ABSTRACT

In this paper, we present a nonlinear curvelet-based sparsity-promoting formulation for three problems in seismic processing and imaging namely, seismic data regularization from data with large percentages of traces missing; seismic amplitude recovery for sub-salt images obtained by reverse-time migration and primary-multiple separation, given an inaccurate multiple prediction. We argue why these nonlinear formulations are beneficial.

In this paper, we report recent developments on the application of the curvelet transform (see e.g. Candes et al., 2005a; Hennenfent and Herrmann, 2006b) to seismic processing and imaging. Our approach banks on two fundamental properties of curvelets, namely the

- **detection of wave-fronts** without requiring *prior* information on the dips or on the velocity model;

- **invariance** under the action of wave propagation.

These two properties make this transform suitable for a robust formulation of problems such as seismic data regularization (Hennenfent and Herrmann, 2006a; Herrmann and Hennenfent, 2007), migration-amplitude recovery (Herrmann et al., 2006b) and primary multiple separation (Herrmann et al., 2006a). All these methods exploit sparsity in the curvelet domain that is a direct consequence of the above two properties and corresponds to a rapid decay for the magnitude-sorted curvelet coefficients.

Curvelets: As can be observed from Fig. 1, curvelets are localized functions that oscillate in one direction and that are smooth in the other directions. They are multiscale and multi-directional and because of their anisotropic shape (they obey the so-called parabolic scaling relationship, yielding a width $\propto 2^{j/2}$ and a length $\propto 2^{j}$ with j the scale), curvelets are optimal for detecting wavefronts. This explains their high compression rates for seismic data and images as reported in the literature.
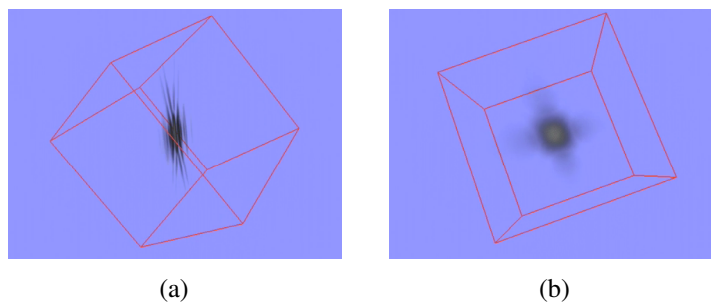

| (a) | (b) |

Figure 1: Example of a 3-D curvelet. Notice the oscillations in one direction and the smoothness in the other two directions.

Sparsity promoting inversion: High compression rates for signal representations are a prerequisite for the robust formulation of stable signal recovery problems and other inverse problems. These compression rates allow for a nonlinear sparsity promoting solution. As such sparsity-promoting norm-one penalty functionals are not new to the geosciences (see for instance the seminal work of Claerbout and Muir (1973), followed by many others), where sparsity is promoted on the model. What is different in the current surge of interest in sparsity-promoting inversion, known as 'compressed sensing' (Candes et al., 2005b; Donoho et al., 2006), is (i) the existence of sparsity promoting transforms such as the curvelet transform; (ii) the deep theoretical understanding on what the conditions are for a successful solution. This work can be seen as the application of these recent ideas to the seismic situation and involves the solution of the following norm-one nonlinear program,

$$\mathbf{P}_\epsilon: \quad \begin{cases} \widetilde{\mathbf{x}} = \arg\min_{\mathbf{x}} \|\mathbf{x}\|_1 \quad \text{s.t.} \quad \|\mathbf{A}\mathbf{x} - \mathbf{y}\|_2 \leq \epsilon \\ \widetilde{\mathbf{d}} = \mathbf{S}^T \widetilde{\mathbf{x}} \end{cases} \tag{1}$$

in which $\mathbf{y}$ is the (incomplete) data, $\mathbf{A}$ the synthesis matrix and $\mathbf{S}^T$ the inverse sparsity transform. Both these matrices consist of the inverse curvelet transform matrix, $\mathbf{S}^T$ (the symbol ^{T} denoting the transpose) compounded with other operators depending on the problem. The above constrained optimization problem is solved to an accuracy of ϵ that depends on the noise level. Finally, $\widetilde{\mathbf{d}}$ stands for the recovered vector with the symbol $^\sim$ reserved for optimized quantities.

CRSI: An important topic in seismic processing is the seismic regularization problem, where attempts are made to recover fully-sampled seismic data volumes from incomplete data, i.e., data with large percentages ($> 80\,\%$) of traces missing. By choosing $\mathbf{A} := \mathbf{RC}^T$, $\mathbf{S} := \mathbf{C}$ and $\mathbf{y} = \mathbf{Rd}$ for the incomplete data, one arrives at the formulation for curvelet recovery by sparsity-promoting inversion (CRSI), which has successfully been applied to the recovery of incomplete seismic data (see e.g. Hennenfent and Herrmann, 2006a). In this formulation, $\mathbf{R}$ is the restriction matrix, selecting the rows from the curvelet transform matrix that correspond to active traces. As opposed to other recovery methods, such as sparse Fourier recovery and plane wave destruction, curvelet-based methods have the advantage of working in situations where there are conflicting dips without stationarity assumptions. The method exploits the high-dimensional continuity of wavefronts and as Fig. 2 demonstrates, recovery results improve when using the 3-D curvelet transform compared to the 2-D transform. For application of this method to real data, refer to other contribution by the authors to the proceedings of this meeting.
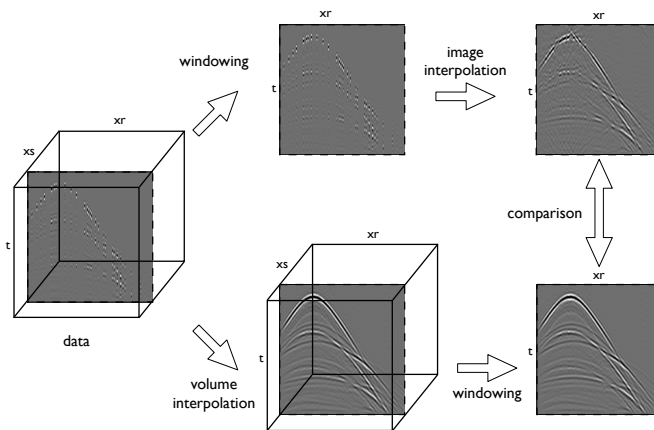

Figure 2: Illustration of sliced versus volumetric interpolation.

Migration amplitude recovery: Because of the 'alleged' invariance of curvelets under wave propagation, there has been a substantial interest in deriving migration operators in the curvelet domain (Douma and de Hoop, 2006; Chauris, 2006). In these approaches, one comes to benefit when strict sparsity is preserved. Strict sparsity is a significant stronger assumption than the preservation of high decay rates for the sorted coefficients. Curvelets are discrete and hence move around on grids and this makes it a challenge to define fast migration operators. Curvelets, however, prove to be very useful for solving the seismic amplitude recovery problem, during which curvelets are being imaged. On theoretical grounds (Herrmann et al., 2006b), one can expect the following identity to approximately hold

$$\mathbf{AA}^T\mathbf{r} \simeq \mathbf{\Psi r} \qquad (2)$$

with $\mathbf{r}$ an appropriately chosen discrete reference vector and $\mathbf{\Psi}$ the discrete normal operator, formed by compounding the discrete scattering and its transpose, the migration

operator. The synthesis operator in this case is defined as $\mathbf{A} := \mathbf{C}^T \mathbf{\Gamma}$ with $\mathbf{\Gamma}$ a diagonal weighting matrix. This identity diagonalizes the normal operator and allows for a stable recovery of the migration amplitudes by setting $\mathbf{y} = \mathbf{K}^T \mathbf{d}$, with $\mathbf{K}^T$ the migration operator and $\mathbf{d}$ the seismic data, and $\mathbf{S}^T := \left(\mathbf{A}^T\right)^\dagger$ with $\dagger$ the pseudo inverse. Results of this procedure on the SEG AA' dataset with a reverse-time migration operator, are summarized in Fig. 3. The resulting image shows a nice recovery of the amplitudes. For more details on this method and on a method to remove remaining artifacts, refer to another contributions by the authors to the proceedings of this meeting.
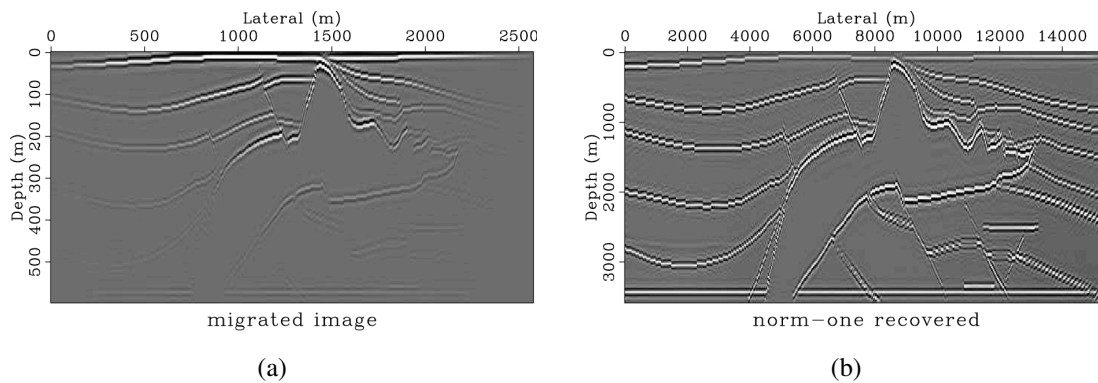

(a)   (b)

Figure 3: Reverse-time migration on the SEG AA' salt model. **(a)** Conventional migrated image $\mathbf{y} = \mathbf{K}^T \mathbf{d}$. **(b)** Seismic amplitude recovery through $\mathbf{P}_\epsilon$. Notice the improved amplitudes. Some minor artifacts remain.

Primary-multiple separation: So far, we looked at exploiting the sparsity of curvelets in the data and image domains for the purpose of recovery. The ability of curvelets to detect wavefronts with conflicting dips, allows for a formulation of a coherent signal separation method that uses inaccurate predictions as weightings. By defining the synthesis matrix as $\mathbf{A} := \begin{bmatrix} \mathbf{C}^T \mathbf{W}_1 & \mathbf{C}^T \mathbf{W}_2 \end{bmatrix}$, $\mathbf{x} = \begin{bmatrix} \mathbf{x}_1 & \mathbf{x}_2 \end{bmatrix}^T$ and $\mathbf{y} = \mathbf{d}$ and by setting the diagonal weighting matrices $\mathbf{W}_{1,2}$ in terms of predictions for the primaries and multiples, the solution of $\mathbf{P}_\epsilon$ separates primaries and multiples (Herrmann et al., 2006a) even for inaccurate predictions for which least-squares subtractions fails (see Fig. 4).

Discussion

The methodology presented in this paper banks on two favorable properties of curvelets, namely their ability to detect wavefronts and their approximate invariance under wave propagation. These properties allow for a formulation of seismic processing and imaging problems that promote sparsity through the nonlinear optimization problem $\mathbf{P}_\epsilon$. We showed that by compounding the curvelet transform with certain matrices each problem can be cast into one and the same optimization problem. The results show that (i) exploiting the multi-dimensional structure of seismic data with 3-D curvelets leads to a recovery scheme that is able to reconstruct fully sampled data volumes from data with $> 80\%$ traces missing; (ii) the invariance of curvelets under the demigration-migration operator can be used to recover the seismic amplitudes by inverting a diagonally weighted curvelet matrix. This latter approach is an extension of recent work by Symes (2006). Finally, we also showed that (iii) the sparsity of curvelet can be used to separate coherent signal components given an (inaccurate) prediction. The successful application to this wide spectrum of problems opens an enticing perspective of extending this framework to multiple prediction and the compression of (migration) operators.
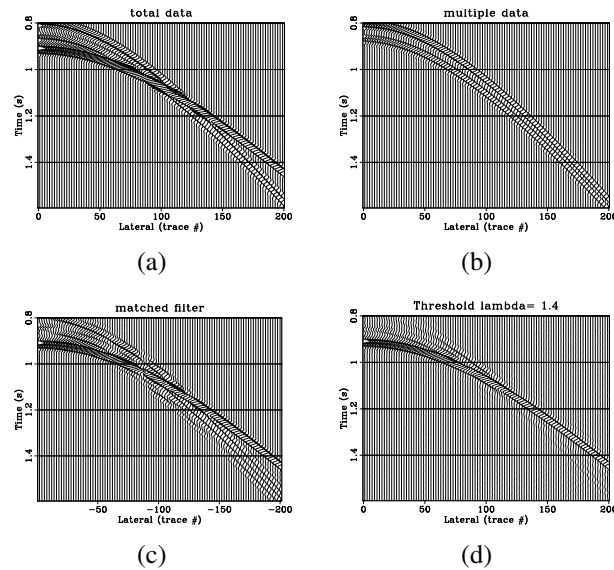
Figure 4: Example of primary-multiple separation through $\mathbf{P}_\epsilon$ for predicted multiples with moveout errors. **(a)** the total data with primaries and multiples. **(b)** the true multiples used for the prediction. **(c)** the result obtained with least-squares adaptive subtraction with localized windows. **(d)** the result obtained with a single curvelet-domain soft thresholding with $\lambda = 1.4$. Notice that least-squares subtractions fails.

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