ABSTRACT

Seismic reflector characterization by a multiscale detection-estimation method

Seismic transitions of the subsurface are typically considered as zero-order singularities (step functions). According to this model, the conventional deconvolution problem aims at recovering the seismic reflectivity as a sparse spike train. However, recent multiscale analysis on sedimentary records revealed the existence of accumulations of varying order singularities in the subsurface, which give rise to fractional-order discontinuities. This observation not only calls for a richer class of seismic reflection waveforms, but it also requires a different methodology to detect and characterize these reflection events. For instance, the assumptions underlying conventional deconvolution no longer hold. Because of the bandwidth limitation of seismic data, multiscale analysis methods based on the decay rate of wavelet coefficients may yield ambiguous results. We avoid this problem by formulating the estimation of the singularity orders by a parametric nonlinear inversion method.

Introduction

The earth's subsurface consists of layers of different materials separated by interfaces, also called transitions. Transitions are characteristic of regions where the acoustic properties of the earth vary rapidly compared to the length-scale of the seismic source wavelet. Extracting information on the locations and nature of the transitions from seismic data has recently received increasing interest, as it provides quantitative information that can be used in various applications, ranging from improving the geological interpretation of the subsurface to detecting changes in the lithology.

Seismic deconvolution is aimed at finding the locations of seismic reflectors and is based on certain assumptions on the reflectivity. For instance, deconvolution methods have been developed for a reflectivity that behaves as a (colored) Gaussian random process (Saggaf & Robinson, 2000). Other approaches are based on the sparsity assumption, where the reflectivity is considered to be given by a sparse spike train. Multiscale analysis on well and seismic data have shown that neither assumption is rich enough to describe the different types of transitions present in sedimentary basins (Herrmann, 2001). These observations have led to the introduction of a new type

of parametrization, where the observed reflectivity is written as a superposition of parametrized waveforms. This parametrization is designed to reflect the presence of transitions other than strictly zero-order transitions (blocked wells) and includes fractional-order transitions. The aim of this paper is two-fold, namely finding the locations of the reflectors, delineating the *stratigraphy*, and extracting information on the *nature* of the transitions. We present a new detection-estimation method, where the reflection events are first detected, then segmented, followed by an estimation based on a descent method (Boyd & Vandenberghe, 2004). The estimated parameters provide information on the transition sharpness that is related to the lithology (Herrmann, 2001; Liner *et al.*, 2004), possibly through a critical point in the elastic moduli (Herrmann & Bernabé, 2004).

The Earth's Model: We represent a vertical 1-D profile of the earth as a superposition of parametrized waveforms of the following type

$$s(z) = \sum_j c_j D^{\alpha_j} \psi(z - z_j), \tag{1}$$

where z is depth, c_j the amplitude for the j^{th} transition, and D^α α-order operator for fractional differentiation ($\alpha > 0$) or integration ($\alpha < 0$). The ψ is some wavelet taken to be the Ricker wavelet. The parameters (attributes) of interest in this case are the location z_j, the amplitude c_j and the order α_j.

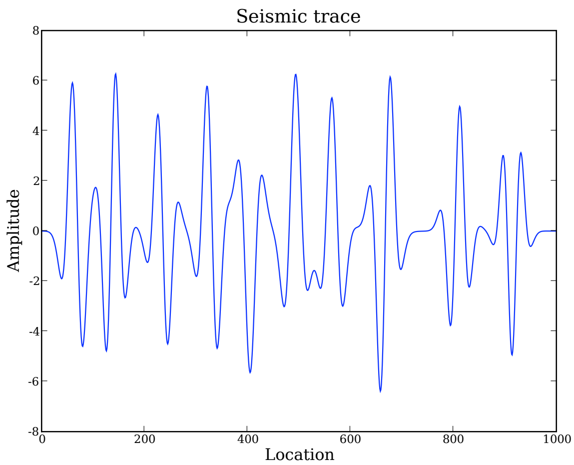
The characterization problem

Given the above signal representation, our task is to recover the different attributes from a seismic trace. We divide this task into a detection stage, where the main events in the data are detected, and an estimation stage, where the parameters of the individual waveforms are estimated. Since our problem does not fit into the classical deconvolution framework, we use a multiscale wavelet technique to locate the main events. After segmentation, the individual waveforms are submitted to a nonlinear inversion procedure to estimate the attributes. This procedure uses rough estimates for the location and scale from the detection stage.

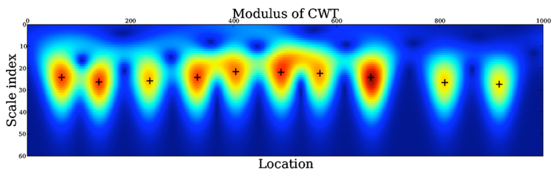
Event location by multiscale edge detection: The variety of different orders of transitions in the subsurface calls for a seismic-event-detection technique that does not make any assumptions regarding the type of transitions. Edge detection based on the multi-scale continuous (complex) wavelet transform modulus maxima (Mallat, 1999) offers an approach that is robust for different waveforms, reflecting different types of transitions. First, the method calculates the forward wavelet transform

$$\mathbb{W}s(\sigma, t) = \big(s * \bar{\psi}_\sigma\big)(t), \tag{2}$$

where $\bar{\psi}_\sigma(t) = \frac{1}{\sqrt{\sigma}}\psi^*(\frac{-t}{\sigma})$. The range of scales σ for the wavelet is adapted to the seismic source function. After forming the modulus maxima lines (MML) from the wavelet coefficients (Mallat, 1999), the maximum points along these lines are calculated, yielding rough estimates for the scale (= band width) and position of the reflection events. The result of this stage is a set of locations and scales $\{\tau^{(n)}, \sigma^{(n)}\}$ with $n = 1 \cdots N$, and N is the number of detected maxima, which corresponds to location and scale. These approximated values are subsequently used as initial guesses as part of the nonlinear inversion during the estimation stage.



(a) Synthetic seismic signal.



(b) Continuous wavelet transform of signal.

Figure 1: A typical example for the detection of a seismic trace **(a)** with seven reflection events. **(b)** The modulus of the continuous wavelet transform with warm colors corresponding to large magnitudes. The local maxima for the wavelet coefficients are used as preliminary estimates for the scale and location of the transitions. The plus signs show modulus maxima.

Partitioning: Given the estimates for the location and scale of the detected events, the trace is segmented into separate events (see Fig. 2). During segmentation, each individual waveform is calculated with

$$s^{(n)}(t) = \mathbf{W}[\tau^{(n)}, \sigma^{(n)}]s(t) \quad \text{with} \quad n = 1 \cdots N, \tag{3}$$

where $\tau^{(n)}$ and $\sigma^{(n)}$ are the locations and scales of the n^{th} detected waveform, $\mathbf{W}[.]$ is the windowing operator centered at $\tau^{(n)}$, and has a support proportional to $\sigma^{(n)}$. The output of this procedure are N vectors with 'isolated' events. Even though this segmentation procedure is somewhat arbitrary, e.g. it depends on a width parameter, we found this method to perform reasonably well for most cases. Sub-wavelength details are not extracted this way and are left to the ensuing estimation stage.

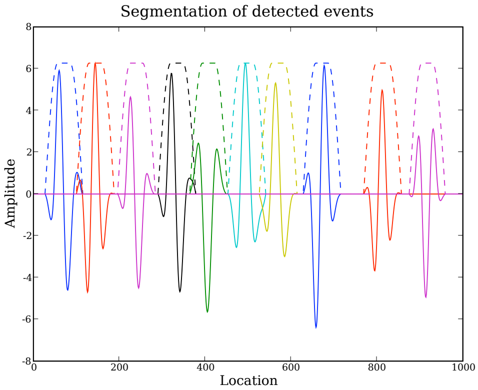


Figure 2: Segmentation of detected events. The solid lines show events and the dashed lines correspond to the action of windowing function.

Estimation: During the second stage of our method, the segmented events are subjected to a nonlinear descent-driven inversion procedure. To setup this inversion procedure, we first need to refine our mathematical model for the parametrized family of waveforms. Given our choice presented earlier, we define an individual waveform as a fractional derivative of a shifted and scaled Gaussian

$$f_\theta(t) = D^\alpha\left(\frac{1}{\sqrt{2\pi\sigma^2}} e^{(t-\tau)^2/2\sigma^2}\right), \tag{4}$$

where the scale is denoted by σ, the location by τ, and the fractional order by α. Motivated by the work of Wakin et al. (2005), we can parametrize the family of waveforms as $M[\theta] = \{f_\theta : \theta \in \mathbf{\Theta}\}$ with $\theta = [\sigma, \tau, \alpha]$. The nonlinear estimation procedure for each segmented waveform consists of minimizing

$$e^{(n)}(\theta) = \left\| s^{(n)} - M[\theta] \right\|_2^2, \tag{5}$$

or

$$\tilde{\theta}^{(n)} = \arg\min_\theta e^{(n)}(\theta), \tag{6}$$

for $\theta \in \mathbf{\Theta}$ with $\mathbf{\Theta}$ a feasible parameter range. To solve the above minimization problem, a descent method is employed that requires differentiability of the forward model with respect to its parameters (θ). Under that assumption, analytical expressions for these partial derivatives can be derived. With these partial derivatives w.r.t. θ_i, $i = 1 \cdots 3$, the descent update can be calculated and is given by

$$J_i^{(n)} = \frac{\partial e^{(n)}}{\partial \theta_i} = 2 \left\langle s^{(n)} - M[\theta], \gamma_{\theta_i} \right\rangle, \tag{7}$$

with $\gamma_{\theta_i} = \frac{\partial f_\theta}{\partial \theta_i}$, and $J_i^{(n)}$ is the projected estimation error for each parameter. During each iteration of the method, $e^{(n)}(t)$ is formed, and $\mathbf{J}^{(n)} = \{J_i^{(n)} : i = 1 \cdots 3\}$ is calculated, followed by the following update

$$\tilde{\theta}^{(n),k+1} \longleftarrow \tilde{\theta}^{(n),k} - \frac{1}{2}\mathbf{J}^{(n)} \tag{8}$$

where k is the number of iterations of the descent method. Our experience using the above scheme have shown that this iterative optimization method provides acceptable solutions to the estimation problem (see Fig. 3).

Discussion

In this paper, we have presented a new characterization method which allows for the estimation of fractional-order discontinuities. These scale attributes may lead in improvement of geological interpretation from the seismic trace. The examples we have presented, indicate that proposed characterization method is leads to accurate results. In addition, our method is well suited for the estimation of the scale exponent attributes from bandwidth limited data. As opposed to wavelet coefficient decay based methods, such as SPICE (Liner *et al.*,2004), our method does not lead to possibly ambiguous estimates since we do not rely on 'infinite' bandwidth.

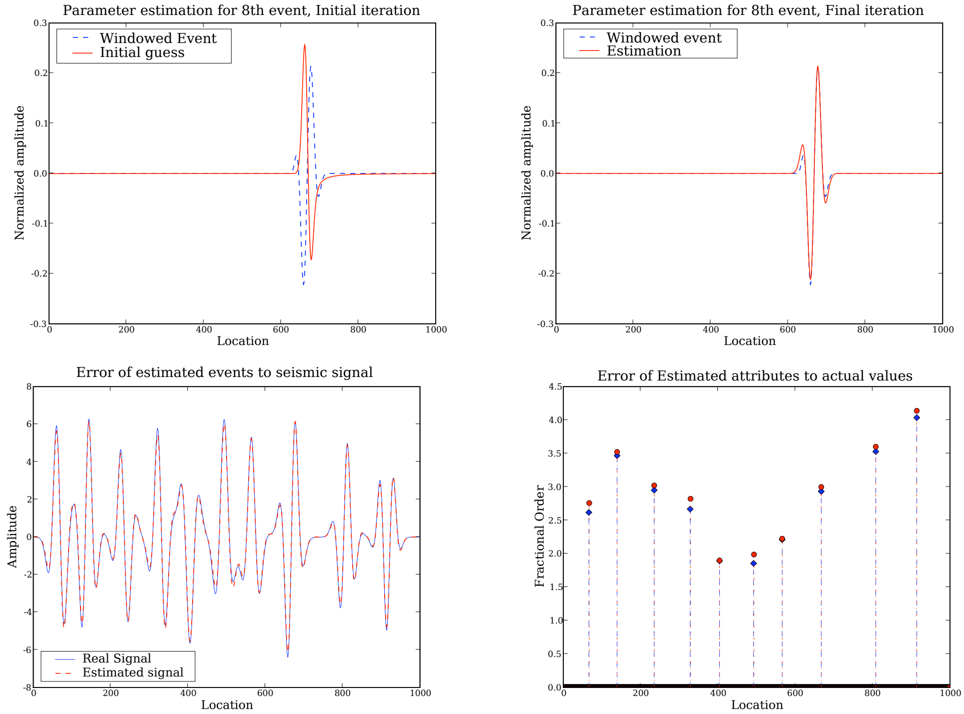


Figure 3: Example of characterization with 10 reflectivity events.**(top)** Initial and final iteration of parameter estimation for one isolated event, where the actual values, initial guess and estimation are $\theta = (12.2, 667, 2.93)$, $\theta_{\mathbf{init}} = (7.81, 668, 0.7)$, and $\tilde{\theta} = (12.72, 667, 3.01)$ respectively. The dashed line shows actual component and the solid line the estimation. **(bottom left)** Estimated seismic signal is formed by superposition of all characterized events and compared with the original seismic trace. **(bottom right)** The estimated attributes of events (τ, α) are compared to their actual values. The Blue diamonds show actual parameters whereas red the circles estimated values.

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References

[1] Boyd S., and Vandenberghe L. [2004] Convex Optimization, Cambridge University Press

[2] Herrmann, F.J., Lyons W.J., and Stark, C. [2001] Seismic facies characterization by monoscale analysis, Geophysical Research Letter, 28(19), 3781-3784.

[3] Herrmann, F.J., and Bernabé Y. [2004] Seismic singularities at upper-mantle phase transitions: a site percolation model. Geophysical Journal International, 159, 949-960

[4] Liner, C., Li, C.-F., Gersztenkorn, A., and Smythe, J. [2004] SPICE: A new general seismic attribute, 72nd Annual International Meeting, SEG, 433436 (Expanded Abstracts).

[5] Mallat, S. [1999] A wavelet tour of signal processing. Academic Press.

[6] Saggaf, M.M., and Robinson, E. A. [2000] A unified framework for the deconvolution of traces of nonwhite reflectivity. Geophysics, 65(5), 16601676.

[7] Wakin, M. B., Donoho, D. L., Choi, H., and Baraniuk R. G. [2005] The Multiscale Structure of Non-Differentiable Image Manifolds. SPIE Wavelets XI, 5914, 413-429, San Diego, (Expanded abstract).