingly *"blue"*

Recovery from 1-2D jittered samplings (25%)

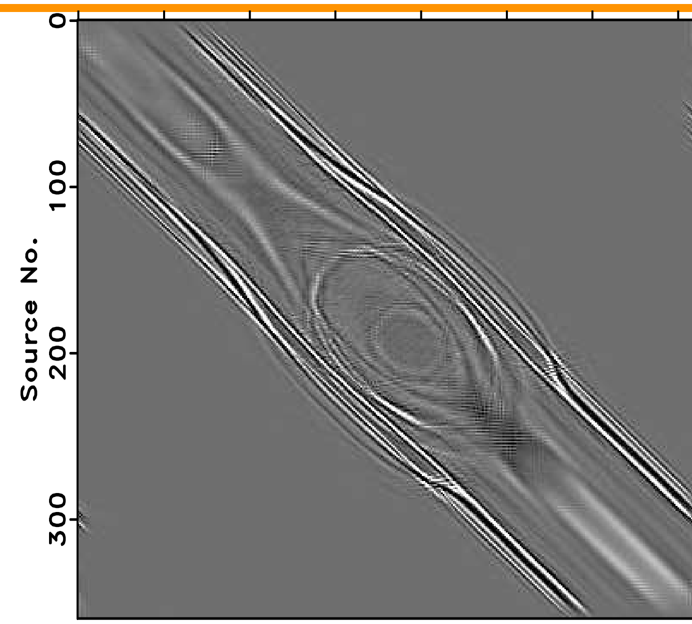
regular
(3.91 dB)



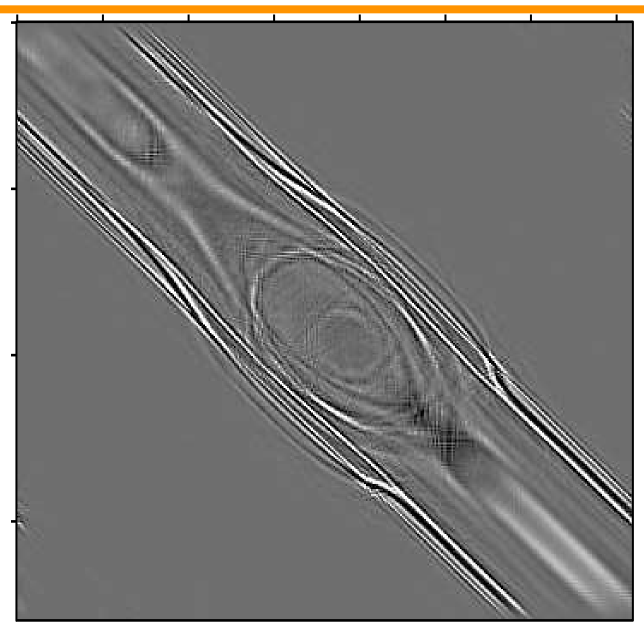
uniform
(7.20 dB)



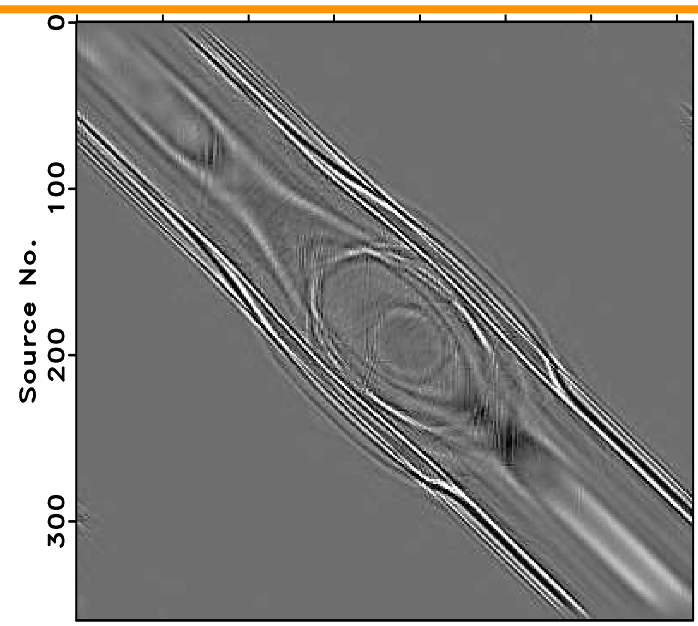
jittered
(8.94 dB)



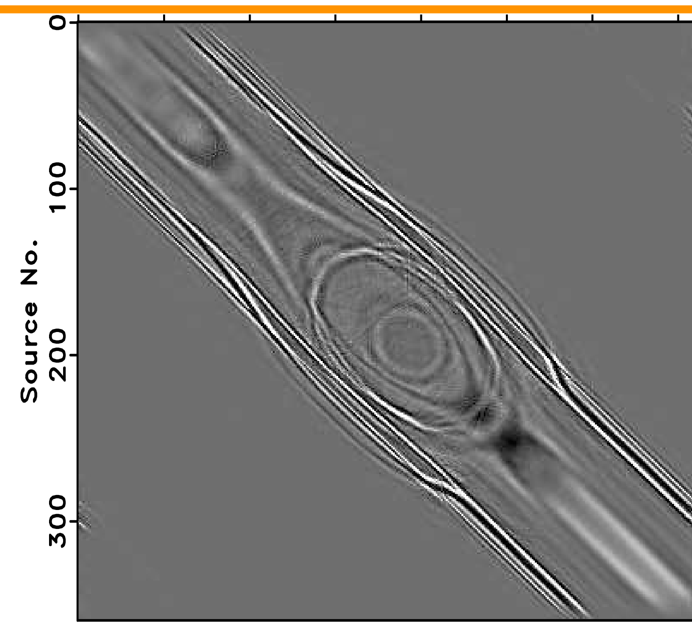
separable
2d jittered (9.45 dB)



non-seperable
2d jittered (10.03 dB)



fully 2d
jittered (10.86 dB)



Spectra become increasingly *"blue"*

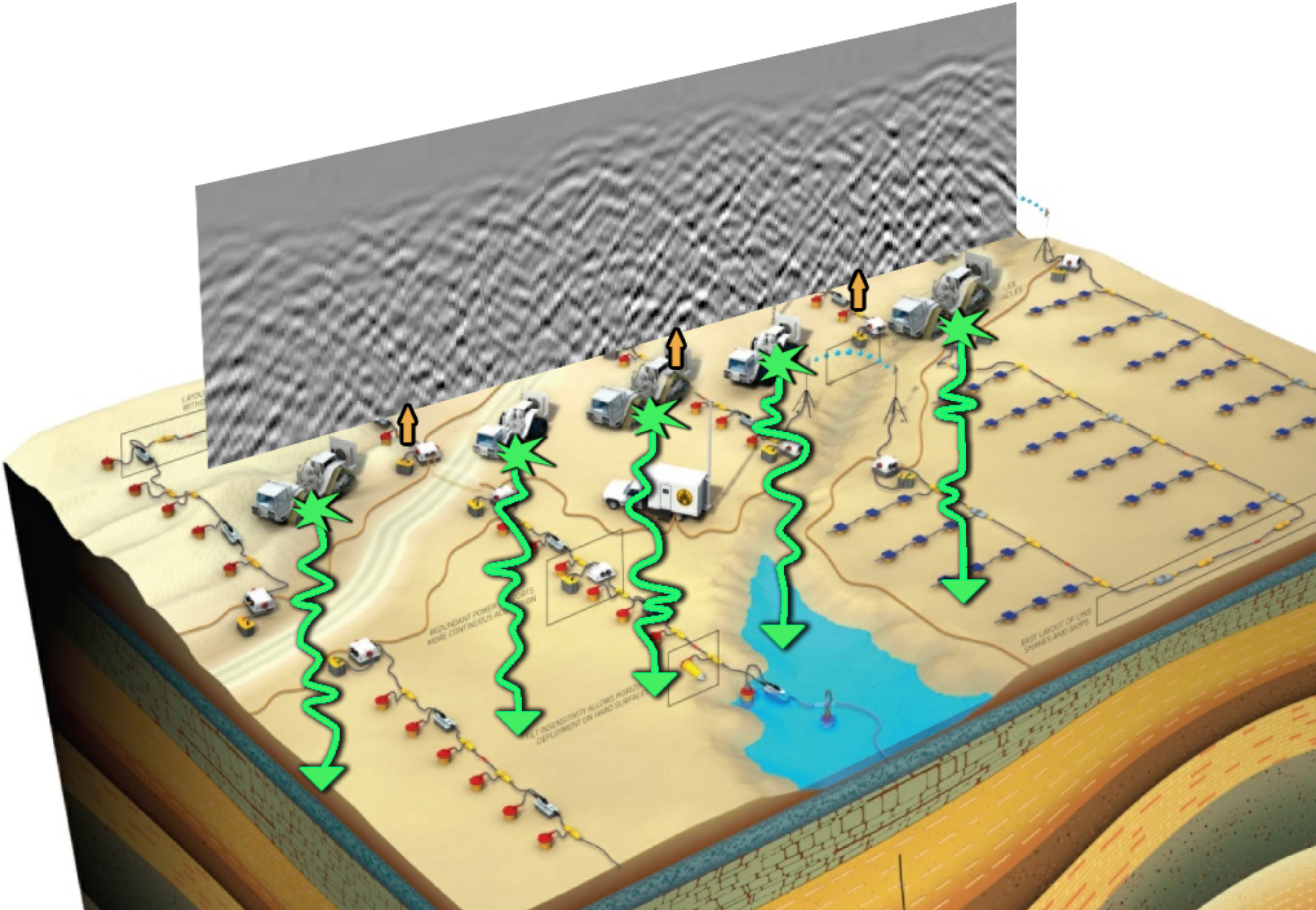
Case study II

[Beasley et. al., '98]
[Berkhout '08]
[Herrmann '09-'10]

Acquisition design according to Compressive Sensing

- *Subsampling with randomized jittered* ***sequential*** sources vs *randomized phase-encoded* ***simultaneous*** sources

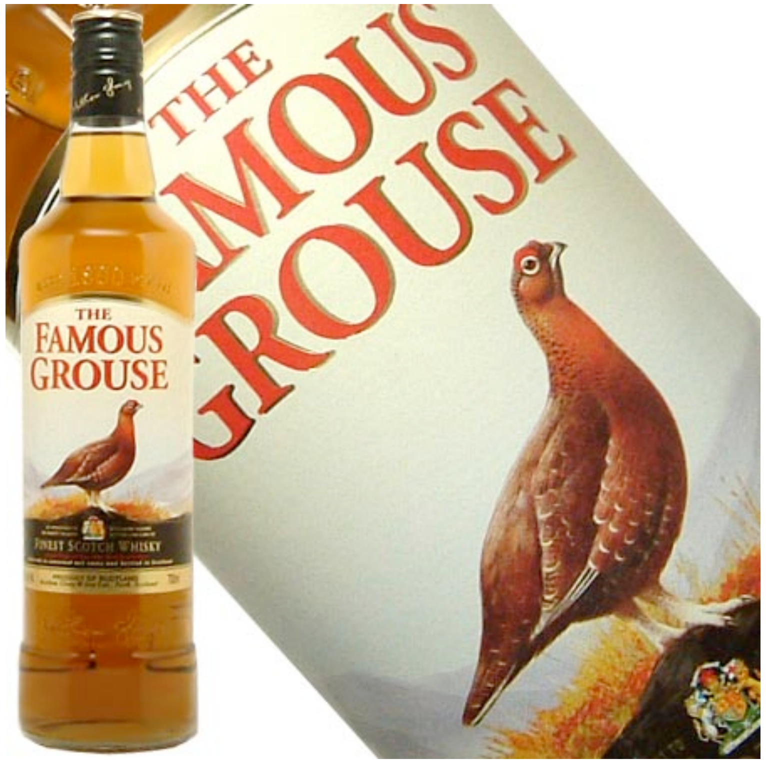
Simultaneous & incoherent sources



Unblending/ Demultiplexing

✓

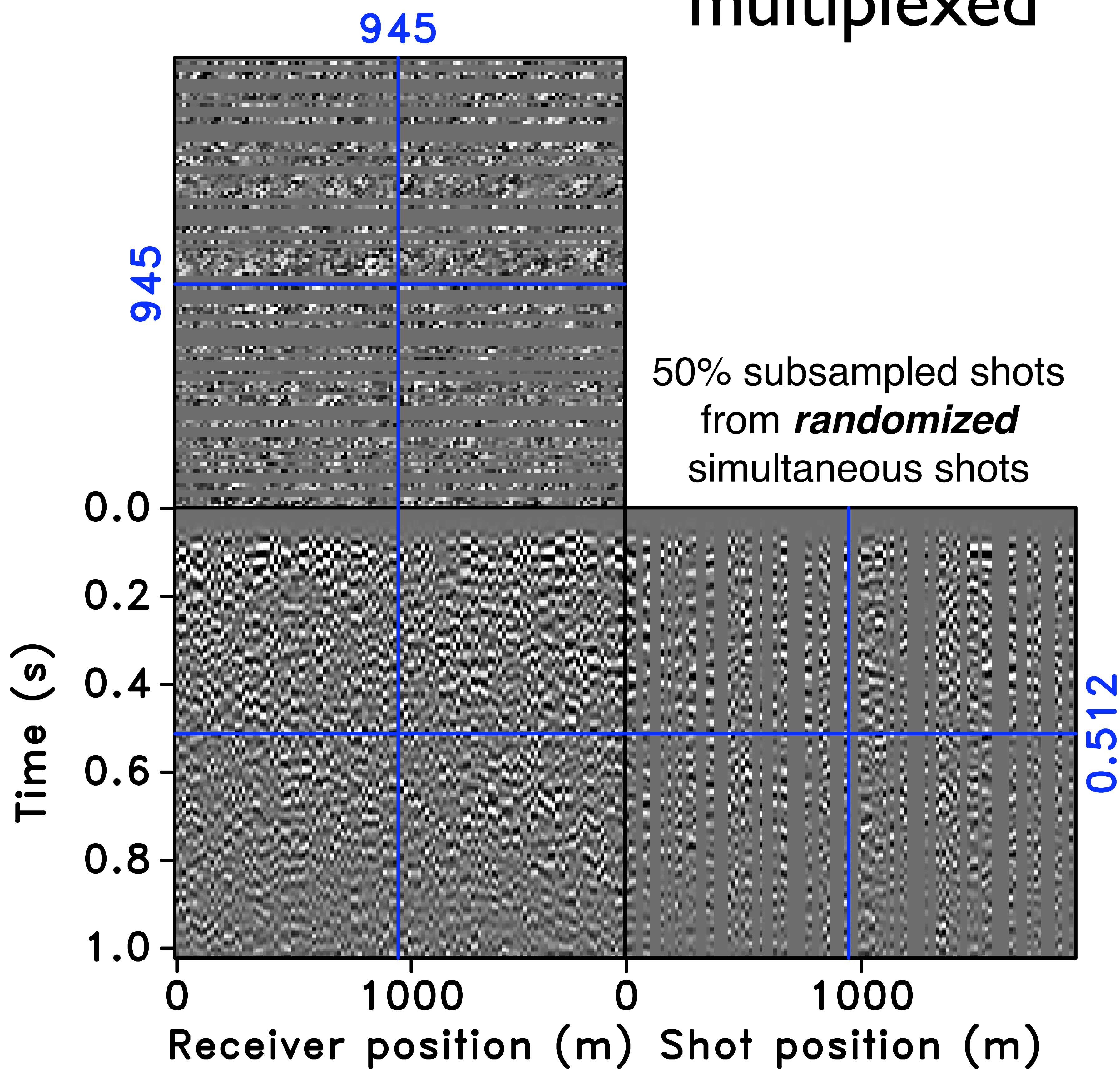
?



$$$$$$$$$$ $

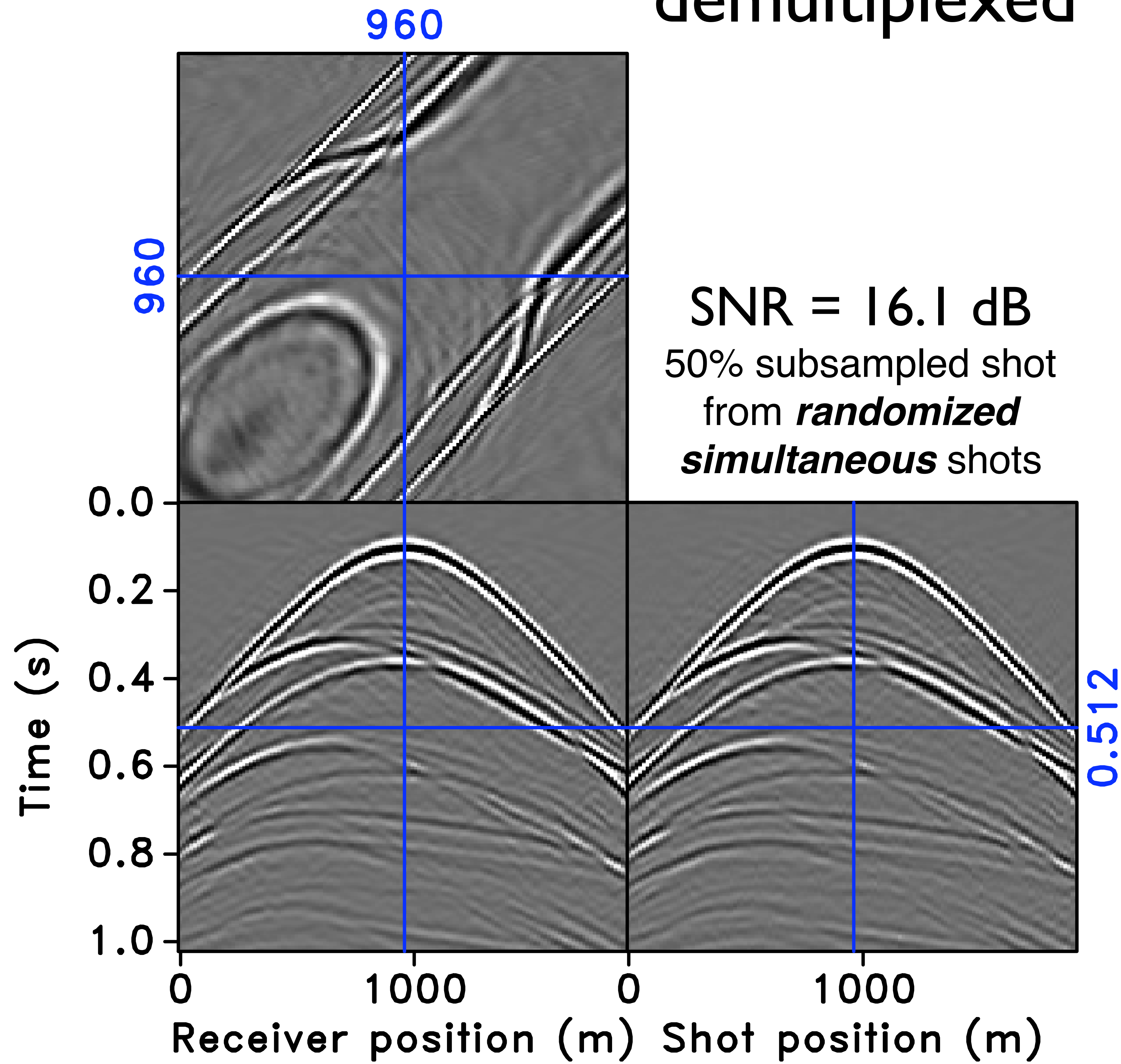
Blending versus *unblending* ...

multiplexed



50% subsampled shots
from ***randomized***
simultaneous shots

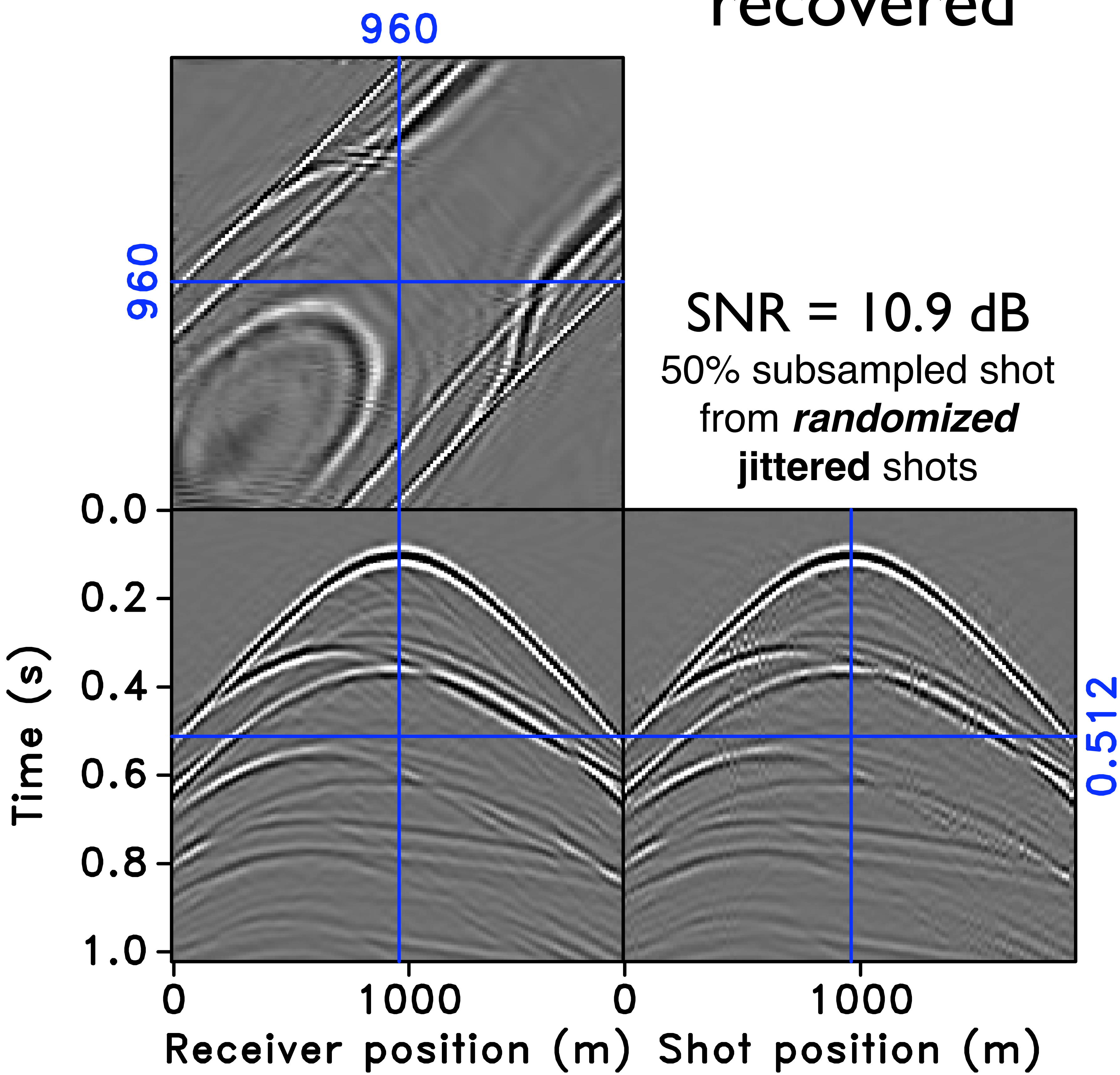
demultiplexed



SNR = 16.1 dB

50% subsampled shot from ***randomized simultaneous*** shots

recovered



SNR = 10.9 dB

50% subsampled shot from ***randomized*** **jittered** shots

Empirical performance analysis

Selection of the appropriate sparsifying transform

- nonlinear approximation error

$$\mathrm{SNR}(\rho) = -20\log\frac{\|\mathbf{f}-\mathbf{f}_\rho\|}{\|\mathbf{f}\|} \quad \text{with} \quad \rho = k/P$$

- recovery error

$$\mathrm{SNR}(\delta) = -20\log\frac{\|\mathbf{f}-\tilde{\mathbf{f}}_\delta\|}{\|\mathbf{f}\|}$$

wi[redacted] $\delta =$

[FJH, '10]

Multiple experiments

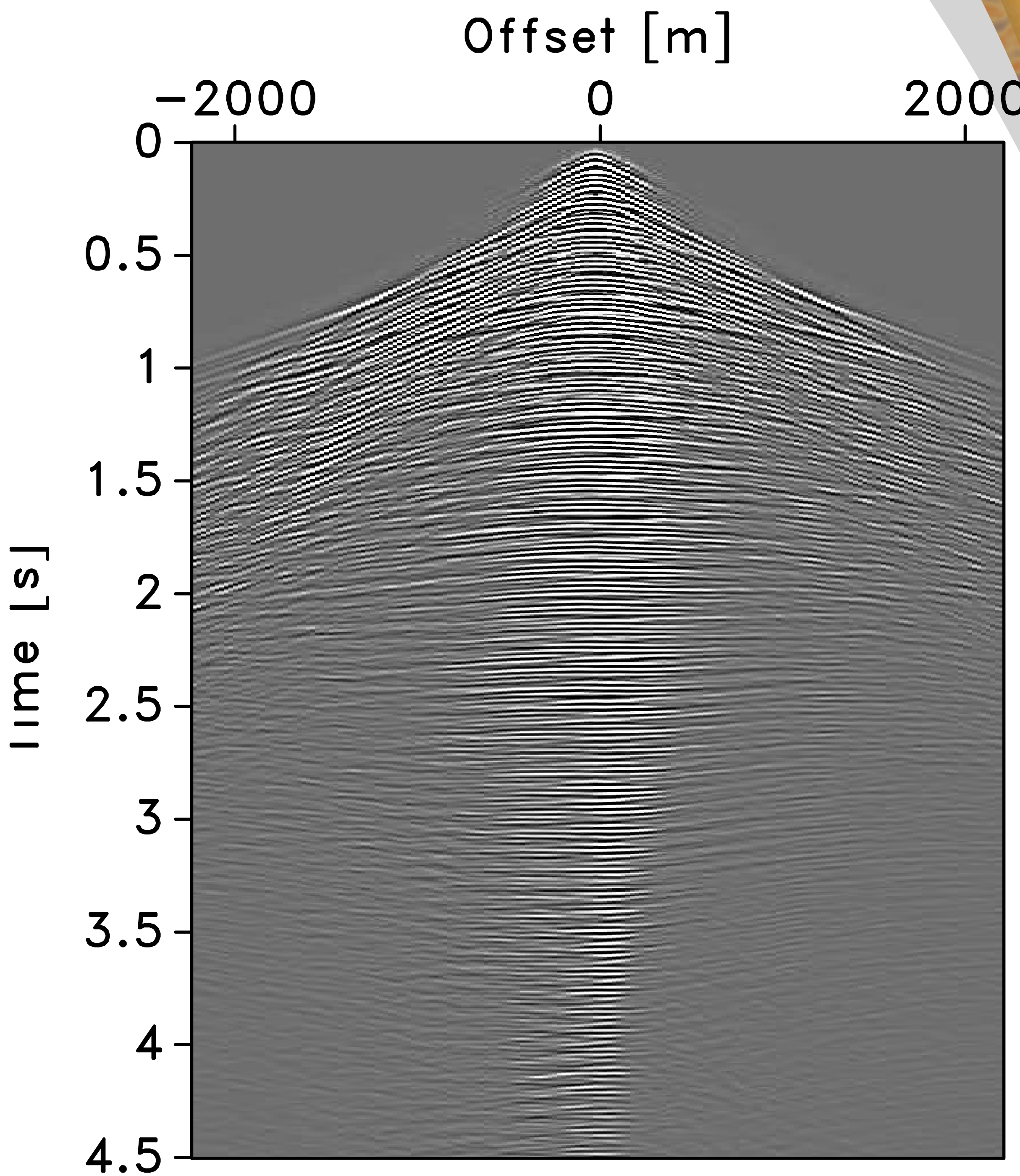
Generate 25 random experiments for varying subsampling ratios:

- sequential sources
- simultaneous sources

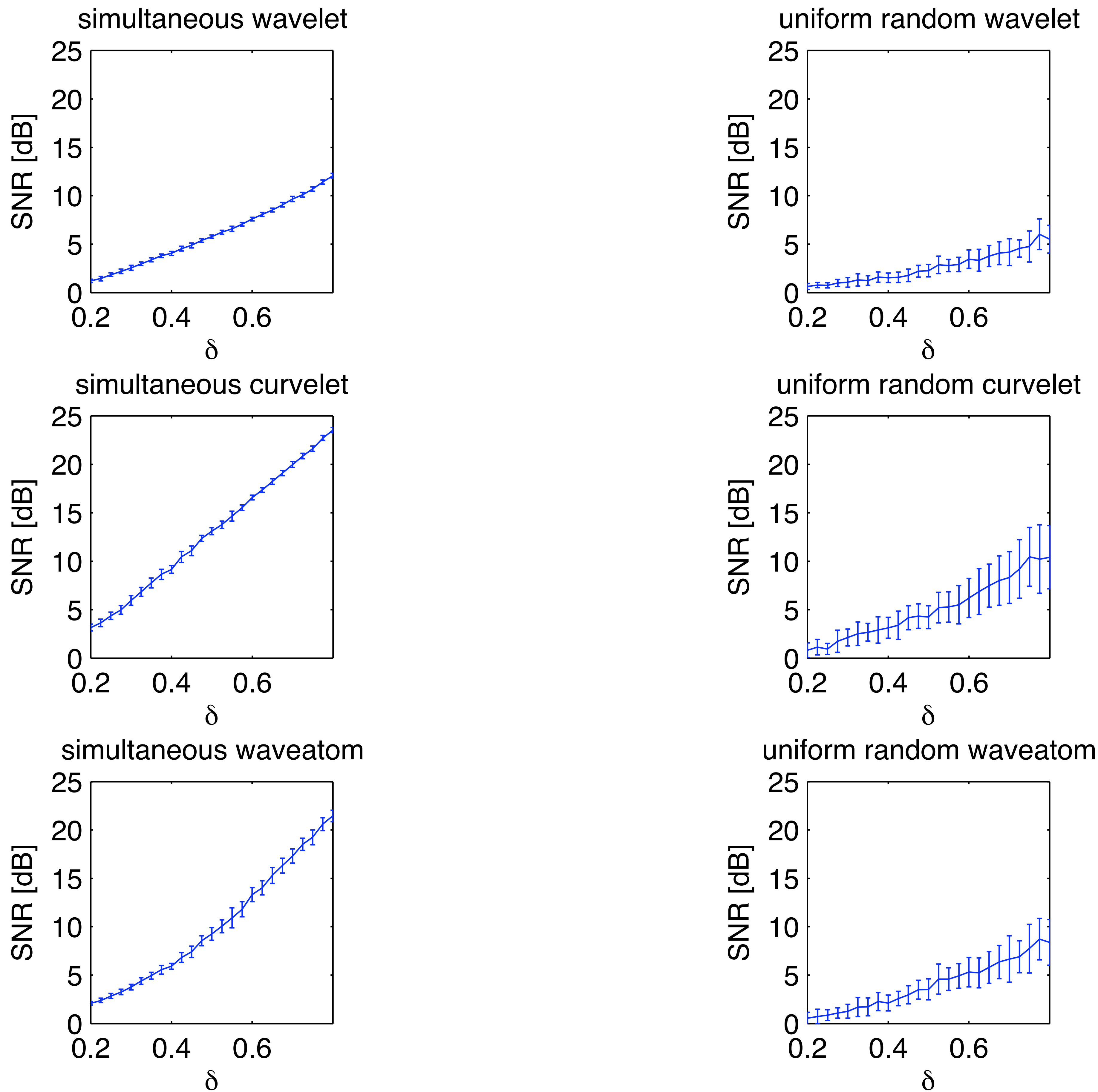
Study recovery errors & oversampling ratios for

- Wavelets
- Curvelets
- Waveatoms

common receiver gather



Multiple experiments



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- recovery error

$$\mathrm{SNR}(\delta) = -20\log\frac{\|\mathbf{f}-\tilde{\mathbf{f}}_\delta\|}{\|\mathbf{f}\|}$$

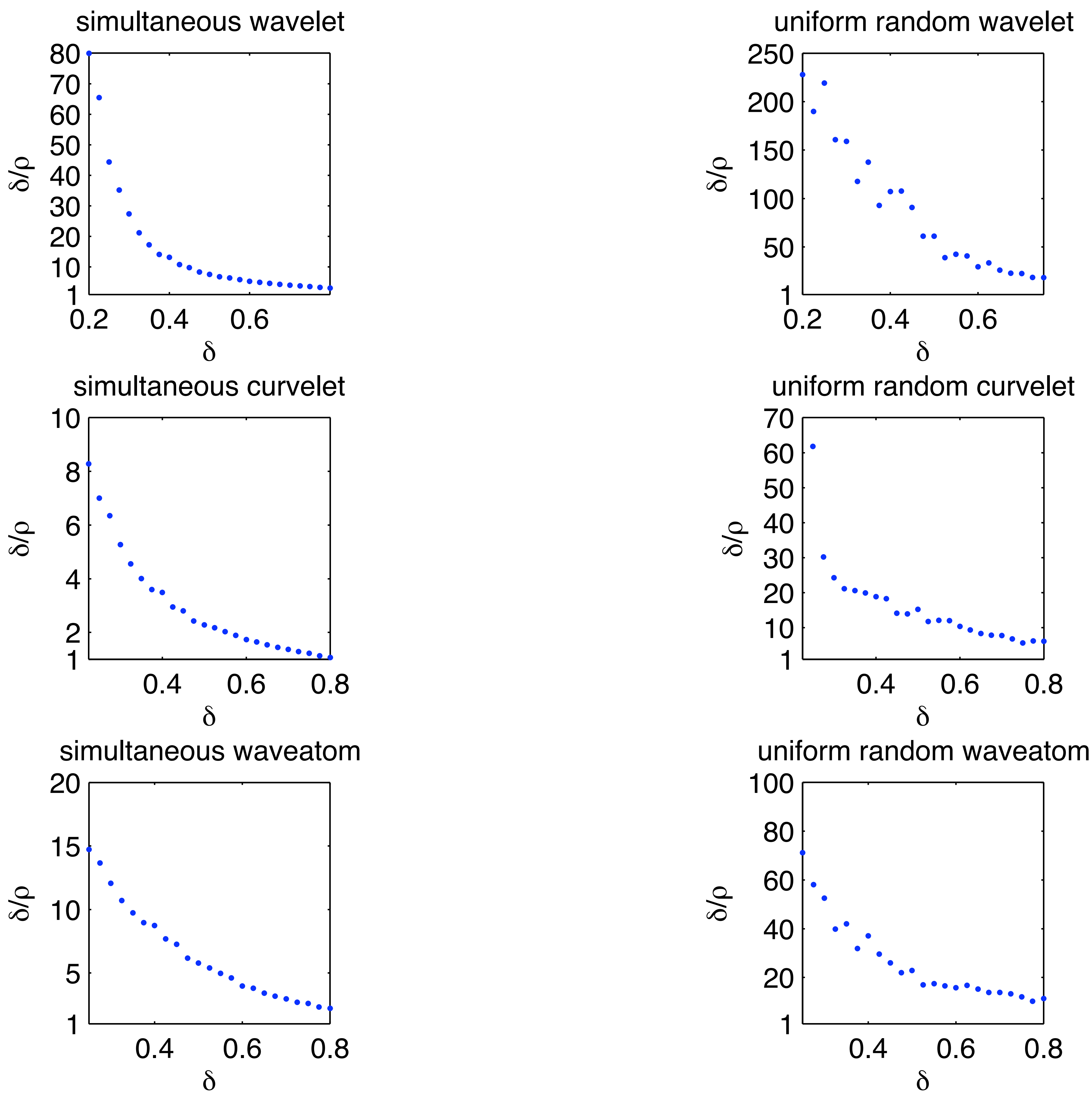
wi[redacted] $\delta =$

- oversampling ratio

$$\delta/\rho \quad \text{with} \quad \rho = \inf\{\tilde{\rho} : \quad \overline{\mathrm{SNR}}(\delta) \leq \mathrm{SNR}(\tilde{\rho})\}$$

[FJH, '10]

Oversampling ratios



Key elements

- [x] *sparsifying transform*

    - typically **localized** in the time-space domain to handle the complexity of seismic data
    - **curvelets**

- [x] *advantageous coarse sampling*

    - generates incoherent random undersampling "noise" in the sparsifying domain
    - does not create large gaps for measurement in the physical domain
    - does not create coherent interferences in simultaneous acquisition

- [ ] *sparsity-promoting solver*

    - requires few matrix-vector multiplications

Reality check

"When a traveler reaches a fork in the road, the l_1-norm tells him to take either one way or the other, but the l_2 -norm instructs him to head off into the bushes."

John F. Claerbout and Francis Muir, 1973

[van den Berg & Friedlander, ’08]
[Hennenfent, FJH, et. al, ‘08]

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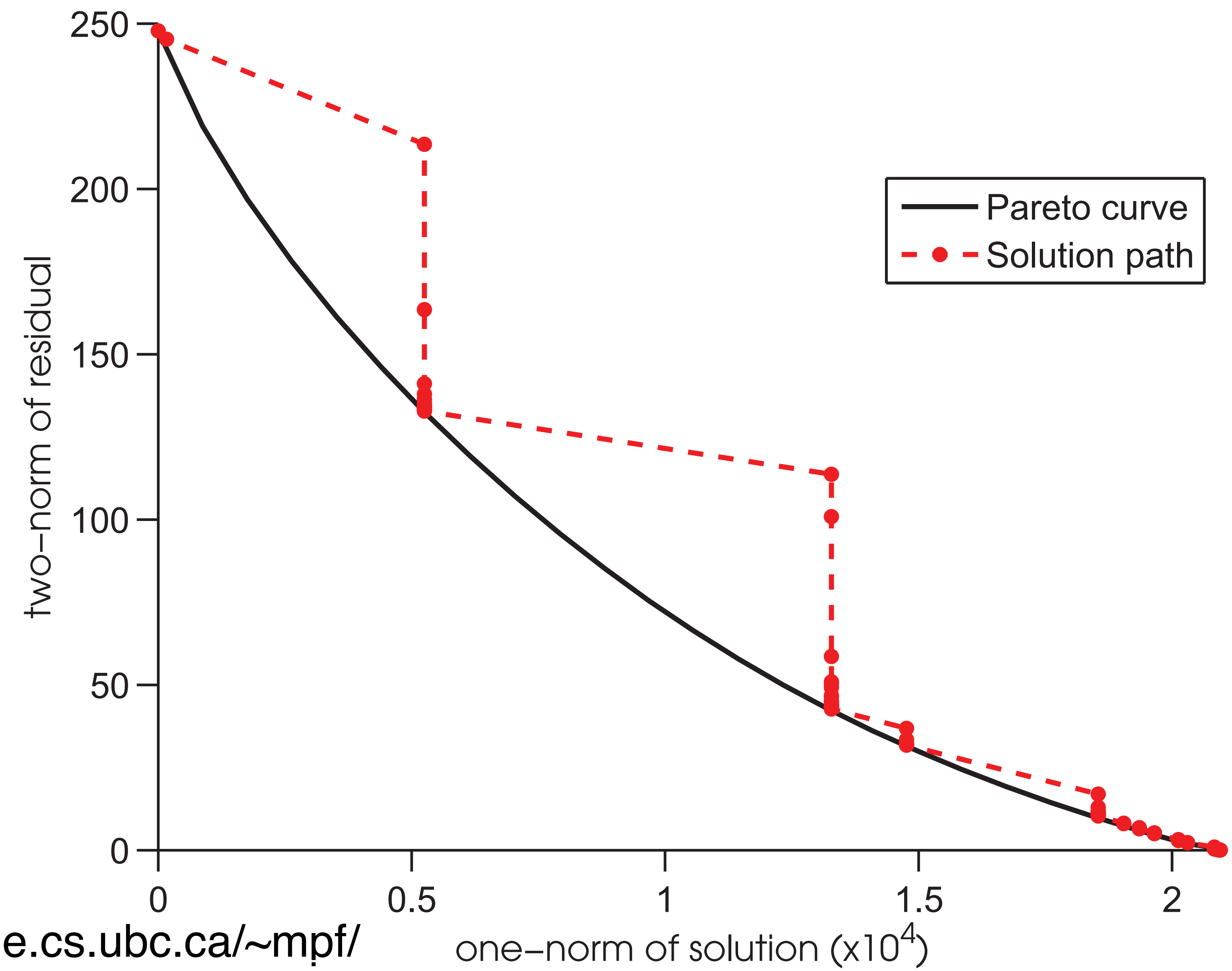
One-norm solver



from http://people.cs.ubc.ca/~mpf/

Key elements

☑ *sparsifying transform*

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☑ *sparsity-promoting solver*

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Observations

Controllable error for reconstruction from *randomized* subsamplings

Curvelets and *simultaneous* acquisition perform the best

Oversampling compared to *conventional compression* is small

Combination of *sampling* & *encoding* into a *single* ***linear*** step has profound implications

- *acquisition* costs ***no*** longer determined by *resolution* & *size*
- *but* by *transform-domain sparsity* & *recovery error*

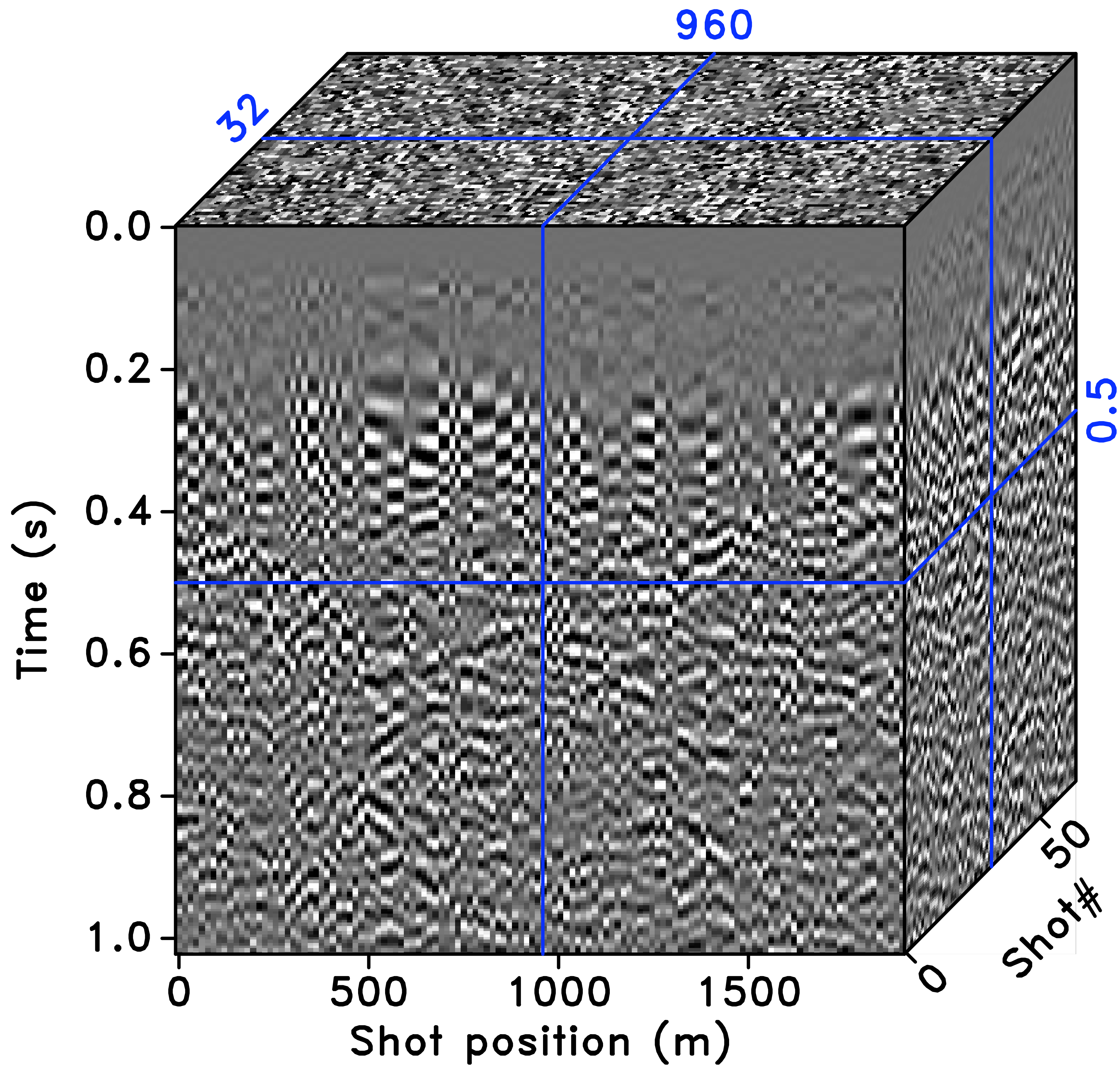
Case study II

Processing according to CS

- CS recovery from *simultaneous* data, followed by *primary* estimation

vs.

- *Primary* estimation *directly* from *simultaneous* data

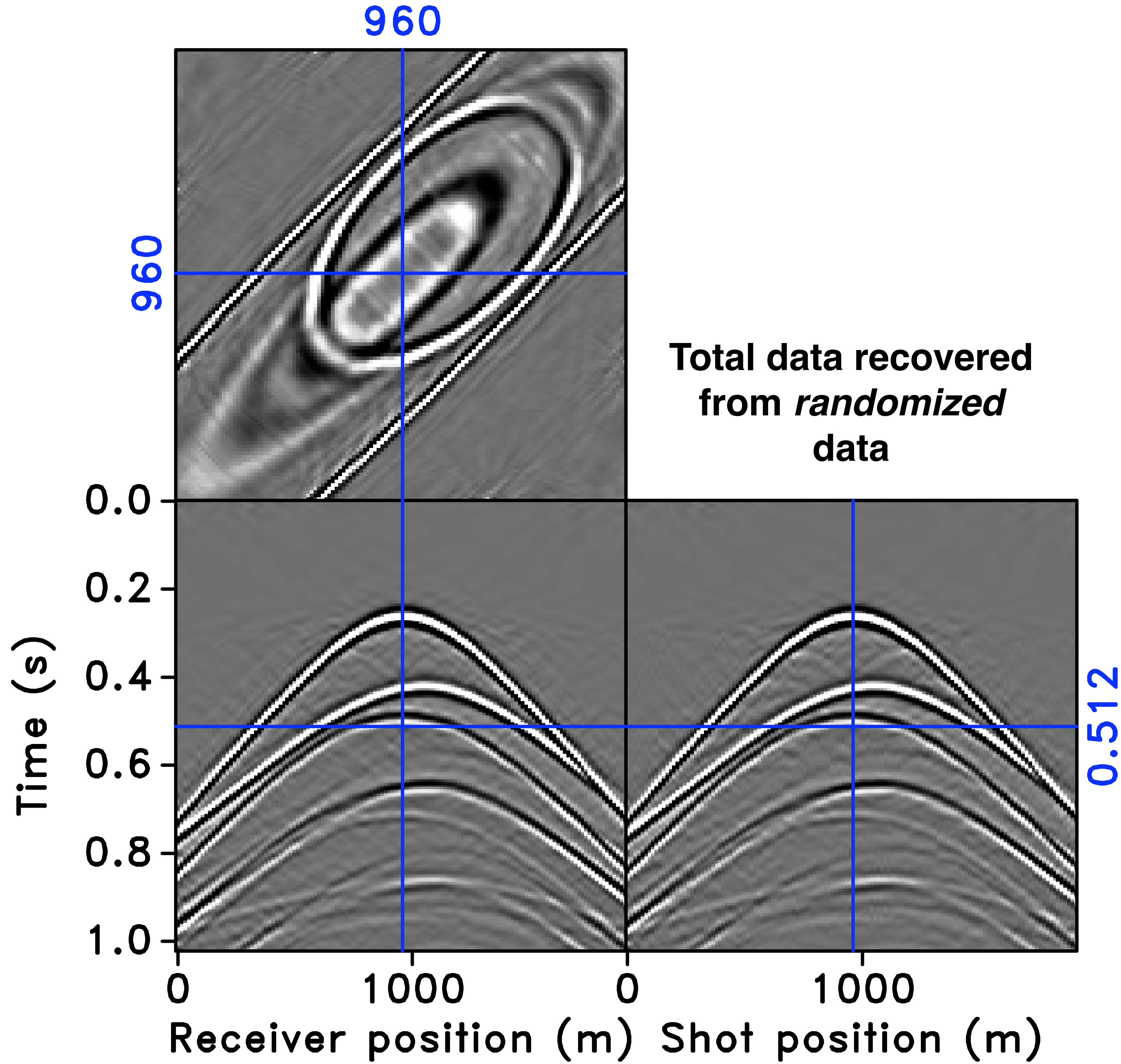

960
32
0.5
0.0
0.2
0.4
0.6
0.8
1.0
Time (s)
0
500
1000
1500
Shot position (m)
0
50
Shot#

CS

Use to demultiplex

$$\mathbf{A} = \left(\mathbf{R}\begin{bmatrix}\textbf{Gaussian}\\ \textbf{matrix}\end{bmatrix} \otimes \mathbf{I}\right)\mathbf{S}^*$$

(Randomized simultaneous sources)



Total data recovered from *randomized* data

[Groenestijn et. al. '09]
[Lin and Herrmann, '09]

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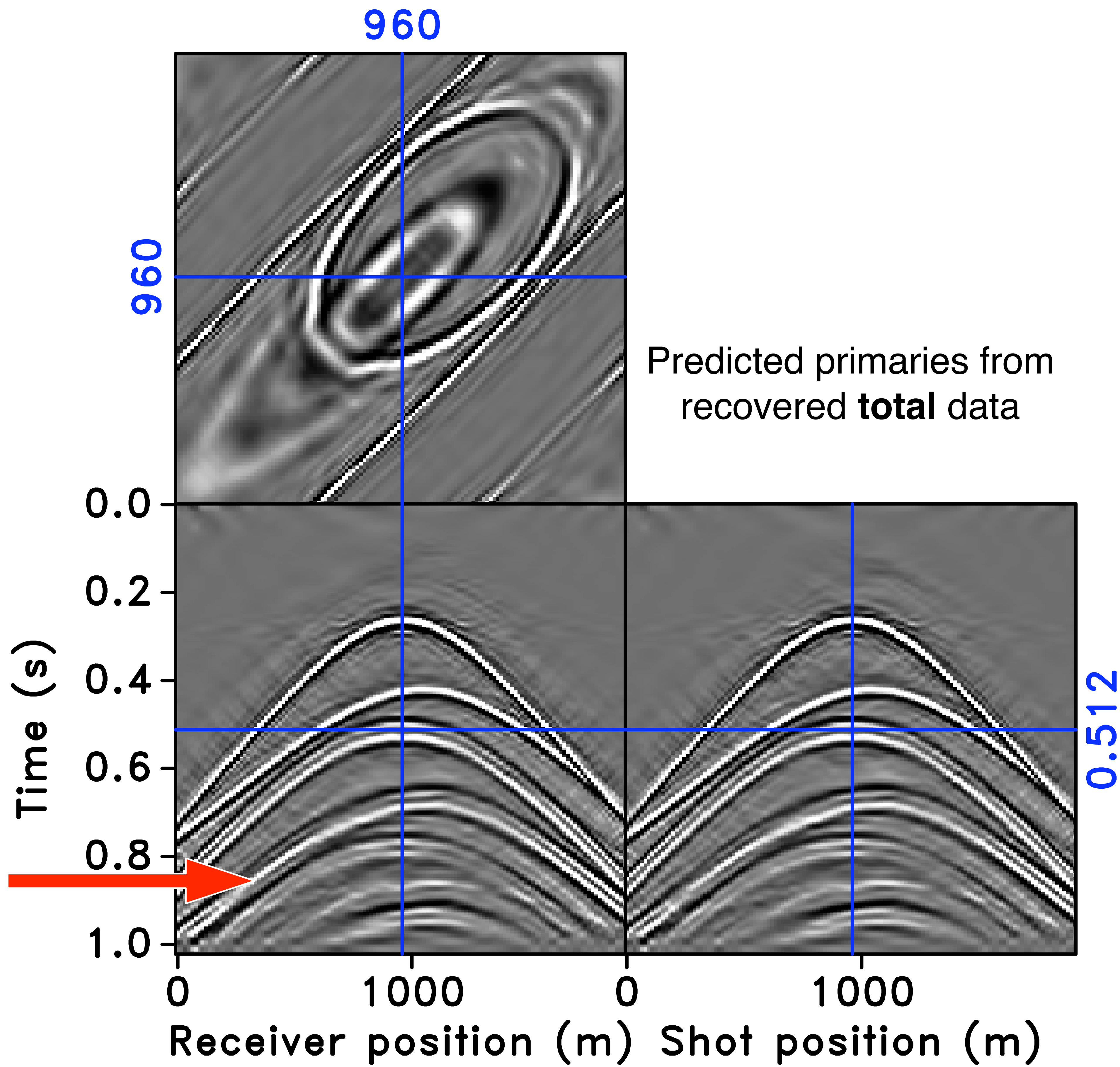
Physical principle

Modeling the surface:

$$\overbrace{\mathbf{P}}^{\text{upgoing wavefield}} \approx \underbrace{\mathbf{G}}_{\text{surface-free impulse response}} \overbrace{[\mathbf{Q}-\mathbf{P}]}^{\text{downgoing wavefield}}$$

Inversion "focusses" multiples onto primaries ...



Predicted primaries from
recovered **total** data

[Lin & FJH, ’09]

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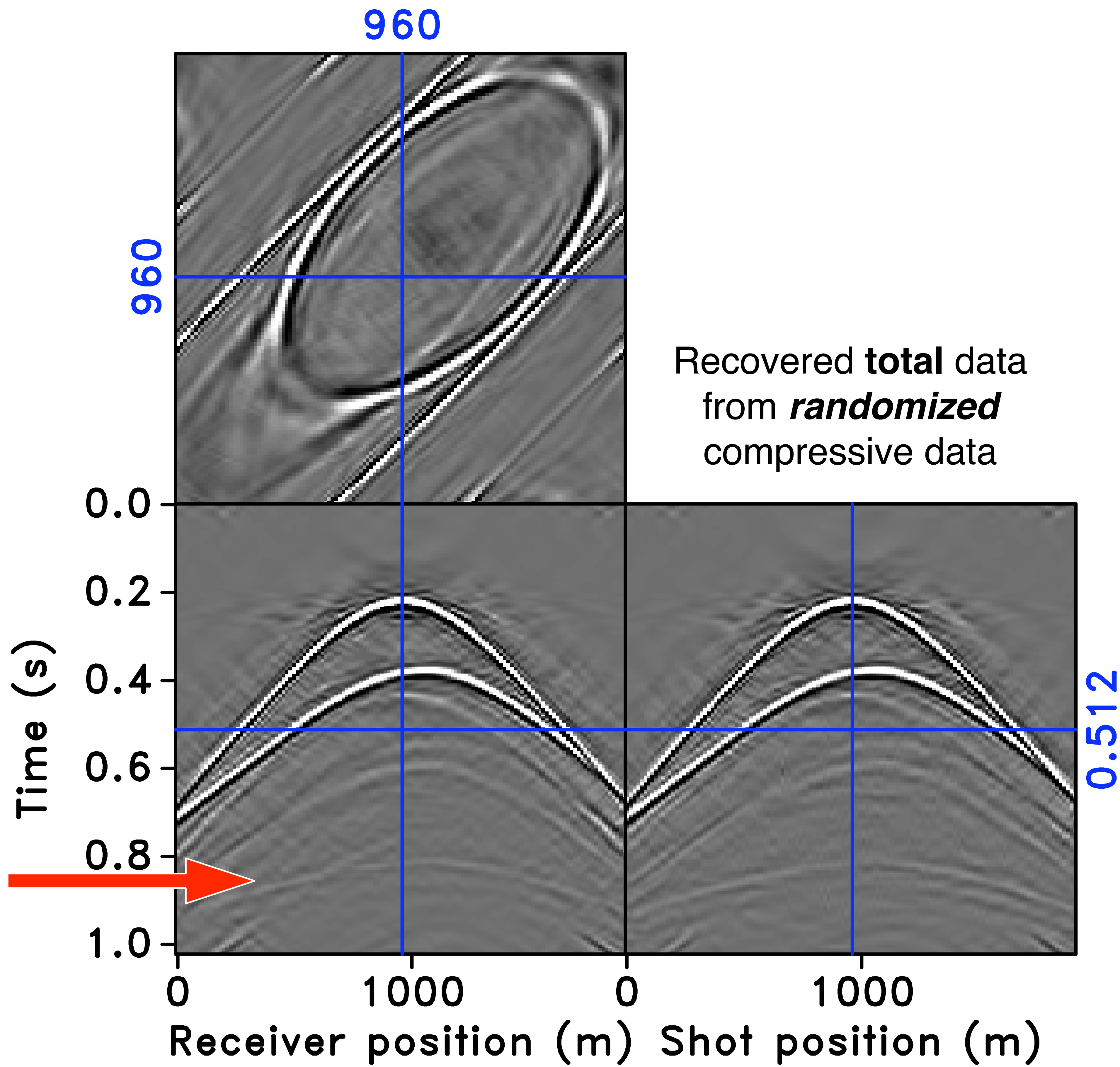
Extension CS

Use to demultiplex & predict

$$\mathbf{A} = \Big[\mathbf{R}\Big]\Big[\text{Gaussian noise}\Big]\overbrace{\Big[\mathbf{M}\Big]}^{\text{randomized physics}}\Big[\mathbf{S}^{\dagger}\Big]$$

(**M** models free surface & source function)


960
960
Recovered **total** data
from ***randomized***
compressive data
0.0
0.2
0.4
0.6
0.8
1.0
Time (s)
0.512
0
1000
0
1000
Receiver position (m)
Shot position (m)

Conclusions

Sparse wavefield recovery benefits from

- *randomization*
- *sparsification*
- *inclusion of physics*

Recovery has a *controlable* error

Leads to acquisition & processing where costs are ***no*** *longer dominated by resolution & size* but by transform-domain sparsity & recovery error

Acknowledgments

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Thank you

slim.eos.ubc.ca

Further reading

Compressive sensing

- *Robust uncertainty principles: Exact signal reconstruction from highly incomplete frequency information by Candes, 06*.
- *Compressed Sensing* by D. Donoho, '06

Simultaneous acquisition

- *A new look at simultaneous sources* by Beasley et. al., '98.
- *Changing the mindset in seismic data acquisition* by Berkhout '08.

Transform-based seismic data regularization

- *Interpolation and extrapolation using a high-resolution discrete Fourier transform* by Sacchi et. al, '98
- *Non-parametric seismic data recovery with curvelet frames* by FJH and Hennenfent.,'07
- *Simply denoise: wavefield reconstruction via jittered undersampling* by Hennenfent and FJH, '08

Estimation of surface-free Green's functions:

- *Estimating primaries by sparse inversion and application to near-offset data reconstruction* by Groenestijn, '09
- *Unified compressive sensing framework for simultaneous acquisition with primary estimation* by T. Lin & FJH, '09

Review

- *Randomized sampling and sparsity: getting more information from fewer samples* by FJH, '09-'10