THE UNIVERSITY OF BRITISH COLUMBIA | VANCOUVER

Irregular (sub-)sampling: from aliasing to noise

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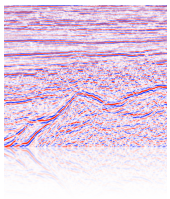
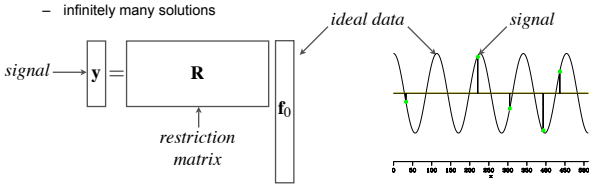


EAGE 69th EAGE Conference & Exhibition
Workshop 6: Noise – The Good, the Bad and the Ugly
Sunday, June 10th, 2007

Interpolation problem

- (severely) underdetermined system of linear equations
  - infinitely many solutions



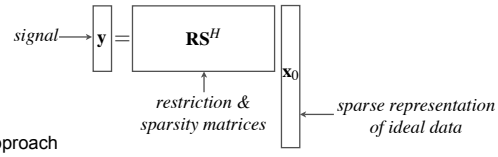
- classical approaches
  - impose smoothness (i.e. quadratic constraint)

$$\tilde{\mathbf{f}} = \arg\min_{\mathbf{f}} \frac{1}{2}\|\mathbf{y} - \mathbf{R}\mathbf{f}\|_2^2 + \lambda\|\mathbf{L}\mathbf{f}\|_2^2$$

* developed by E. Candès, D. Donoho, T. Tao

Compressive sampling theory*

- reformulation
  - explore sparsity of ideal data in some domain



- new approach
  - impose sparsity in some domain

$$\tilde{\mathbf{x}} = \arg\min_{\mathbf{x}} \|\mathbf{x}\|_1 \quad \text{s.t.} \quad \mathbf{R}\mathbf{S}^H\mathbf{x} = \mathbf{y}$$

interpolated data $\longrightarrow \tilde{\mathbf{f}} = \mathbf{S}^H\tilde{\mathbf{x}}$

  - performance
    - **perfect recovery far beyond Shannon sampling theory** under specific conditions

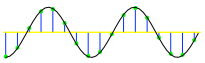
* Simply denoise: wavefield reconstruction via coarse stochastic sampling (Hennenfent and Herrmann, 2007)

Experiment*: superposition of sinusoids

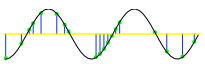
$$\min_{\mathbf{x}} \|\mathbf{x}\|_1 \quad \text{s.t.} \quad \mathbf{A}\mathbf{x} = \mathbf{y} \quad \text{with} \quad \mathbf{A} := \mathbf{R}\mathbf{S}^T$$

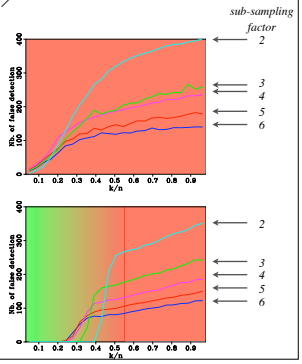
$\mathbf{S}^T$ ← discrete cosine transform
$\mathbf{R}$ ← restriction

- regular sub-sampling below Nyquist rate
  - example



- irregular sub-sampling below Nyquist rate
  - example





Explanation

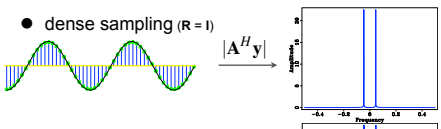
- sub-sampling below Nyquist rate introduces *error* due to
  - indeterminacy
  - $\mathbf{A}^T\mathbf{A} \neq \mathbf{I}$
- indeterminacy noise: $\mathbf{A}^T\mathbf{A}\mathbf{x}_0$-ß$\mathbf{x}_0$ = $\mathbf{A}^T$**y**-ß$\mathbf{x}_0$
  - noise characteristics depends upon **R**
  - noise level depends upon under-determinacy of the system, i.e. shape of **A**
- sparsity-promoting methods
  - assume
    - solution sparse
    - **indeterminacy noise non-sparse** ← *not true for regular sub-sampling!!!*

* Simply denoise: wavefield reconstruction via coarse stochastic sampling (Hennenfent and Herrmann, 2007)

Example*

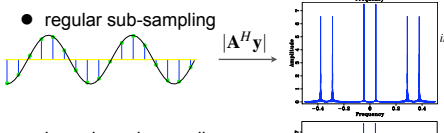
S defined as the Fourier transform

- dense sampling (**R** = **I**)



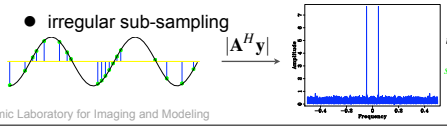
no indeterminacy noise (**A**^{T}**A** = **I**)

- regular sub-sampling



indeterminacy noise strong & coherent (aliases)
sparsity argument fails to separate signal from noise

- irregular sub-sampling



indeterminacy noise weak & broadband
sparsity argument succeed to separate signal from noise

How irregular is irregular enough?

- Wednesday, June 13th
  - Seismic Signal Processing and Regularisation (Lecture room 2, 13:30)

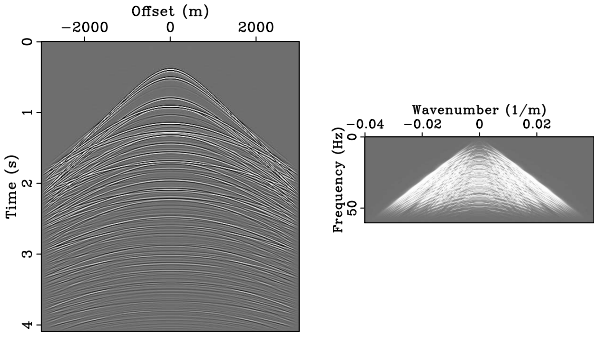
Application to seismic data interpolation

- Curvelet Reconstruction with Sparsity-promoting Inversion (CRSI)
  - Monday, June 11th
    - workshop 8: Curvelets, Contourlets, Seislets, ... in seismic data processing - where are we and where are we going? (Lecture room 6, 11:20)

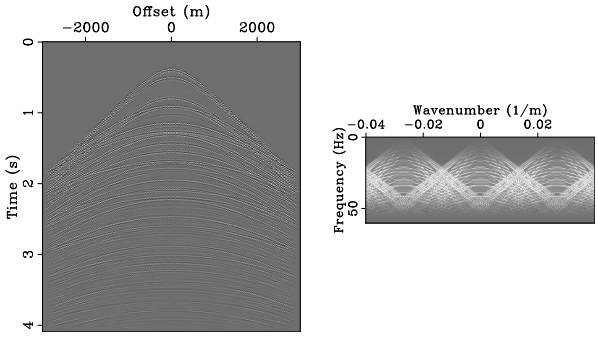
$$(P_1)\begin{cases} \tilde{\mathbf{x}} = \arg \overbrace{\min_{\mathbf{x}} \|\mathbf{W}\mathbf{x}\|_1}^{\text{sparsity constraint}} \quad \text{s.t.} \quad \overbrace{\|\mathbf{y} - \mathbf{R}\mathbf{C}^H\mathbf{x}\|_2}^{\text{data misfit}} \leq \varepsilon \\ \tilde{\mathbf{f}} = \mathbf{C}^H\tilde{\mathbf{x}} \end{cases}$$

- Surface-related multiple prediction from incomplete data
  - Thursday, June 14th
    - multiple attenuation (Lecture room 2, 9:20)

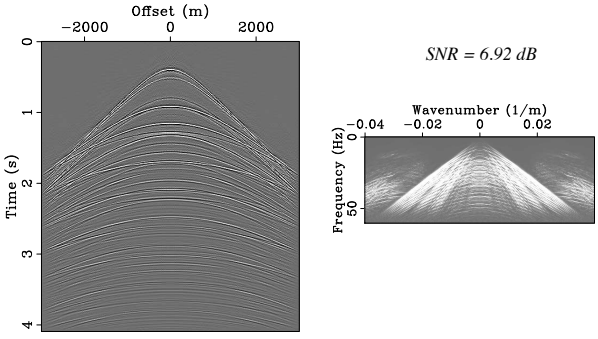
Model



Regular sub-sampling

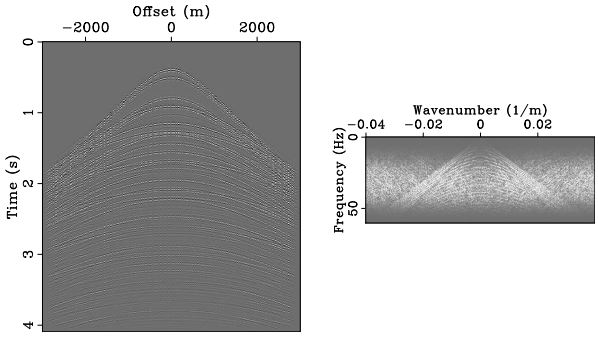


CRSI from regular sub-sampling

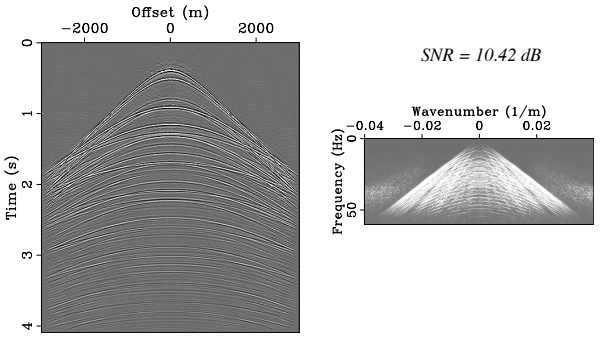


$$\text{SNR} = 20 \times \log_{10}\left(\frac{\|\text{model}\|_2}{\|\text{reconstruction error}\|_2}\right)$$

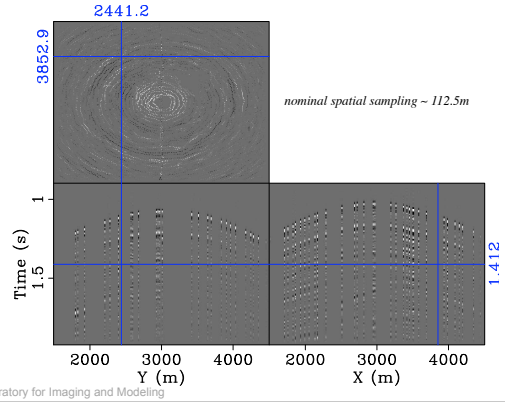
Irregular sub-sampling



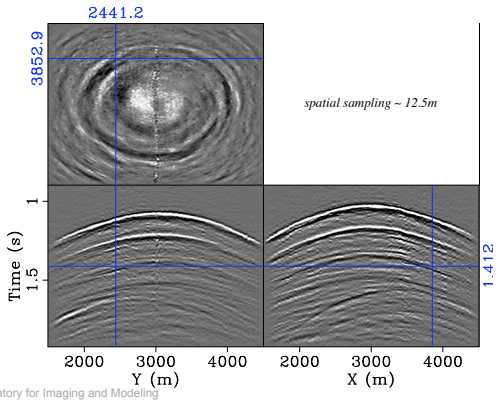
CRSI from irregular sub-sampling



Data



CRSI



Conclusions

- *sparsity* is a powerful property that offers striking benefits for signal reconstruction BUT it is not enough
- in the sparse domain, *interpolation is a denoising problem*
  - remove indeterminacy noise
  - noise level & characteristics depends upon sub-sampling
- *irregular sub-sampling* turns aliasing into easy-to-remove noise
  - aliases look like signal => sparsity-promoting methods fail

Acknowledgments

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