Introduction

Seismic data acquisition forms one of the main bottlenecks in seismic imaging and inversion. Acquisition by itself is the most costly amongst the different steps that lead to images of the Earth's subsurface. Moreover, acquisition is also hampered by mundane physical and financial constraints that are related to practically feasible designs of acquisition grids and seismic sources. Physical constrains, for instance, typically lead to data with missing traces that require interpolation, a post-processing step called seismic regularization, and deconvolution for the (sweep) source functions. Similarly, simultaneous acquisition—aimed at improving the performance of marine- and land-acquisition crews—calls for development of a new set of design principles and post-processing tools. In this paper, we focus on new techniques to separate (demultiplex) simultaneously acquired data (Beasley, 2008; Krohn and Neelamani, 2008; Berkhout, 2008; Neelamani et al., 2008; Herrmann et al., 2009). Because multiple sources are fired continuously and simultaneously, blended acquisition should in principle reduce the acquisition costs at the expense of minor costs in post processing to obtain conventional source-separated (and deconvolved) data volumes.

In our approach, we leverage developments from the field of compressive sensing (CS in short throughout the paper, Candès et al., 2006; Donoho, 2006)—where the argument is made, and rigorously proven—that compressible signals can be recovered from severely sub-Nyquist sampling by solving a sparsity promoting program. The CS approach differs from most simultaneous acquisition/recovery schemes because it combines three indispensable components namely, (i) the design of sub-sampling schemes that turn coherent sub-sampling interferences into harmless Gaussian-like noise (see e.g. Hennenfent and Herrmann (2008), where this principle is used to recover seismic data volumes from missing traces with curvelet-domain sparsity), (ii) the selection of a sparsifying domain (such as curvelets) in which the data can be represented parsimoniously, and (iii) the use of sparsity-promoting programs to recover the source-separated data volumes. As long as a sampling/recovery scheme adheres to this principles (this is not true for the majority of the current simultaneous acquisition strategies where typically one of the components is missing), CS guarantees recovery to high fidelity as long as the degree of subsampling is commensurate with the transform-domain sparsity. This means that sparser signals allow for larger degrees of subsamplings as long as these subsamplings are carried out according to CS principles.

This finding has profound implications for acquisition because the costs are now no longer determined by the size of the acquisition grid but by the transform-domain sparsity instead. By using CS principles, we present a rigorous framework for simultaneous acquisition and subsequent recovery by sparsity promotion. For the actual design of the subsampling scheme, we are motivated by recent work of Neelamani et al. (2008), and Herrmann et al. (2009) where phase-encoded simultaneous sources were used to reduce the computational cost of wavefield simulations. Difference, here is that we include a sweep function that makes our methodology relevant for land acquisition with vibroseis trucks. We use the principle of superposition, which allows us to work with simultaneous sources where all sources are firing.

Simultaneous sources as a case of compressive sensing

Compressive sensing (CS) theory proves that recovery through sparsity promotion is possible from a sample size m that is proportional to the signal's sparsity (here, the number of nonzeros, k) as opposed to the signal length N. The main contribution of this paper is to recognize simultaneous acquisition as an instance of compressive sensing and taking advantage of it in our formulation of the simultaneous acquisition problem. The benefits afforded by a well-designed subsampling scheme designed according to the compressive sensing framework are:

(i) improved demultiplexing into source-separated data volumes by recovery through transform-domain sparsity promotion

(ii) compression of imaging operators through a reduction of the number of sources (i.e., number of right-hand sides for the wavefield simulators) and number of frequencies per simultaneous source (i.e., the number of block-diagonals in the discretization of the Helmholtz equation) (Herrmann et al., 2009).

Before discussing an example of practical source design for this subsampling scheme, let us first review the different components that allow us to recover from simultaneous data.

The compressive-sampling matrix: The success of compressive simulation depends on devising a subsampling of the physically distinct source and frequency axes where coherent interferences are turned into random noise (Hennenfent and Herrmann, 2008). Since speed is of the essence for the recovery, we follow recent work by Romberg (2008) and implement the CS matrix through a random phase encoder in Fourier space. To maximize independence amongst the sources, we apply different restrictions for each of the n'_s simultaneous shots—i.e., we have

$$\mathbf{RM} = \overbrace{\begin{bmatrix} \mathbf{R}_1^{\Sigma} \otimes \mathbf{I} \otimes \mathbf{R}_1^{\Omega} \\ \vdots \\ \mathbf{R}_{n_{s'}}^{\Sigma} \otimes \mathbf{I} \otimes \mathbf{R}_{n_{s'}}^{\Omega} \end{bmatrix}}^{\text{sub sampler}} \overbrace{\left(\mathbf{F}_2^* \mathrm{diag}\left(e^{\hat{i}\boldsymbol{\theta}}\right) \otimes \mathbf{I} \right) \mathbf{F}_3}^{\text{random phase encoder}}, \tag{1}$$

with $\mathbf{F}_{2,3}$ the 2,3-D Fourier transforms, and $\boldsymbol{\theta} = \mathrm{Uniform}([0, 2\pi])$ a random phase rotation. The matrices $\mathbf{R}^{\Omega}$ and $\mathbf{R}^{\Sigma}$ represent CS-subsampling operators acting along the rows (frequency coordinate) and columns (shot coordinate) of the data volume, respectively. As shown by Herrmann et al. (2009) application of this CS-sampling matrix, $\mathbf{RM}$, to the data is equivalent to applying it to the source wavefields directly turning single-source shots into a subset ($n'_s \ll n_s$ with n_s the number of separated single-source shots) of time-harmonic simultaneous sources that are randomly phase encoded and that have for each simultaneous shot a different set of angular frequencies missing—i.e., there are $n'_f \ll n_f$ (with n_f the number of frequencies of fully sampled data) frequencies non-zero.

The sparsfying transform: Aside from proper CS sampling the recovery from simultaneous simulations depends on a sparsifying transform that compresses seismic data, is fast, and reasonably incoherent with the CS sampling matrix. We accomplish this by defining the sparsity transform as the Kronecker product between the 2-D discrete curvelet transform (Candès et al., 2006) along the source-receiver coordinates, and the discrete wavelet transform along the time coordinate—i.e., $\mathbf{S} := \mathbf{C} \otimes \mathbf{W}$ with $\mathbf{C}$, $\mathbf{W}$ the curvelet- and wavelet-transform matrices, respectively.

Recovery by sparsity promotion: We reconstruct the seismic wavefield by solving the following nonlinear optimization problem

$$\widetilde{\mathbf{x}} = \arg\min_{\mathbf{x}} \|\mathbf{x}\|_1 \quad \text{subject to} \quad \mathbf{A}\mathbf{x} = \mathbf{y}, \tag{2}$$

with $\widetilde{\mathbf{d}} = \mathbf{S}^*\widetilde{\mathbf{x}}$ recovered data, $\mathbf{A} := \mathbf{RMS}^*$ the CS matrix, and $\mathbf{y}$ the compressive simulation. This is solved with SPGℓ_1, a projected-gradient algorithm with root finding (Berg and Friedlander, 2008).

Phase-encoded vibroseis sweeps

Even though the above sampling scheme is viable when reducing simulation cost during forward modelling, straightforward application of this scheme to seismic acquisition is impractical. For instance, vibroseis trucks have physical constraints that limit the type of source functions they can emit. To incorporate this limitation, we slightly refine the CS-sampling matrix defined in

Eq. 1 by replacing the identity matrix, $\mathbf{I}$ in the phase encoder, by $\mathbf{\Psi} = \mathrm{diag}\{\psi\}$, where ψ is a discretization of a typical frequency sweep defined according to $\psi(t) = \cos\left(2\pi(f_b t + \frac{f_e}{2}t^2)\right)$ with $f_b = 10\,\mathrm{Hz}$ and $f_e = 80\,\mathrm{Hz}$.

Simulation experiments

To illustrate CS-recovery quality, we conduct a series of experiments for two velocity models, namely the complex model used in Herrmann et al. (2007), and a simple single-layer model. These models generate seismic lines that differ in complexity. During these experiments, we vary the subsampling ratio and the frequency-to-shot subsampling ratio. All simulations are carried out with a fully parallel Helmholtz solver with $\beta = 0.5$, for a spread with 128 col-located shots and receivers sampled at a 15 m interval, convolved with a Ricker wavelet with a central frequency of 10Hz to simulate natural earth wave dispersion. The time sample interval is $0.004\,\mathrm{s}$ and the source function is a frequency sweep from 1Hz to 250Hz. By solving a compressive sensing problem, we recover the full simulation for the two datasets with $80\,\%$ of the data missing ($90\,\%$ for the case of the simple model) while in effect simultaneous deconvolving for the frequency sweep. As expected, we can get similar results with the simple model using less of the shot and frequency information is better because of the reduced complexity. Because the speedup of the solution is roughly proportional to the subsampling ratio, we can conclude that speedups of four to six times are possible at the expense of a minor drop image quality.

Discussion and conclusions

Compressive sampling is considered a paradigm shift, and we have shown that simultaneous acquisition is a natural candidate for the application of its principles. Savings in acquisition time can be achieved through a deliberate reduction in the number of sweeps recorded, the extent of which we know through CS is commiserate with the complexity of the wavefield rather than the number of shot positions. This method is promising in cases where we have control over the sources, such as land surveys with vibroseis sources. Furthermore, there is an opportunity to further reduce data transmission and processing cost achieved through an additional subsampling in the frequency domain. Our proof of principle in our opinion, is encouraging and invites further investigation into the design and implementation of new acquisition schemes based on the principles from compressive sensing.

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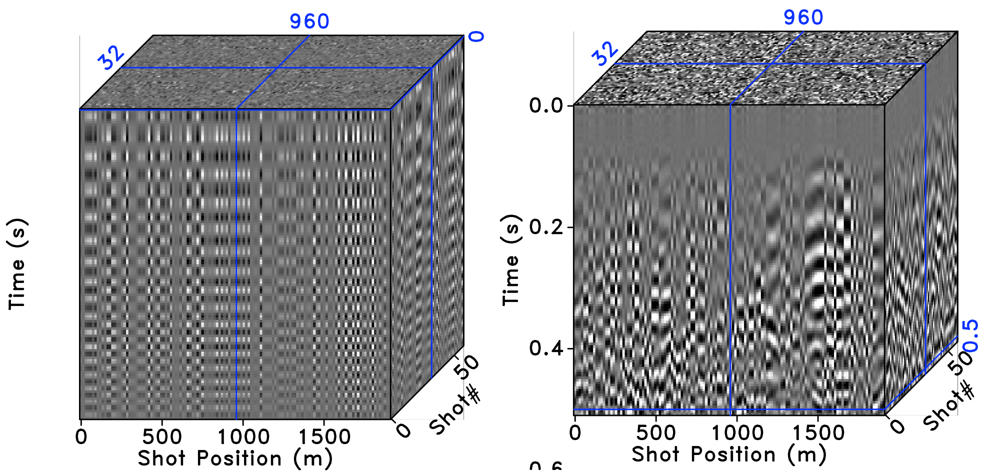

Figure 1: Acquisition design with compressive sensing. **(left)** Seismic source cube containing randomly phase encoded frequency sweeps at all shot locations. Note that there are fewer total shots compared to the number of shot locations. **(right)** Simulated data for the simultaneous sources.

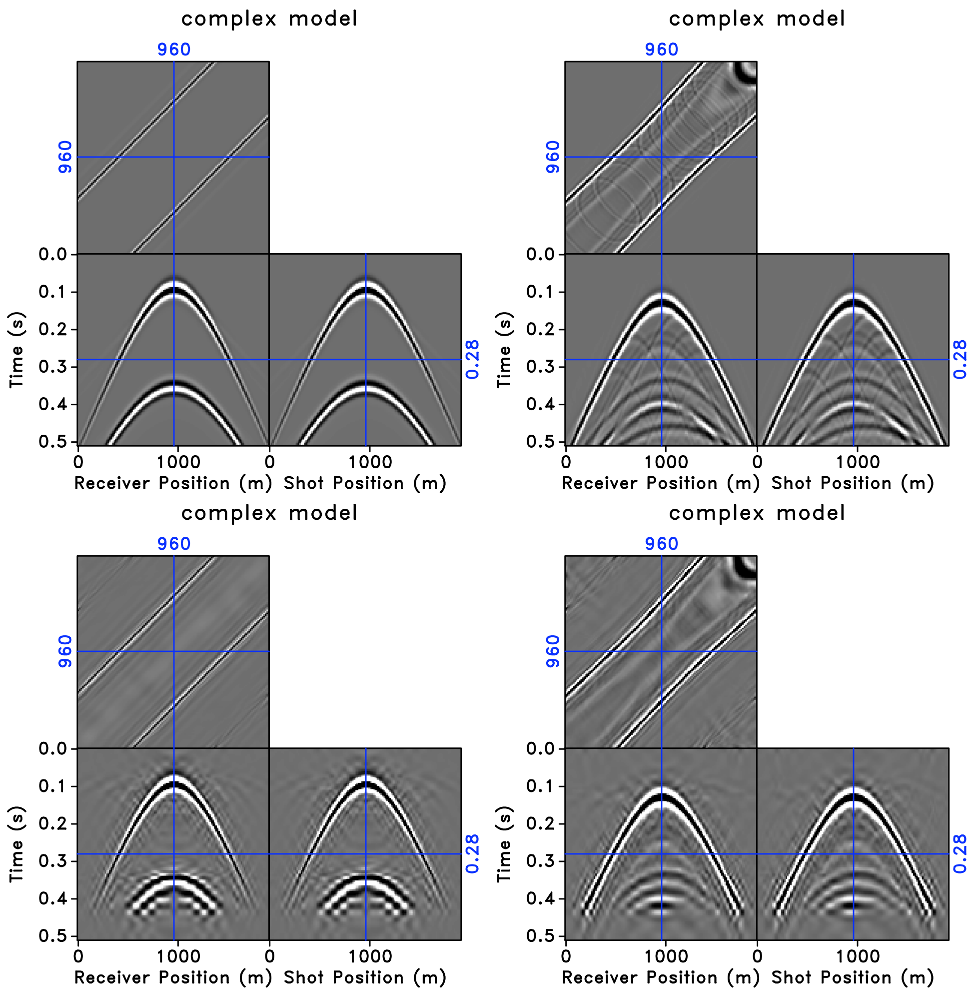

Figure 2: Numerical simulation comparison between conventional and compressive acquisition for simple and complex velocity models. **(tl)** Seismic line for the simple model. **(tr)** The same for the complex model. **(bl)**. Recovered simulation for the simple model from 10 % of the samples. **(br)** The same but for the complex model from 20 % of the samples..