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Surface-related multiple prediction from incomplete data

Felix J. Herrmann

joint work with Deli Wang and Gilles Hennenfent.

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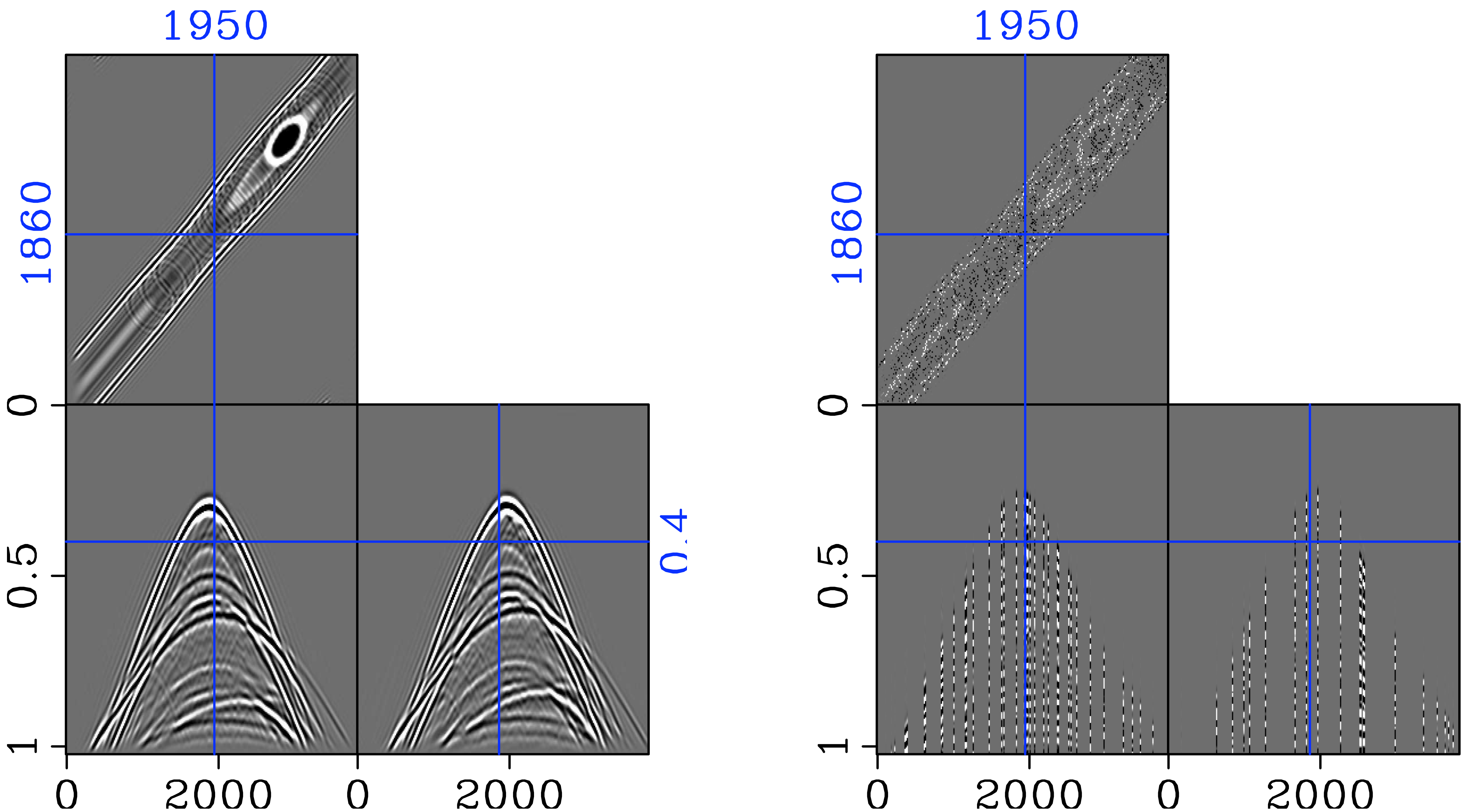
The problem



Total data

85 % traces missing

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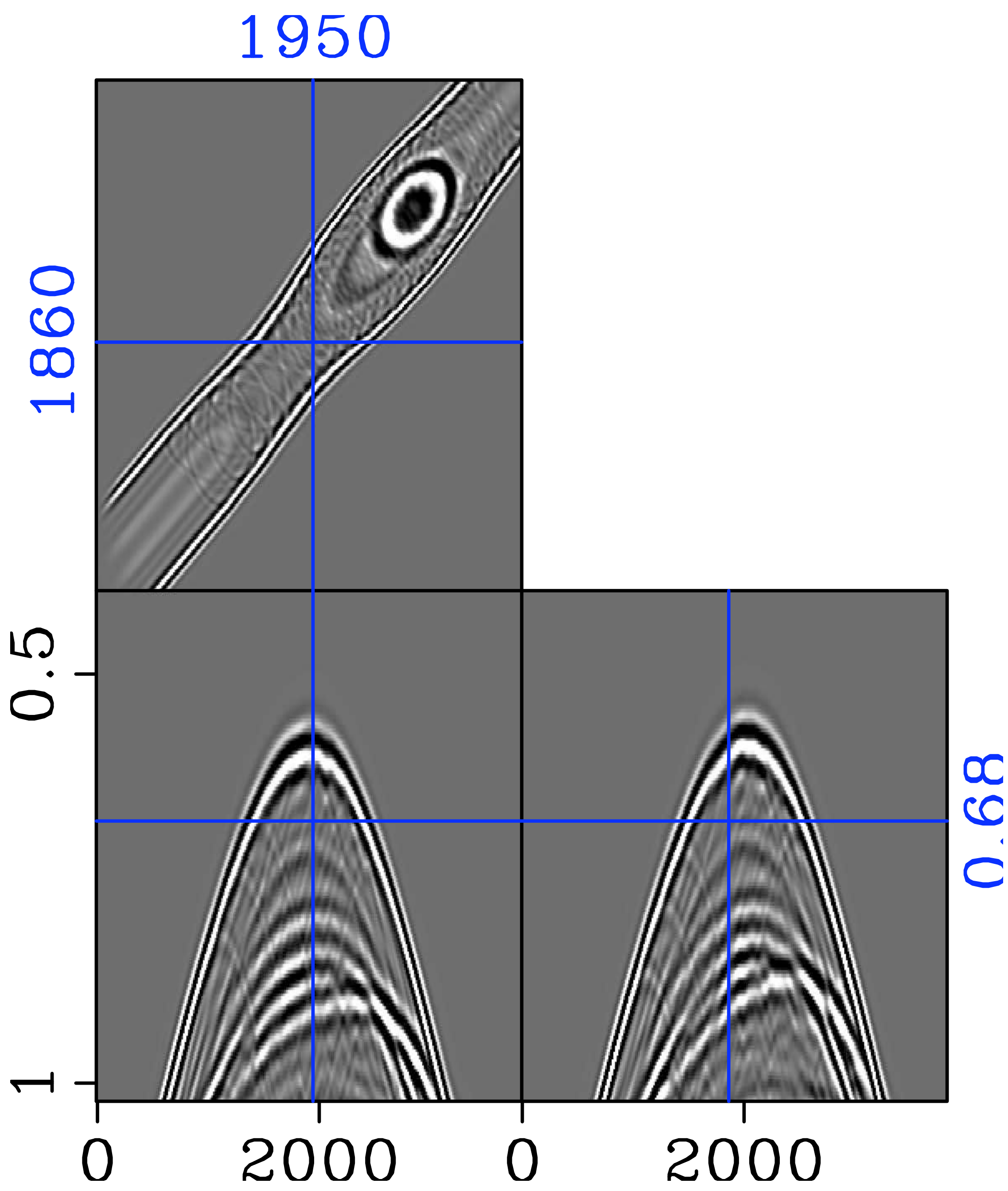
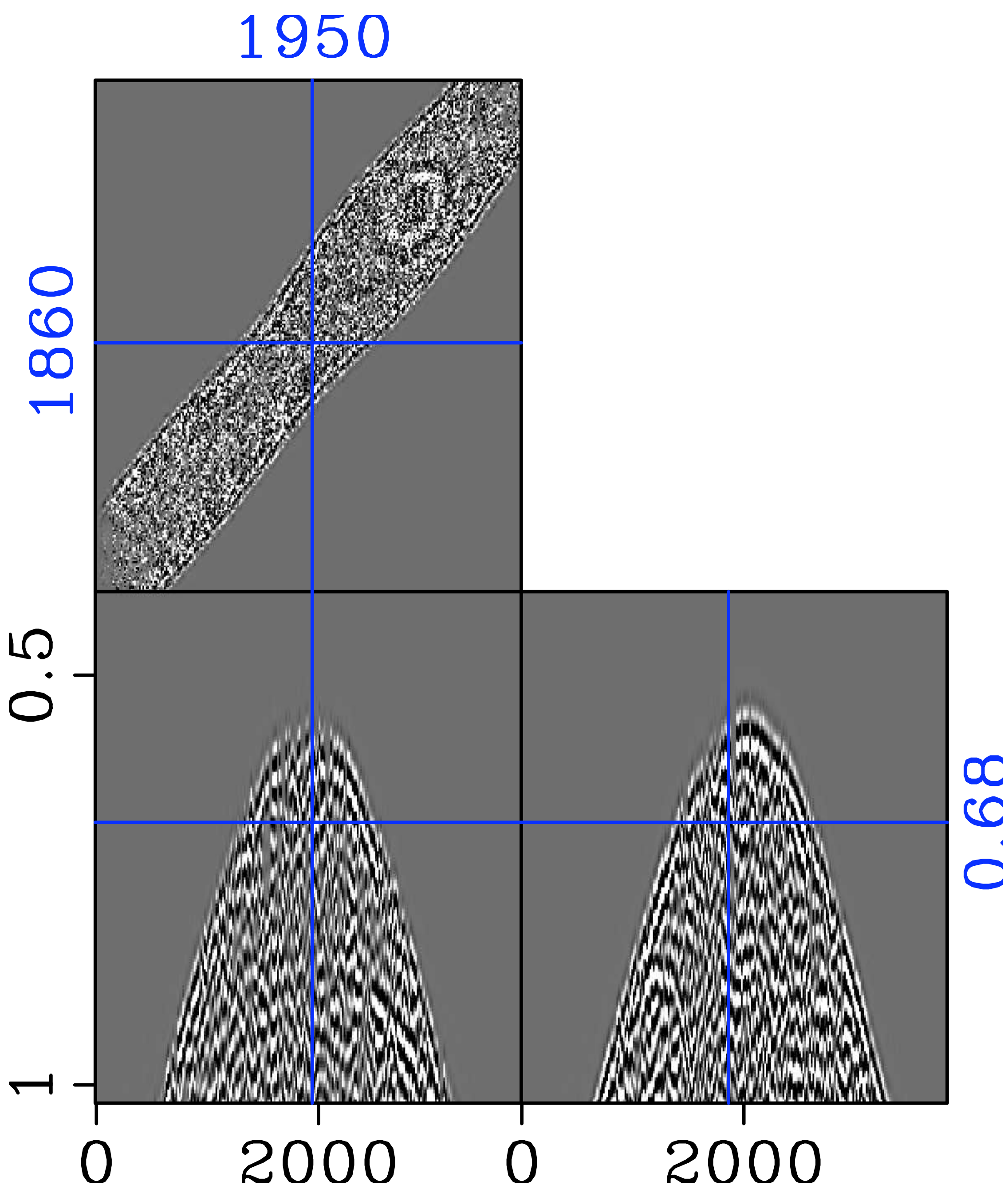
The problem cont'd

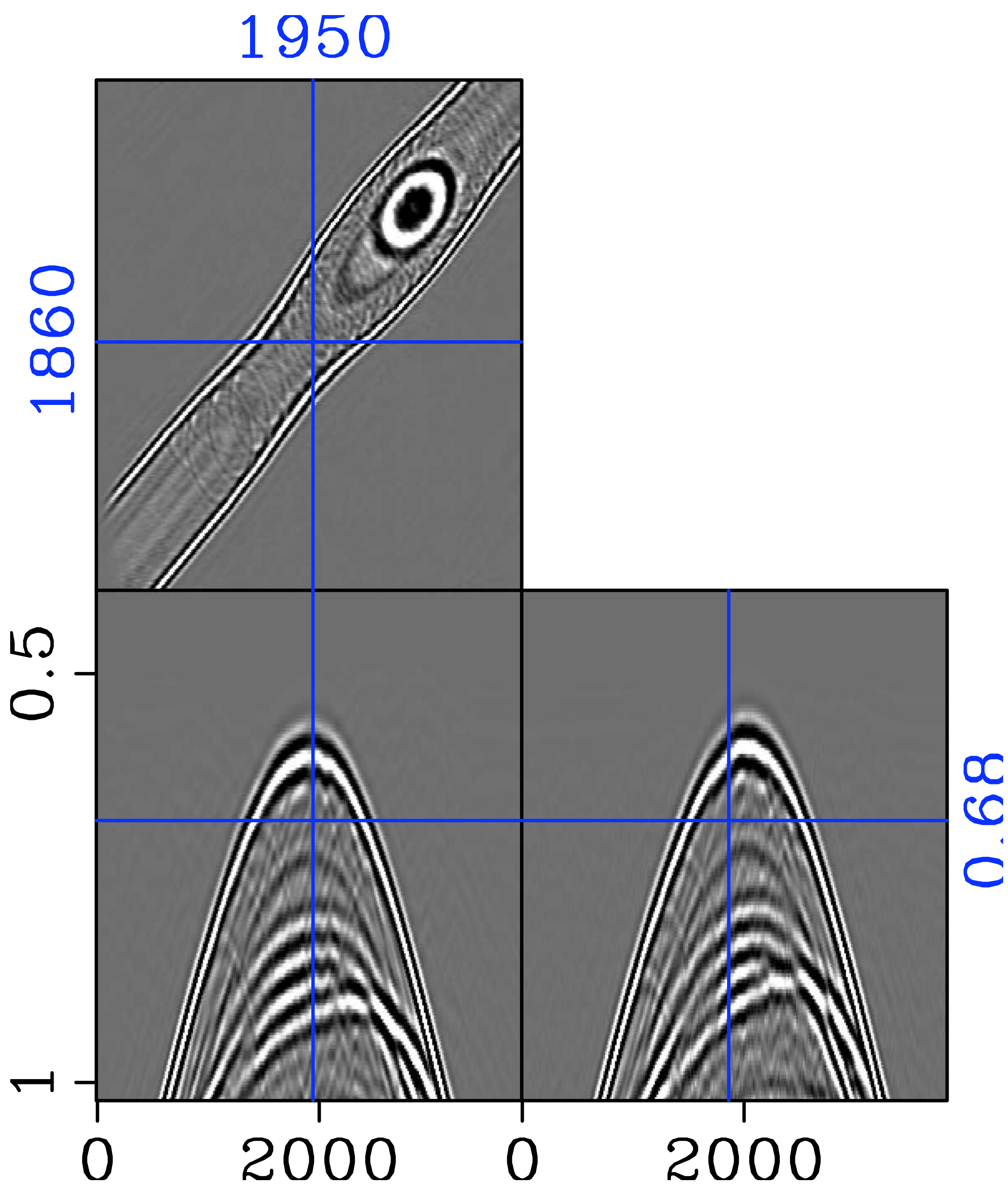


SRME from complete data

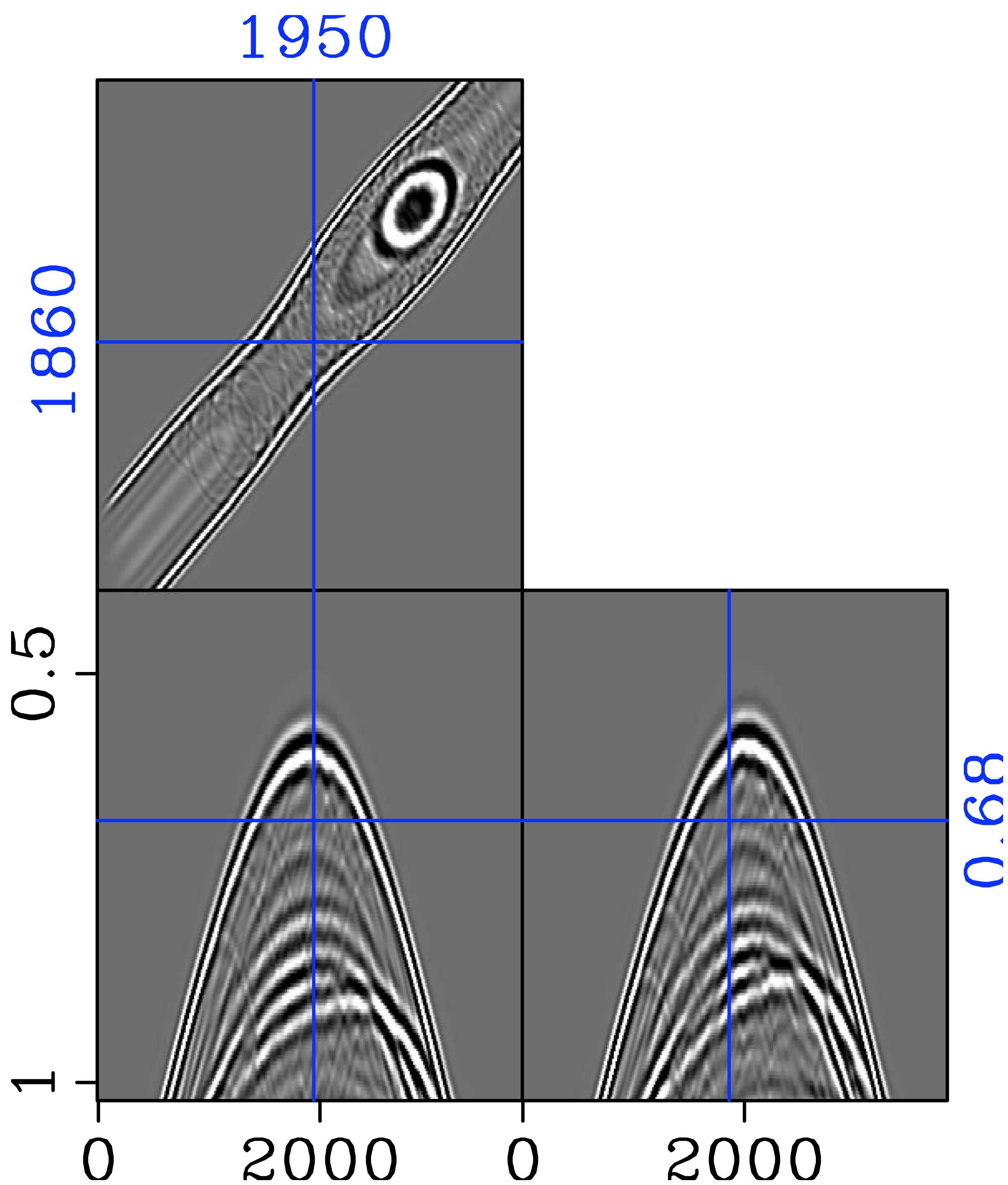


SRME from missing data

Our solution



SRME from recovered data



SRME from original data

Motivation

Data-driven (SRME) multiple prediction requires fully sampled data.

The Focal transform (Berkhout & Verschuur ‘06) allows for

- mapping of multiples => primaries
- incorporation of *prior* information in the recovery

Present a curvelet-based scheme for sparsity-promoting

- recovery of the data
- prediction of primaries and surface-related multiples

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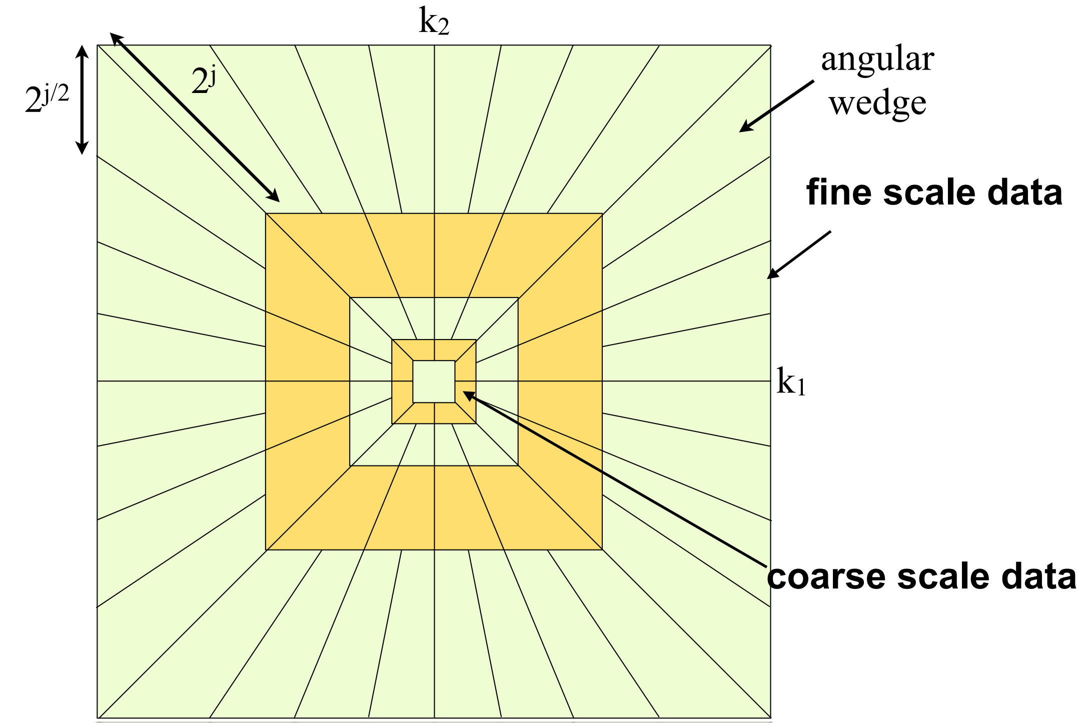
The curvelet transform

Representations for seismic data

Transform	Underlying assumption
FK	plane waves
linear/parabolic Radon transform	linear/parabolic events
wavelet transform	point-like events (1D singularities)
curvelet transform	**curve-like events (2D singularities)**

Properties curvelet transform:

- **multiscale:** tiling of the FK domain into dyadic coronae
- **multi-directional:** coronae sub-partitioned into angular wedges, # of angle doubles every other scale
- **anisotropic:** parabolic scaling principle
- **Rapid decay space**
- **Strictly localized in Fourier**
- **Frame with moderate redundancy (8 X in 2-D and 24 X in 3-D)**



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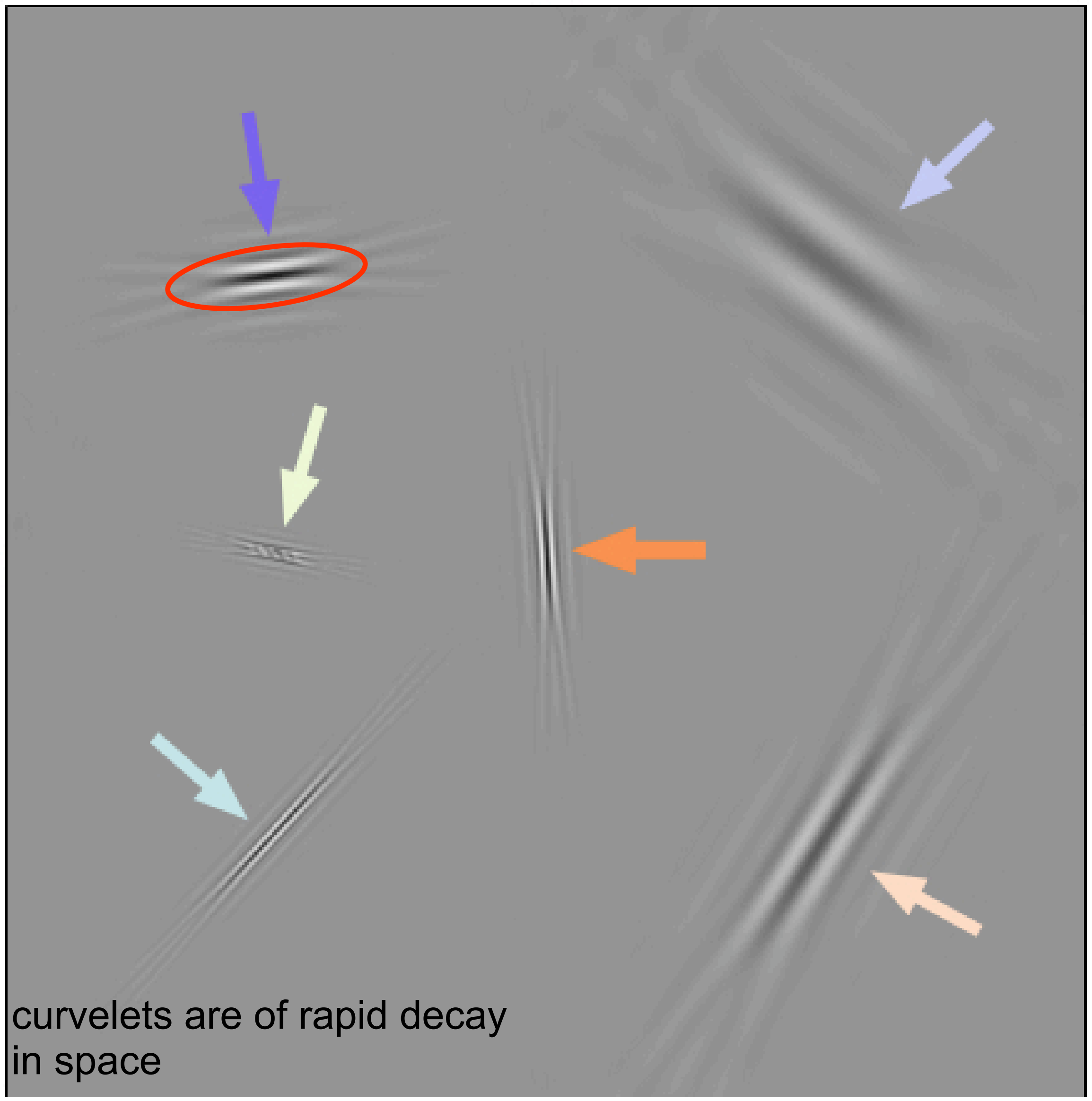
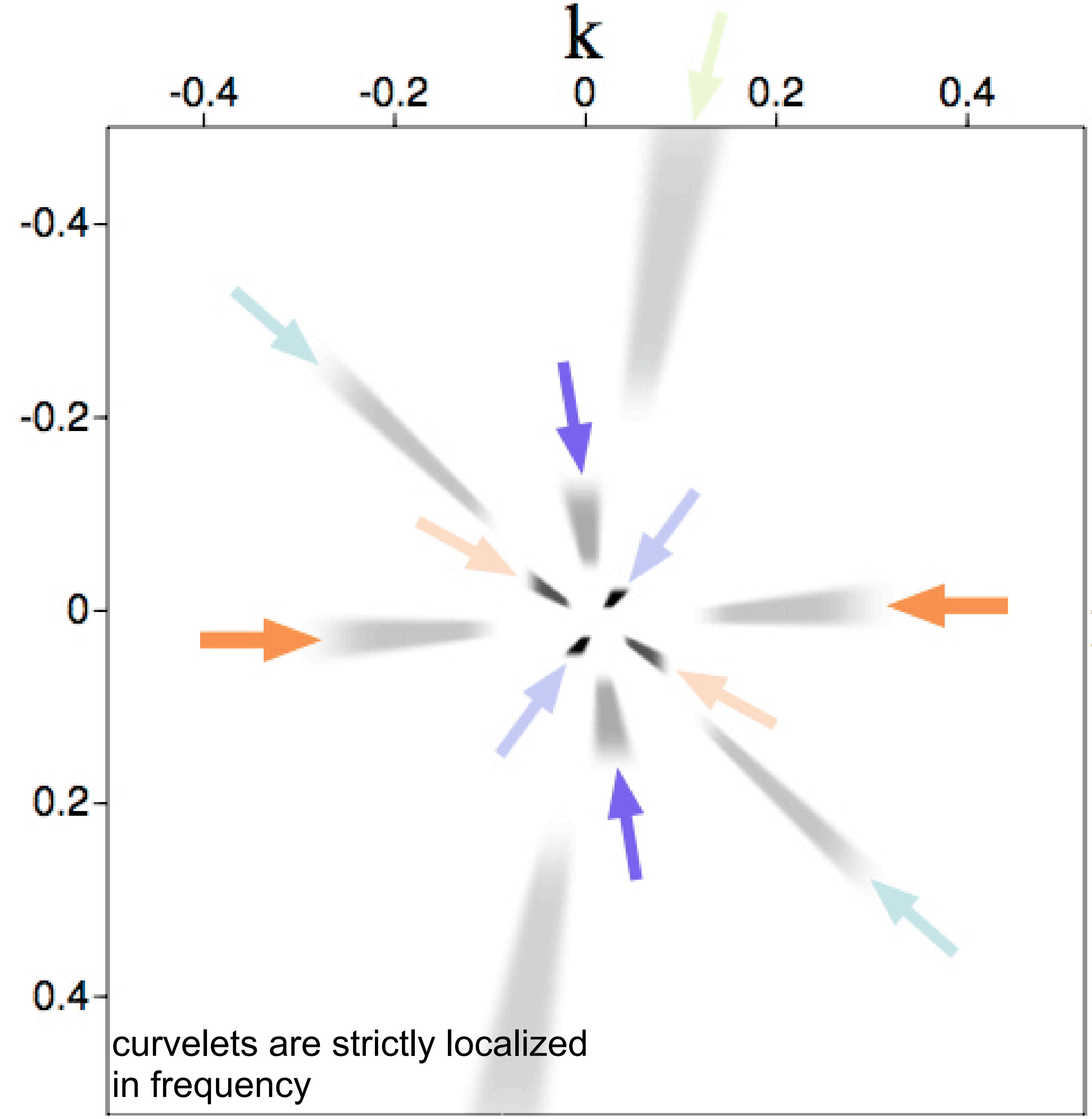
2-D curvelets



x-t



f-k

Oscillatory in one direction and smooth in the others!

Obey *parabolic* scaling relation $\text{length} \approx \text{width}^2$

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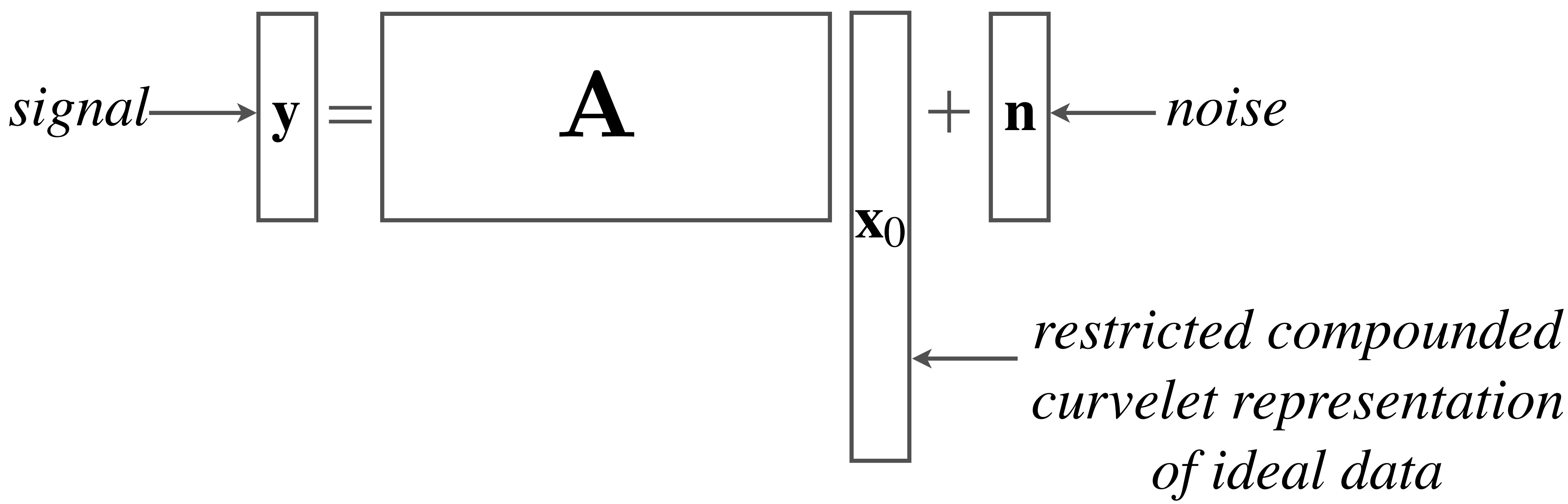
Curvelet sparsity promotion

Sparsity-promoting program

Solve for $\mathbf{x}_0$



with

$$\mathbf{P}_\epsilon : \begin{cases} \tilde{\mathbf{x}} = \arg\min_{\mathbf{x}} \|\mathbf{x}\|_1 \quad \text{s.t.} \quad \|\mathbf{A}\mathbf{x} - \mathbf{y}\|_2 \leq \epsilon \\ \tilde{\mathbf{m}} = \mathbf{C}^T \tilde{\mathbf{x}} \end{cases}$$

- exploit sparsity in the curvelet domain as a prior.
- find the sparsest set of curvelet coefficients that match the data.
- invert an underdetermined system.

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Focused recovery with curvelets

joint work with Deli Wang (visitor from Jilin university) and Gilles Hennenfent

Focused recovery

Non-data-adaptive Curvelet Reconstruction with Sparsity-promoting Inversion (CRSI) derives from ***sparsity*** of seismic data.

Berkhout and Verschuur's ***data-adaptive*** Focal transform derives from ***focusing*** of seismic data by the major primaries.

Both approaches entail the ***inversion*** of a linear operator.

Combination of the two yields

- improved focusing => more sparsity
- curvelet sparsity => better focusing

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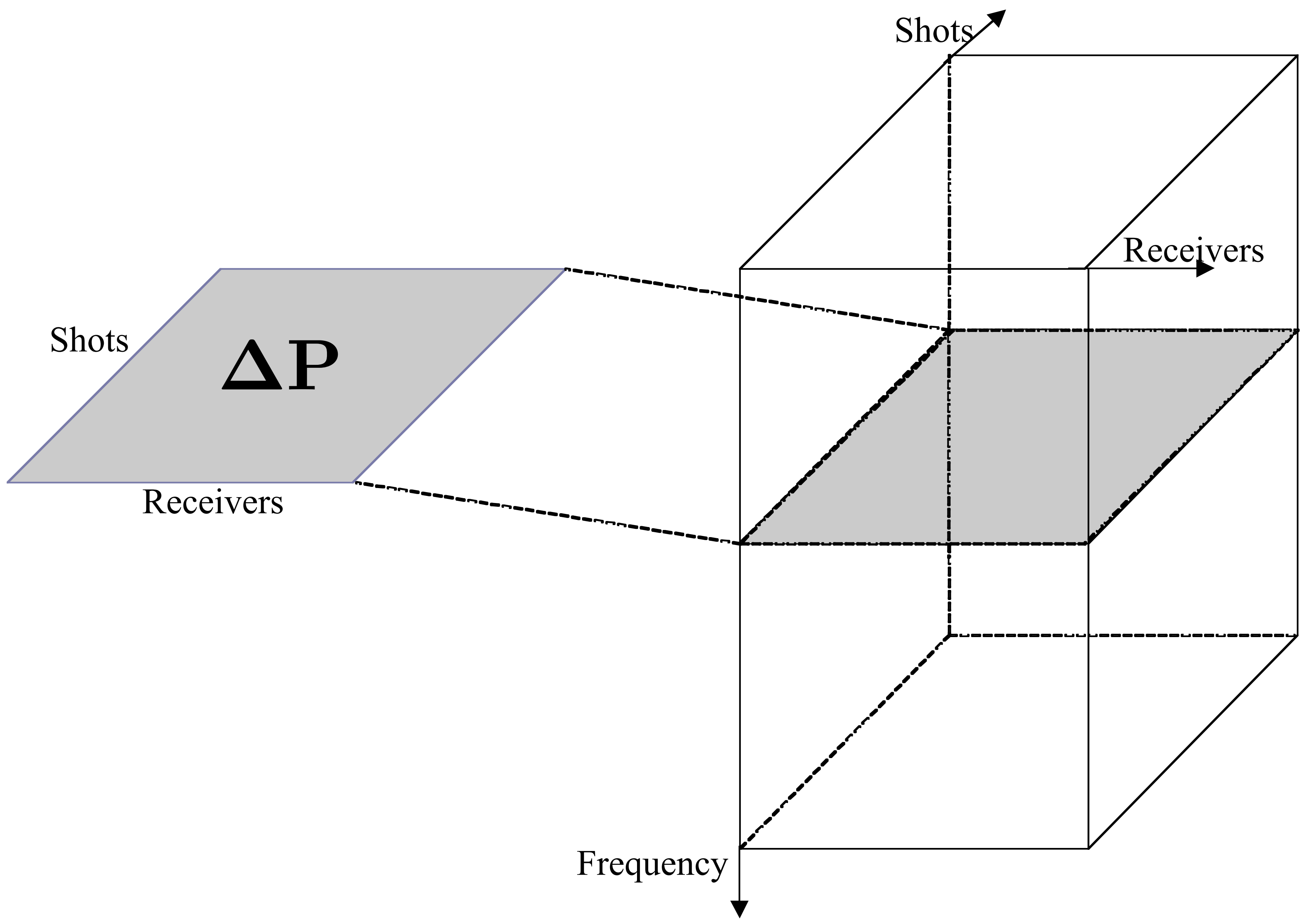
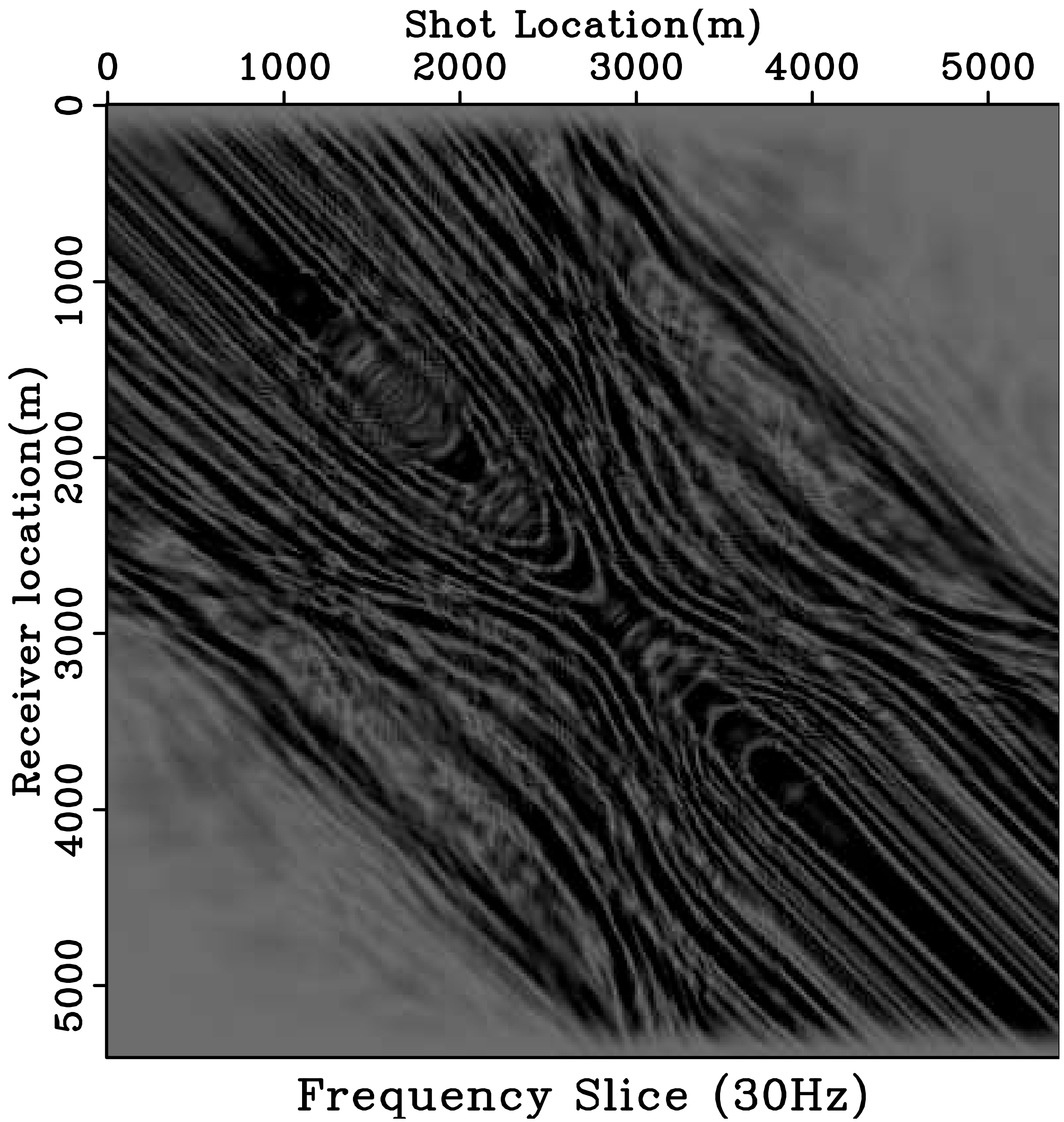
Primary operator



Frequency slice from data matrix with dominant primaries.

Primary operator



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Primary operator

Primaries to first-order multiples:

$$\mathbf{\Delta p} \mapsto \mathrm{m}^1 = (\mathbf{\Delta P}\boldsymbol{\mathcal{A}} *_{t,x} \mathbf{\Delta p})$$

First-order multiples into primaries:

$$\mathrm{m}^1 \mapsto \mathbf{\Delta p} \approx (\mathbf{\Delta P}\boldsymbol{\mathcal{A}} \otimes_{t,x} \mathbf{\Delta p})$$

with the acquisition matrix

$$\boldsymbol{\mathcal{A}} = \left(\boldsymbol{\mathcal{S}}^{\dagger}\mathbf{R}\boldsymbol{\mathcal{D}}^{\dagger}\right)$$

“inverting” for source and receiver wavelet wavelets geometry and surface reflectivity.

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Curvelet-based Focal transform

Solve with 3-D curvelet transform

$$\mathbf{P}_\epsilon : \quad \begin{cases} \widetilde{\mathbf{x}} = \arg\min_{\mathbf{x}} \|\mathbf{x}\|_1 \quad \text{s.t.} \quad \|\mathbf{A}\mathbf{x} - \mathbf{y}\|_2 \le \epsilon \\ \widetilde{\mathbf{f}} = \mathbf{S}^T \widetilde{\mathbf{x}} \end{cases}$$

with

$$\begin{aligned} \mathbf{A} &:= \boldsymbol{\Delta}\mathbf{P}\mathbf{C}^T \quad \text{and } \boldsymbol{\Delta}\mathbf{P} := \mathbf{F}^H \text{block diag}\{\boldsymbol{\Delta}\mathbf{p}\}\mathbf{F} \\ \mathbf{S} &:= \mathbf{C} \\ \mathbf{y} &= \mathbf{P}(:) \\ \mathbf{P} &= \text{total data.} \end{aligned}$$

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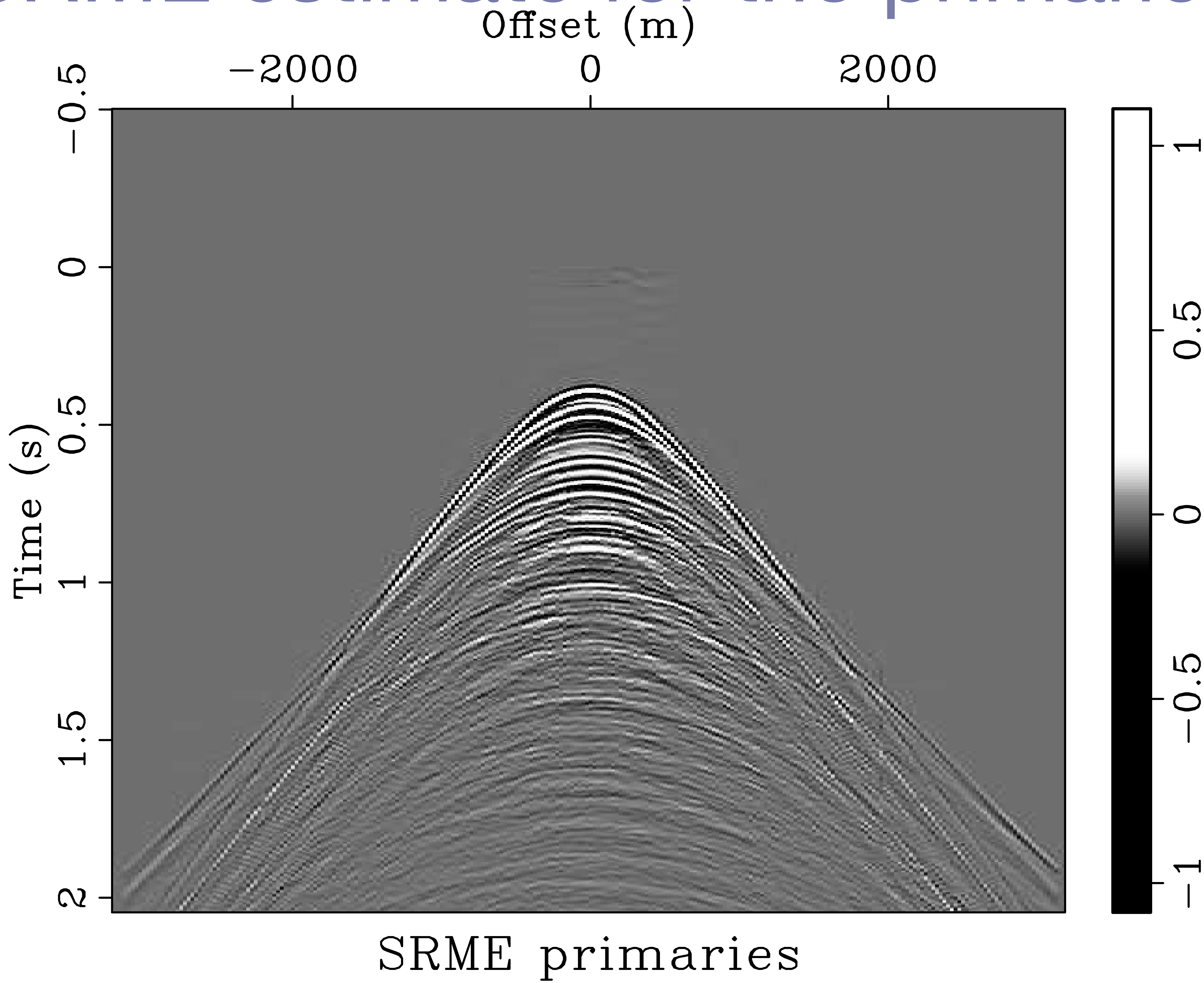
SRME estimate for the primaries



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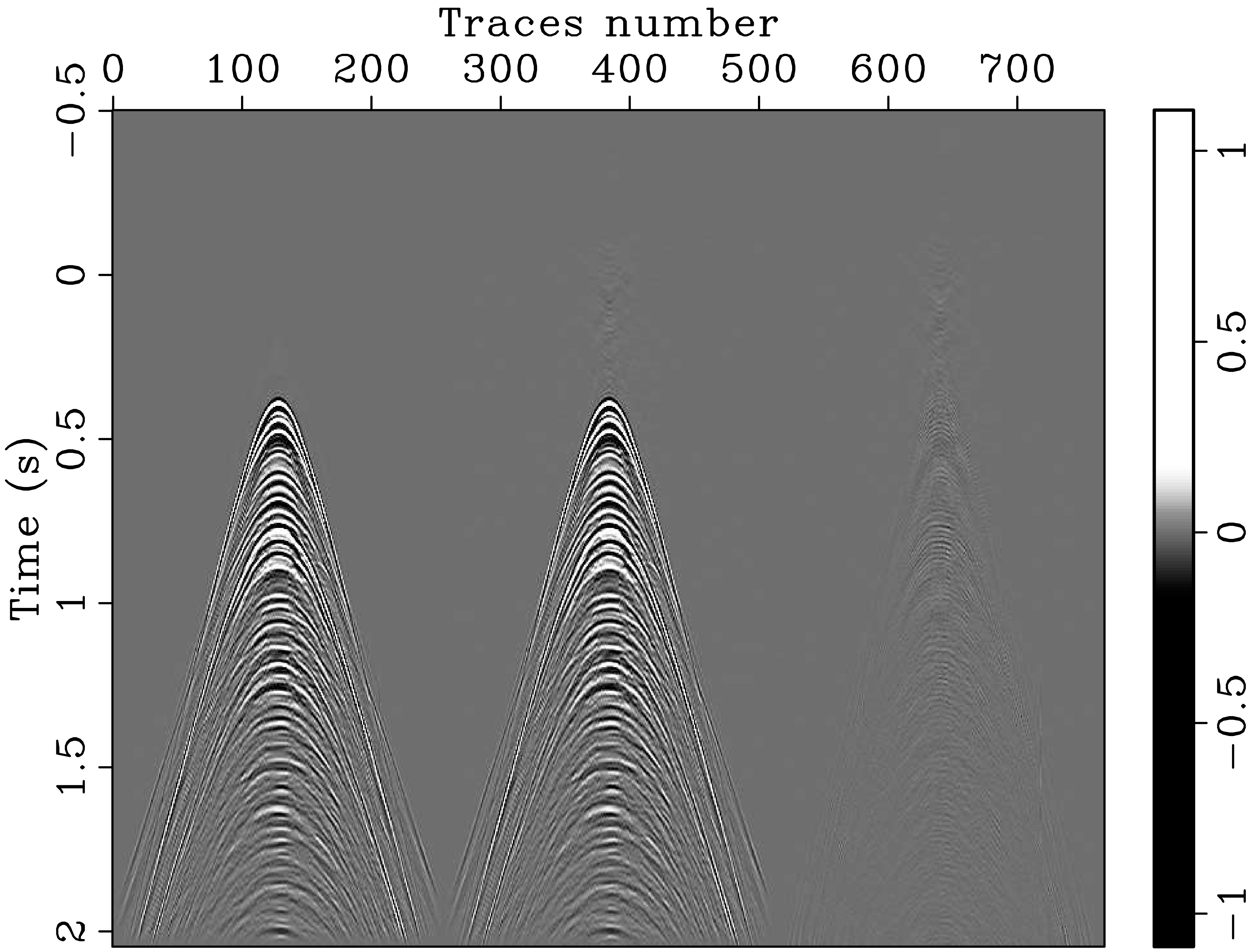
Difference



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Recovery with focussing

Solve

$$\mathbf{P}_\epsilon : \quad \begin{cases} \widetilde{\mathbf{x}} = \arg\min_{\mathbf{x}} \|\mathbf{x}\|_1 \quad \text{s.t.} \quad \|\mathbf{A}\mathbf{x} - \mathbf{y}\|_2 \leq \epsilon \\ \widetilde{\mathbf{f}} = \mathbf{S}^T \widetilde{\mathbf{x}} \end{cases}$$

with

$$\begin{aligned} \mathbf{A} &:= \mathbf{R}\boldsymbol{\Delta}\mathbf{P}\mathbf{C}^T \\ \mathbf{S}^T &:= \boldsymbol{\Delta}\mathbf{P}\mathbf{C}^T \\ \mathbf{y} &= \mathbf{R}\mathbf{P}(:) \\ \mathbf{R} &= \text{picking operator.} \end{aligned}$$

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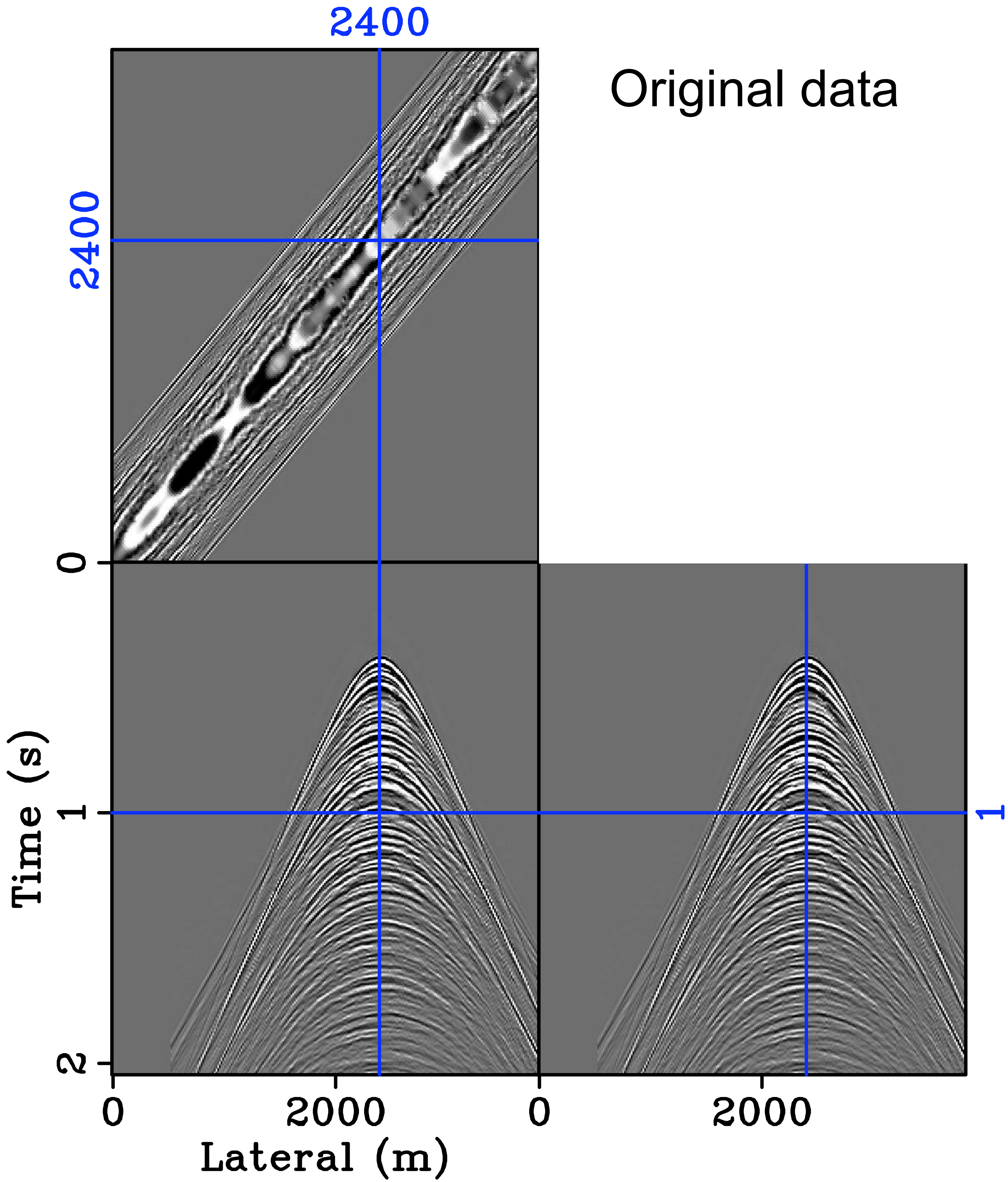

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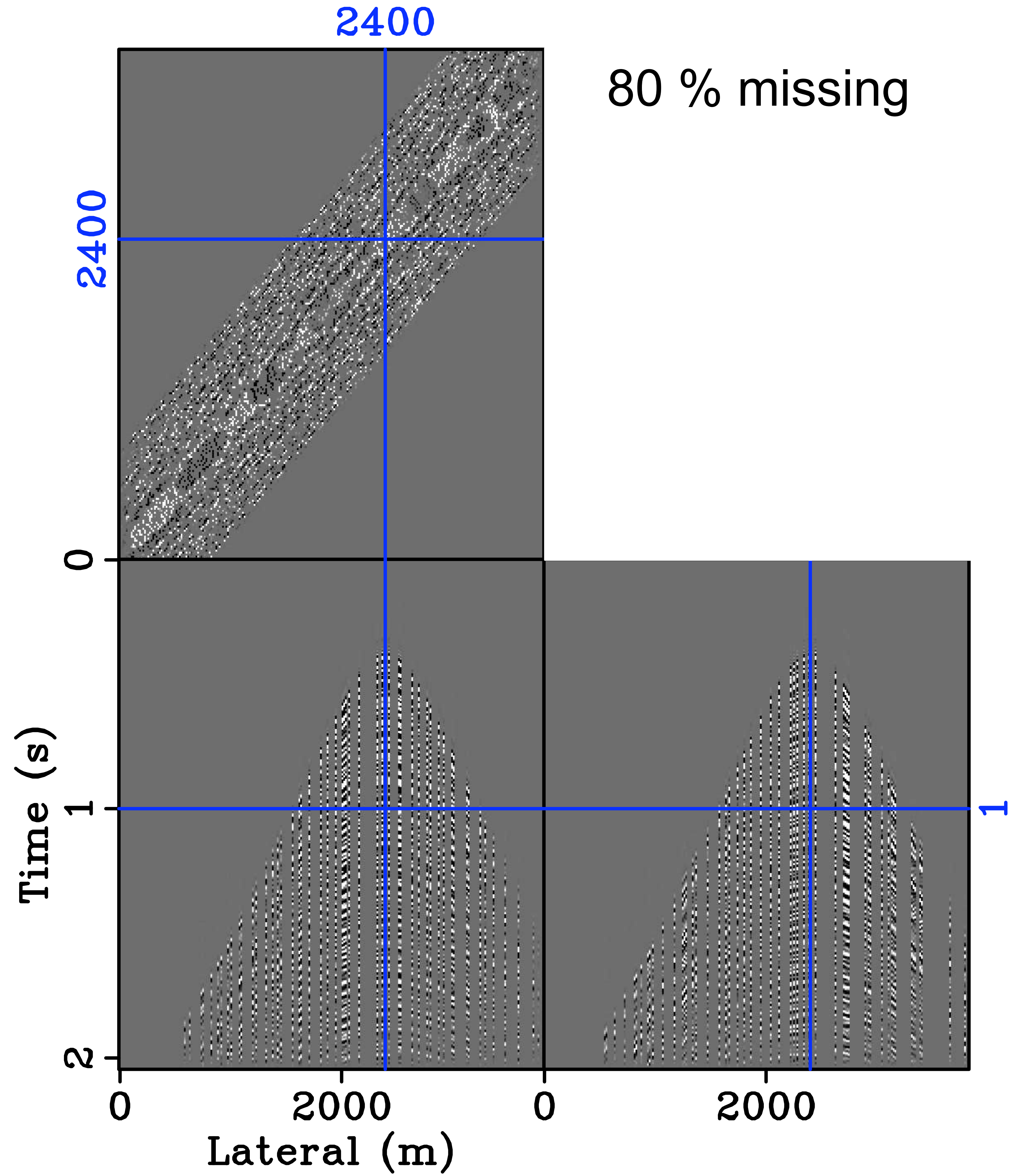
80 % missing



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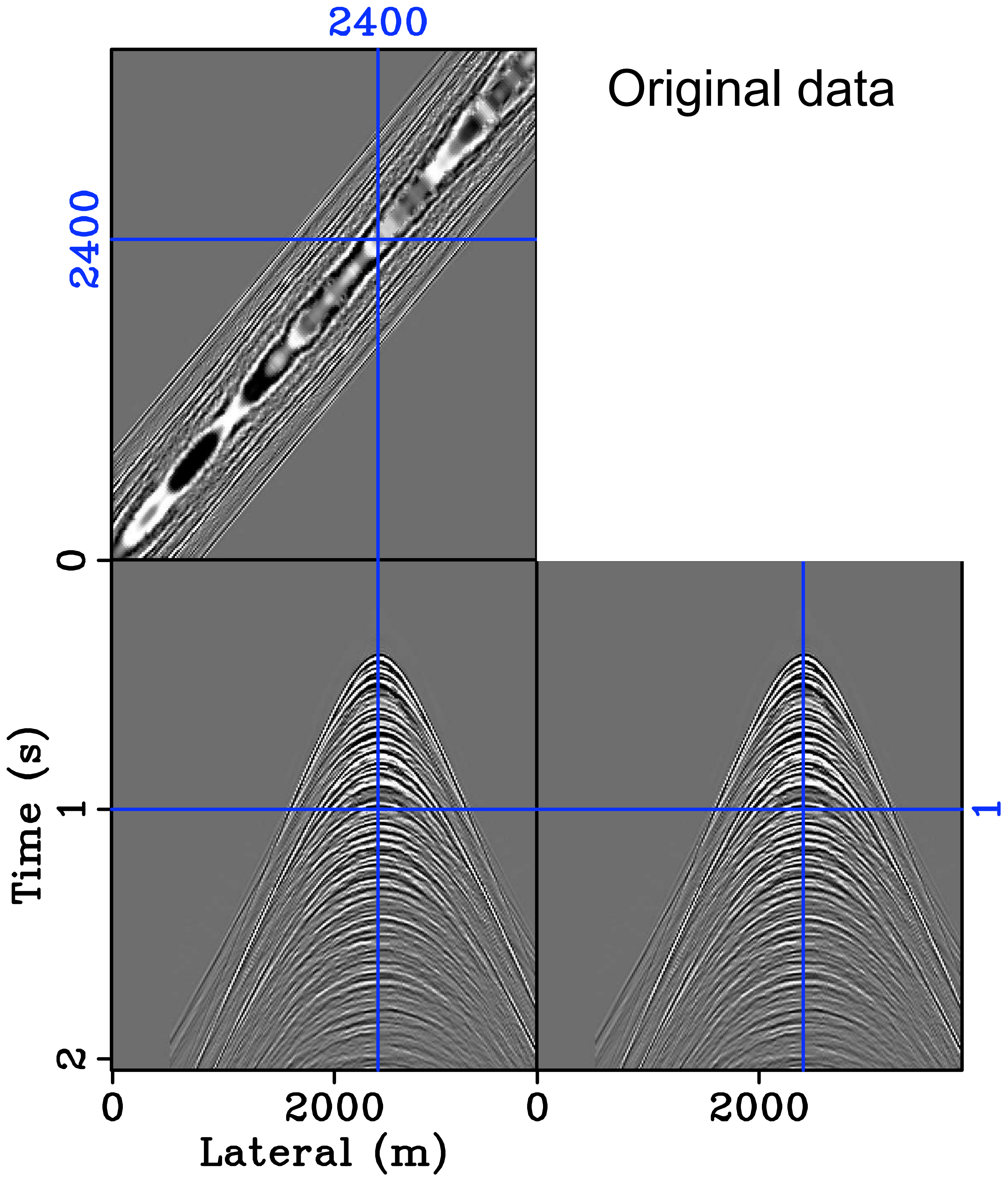
Original data



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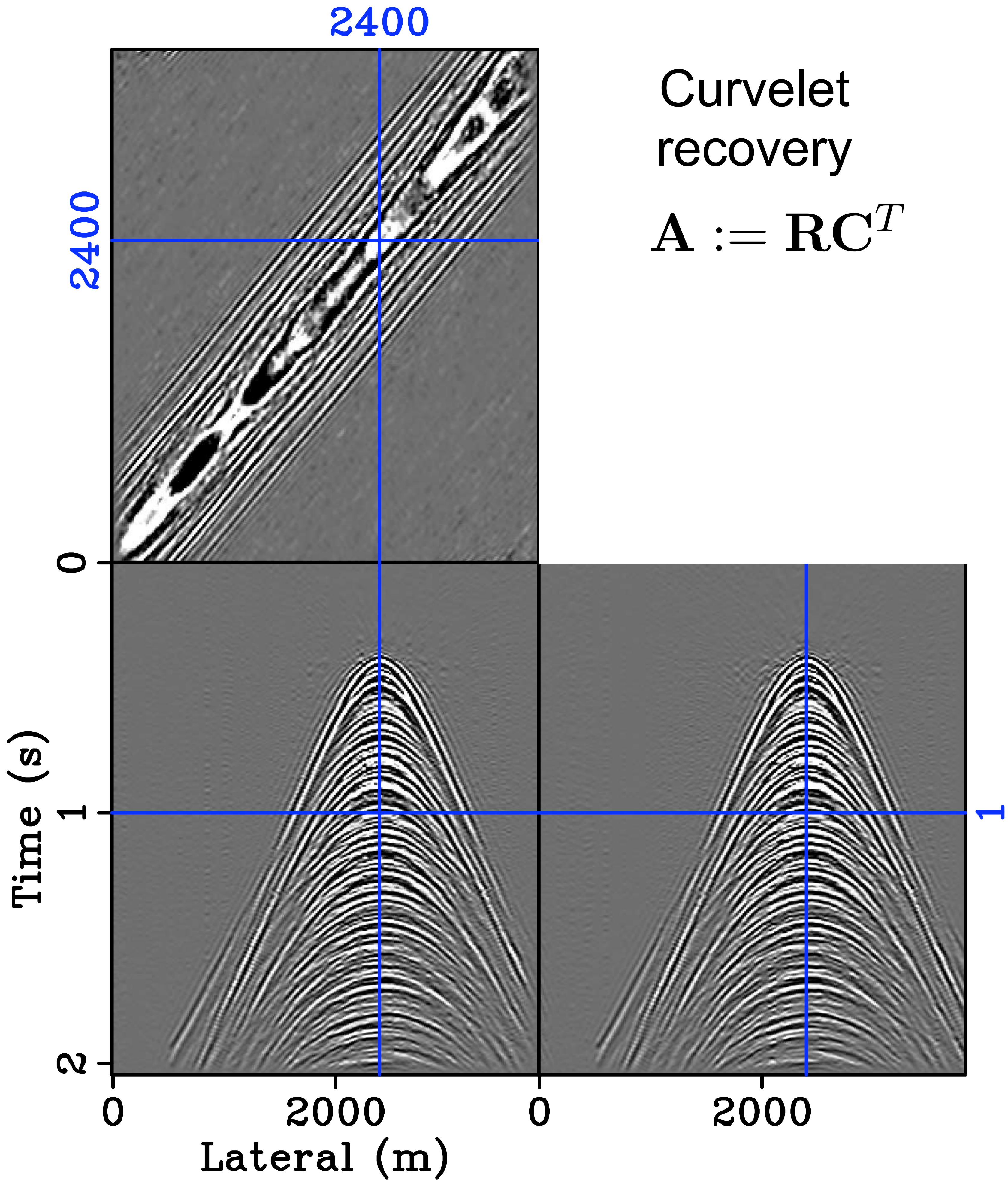
Curvelet recovery

$$\mathbf{A} := \mathbf{R}\mathbf{C}^T$$



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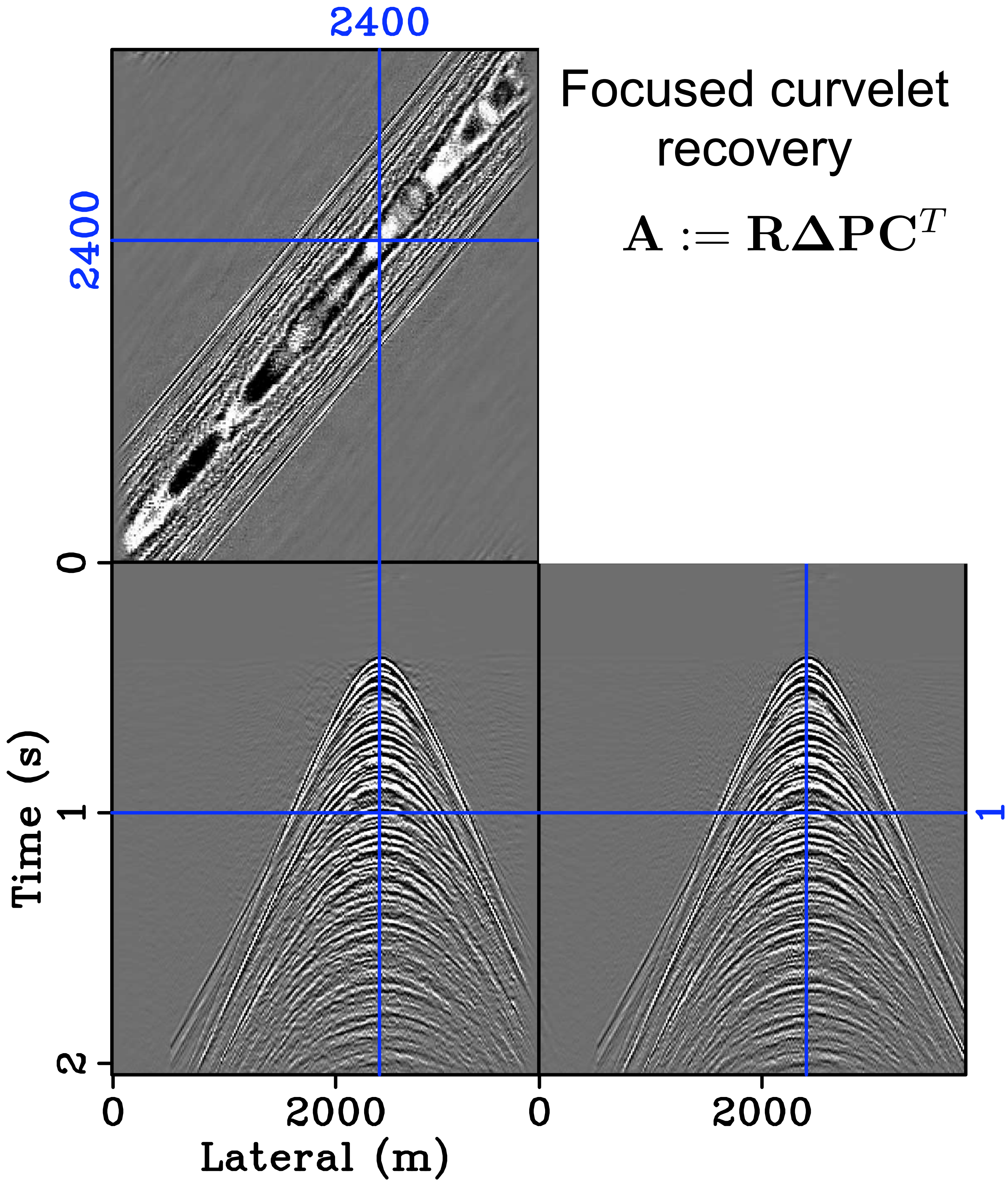
Focused curvelet recovery

$$\mathbf{A} := \mathbf{R}\boldsymbol{\Delta}\mathbf{P}\mathbf{C}^T$$



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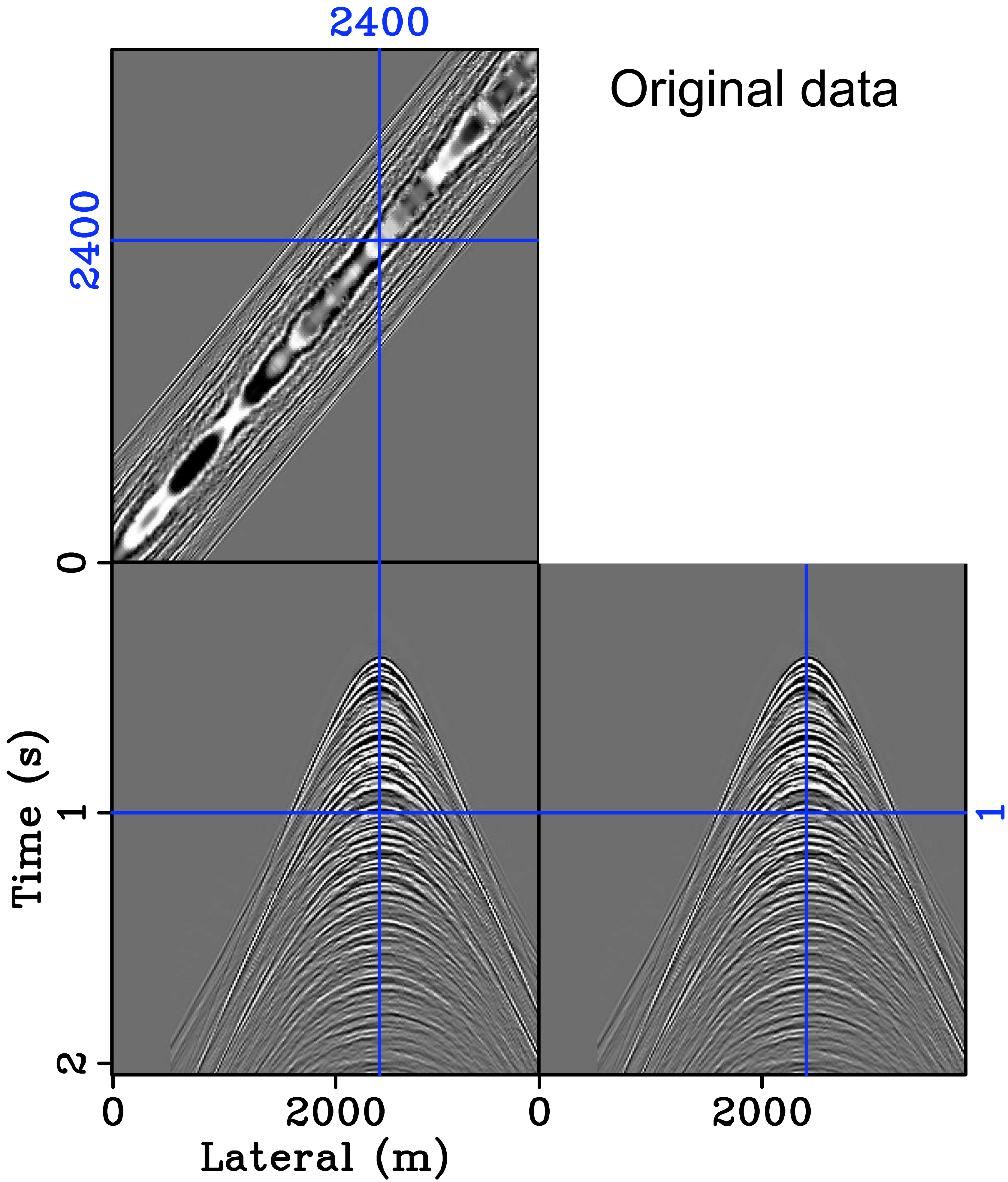
Original data



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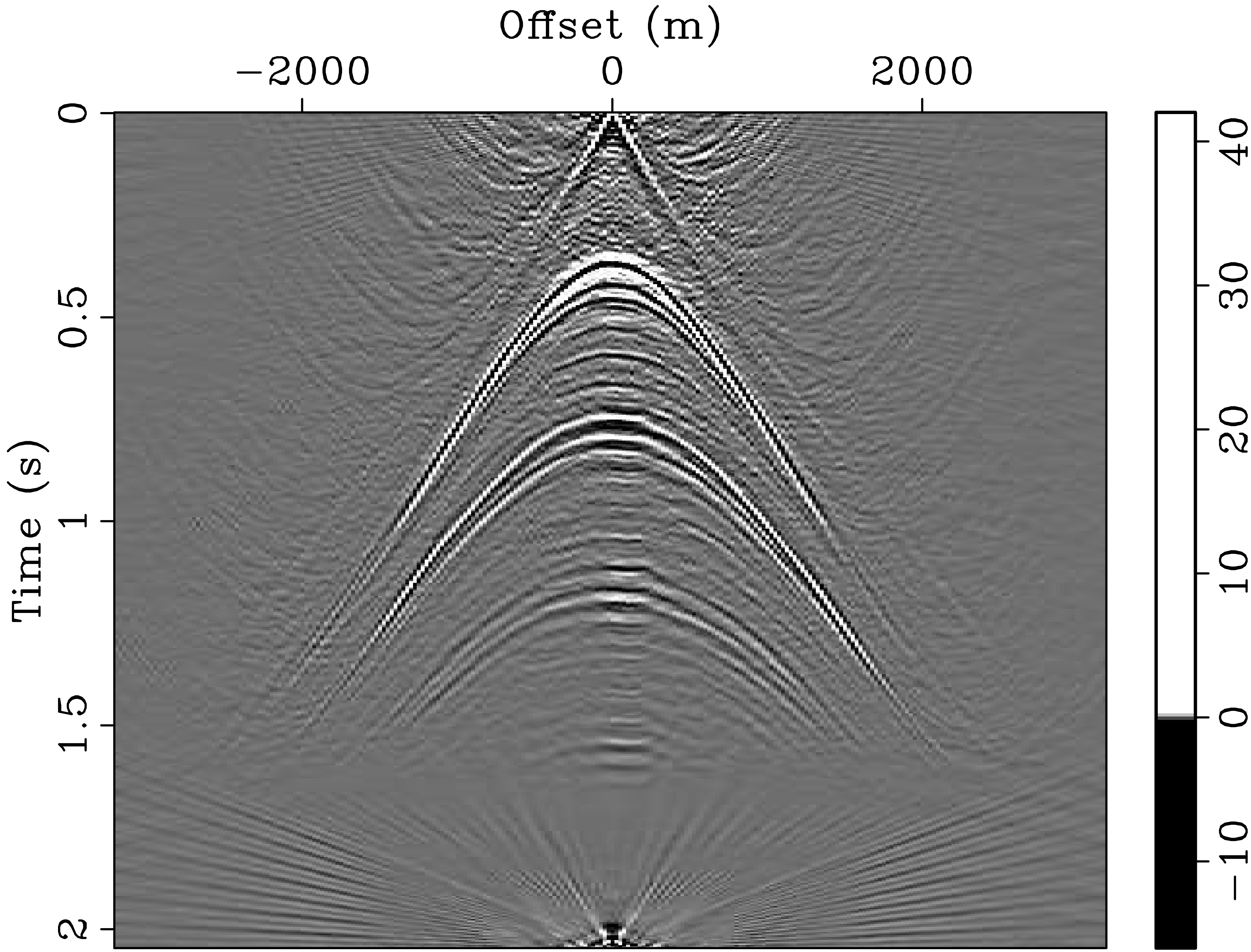

Recovered Data in Focal Domain

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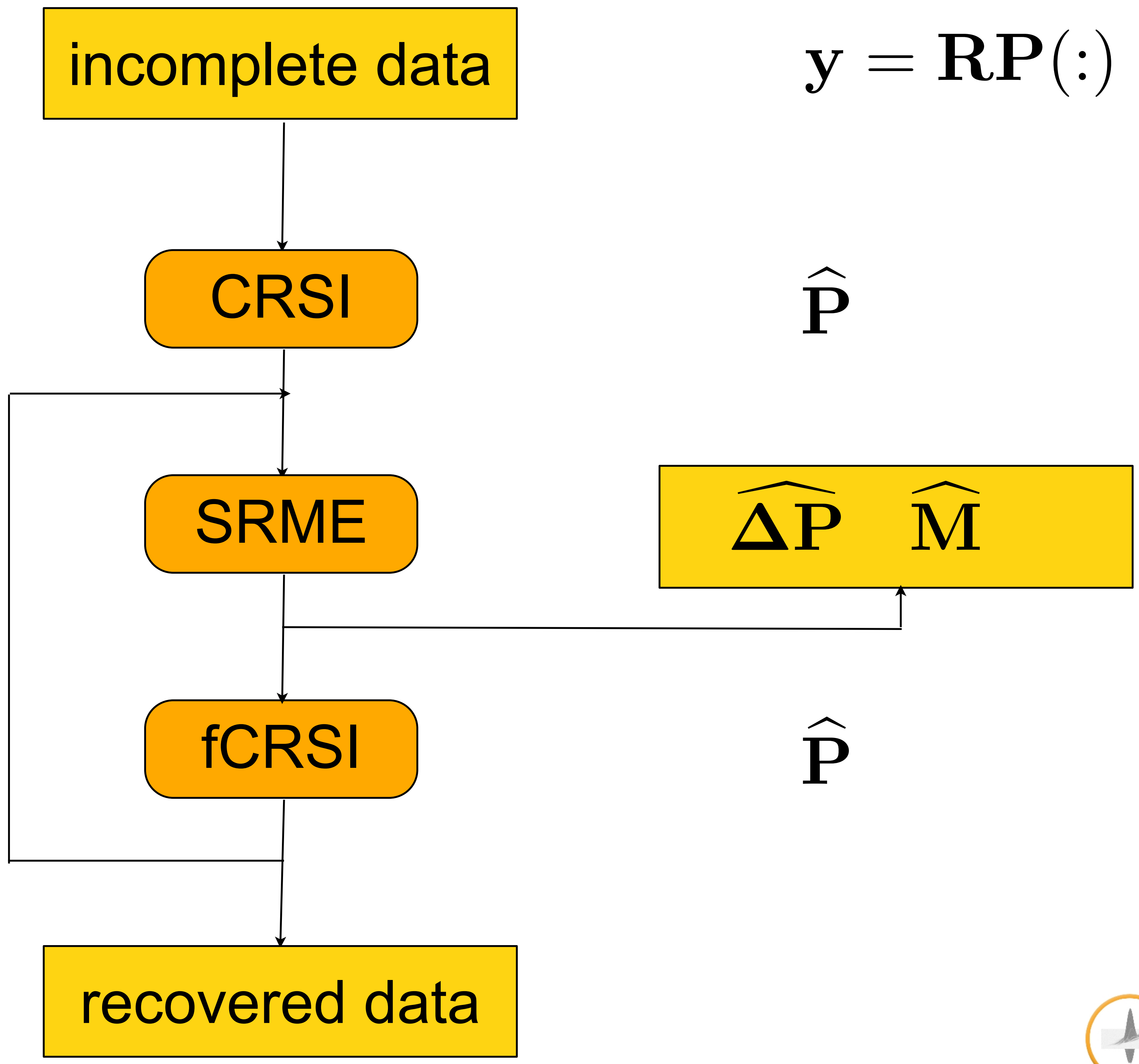
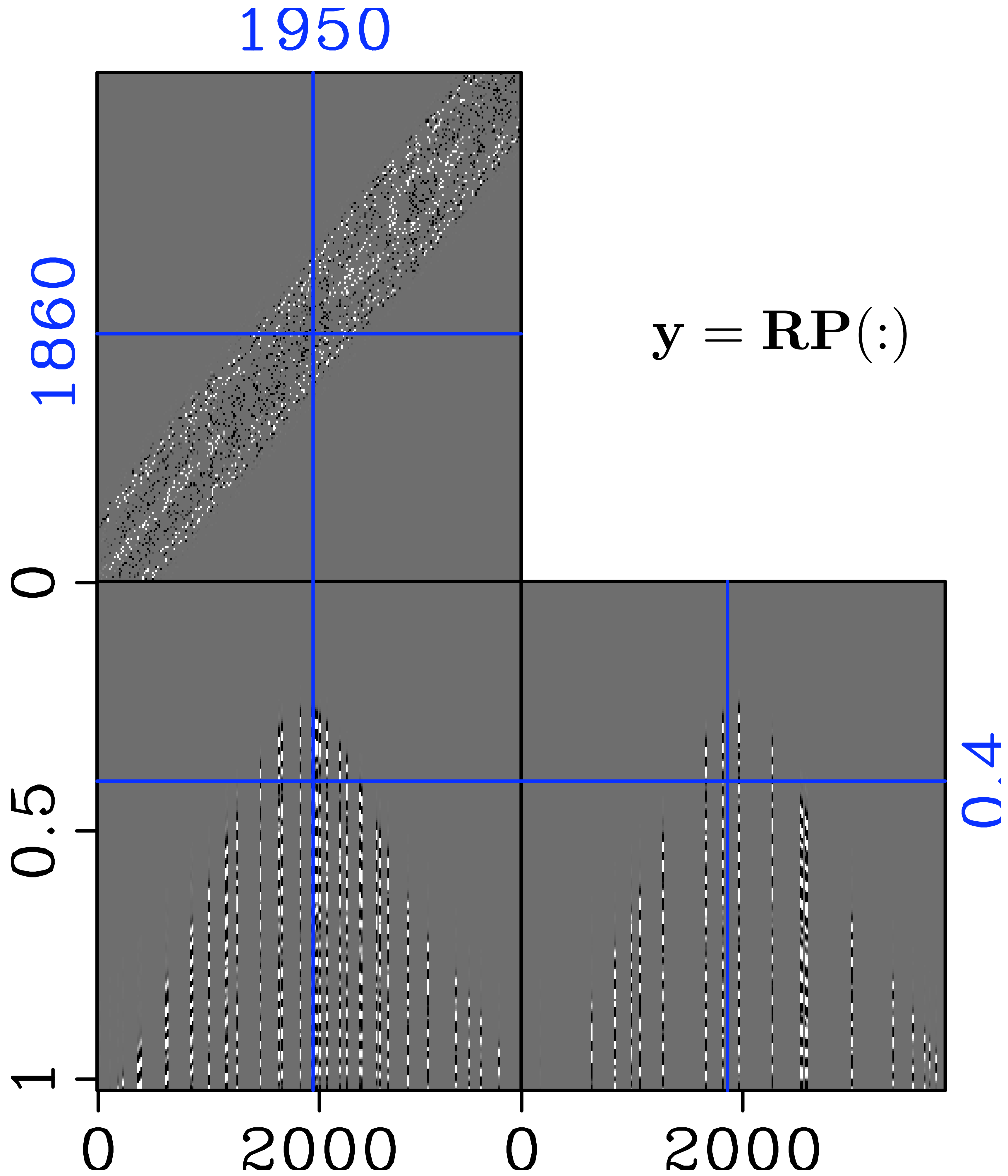
Nonlinear primary-multiple prediction

joint work with Deli Wang (visitor from Jilin university) and Eric Verschuur

Multiple prediction with fCRSI




1950
1860
0.4
0
0.5
1
0
2000
0
2000
$\mathbf{y} = \mathbf{RP}(:)$
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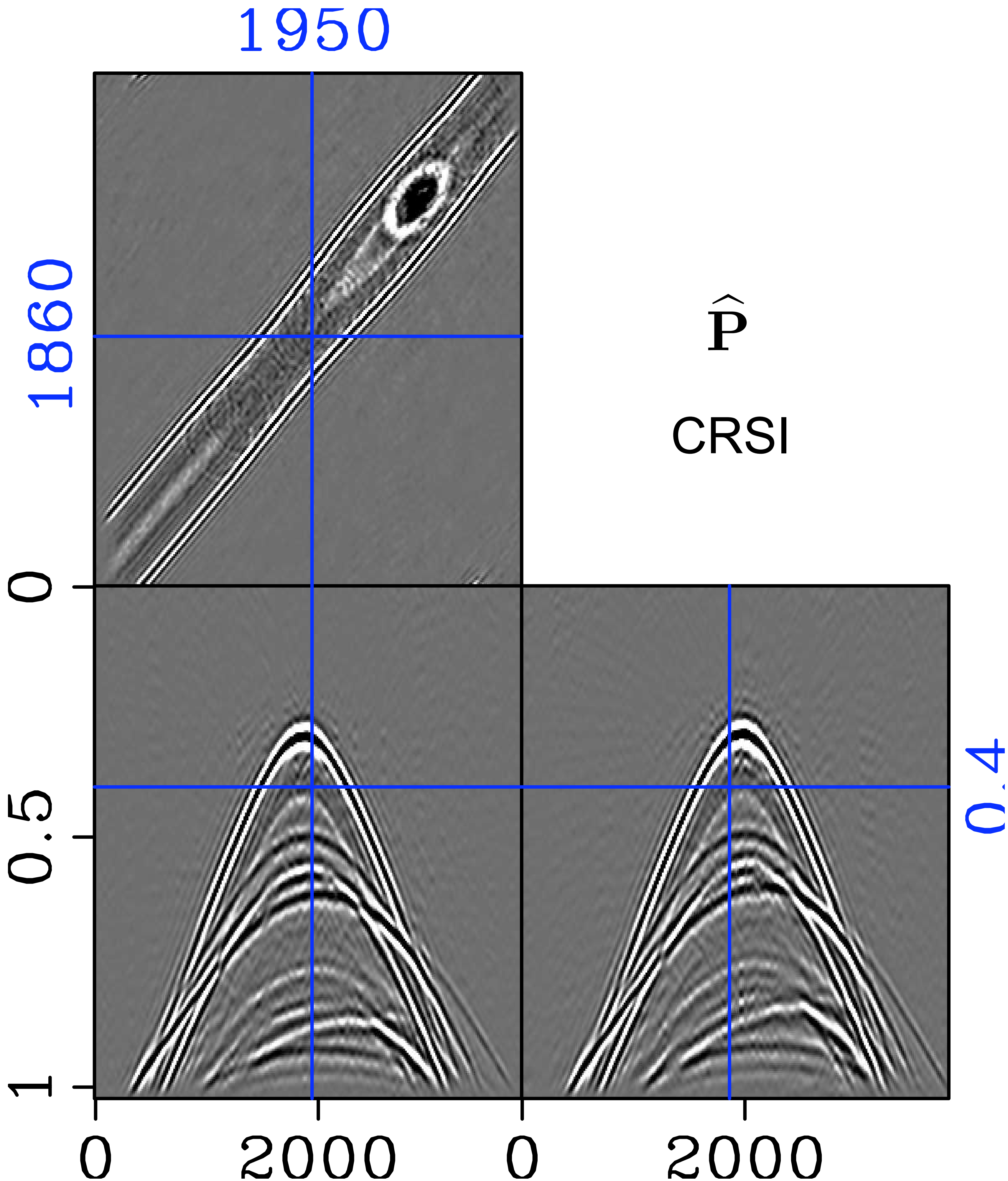

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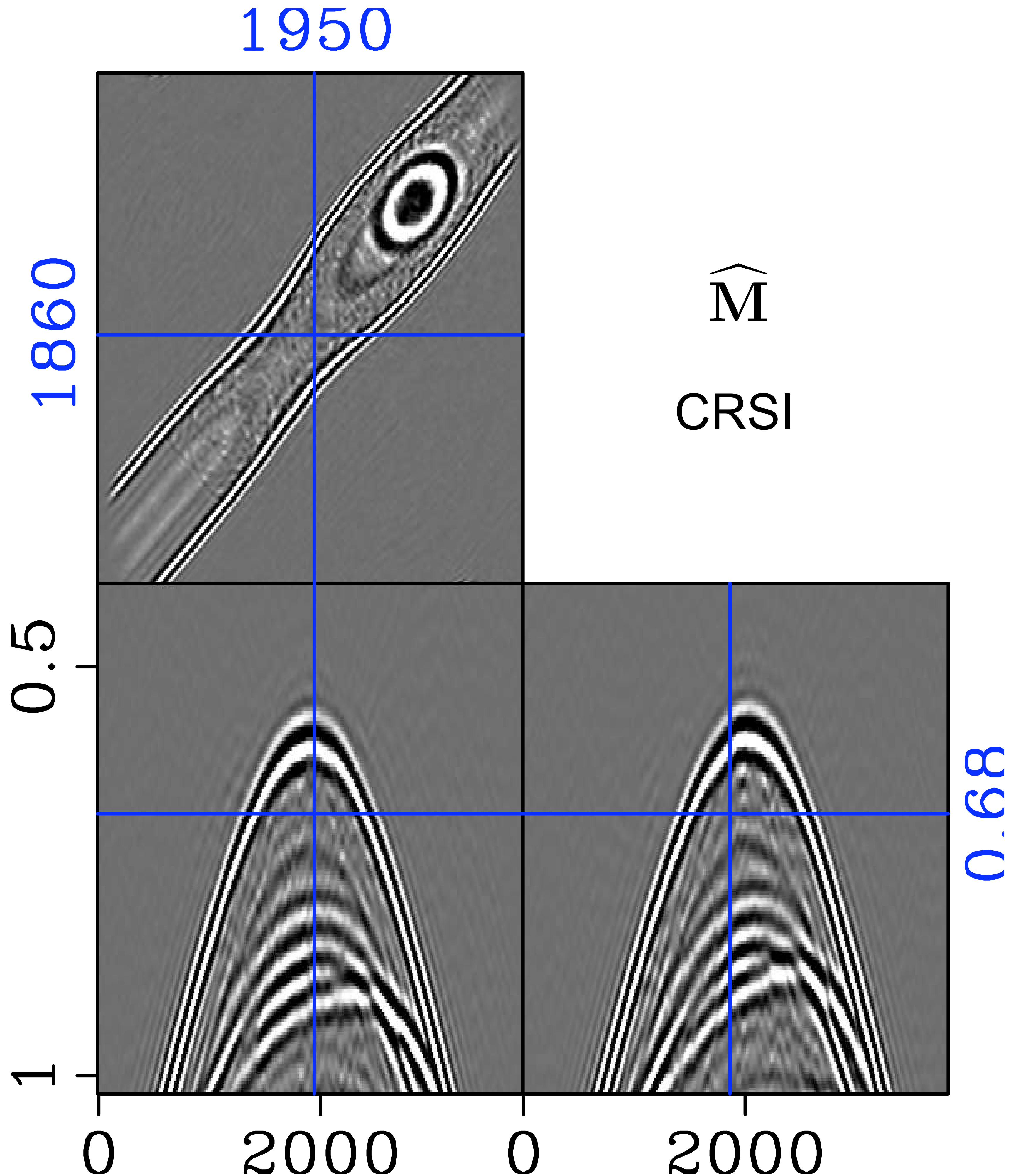
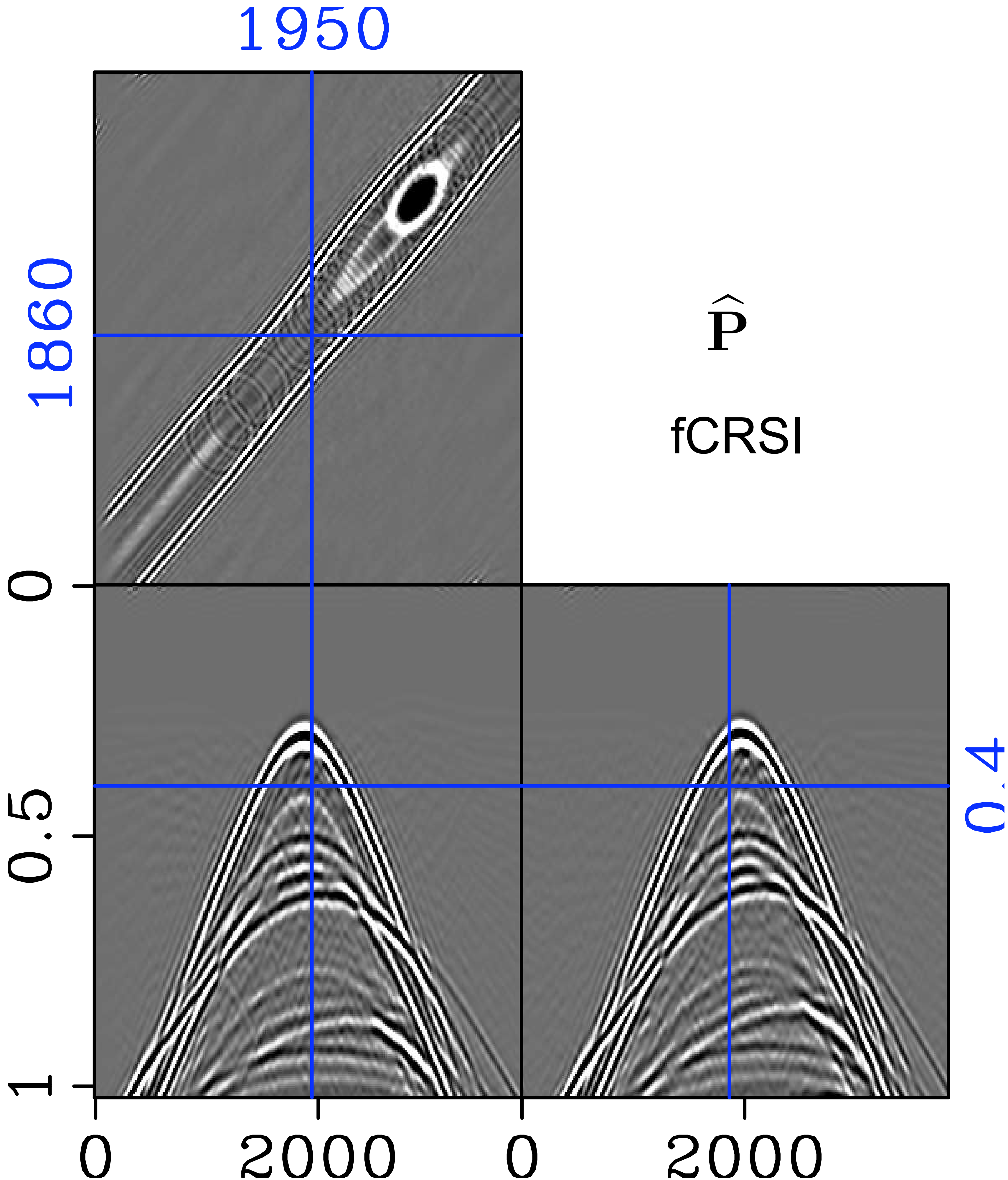

1950
1860
0.5
1
0.68
0
2000
0
2000
$\widehat{\mathbf{M}}$
CRSI



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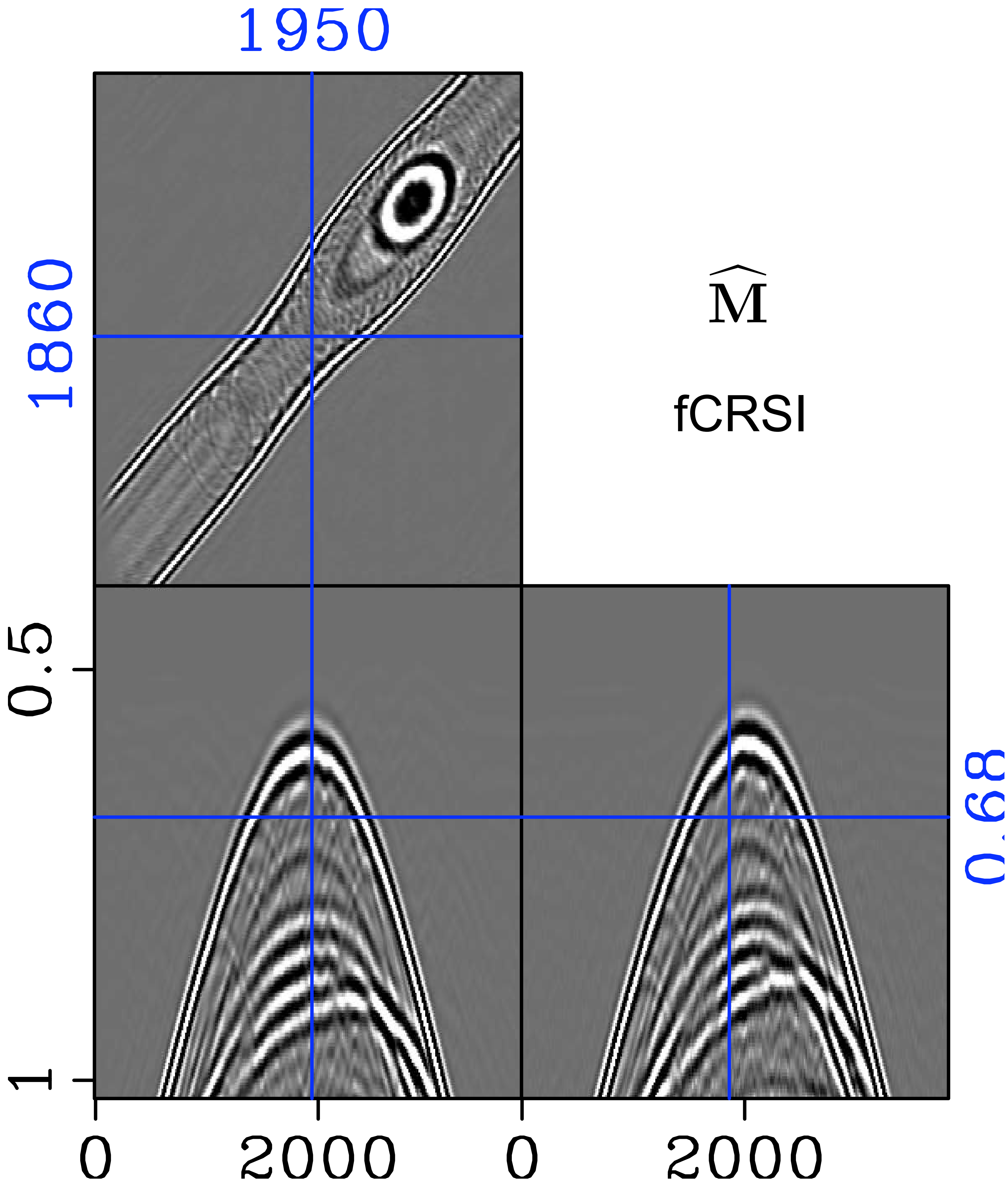

$\widehat{\mathbf{M}}$

fCRSI

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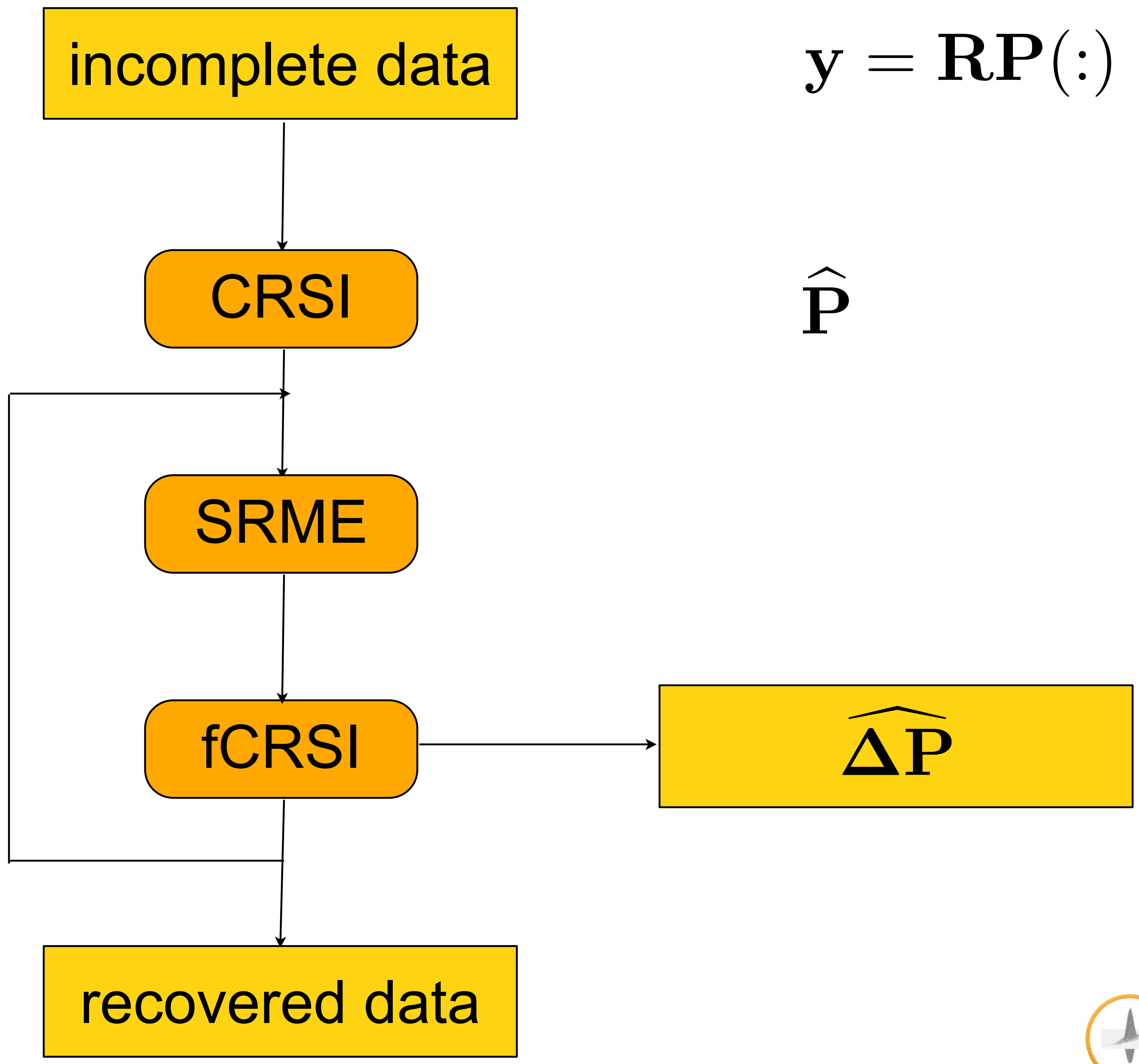
Primary prediction with fCRSI



$$\mathbf{y} = \mathbf{RP}(:)$$

$$\widehat{\mathbf{P}}$$

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Curvelet-based Focal transform

Solve

$$\mathbf{P}_\epsilon : \quad \begin{cases} \widetilde{\mathbf{x}} = \arg\min_{\mathbf{x}} \|\mathbf{x}\|_1 \quad \text{s.t.} \quad \|\mathbf{A}\mathbf{x} - \mathbf{y}\|_2 \leq \epsilon \\ \widetilde{\mathbf{f}} = \mathbf{S}^T \widetilde{\mathbf{x}} \end{cases}$$

with

$$\begin{aligned} \mathbf{A} &:= \boldsymbol{\Delta}\mathbf{P}\mathbf{C}^T \\ \mathbf{S} &:= \mathbf{C} \\ \mathbf{y} &= \mathbf{P}(:) \\ \mathbf{P} &= \text{total data} \\ \tilde{\mathbf{f}} &= \text{focused data.} \end{aligned}$$

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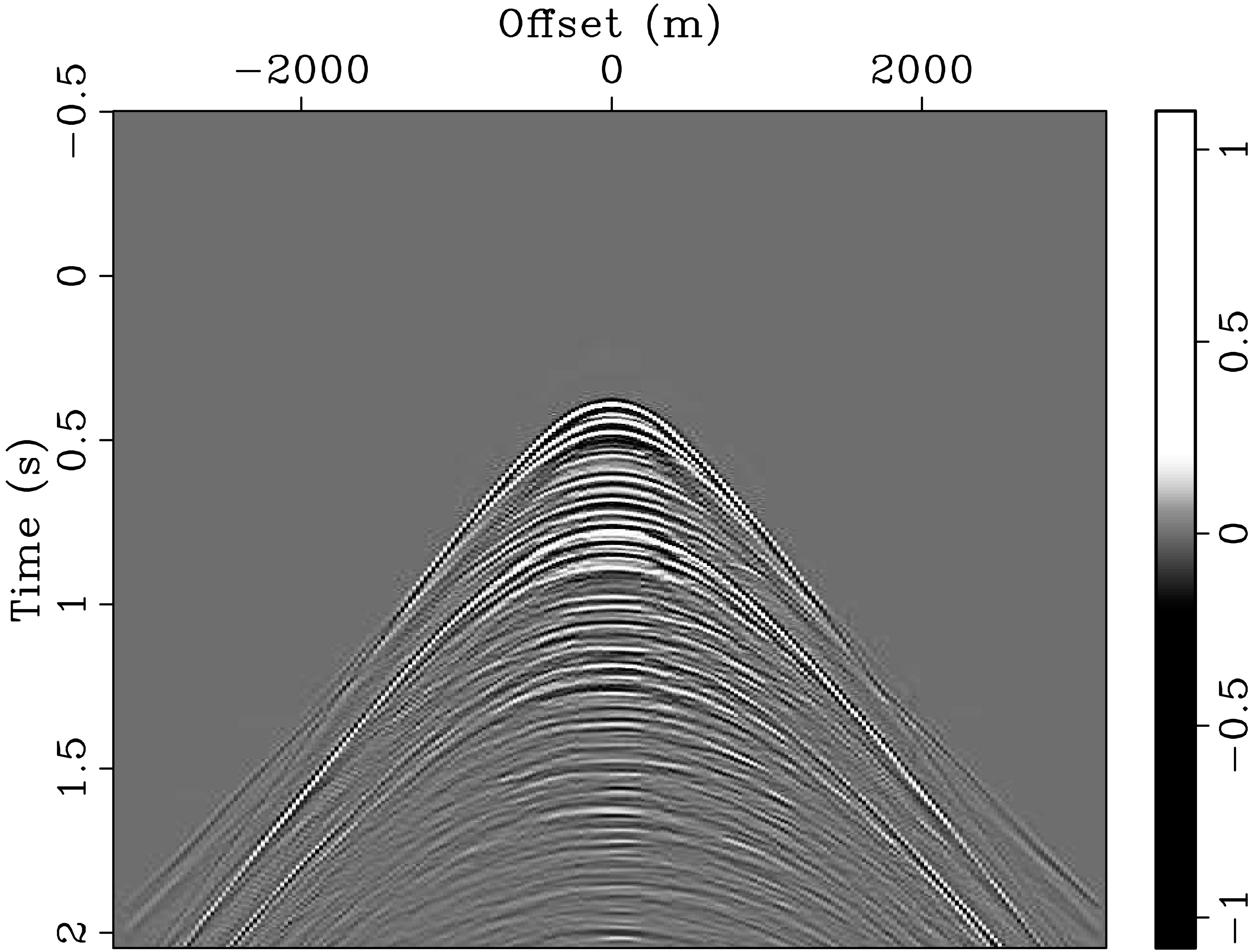
Total data



$\widehat{\mathbf{P}}$

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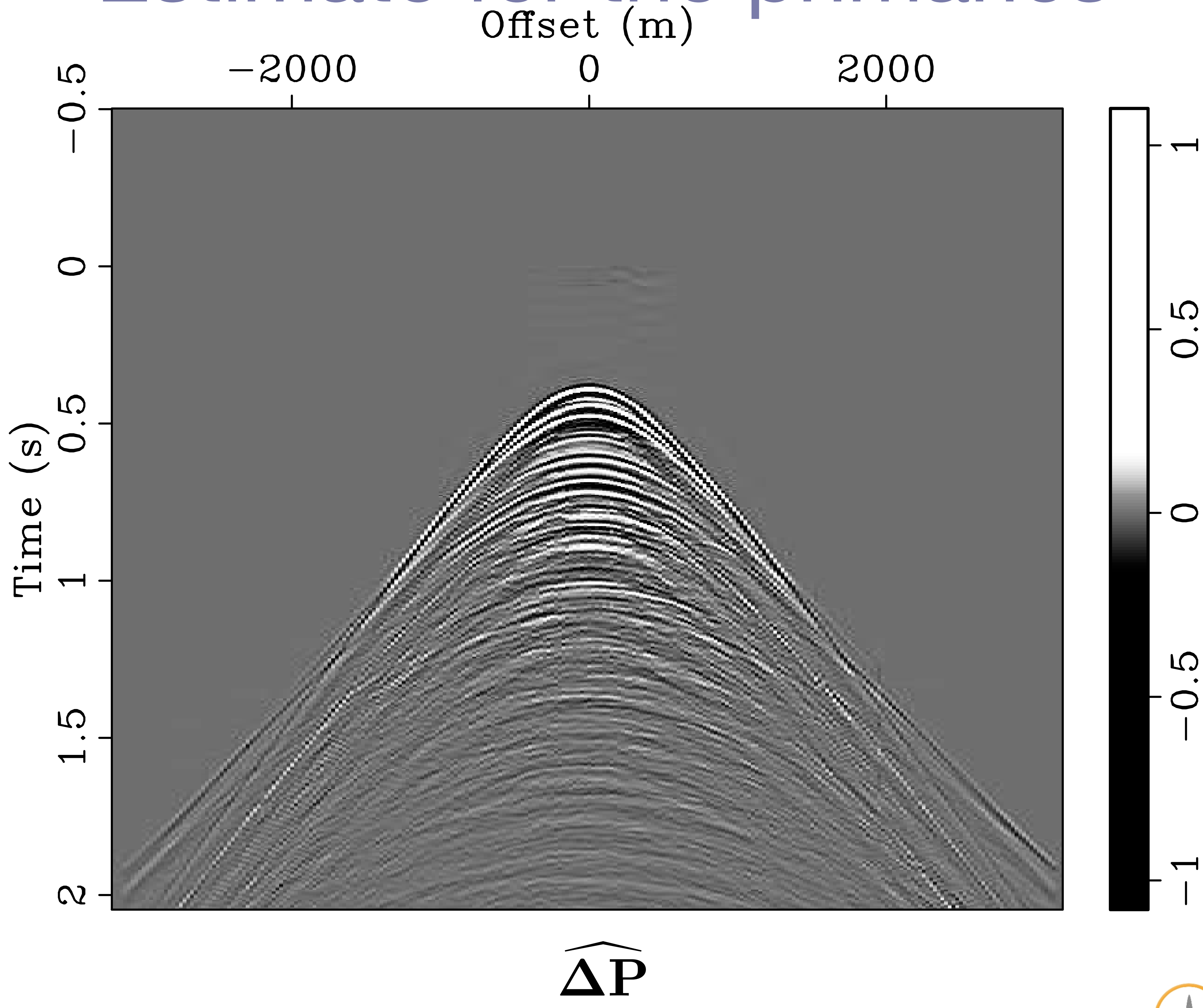
Estimate for the primaries



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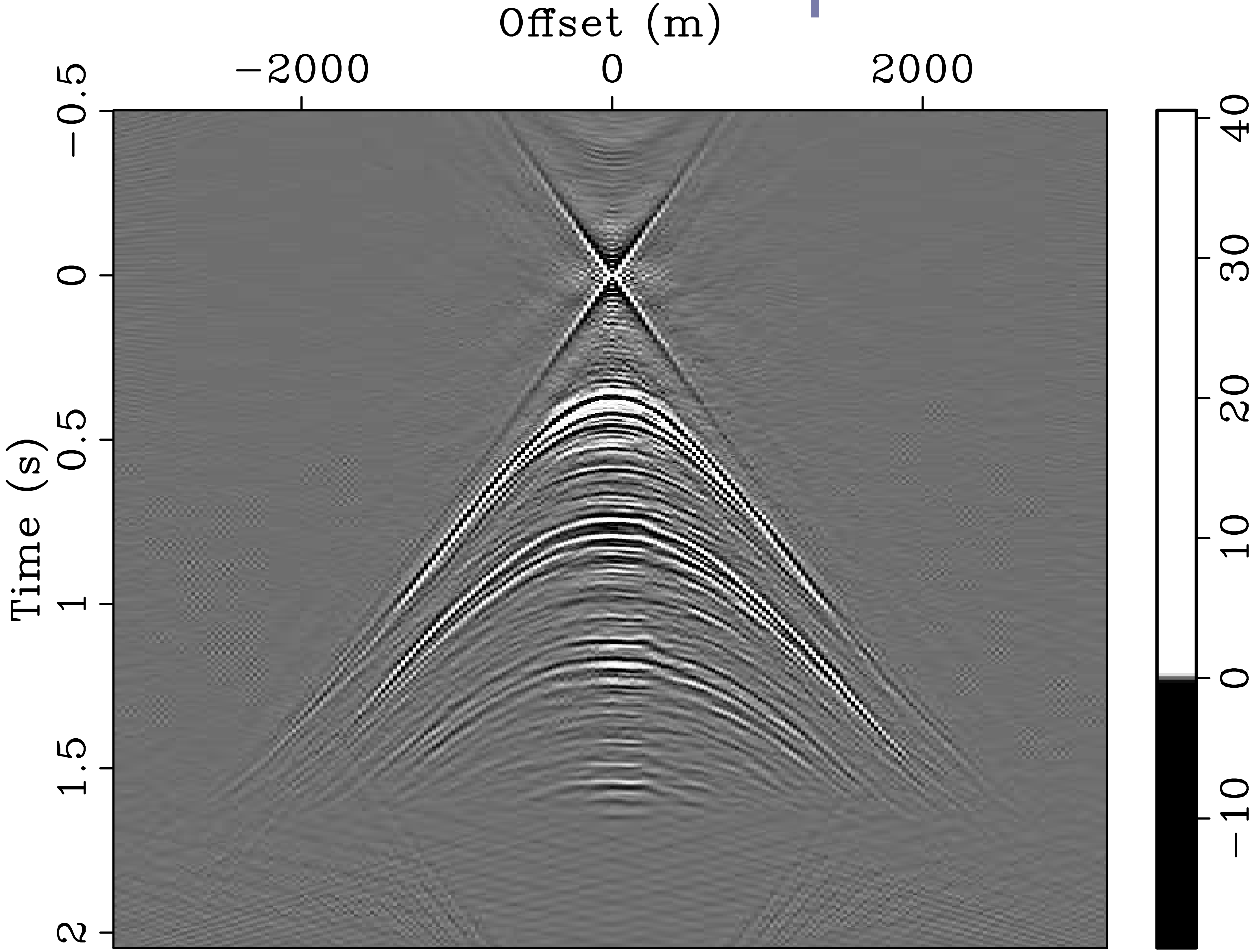
Focused with the primaries



$\tilde{f}$

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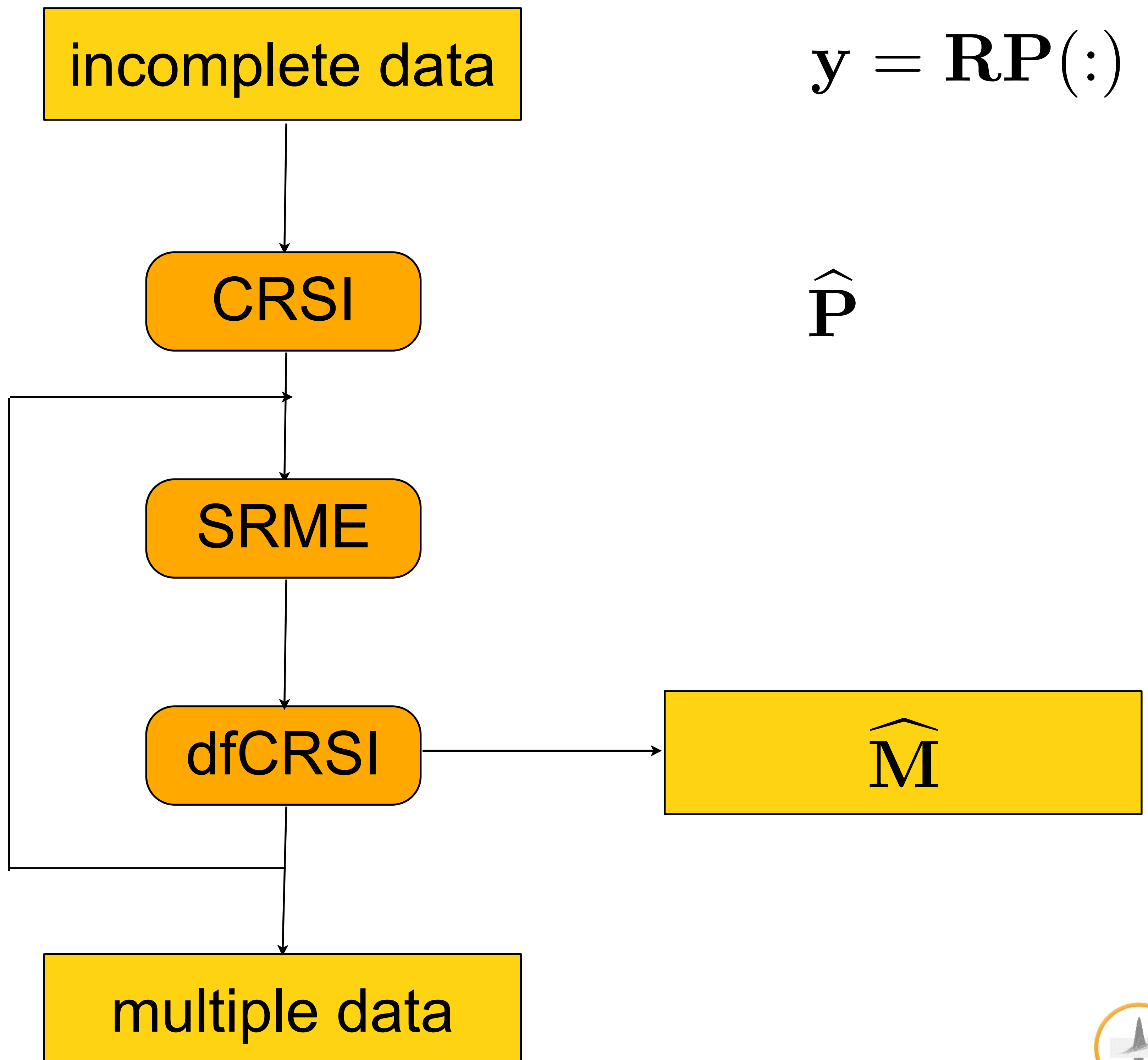
Multiple prediction with dfCRSI



Curvelet-based deFocal transform

Solve

$$\mathbf{P}_\epsilon : \qquad \begin{cases} \widetilde{\mathbf{x}} = \arg\min_{\mathbf{x}} \|\mathbf{x}\|_1 \quad \text{s.t.} \quad \|\mathbf{A}\mathbf{x} - \mathbf{y}\|_2 \leq \epsilon \\ \widetilde{\mathbf{f}} = \mathbf{S}^T \widetilde{\mathbf{x}} \end{cases}$$

with

$$\begin{aligned} \mathbf{A} &:= \boldsymbol{\Delta}\mathbf{P}^T\mathbf{C}^T \text{ and } \boldsymbol{\Delta}\mathbf{P} := \mathbf{F}^H \text{block diag}\{\text{conj}(\boldsymbol{\Delta}\mathbf{p})\}\mathbf{F} \\ \mathbf{S} &:= \mathbf{C} \\ \mathbf{y} &= \mathbf{P}(:) \\ \mathbf{P} &= \text{total data} \\ \tilde{\mathbf{f}} &= \text{defocussed data.} \end{aligned}$$

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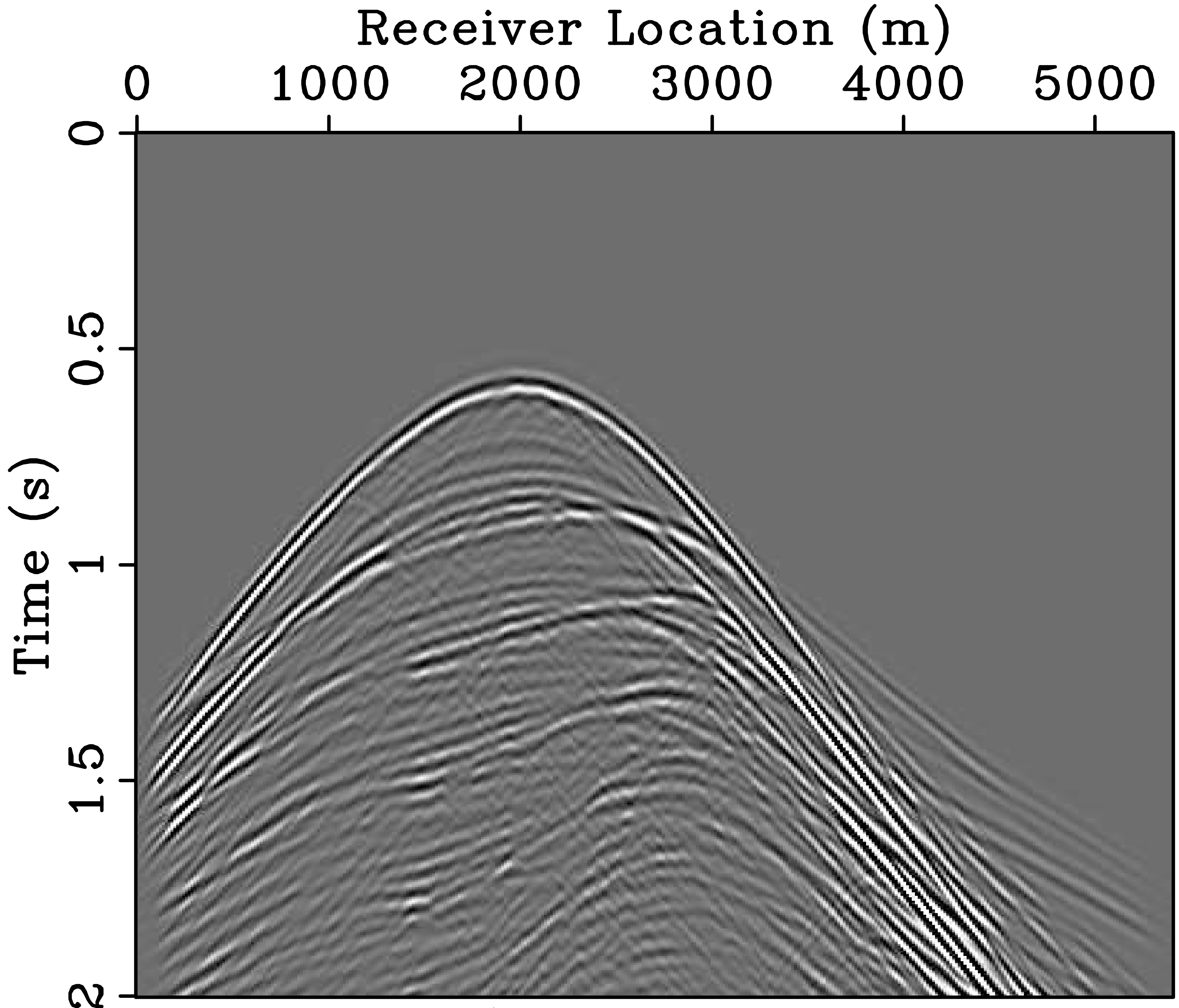
SRME predicted multiples



$$\widehat{\mathbf{P}} = \mathbf{P}\mathbf{P}$$

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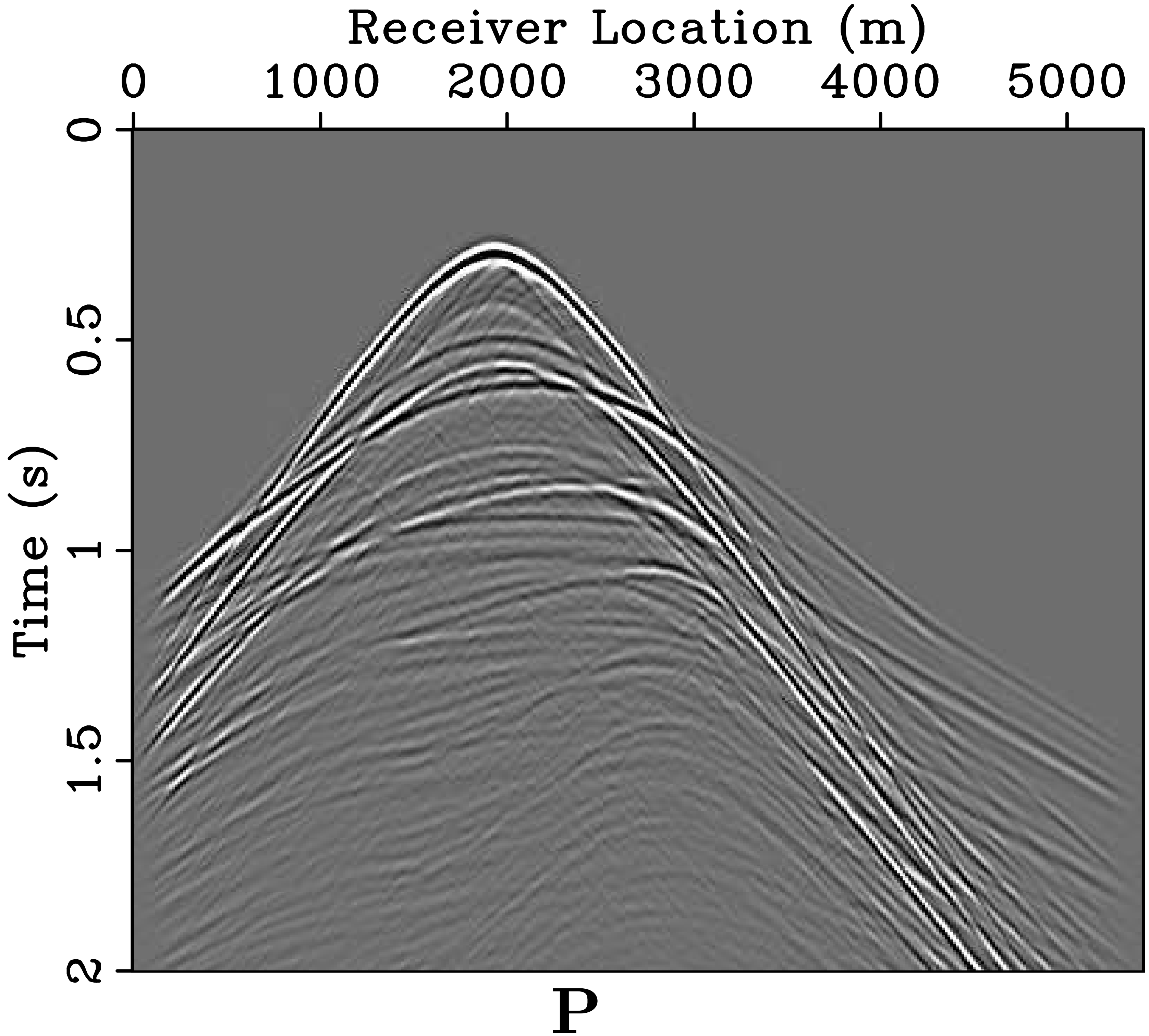
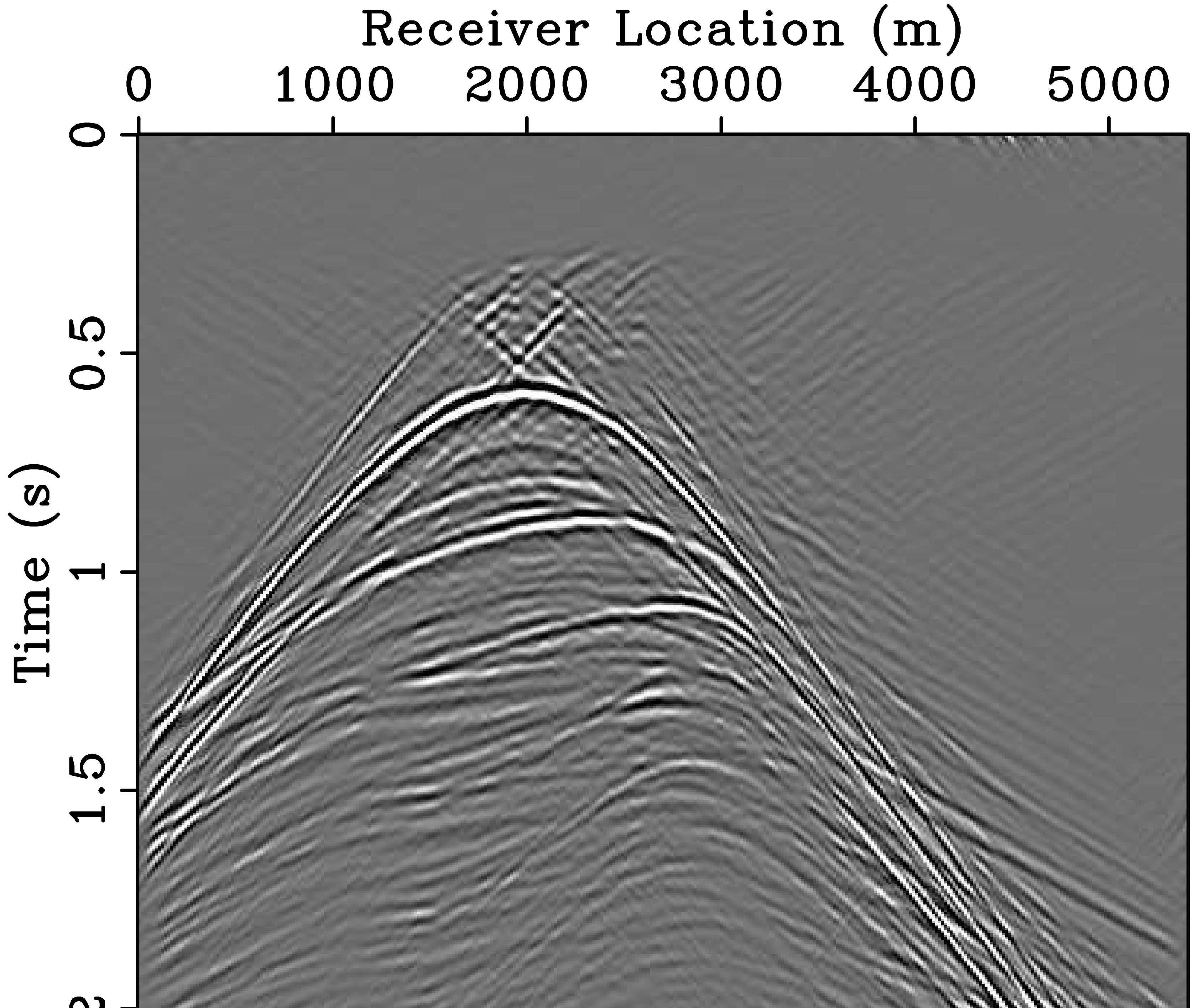
Original data



Multiple estimate by dfCRSI



$\tilde{f}$

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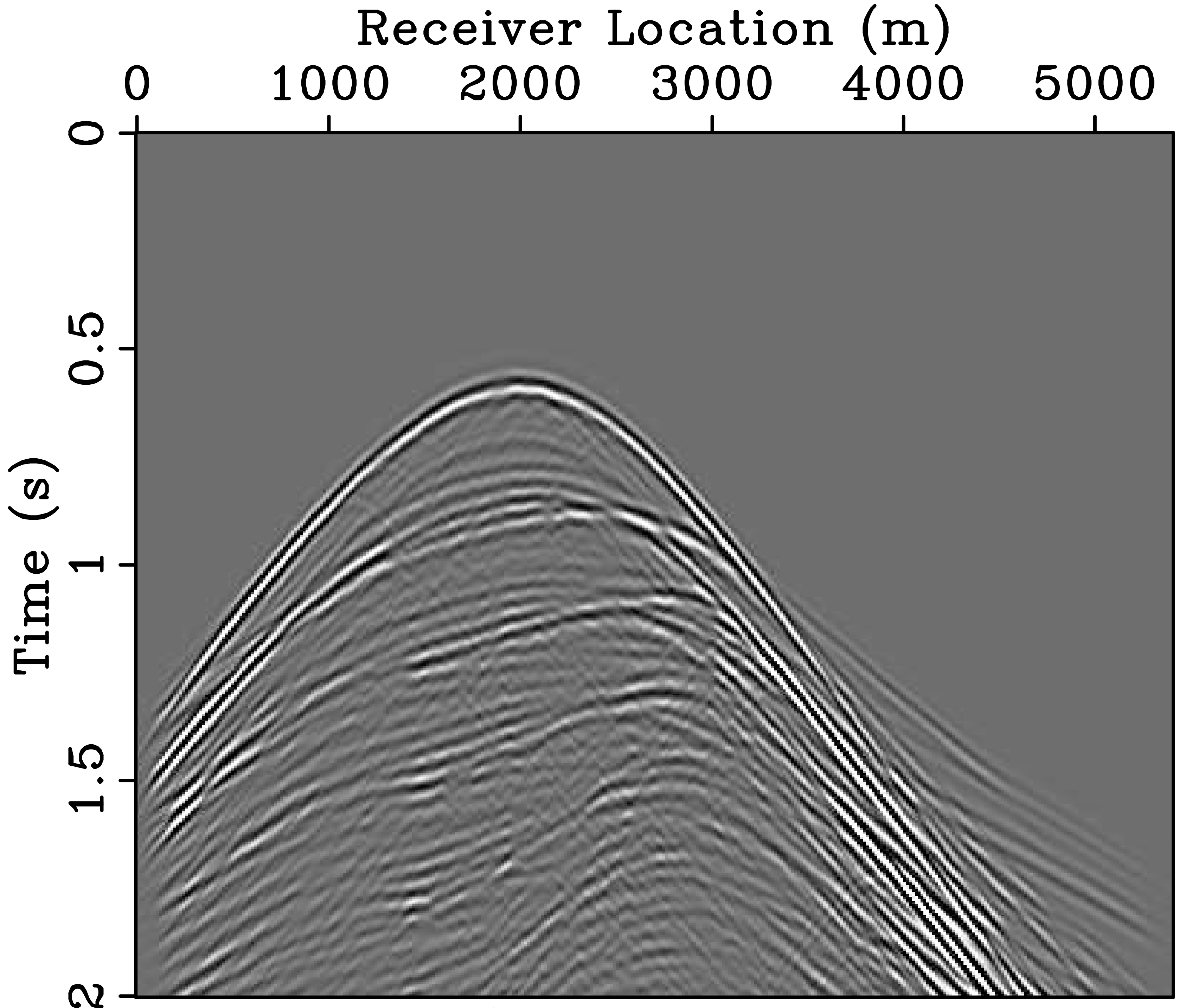
SRME predicted multiples



$$\widehat{\mathbf{P}} = \mathbf{P}\mathbf{P}$$

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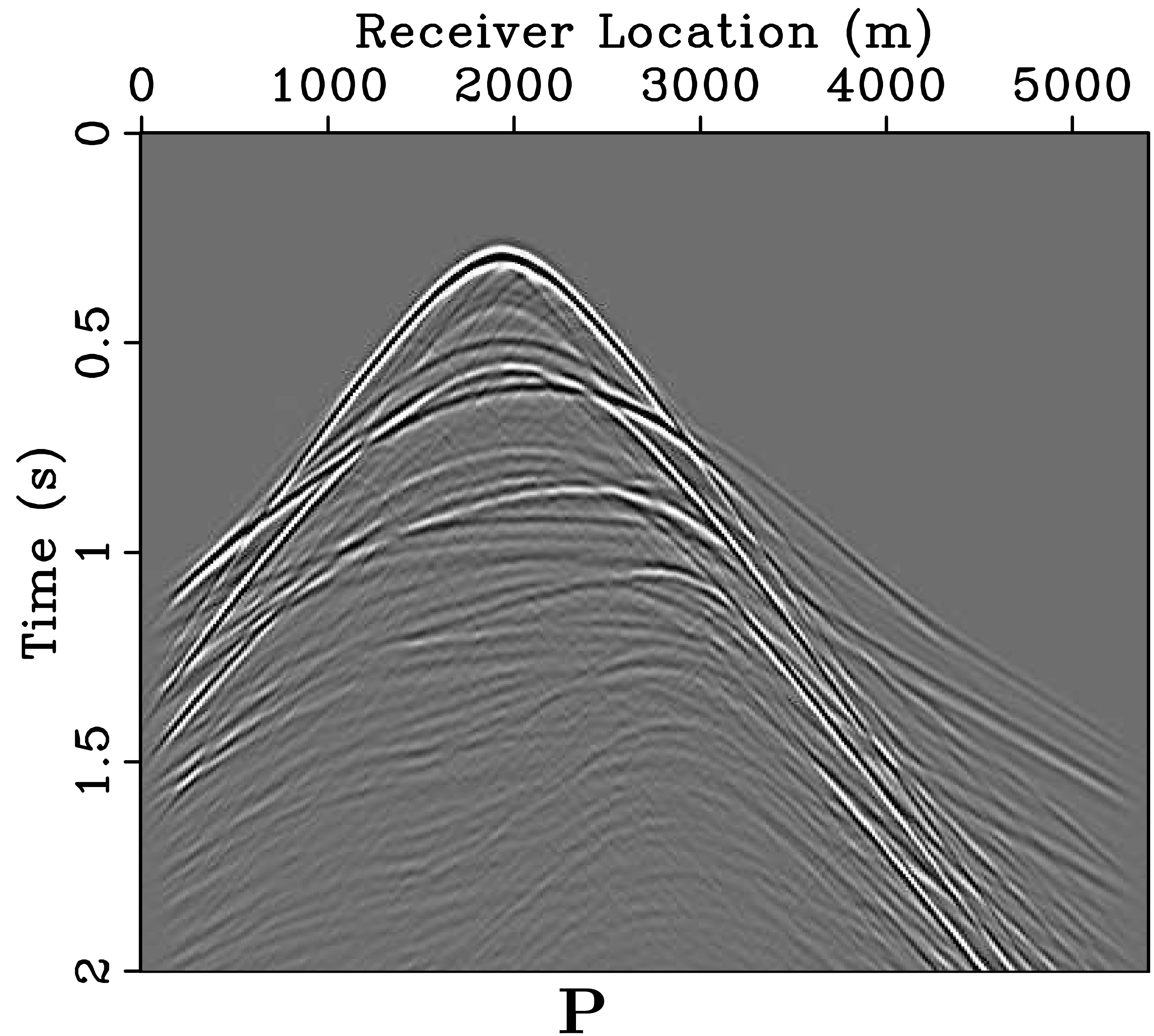
Original data



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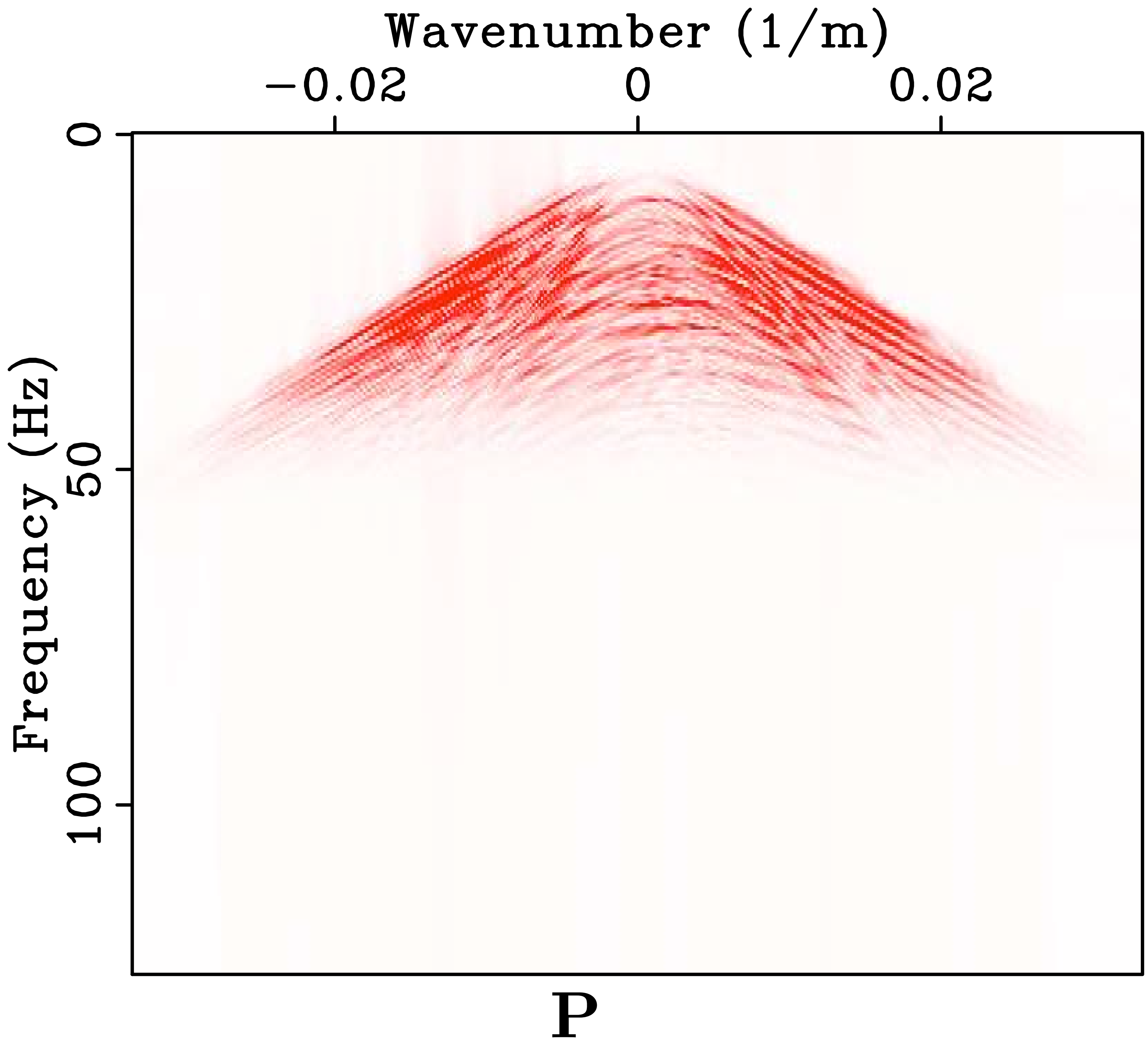
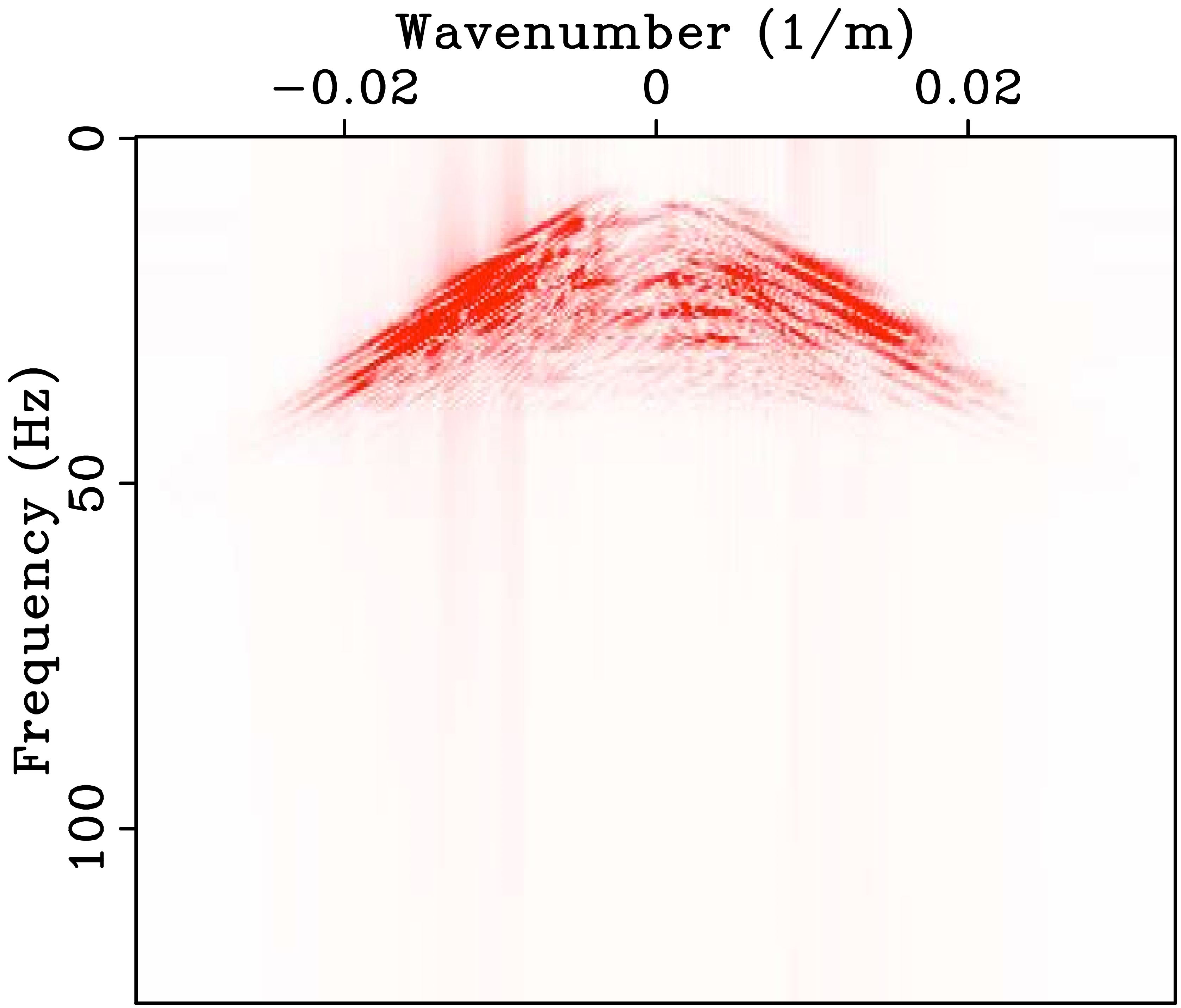
Original data



SRME predicted multiples



$$\widehat{\mathbf{P}} = \mathbf{P}\mathbf{P}$$

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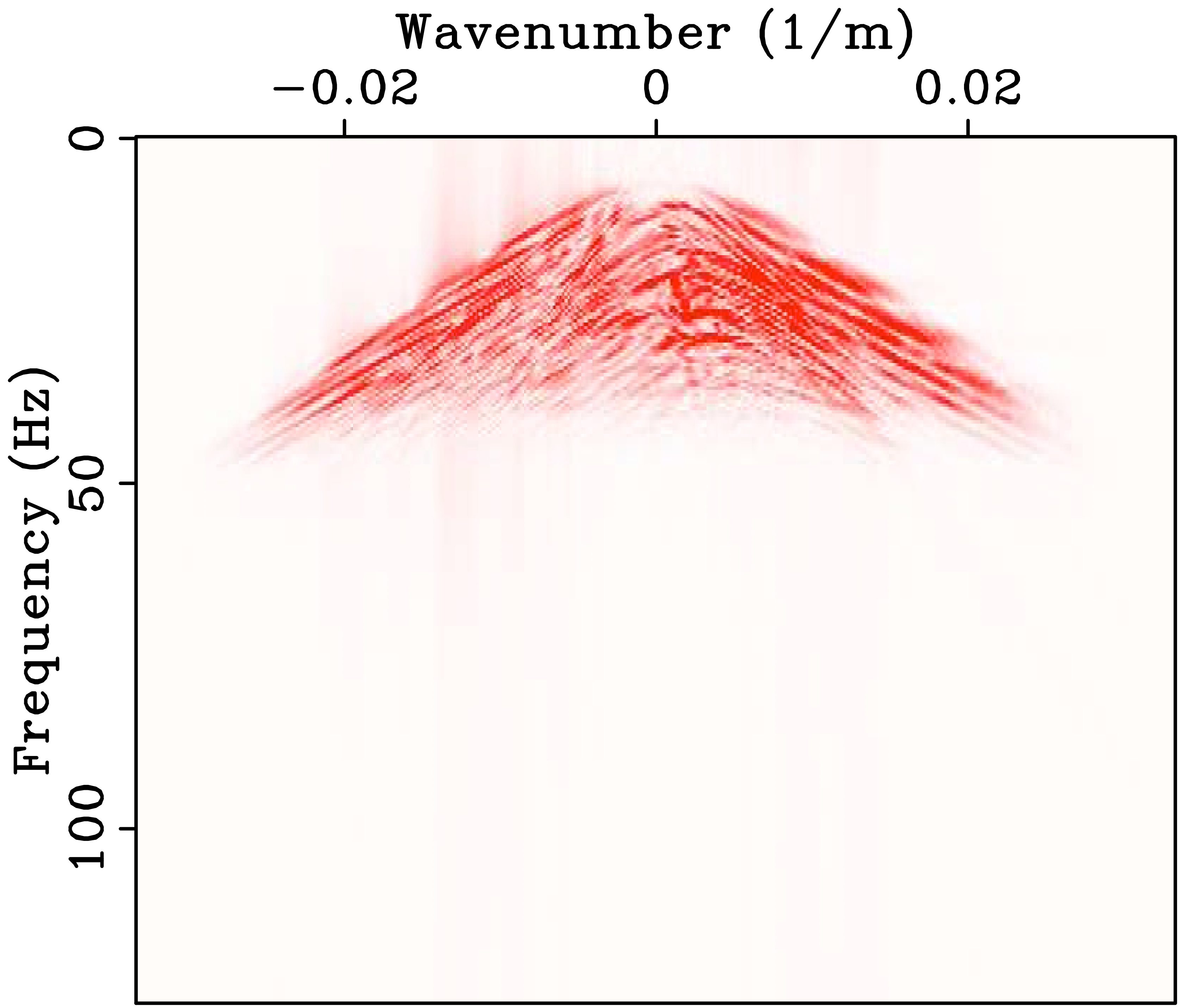
Multiple estimate by dfCRSI



$\tilde{f}$

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Conclusions

Focused CRSI

- improves the recovery and hence predicted multiples
- precursor of migration-based CRSI
- primary estimates have higher bandwidth (deconvolution of the source)

deFocused CRSI

- improves the band width
- contains artifacts due to remnant multiple energy & X-terms

Curvelet-based approach improves the primary-multiple prediction.

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Acknowledgments

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