Just diagonalize: a curvelet-based approach to seismic amplitude recovery

Felix J. Herrmann*,
Peyman Moghaddam* & Chris Stolk (Universiteit Twente)
*Seismic Laboratory for Imaging and Modeling
slim.eos.ubc.ca

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Motivation

Migration generally does not correctly recover the amplitudes.

Least-squares migration is computationally unfeasible.

Amplitude recovery (e.g. AGC) lacks robustness w.r.t. noise.

Existing diagonal amplitude-recovery methods

- do not always correct for the order (1 - 2D) of the Hessian [see Symes '07]
- do not invert the scaling robustly

Moreover, these (scaling) methods assume that there

- are no conflicting dips (conormal) in the model
- is infinite aperture
- are infinitely-high frequencies
- etc.

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Curvelets & seismology

Wish list

A transform that

- detects the reflectors without ***prior*** information on the ***geologic*** dips
- is ***sparse***, i.e. the magnitude-sorted coefficients decay ***fast***
- is relative ***invariance*** under the ***demigration-migration***, i.e. sparse on migrated images

Curvelets

- were "born" from studying high-frequency solution operators for wave propagation*
- diagonalization of migration operators**

***See work by Stein, Smit, Donoho, Candes & Demanet**
**** Main motivation for Douma & de Hoop and Chauris**

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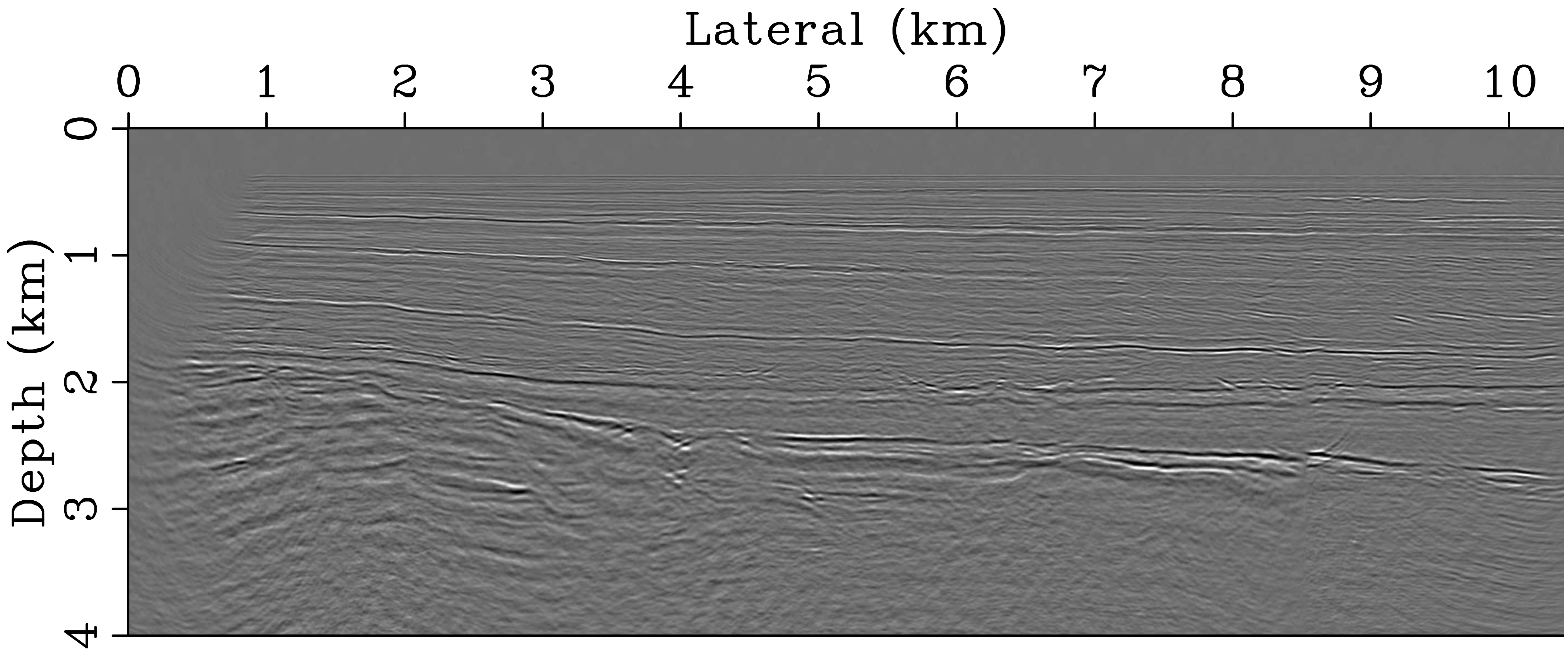
Nonlinear approximation

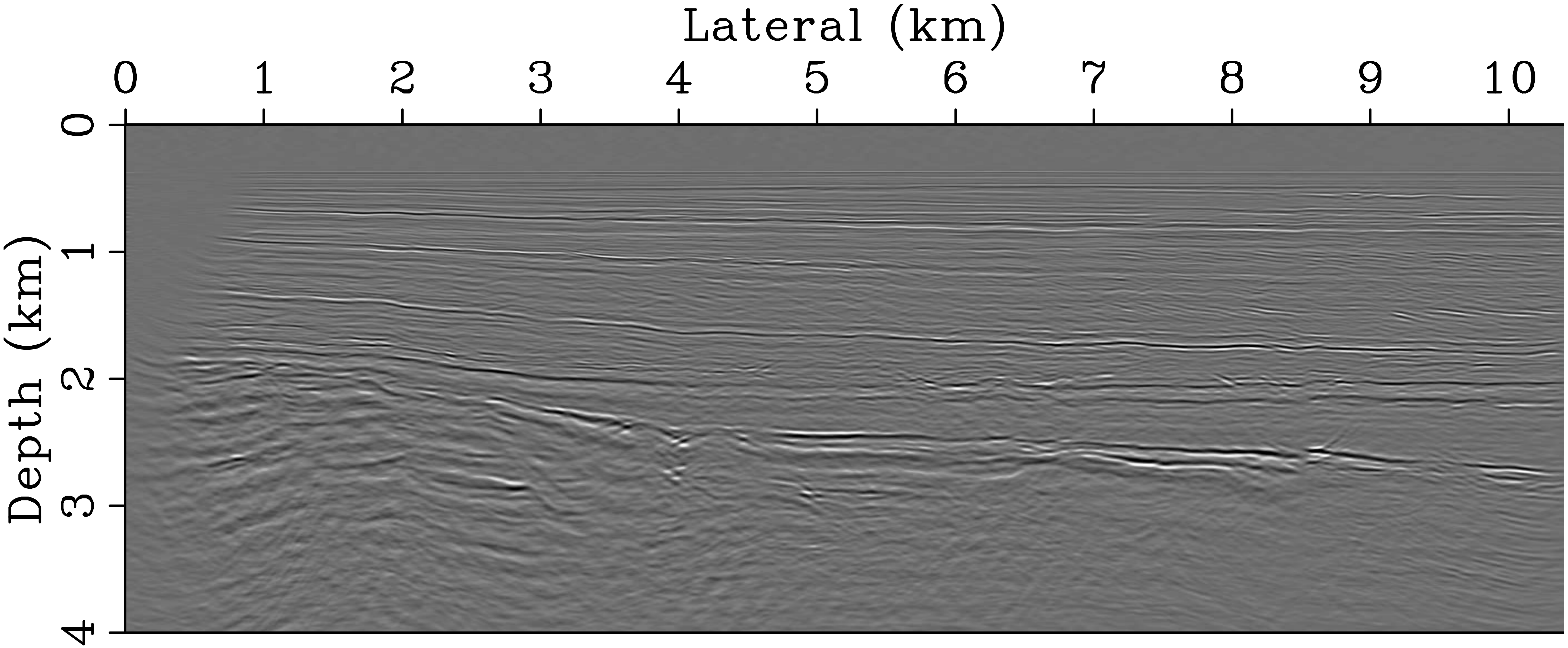
Migrated mobil data set



reconstructed data with p=99

Nonlinear approximation

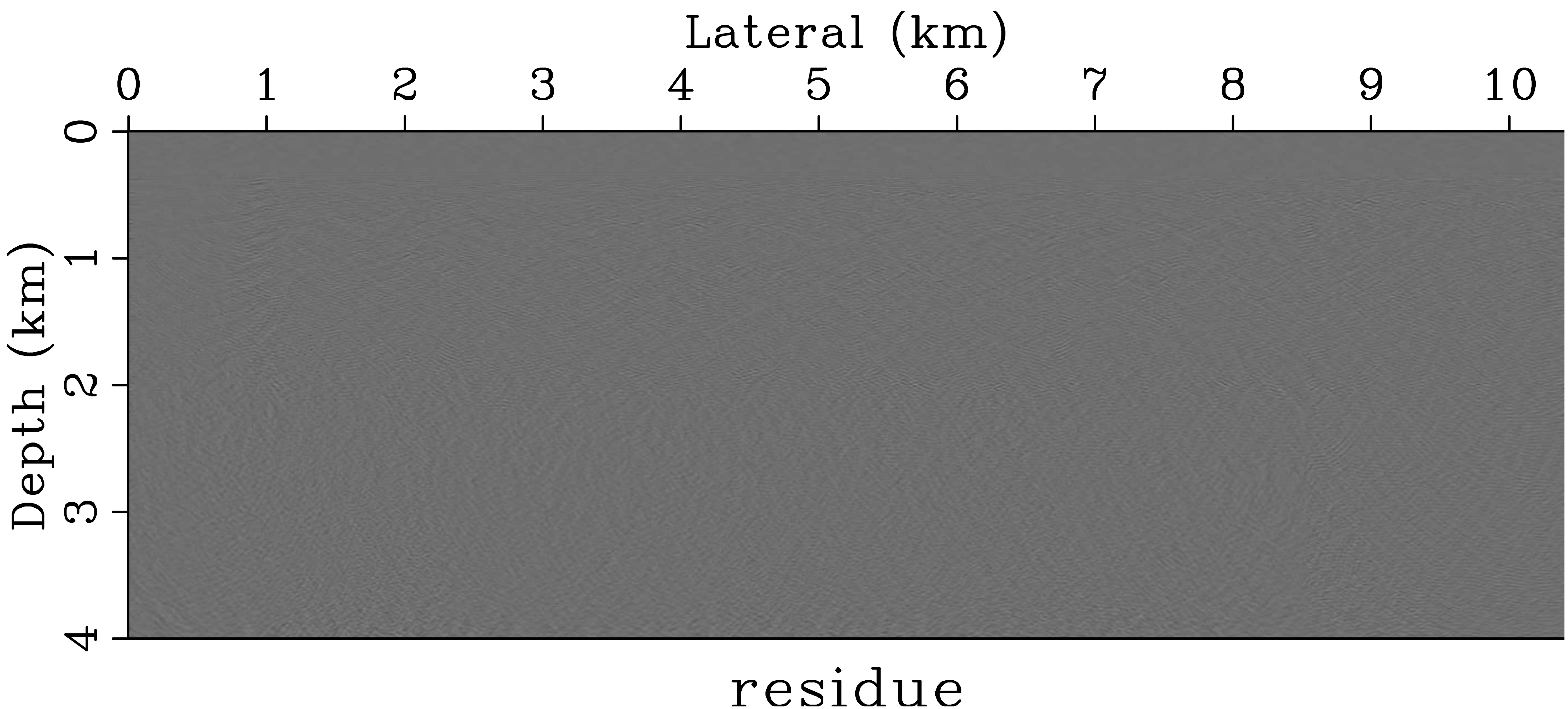
Recovery from largest 3 %



reconstructed data with p=3

Nonlinear approximation

Difference



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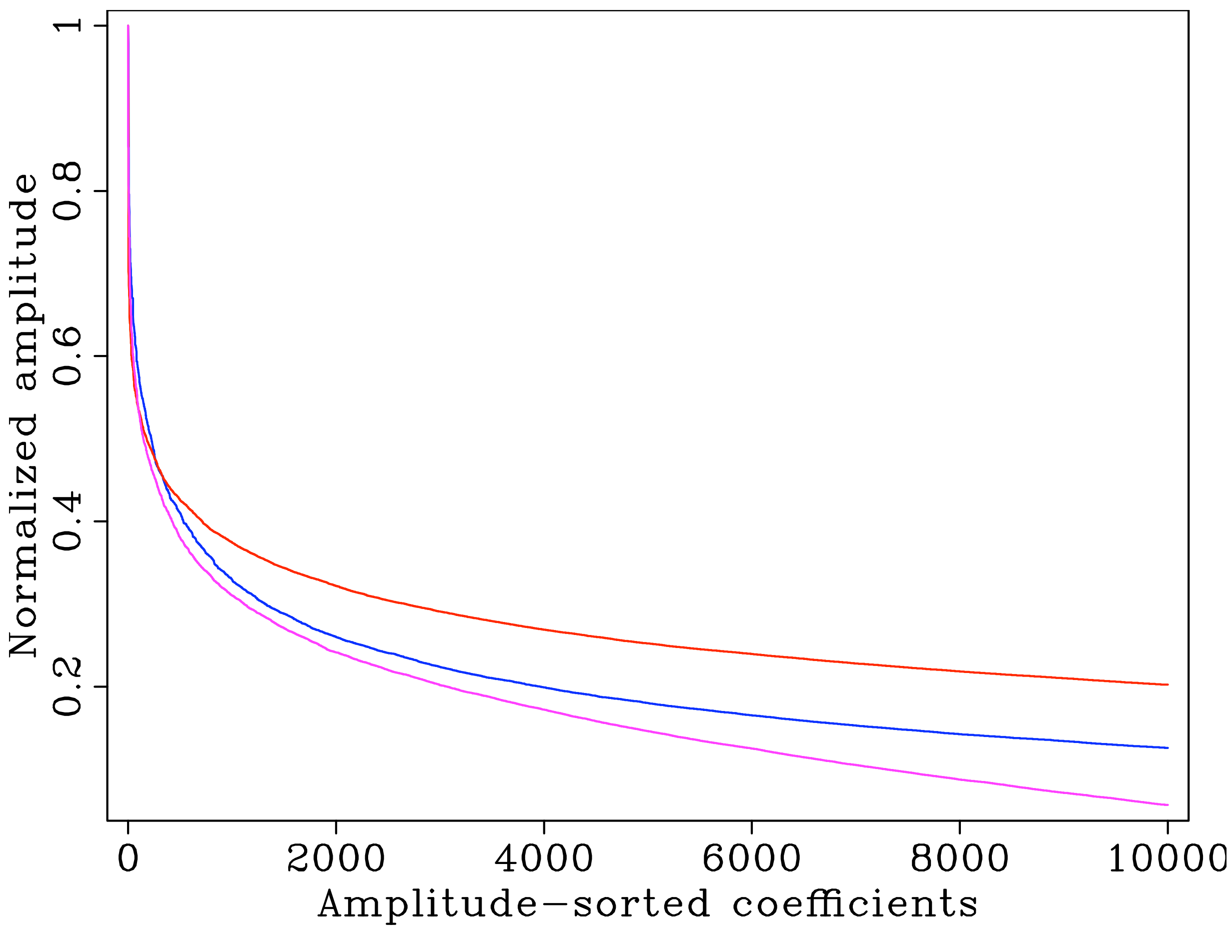
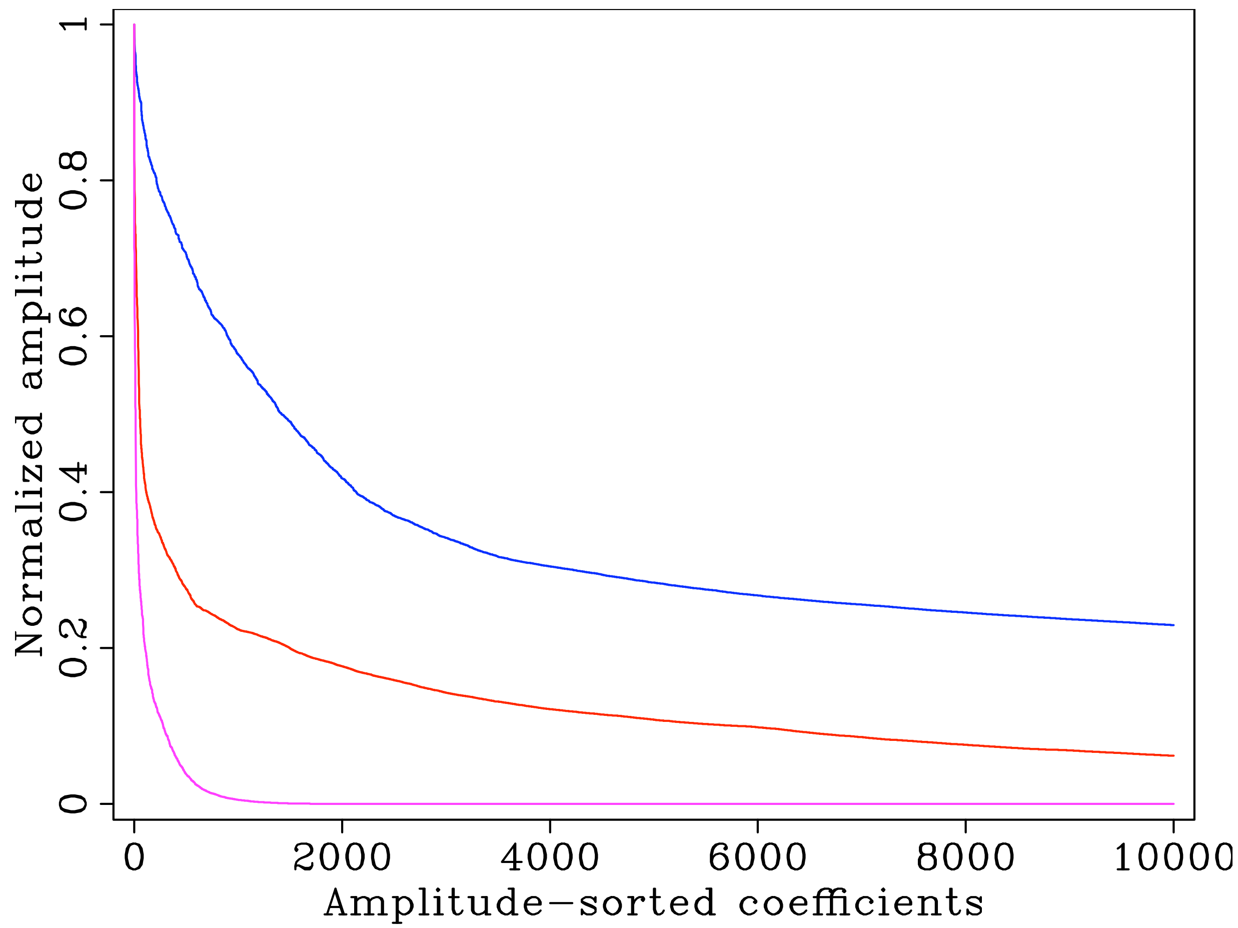
Nonlinear approximation rates

Imaged Mobil data



Reflectivity SEG AA’



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Curvelets & wave propagation

Theoretical results that claim that curvelets near diagoanalize migration operators [Demanet et. al, de Hoop]

Encouraging results for constant velocity media [Douma & de Hoop; Chauris]

Challenge: discrete curvelets move off the grid

- interpolation
- definition of curvelet molecules [Demanet et. al, de Hoop]

In not so smooth media curvelets spread significantly ....

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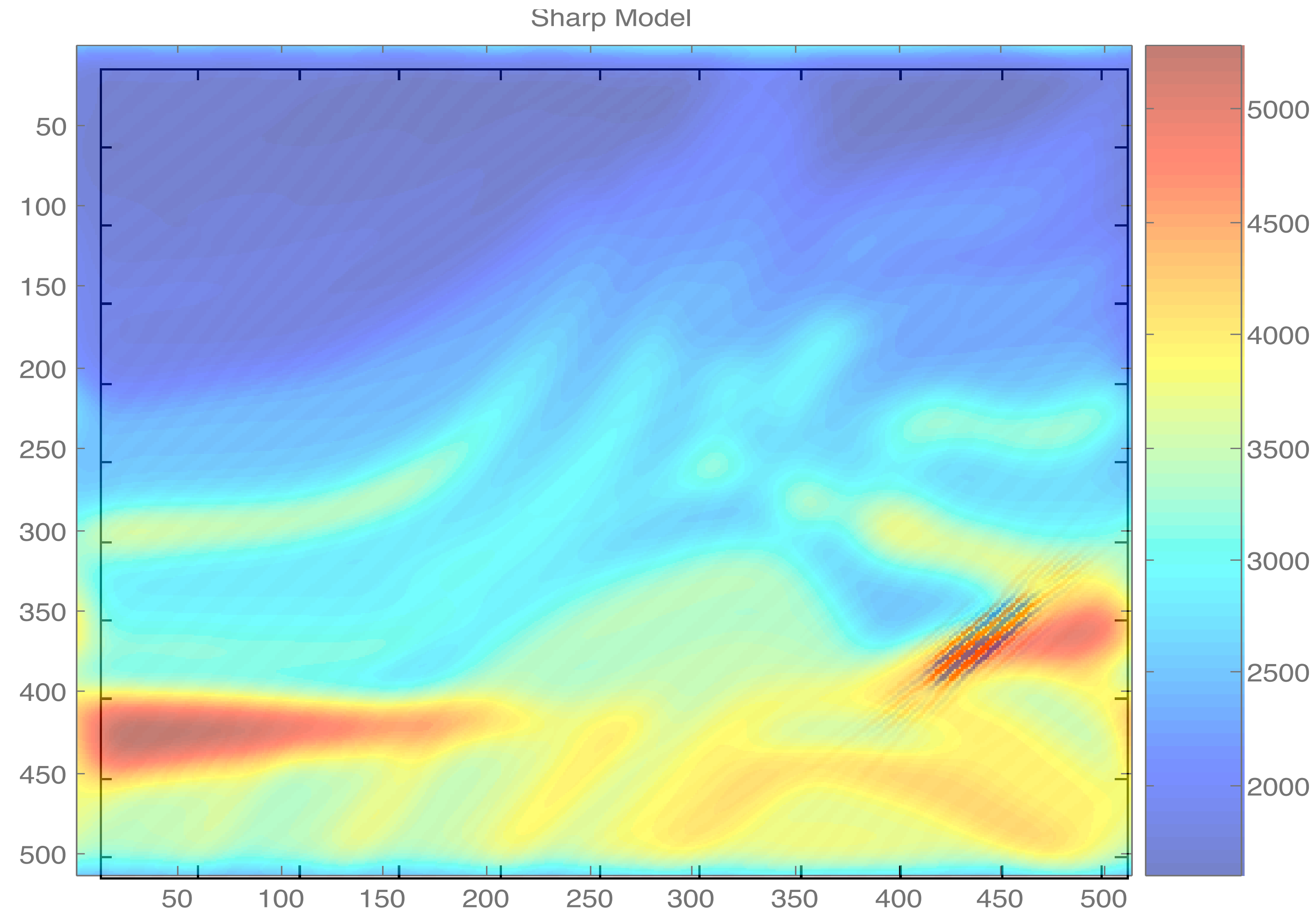
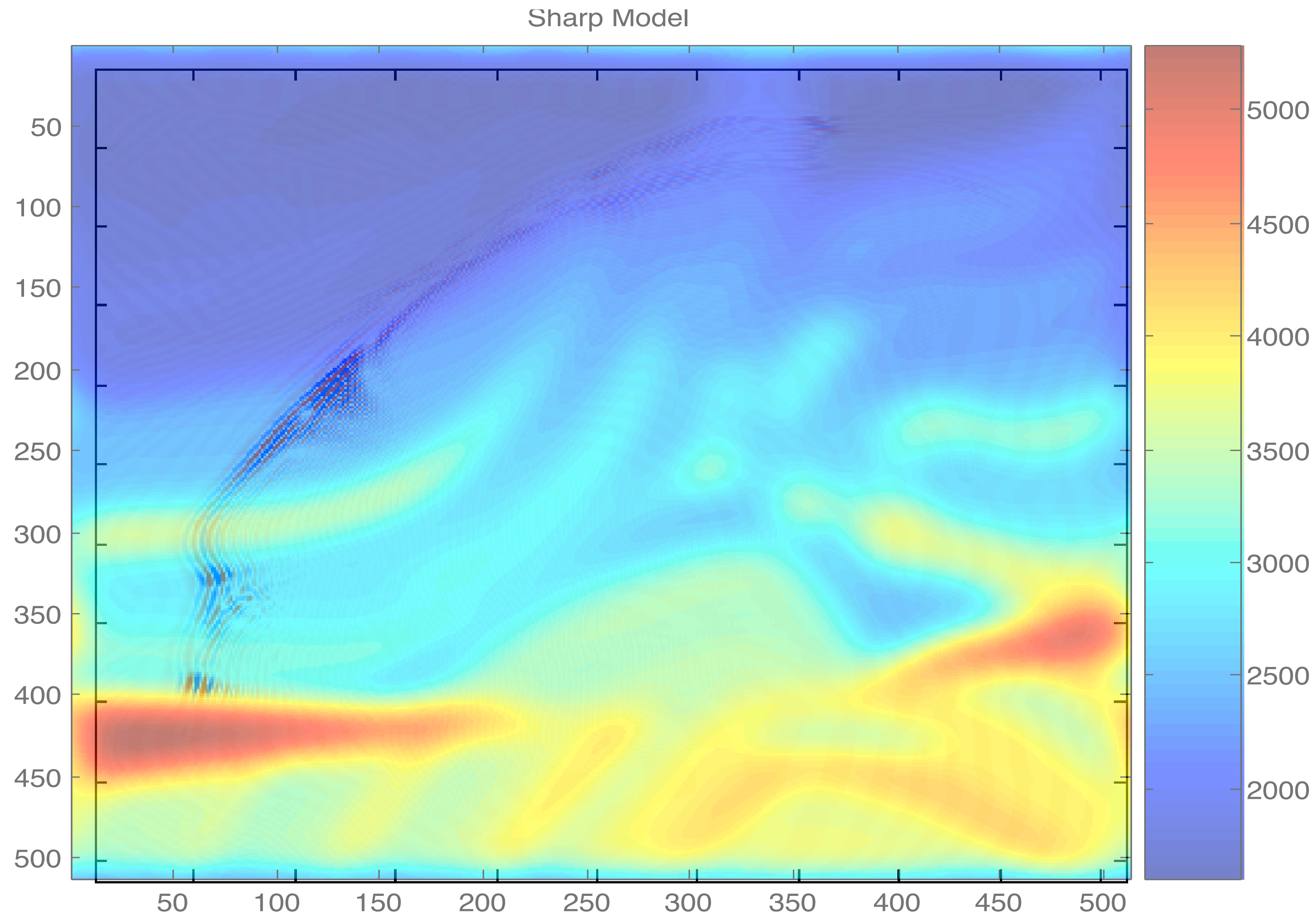
Curvelet propagation

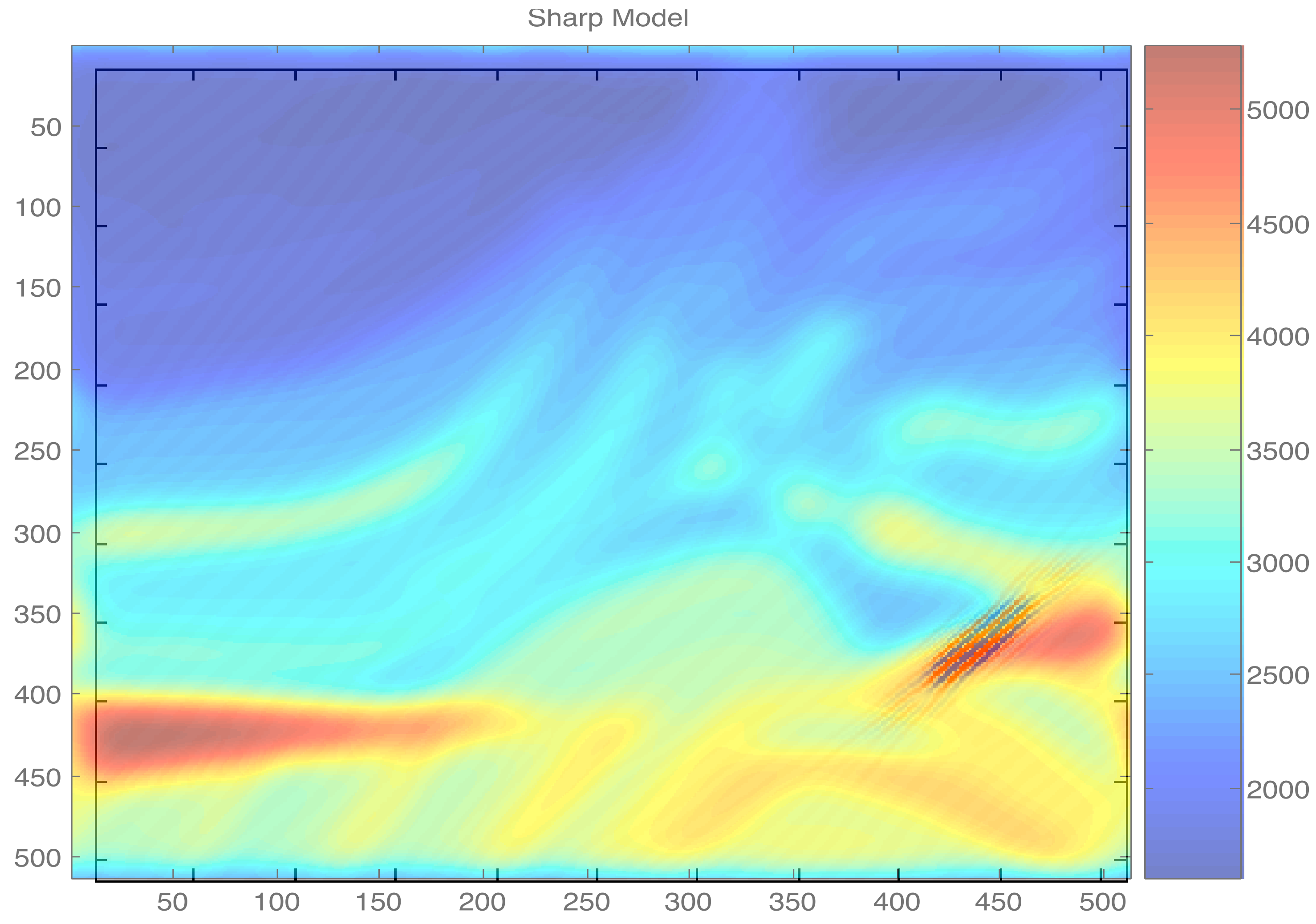


Curvelet propagation



Major challenge. Limit ourselves to migration amplitude recovery!

“Imaged” curvelet



Hessian/Normal operator

[Stolk 2002, ten Kroode 1997, de Hoop 2000, 2003]

Alternative to expensive least-squares migration.

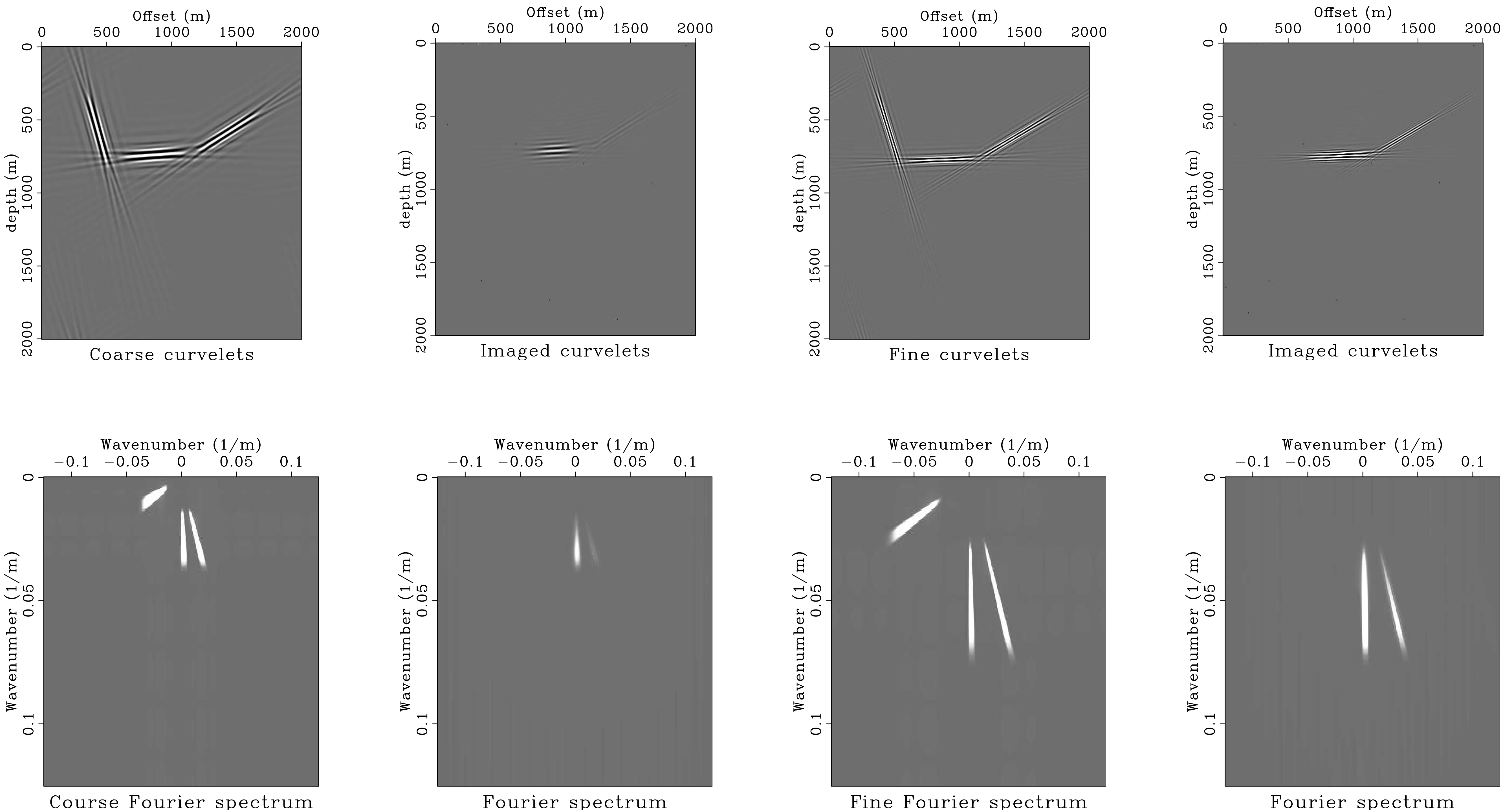
In high-frequency limit Ψ is a PsDO

$$\big(\Psi f\big)(x) := \big(K^T K f\big)(x) = \int_{\mathbb{R}^d} e^{-ix\cdot\xi} a(x,\xi) \hat{f}(\xi) \mathrm{d}\xi$$

- pseudolocal
- singularities are preserved

Corresponds to a spatially-varying dip filter after appropriate preconditioning (=> zero order).

Invariance under Hessian matrix



- curvelets remain invariant
- approximation improves for higher frequencies

Diagonal approximation of the Hessian

Existing scaling methods

Methods are based on a diagonal approximation of Ψ.

- Illumination-based normalization (Rickett '02)
- Amplitude preserved migration (Plessix & Mulder '04)
- Amplitude corrections (Guitton '04)
- Amplitude scaling (Symes '07)

We are interested in an 'Operator and image adaptive' scaling method which

- estimates the action of Ψ from a reference vector close to the actual image
- assumes a smooth symbol of Ψ in space and angle
- does not require the reflectors to be conormal <=> allows for conflicting dips
- stably inverts the diagonal

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Approximation

Theorem 1. *The following estimate for the error holds*

$$\|(\Psi(x,D) - C^T \mathbf{D}_\Psi C)\varphi_\mu\|_{L^2(\mathbb{R}^n)} \leq C'' 2^{-|\mu|/2},$$

where C'' *is a constant depending on* Ψ*.*

Allows for the decomposition

$$\begin{aligned} \big(\Psi\varphi_\mu\big)(x) &\simeq \big(C^T \mathbf{D}_\Psi C \varphi_\mu\big)(x) \\ &= \big(AA^T \varphi_\mu\big)(x) \end{aligned}$$

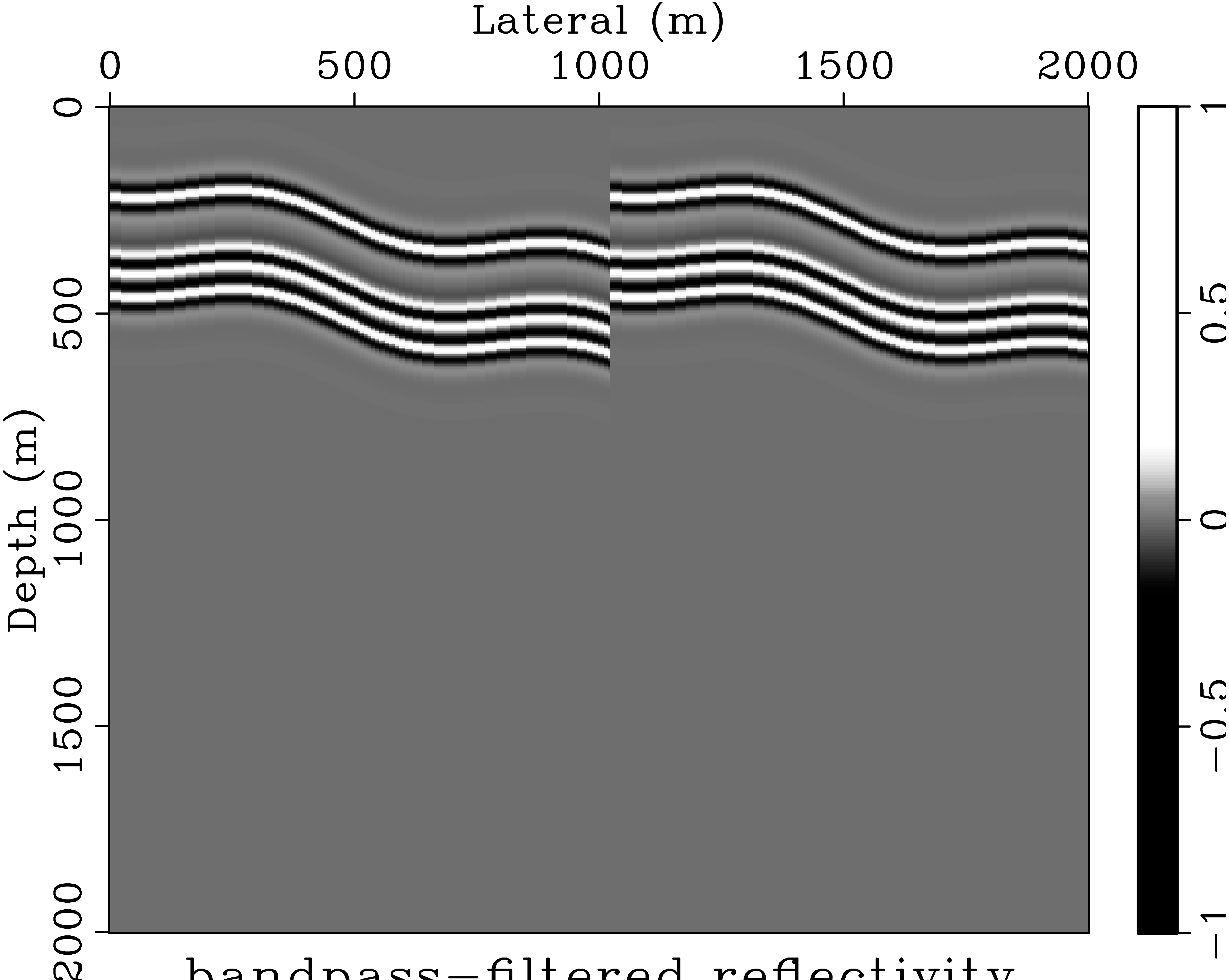
with $A := \sqrt{\mathbf{D}_\Psi} C$ and $A^T := C^T \sqrt{\mathbf{D}_\Psi}$.

Approximation

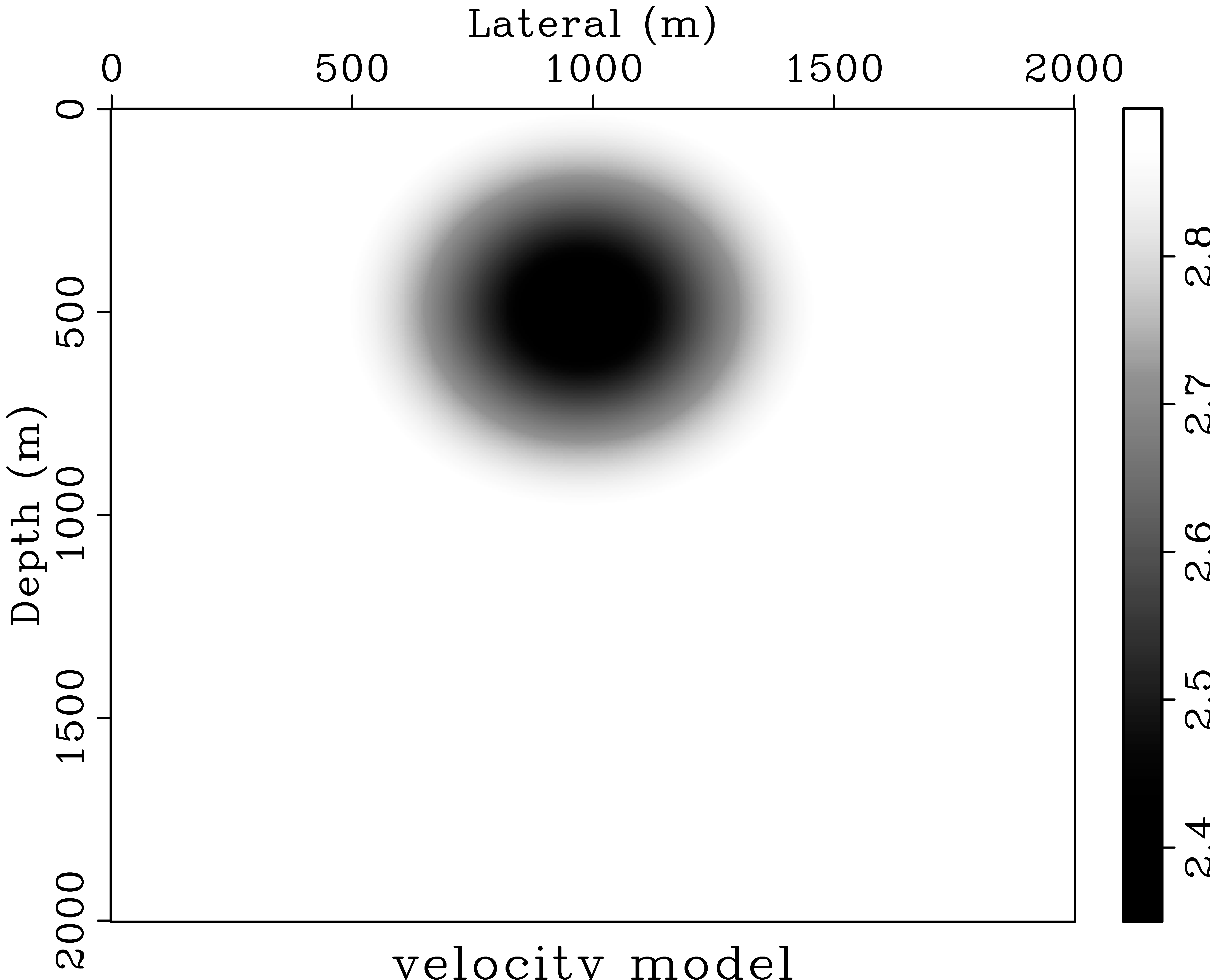
$$\begin{aligned} y(x) &= \big(\Psi m\big)(x) + e(x) \\ &\simeq \big(AA^T m\big)(x) + e(x) \\ &= Ax_0 + e, \end{aligned}$$

- Wavelet-vagulette like [Donoho, Candes]
- Amenable to nonlinear recovery

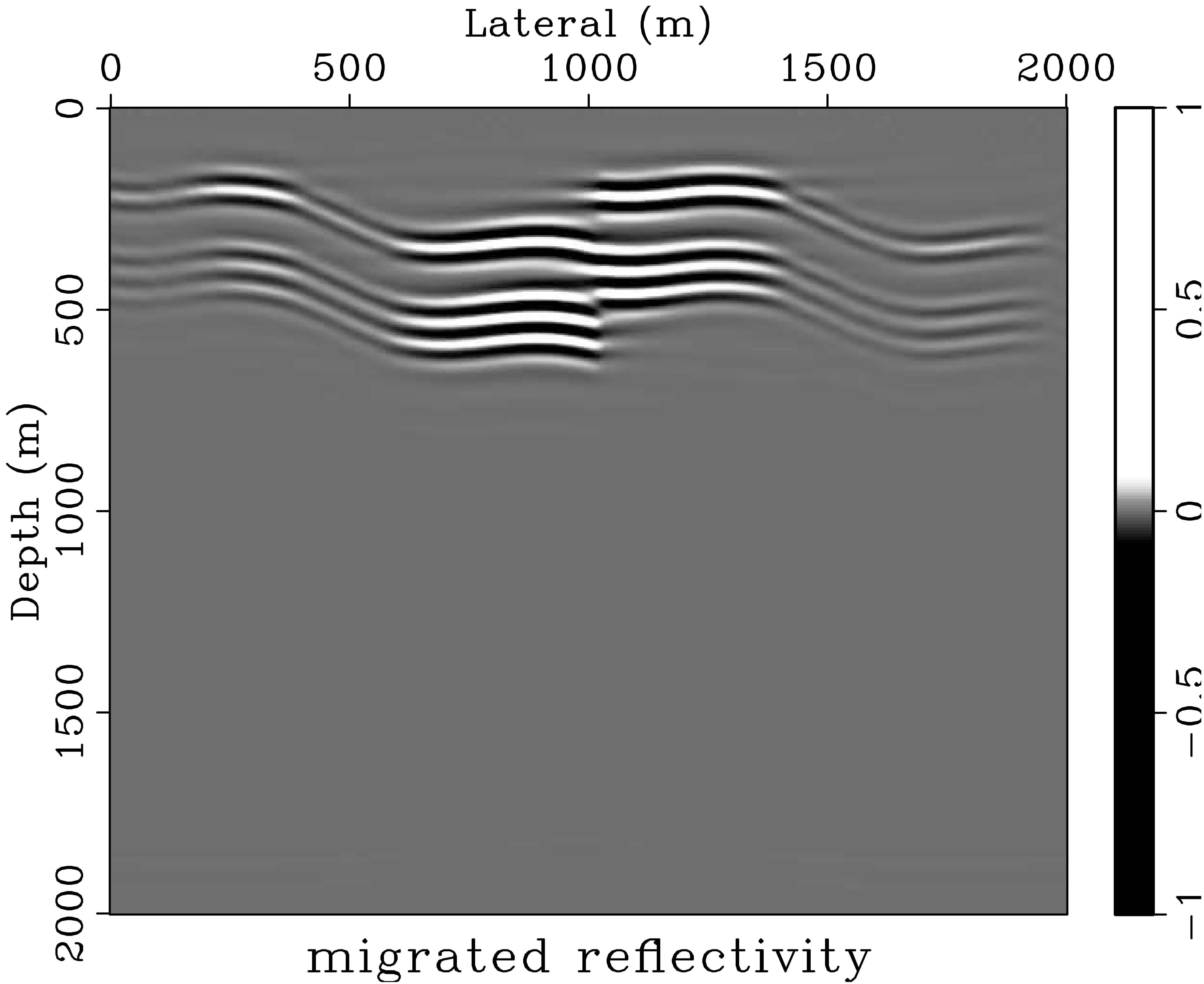
Estimation of the diagonal scaling



bandpass−filtered reflectivity

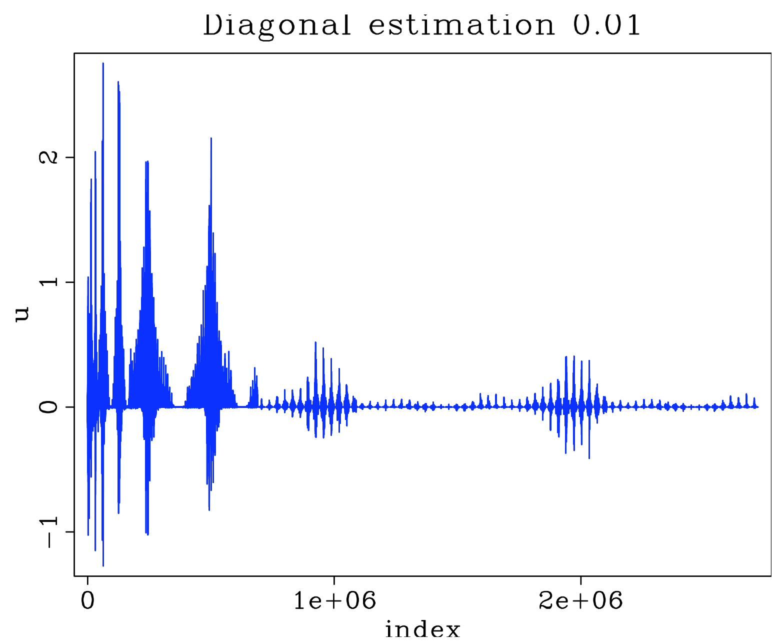


Lateral (m)
0
500
1000
1500
2000
Depth (m)
0
500
1000
1500
2000
2.4
2.5
2.6
2.7
2.8
velocity model

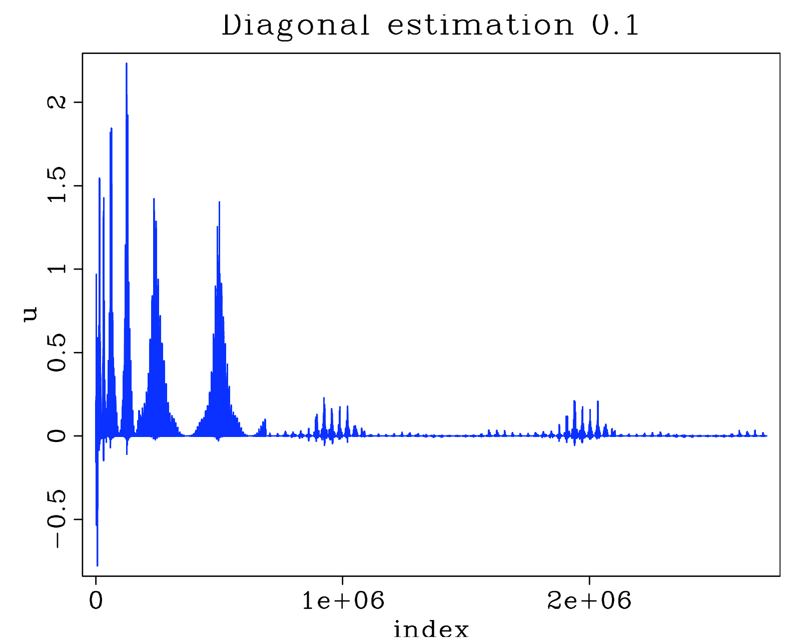


Lateral (m)
0
500
1000
1500
2000
Depth (m)
0
500
1000
1500
2000
1
0.5
0
−0.5
−1
migrated reflectivity

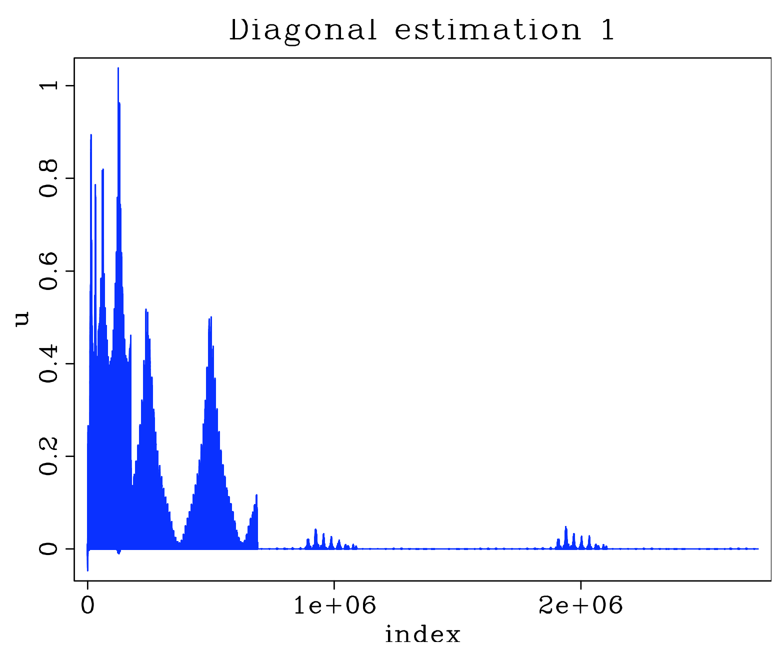
Diagonal estimation

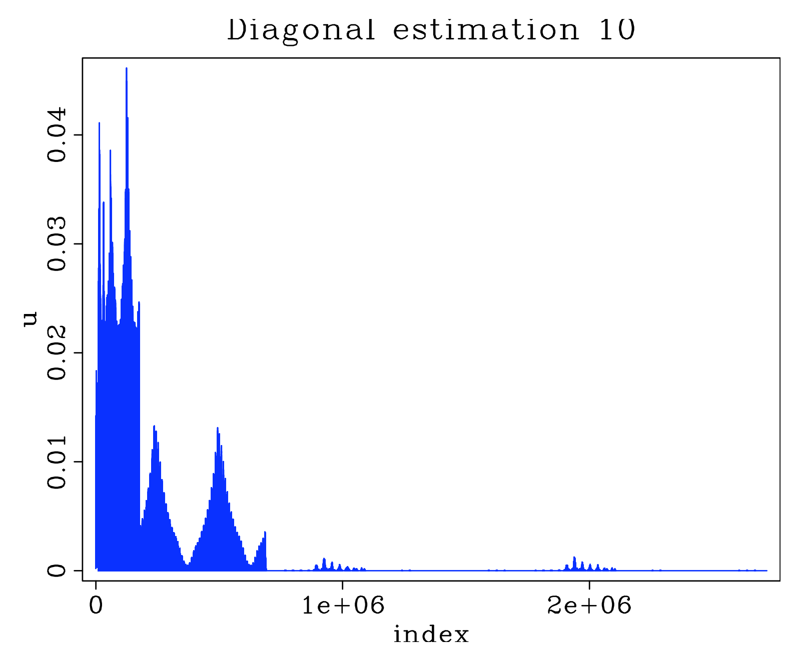


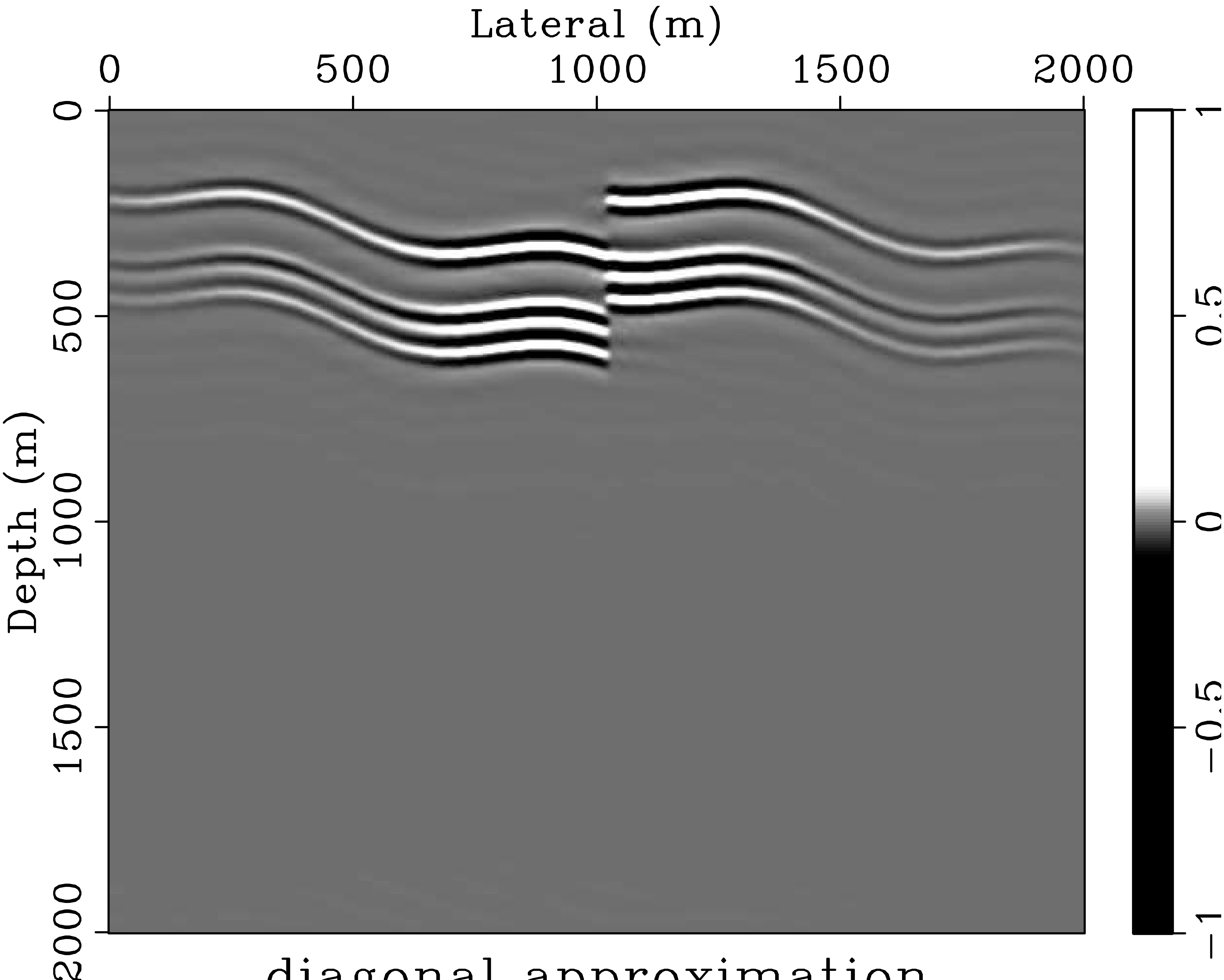
(a)



(b)







diagonal approximation

Seismic amplitude recovery

Recovery

Final form

$$\mathbf{y} = \mathbf{A}\mathbf{x}_0 + \boldsymbol{\varepsilon}$$

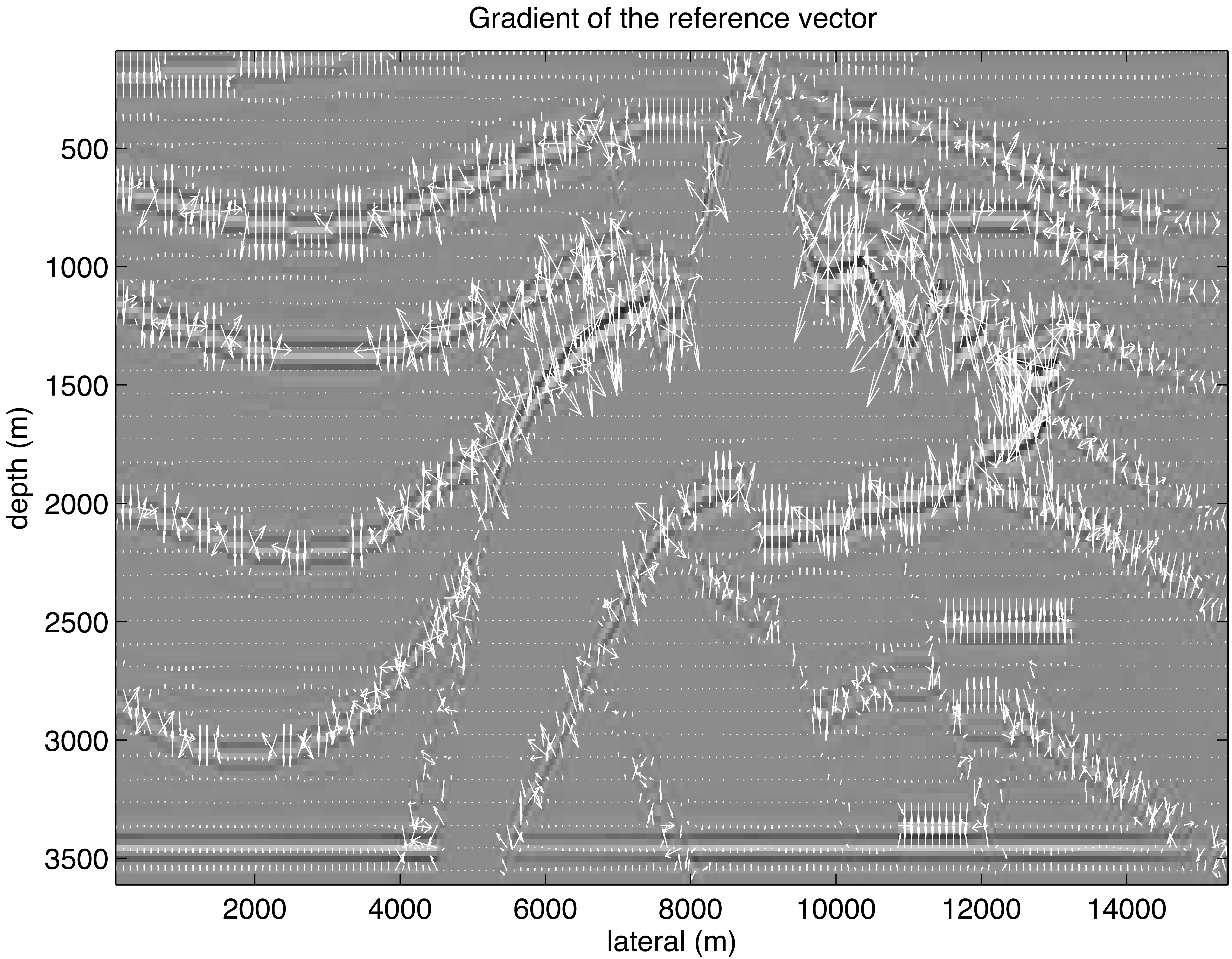
with $\mathbf{x}_0 = \mathbf{\Gamma}\mathbf{C}\mathbf{m}$ and $\boldsymbol{\epsilon} = \mathbf{A}\mathbf{e}$.

Solve

$$\mathbf{P}: \quad \begin{cases} \min_{\mathbf{x}} J(\mathbf{x}) \quad \text{subject to} \quad \|\mathbf{y} - \mathbf{A}\mathbf{x}\|_2 \leq \epsilon \\ \\ \tilde{\mathbf{m}} = (\mathbf{A}^{\mathbf{H}})^{\dagger}\tilde{\mathbf{x}} \end{cases}$$

with

$$J(\mathbf{x}) = \overbrace{\alpha\|\mathbf{x}\|_1}^{\text{sparsity}} + \beta \underbrace{\|\mathbf{\Lambda}^{1/2}\left(\mathbf{A}^{H}\right)^{\dagger}\mathbf{x}\|_p}_{\text{continuity}}.$$


Gradient of the reference vector
depth (m)
500
1000
1500
2000
2500
3000
3500
lateral (m)
2000
4000
6000
8000
10000
12000
14000

Application to the SEG AA' model

Example

SEGAA' data:

- "broad-band" half-integrated wavelet [5-60 Hz]
- 324 shots, 176 receivers, shot at 48 m
- 5 s of data

Modeling operator

- Reverse-time migration with optimal check pointing (Symes '07)
- 8000 time steps
- modeling 64, and migration 294 minutes on 68 CPU's

Scaling requires 1 extra migration-demigration

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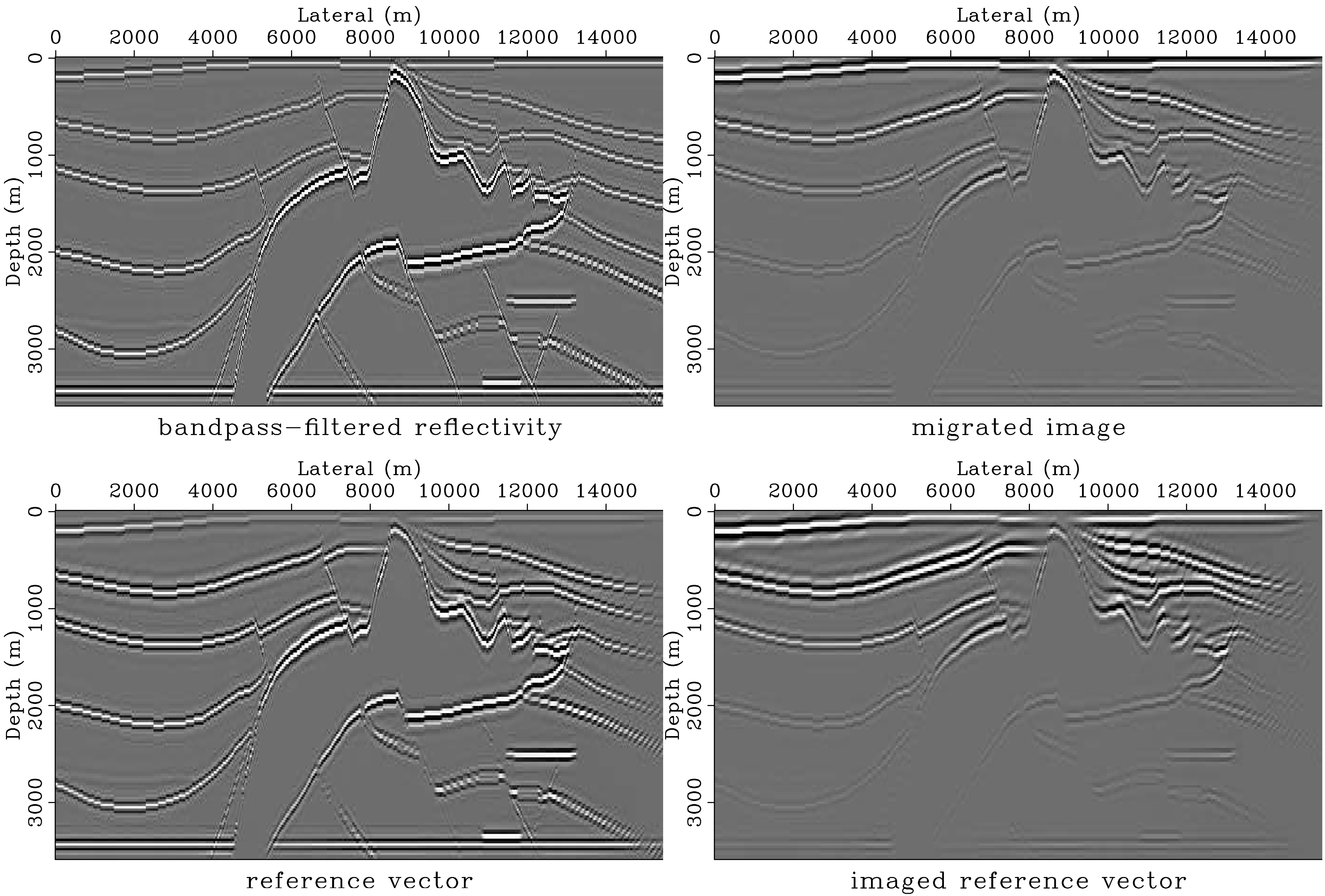
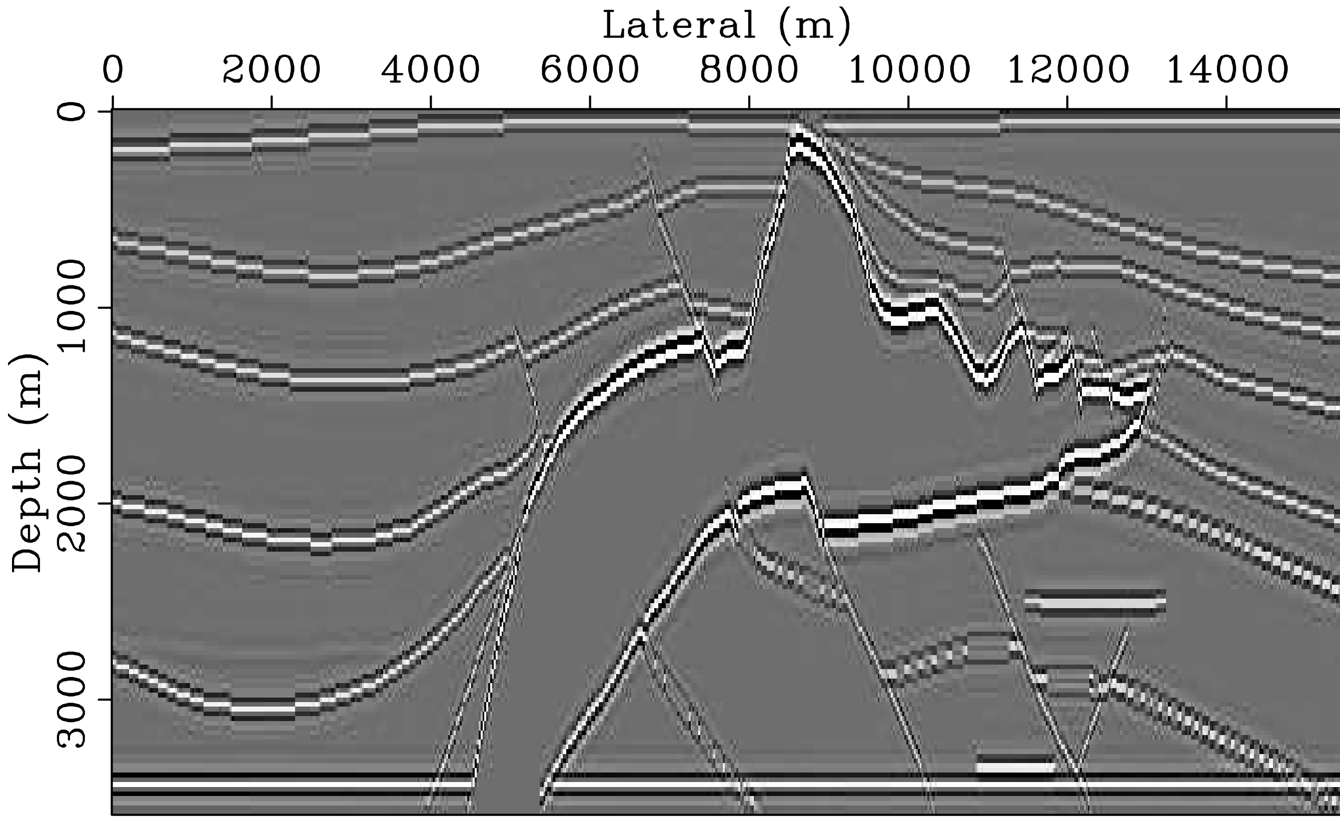
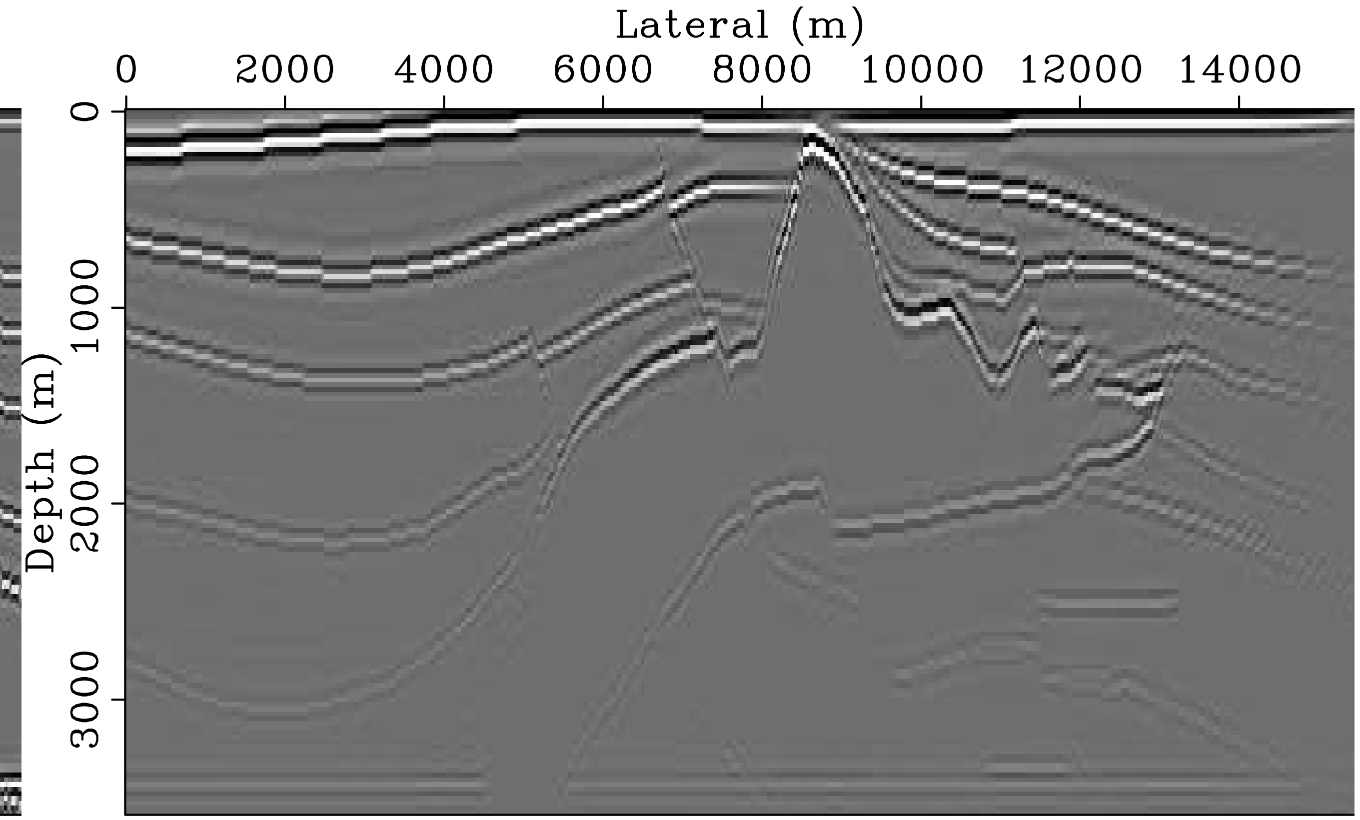
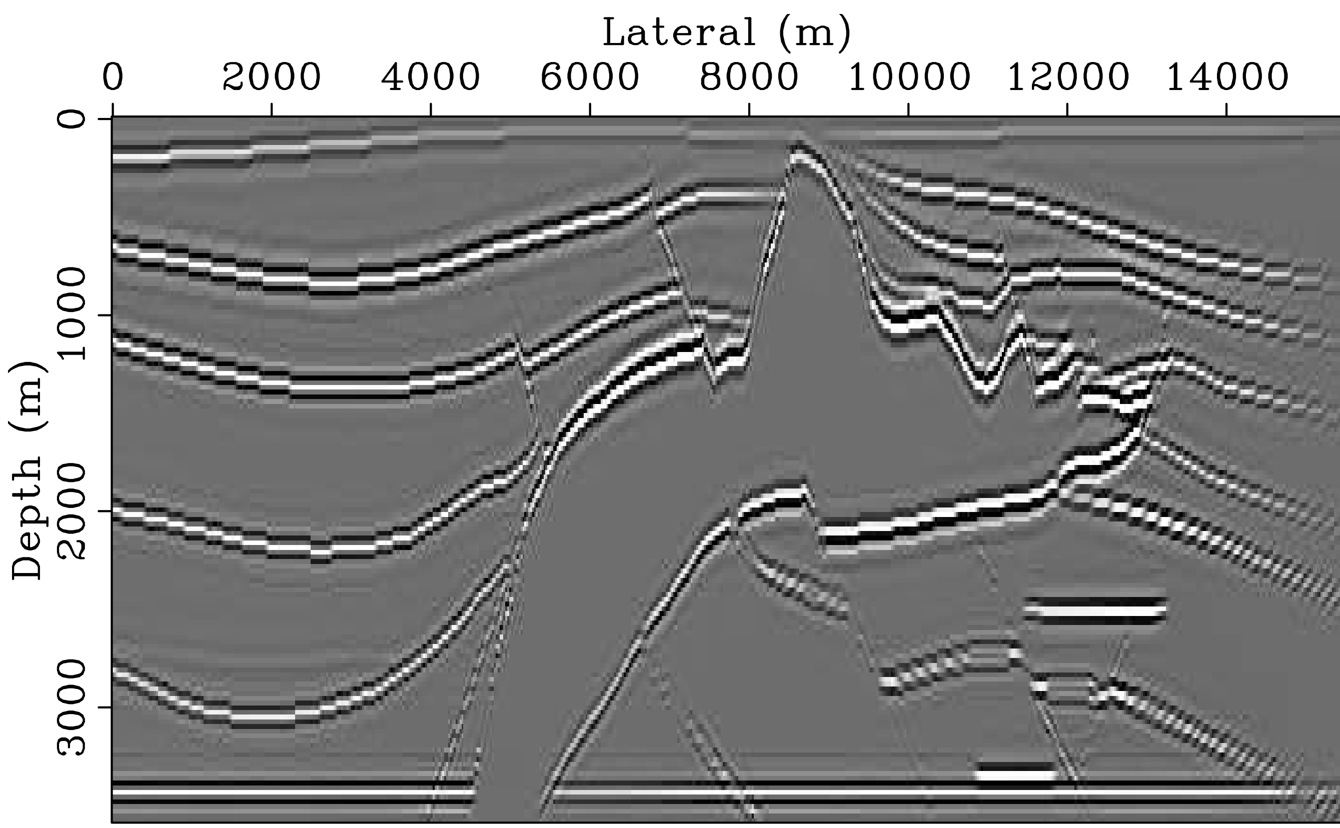

Lateral (m)
0 2000 4000 6000 8000 10000 12000 14000
Depth (m)
0 1000 2000 3000
bandpass−filtered reflectivity
migrated image
reference vector
imaged reference vector



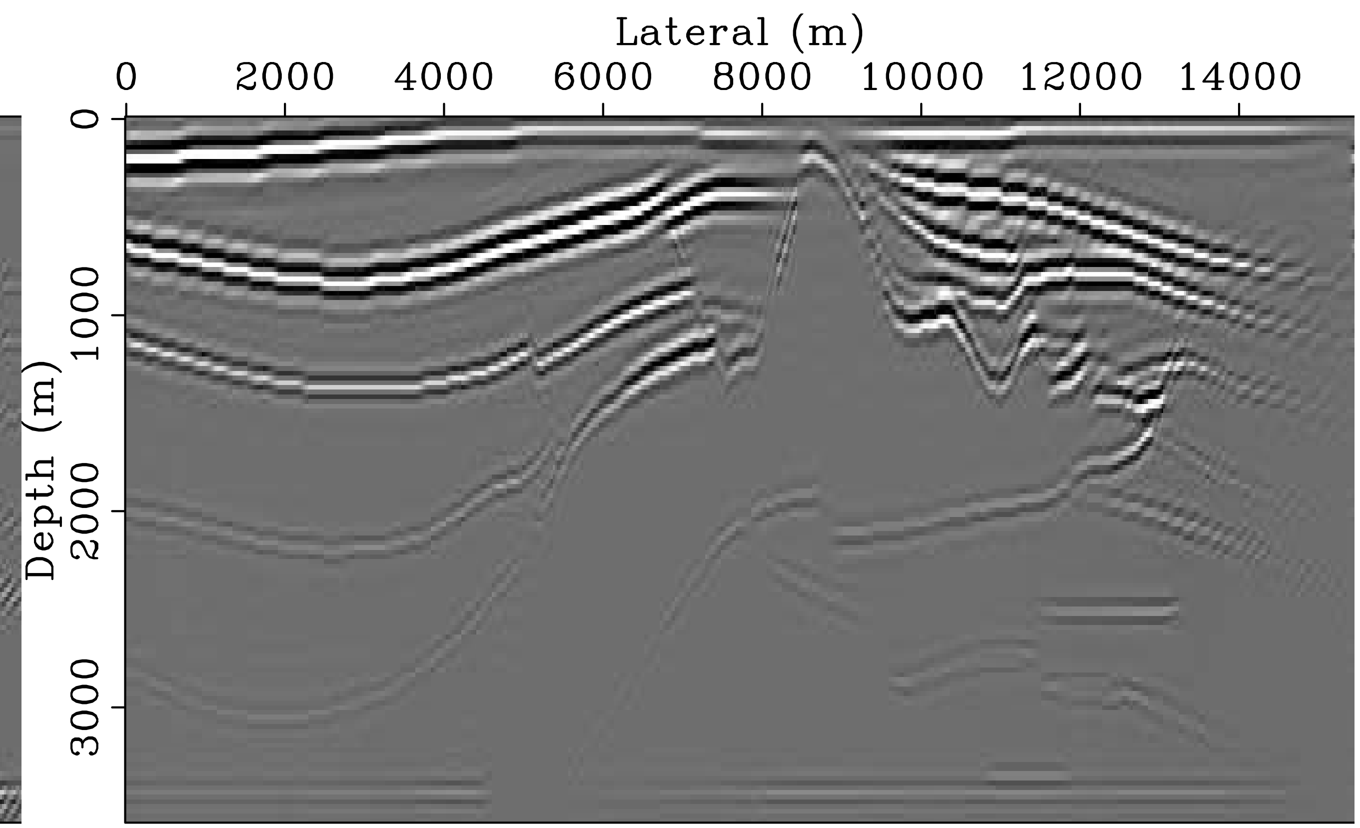
bandpass-filtered reflectivity



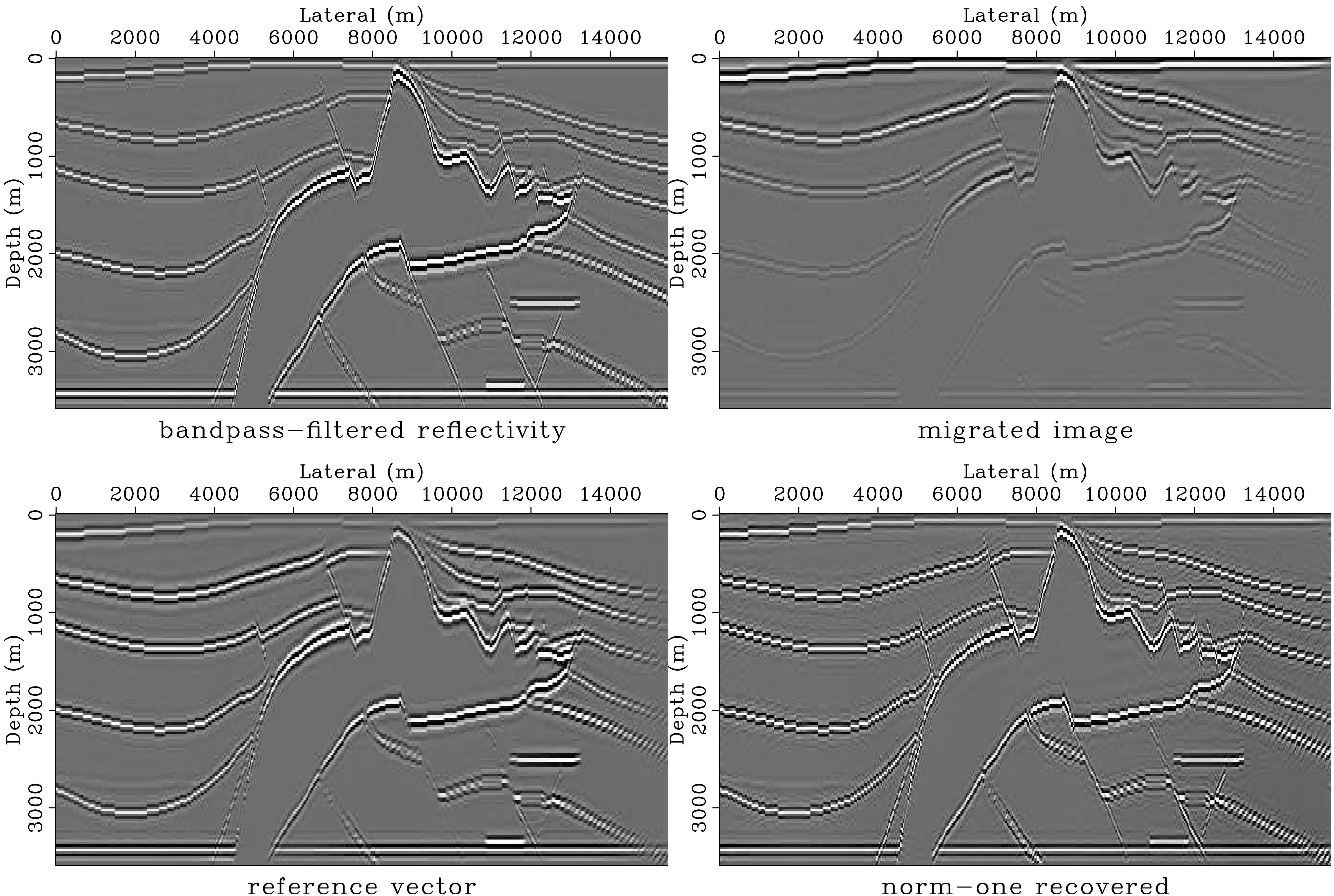
migrated image

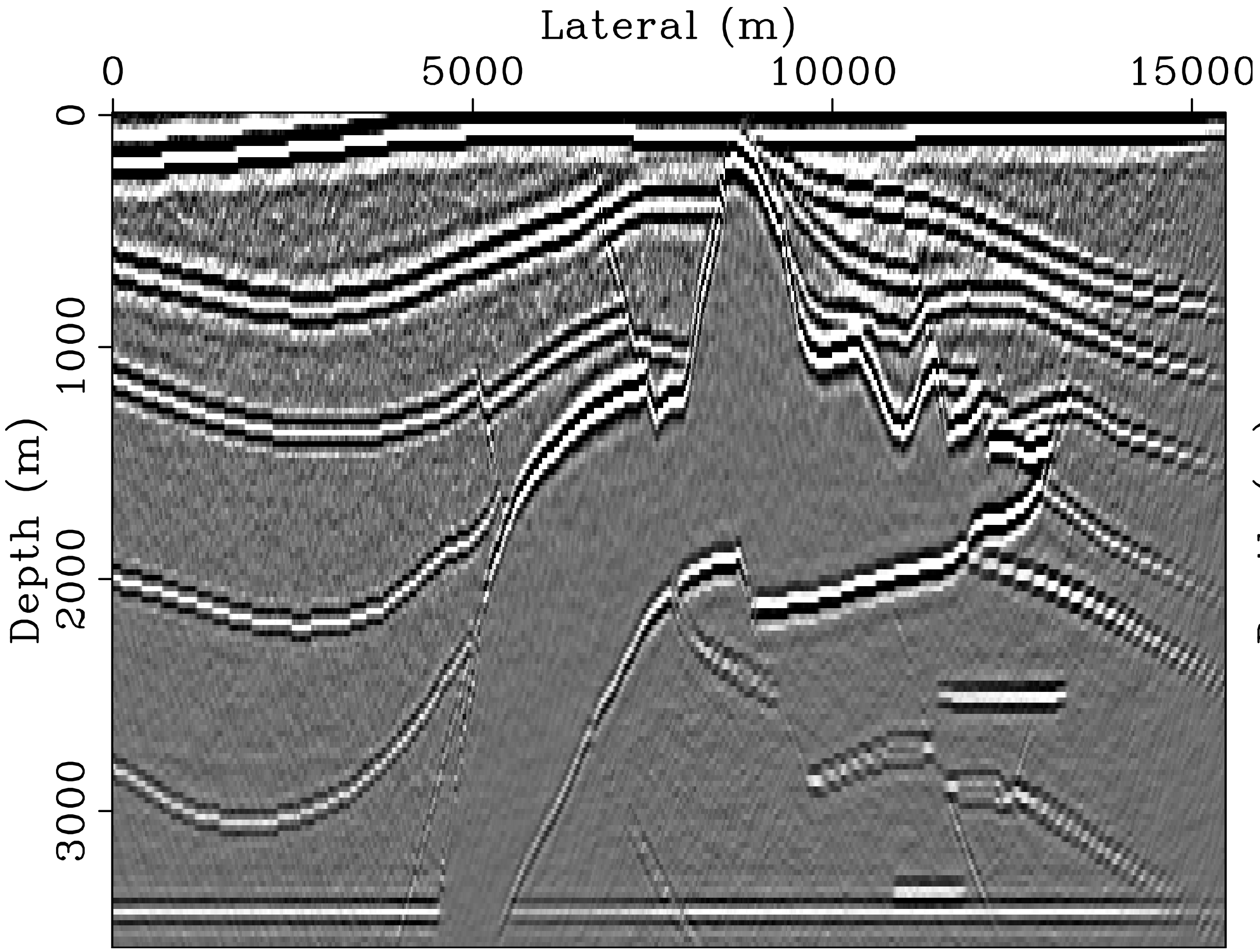


reference vector

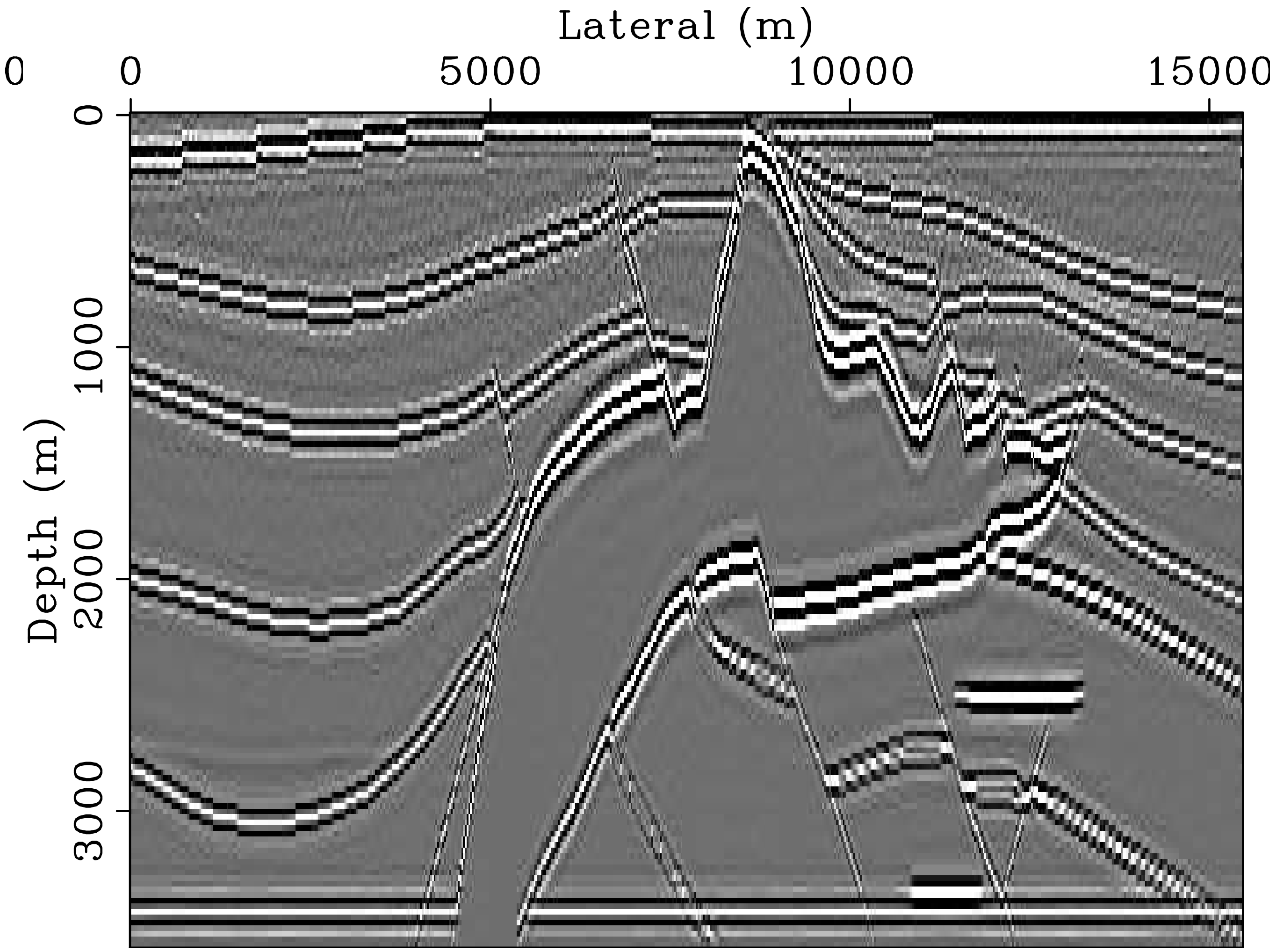


diagonal approximation

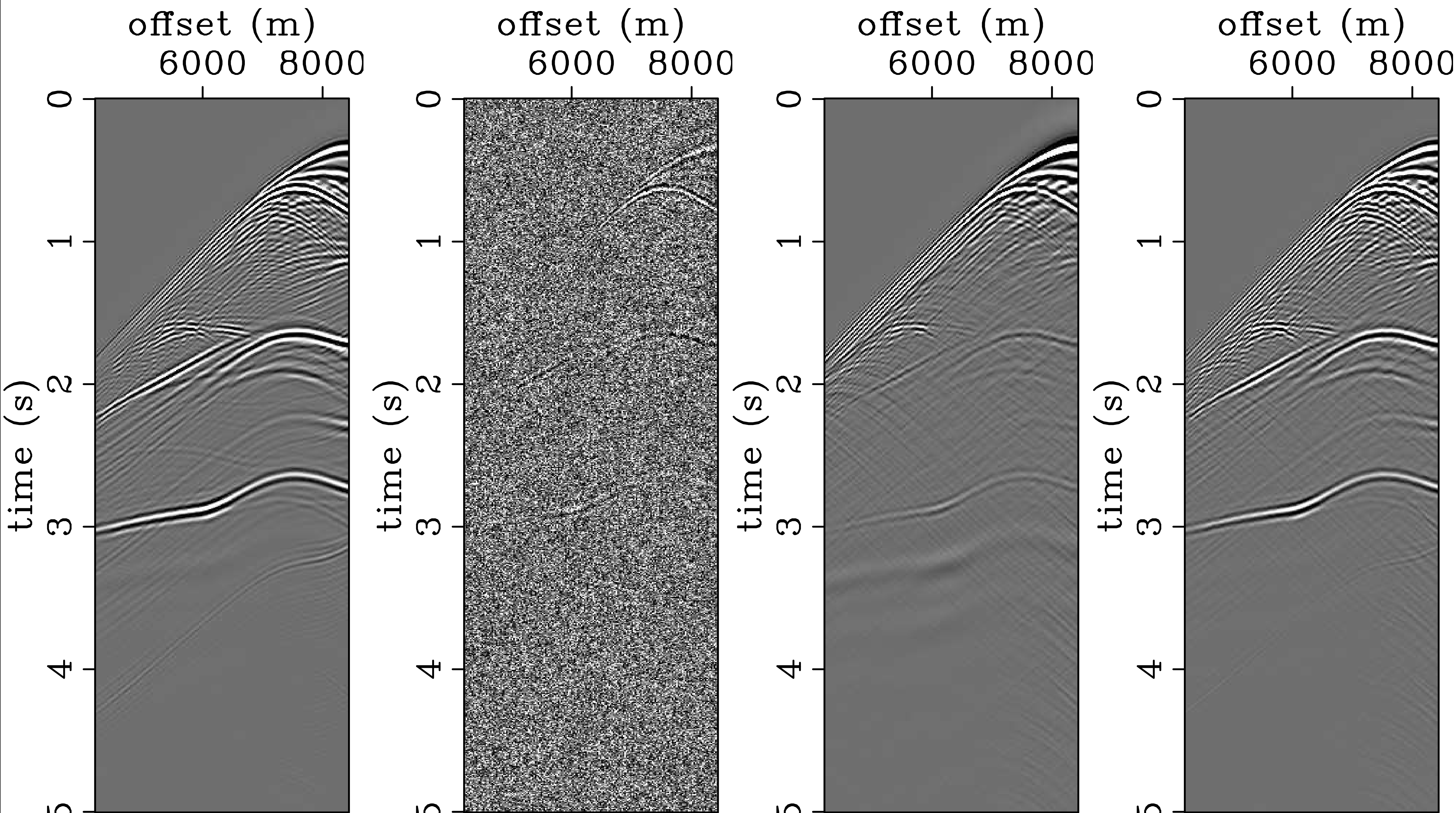

Lateral (m)
0 2000 4000 6000 8000 10000 12000 14000
Depth (m)
0 1000 2000 3000
bandpass−filtered reflectivity
migrated image
reference vector
norm−one recovered



Migrated data



Amplitude-corrected & denoised migrated data



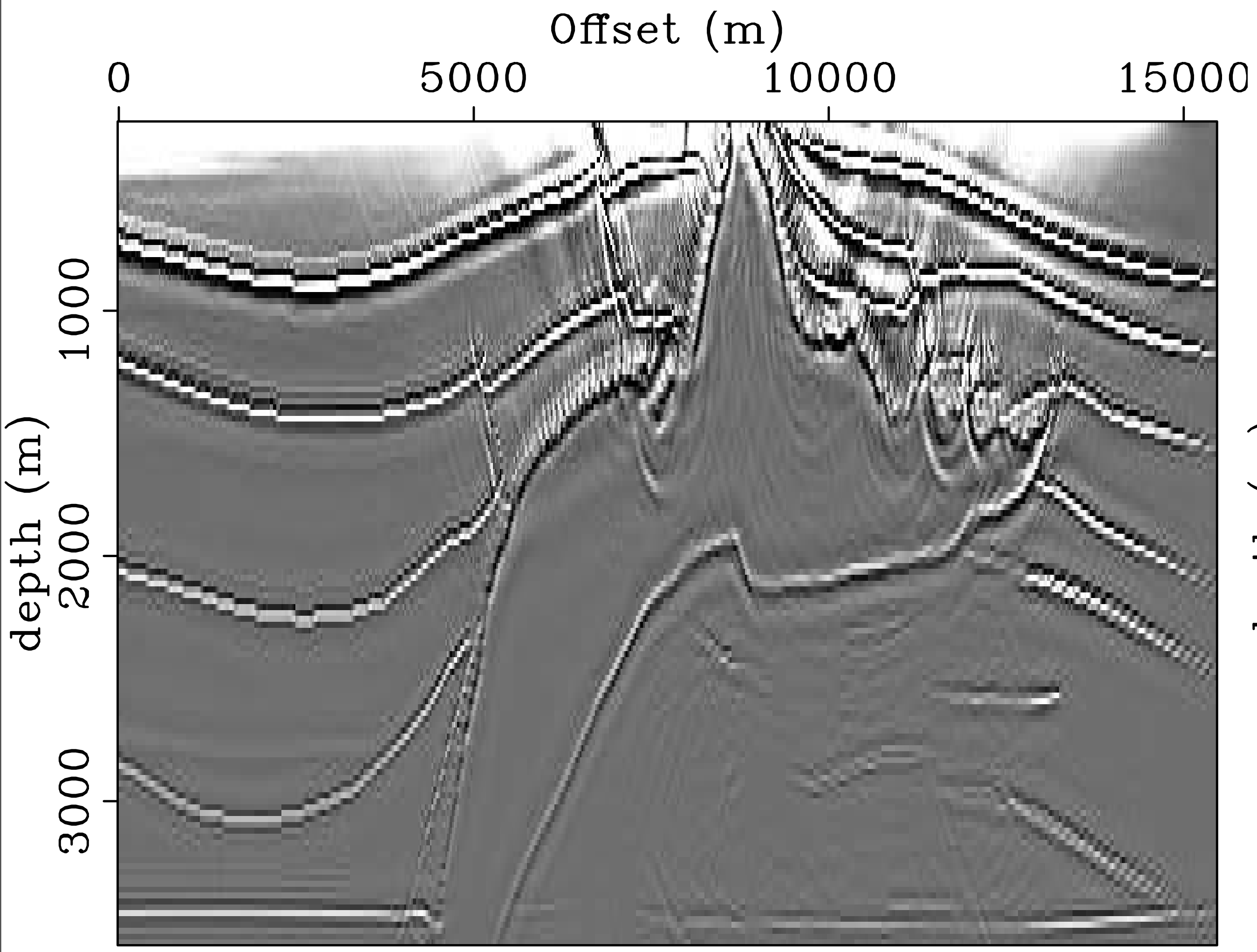
Noise-free data

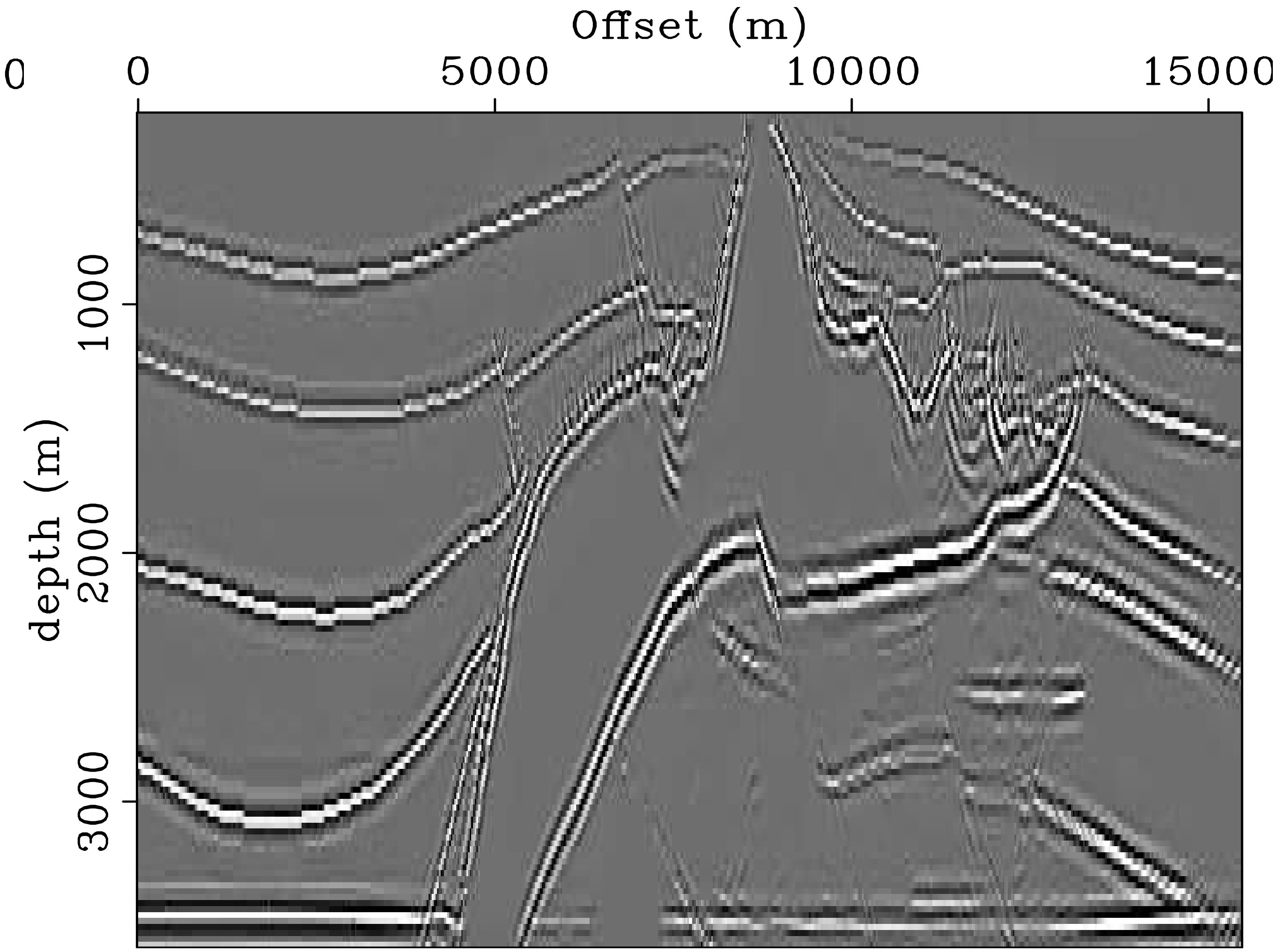
Noisy data (3 dB)

Data from migrated image

Data from amplitude-corrected & denoised migrated image

Nonlinear data





Conclusions

Curvelet-domain scaling

- handles conflicting dips (conormality assumption)
- exploits invariance under the PsDO
- robust w.r.t. noise

Diagonal approximation

- exploits smoothness of the symbol
- uses "neighbor" structure of the curvelet transform

Results on the SEG AA' show

- recovery of amplitudes beneath the Salt
- successful recovery of clutter
- improvement of the continuity

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Acknowledgments

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