Recent developments in curvelet-based seismic processing

Felix J. Herrmann

Seismic Laboratory for Imaging and Modeling

slim.eos.ubc.ca

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Combinations of **parsimonious** *signal representations with* **nonlinear** *sparsity promoting programs hold the* **key** *to the next-generation of seismic data processing algorithms ...*

Since they

- *allow for a formulation that is* ***stable*** *w.r.t. noise & incomplete data*
- *do not require* ***prior*** *information on the* ***velocity*** *or* ***locations*** *&* ***dips*** *of the events*

Seismic data and images are complicated because

- wavefronts & reflectors are multiscale & multi-directional
- the presence of caustics, faults and pinchouts

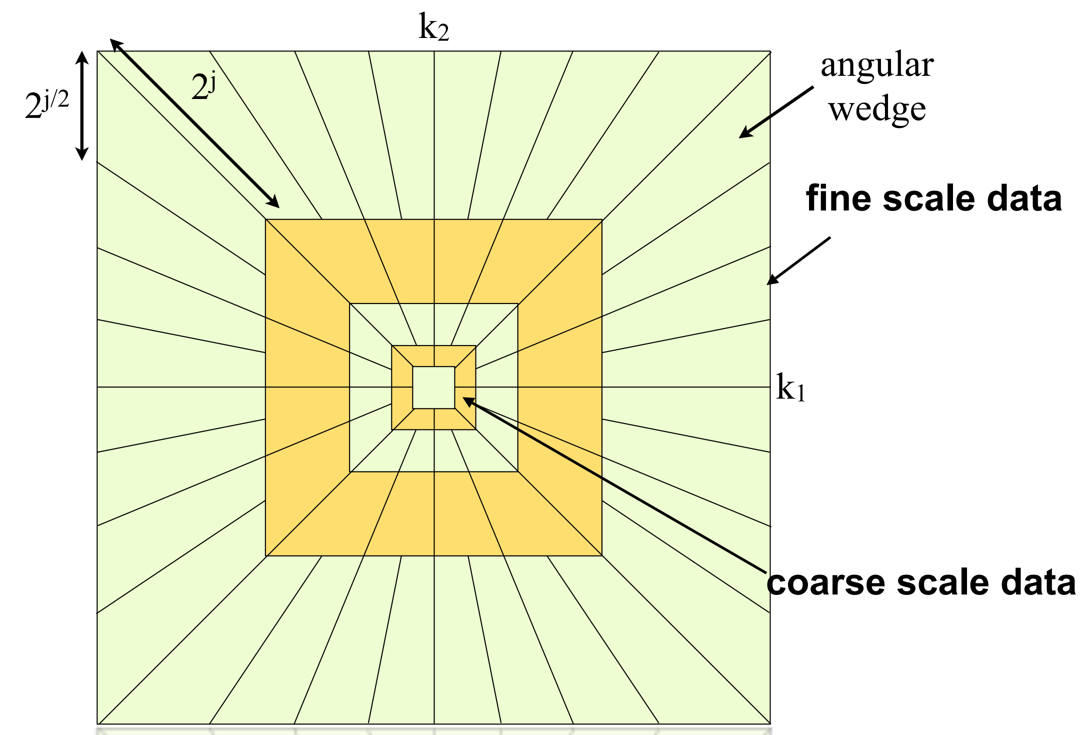
Curvelets

Representations for seismic data

Transform	Underlying assumption
FK	plane waves
linear/parabolic Radon transform	linear/parabolic events
wavelet transform	point-like events (1D singularities)
curvelet transform	**curve-like events (2D singularities)**

Properties curvelet transform:

- **multiscale:** tiling of the FK domain into dyadic coronae
- **multi-directional:** coronae sub-partitioned into angular wedges, # of angle doubles every other scale
- **anisotropic:** parabolic scaling principle
- **Rapid decay space**
- **Strictly localized in Fourier**
- **Frame with moderate redundancy**



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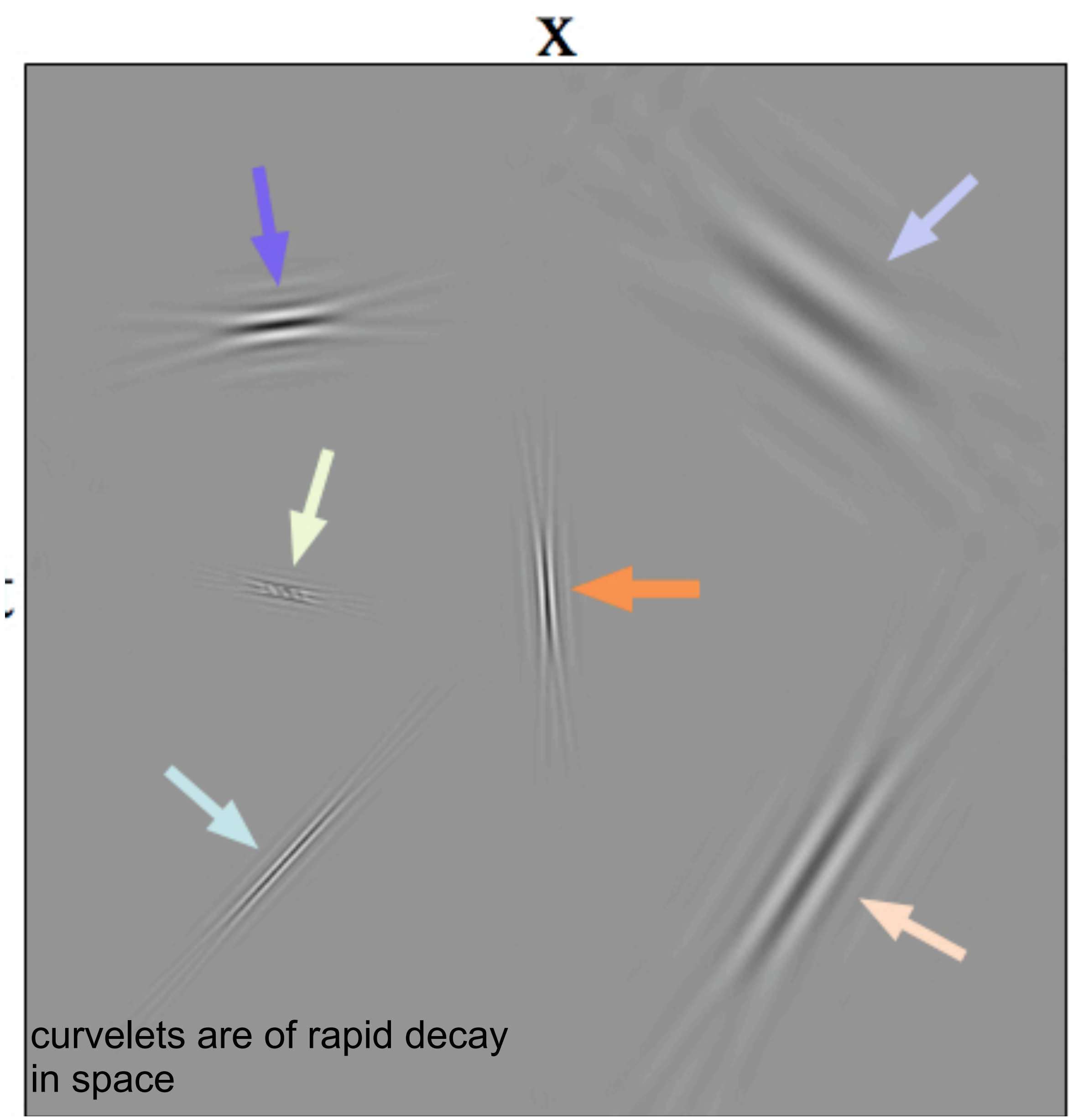
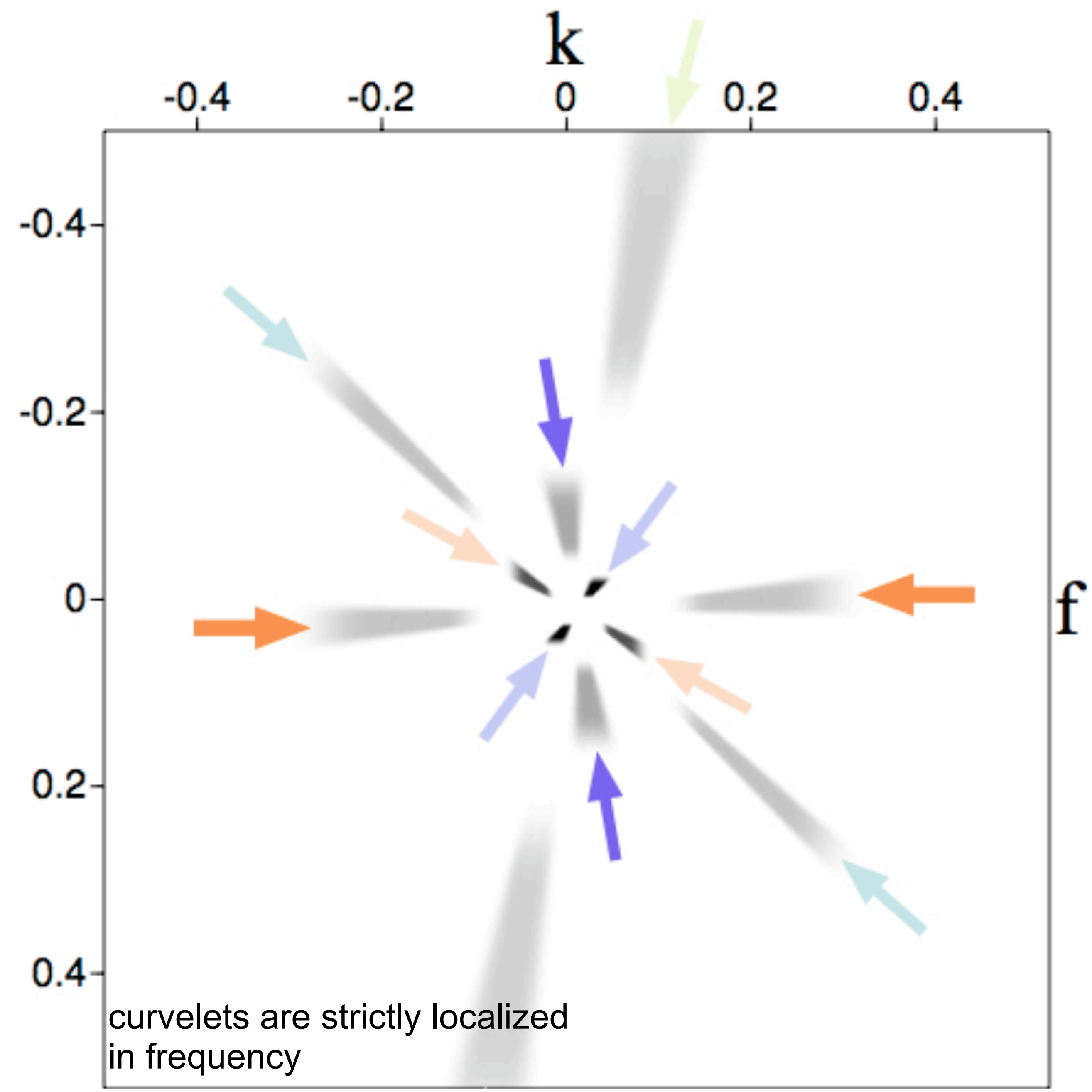
2-D curvelets



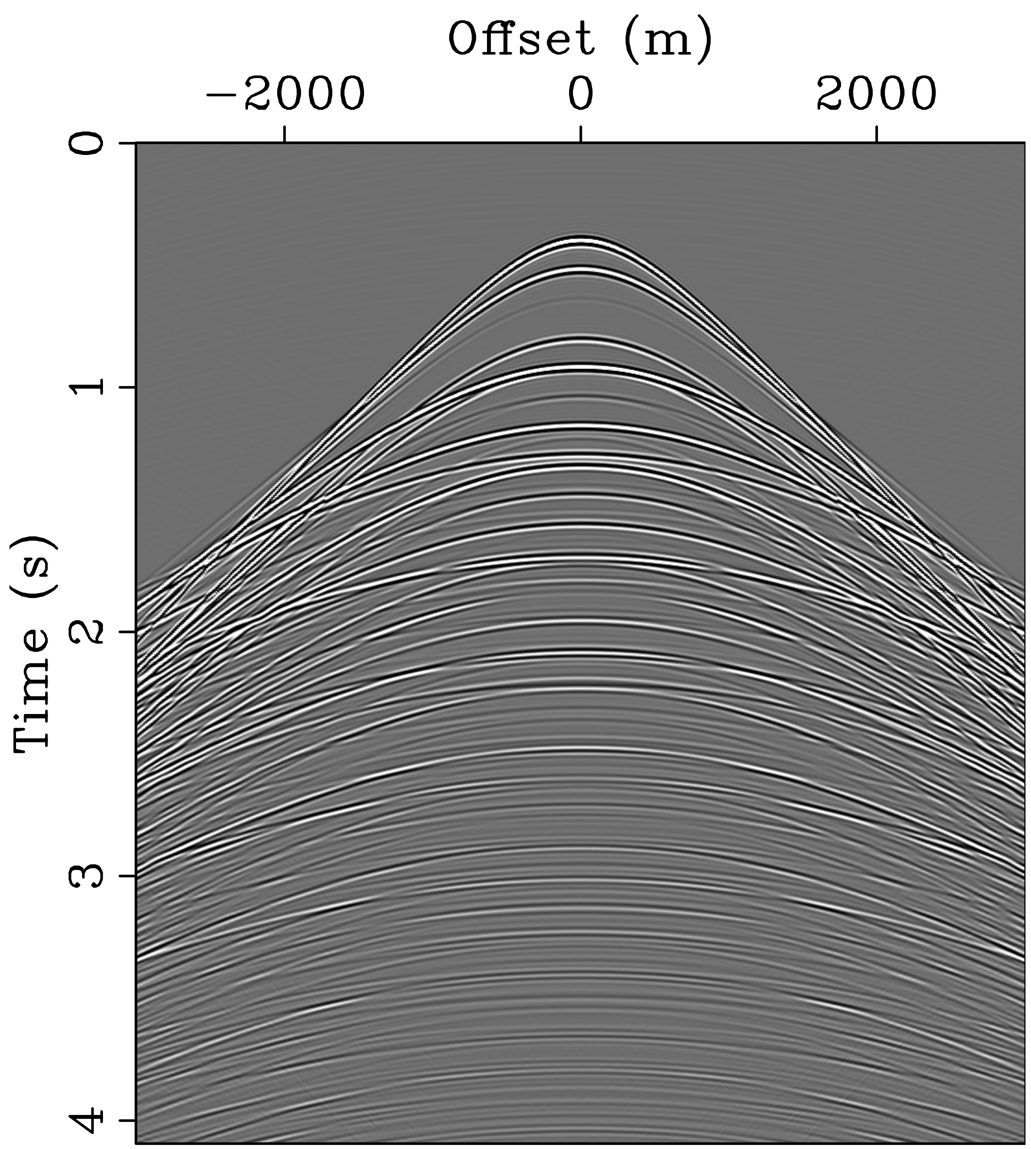
x-t



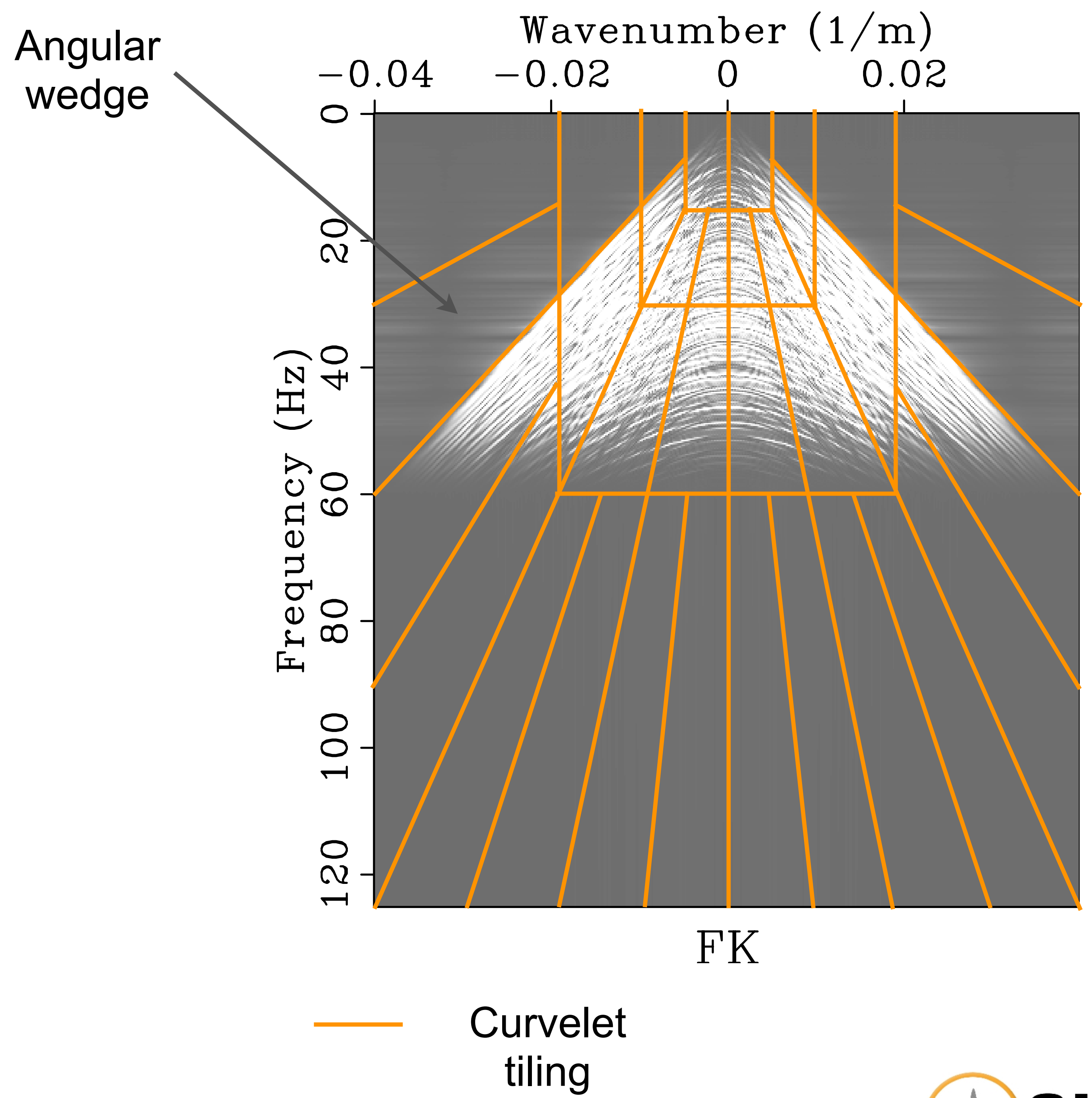
f-k

Oscillatory in one direction and smooth in the others!

Curvelet tiling & seismic data

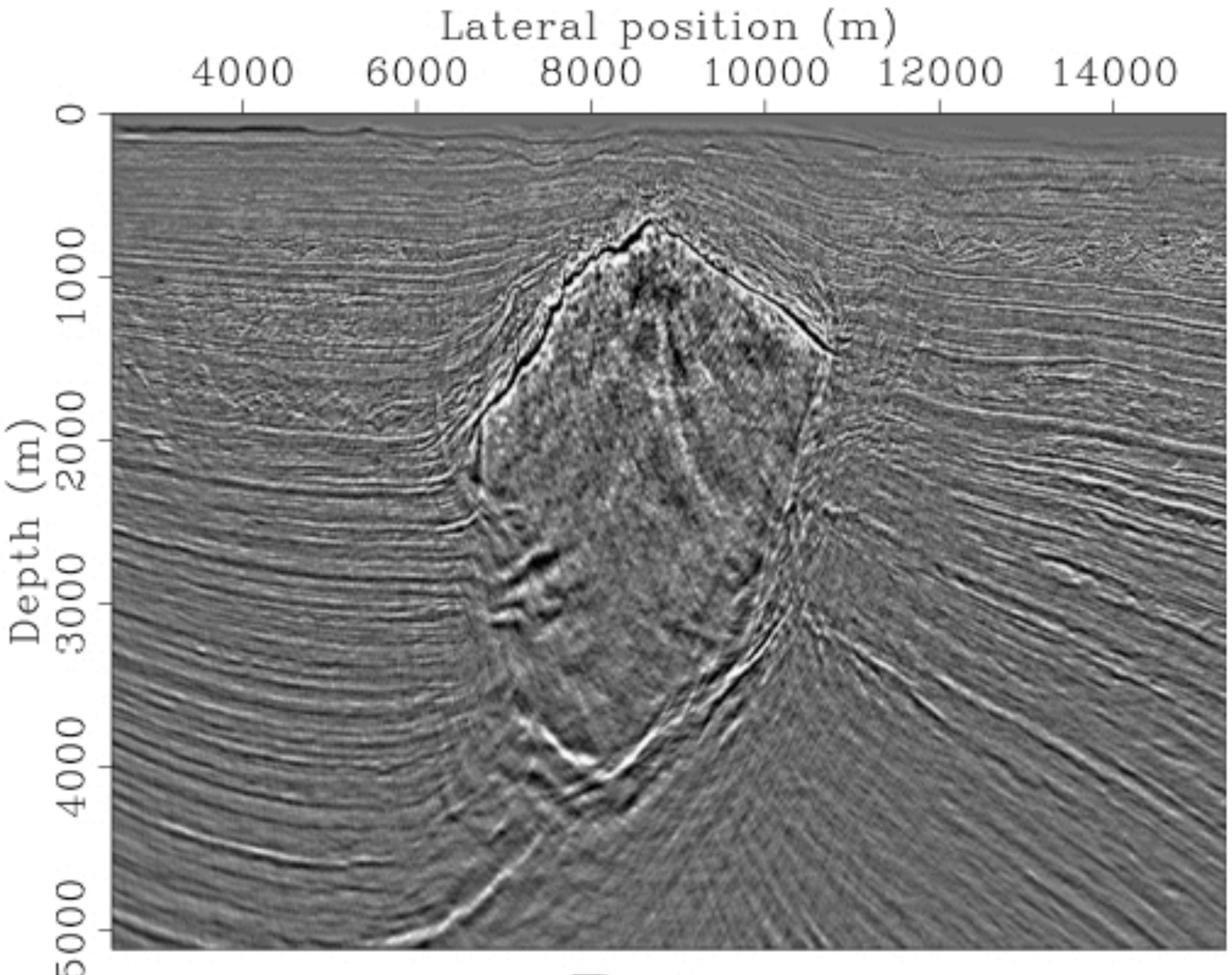


Data



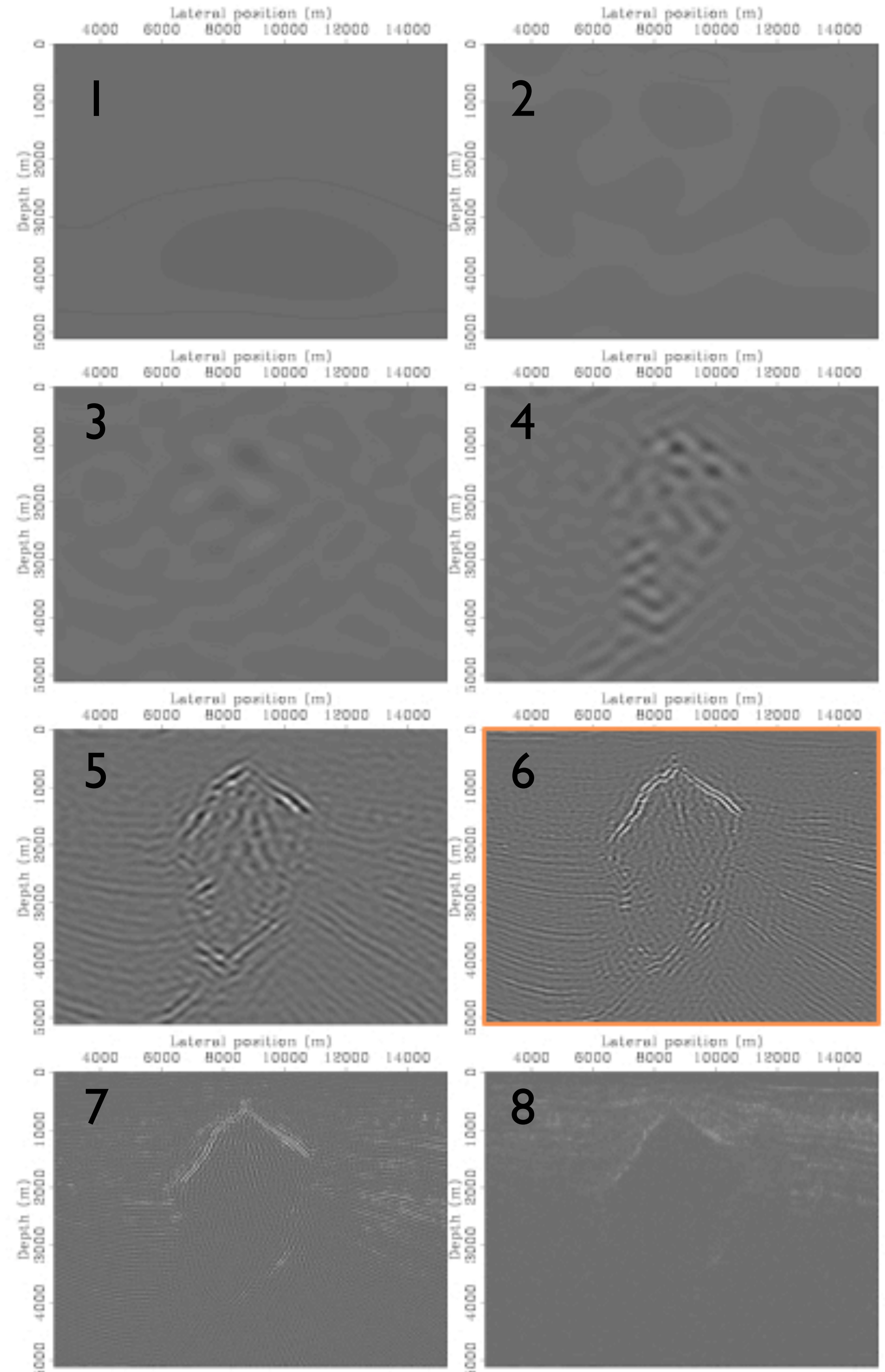
FK

Real data frequency bands example



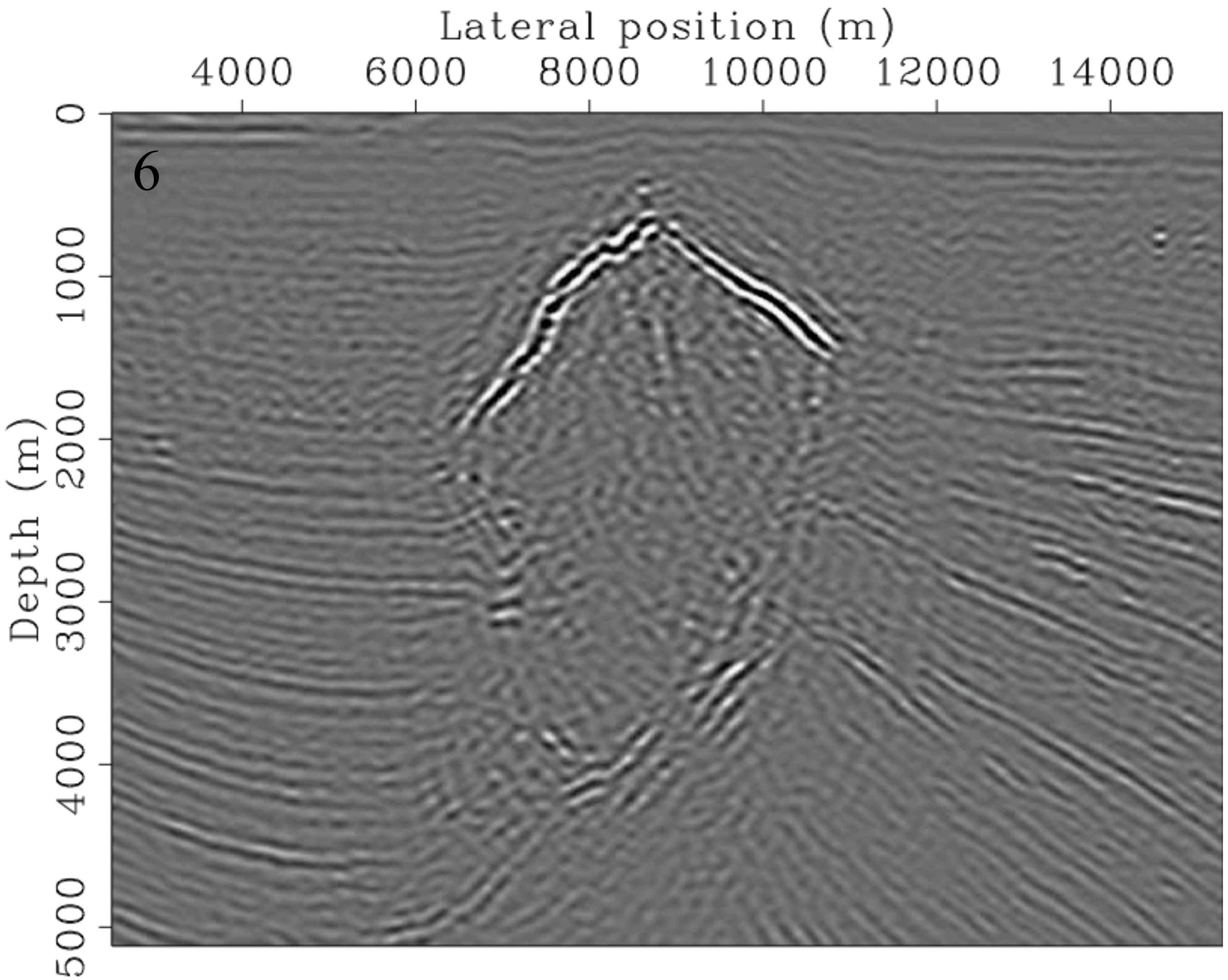
Data

Data is multiscale!



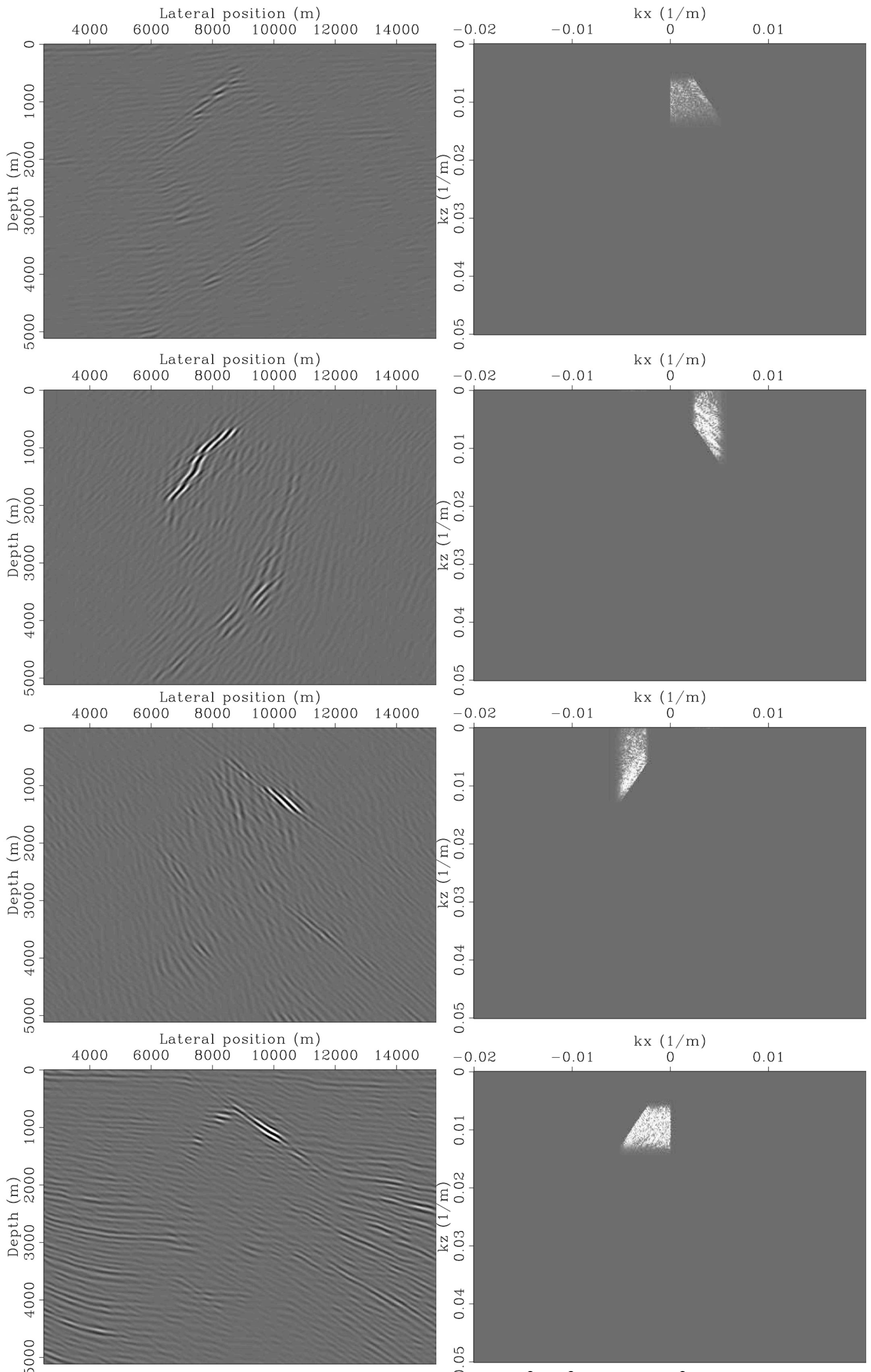
Decomposition in frequency bands

Single frequency band angular wedges



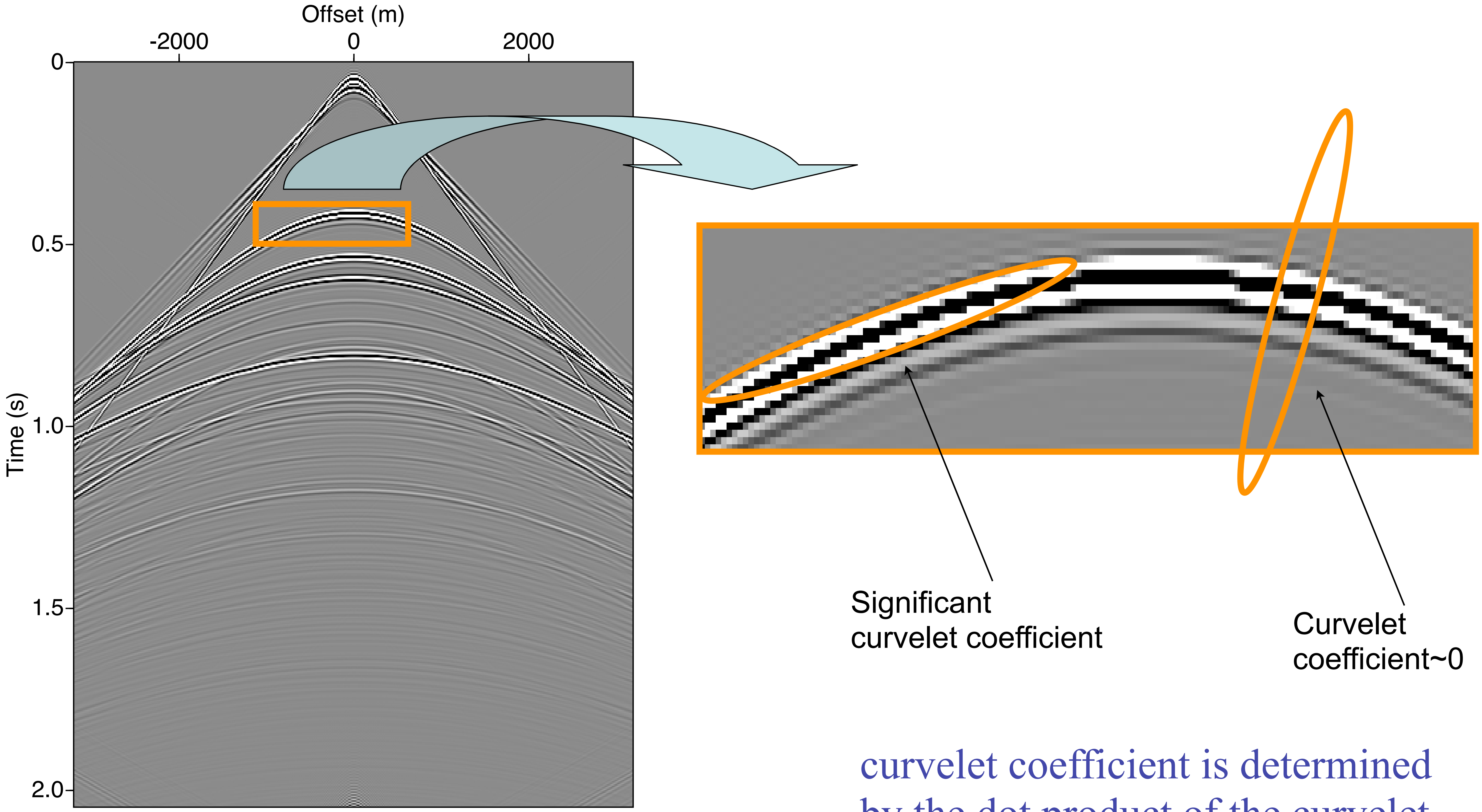
6th scale image

Data is multidirectional!



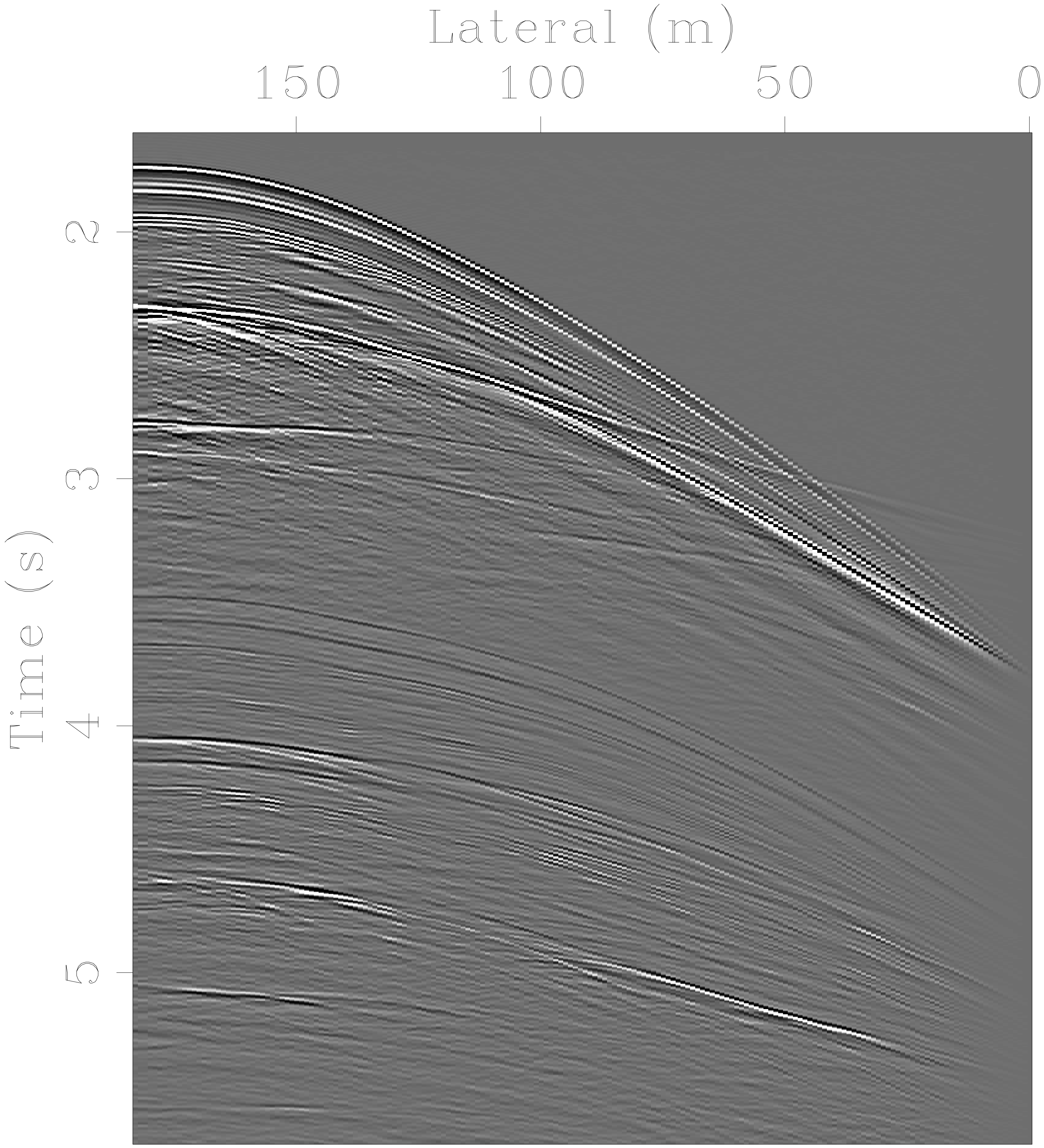
Decomposition in angular wedges

Wavefront detection



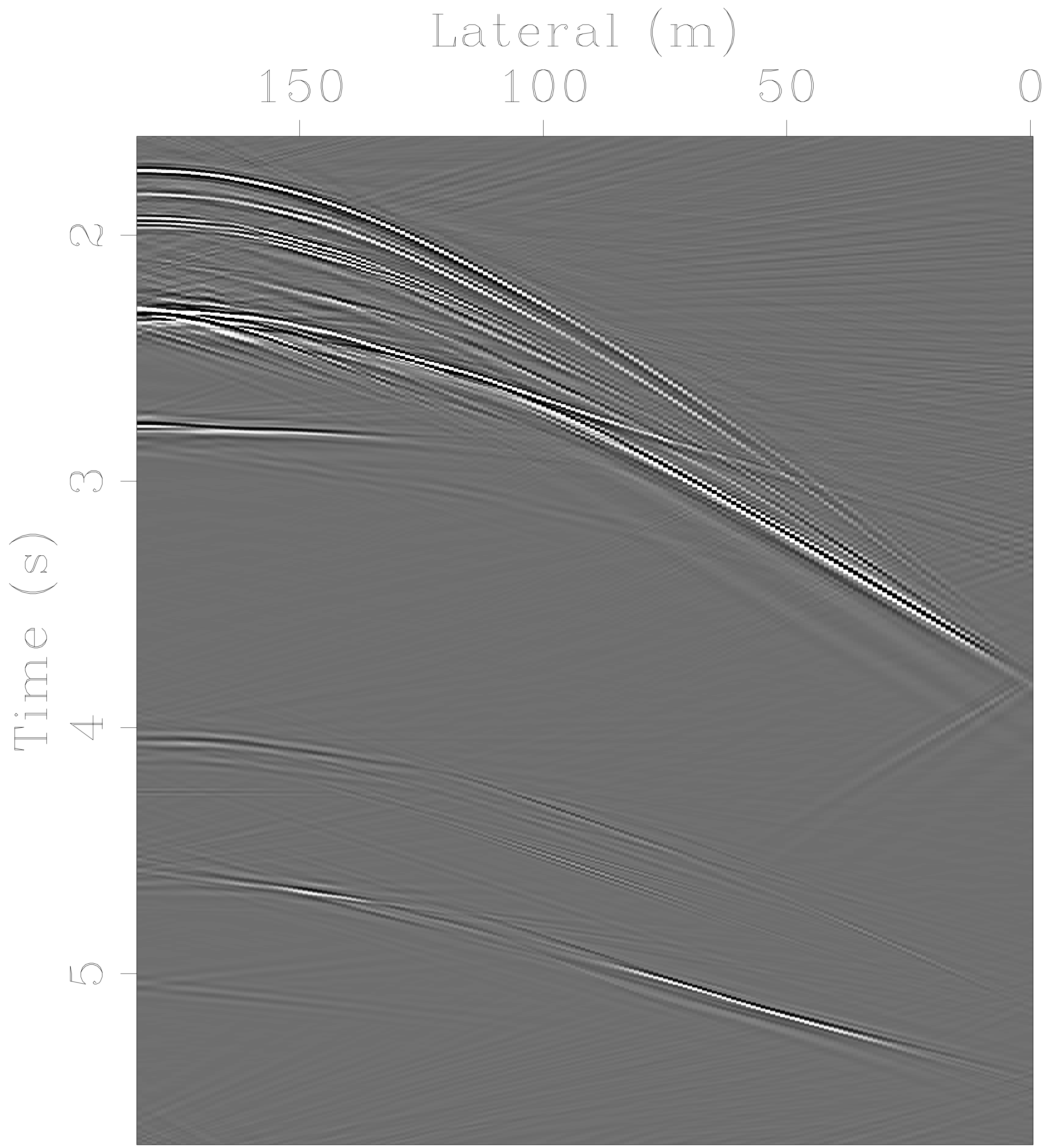
curvelet coefficient is determined by the dot product of the curvelet function with the data

Nonlinear approximation



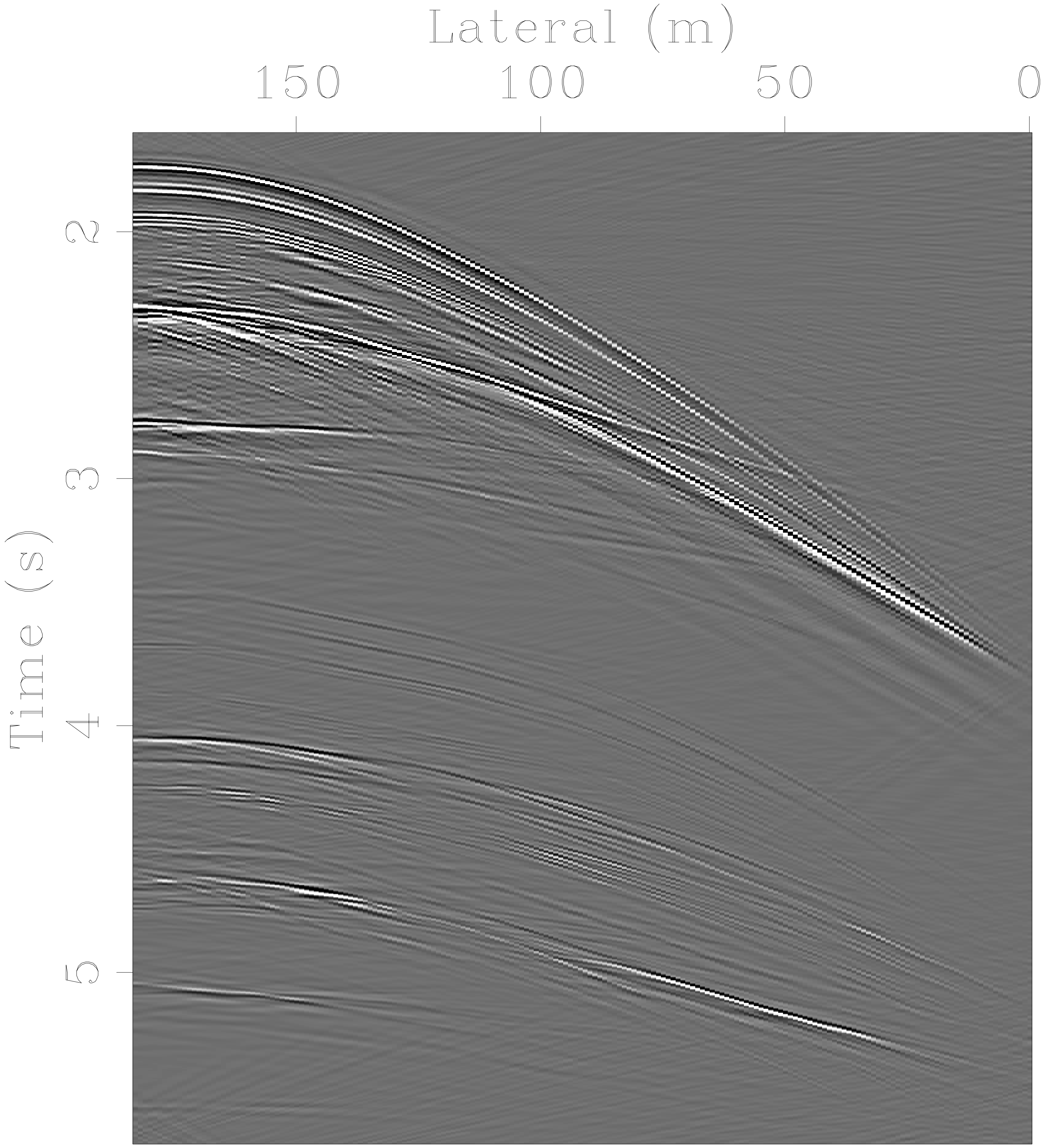
reconstructed data with p=99

Nonlinear approximation



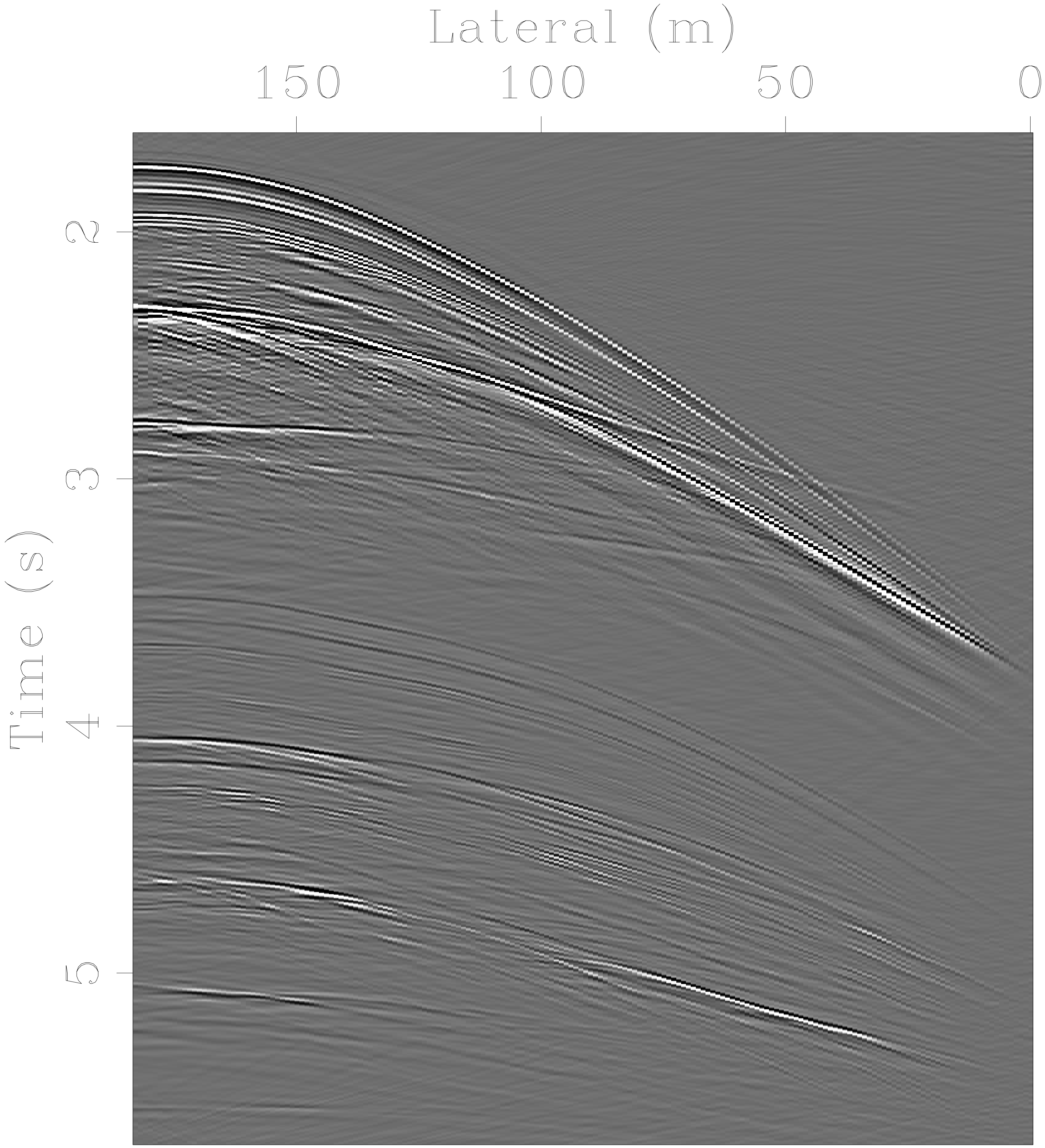
reconstructed data with p=1

Nonlinear approximation



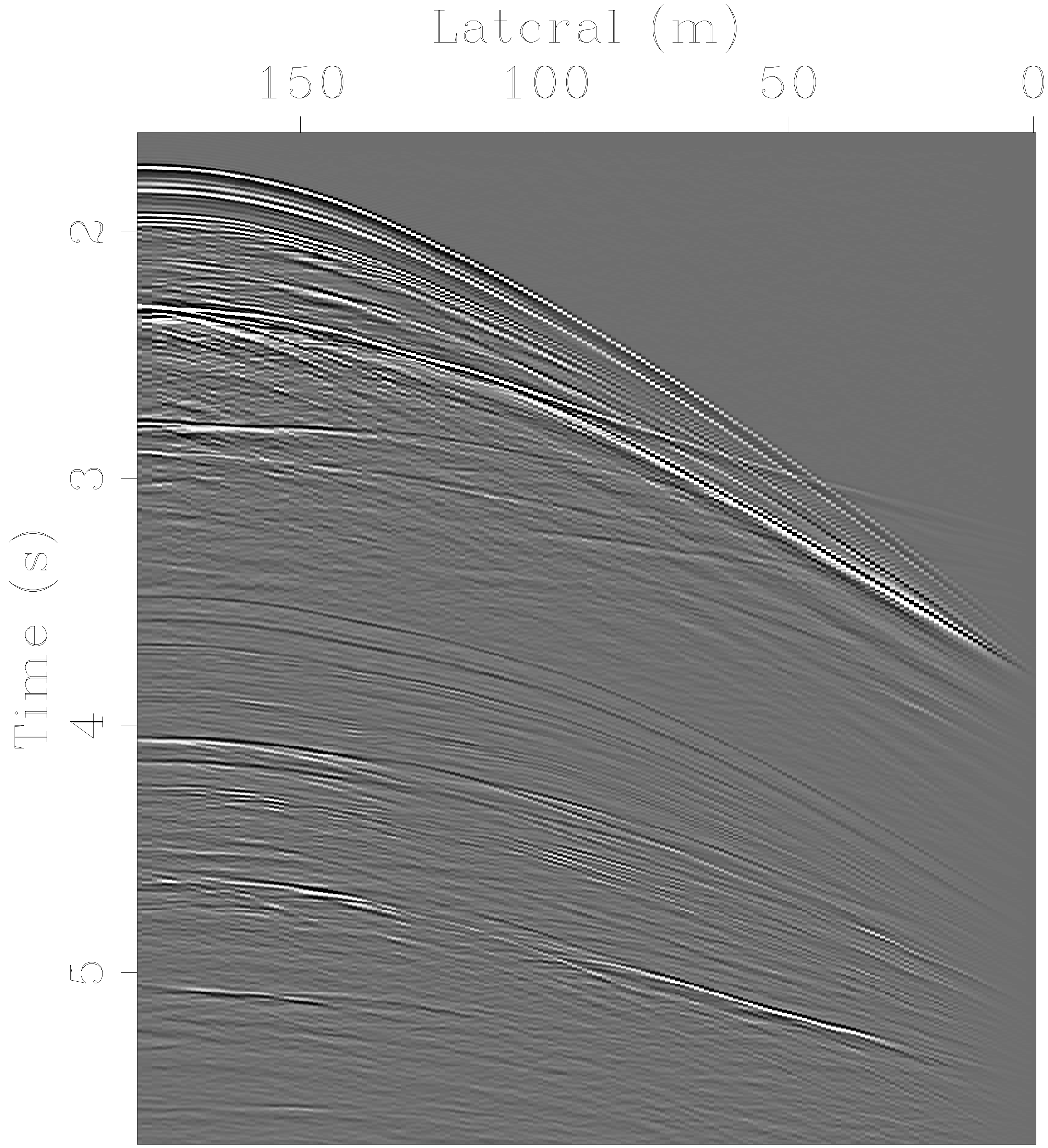
reconstructed data with p=5

Nonlinear approximation



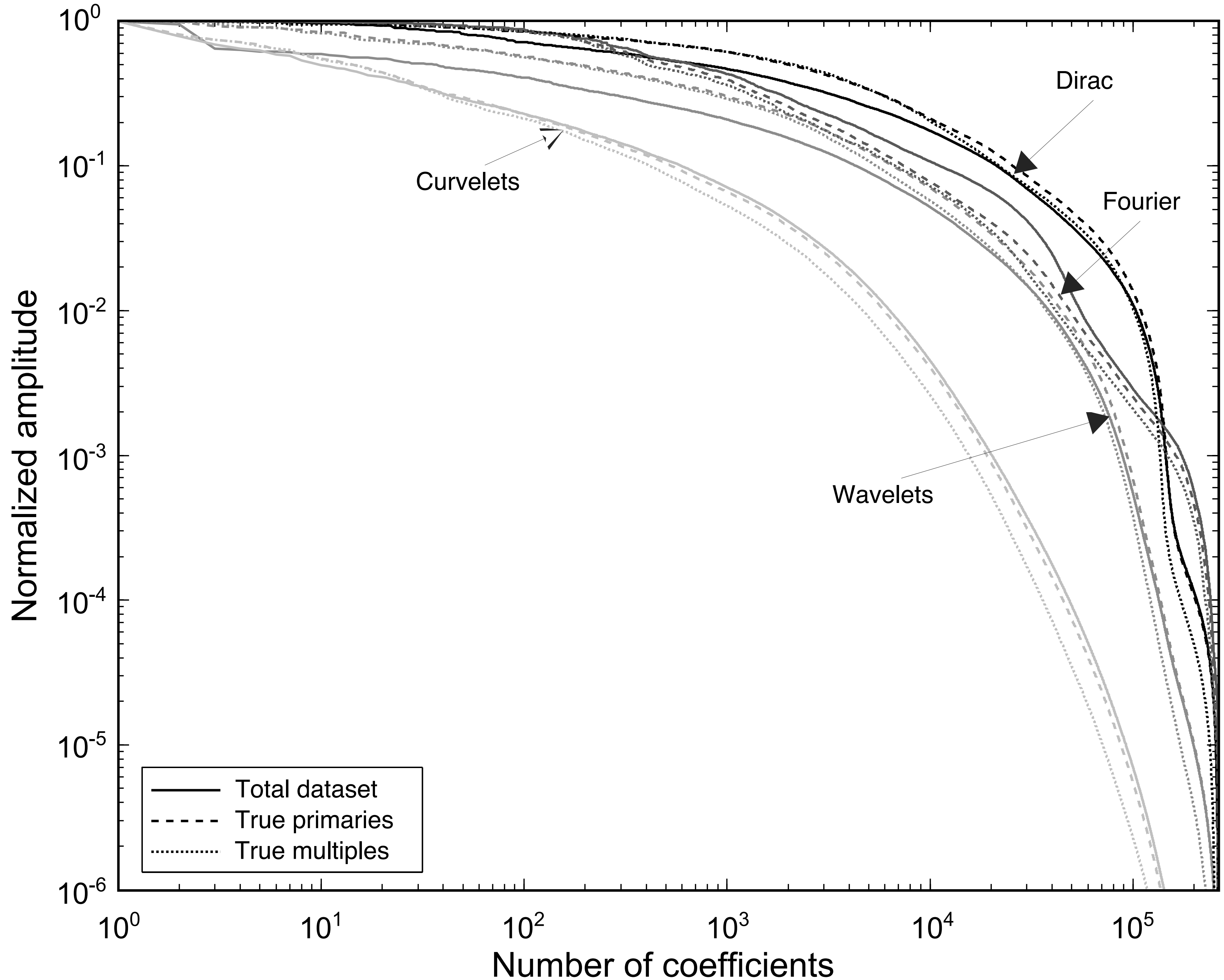
reconstructed data with p=10

Nonlinear approximation



reconstructed data with p=99

Nonlinear approximation rates



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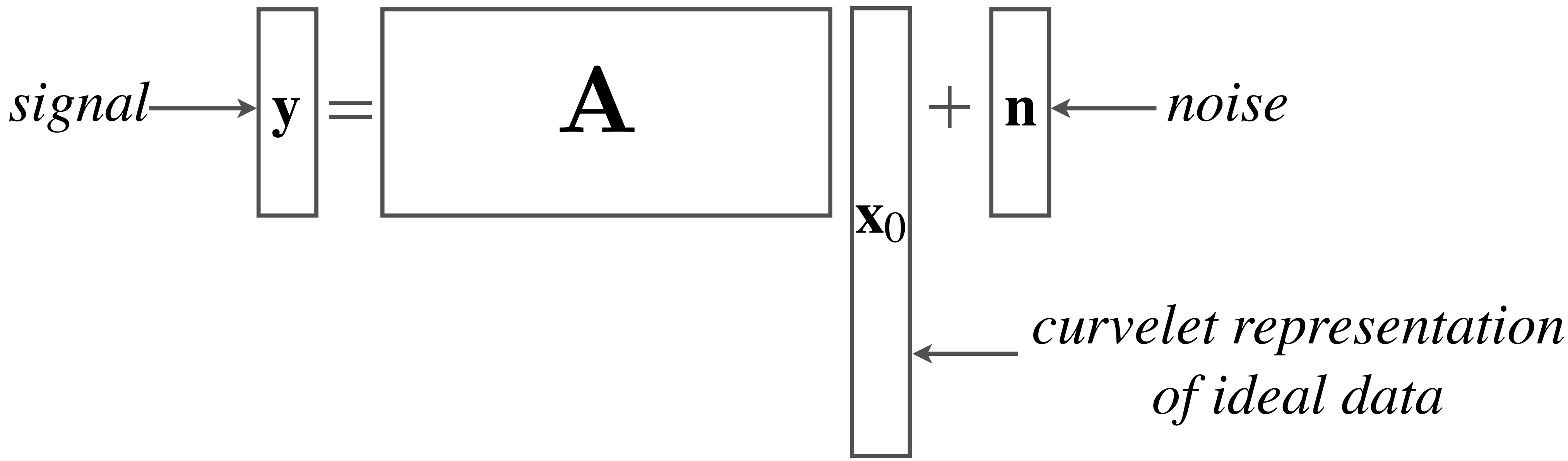
Sparsity promoting inversion

Key idea



$$\tilde{\mathbf{x}} = \arg\min_{\mathbf{x}} \|\mathbf{x}\|_1 \quad \text{s.t.} \quad \|\mathbf{A}\mathbf{x} - \mathbf{y}\|_2 \leq \epsilon$$

sparsity enhancement (↑ $\|\mathbf{x}\|_1$)

data misfit (↑ $\|\mathbf{A}\mathbf{x} - \mathbf{y}\|_2$)

When a traveler reaches a fork in the road, the l1 -norm tells him to take either one way or the other, but the l2 -norm instructs him to head off into the bushes.

John F. Claerbout and Francis Muir, 1973

New field “compressive sampling”: D. Donoho, E. Candes et. al., M. Elad etc.

Preceded by others in geophysics: M. Sacchi & T. Ulrych and co-workers etc.

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Applications

Sparsity promotion can be used to

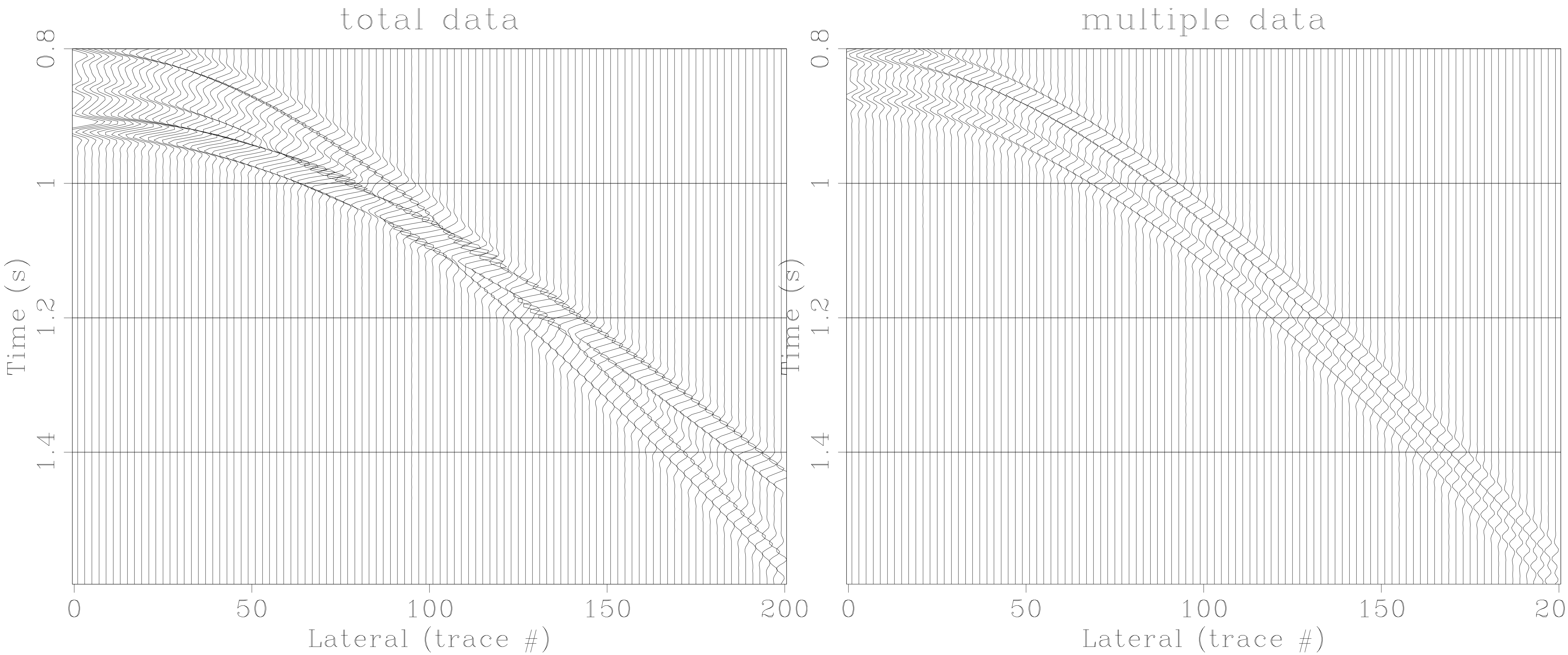
- recovery from incomplete data: “Curvelet reconstruction with sparsity promoting inversion: successes & challenges and “Irregular sampling: from aliasing to noise”
- migration amplitude recovery: “Just diagonalize: a curvelet-based approach to seismic amplitude recovery
- ground-roll removal: “Curvelet applications in surface wave removal”
- multiple prediction: “Surface related multiple prediction from incomplete data”
- seismic processing: “Seismic imaging and processing with curvelets”

Primary-multiple separation

Joint work with Eric Verschuur, Deli Wang, Rayan Saab and Ozgur Yilmaz

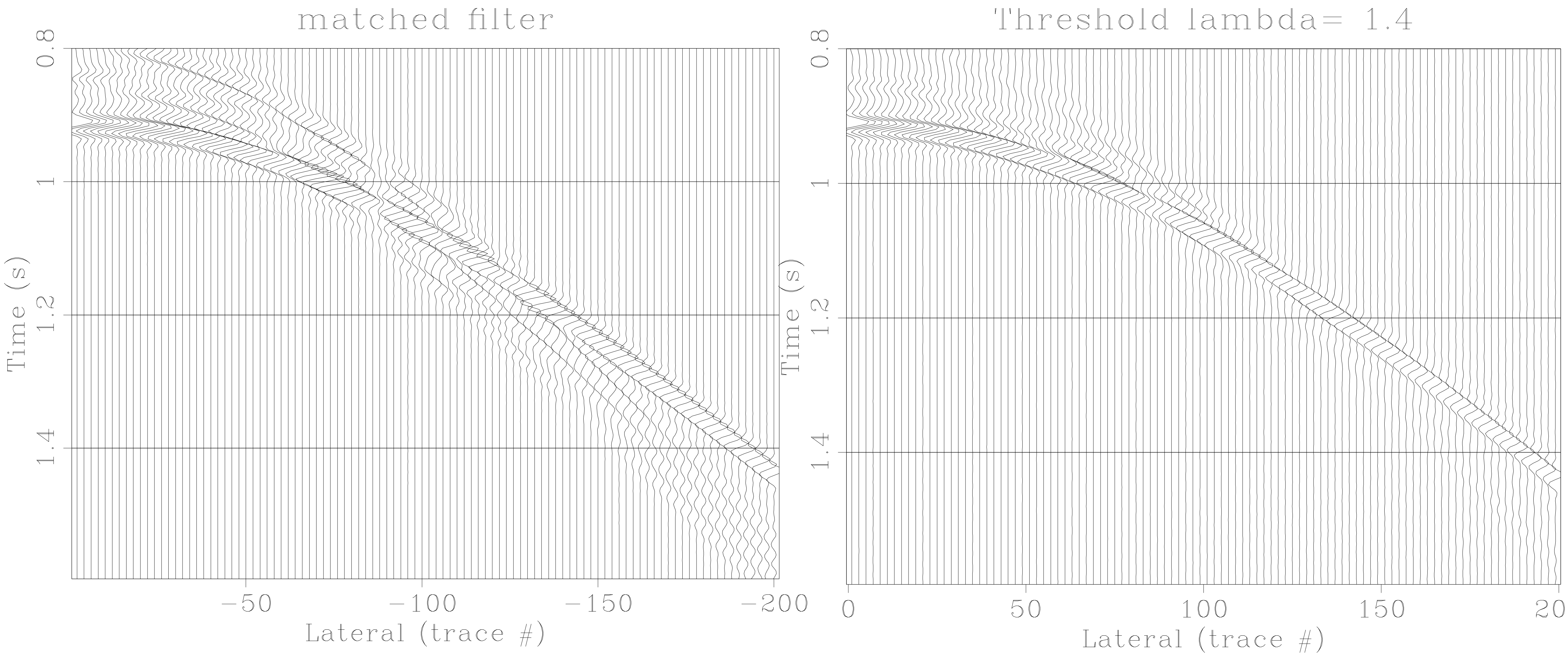



Move-out error



Multiple prediction with erroneous move out.

Move-out error



Curvelet-based result obtained by single soft threshold given by the predicted multiples

$$\tilde{\mathbf{s}}_1 = \mathbf{C}^T T_{\lambda|\mathbf{C}\breve{\mathbf{s}}_2|}(\mathbf{C}\mathbf{s})$$

Approach

Bayesian formulation of the primary-multiple separation problem

- promotes ***sparsity*** on estimated primaries & multiples
- minimizes ***misfit*** between total data and sum of estimated ***primaries*** and ***multiples***
- exploits decorrelation in the curvelet domain
- **new:** minimizes ***misfit*** between estimated and (SRME) predicted ***multiples***

Separation formulated in terms of a sparsity promoting program robust under

- moderate timing and phase errors
- noise

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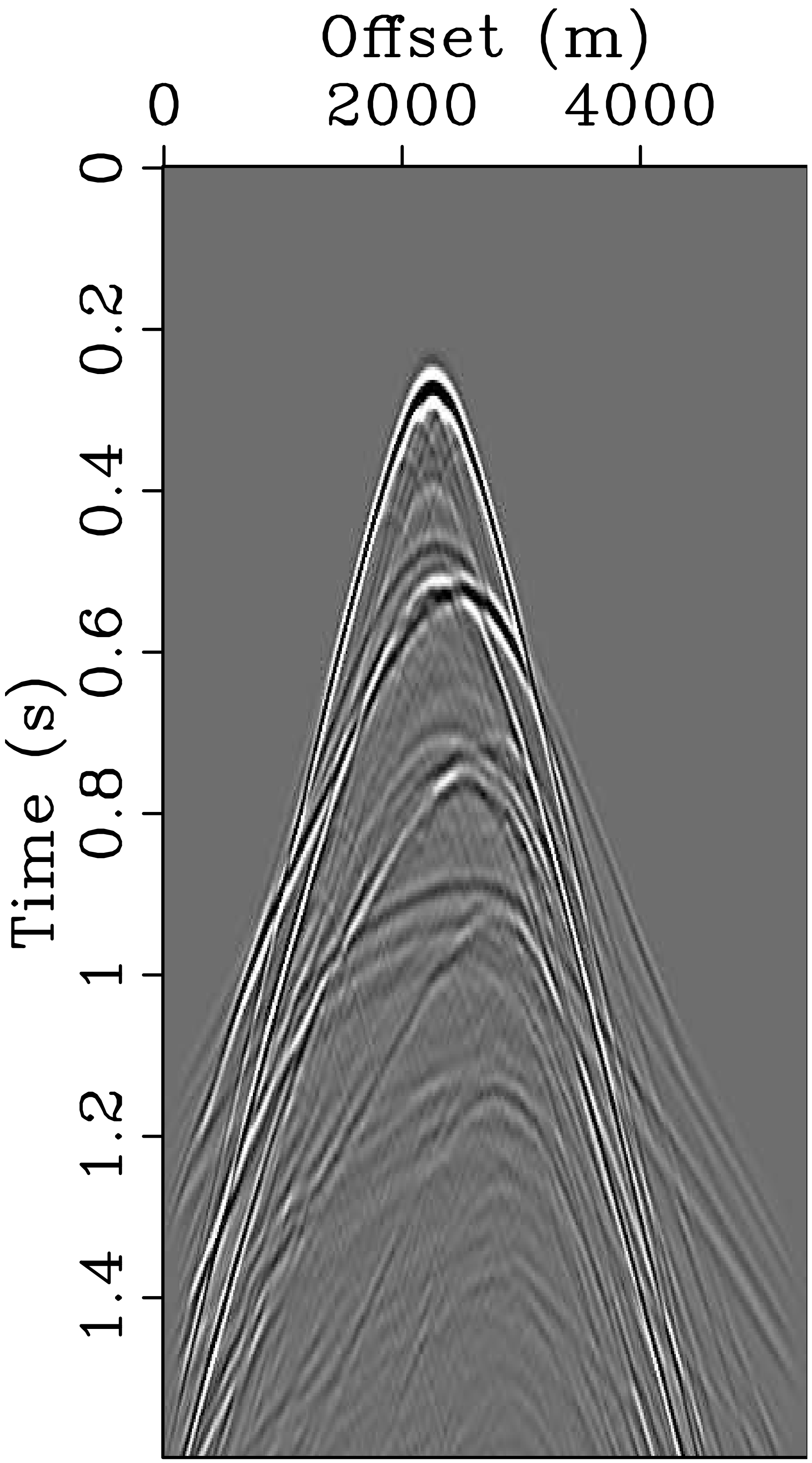
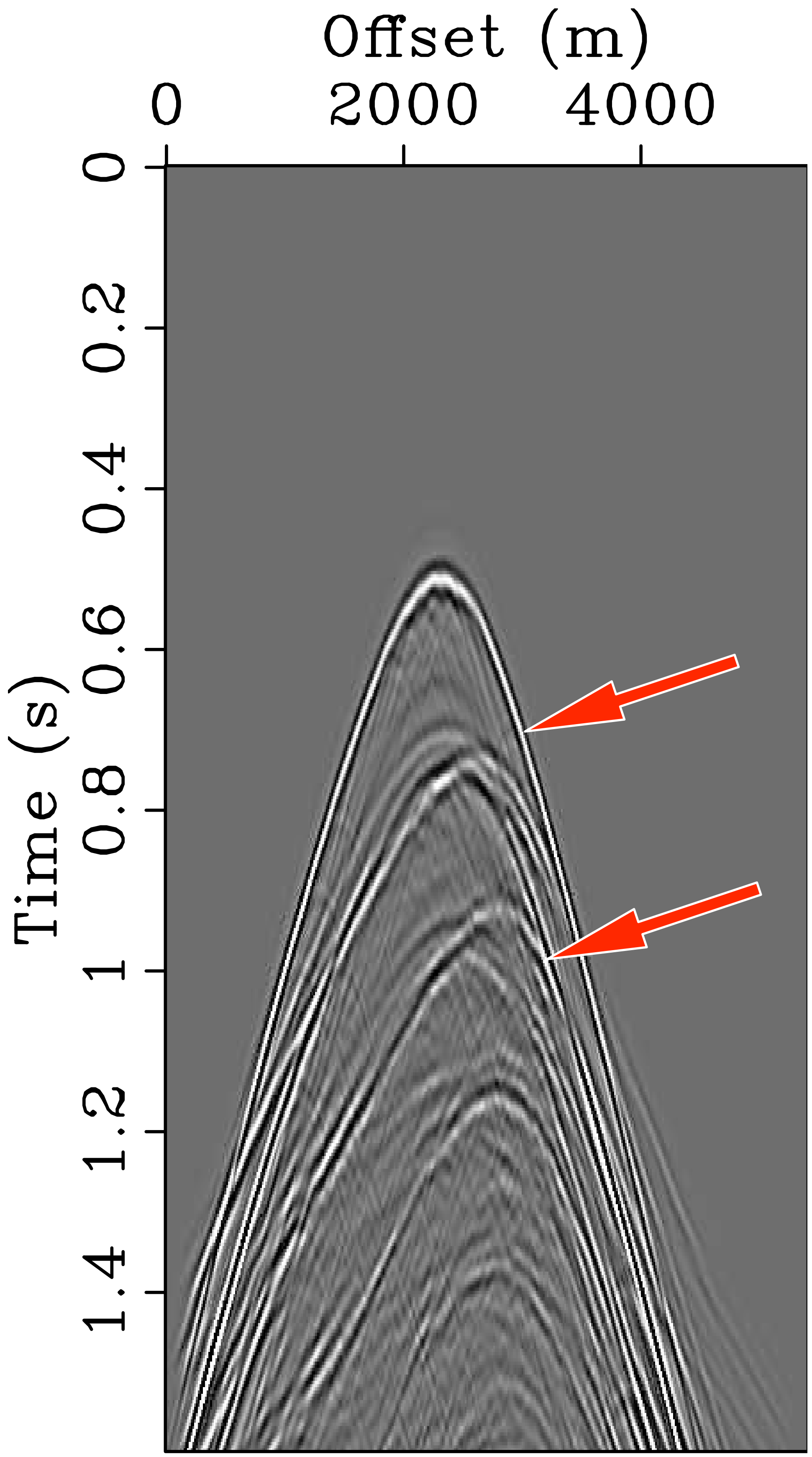
Synthetic example



total data



SRME predicted multiples

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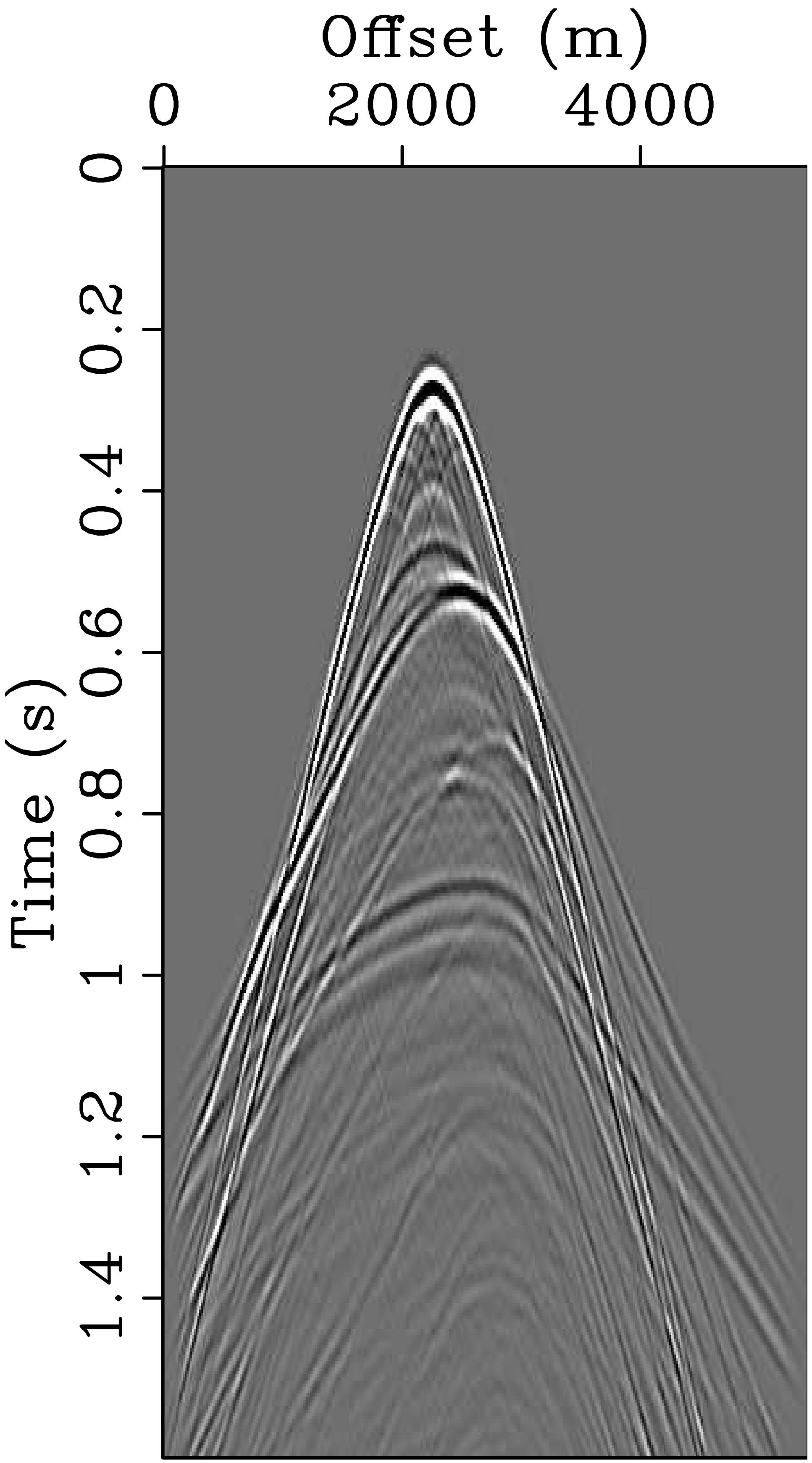
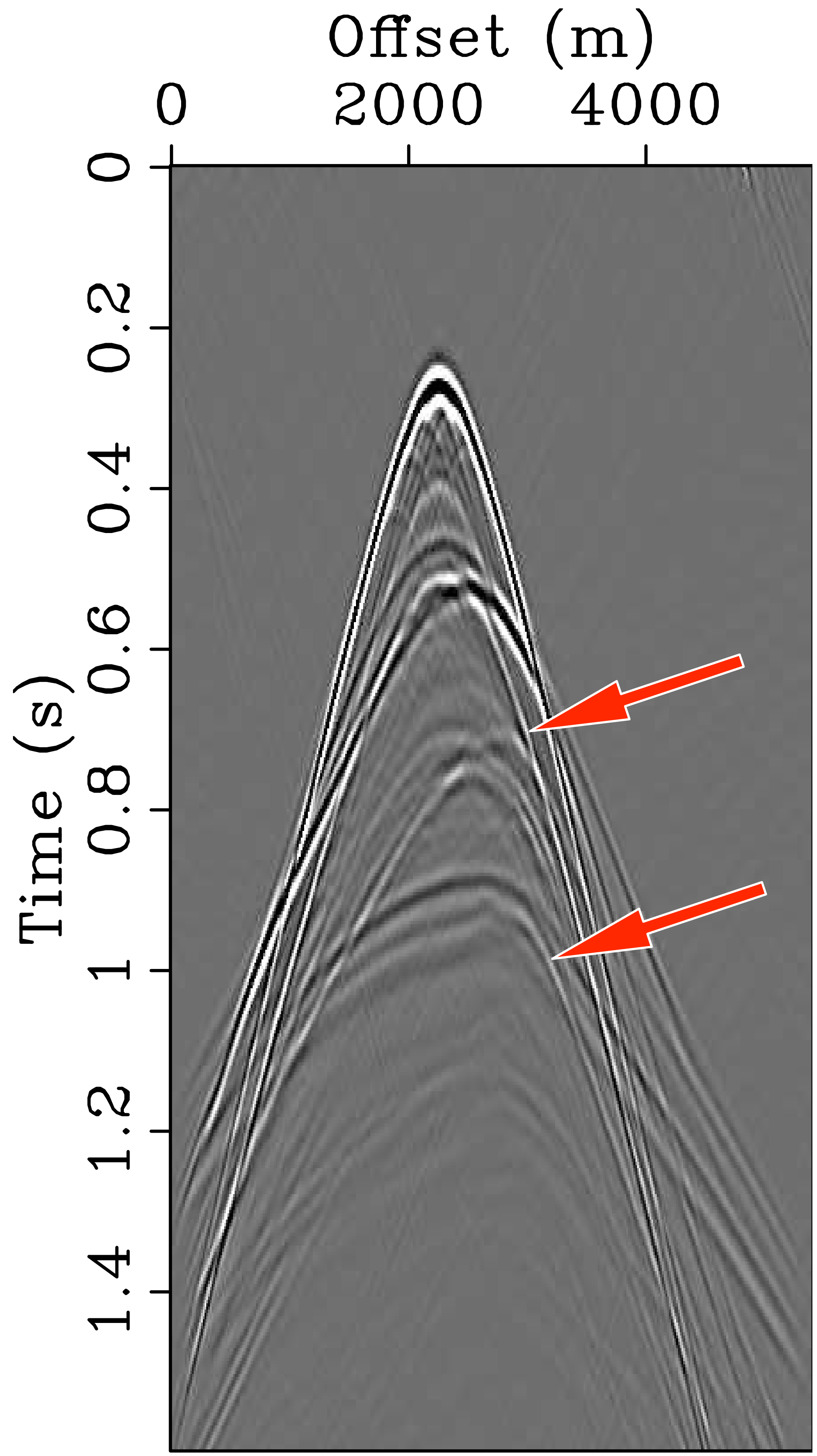
Synthetic example



SRME predicted primaries



curvelet-thresholded

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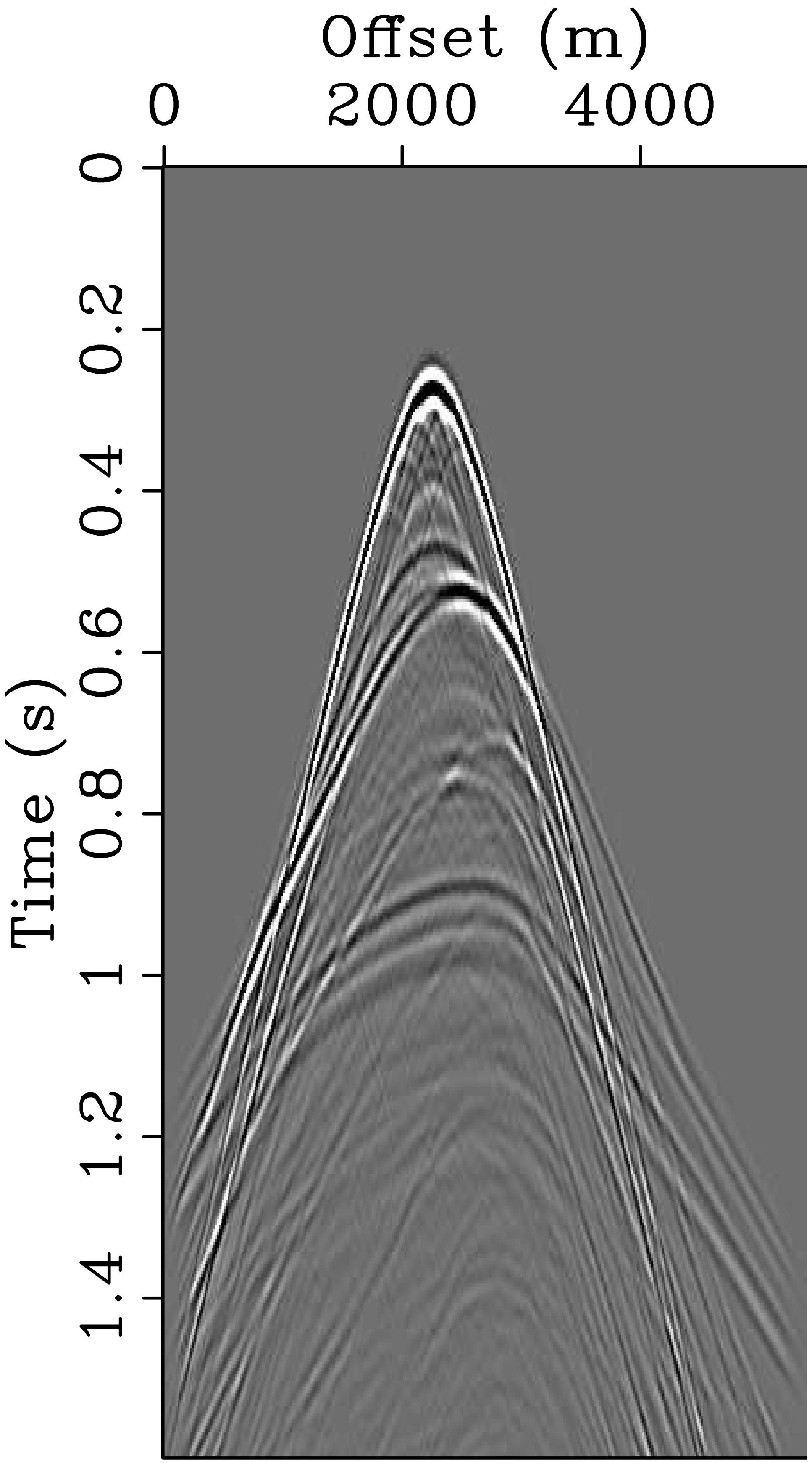
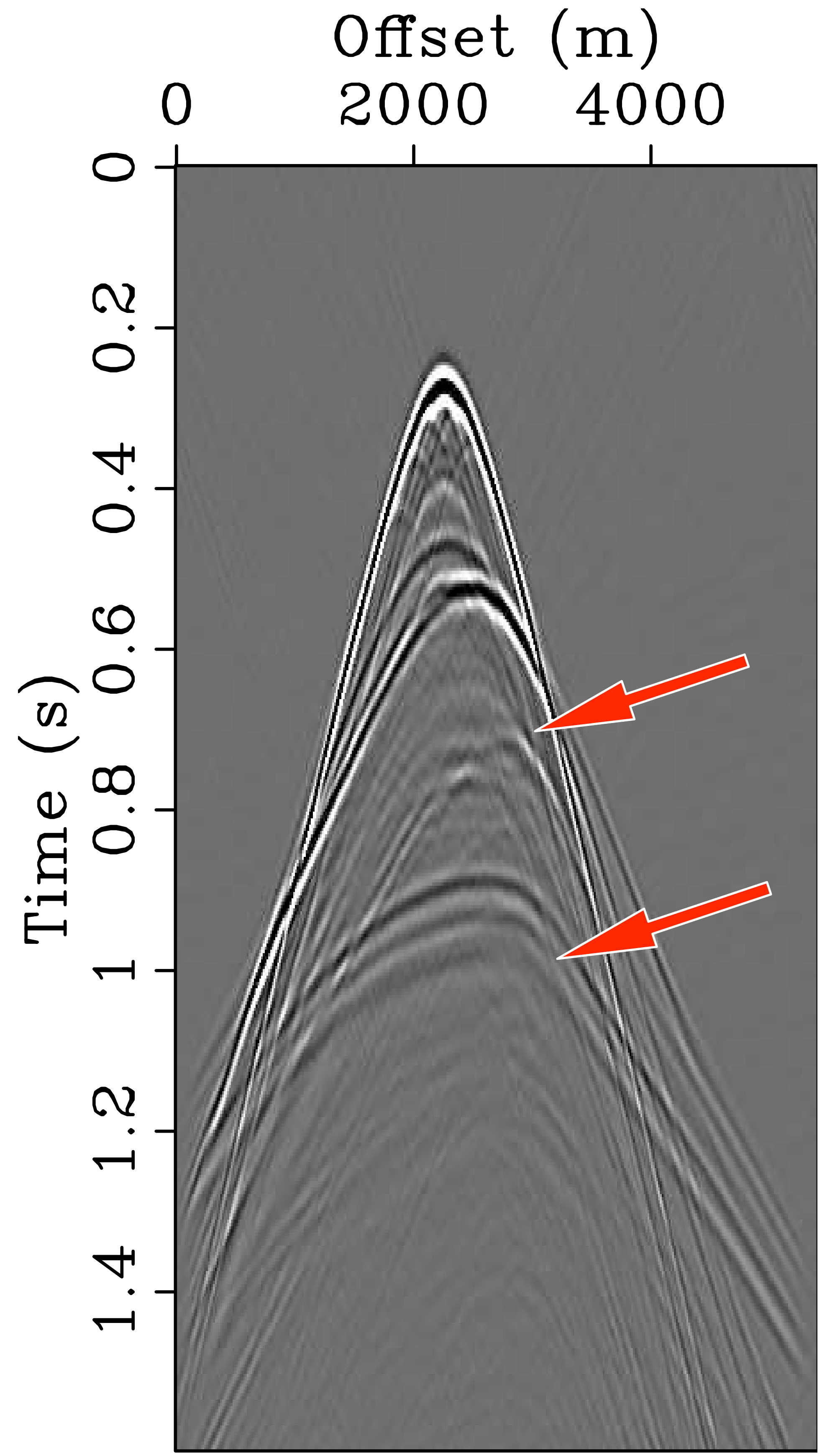
Synthetic example



SRME predicted primaries



estimated

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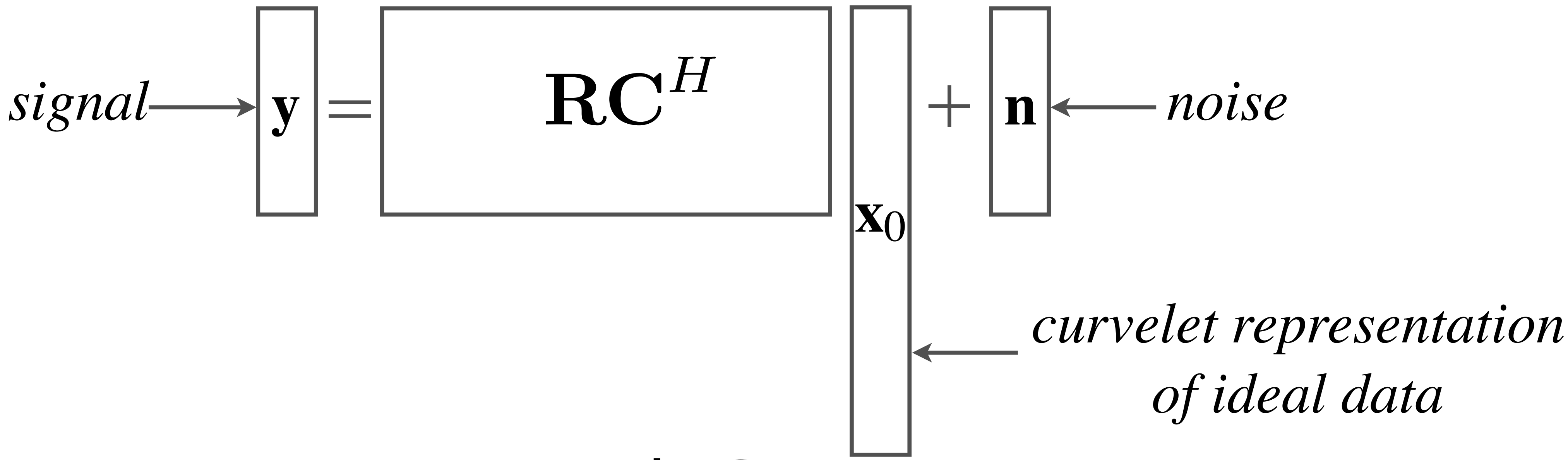
Curvelet-based recovery

joint work with Gilles Hennenfent

Sparsity-promoting inversion*

Reformulation of the problem



Curvelet Reconstruction with Sparsity-promoting Inversion (CRSI)

- look for the **sparsest/most compressible**, **physical** solution

KEY POINT OF THE RECOVERY

$$\mathbf{P}_\epsilon : \begin{cases} \tilde{\mathbf{x}} = \arg\min_{\mathbf{x}} \overbrace{\|\mathbf{W}\mathbf{x}\|_1}^{\text{sparsity constraint}} \quad \text{s.t.} \quad \overbrace{\|\mathbf{A}\mathbf{x} - \mathbf{y}\|_2}^{\text{data misfit}} \leq \epsilon \\ \tilde{\mathbf{f}} = \mathbf{C}^T \tilde{\mathbf{x}} \end{cases}$$

* inspired by *Stable Signal Recovery (SSR) theory* by E. Candès, J. Romberg, T. Tao, *Compressed sensing* by D. Donoho & *Fourier Reconstruction with Sparse Inversion (FRSI)* by P. Zwartjes

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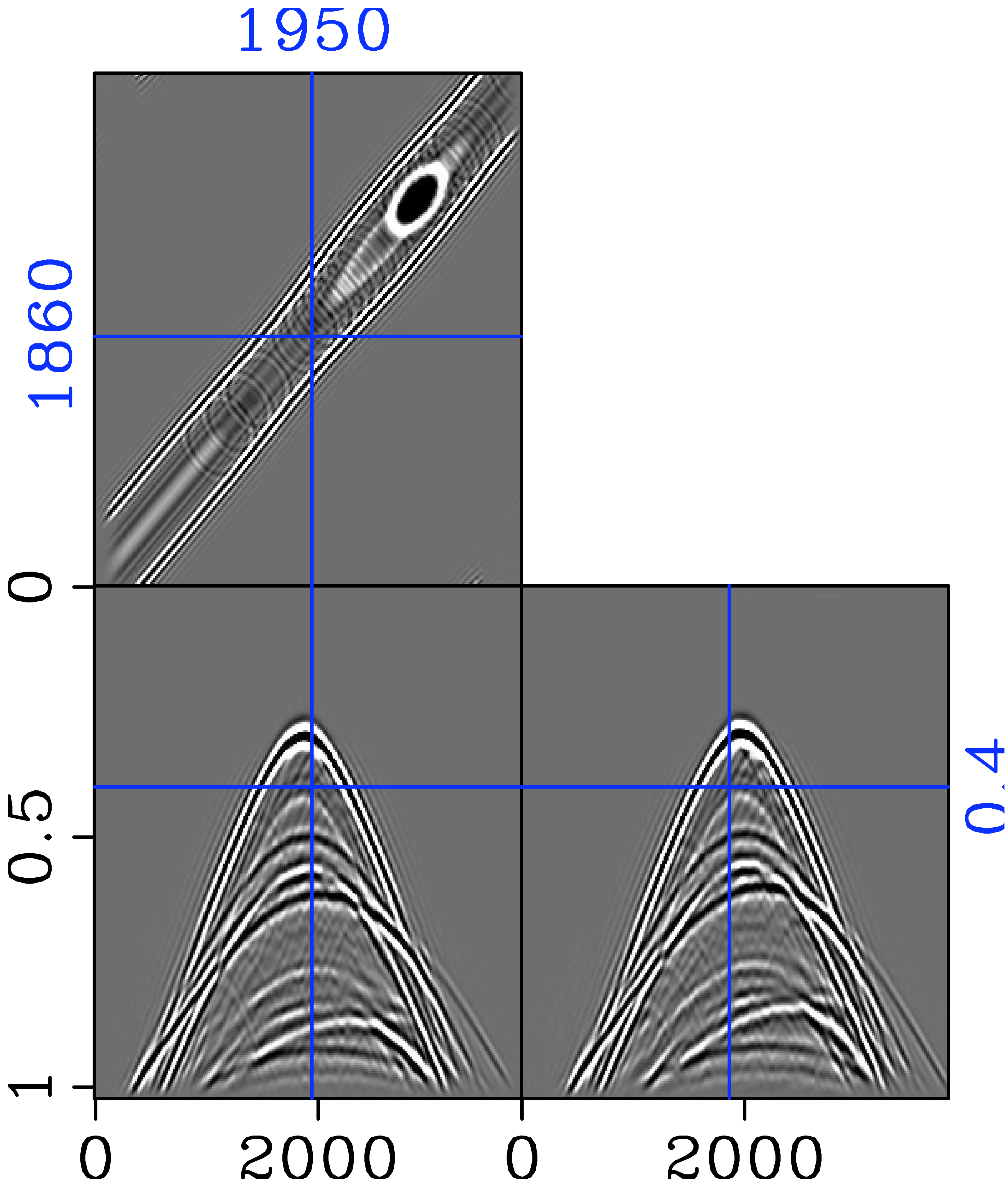

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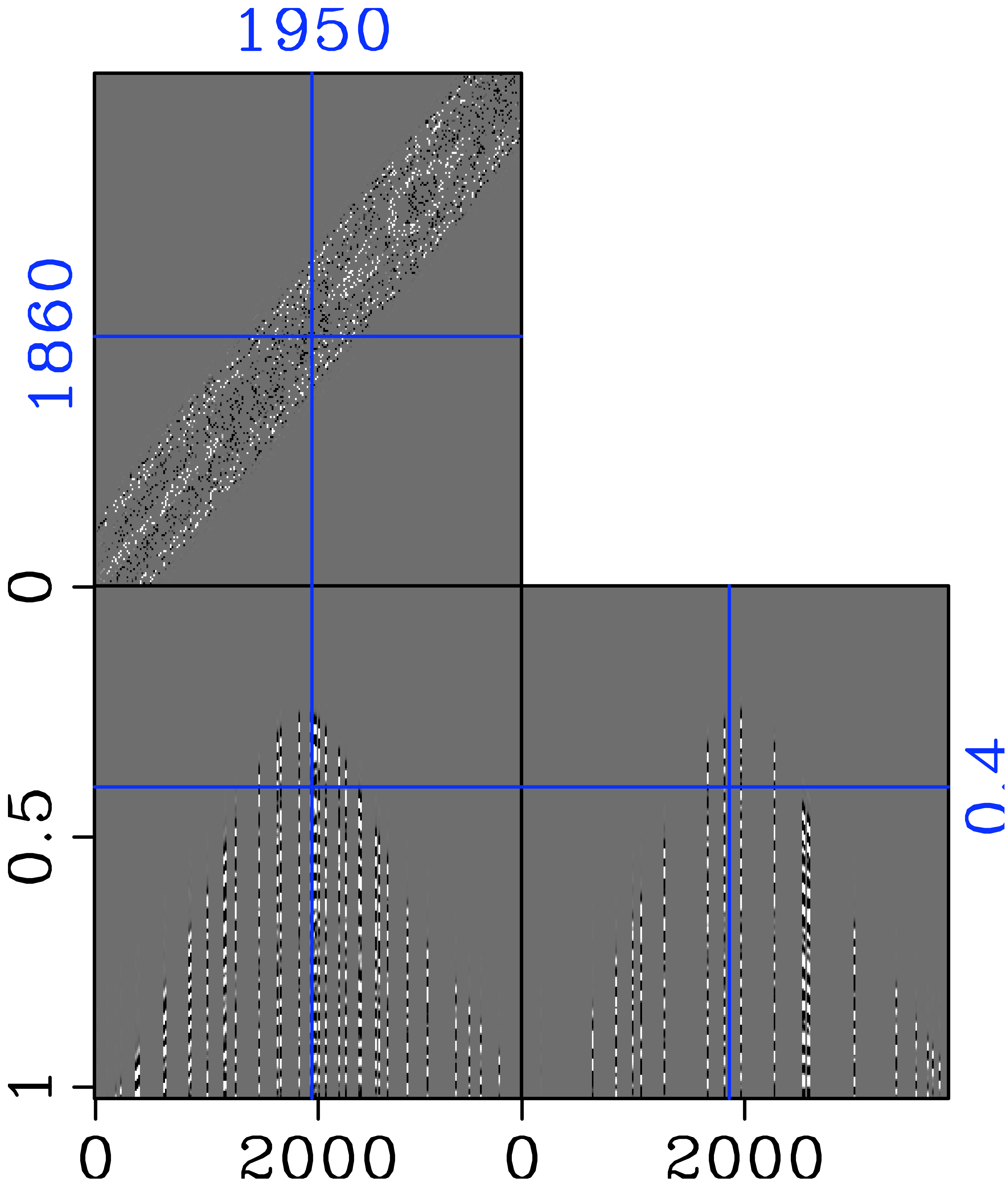
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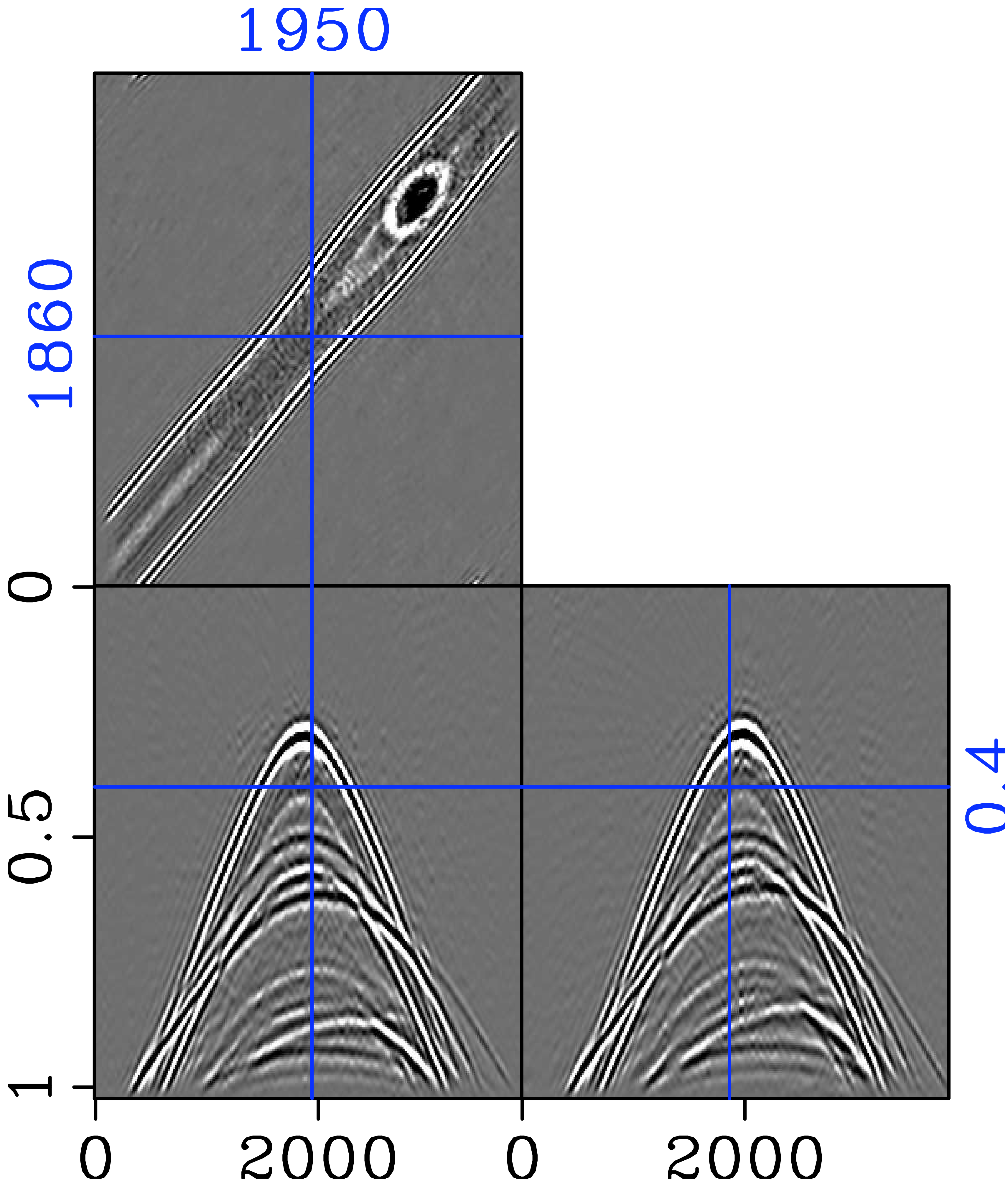
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Focused recovery with curvelets

joint work with Deli Wang (visitor from Jilin university) and Gilles Hennenfent

Motivation

Can the recovery be extended to “migration-like” operators?

How can we incorporate ***prior*** information on the wavefield, e.g. information on major primaries from SRME?

How can we compress extrapolation operator?

Compound primary operator with inverse curvelet transform.

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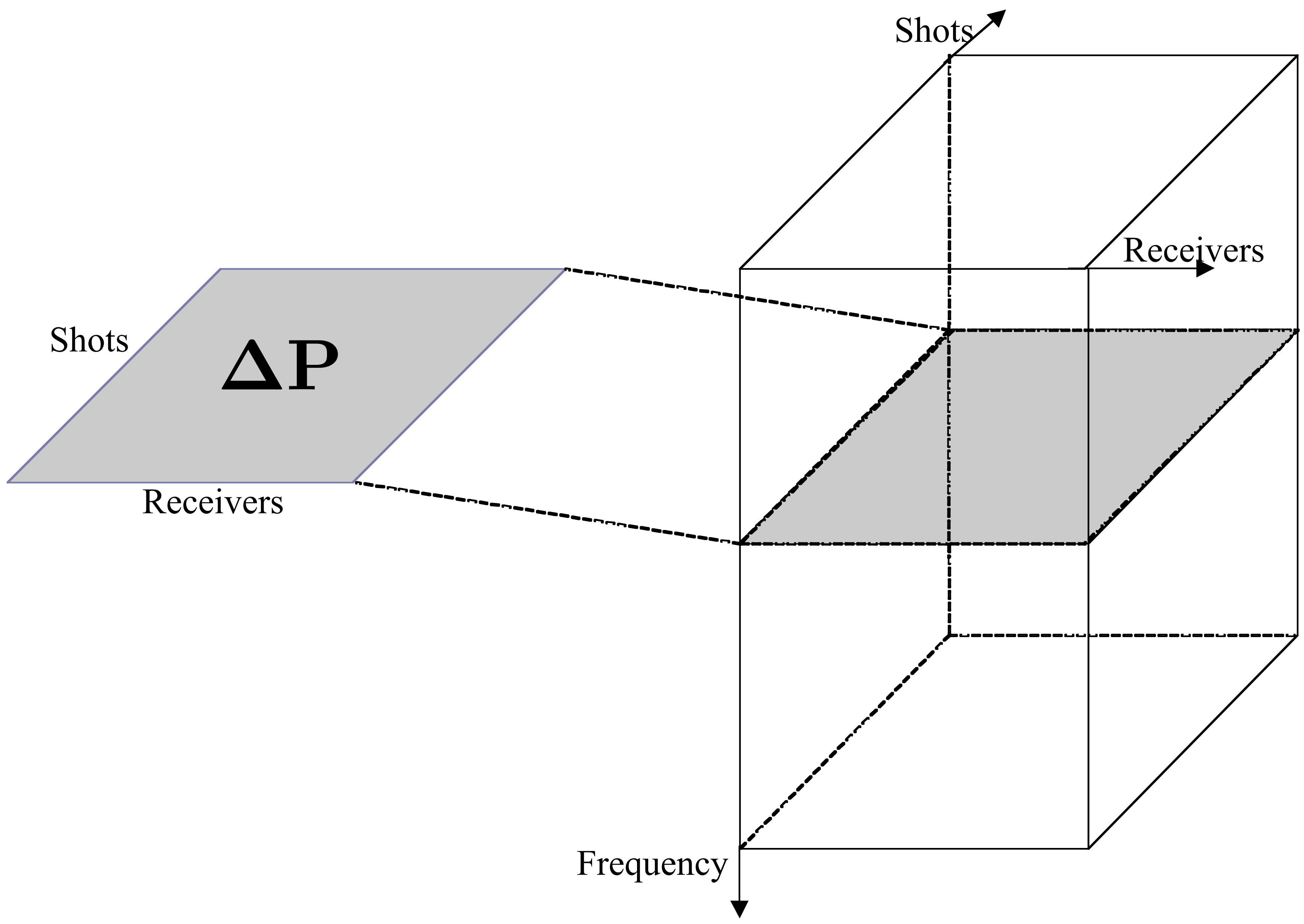
Primary operator

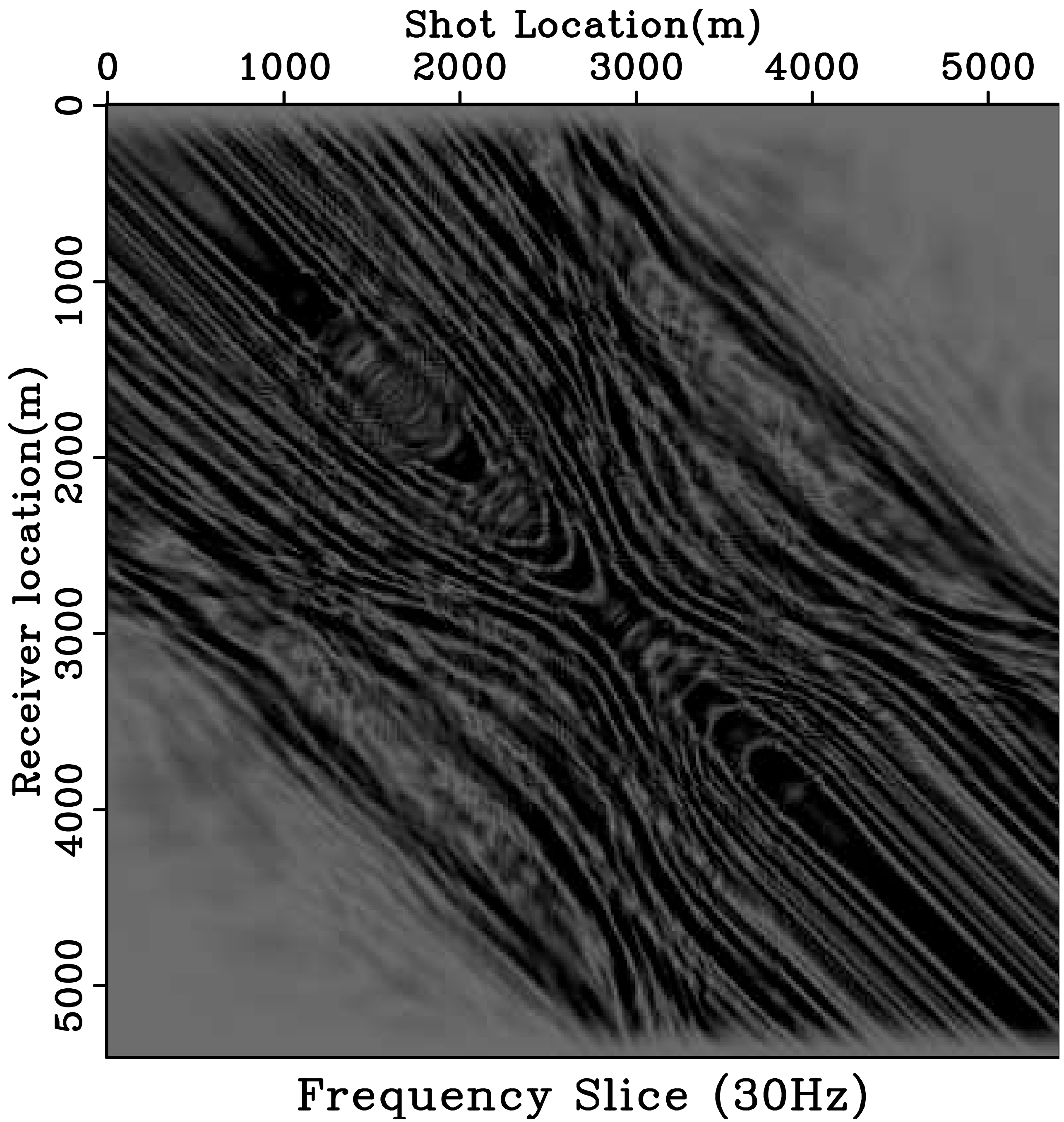
[Berkhout & Verschuur '96]



Frequency slice from data cube

Primary operator

[Berkhout & Verschuur '96]



Maps primaries into first-order multiples. So its inverse focuses ....

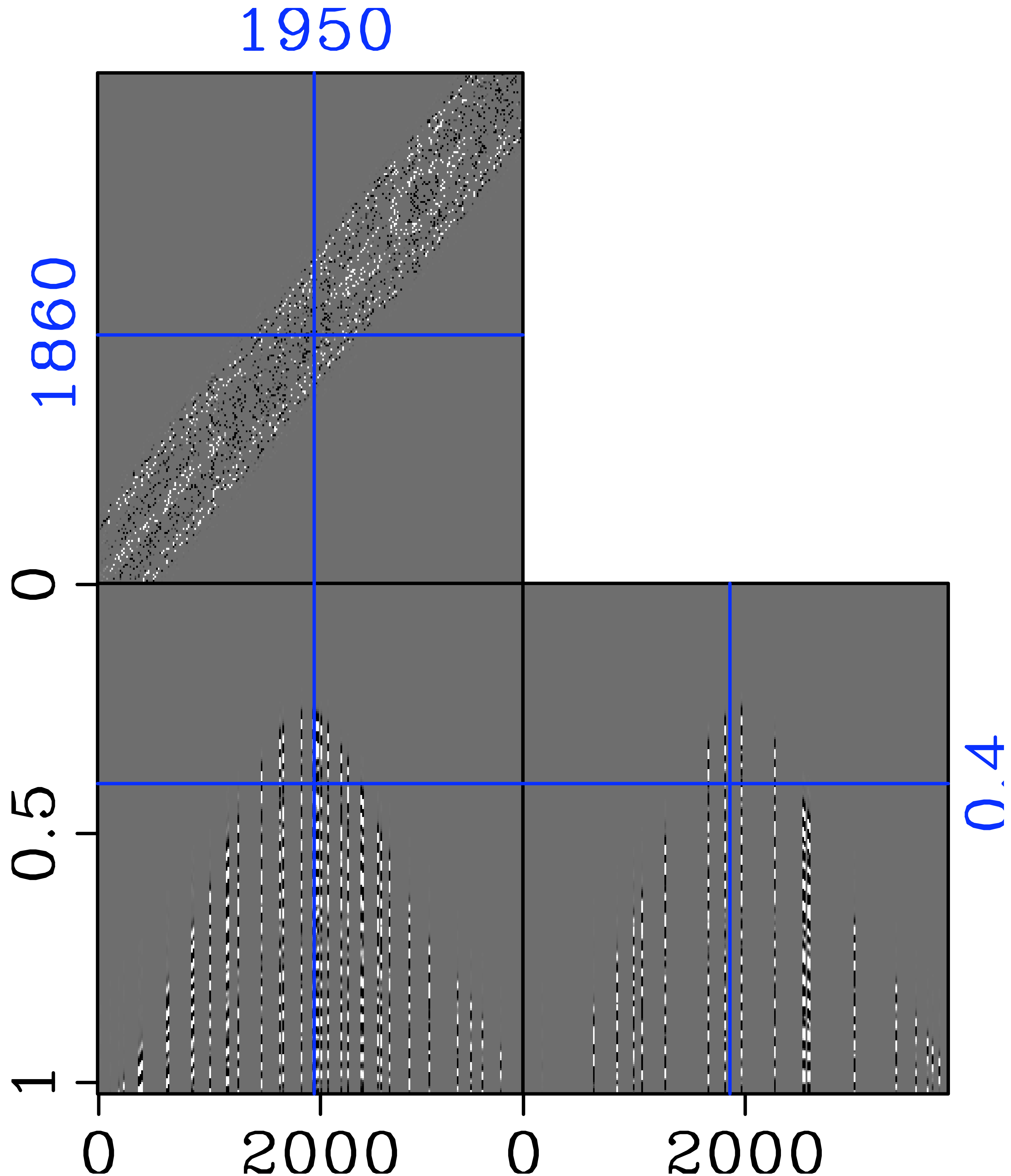
Recovery with focussing

Solve

$$\mathbf{P}_\epsilon : \quad \begin{cases} \widetilde{\mathbf{x}} = \arg\min_{\mathbf{x}} \|\mathbf{x}\|_1 \quad \text{s.t.} \quad \|\mathbf{A}\mathbf{x} - \mathbf{y}\|_2 \leq \epsilon \\ \widetilde{\mathbf{f}} = \mathbf{S}^T \widetilde{\mathbf{x}} \end{cases}$$

with

$$\begin{aligned} \mathbf{A} &:= \mathbf{R}\mathbf{\Delta}\mathbf{P}\mathbf{C}^T \text{and } \mathbf{\Delta}\mathbf{P} := \mathbf{F}^H \text{block diag}\{\Delta p\}\mathbf{F} \\ \mathbf{S}^T &:= \mathbf{\Delta}\mathbf{P}\mathbf{C}^T \\ \mathbf{y} &= \mathbf{R}\mathbf{P}(:) \\ \mathbf{R} &= \text{picking operator.} \end{aligned}$$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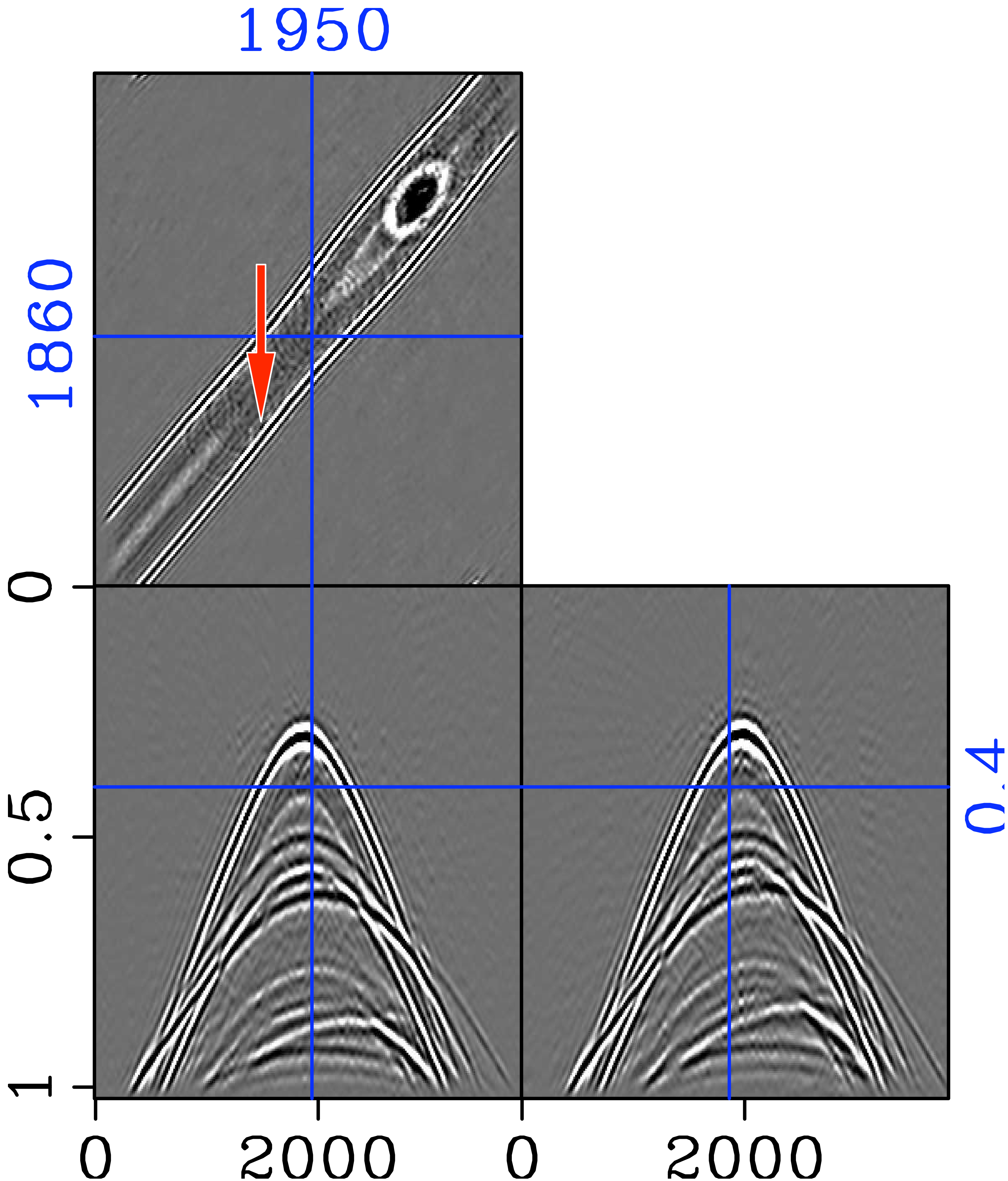
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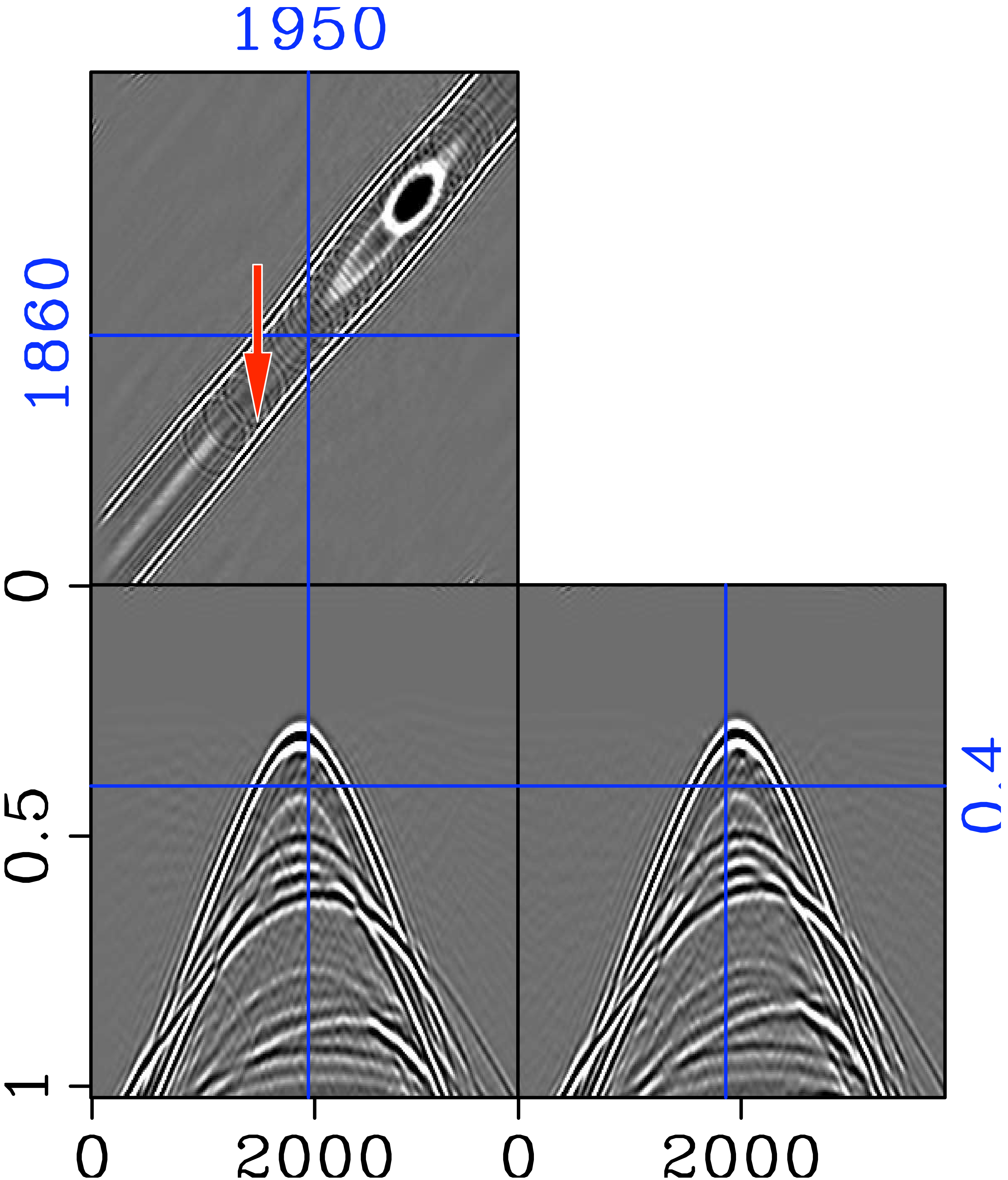

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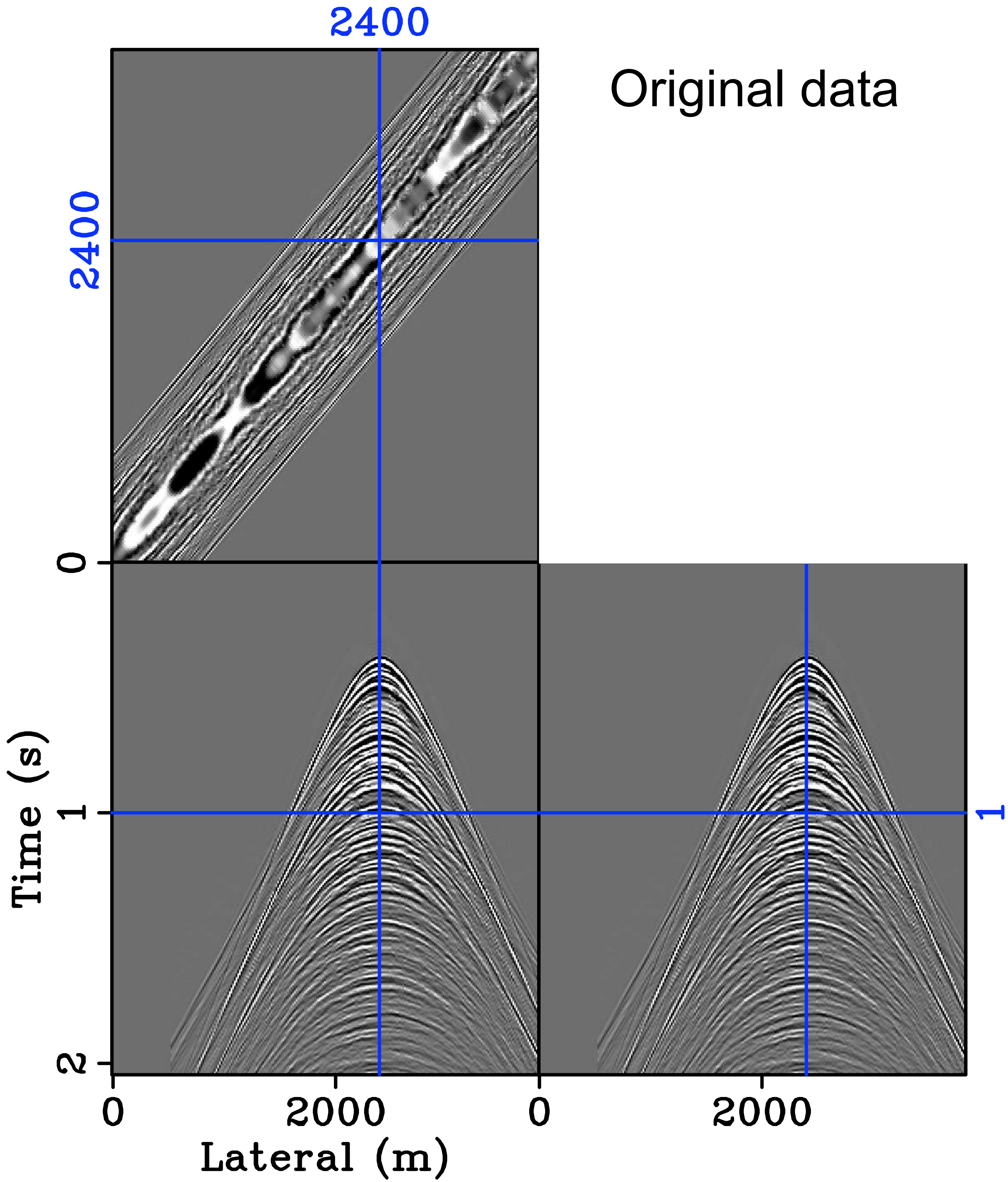
Original data



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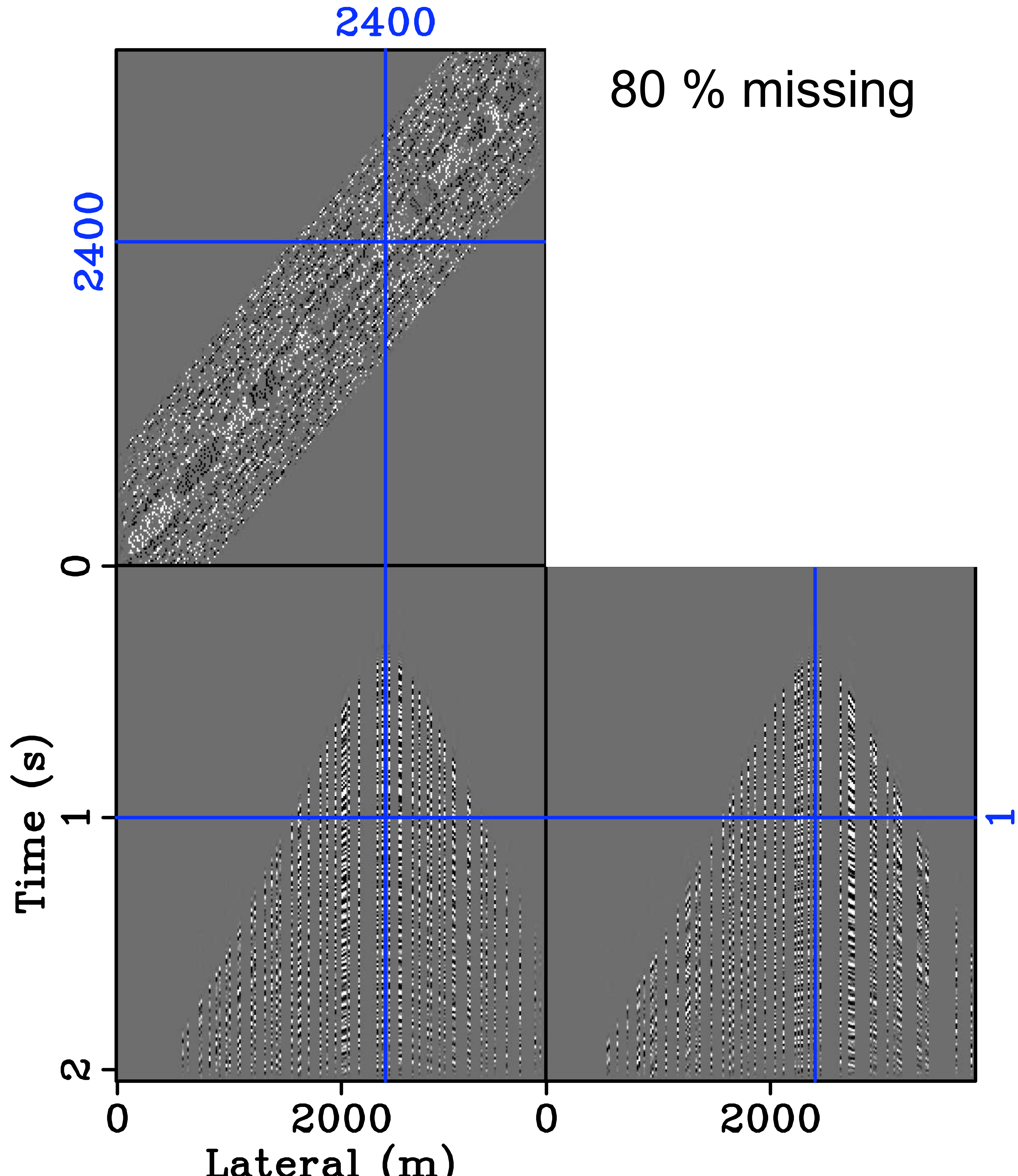
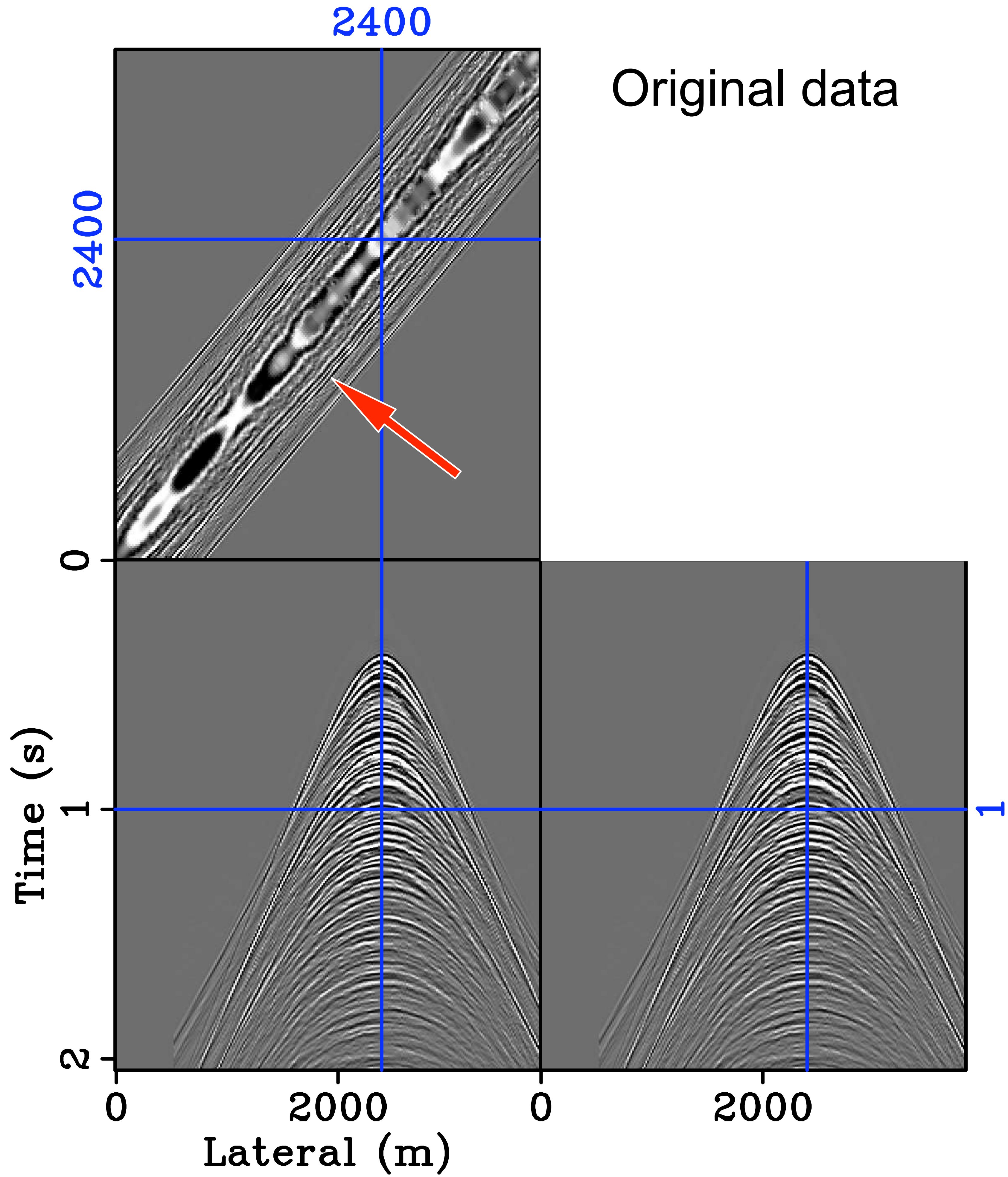
80 % missing

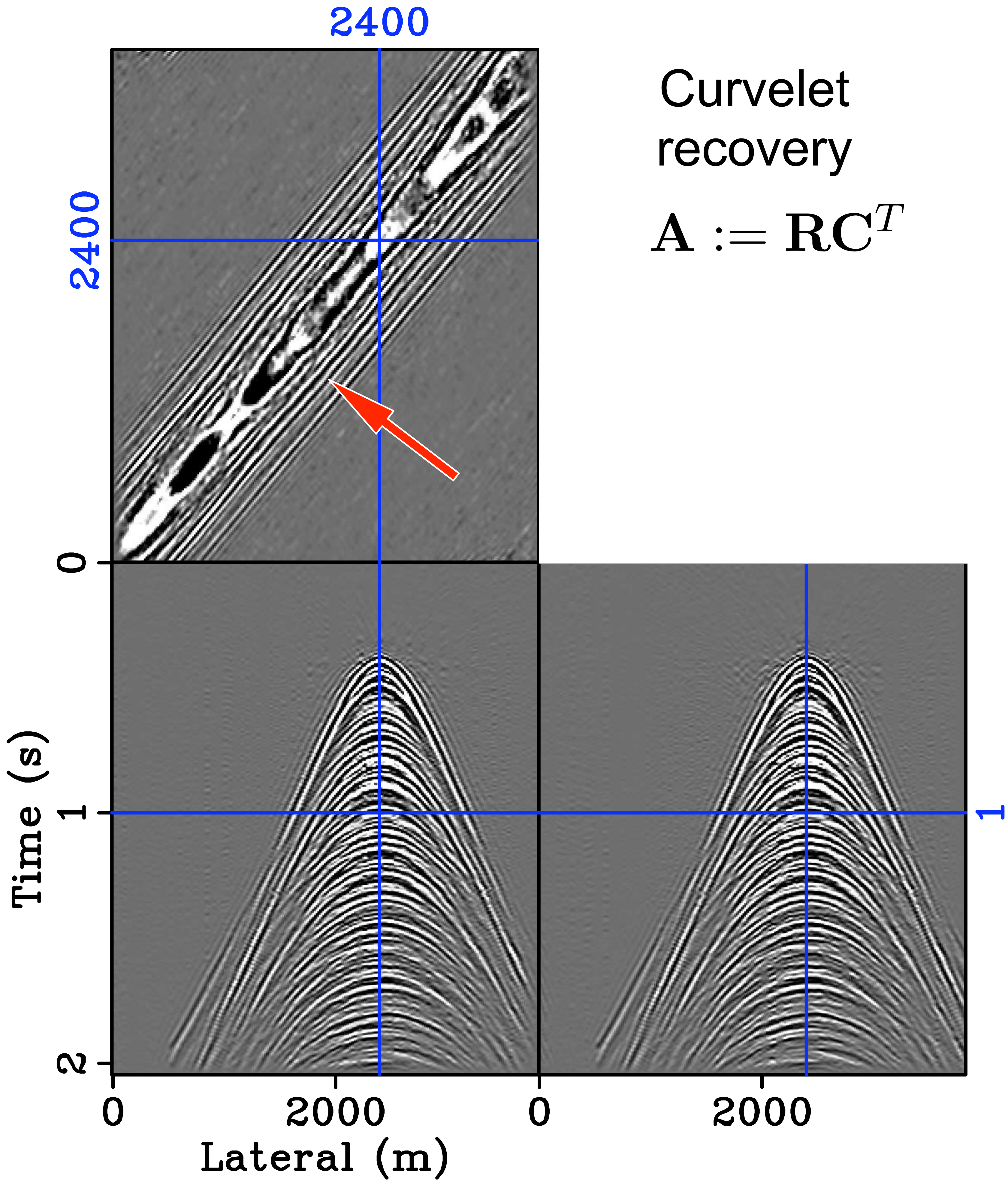




Original data

Curvelet recovery

$$\mathbf{A} := \mathbf{R}\mathbf{C}^T$$



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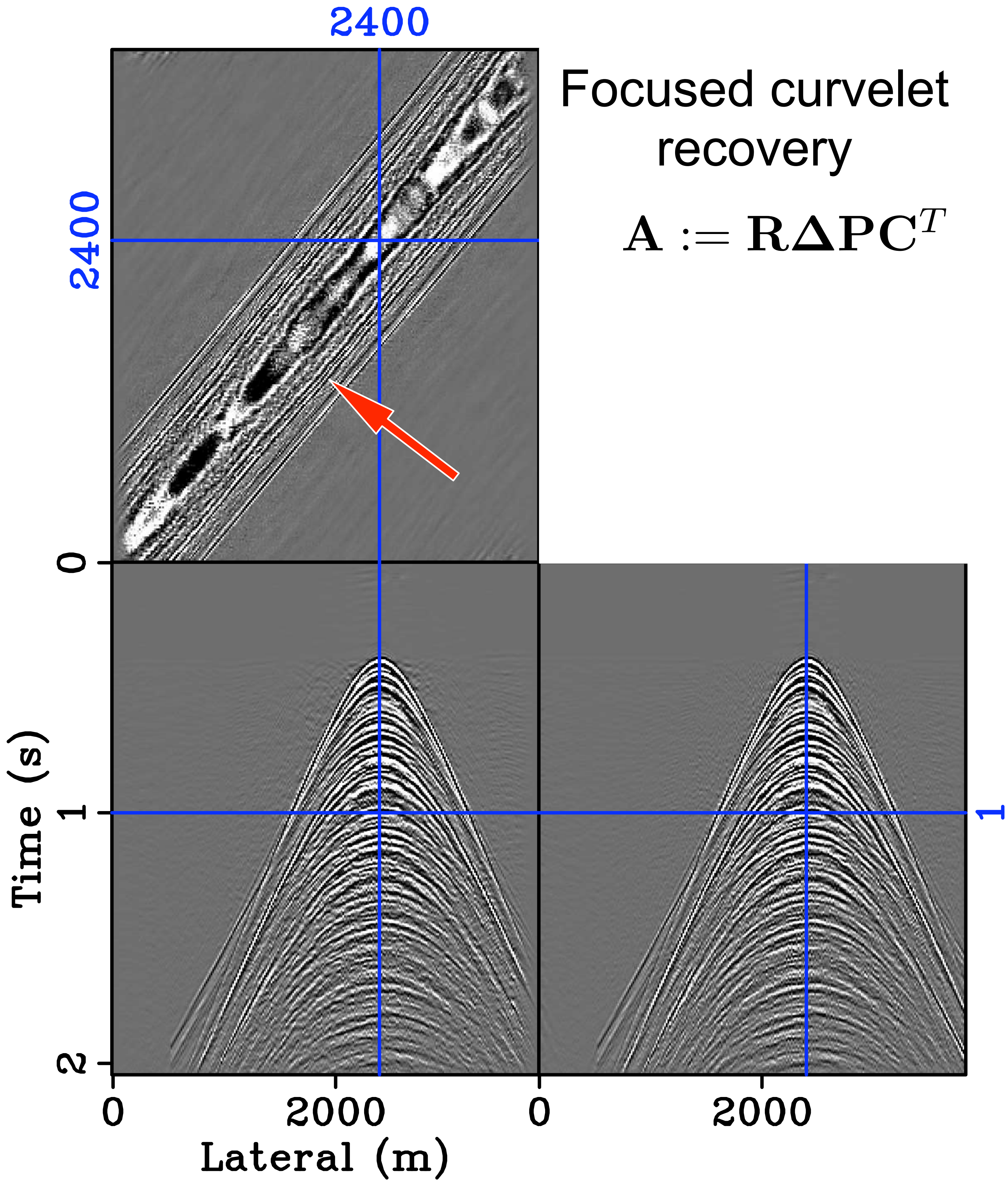
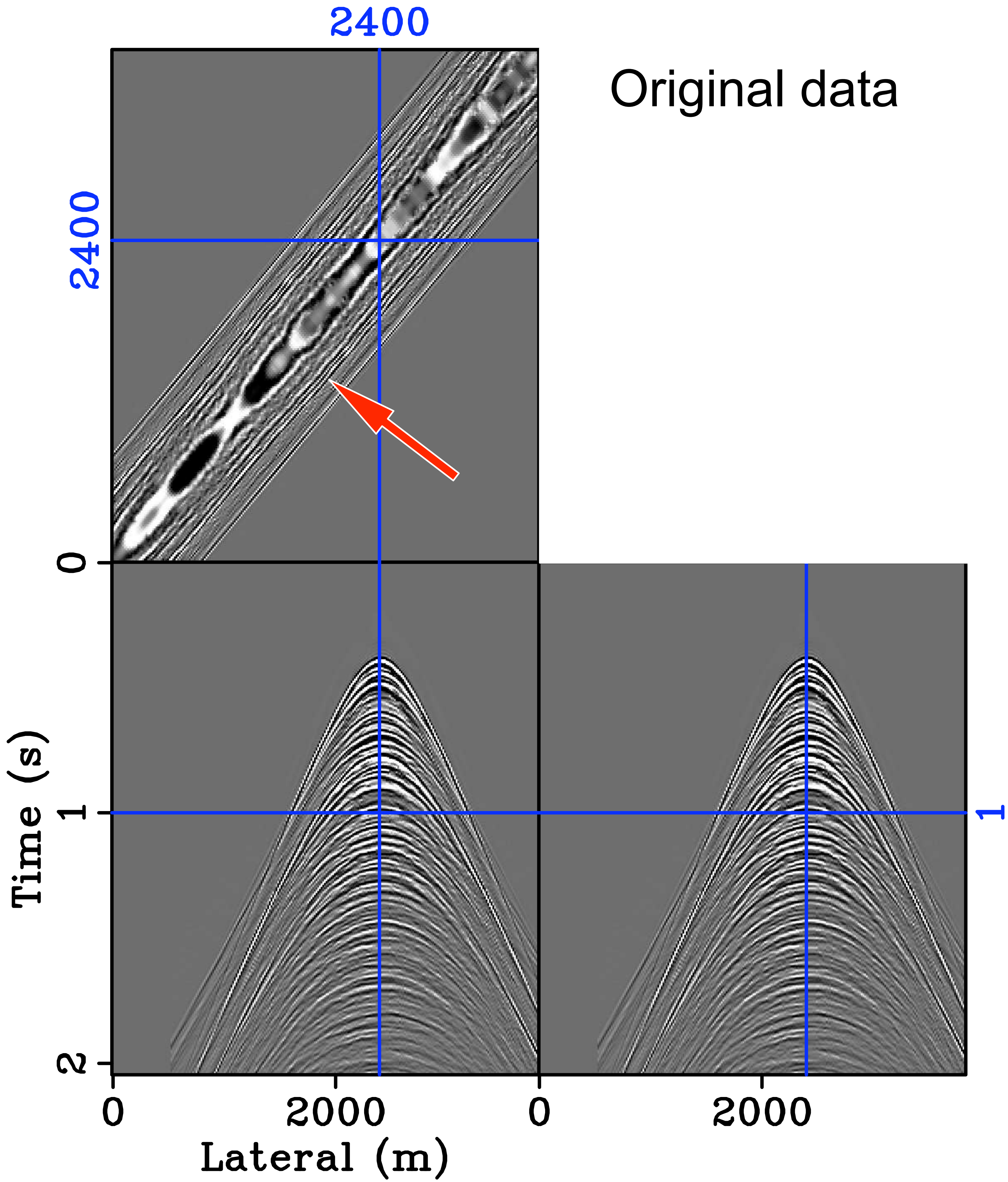
Focused curvelet recovery

$$\mathbf{A} := \mathbf{R}\boldsymbol{\Delta}\mathbf{P}\mathbf{C}^T$$



Original data



Conclusions

Curvelets represent a versatile transform that

- brings robustness w.r.t. moderate shifts and phase rotations to primary multiple separation
- allows for the nonlinear recovery for severely sub-Nyquist data
- leads to an improved recovery when compounded with "migration like" operators

Opens tentative perspectives towards a new sampling theory

- for seismic data
- that includes migration operators ...

Acknowledgments

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Eric Verschuur for providing us with the synthetic and real data examples.

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