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Curvelet reconstruction with sparsity-promoting inversion: successes and challenges

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SUMMARY

In this overview of the recent Curvelet Reconstruction with Sparsity-promoting Inversion (CRSI) method, we present our latest 2-D and 3-D interpolation results on both synthetic and real datasets. We compare these results to interpolated data using other existing methods. Finally, we discuss the challenges related to sparsity-promoting solvers for the large-scale problems the industry faces.

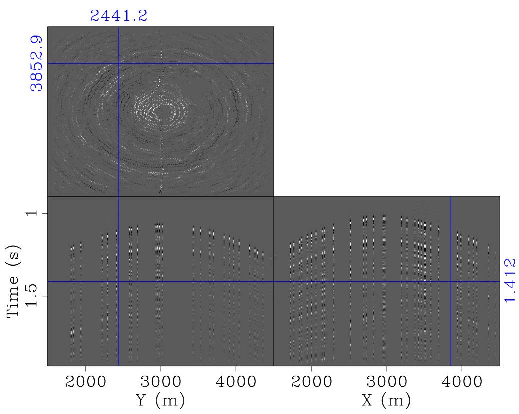
In seismic imaging, most of the commonly-used multi-trace processing algorithms need a dense and regular coverage of the survey area since irregularities and aliasing in the acquired data often transform into artifacts and poor spatial resolution in the image of the subsurface. In the field, however, seismic data is frequently irregularly and/or sparsely sampled along spatial coordinates due to practical and economical constraints. Interpolation of the acquired data to a dense and regular grid is thus a crucial step of the seismic processing and imaging work flow.

The approach we advocate for interpolation is to view seismic data from a geometrical perspective. Indeed, seismic data presents two key features:

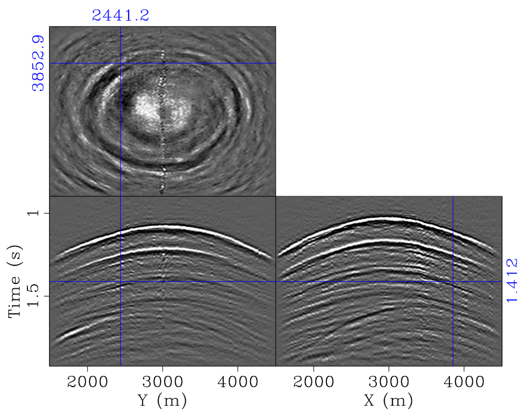
- high dimensionality (typically 5-D for a 3-D survey – time, 2 spatial coordinates for the source, and 2 spatial coordinates for the receiver),
- strong geometrical structure (seismic data provides a spatio-temporal sampling of the reflected wave field, which contains different arrivals – i.e. wavefronts – that correspond to different interactions of the incident wave field with inhomogeneities in the Earth's subsurface).

Our interpolation algorithm (Hennenfent and Herrmann, 2005; Herrmann and Hennenfent, 2007), categorized as transform-based method and named Curvelet Reconstruction with Sparse Inversion (CRSI), explores this high multi-dimensional structure of the dataset, through modern harmonic analysis tools called curvelets (see e.g. Candès and Donoho, 2004).

CRSI was successfully applied to both synthetic and real 2-D and 3-D data. Fig. 1 shows the reconstruction of the Oseberg 3-D walkaway VSP survey. The data (Fig. 1(a)) has 90% of the traces missing. The reconstructed data using 3-D CRSI is presented in Fig. 1(b).



(a)



(b)

Figure 1: Oseberg 3-D walkaway VSP survey and its reconstruction using 3-D CRSI.

The strength of CRSI compared to other existing methods lies in the fact that curvelets act as natural wavefront detectors. By construction of the curvelets, this detection and discrimination is a function of the location in the time-space domain, the frequency content and the angle of the wavefronts. CRSI has thus no problem dealing with e.g. curved wavefronts, or conflicting dips. This typically translates into several dB of improvement compared to other methods when interpolating complicated datasets as was shown in Herrmann and Hennenfent (2007) and will be further illustrated in this overview.

The non-linear optimization involved in CRSI is derived from an iterative thresholding algorithm (Daubechies et al., 2005). Although not as fast as other approximate ℓ_1-solvers, e.g. conjugate gradient on the normal equation (CGNE) combined with iterated least-squares (IRLS), our experiments show so far that our solver provide sparser and more accurate solutions. We discuss the challenges involved in using such solver for industry-size interpolation problems and debate the necessity of working towards a new large-scale sparsity-promoting solver.

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REFERENCES

Candès, E. and D. Donoho, 2004, New tight frames of curvelets and optimal representations of objects with piecewise C^2 singularities: Communications on Pure and Applied Mathematics, **57**, 219–266.

Daubechies, I., M. Defrise, and C. de Mol, 2005, An iterative thresholding algorithm for linear inverse problems with a sparsity constrains: Communications on Pure and Applied Mathematics, **57**, 1413–1457.

Hennenfent, G. and F. Herrmann, 2005, Sparseness-constrained data continuation with curvelet frames: Applications to missing traces and aliased signals in 2/3-D: Presented at the SEG International Exposition and 75th Annual Meeting.

Herrmann, F. and G. Hennenfent, 2007, Non-parametric seismic data recovery with curvelet frames. (submitted).