Amsterdam '09

Designing simultaneous acquisitions **with** Compressive Sensing

Tim T.Y. Lin *jointly with Felix J. Herrmann*

SLIM
University of British Columbia

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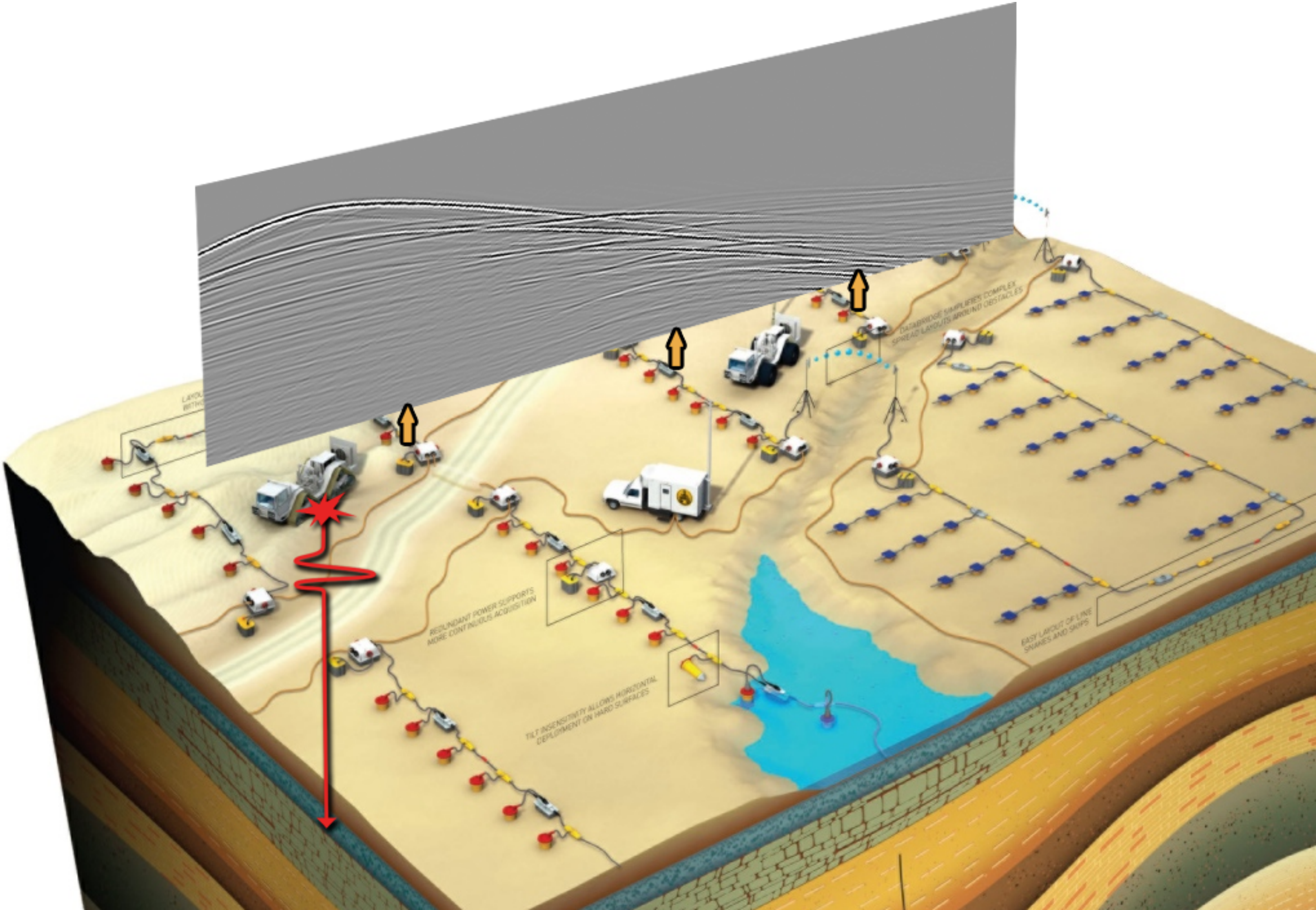
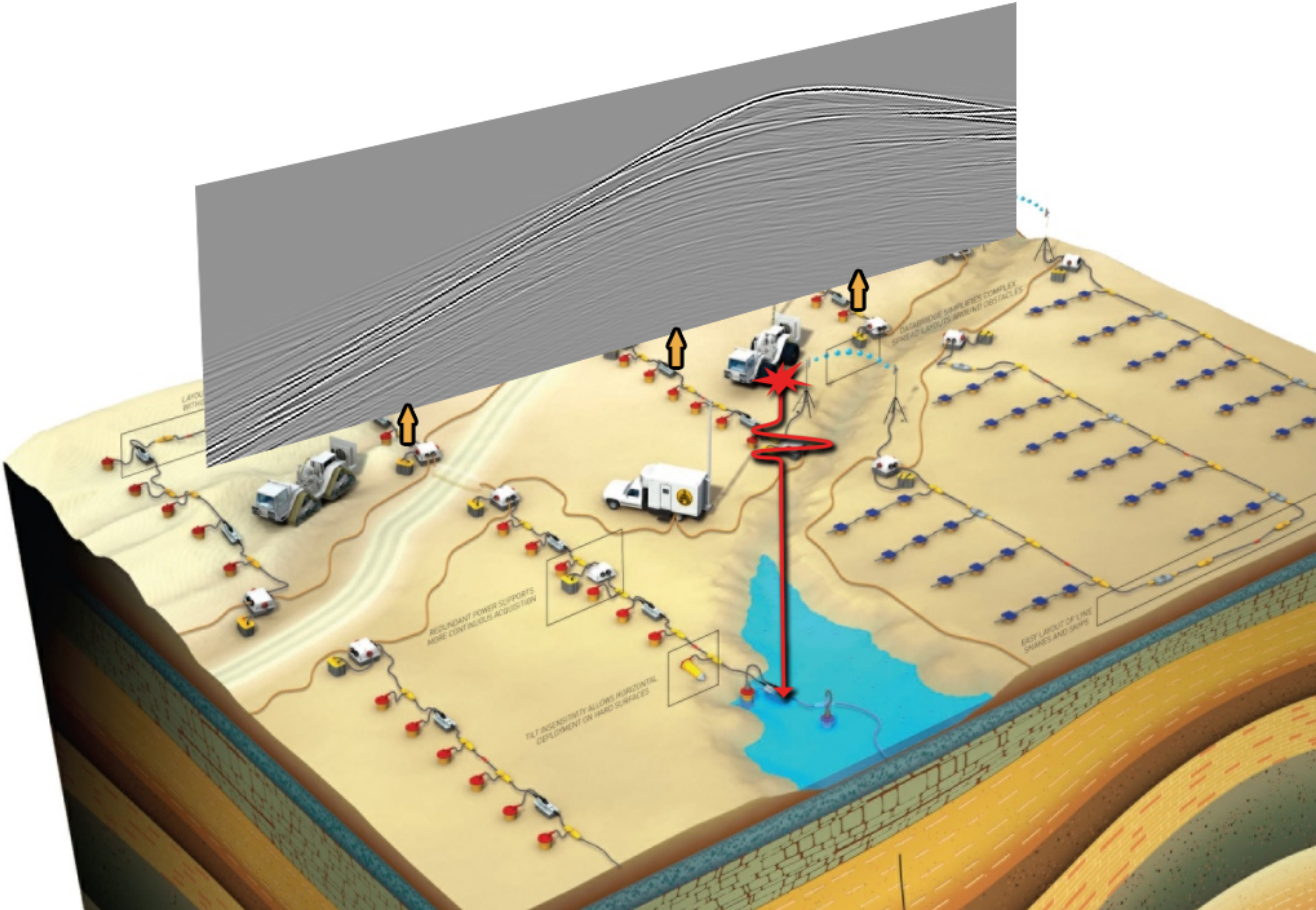
meet...

...the blended shot record


DATABRIDGE SIMPLIFIES COMPLEX SPREAD LAYOUTS AROUND OBSTACLES
REDUNDANT POWER SUPPORTS MORE CONTINUOUS ACQUISITION
TILT INSENSITIVITY ALLOWS HORIZONTAL DEPLOYMENT ON HARD SURFACES
EASY LAYOUT OF LINE SNAKES AND SKIPS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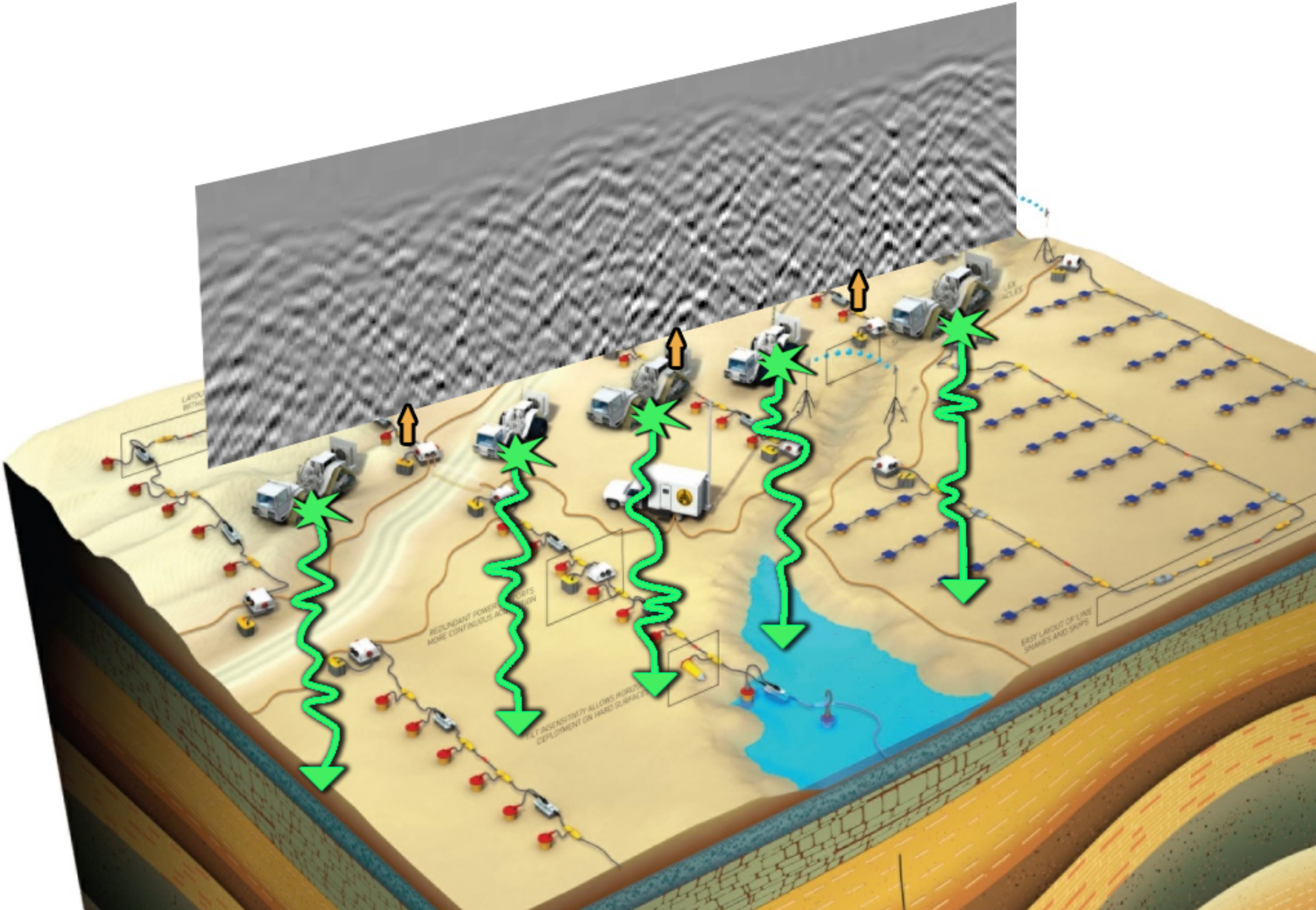
EASY LAYOUT OF LINE
SNAKES AND SKIPS

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fact:

blended shots will never stop looking ugly

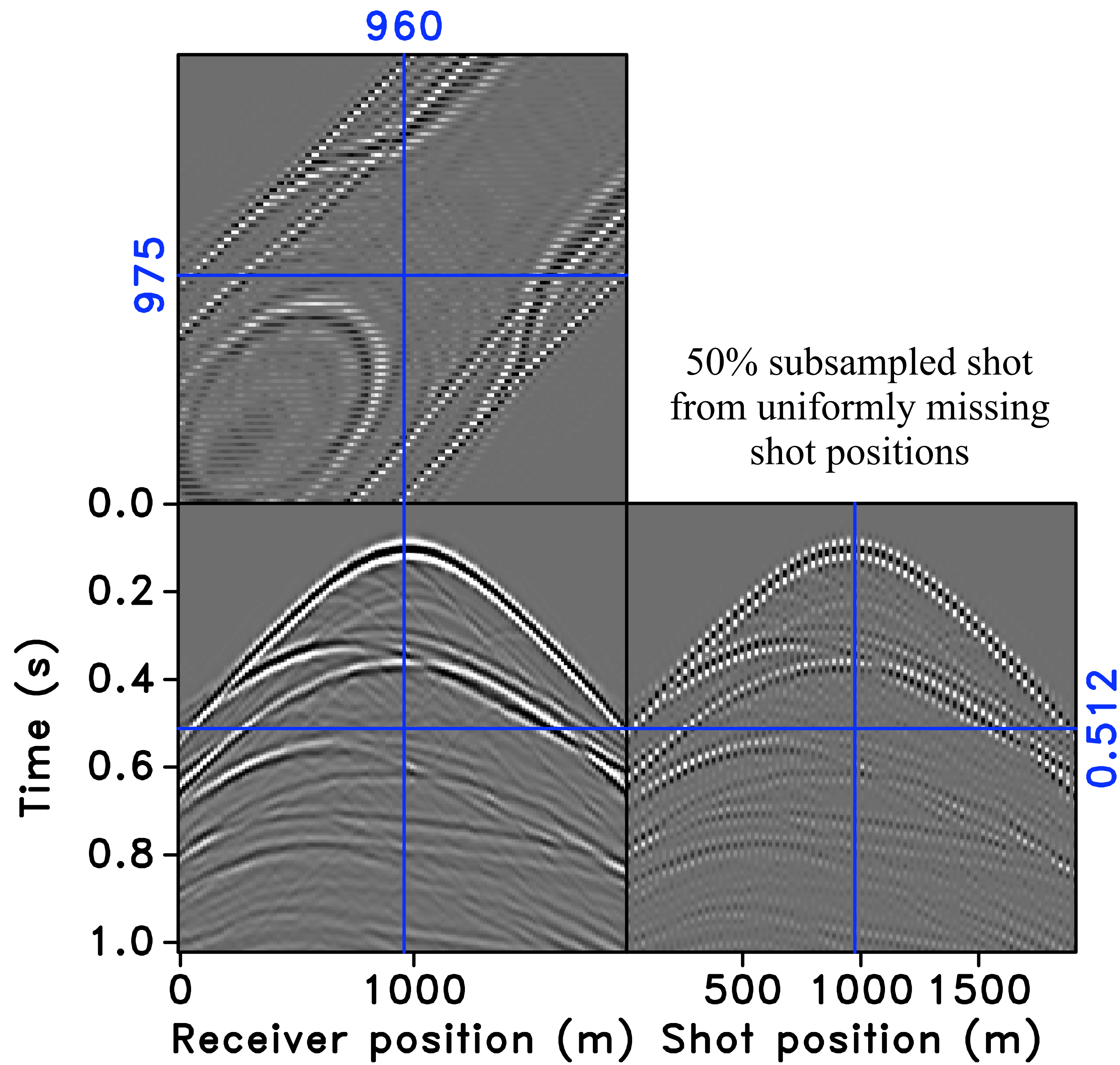
but

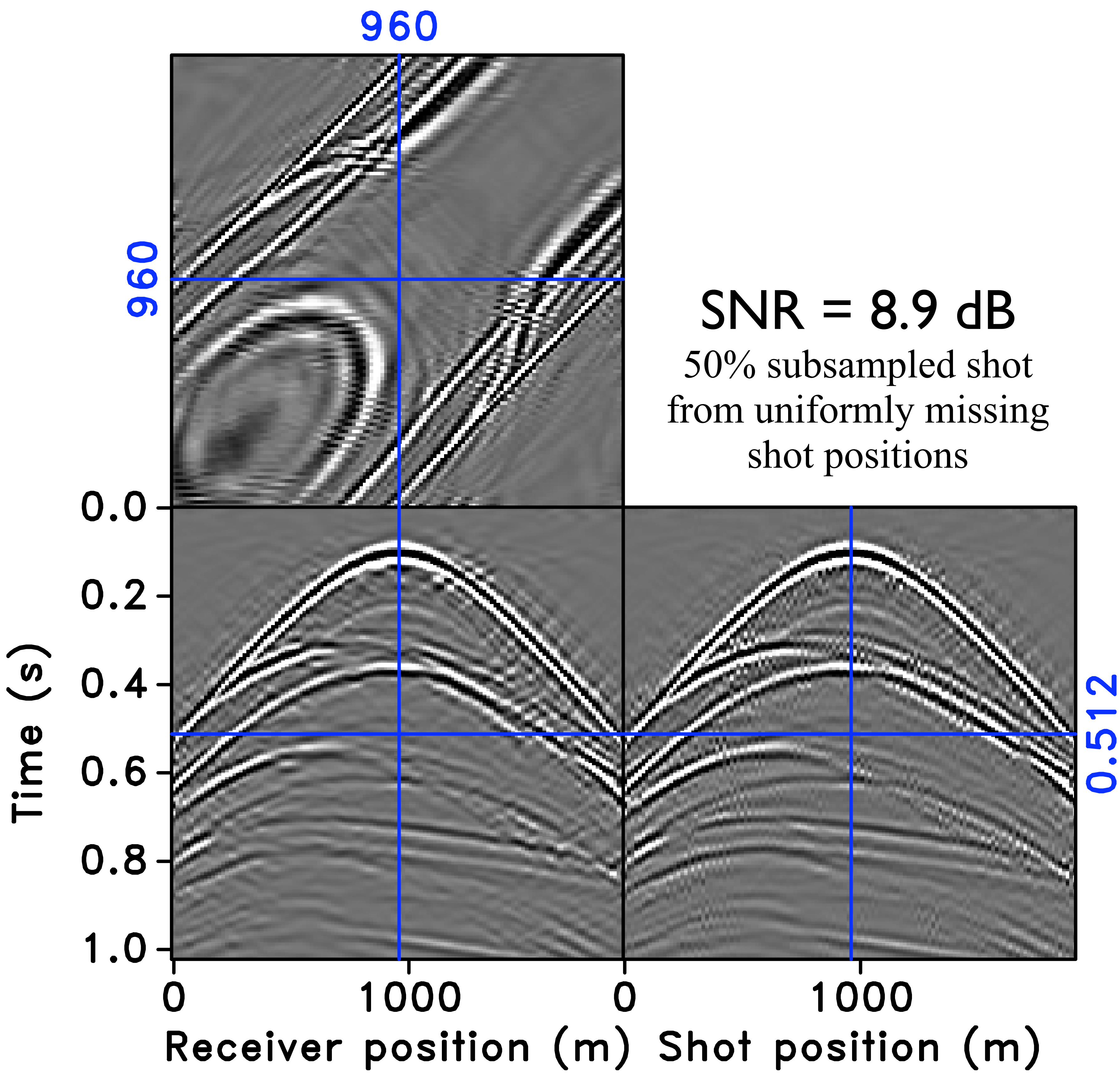
you know... *never* judge a book by its cover

pathology

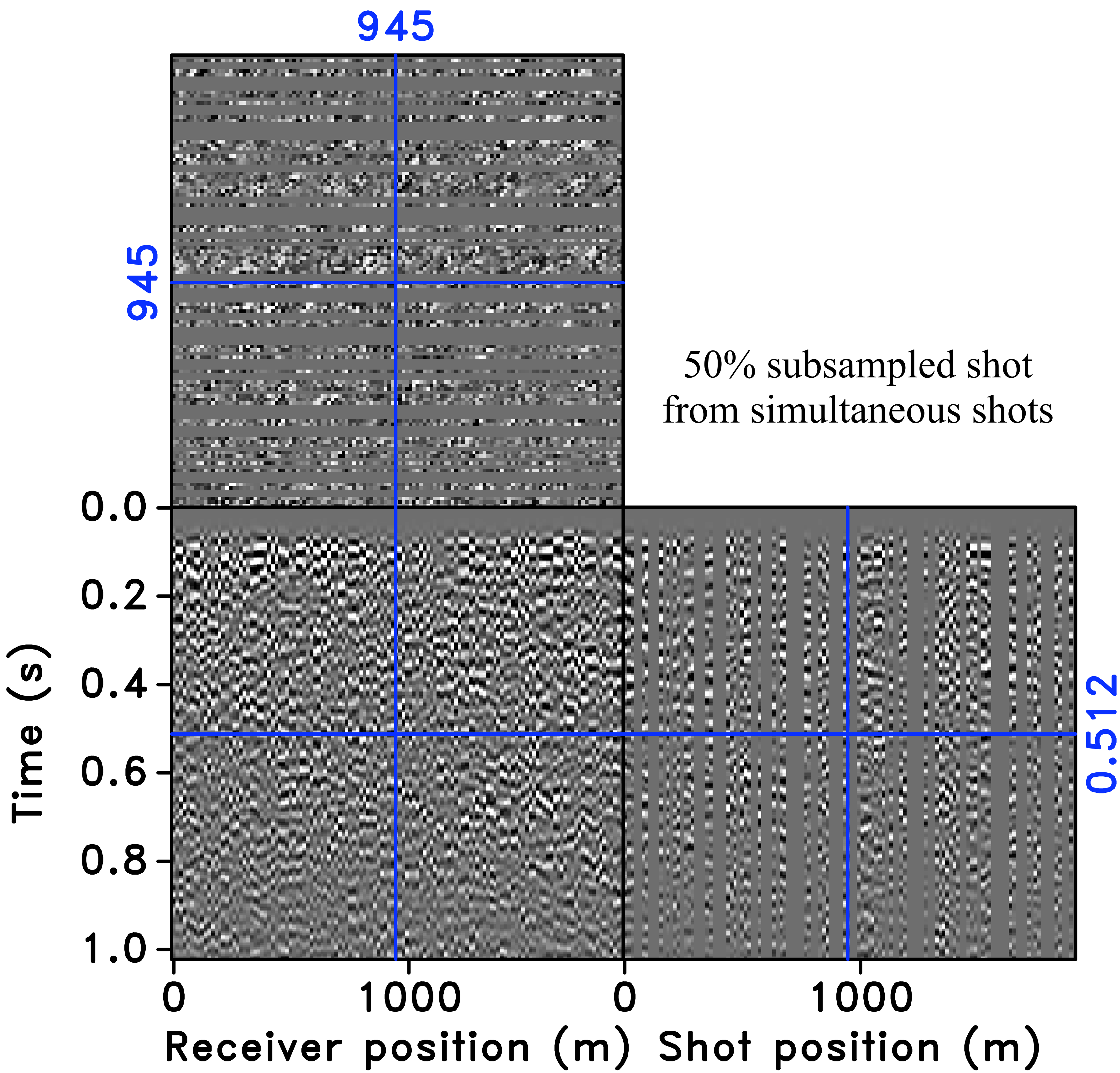
shot interpolation

12.5m to 25m

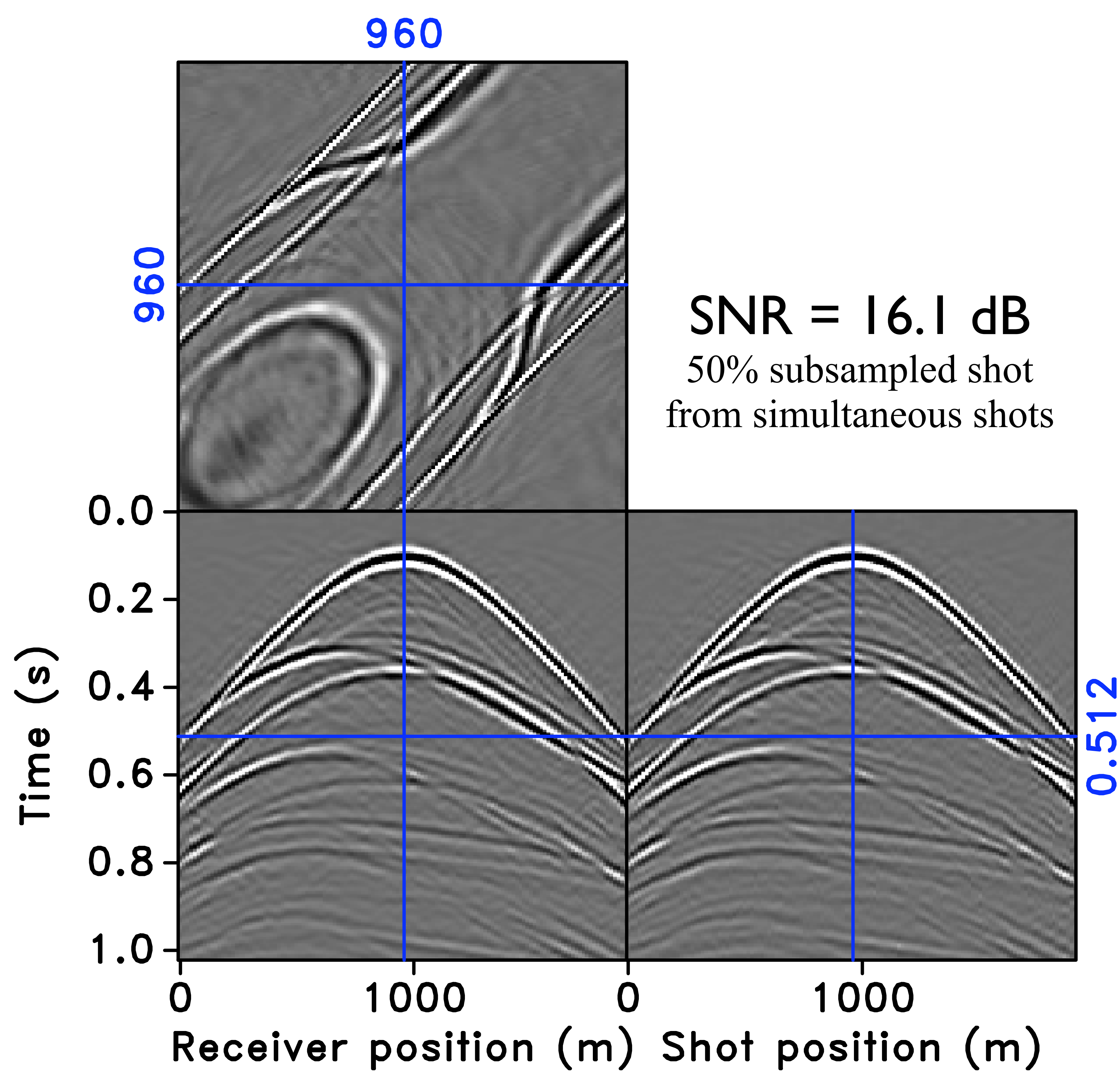

960
975
50% subsampled shot from uniformly missing shot positions
0.0
0.2
0.4
0.6
0.8
1.0
Time (s)
0.512
0
1000
500
1000
1500
Receiver position (m)
Shot position (m)



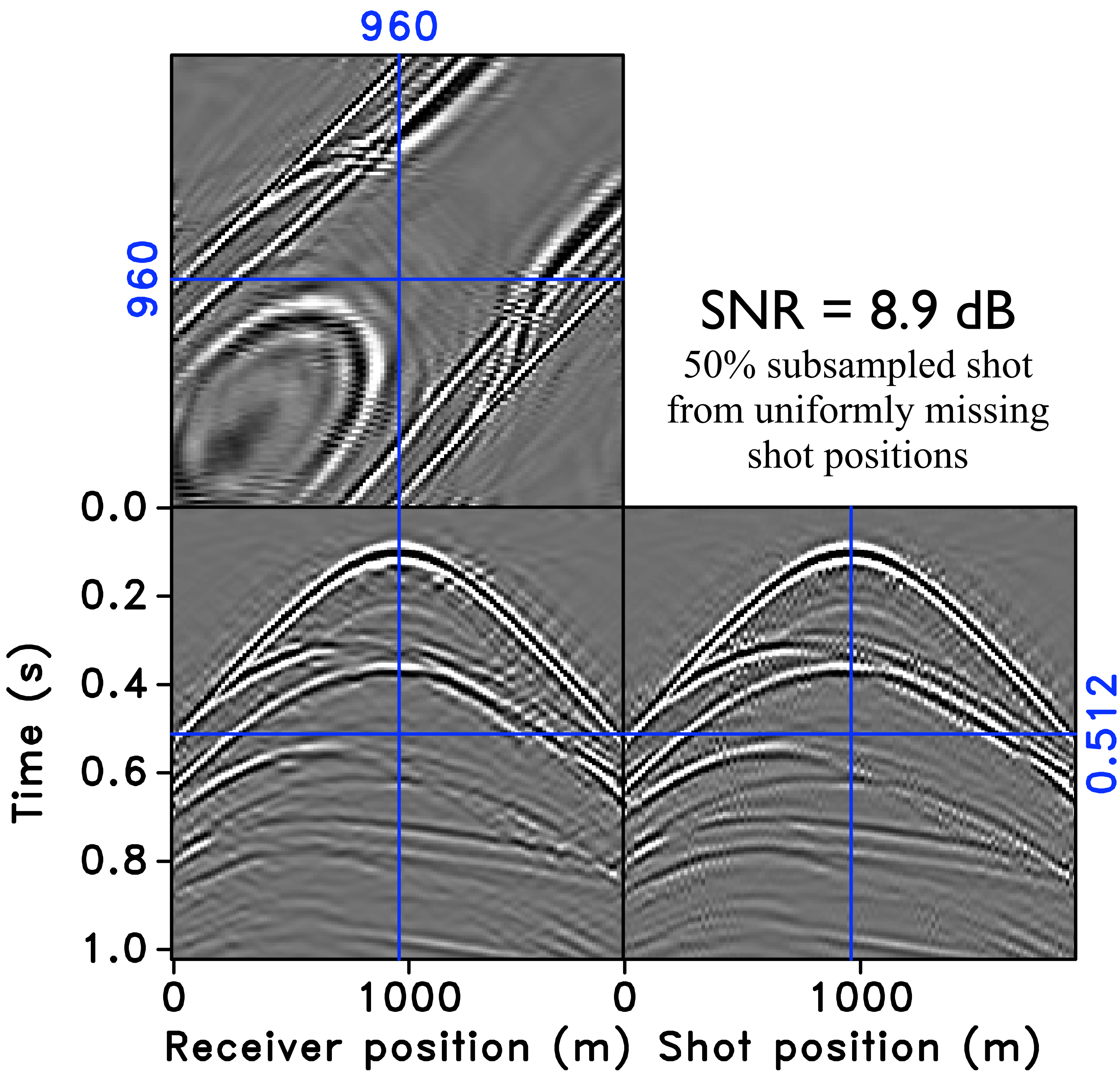
960
960
SNR = 8.9 dB
50% subsampled shot
from uniformly missing
shot positions
0.512
0.0
0.2
0.4
0.6
0.8
1.0
Time (s)
0
1000
0
1000
Receiver position (m)
Shot position (m)



945
945
0.512
50% subsampled shot
from simultaneous shots
Time (s)
0.0
0.2
0.4
0.6
0.8
1.0
0
1000
0
1000
Receiver position (m)
Shot position (m)



960
960
SNR = 16.1 dB
50% subsampled shot
from simultaneous shots
0.0
0.2
0.4
0.6
0.8
1.0
Time (s)
0.512
0
1000
0
1000
Receiver position (m)
Shot position (m)



SNR = 8.9 dB
50% subsampled shot
from uniformly missing
shot positions

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how? need to know **why**

why? need to know **how**

HOW? part one

it's a matrix

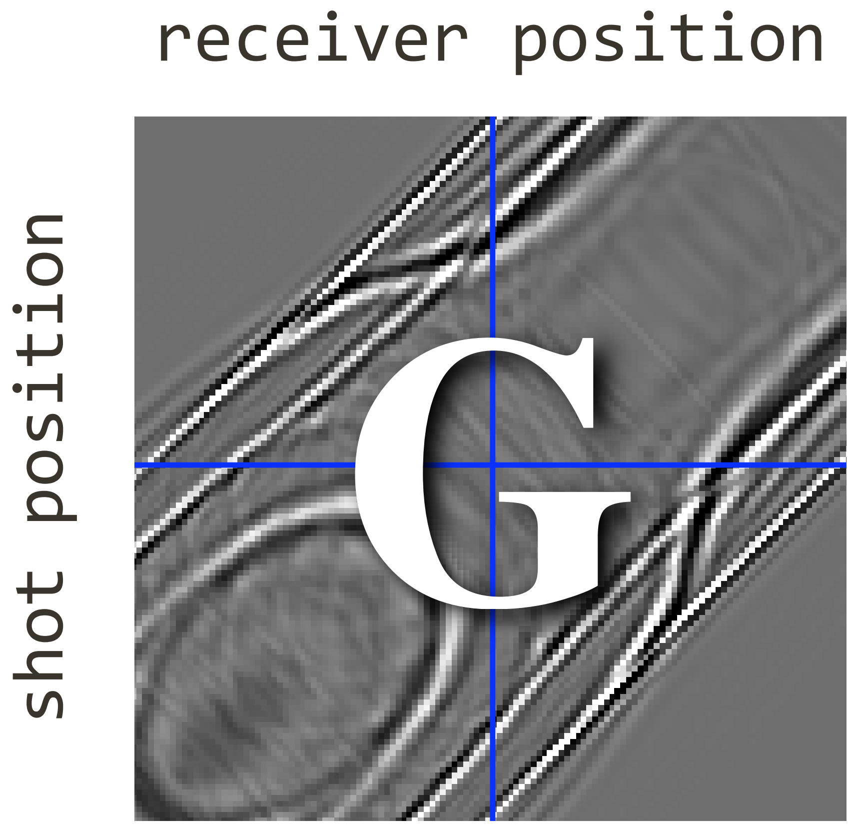
receiver position

shot position

$$\left[\;\mathbf{G}\;\right]$$

Green's function

it's a matrix



Green's function

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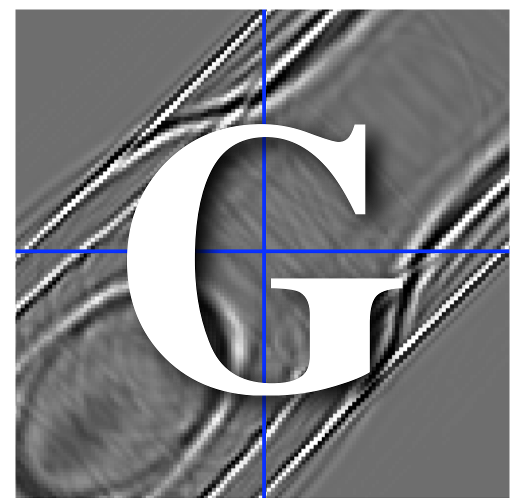
it’s linear algebra

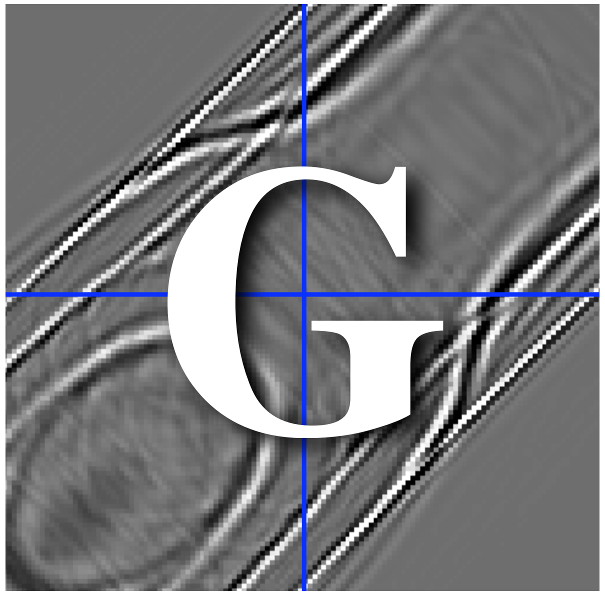
$$\mathbf{D} = \left[\;\mathbf{Q}\;\right]_{\textbf{Shot}} \mathbf{G} \left[\qquad\right]_{\textbf{Recv}}$$



represents acquisition of data

eg: ideal coverage

$$\mathbf{D} = \begin{bmatrix} 1 & & & & & \\ & 1 & & & & \\ & & 1 & & & \\ & & & 1 & & \\ & & & & 1 & \\ & & & & & 1 \end{bmatrix}$$



identity matrix

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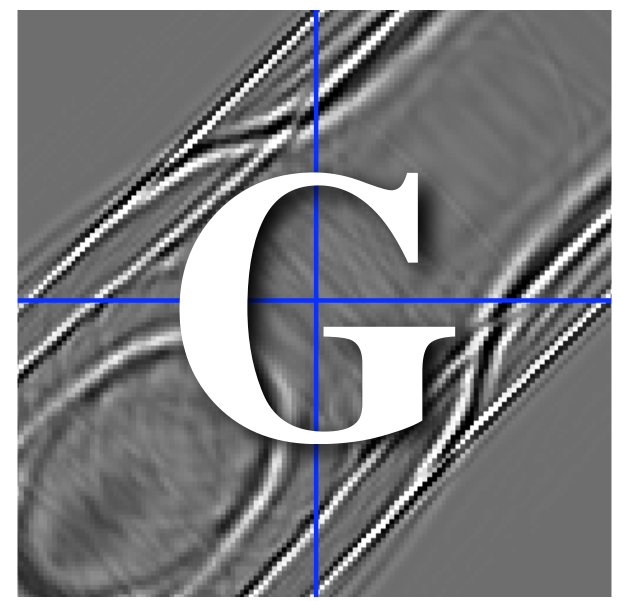
eg: 2x undersampled shots

$$\mathbf{D} = \begin{bmatrix} 1 & & & & & \\ & 0 & & & & \\ & & 1 & & & \\ & & & 0 & & \\ & & & & 1 & \\ & & & & & 0 \end{bmatrix}$$



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now, D ← G ✓

D → G ?

simplest formulation

$$\min \|\mathbf{D} - \mathbf{QG}\|_2$$

solve least-squares for **G**

but wait...

I know *geophysics*! **G** has some sort of structure

it's information theory

I know... a *compressive* representation **S**

$$\mathbf{G} = \mathbf{S}^{\dagger}\mathbf{x}$$

(x is compressible or sparse)

it's statistics

$\mathbf{x}_{\mathrm{ml}}$ for a given energy mismatch σ

$$\begin{array}{ll} \min & \mathrm{NNZ}(\mathbf{x}) \\ \text{s.t.} & \|\mathbf{D} - \mathbf{Q}\mathbf{S}^{\dagger}\mathbf{x}\|_2 \leq \sigma \end{array}$$

SLIM

it's statistics

$\mathbf{x}_{\mathrm{ml}}$ for a given energy mismatch σ

$$\begin{aligned} \min \quad & \mathrm{NNZ}(\mathbf{x}) \\ \text{s.t.} \quad & \|\mathbf{D} - \underbrace{\mathbf{Q}\mathbf{S}^{\dagger}}_{\mathbf{A}}\mathbf{x}\|_2 \leq \sigma \end{aligned}$$

WHY?

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when does...

$$\mathbf{S}^{\dagger}\mathbf{x}_{\mathrm{ml}} \approx \mathbf{G}$$

how do we know?

talk to ~~strangers~~ mathematicians

Candes

Tao

Donoho

Romberg

talk to ~~strangers~~ mathematicians

“Look at A!”

(Compressive Sensing)

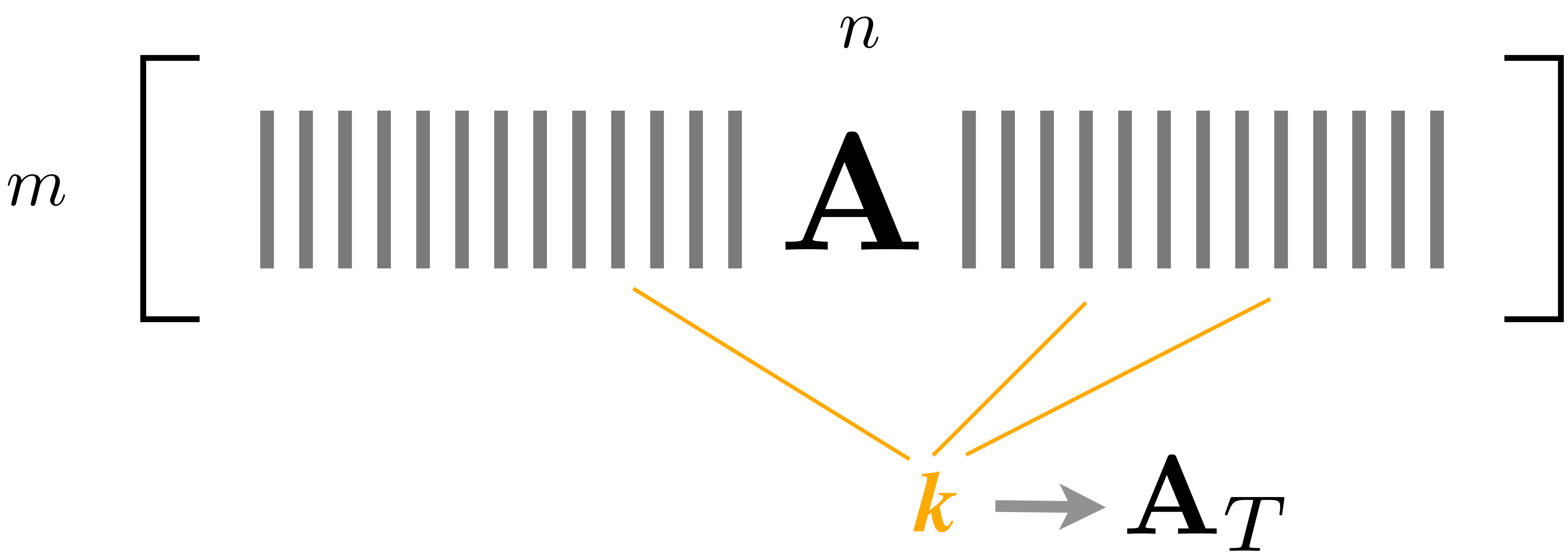



talk to ~~strangers~~ mathematicians

$$(1-\delta_k)\|\mathbf{x}_T\|_{\ell 2} \leq \|\mathbf{A}_T\mathbf{x}\|_{\ell 2} \leq (1+\delta_k)\|\mathbf{x}_T\|_{\ell 2}$$

(Restricted Isometry Property)

RIP for $k \leq m \ll n$



RIP for $k \leq m \ll n$

$\mathbf{A}_T$ how close is it to an orthonormal basis?

(if close enough, then if $\mathrm{NNZ}(\mathbf{x}) \leq k/2$,
$\mathbf{S}^{\dagger}\mathbf{x}_{\mathrm{ml}} = \mathbf{G}$ with overwhelming probability)

only downside...

you can't actually *calculate* whether RIP is satisfied

"that's NP hard!"




bad, bad examples

$$\mathbf{A} = \begin{bmatrix} 1\;0 & & \\ & 1\;0 & \\ & & 1\;0 \end{bmatrix} \begin{bmatrix} \mathbf{S}^{\dagger} \end{bmatrix}$$

(2x shot undersampling)

bad, bad examples

$$\mathbf{A} = \begin{bmatrix} 1\;1 & & \\ & 1\;1 & \\ & & 1\;1 \end{bmatrix} \begin{bmatrix} \mathbf{S}^{\dagger} \end{bmatrix}$$

(Blend every-other shot)

good example

$$\mathbf{A} = \begin{bmatrix} \textbf{Gaussian} \\ \textbf{noise} \end{bmatrix} \begin{bmatrix} \mathbf{S}^{\dagger} \end{bmatrix}$$

(Completely blended shots)

that’s why

you can *fundamentally* expect to get more info from fully blended shots

intermediate conclusions

1) **Random** is *good*

intermediate conclusions

2) the *sparser* the signal...
... the **more** you can *subsample*

intermediate conclusions

3) the **more** you do with **A**…
… the **more** "random" it is
… the **more** *likely* it holds RIP

“more”? e.g.

$$\mathbf{A} = \begin{bmatrix} \text{Gaussian} \\ \text{noise} \end{bmatrix} \begin{bmatrix} \mathbf{M} \end{bmatrix} \begin{bmatrix} \mathbf{S}^{\dagger} \end{bmatrix}$$

(M maps primaries to the data, *ie* in SRME and EPSI)

HOW? part two

oops...

$$\begin{array}{ll} \min & \mathrm{NNZ}(\mathbf{x}) \\ \text{s.t.} & \|\mathbf{D} - \mathbf{A}\mathbf{x}\|_2 \leq \sigma \end{array}$$

... is NP-hard

let me fix that

$$\begin{array}{ll} \min & \|\mathbf{x}\|_{\ell 1} \\ \text{s.t.} & \|\mathbf{D} - \mathbf{A}\mathbf{x}\|_2 \leq \sigma \end{array}$$

is a very good *convex* relaxation

pulled a fast one!

the results of
compressive sensing
specifically assumes L1
relaxation

no worries!

any good solvers?

dsp.ece.rice.edu/cs

Feb 2006: 40 papers

June 2007: 100 papers

June 2009: 500 papers

dsp.ece.rice.edu/cs

l1-Magic
SparseLab
GPSR
ell-1 LS
sparsify

solvers, Jun 2007

MPTK
Bayesian
SPGL1
sparseMRI
FPC
Chaining Pursuit
Regularized OMP
TwIST
Fast CS using
SRM
FPC_AS
Fast Bayesian
Matching Pursuit
SL0
PPPA
CoSAMP
CS via belief prop
SpaRSA
KF-CS: Kalman
Filtered CS
Fast Bayesian CS

dsp.ece.rice.edu/cs

solvers, Jun 2009

our experience?

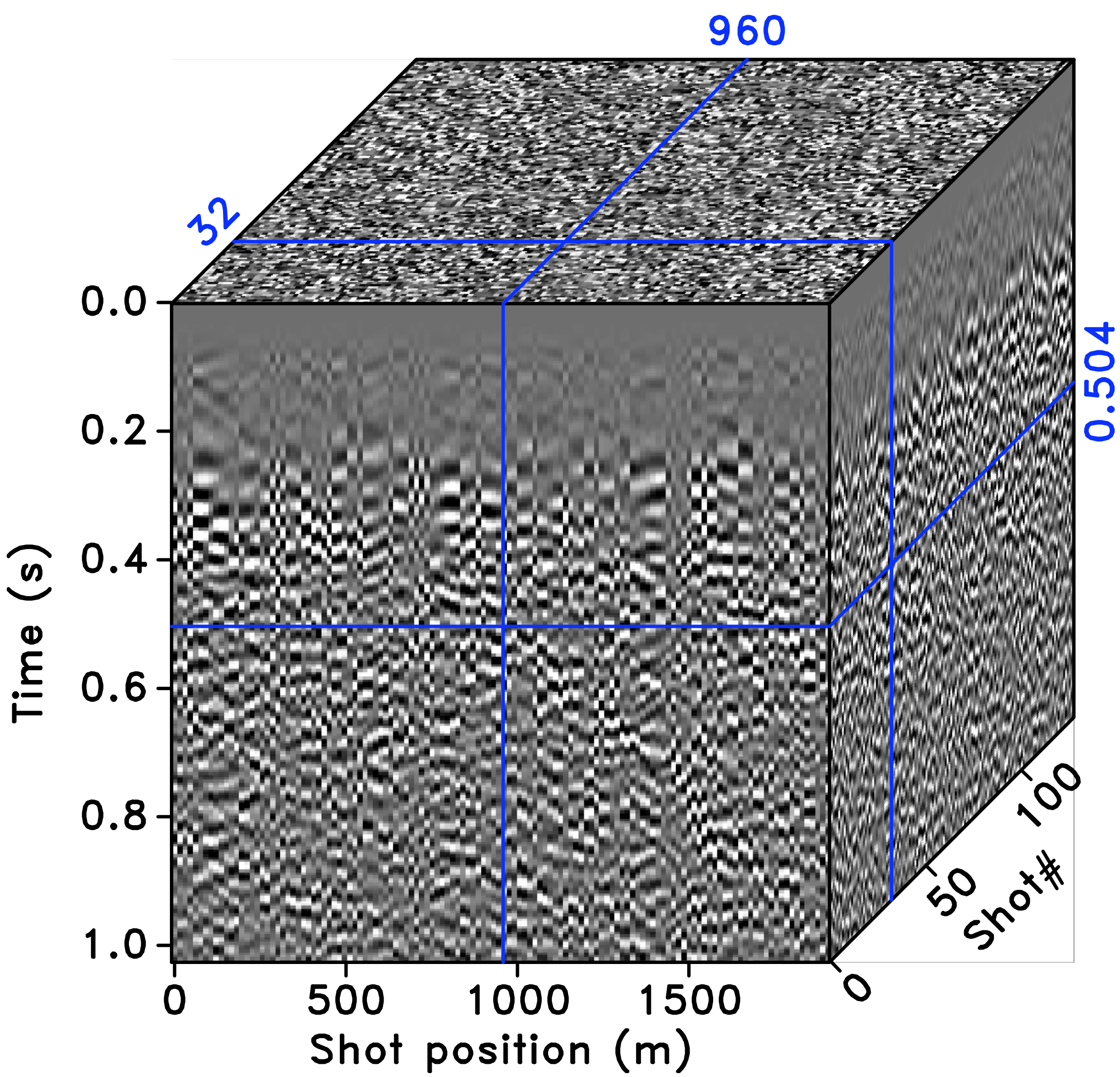
some spectral projected gradient-based methods...

the cost of calculating $\mathbf{A}^T\mathbf{A}\mathbf{x}$

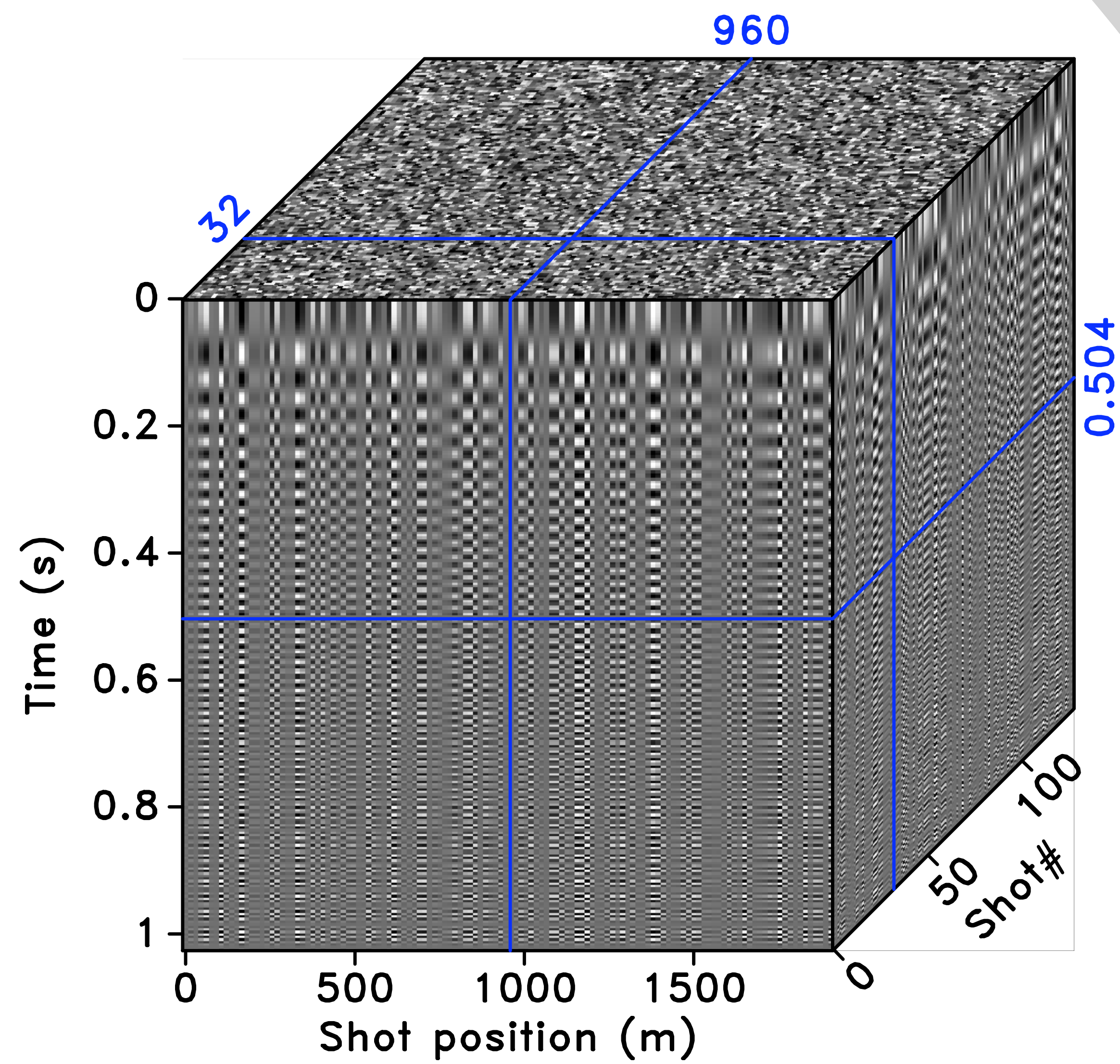
30x to 90x

that buys you...

- source signature deconvolution
- separate blended data
- wavelet estimation
- primary estimation

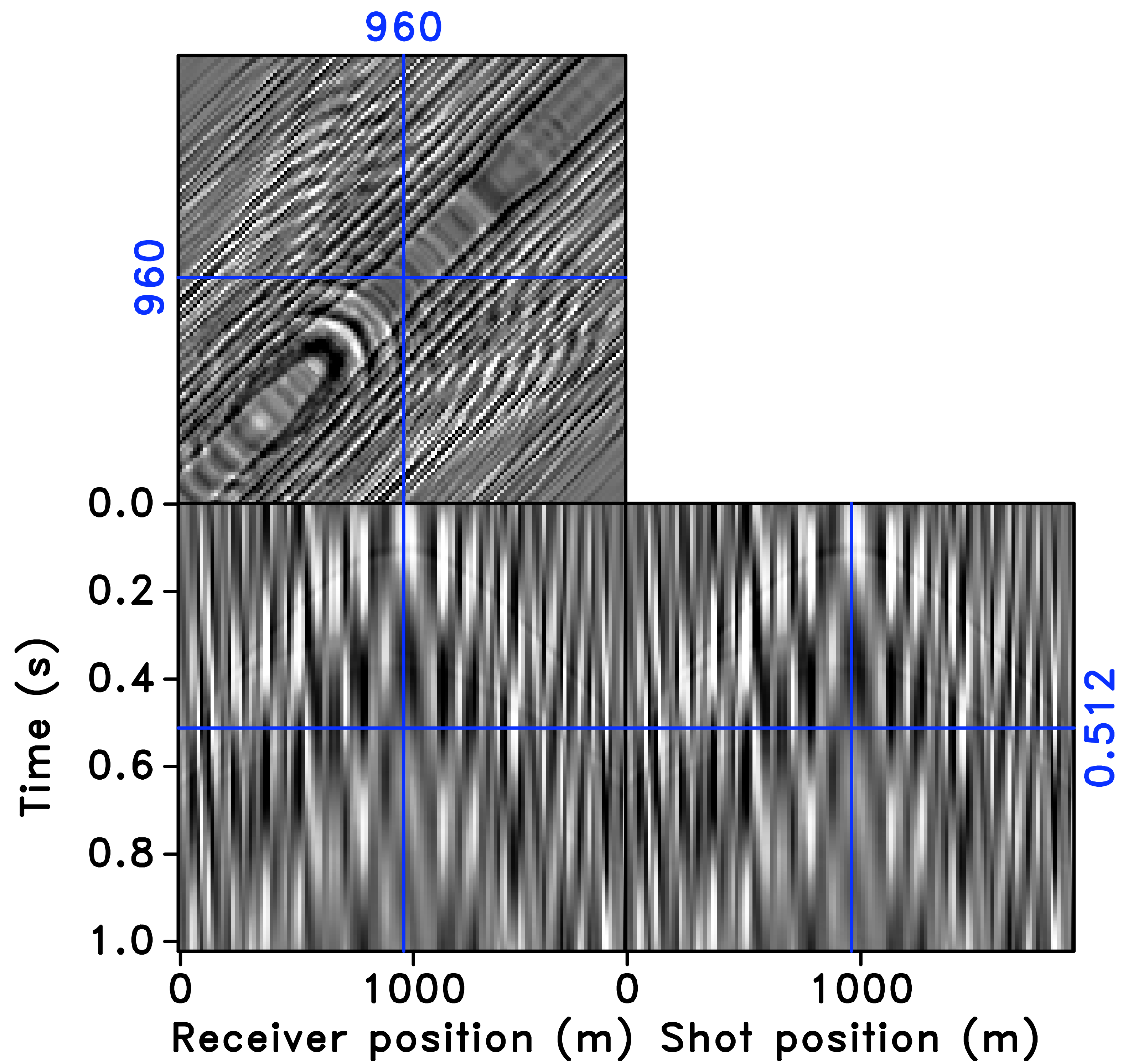

960
32
0.504
0.0
0.2
0.4
0.6
0.8
1.0
Time (s)
0
500
1000
1500
Shot position (m)
0
50
100
Shot#

D

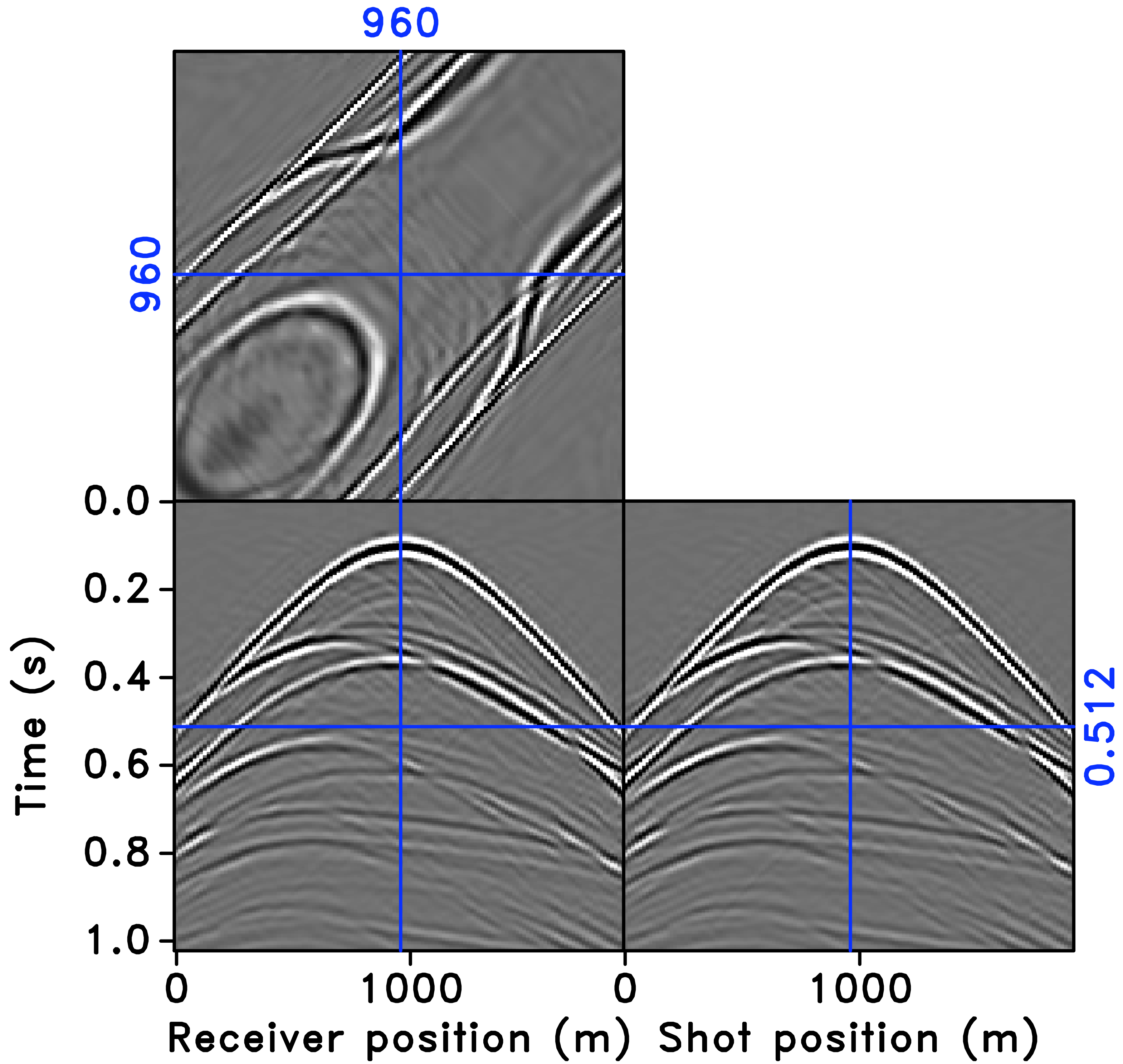

960
32
0.504
0
0.2
0.4
0.6
0.8
1
Time (s)
0
500
1000
1500
Shot position (m)
0
50
100
Shot#

Q

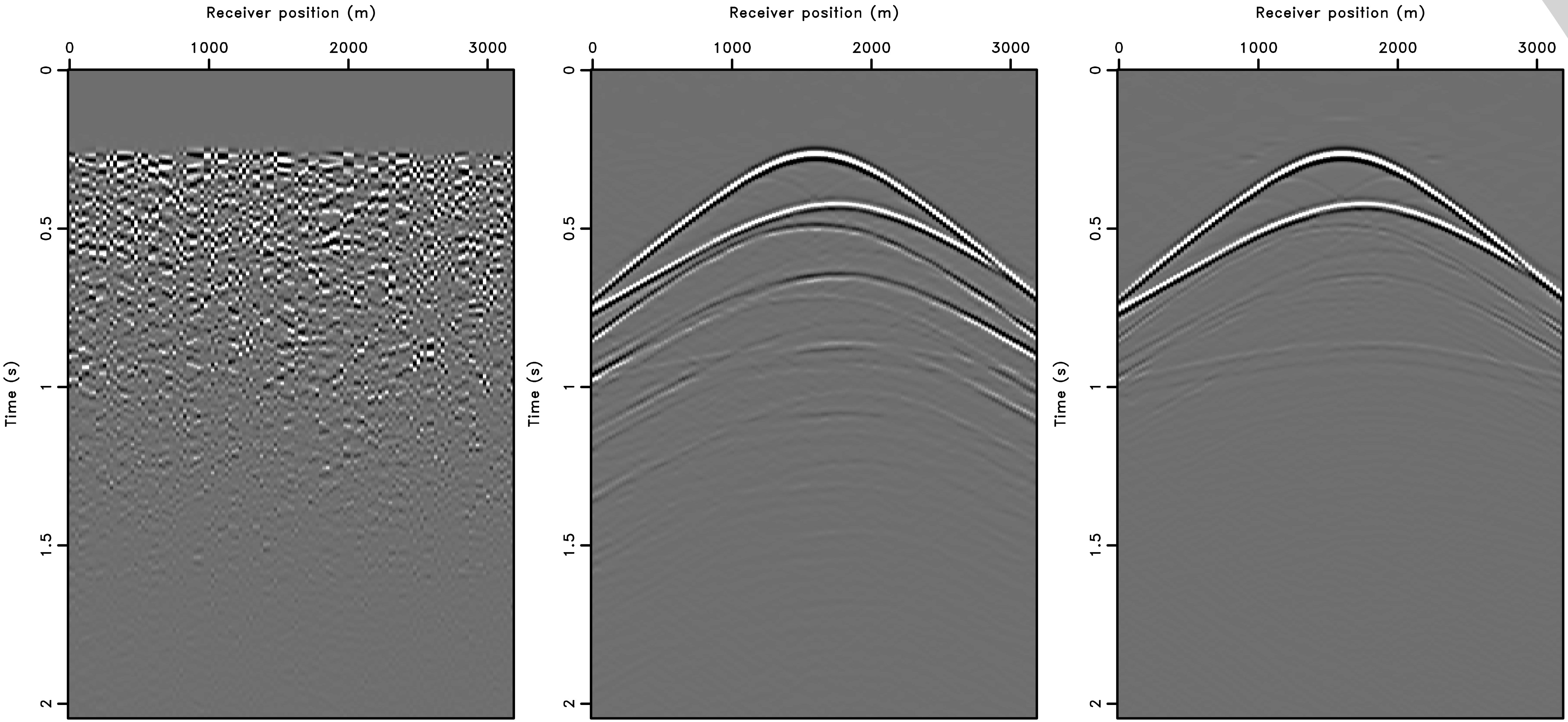
matched filter



L1 inversion

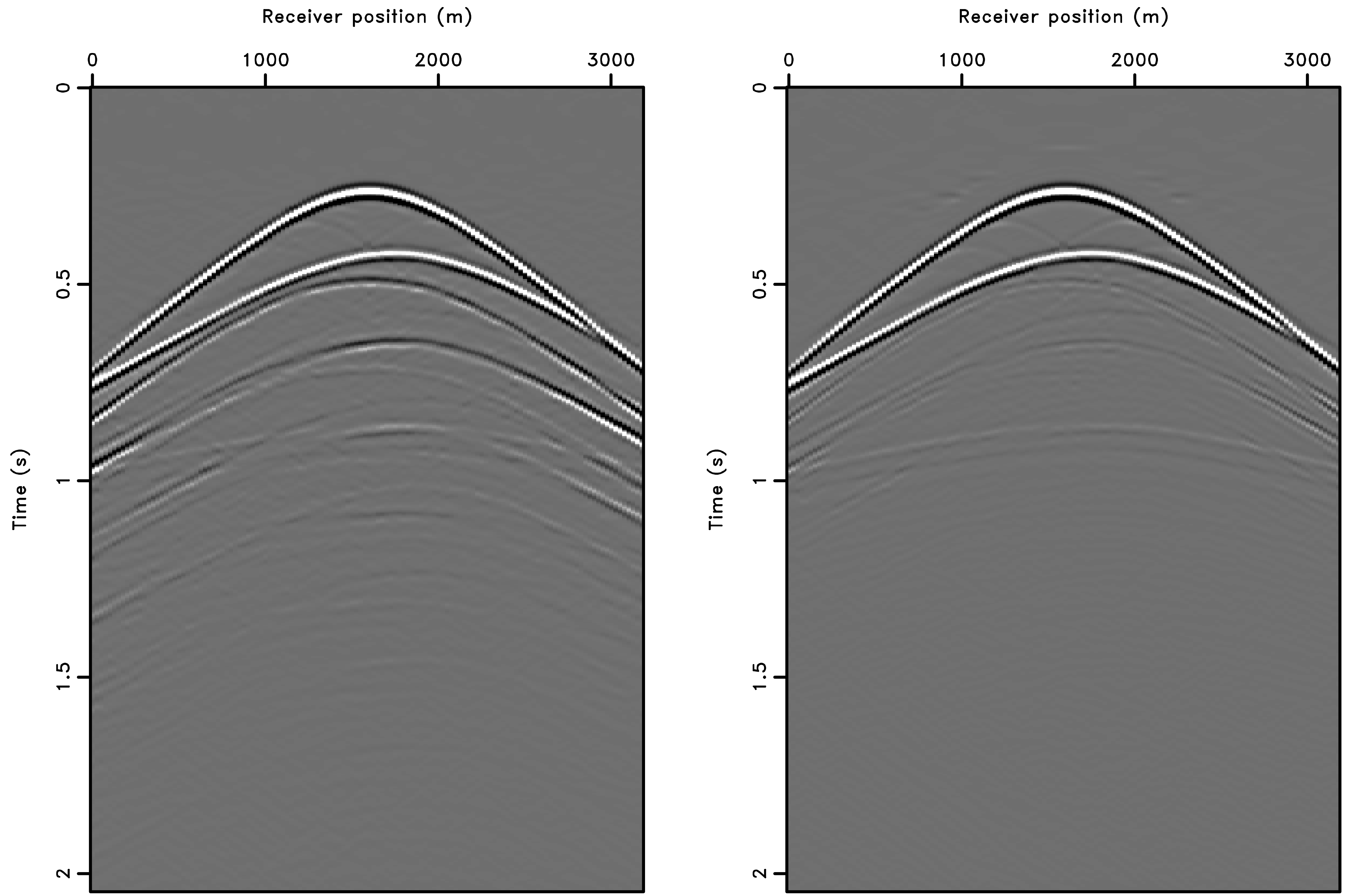


75x


Receiver position (m)
0
1000
2000
3000
Time (s)
0
0.5
1
1.5
2
Receiver position (m)
0
1000
2000
3000
Time (s)
0
0.5
1
1.5
2
Receiver position (m)
0
1000
2000
3000
Time (s)
0
0.5
1
1.5
2

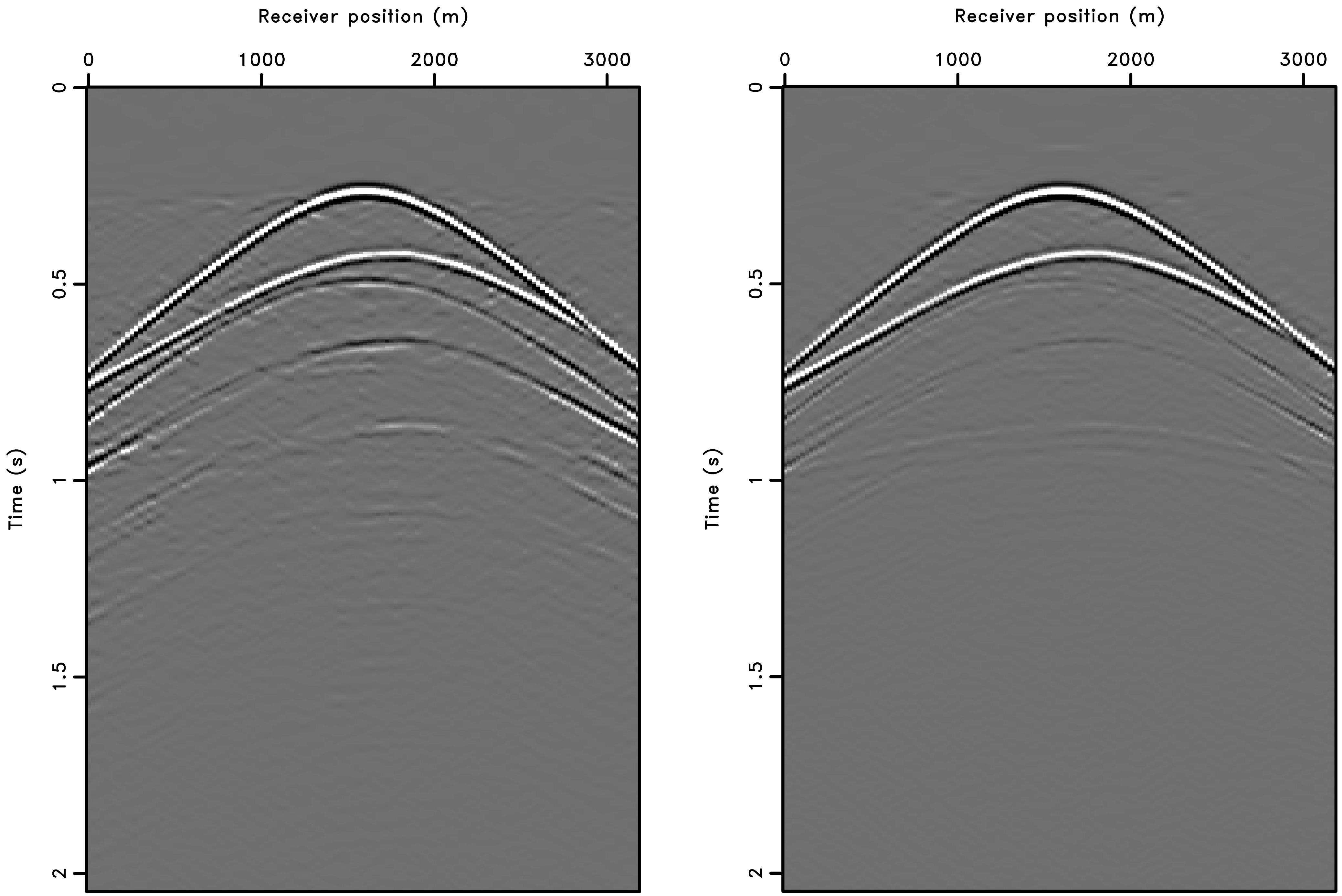
60x

2x downsample



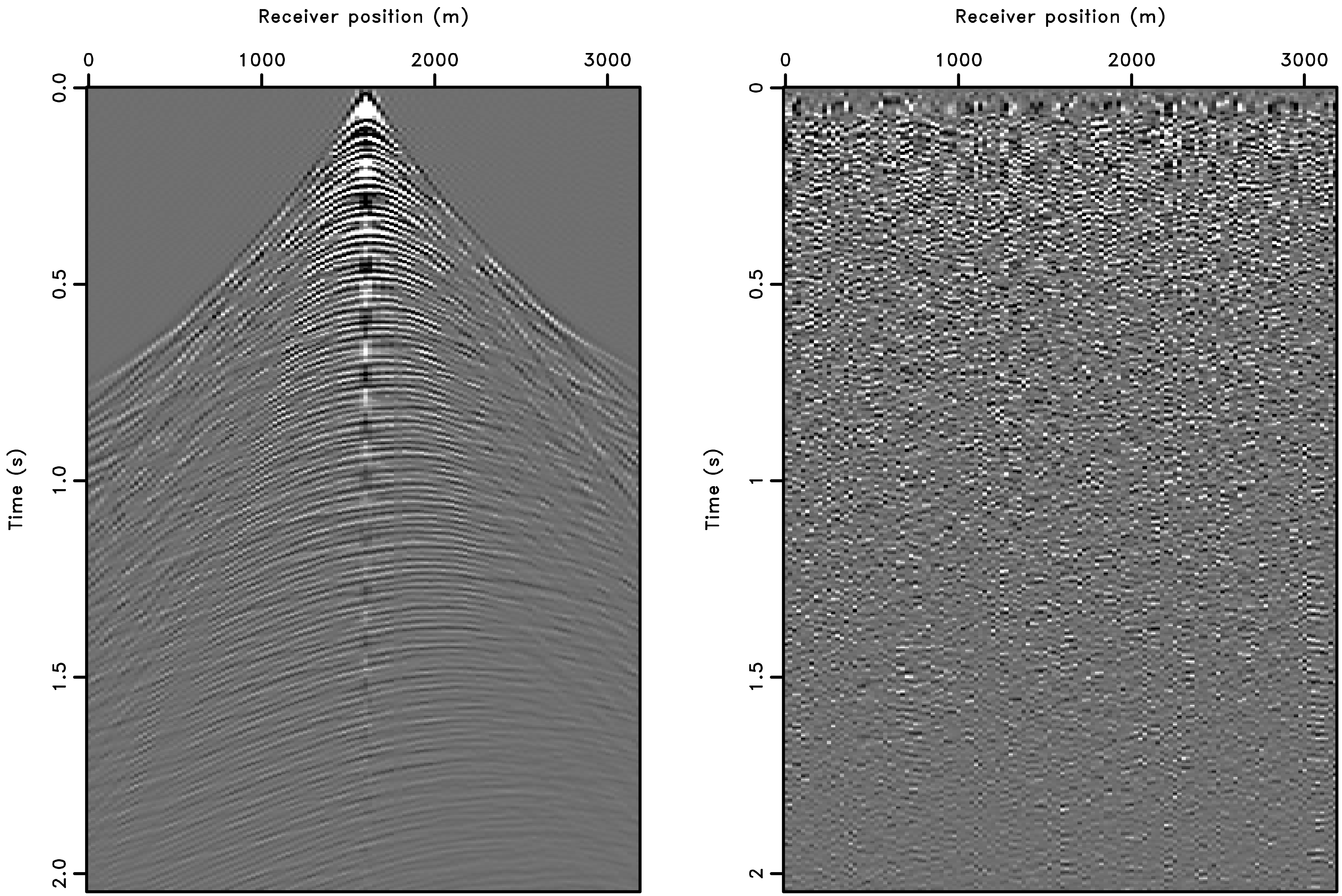
60x

5x downsample

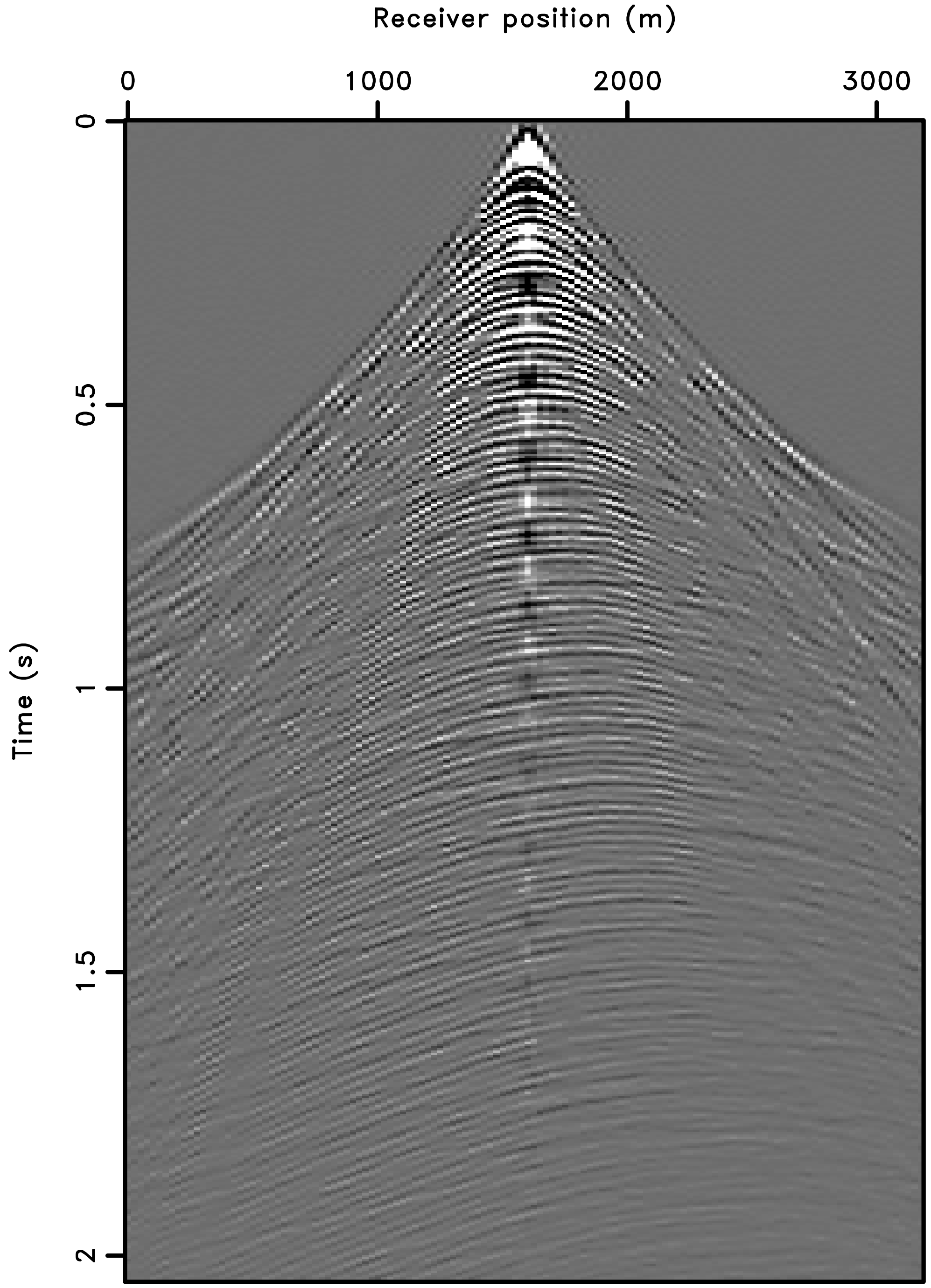


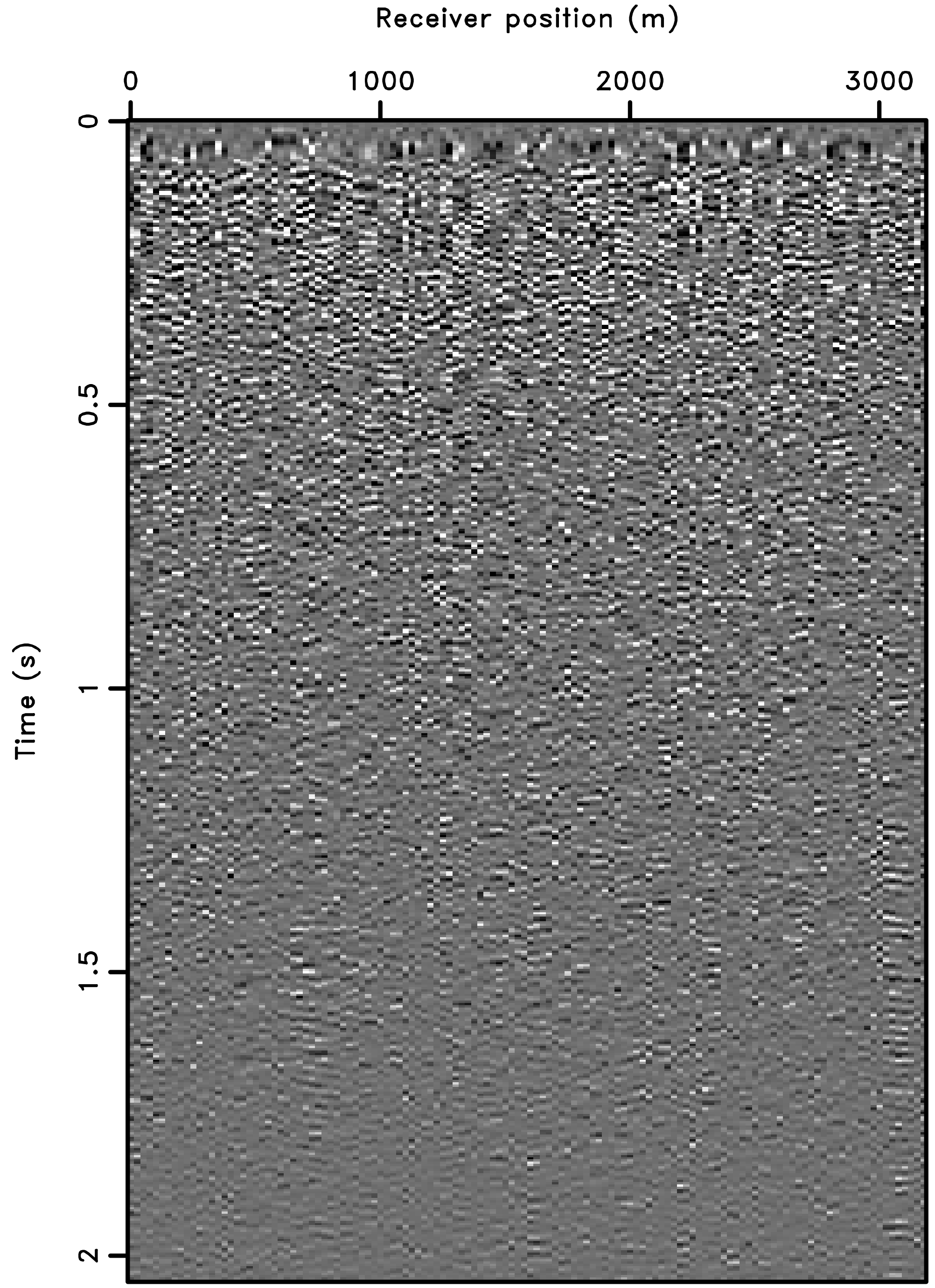
60x

real marine data



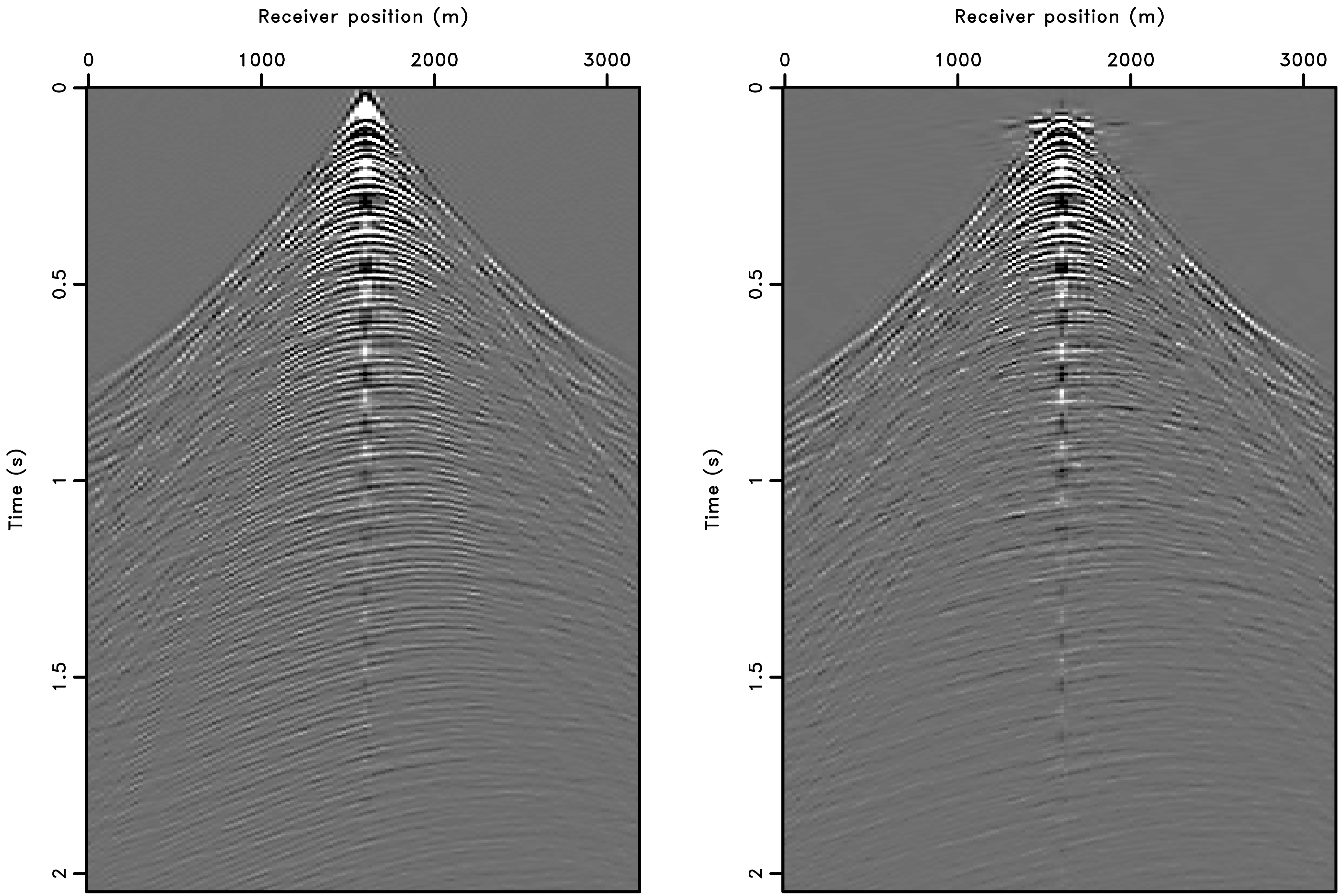
separate





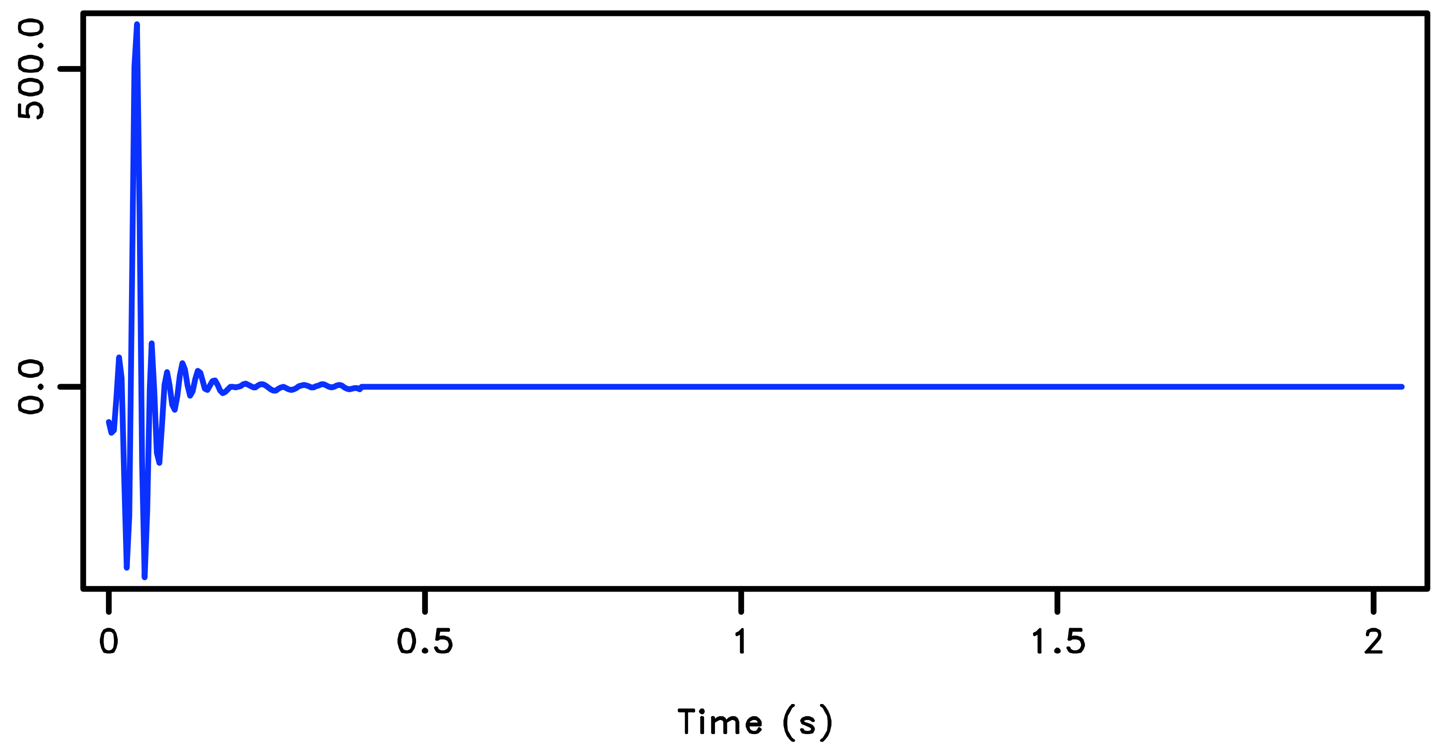
85x

primary estimation

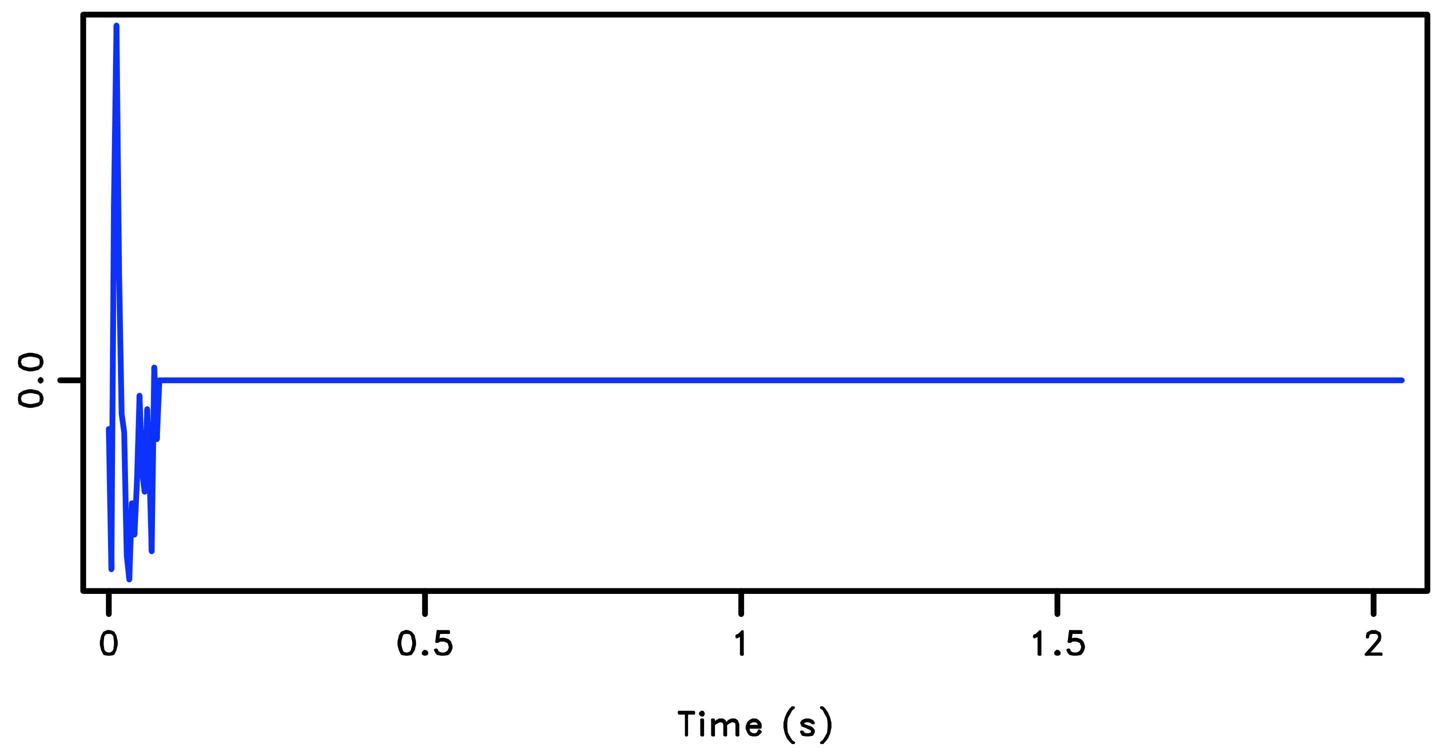


85x

wavelet estimation



synth



marine

conclusions

Acquisition is ***sampling***

Acquisition is *compressive sampling*

conclusions

sampling is tied to *sparsity*

conclusions

sparse inversion gets your data

conclusions

Compressive sensing

is the framework that ties it together

bedankt

-The DELPHI team

-BP

-Current sponsors of SLIM