THE UNIVERSITY OF BRITISH COLUMBIA | VANCOUVER

Curvelet Reconstruction with Sparsity-promoting Inversion

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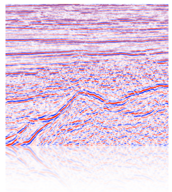


EAGE 69th EAGE Conference & Exhibition
Workshop 8: Curvelets, Contourlets, Seislets, ... in Seismic Data Processing - Where are We and Where Are We Going?
Monday, June 11th, 2007

Motivation

- field data are often spatially sparse and/or irregular
  - spatial aliasing degrades
    - stack
    - multiple prediction & attenuation
  - acquisition irregularities
    - not well handled by most commonly-used multi-trace algorithms
    - turn into image artifacts
- seismic data regularization
  - necessary for many applications
  - impacts
    - quality of 3D algorithms
    - acquisition decision & survey costs

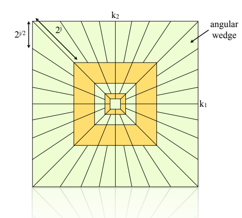
Approach

- view seismic data from a geometrical perspective
  - high dimensional (typ. 5-D for a 3-D survey)
  - strong geometrical structure (spatio-temporal sampling of reflected wavefield)

Representations for seismic data

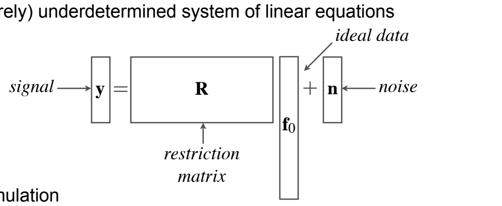
Transform	Underlying assumption
FK	plane waves
linear/parabolic Radon transform	linear/parabolic events
wavelet transform	point-like events (1D singularities)
curvelet transform	**curve-like events (2D singularities)**

- curvelet transform
  - **multi-scale:** tiling of the FK domain into dyadic coronae
  - **multi-directional:** coronae sub-partitioned into angular wedges, # of angle doubles every other scale
  - **anisotropic:** parabolic scaling principle
  - **local**

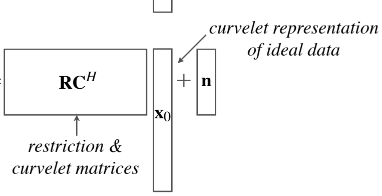


Interpolation problem

- (severely) underdetermined system of linear equations

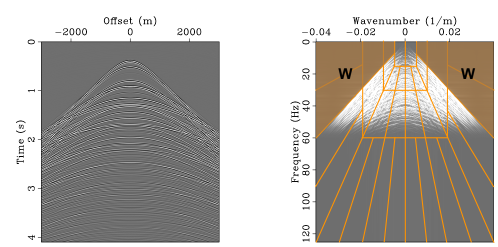


- reformulation



Curvelet Reconstruction with Sparsity-promoting Inversion (CRSI)

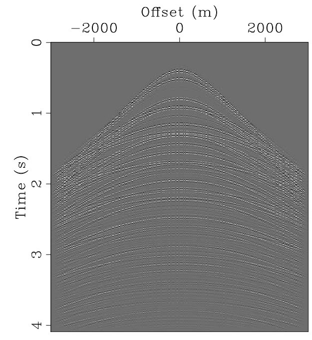
$$(P_1)\begin{cases}\tilde{\mathbf{x}} = \arg\overbrace{\min_{\mathbf{x}} \|\mathbf{W}\mathbf{x}\|_1}^{\text{sparsity constraint}} \quad \text{s.t.} \quad \overbrace{\|\mathbf{y}-\mathbf{R}\mathbf{C}^H\mathbf{x}\|_2 \le \varepsilon}^{\text{data misfit}} \\ \tilde{\mathbf{f}} = \mathbf{C}^H\tilde{\mathbf{x}}\end{cases}$$

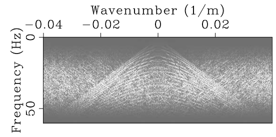


Successes

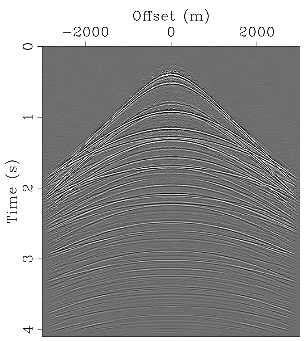
- going from l_1-norm to weighted l_1-norm to impose maximum dip constraint in the curvelet domain
  - equivalent to a minimum velocity constraint

Input data (67% traces missing)

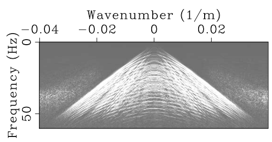




CRSI – sparsity only

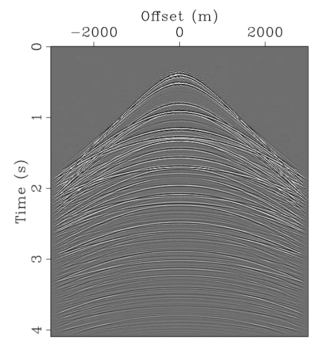


SNR = 10.42 dB

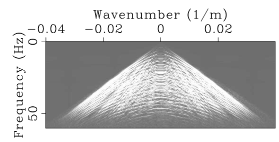


l_1-norm on curvelet representation

CRSI – weighted sparsity



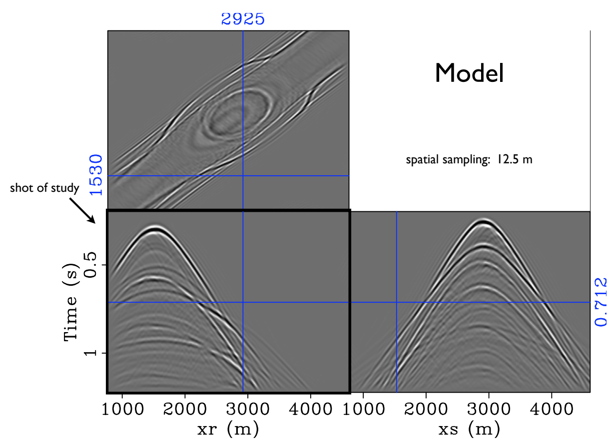
SNR = 14.44 dB

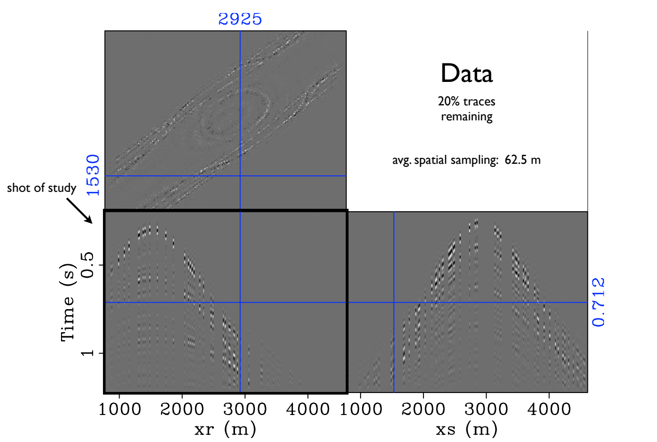


weighted l_1-norm on curvelet representation

Successes

- going from l_1-norm to weighted l_1-norm to impose maximum dip constraint in the curvelet domain
  - equivalent to a minimum velocity constraint
- increasing dimensionality of the problem
  - higher-dimensional structure exploited
  - significant uplift from image to volume interpolation

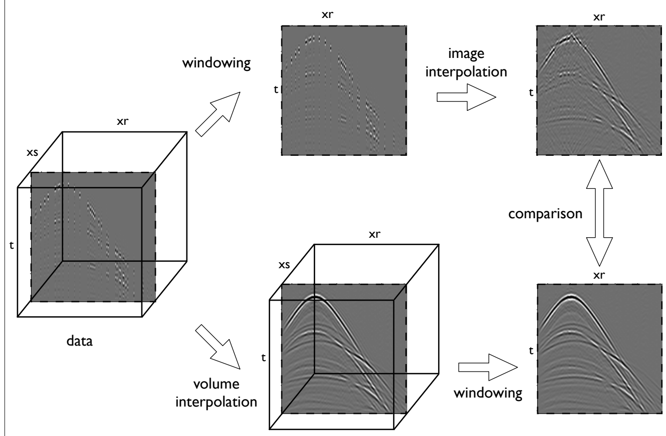


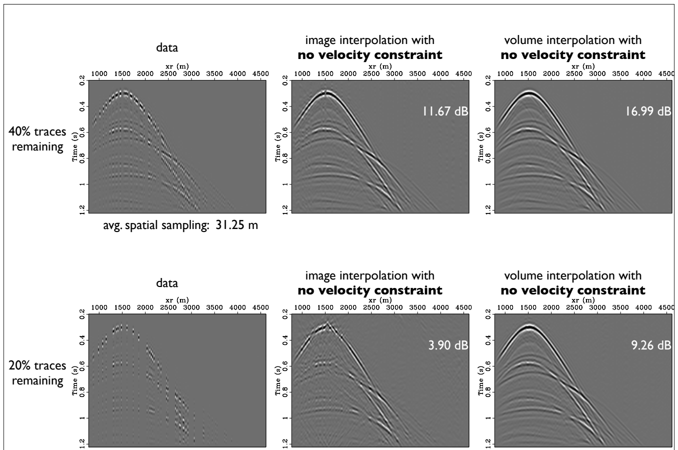


Data

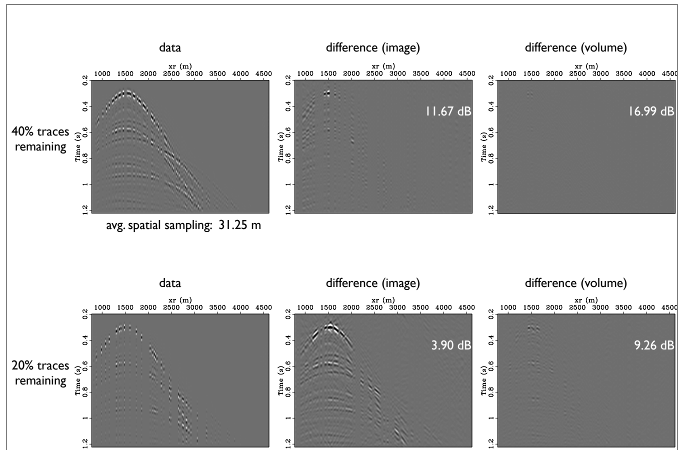
20% traces remaining

avg. spatial sampling: 62.5 m





$$\mathrm{SNR} = 20 \times \log_{10}\left(\frac{\|\mathrm{model}\|_2}{\|\mathrm{reconstruction\ error}\|_2}\right)$$



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- increasing dimensionality of the problem
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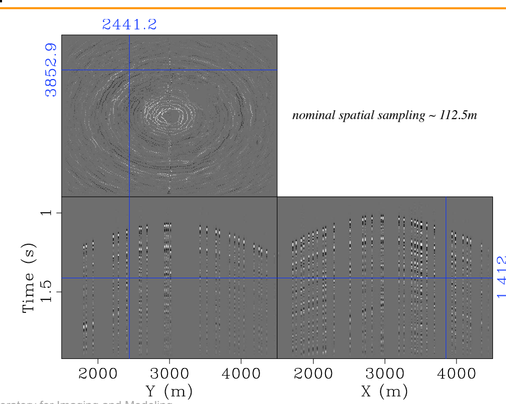
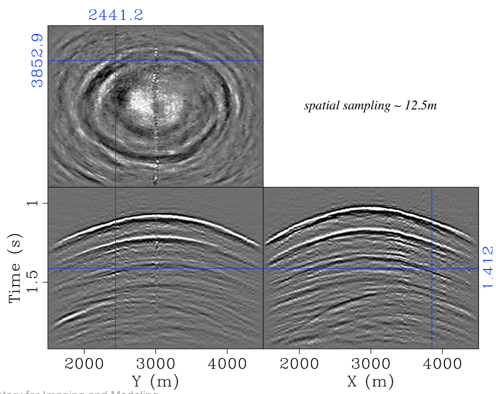
  - significant uplift from image to volume interpolation
- interpolating irregularly sub-sampled data
  - Wednesday, June 13th
    - Seismic Signal Processing and Regularisation (Lecture room 2, 13:30)

Input data



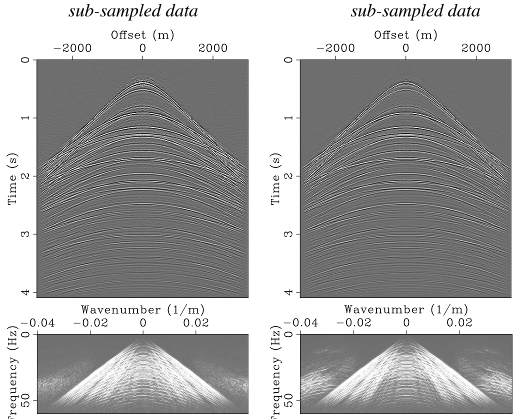
CRSI - sparsity only



Challenges

- interpolating regularly sub-sampled data
  - Wednesday, June 13th
    - Seismic Signal Processing and Regularisation (Lecture room 2, 13:30)

CRSI from irregularly sub-sampled data | *CRSI from regularly sub-sampled data*



SNR = 10.42 dB | *SNR = 6.92 dB*

Challenges

- interpolating regularly sub-sampled data
  - Wednesday, June 13th
    - Seismic Signal Processing and Regularisation (Lecture room 2, 13:30)
- efficiently solving large-scale sparsity-promoting optimization
  - opportunistic/greedy solvers
    - (Stagewise) Orthogonal Matching Pursuit – (St)OMP
    - Heavy Hitters on Steroid – HHS
    - etc.
  - general solvers
    - Primal-Dual Barrier Method for Convex Objectives – PDCO
    - Gradient Projection for Sparse Reconstruction – GPSR
    - Conjugate Gradient on the Normal Equation + Iterative Re-weighted Least-Squares – CGNE+IRLS
    - **Iterative Thresholding + Cooling – ITC**
    - etc.

Conclusions

- CRSI
  - uses **curvelets** to exploit the very strong geometrical structure of seismic data
  - performs best for
    - high dimensional data
    - irregularly sub-sampled data
- main challenge when using sparsity as a prior is the solver

Acknowledgments

- SLIM team members
- E. Candès, L. Demanet, and L. Ying for CurveLab
- S. Fomel and the developers of Madagascar

This presentation was carried out as part of the SINBAD project with financial support, secured through ITF, from the following organizations: BG, BP, Chevron, ExxonMobil, and Shell. SINBAD is part of the collaborative research & development (CRD) grant number 334810-05 funded by the Natural Science and Engineering Research Council (NSERC).