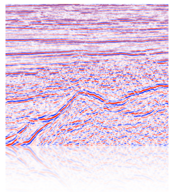
THE UNIVERSITY OF BRITISH COLUMBIA | VANCOUVER

Irregular (sub-)sampling: from aliasing to noise

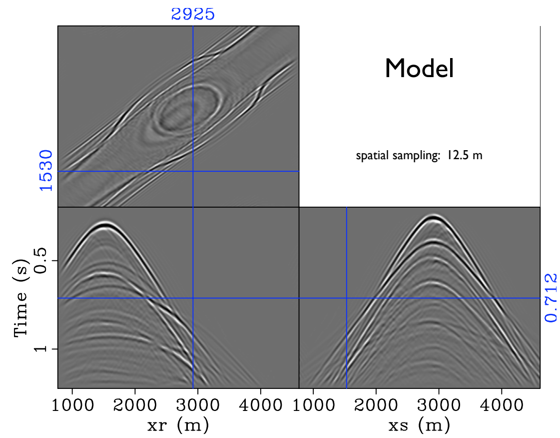
Gilles Hennenfent
ghennenfent@eos.ubc.ca
http://wigner.eos.ubc.ca/~hegilles

Felix J. Herrmann
fherrmann@eos.ubc.ca
http://slim.eos.ubc.ca/

Seismic Laboratory for Imaging & Modeling
Department of Earth & Ocean Sciences
The University of British Columbia

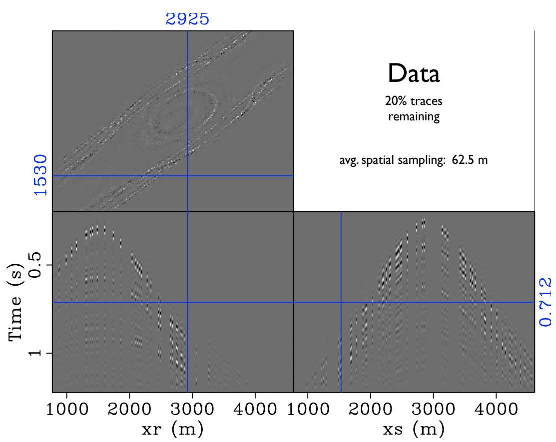


EAGE 69th EAGE Conference & Exhibition
Seismic signal processing & regularization (Lecture room 2)
Wednesday, June 13th, 2007



Model

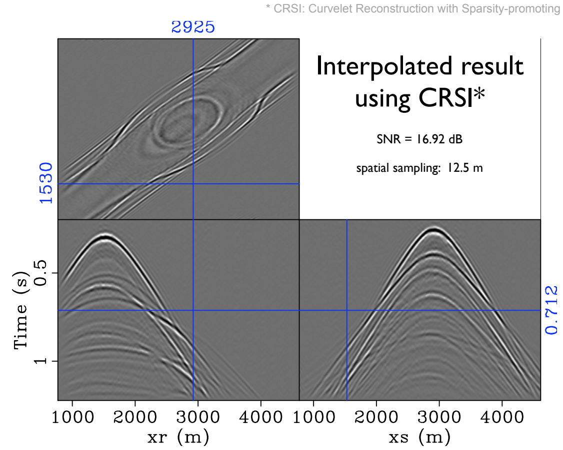
spatial sampling: 12.5 m



Data

20% traces remaining

avg. spatial sampling: 62.5 m

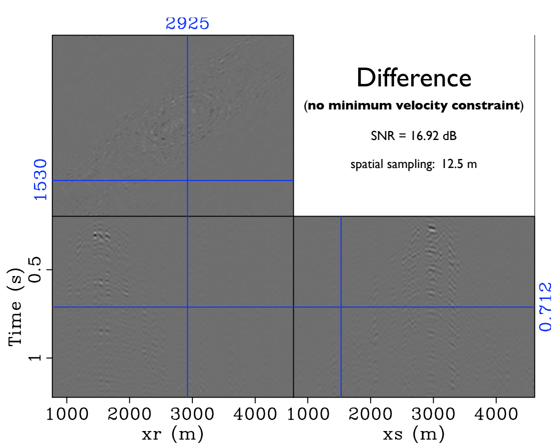


Interpolated result using CRSI*

SNR = 16.92 dB

spatial sampling: 12.5 m

* CRSI: Curvelet Reconstruction with Sparsity-promoting Inversion

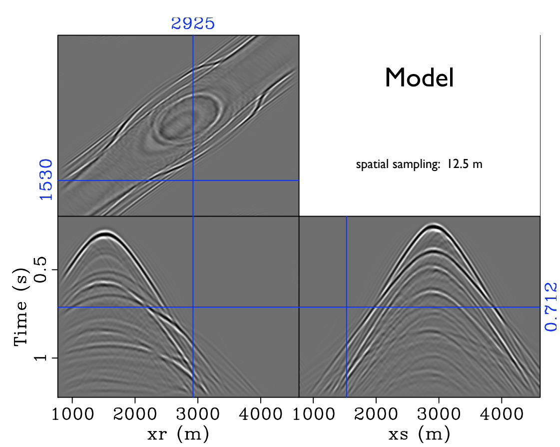


Difference

(no minimum velocity constraint)

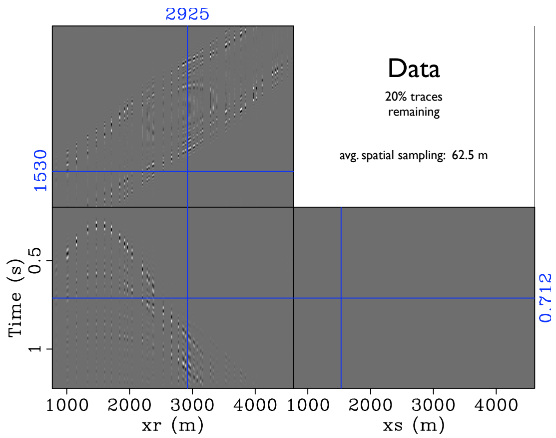
SNR = 16.92 dB

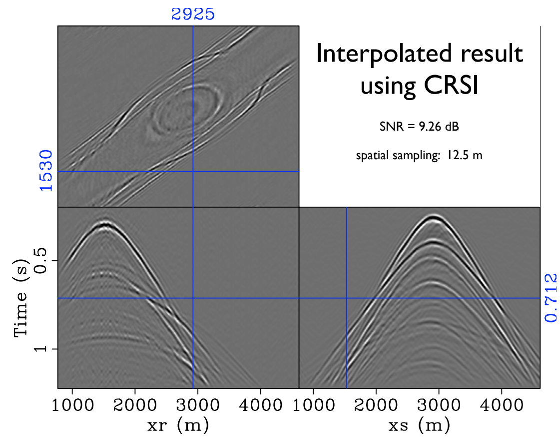
spatial sampling: 12.5 m

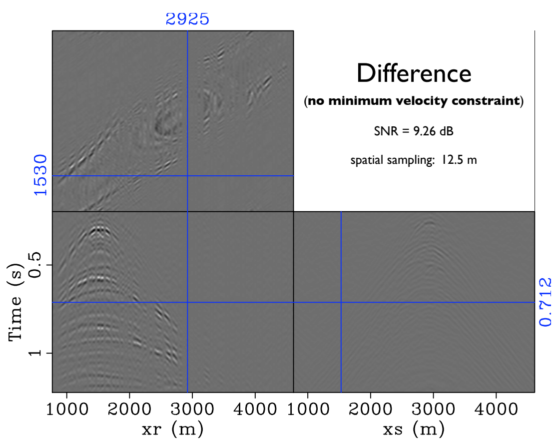


Model

spatial sampling: 12.5 m





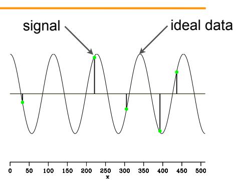


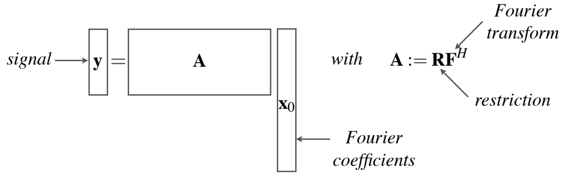
Motivation

- observations
  1. *missing traces at irregular locations* along source and receiver coordinates
     - reconstruction using CRSI: 16.92 dB
  2. *missing receiver at irregular locations* along receiver coordinate
     - reconstruction using CRSI: 9.26 dB
- questions
  - what makes (1) more favorable for reconstruction using CRSI (and possibly other methods) than (2)?
  - from the acquisition geometry, can we predict the success of CRSI (and possibly other methods)?
  - can we design CRSI- (and possibly other methods) friendly acquisition geometries if we know we need to interpolate before further processing?

Experiment

- setting
  - $\mathbf{x}_0$ is of length N
  - $\mathbf{x}_0$ has $k<<N$ nonzero entries
  - ideal data $\mathbf{f}_0 = \mathbf{F}^H\mathbf{x}_0$
  - $\mathbf{R}$ sub-samples $\mathbf{f}_0$ either regularly or irregularly
  - signal $\mathbf{y}$ is a length $n<<N$

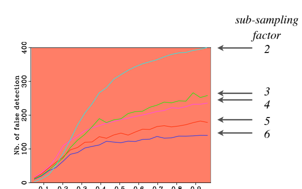




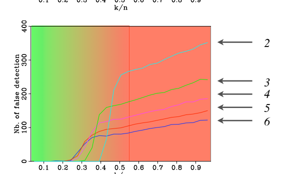
Sparsity-promoting reconstruction

$$\min_{\mathbf{x}} \|\mathbf{x}\|_1 \quad \text{s.t.} \quad \mathbf{A}\mathbf{x} = \mathbf{y}$$

- regular sub-sampling below Nyquist rate



- irregular sub-sampling below Nyquist rate

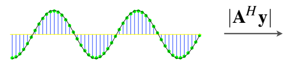


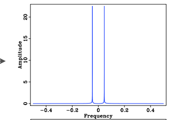
Explanation

- sub-sampling below Nyquist rate introduces *error* due to
  - indeterminacy
  - $\mathbf{A}^H\mathbf{A} \neq \mathbf{I}$
- indeterminacy noise: $\mathbf{A}^H\mathbf{A}\mathbf{x}_0 - ß\mathbf{x}_0 = \mathbf{A}^H\mathbf{y} - ß\mathbf{x}_0$
  - noise characteristics depends upon **R**
  - noise level depends upon under-determinacy of the system, i.e. shape of **A**
- sparsity-promoting methods
  - assume
    - solution sparse
    - **indeterminacy noise non-sparse** ← *not true for regular sub-sampling!!!*

Indeterminacy noise

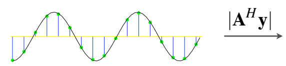
- dense sampling (**R** = **I**)

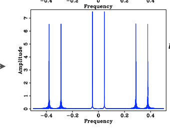




no indeterminacy noise ($\mathbf{A}^H\mathbf{A} = \mathbf{I}$)

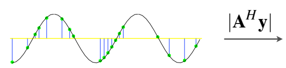
- regular sub-sampling

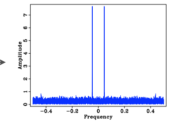




indeterminacy noise strong & coherent (aliases)
sparsity argument fails to separate signal from noise

- irregular sub-sampling





indeterminacy noise weak & broadband
sparsity argument succeed to separate signal from noise

How irregular is irregular enough?

- fine sampling

$$x_n = n \cdot \delta_x \quad \text{for} \quad n \in [\![0, N-1]\!]$$

$$s(x) = \sum_{n=0}^{N-1} \delta(x - x_n) = \mathbf{s}$$

$$|\hat{\mathbf{s}}_k|^2 = \begin{cases} N^2, & k = 0 \\ 0, & \text{otherwise} \end{cases}$$

- coarse sampling

$$x_n = n \cdot \Delta_x \quad \text{for} \quad n \in [\![0, N/\gamma - 1]\!] \quad \text{with } \Delta_x = \gamma\delta_x.$$

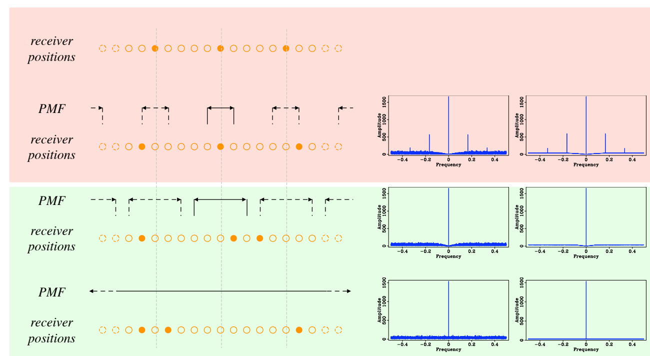
$$|\hat{\mathbf{s}}_k|^2 = \begin{cases} (N/\gamma)^2, & k = 0, \ldots, \frac{(\gamma-1)N}{\gamma} \\ 0, & \text{otherwise} \end{cases}$$

- jittered coarse sampling

$$x_n = n \cdot \Delta_x + \epsilon_n\delta_x \quad \text{for} \quad n \in [\![0, N/\gamma - 1]\!]$$

$$\mathrm{E}[|\hat{\mathbf{s}}_k|^2] = \begin{cases} (1 - |\hat{\mathbf{p}}_k|^2) \cdot N/\gamma + |\hat{\mathbf{p}}_k|^2 \cdot (N/\gamma)^2, & k = 0, \ldots, \frac{(\gamma-1)N}{\gamma} \\ (1 - |\hat{\mathbf{p}}_k|^2) \cdot N/\gamma, & \text{otherwise} \end{cases}$$

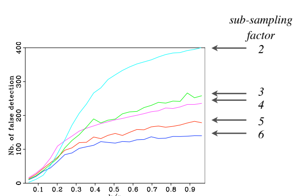
How irregular is irregular enough?



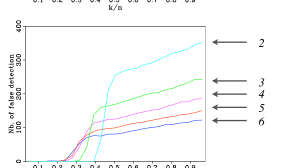
Sparsity-promoting reconstruction

$$\min_{\mathbf{x}} \|\mathbf{x}\|_1 \quad \text{s.t.} \quad \mathbf{A}\mathbf{x} = \mathbf{y}$$

- regular sub-sampling below Nyquist rate



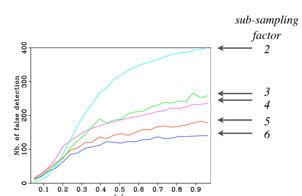
- irregular sub-sampling below Nyquist rate



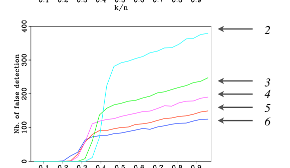
Sparsity-promoting reconstruction

$$\min_{\mathbf{x}} \|\mathbf{x}\|_1 \quad \text{s.t.} \quad \mathbf{A}\mathbf{x} = \mathbf{y}$$

- regular sub-sampling below Nyquist rate

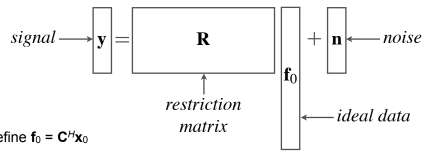


- optimal jittered sub-sampling below Nyquist rate



CRSI overview

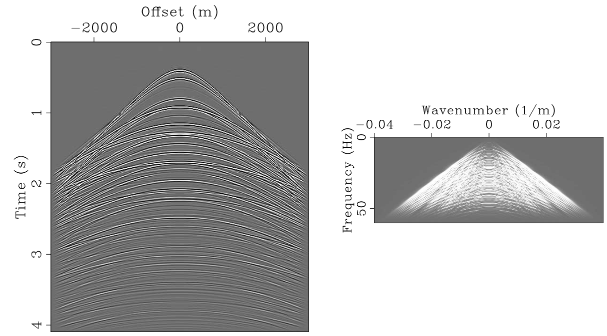
- transform-based method
  - uses curvelets to exploit strong geometrical structure of seismic data volume
- sparsity-promoting algorithm (see also FRSI, ALFT, etc.)
  - linear forward model



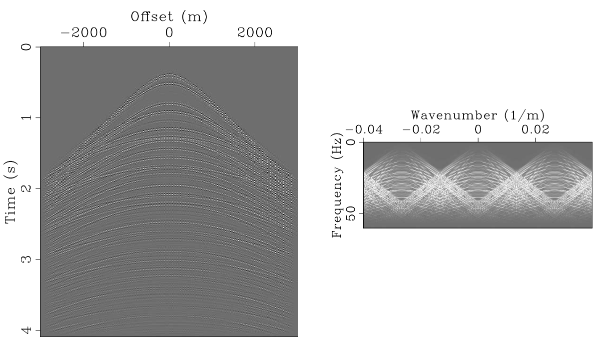
  - define $\mathbf{f}_0 = \mathbf{C}^H\mathbf{x}_0$

$$(P_1)\begin{cases}\tilde{\mathbf{x}} = \arg \overbrace{\min_{\mathbf{x}} \|\mathbf{W}\mathbf{x}\|_1}^{\text{sparsity constraint}} \quad \text{s.t.} \quad \overbrace{\|\mathbf{y}-\mathbf{R}\mathbf{C}^H\mathbf{x}\|_2 \le \varepsilon}^{\text{data misfit}} \\ \tilde{\mathbf{f}} = \mathbf{C}^H\tilde{\mathbf{x}}\end{cases}$$

Model

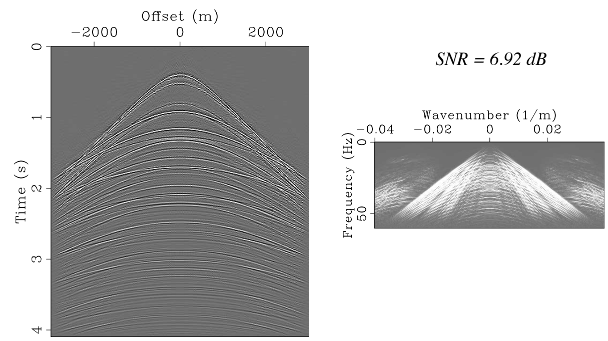


Regular sub-sampling



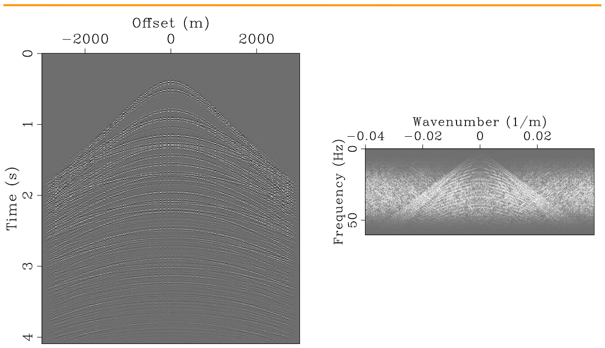
CRSI from regular sub-sampling

SNR = 6.92 dB



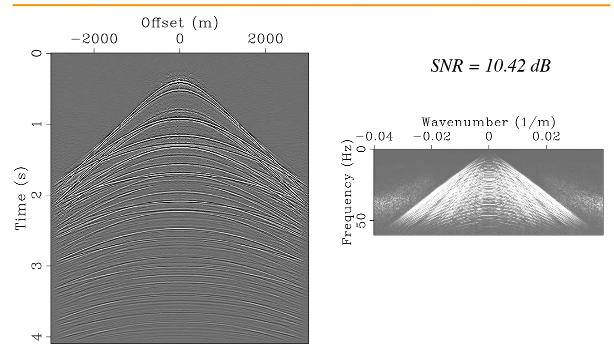
$$\text{SNR} = 20 \times \log_{10}\left(\frac{\|\text{model}\|_2}{\|\text{reconstruction error}\|_2}\right)$$

Optimal jittered sub-sampling

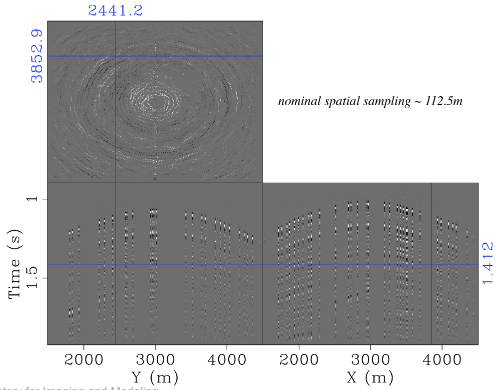


CRSI from optimal jittered sub-sampling

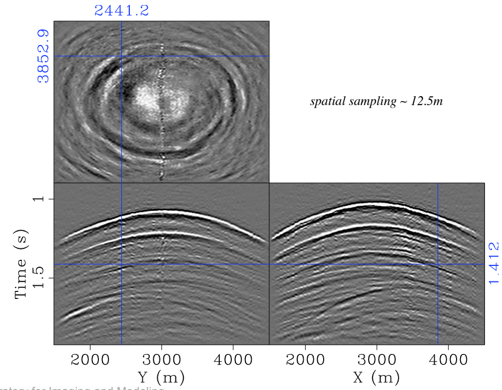
SNR = 10.42 dB



Data



CRSI



Conclusions

- *sparsity* is a powerful property that offers striking benefits for signal reconstruction BUT it is not enough
- in the sparse domain, *interpolation is a denoising problem*
  - remove indeterminacy noise
  - noise level & characteristics depends upon sub-sampling
- *irregular & jittered sub-samplings* turn aliasing into easy-to-remove noise
  - aliases look like signal => sparsity-promoting methods fail

Acknowledgments

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- Norsk Hydro for the real dataset
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