Introduction

Wave equation-based multiple removal methods such as Surface-Related Multiple Elimination (SRME), and wave-field extrapolation methods (WEM) aim to first generate an estimate of the multiples before using an adaptive subtraction to deal with imperfections caused by the approximations used during multiple prediction. For SRME, these imperfections are directly related to the acquired data. SRME requires a dense grid of point sources and point receivers, where the effective source signature is needed to compensate for the amplitude and phase discrepancies that arise from the (iterative) convolutional process of predicting multiples. For WEM, an additional requirement is the utilization of an exact reflectivity model of the subsurface geology.

Because these approximations are at most only partly accounted for, the predicted multiples exhibit amplitude and phase errors when compared to the multiples present in the input data. As a result, straight subtraction of the predicted multiples will not result in optimal results. Hence, there is a need for a second step, where the predicted multiples are adaptively subtracted from the input data.

Several different techniques exist for this purpose. A method commonly used is based on an L2-norm energy minimization, where the predicted multiples or noise are subtracted from the input data using the minimum energy criterion, which states that the total energy after subtraction of the multiples should be minimized. The predicted multiples are removed by designing Wiener filters in overlapping windows. Although very good results can be obtained using such an approach, this is likely to perform suboptimal when no filter can be found that can correct for all amplitude and phase discrepancies that exist within one window. This is particularly the case when there is:

1) Interference between multiples and primaries. This leads to problems for the minimum energy criterion: when multiples interfere with primaries, the removal of multiples will reveal the primaries, and should therefore locally increase the energy in the data window. It is remarked that this problem can be mitigated somewhat by choosing the windows, over which the energy is minimized, large enough such that significant non-interfering events are present.

2) Interference of multiples of different orders. Due to the approximations used during multiple prediction, different amplitude and phase corrections may be needed for different orders of multiples. Conventional adaptive subtraction ignores this by finding only a global solution that minimizes the energy for all orders of multiples at the same time.

The impact of the shortcomings in conventional adaptive subtraction is illustrated in Figure 1. The input stack shown in the left part of Figure 1 displays two types of interference patterns: interfering primaries and multiples between 0.9 and 1.1 seconds, and interfering multiples of different orders between 1.4 and 1.6 seconds. The latter set consists of a first-order peg-leg and second-order water bottom multiple. The stacked input and prediction in Figure 1 already prelude the remnant multiple energy in the stack of conventional adaptive subtraction results on the right of Figure 1 around one second. Despite accurate predictions of both the 1st order pegleg multiple and 2nd order water bottom multiple neither is fully removed, the former most notably so.

An alternative approach utilizes the curvelet transform to separate primaries and multiples prior to adaptive subtraction of the predicted multiples from the primaries. The adaptive subtraction remains to be based on an L2-norm energy minimization. In the next section, a brief summary of the method is described, before a comparison is made between the results from using adaptive subtraction in the real curvelet domain and those obtained using conventional adaptive subtraction in the spatial domain.

Adaptive subtraction in the curvelet domain

The curvelet transform breaks up seismic sections into small, localized events called curvelets. Different curvelets sample varying parts of the seismic section, with varying frequencies (or

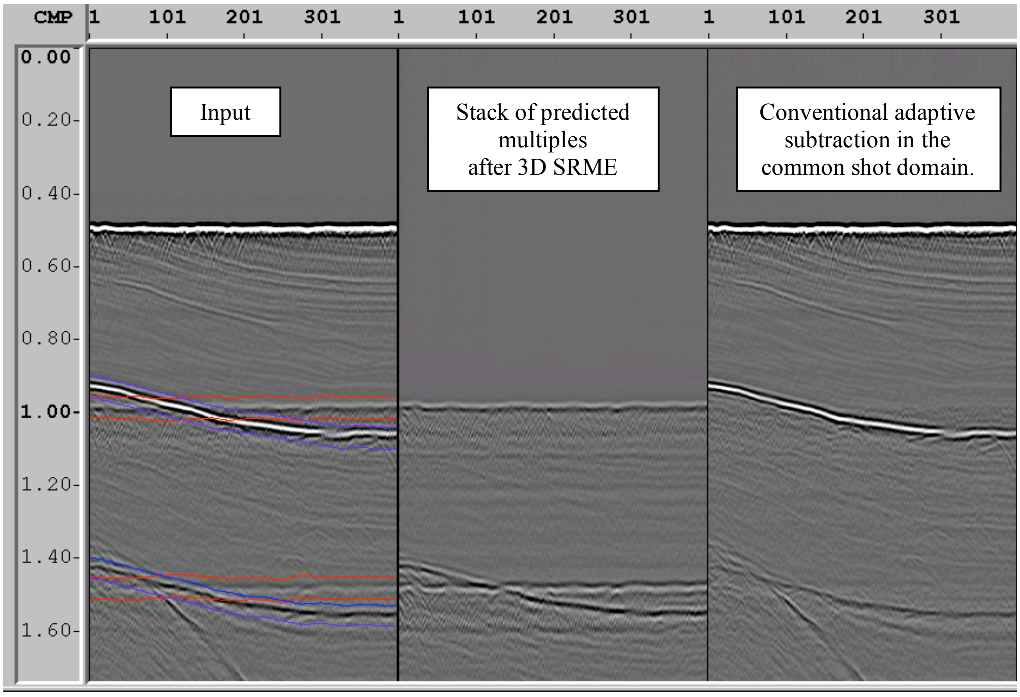

Figure 1: Stacks of input (left), 3D SRME prediction (middle) and conventional adaptive subtraction on common shots (right). The coloured curves in the input stack indicate RMS windows for two pairs of interfering events. The first-order water-bottom multiple (red) and strong subsurface primary (blue) make up the first pair. The peg-leg multiple (blue) associated with these two events and the second-order water bottom multiple (red) are the second pair. The corresponding input RMS values are used for Figure 3 to normalize the adaptive subtraction RMS values.

alternatively scales) and dip. The last property makes it likely that events with different dips are mapped to different curvelets. Various approaches for curvelet-domain adaptive subtraction have been formulated and implemented. An early approach (Herrmann and Verschuur 2004) deploys real curvelets to perform the adaptive subtraction implicitly by shrinking the magnitudes of the input's curvelet coefficients. Neelamani, Baumstein and Ross (2008) subtract predicted multiples from the input in an adaptive fashion, but use complex curvelets to transform both wavefields. For each individual curvelet, amplitude and phase factors are determined that minimize the difference between coefficients of input and predicted multiples. This approach works locally, meaning that it neglects the interdependence of curvelets. In this paper, the curvelet-domain approach described in Herrmann et al (2008) is used, which works with real curvelets and only adjusts the magnitudes, but does take the interdependence of curvelets into account. The second stage of the curvelet-domain approach used in this paper (Saab et al 2008) deploys complex curvelets and deals with timing and phase differences, again considering curvelet-interdependence. Here, only the final result after the second stage is shown.

Comparison of Results

Similar to conventional adaptive subtraction, the curvelet-domain approach can be applied to common offset gathers as well as common shot gathers, leading to the four different stacks in Figure 2. In each of the four stacks of Figure 2 root mean square values were calculated in the four windows highlighted on the left in Figure 1. Normalized by the input RMS values, the adaptive subtraction

RMS values are plotted in Figure 3; grey RMS-curves correspond to the curvelet-domain approach, black RMS-curves correspond to the conventional result. Visual inspection of Figure 2 shows that the curvelet-domain approach removes slightly more energy of the first-order water bottom multiples than the conventional result, which is quantified by the curves in the upper right quadrant of Figure 2. Unfortunately, the curvelet-domain method applied to common offset gathers leads to significant primary leakage, which comes out clearly in the upper left quadrant of Figure 3. A possible explanation could be that the same parameters were used for the application to both common shot and common offset gathers; more favourable results may be obtained by independent parameter selection for common offset gathers. However, the curvelet-domain approach attenuates both the peg-leg multiple and second-order water bottom multiple between 1.4 and 1.6 seconds consistently better than standard adaptive subtraction.

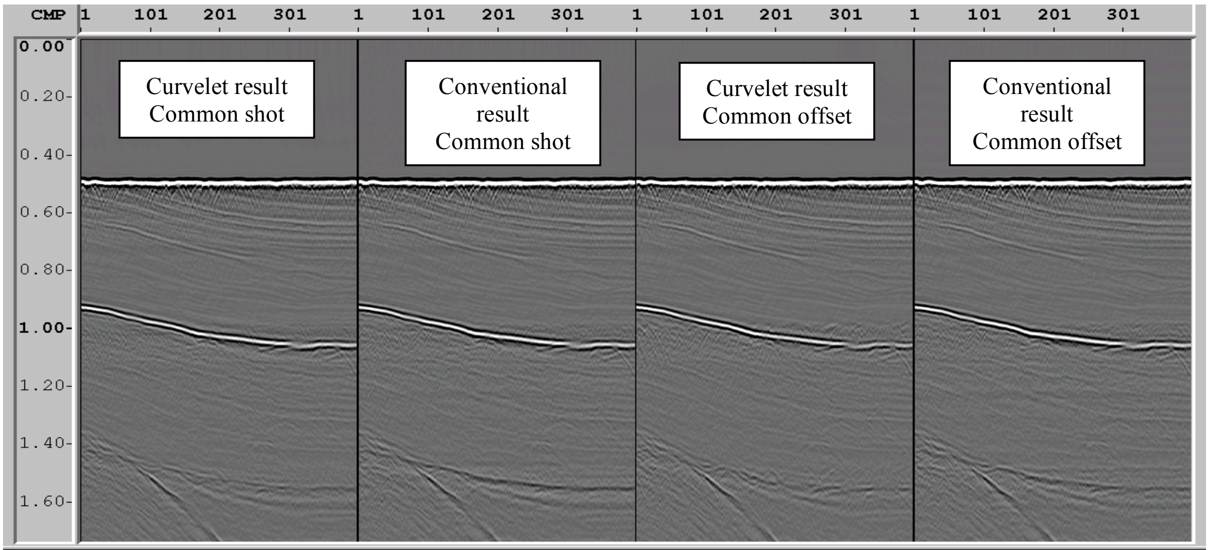

Figure 2: The stack on the far left resulted from using the curvelet-domain approach on shot gather predictions, the stack to the left of the middle is based on standard adaptive subtraction of shot gather predictions. The stack to the right of the middle resulted from the curvelet-domain approach on common offset gather predictions. The stack on the far right is based on conventional adaptive subtraction of common offset gathers. The RMS ratios for the events highlighted in Figure 1 are shown in Figure 3.

Conclusions

In this paper, a comparison was made between a conventional adaptive subtraction approach and a method working in the curvelet domain, in terms of their abilities to separate interfering primaries and first-order multiples and to suppress interfering multiples of different orders.
In case of interfering primaries and first-order multiples, the curvelet-domain approach attenuates the multiples slightly better. The application to common offset gathers suffers from primary degradation, whereas the common shot application preserves the primary satisfactory. For the application to both common shot and common offset gathers, interfering multiples of different orders are attenuated significantly better by the curvelet-domain approach.

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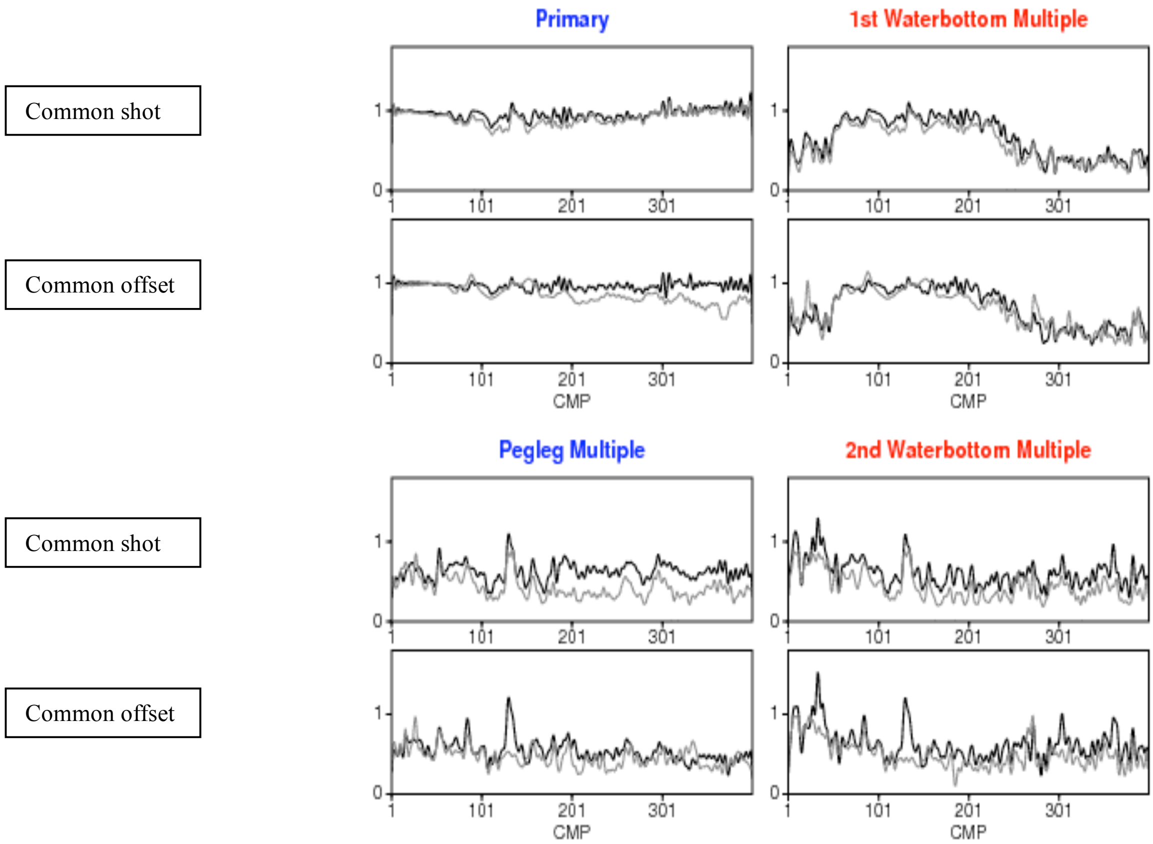

Figure 3: All curves in this figure correspond to ratios of adaptive subtraction RMS values over input RMS values. Black corresponds to conventional L2-norm adaptive subtraction, grey corresponds to the curvelet domain approach. The curves in the upper left quadrant of Figure 3 correspond to the primary highlighted with blue in Figure 1 between 0.9 and 1.1 seconds. The ratios plotted in the top of this quadrant are based on adaptive subtraction per shot as shown in the left two stacks in Figure 2, the ratios plotted in the bottom of this quadrant are based on adaptive subtraction applied to common offset gathers as shown in the right two stacks in Figure 2. The curves in the upper right quadrant correspond to the first water bottom multiple at 1 second (red in Figure 1). The curves in the lower left quadrant correspond to the peg-leg multiple between 1.4 and 1.6 seconds (blue in Figure 1). The curves in the lower right quadrant correspond to the second water bottom multiple at 1.5 seconds (red in Figure 1).