INTRODUCTION

Removal of surface waves is an integral step in seismic processing. There are many standard techniques for removal of this type of coherent noise, such as f-k filtering, but these methods are not always effective. One of the common problems with removal of surface waves is that they tend to be aliased in the frequency domain. This can make removal difficult and affect the frequency content of the reflector signals, as this signals will not be completely separated. As seen in Hennenfent and Herrmann (2006) interpolation can be used effectively to re-sample the seismic record thus dealiasing the surface waves. This separates the signals in the frequency domain allowing for a more precise and complete removal.

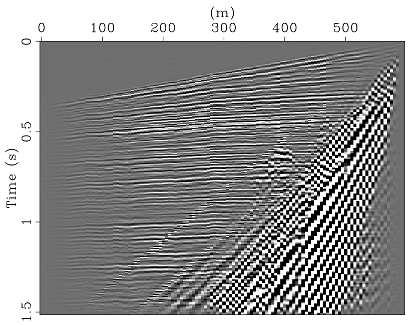

Figure 1: Original data set containing a high amplitude aliased surface wave. This data also contains the air blast as another linear signal.

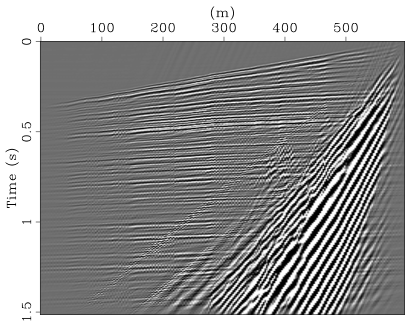

Figure 2: Data set after CRSI to remove aliasing. The spacial sampling of the signal has been increased by synthetically doubling the amount of geophones in the survey.

We aim to remove surface waves while preserving the frequency content of the reflected seismic signal. To accomplish this, we will use a two step method that includes generating a prediction for the surface wave and then facilitating the separation with a Curvelet based block solver method. This will allow us to minimize the effect of surface wave removal on reflector signals and their frequency content.

We will use the properties of the image to generate a surface wave prediction, which will be used in the separation. A prediction will then be generated by exploiting the sparsity of wave fields in the Curvelet domain. The benefits of these methods are that they do not require any prior knowledge relating to a velocity model or survey design.

From a calculated prediction, we will use an iterative Curvelet based block solver to facilitate the separation between the predicted surface wave and the original data set. By performing a block-coordinate relaxation method in this domain, we generate a more robust and accurate separation scheme that allows us to take advantage of the sparsity of seismic waveforms in the Curvelet domain. By allowing our solver to match information with the data set and the prediction in the Curvelet domain instead of simply performing an f-k filtering scheme, the specific waveforms and their characteristics are identified and removed instead of wide spanning frequency content which may degrade reflector signal.

PREDICTION

The Curvelet transform has several properties that make it useful for producing surface wave estimations. The multi-scale nature of the Curvelet transform allows the separate components of the surface wave and reflector signals to be mapped do individual parts of the Curvelet vector. The velocity and thus the angle of the wave front also affects the mapping to the Curvelet domain as it appears in the data. By knowing the location of the coefficients that relate to specific scales and angles, different components of the wave fronts can be identified. By thresholding out the components that are not associated with the surface waves, and then reconstructing the image, which can generate a prediction that may be used in the block solver.

Surface waves can be found in a variety of forms. Generally what we expect is a low frequency, high amplitude wave travelling at low velocity. This would map to high angles and low scales in the Curvelet domain. If the surface wave is poorly sampled, we my see surface waves that have considerable amounts of aliasing. This may give the appearance of a higher frequency wave. In this case, we expect the surface wave to appear still at high angles but also at high scales. By utilizing the scale and angle separation in the Curvelet domain, we do not limit ourselves to simply frequency-based prediction methods and even highly aliased surface waves may be predicted.

CURVELET-BASED BLOCK COORDINATE RELAXATION

The separation method is based on a Curvelet-based block solver Yarham et al. (2006) that solves a non-linear optimization problem for an unknown augmented vector $x_0 = [x_1, x_2]^T$ with two signal components corresponding to the reflector and surface wave signals. We frame this simply as $y = Ax_0$, with y representing the total data set including coherent and white noise components as well as reflectors. $A = [C^T, C^T]$ with the Curvelet synthesis matrix and C^T representing the inverse Curvelet transform. The strength behind this method is that it transforms the different wave forms into the Curvelet domain where they are sparsely represented by separate coefficients. We use our first estimation for the surface wave $\tilde{s_2}$, which generates our first estimate for our reflector signal as $\tilde{s_1} = s - \tilde{s_2}$ with s as representing the full data set. By using these estimates as weights in the following optimization problem

$$\hat{x_j} = argmin_x \frac{1}{2} ||y - A_j x_j - \sum_{i \neq j} A_i x_i||_2^2 + ||x_j||_{1,\lambda_m \cdot w_j} \; j = 1, \cdots, 2 \tag{1}$$

an estimate for x is obtained. From this estimate the reflected signal and surface wave follow according to $\hat{s_1} = C^H \hat{x_1}$ and $\hat{s_2} = C^H \hat{x_2}$. The weights in the above expression are set according to

$$\begin{cases} w_1 := C_1 |\tilde{s_2}| \\ w_2 := C_2 |\tilde{s_1}| \end{cases} \tag{2}$$

The optimization problem forms an iterative loop with the constants of C_1 and C_2 controlled by the L_1 norms of the weighted predictions. This loop repeats as long as $\epsilon > \beta$ with $\epsilon = \hat{s} - (\hat{e}s_1 + \hat{s_2})$ and β representing and acceptable separation error. By working in this fashion, we have control over the accuracy and amount of resources allocated to this problem and can ensure an acceptable separation.

One of the benefits of using a block solver approach is the ability to connect separate predictions and removal schemes without affecting reflector information. It also allows for a fairly wide tolerance of location, phase, amplitude and angle of the prediction. In general, as long as the prediction does not contain information pertaining to reflectors, the reflectors will be preserved. This allows for 'chaining' of this method where a prediction is made and used in a separation that may not have removed all the surface wave information.

METHOD APPLICATION

There are several benefits to our removal scheme. First, they allow completely individual separations of the signal components. It has been shown by Starck et al. (2004) that this iterative soft thresholding is equivalent to solving an L_1 minimization. Secondly, the two output signals are separated from each other allowing for a step-wise removal scheme. Third, the precision of separation may be controlled allowing for a balance between computational resource use and quality of the final separation to be formed.

To begin, a prediction is generated in some fashion. The method of prediction generation that we will utilize will focus around Curvelet based predictions but any effective method that produces a prediction mimicking the properties of the surface wave may be used. This prediction is used to define the weighted soft thresholding which facilitates the signal separation.

The block solver works in a similar manner to that developed by Elad et al. (2005) with a few adaptations. In the case, two separate transforms were utilized to separate out two separate signals, both of which were sparse in there respective domains. In our case, we use only the Curvelet transform. This is due to the fact that both of the seismic waveforms are sparse in the Curvelet domain. To facilitate signal separation, an appropriate weighting scheme is employed that will identify the coefficients of the separate signal components.

We use our surface wave prediction, $\tilde{s_2}$ to generate our second estimate,$\tilde{s_1}$, as above. From here, the first estimation is fixed while the second estimation is updated by a weighted thresholding scheme. Following that, the second estimation is held fixed and the first is updated. The process repeats while a λ is reduced after every pair to facilitate a cooling method. This method drives the two signals apart iteratively leaving us with our calculated noise based on our estimate and our predicted reflector signals.

NUMERICAL RESULTS

We will use the data set shown in figure 2 as an example. This data set has been dealiased by re-sampling and interpolation as done through Curvelet Reconstruction with Sparsity promoting Inversion (CRSI) from the data set shown in figure 1.

A series of predictions was made based on the scale and angles of the surface wave in the Curvelet domain. These were then removed with the Curvelet based block solver. The final result is shown in figure 3. For comparison, a standard f-k filtering was also done and this result is shown in Figure 5. They look comparable until we examine the frequency spectrum. As we would expect, f-k filtering will eliminate large sections of the frequency spectrum and eliminate all low frequency components regardless of if they belong to the reflector or the surface waves. Another thing that we notice is that there is considerable smoothing associated with the f-k filtering which we do not see with the Curvelet denoised data.

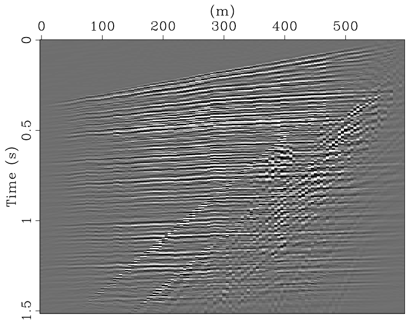

Figure 3: Curvelet denoised data set. The majority of the surface wave has been removed and the reflector data has been preserved.

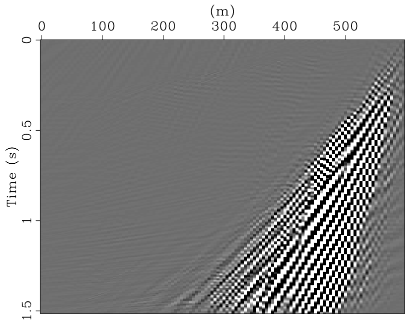

Figure 4: Curvelet removed surface wave. Notice how no reflector information has been removed.

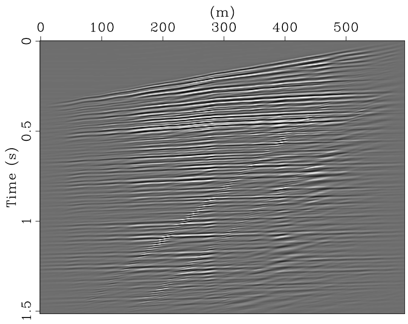

Figure 5: Standard f-k filtered data set.

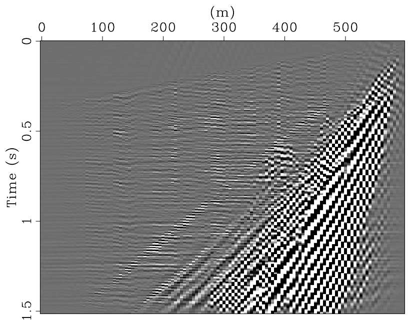

Figure 6: F-K filtered surface wave. Due to smoothing, some reflector information has been removed.

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