Irregular sampling: from aliasing to noise

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ABSTRACT

Seismic data is often irregularly and/or sparsely sampled along spatial coordinates. We show that these acquisition geometries are not necessarily a source of adversity in order to accurately reconstruct adequately-sampled data. We use two examples to illustrate that it may actually be better than equivalent regularly subsampled data. This comment was already made in earlier works by other authors. We explain this behavior by two key observations. Firstly, a noise-free underdetermined problem can be seen as a noisy well-determined problem. Secondly, regularly subsampling creates strong coherent acquisition noise (aliasing) difficult to remove unlike the noise created by irregularly subsampling that is typically weaker and Gaussian-like.

Adequate sampling of seismic data is traditionally understood as evenly-distributed time and space measurements of the reflected wavefield. Moreover, the sampling rate along each axis must be equal to or above twice the highest frequency/wavenumber of the continuous signal being discretized (Shannon/Nyquist sampling theorem). In practice, however, seismic data is often irregularly and/or sparsely sampled along spatial coordinates, which is generally considered as a nuisance since it breaks one or both previously-stated conditions of adequate sampling. It turns out that these acquisition geometries are not necessarily a source of adversity to accurately reconstruct adequately-sampled data. This new theory, developed in the information theory community, is referred to in the literature by the terms "compressed sensing" or "compressive sampling" (see e.g. Donoho, 2006, and the compressed sensing resources of the Rice University DSP group at www.dsp.ece.rice.edu/cs for more references).

Undersampled seismic data: Fig. 1 depicts two sub-Nyquist spatial samplings (time is adequately sampled) of some adequately-sampled synthetic seismic data (Fig. 1(a)) and their effect in the f-k domain. When the data of Fig. 1(a) is regularly subsampled by a factor of two (Fig. 1(b)), the well-known phenomenon of aliasing occurs. The spectrum of the adequately-sampled data (Fig. 1(d)) is corrupted by strong aliases beyond 30 Hz (Fig. 1(e)) leaving the low frequencies unaltered. In the case of irregular subsampling by a factor of two (Fig. 1(c)), the spectrum is globally altered by some weak noise (Fig. 1(f)).

From an interpolation perspective, the data of Fig. 1(b) is typically perceived as easier to handle than the data of Fig. 1(c) and a variety of algorithms are available (see e.g. Gulunay, 2003; Abma and Kabir, 2005, for an overview). The key idea is to use the non-aliased lower frequencies to de-alias higher frequencies (Spitz, 1991; Gulunay, 2003), which is not immediately possible in the case of irregularly sampled data since the whole spectrum is degraded. Besides, as rightly pointed out by Abma and Kabir (2005), most interpolation algorithms assume regular spatial sampling (e.g. f-k, t-x, and f-x prediction filter algorithms).

Inspired by Donoho et al. (2006), we propose to look at the interpolation problem as a denoising problem or, in other words, to turn a noise-free underdetermined problem into a noisy well-determined problem (Donoho's slogan). Indeed, Figs. 1(e) and 1(f) are noisy versions of Fig. 1(d). In the case of regular subsampling, the noise is highly structured and has strong amplitudes, which makes it difficult to remove. Conversely, irregular subsampling generates lower amplitude noise that looks Gaussian, which is easier to remove. It thus seems that irregular subsampling may actually help to recover more accurately seismic data in a much wider frequency band than would be possible using equivalent regular subsampling or any subsampling that generates structured noise. This comment, already made by other authors in earlier works, is illustrated in the next section using the Curvelet Recovery with Sparsity-Promoting Inversion (CRSI) algorithm (Herrmann and Hennenfent, 2007).

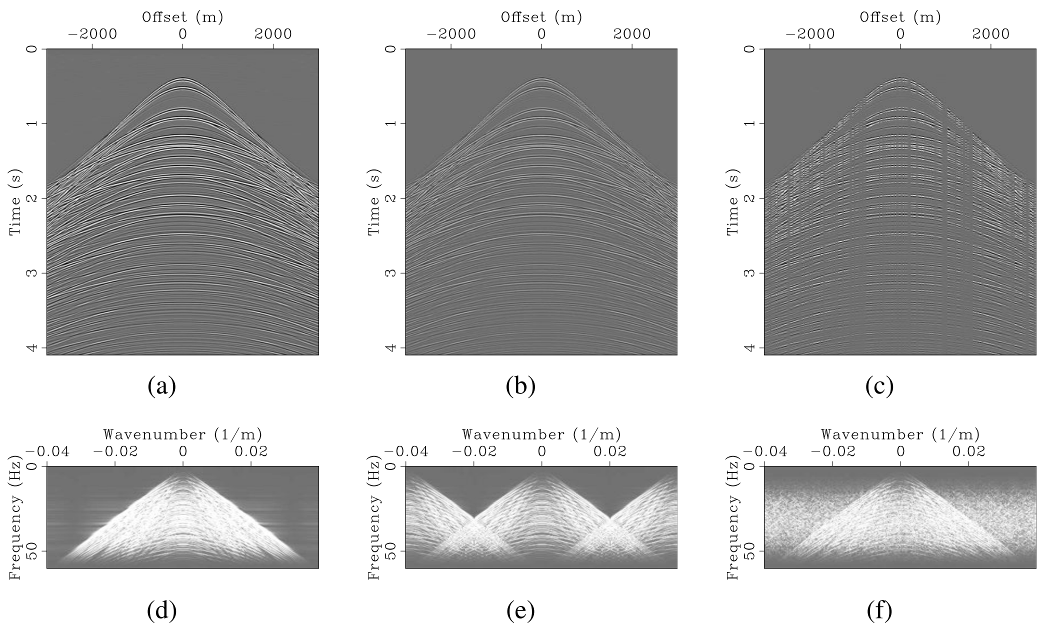

Figure 1: Seismic data with different spatial samplings (a,b,c) and their corresponding amplitude spectrum (d,e,f).

Reconstruction using CRSI: Among the many applications of multiscale (and multidirectional) transforms (e.g. wavelets, seislets, curvelets, and other *lets) denoising is probably the most popular. The crucial point is to find a transform which can represent the noise-free signal with few atoms (this argument is called sparsity) and at the same time poorly represents the contaminating noise. In such a transform domain the signal's energy is concentrated in few significant coefficients whereas the noise's energy is spread across the transform domain. Hence, it is easy to discriminate between noise and signal and to recover the noise-free signal (see e.g. Mallat, 1998, for more details). In fact, the sparser the representation of the noise-free signal in the transform domain, the easier the separation.

CRSI uses curvelets, which arguably provide the sparsest non-adaptive representation for seismic data (see e.g. Herrmann and Hennenfent, 2007, and other curvelet-related contributions to the proceedings of this meeting), to denoise (read: interpolate) seismic data with missing traces and thus separate the seismic signal from the acquisition noise.

Synthetic data example: We consider two different sparse acquisition geometries for a 2D survey (one 3D shot with these acquisition patterns would lead to similar results). Adequately-sampled data is reconstructed using 3D CRSI on the prestack volume. No minimum velocity constraint and no frequency restriction are used in CRSI to better single out the effect of the acquisition geometry on the reconstruction. Fig. 2(a) shows the prestack volume where every shot was recorded by a different subset of receivers (20% of the total number of receivers), Fig. 2(c) the reconstructed data (signal-to-noise ratio (SNR) = 16.92 dB), and Fig. 2(e) the difference cube. In comparison, Fig. 2(b) shows the prestack volume where the same subset of receivers (also 20% of the total number of receivers) recorded all the shots, Fig. 2(d) the reconstructed data (SNR = 9.26 dB), and Fig. 2(f) the difference cube. This experiment clearly illustrates our comment that irregular sampling favors in fact the recovery of seismic data over sampling that exhibits structure for the same amount of data collected.

Real data example: We examine the Oseberg 3D walkaway VSP survey to further illustrate the benefit of irregular sampling. The data (Fig. 3(a)) only contains 10% of the traces we are interested in. The irregular sampling geometry comes from the fact that the source vessel sailed in circles. The reconstructed data using 3D CRSI is presented in Fig. 3(b). It turns out that, even in this vastly underdetermined problem, the acquisition noise generated by the irregular sampling geometry can be removed and the data fairly accurately recovered.

Conclusions: We proposed to look at the seismic data interpolation problem from a denoising perspective. From this standpoint, we noted that regular subsampling geometries generate acquisition noise more difficult to remove than irregular subsampling geometries do for the same amount of data collected. It thus seems that irregular subsampling may actually help to recover seismic data in a much wider frequency band than would be possible using equivalent regular subsampling or any subsampling that generates structured acquisition noise. We believe this new insight may lead to new acquisition strategies. On land, for example, a regular sampling may lead to aliased ground-roll that needs to be interpolated to a finer grid in order to be removed. Our observations suggest one should irregularly sample on the finer grid instead. This could lead to a better ground-roll interpolation and hence ground-roll removal.

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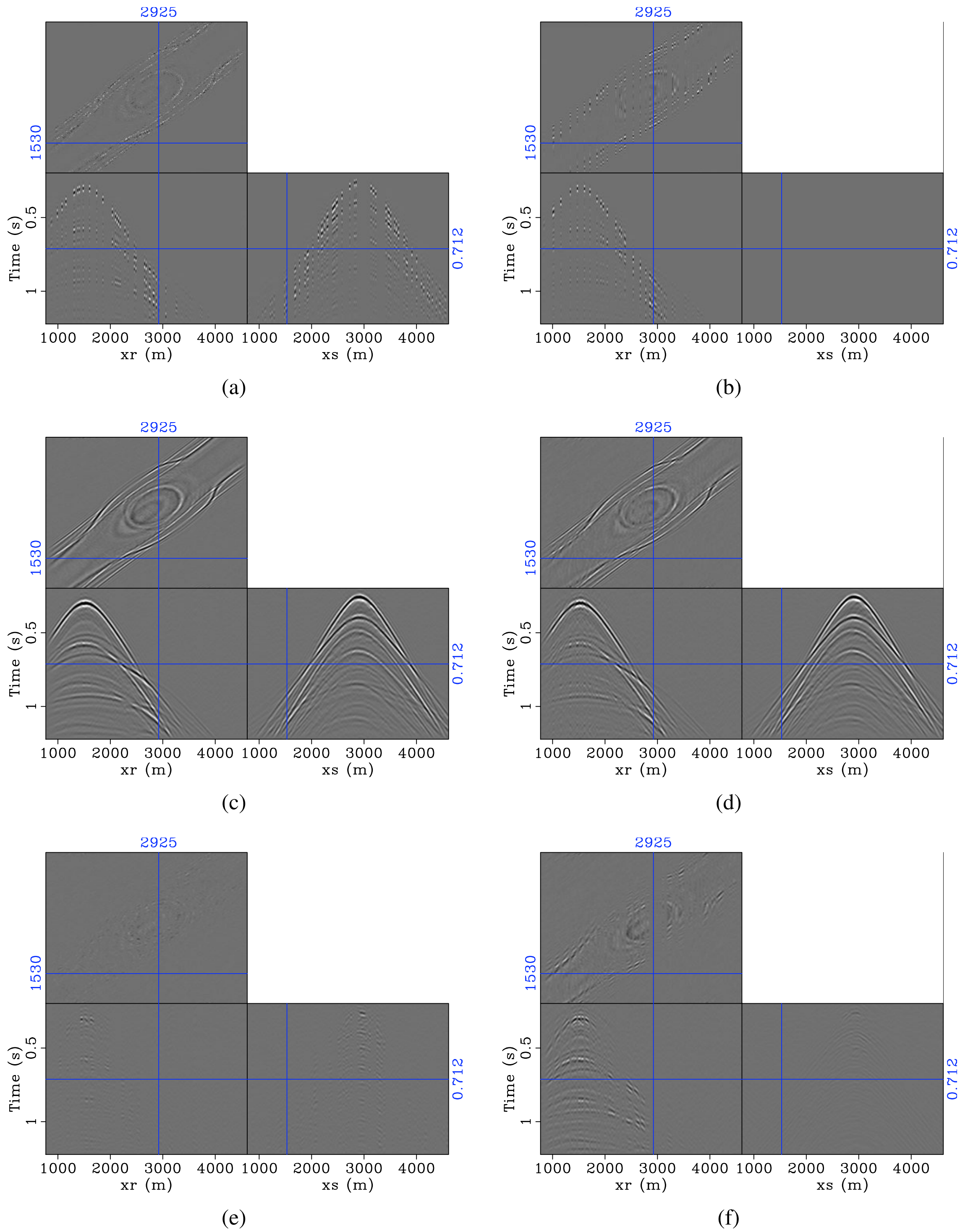

Figure 2: 2D prestack data volumes with different acquisition geometries (a,b), their reconstruction using 3D CRSI (c,d), and the corresponding difference cubes (e,f).

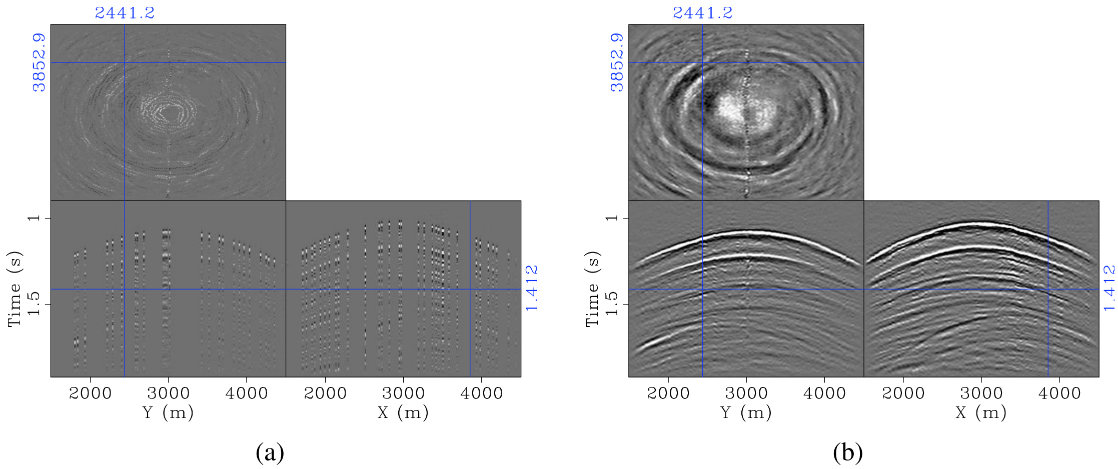

Figure 3: Oseberg 3D walkaway VSP survey and its reconstruction using 3D CRSI.

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