ABSTRACT

Incomplete data, unknown source-receiver signatures and free-surface reflectivity represent challenges for a successful prediction and subsequent removal of multiples. In this paper, a new method will be represented that tackles these challenges by combining what we know about wavefield (de-)focussing, by weighted convolutions/correlations, and recently developed curvelet-based recovery by sparsity-promoting inversion (CRSI). With this combination, we are able to leverage recent insights from wave physics towards a nonlinear formulation for the multiple-prediction problem that works for incomplete data and without detailed knowledge on the surface effects.

Surface-related multiple prediction and seismic interferometry are examples where weighted multi-dimensional cross-convolutions and cross-correlations of seismic data volumes provide information on Green's functions that describe the Earth response at the surface. For instance, surface-related multiples can approximately be predicted through a weighted multidimensional convolution of the data with itself, while 'daylight imaging' techniques extract the Green's function by cross-correlation of wavefields (see e.g. Wapenaar et al., 2006, which contains a collection of the most recent papers on this topic). Recently, new approaches have been proposed, where the Green's functions are extracted through *inversion* or *deconvolution* (See the contributions of Snieder *et.al*, Schuster *et.al.* and Berkhout and Verschuur in Wapenaar et al., 2006). We follow a similar approach, where we are interested in finding an alternative formulation for the following two operations:

- **wavefield defocusing**, where the wavefield is *convolved* with the 'primary' wavefield. This convolution maps the primaries into first-order multiples and first-order multiples into second-order multiples etc., i.e., we have the mapping

$$p^{(m)}(x,t) \mapsto p^{(m+1)}(x,t) = \big(G\big[\tilde{p}^0,\, \mathcal{A}\big]p^m\big)(x,t) \tag{1}$$

  with $G\big[\tilde{p}^0,\, \psi\big]\cdot := \mathcal{A}^\dagger *_{x,t} \tilde{p}^0 *_{x,t} \cdot$, the symbol $*_{x,t}$ denoting multi-dimensional cross-convolution and $\mathcal{A}^\dagger$ the weighting;

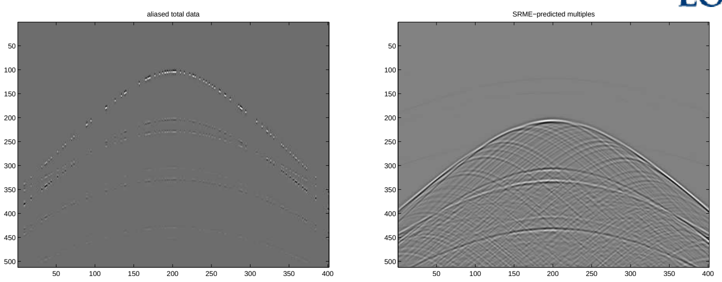
- **wavefield focusing**, where the wavefield is **correlated** with the 'primary' wavefield. This correlation maps the primaries into first order multiples and first-order multiples into second-order multiples etc., i.e., we have the mapping

$$p^{(m+1)}(x,t) \mapsto p^{(m)}(x,t) = \big(F\big[\tilde{p}^0, \mathcal{B}\big]p^{m+1}\big)(x,t) \tag{2}$$

  with $P\big[\tilde{p}^0, \mathcal{B}\big]\cdot := \tilde{p}^0 \otimes_{x,t} \mathcal{B}^\dagger *_{x,t} \cdot$, the symbol $\otimes_{x,t}$ denoting multi-dimensional cross-correlation and $\mathcal{B}^\dagger$ another weighting.

In these expressions, p^m refers to the $(m)^{th}$-order multiple in the data (p^0 represents the primary wavefield) and $\tilde{p}^0$ represents an estimate for the 'primaries' that is assumed to be given (we used " to indicate that in practice we only have approximate knowledge of the primaries since the sole purpose of this work is to estimate these primaries). G and F are the defocusing and focussing operators that map the m^{th}-order component to the $(m+1)^{th}$-order component and back. The defocusing operator consists of a multi-dimensional weighted cross-convolution between the 'primaries', $\tilde{p}^0$, and the wavefield, followed by a *deconvolution* by $\mathcal{A}^\dagger$ that contains the surface reflectivity and the pseudo inverse (denoted by the †) of the source and receiver directivity and time signatures. After cross-convolution, the wavefield has a tendency to spread out, i.e., we added a 'travel path', hence the name defocusing. The wavefield focusing operator, on the other hand, constitutes a weighted **cross-correlation** of the wavefield with the 'primaries', removing a 'travel path'. This weighting by $\mathcal{B}^\dagger$ is defined by the damped pseudo inverse of the 'autocorrelation' of the 'primary' operator.

In this paper, we seek an alternative formulation, where (i) **no** information is required on $\mathcal{A}$; (ii) that is stable w.r.t. incomplete data and (iii) where focusing is accomplished by sparsity-promoting inversion (replacing $\mathcal{B}^\dagger$). Fig. 1 illustrates the effect of missing data on the prediction of multiples. To accomplish these goals, we combine the focusing property and the sparsity of curvelets. After discretization (lower- and upper-case bold symbols refer to discretized vectors and matrices), define the following



(a) (b)

Figure 1: SRME multiple prediction **(b)** from incomplete data **(a)** with $80\,\%$ of the traces missing. Notice the artifacts due to the missing data leading to a deterioration of the multiple prediction.

incomplete data representation

$$\mathbf{y} = \mathbf{A}\mathbf{x} \quad \text{with} \quad \mathbf{A} := \mathbf{R}\mathbf{P}\mathbf{C}^T \tag{3}$$

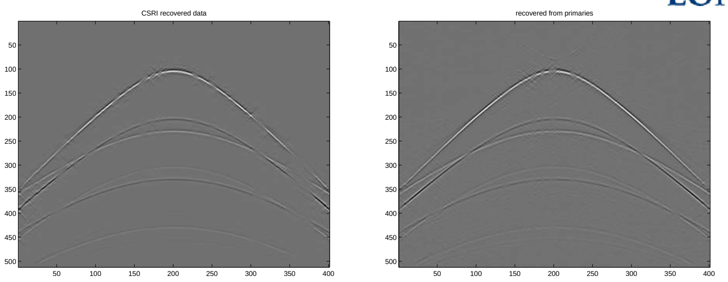
with $\mathbf{R}$ the restriction operator; $\mathbf{P} := \widetilde{\mathbf{p}}^0 *_{x,t}$ the 'primary' operator; $\mathbf{C}^T$ the transpose (inverse) of the curvelet transform (see e.g. Hennenfent and Herrmann, 2006b, and the references therein), $\mathbf{x}$ the curvelet coefficient vector and $\mathbf{y} = \mathbf{R}\mathbf{d}$ the incomplete data. This signal representation differs from standard-CRSI (Hennenfent and Herrmann, 2006a; Herrmann and Hennenfent, 2007) by including the 'primary' operator $\mathbf{P}$.

CRSI with focusing: By inverting the defocusing, seismic data is focused, bootstrapping the sparsity obtained by the curvelet transform. Using this property, the recovery from incomplete data can be written as follows

$$\mathbf{F}: \quad \begin{cases} \widetilde{\mathbf{x}} = \arg\min_{\mathbf{x}} \|\mathbf{x}\|_1 \quad \text{subject to} \quad \mathbf{A}\mathbf{x} = \mathbf{y} \\ \widetilde{\mathbf{d}}^{(m-1)} = \mathbf{C}^T\widetilde{\mathbf{x}} \\ \widetilde{\mathbf{d}} = \mathbf{P}\mathbf{C}^T\widetilde{\mathbf{x}}. \end{cases} \tag{4}$$

In words, the solution of $\mathbf{F}$ involves finding the sparsest set of curvelet coefficients that matches the incomplete data when convolved with the primaries. The data, $\mathbf{d}$, in this case includes primaries and multiples (see Fig. 1). As such, the estimated coefficients represent an estimate for the *focused* data since they are converted back into data by the 'primary' operator during the optimization. Eq. 4 corresponds to a curvelet-sparsity regularized inverse of Berkhout's focusing matrix and of the convolution operator in interferometric imaging (Vasconcelos and Snieder, 2006). The symbol $\mathbf{d}^{(m-1)}$ refers to focused data with primaries mapped to the focal point and m^{th}-order multiples mapped to $(m-1)^{th}$-order. The result of the sparse recovery from the incomplete data using standard-CRSI (Herrmann and Hennenfent, 2007) and CRSI + focusing are summarized in Fig 2. Expectedly, the curvelet transform compounded with the primary operator improves the recovery.

Defocusing with CRSI: After successful recovery of the incomplete data, multiples can be predicted using the nonlinear mapping defined in Eq. 1. This mapping through multi-dimensional convolution, however, has the disadvantage that an estimate is needed for $\mathcal{A}$. By defining $\mathbf{A} := \mathbf{P}^*\mathbf{C}^T$ with $\mathbf{P}^* = \widetilde{\mathbf{p}}^0 \otimes_{x,t}$ the adjoint of the primary



(a) (b)

Figure 2: Comparison between CSRI **(a)** and CSRI + focusing **(b)** for data with $80\,\%$ of the traces missing. Notice the significant uplift from compounding the inverse curvelet transform with the focusing 'primary' operator.

operator, multiples can be predicted by solving

$$\mathbf{G}: \qquad \begin{cases} \widetilde{\mathbf{x}} = \arg\min_{\mathbf{x}} \|\mathbf{x}\|_1 \quad \text{subject to} \quad \mathbf{A}\mathbf{x} = \mathbf{y} \\ \widetilde{\mathbf{d}}^{(m+1)} = \mathbf{C}^T\widetilde{\mathbf{x}} \end{cases} \tag{5}$$

with $\mathbf{y} = \widetilde{\mathbf{d}}$ (estimated above). In words, the solution of $\mathbf{G}$ corresponds to finding the sparsest set of curvelet coefficients that matches the data when cross-correlated with the primaries. As such the estimated coefficients represent an estimate for the multiples, since this estimate for the multiples is converted to the primaries after applying the correlation during the optimization. More precisely, this formulation corresponds to a sparse inversion of the operator that maps multiples to primaries. The advantage of this formulation is that it does not require information on $\mathcal{A}$ as can be observed from Fig. 3.

Examples: Without loss of generality, we considered the acoustice reflection response of a 1-D medium consisting of three layers and a free surface. For this type of medium, the multi-dimensional cross-convolutions and correlations become simple convolutions and correlations that are diagonal in the $f-k$ domain. In practice, the primaries are not known and the data itself is used instead, as part of an iterative procedure. In this case, spurious non-physical events may occur an observation reported in the literature (Snieder *et.al.* in Wapenaar et al., 2006).

Discussion

The methodology presented in this paper banks on two complementary aspects of wave phenomena, namely, (i) the focusing and defocusing by multidimensional cross-convolutions/correlations, reflecting certain physical relations, and (ii) the existence of a multiscale and multi-directional curvelet transform that sparsely represents high-frequency solutions of wave equations. Pairing these two aspects leads to a new formulation for the prediction of multiples from incomplete data, without knowledge on the surface effects. The focusing is found to improve the recovery because the data becomes sparser in the curvelet domain after focusing and this explains the improvement over curvelet-only CRSI. We also observed that $\mathbf{F}$ corresponds to the focal transform and interferometric imaging by deconvolution formalisms, opening the interesting new perspective of adding more robustness. The prediction for the multiples after the recovery also benefited from the sparsity promotion. Again the sparsity of curvelets, that is

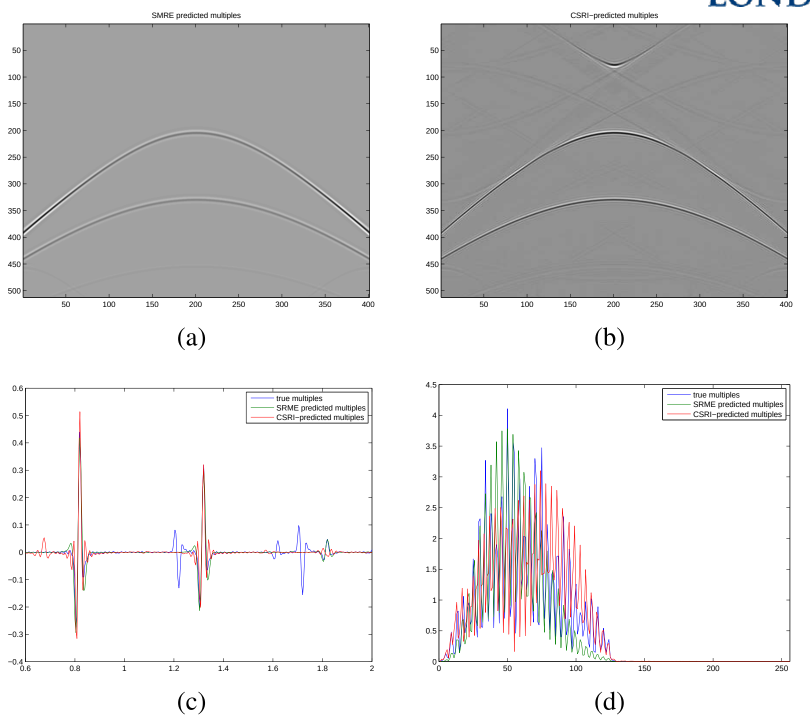


Figure 3: Comparison between convolution-based multiple prediction **(a)** and sparsity-based multiple prediction **(b)**. Aside from the acausal artifact, the sparsity-promoting multiple prediction according to Eq.5 improves the frequency content and makes it closer to the spectrum of the true multiples. Trace-wise comparisons in **(c-d)** between the true (including internal) mutiples (blue), the multiples predicted with conventional (green) and sparsity promoted predicted multiples (red) confirm this observation. The difference in the spectrum are partially due to the fact that we only predicted the surface-related multiples.

related to the invariance of curvelets under wave propagation, leads to an improved prediction. This improvement can be understood because the method inverts the adjoint of the primary operator that contains the surface effects. The improved predictions will in turn improve curvelet-based primary-multiple separation (Herrmann et al., 2006). Interferometric prediction of ground roll will be discussed elsewhere in these proceedings.

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