

Abstract

A non-linear primary-multiple separation method using curvelets frames is presented. The advantage of this method is that curvelets arguably provide an optimal sparse representation for both primaries and multiples. As such curvelets frames are ideal candidates to separate primaries from multiples given inaccurate predictions for these two data components. The method derives its robustness regarding the presence of noise; errors in the prediction and missing data from the curvelet frame's ability (i) to represent both signal components with a limited number of multi-scale and directional basis functions; (ii) to separate the components on the basis of differences in location, orientation and scales and (iii) to minimize correlations between the coefficients of the two components. A brief sketch of the theory is provided as well as a number of examples on synthetic and real data.

Separation by matched filtering

In many complex areas, multiple suppression techniques based on move-out filtering fail due to the fact that the underlying assumptions are not met, i.e. hyperbolic move-out in the CMP-offset domain, separation of primaries and multiples based on move-out differences. Several attempts have been made to extend the move-out discrimination methods towards handling 3D complexities, e.g. by introducing apex-shifted hyperbolic transforms [5]. In many cases where the move-out filtering based methods fail, wave-equation based predictive methods may provide a solution. However, prediction methods based on 2D input data [see e.g. 18, 1] are often not capable of correctly predicting the surface-related and internal multiples in a complex 3D world.

In general, wave-equation based methods consist of two main steps: a prediction step and an adaptive subtraction step. In the latter, imperfections in the prediction stage are compensated, such as the water bottom reflectivity [see e.g. 2, 20, 11] or the source and receiver characteristics [18, 1, 8]. A straightforward solution, as described in [17], is based on an adaptive subtraction using a least-squares matched filtering approach:

$$\mathbf{d}_0(x_r, x_s, t) = \mathbf{d}(x_r, x_s, t) - \mathbf{a}(t) * \mathbf{m}(x_r, x_s, t), \quad (1)$$

where the multiples $\mathbf{m}(x_r, x_s, t)$ are matched to the true multiples in the input data $\mathbf{d}(x_r, x_s, t)$ by a matching filter $\mathbf{a}(t)$, which minimizes

$$\hat{\mathbf{a}}(t) = \arg \min_{\mathbf{a}(t)} \frac{1}{2} \|\mathbf{d} - \mathbf{a} * \mathbf{m}\|_p^p \quad (2)$$

for $p = 1$ or $p = 2$. Given an estimate for $\mathbf{a}(t)$, the primaries $\mathbf{d}_0(x_r, x_s, t)$ are calculated according Eq. 1. The adaptive subtraction is usually conducted in overlapping windows in source (x_s), receiver (x_r) and time (t) coordinates with a window size in the order of a few hundred milliseconds in the time and a few tens of traces in the space directions. Within each window a separate matching filter $\mathbf{a} = \mathbf{a}(x_r, x_s, t)$ is estimated. Even though the above matched filtering procedure is able to absorb certain errors in the multiple predictions, the approach is bound to fail when the predictions are too far off w.r.t. the true multiples or when noise is present. This failure may lead to an erroneous treatment of the primaries and spurious multiples may remain in the data. Several attempts have been made to improve the situation by either making the multiple predictions more accurate or by devising a more robust subtraction/separation scheme. Methods based on model-driven time delays, as proposed by [12] and [13], or based on data-driven time delays [9] are examples of the first category. As an example of the second category, [10] decomposes the predicted multiples into coherent and incoherent parts, the latter mostly contain events related to diffracted multiples. Both parts are simultaneously subtracted from the input data. [19] also exploits extended subtraction techniques to better adapt to the true multiples without distorting the primaries by introducing an adaptation based on local time and phase shifts.

In practice, extending the adaptive subtraction process to include non-stationarity of the matched filter may give rise to improved suppression results. However, the method is bound by intrinsic limits on matched filtering. These limits concern: (i) the allowed degree of non-stationarity for the error in the multiple prediction; (ii) the inherent sensitivity of matched filtering with regard to errors in the phase, location, dip and frequency characteristics of the predicted multiples; (iii) the presence of noise and problems with edge effects for the local filter.

Given these limitations, we arrive at a point where the matched filtering techniques cannot deliver satisfactory results and the purpose of this paper is to come with a robust method that is able to separate primaries from multiples in the presence of noise and for predictions that exhibit non-stationary errors in the phase, location, dip and frequency content. With our method, we aim to (i) remove the relative strong sensitivity to the accuracy of the predicted multiples; (ii) avoid the creation of spurious artifacts or worse and (iii) avoid possible distortions of the primary energy.

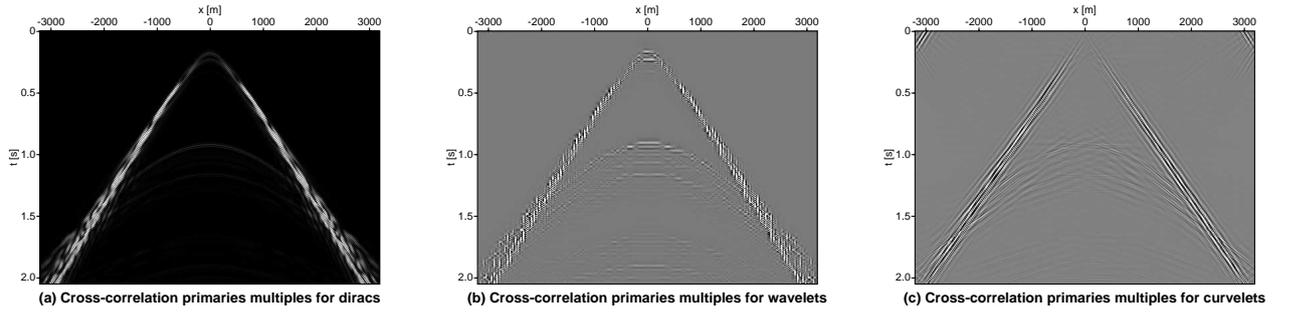


Figure 1: Correlations between the coefficients of the primaries and multiples are computed by multiplying the coefficients, followed by an inverse transform. Clearly, curvelets potentially separate the best because they are the least correlated.

Curvelet-domain primary-multiple separation

We propose a transform-domain method to separate primaries and multiples in the presence of noise. Our signal model reads

$$\mathbf{s} = \mathbf{s}_1 + \mathbf{s}_2 + \mathbf{n} \quad (3)$$

and the question is to separate the data \mathbf{s} into its two components \mathbf{s}_1 (primaries) and \mathbf{s}_2 (multiples) in the presence of a white Gaussian noise term \mathbf{n} of strength σ . Following ideas close in spirit to [16, 5] and [15], we formulate the answer to the above signal separation problem as

$$\hat{\mathbf{c}} = \arg \min_{\mathbf{c}} \frac{1}{2} \|\mathbf{s} - \mathbf{T}\mathbf{c}\|_2^2 + \|\mathbf{c}\|_{p,w}, \quad (4)$$

in which

$$\mathbf{T} = [\mathbf{T}_1 \quad \mathbf{T}_2], \quad \mathbf{c} = [\mathbf{c}_1 \quad \mathbf{c}_2]^T \quad \text{and} \quad \mathbf{w} = [\mathbf{w}_1 \quad \mathbf{w}_2]^T \quad (5)$$

refer to the augmented system, with two frame compositions (\mathbf{T}_1 and \mathbf{T}_2): one for the primaries and one for the multiples, and to the frame coefficient vector \mathbf{c} . $\|\mathbf{c}\|_{p,w}$ stands for the ℓ^p -norm on the coefficients weighted by the vector $\mathbf{w} \geq 0$. Typical examples of the above formalism are (i) linear $(f - k)$ -filtering with \mathbf{T} the inverse Fourier transform and $p = 2$ [21]; (ii) sparseness enhanced ($p = 1$) Radon filtering with \mathbf{T} the adjoint of the linear or parabolic Radon transform (or combinations thereof) [16] and (iii) Morphological Component Analysis (MCA) where the ℓ^1 -norm is minimized for a redundant dictionary \mathbf{T} [15]. This dictionary consists of mutually incoherent basis functions such as wavelets, curvelets and/or discrete cosines.

As opposed to standard MCA – where texture- and edge-like signal components are separated by choosing transforms that are sparse for only one of the signal components and not for the other – similarities between primaries and multiples makes primary-multiple separation a challenge. This observation explains the partial success of $(f - k)$ or Radon methods, even when predictions for the multiples $\check{\mathbf{s}}_2$ and primaries $\check{\mathbf{s}}_1 = \mathbf{s} - \check{\mathbf{s}}_2$ are available [21]. Lack of sparseness in the transformed domain, non-locality of Fourier-like basis functions [21] are partially to blame for the less than optimal signal separation, which may give rise to inadvertent primary energy removal and remaining spurious multiple energy.

Unfortunately, primaries and multiples are both solutions to the wave equation admitting a sparse representation by curvelets but no separation. Inspired by our earlier work [7, 6], separation of the two components can be promoted by minimizing the correlation between the curvelet coefficients of the primaries and multiples. Since we are given predictions for the two components, we can separate the primaries and multiples by weighting the ℓ^1 -norms in Eq. 4 by the predictions, i.e. $\mathbf{w}_1(\lambda) = \lambda|\check{\mathbf{c}}_2|$ and $\mathbf{w}_2 = \lambda|\check{\mathbf{c}}_1|$. This weighting in Eq. 4 drives the signal separation because the minimization attempts to minimize the weighted ℓ^1 -norms, which corresponds to jointly minimizing correlations between the estimated coefficients for the primaries and predicted multiples on the one hand and the estimated coefficients for the multiples and predicted primaries on the other hand. The success of this signal-separation procedure depends largely on the correlation between coefficients of the signal components in the basis. The smaller this correlation the better the performance will be of a signal separation based on Eq. 4 which maximizes the decorrelation. As Fig. 1 suggests, curvelet frames are the appropriate choice since these frames clearly accomplish the largest degree of decorrelation between primaries and multiples. The actual primary-multiple separation is carried out by using an iterative block solver [see e.g. 15, and the references therein] to compute a solution for Eq. 4. During each iteration, estimates for the coefficients of the j^{th} signal component are calculated with

$$\mathbf{c}_j^m = S_{w_j(\lambda_m)}^s \left(\mathbf{c}_j^{m-1} + \mathbf{T}_j^* \left(\mathbf{s} - \sum_i \mathbf{T}_i \mathbf{c}_i^{m-1} \right) \right) \quad \text{for } j = 1, 2, \quad (6)$$

where $S_{w_j(\lambda_m)}^s$ is a soft thresholding operator with a threshold $w_j(\lambda_m)$. The control parameter λ_m is decreased for each iteration. For large enough number of iterations m , Eq. 6 converges to the solution of Eq. 4 [see e.g. 15, 4].

Examples

The above iterative primary-multiple separation method is applied to synthetic as well as real data. Fig. 2 demonstrates our method applied to noise-free (top-row) and noisy (bottom-row) data. Our method is clearly able to achieve good results on synthetic data

even for the noisy case. Encouraging results are also obtained for real data. Fig. 3 demonstrates the effectiveness of multiple removal for a time slice from a 2D line from the Mississippi Canyon (Gulf of Mexico). Note the recovery of primaries around shot 100 and shot 900.

Discussion and conclusions

The success of our approach can heuristically be explained by arguing that curvelets act as an “unconditional basis” for seismic data (both primaries and multiples). Whenever a basis is unconditional, one finds that (i) the data’s covariance is near diagonal in the basis and (ii) the norm (say ℓ^p) shrinks when shrinking the coefficients. Our argument that curvelets can be considered to be close to an unconditional basis for seismic data derives from the work on function spaces by [14] and from Theorem 1.1 of [3] which states that Green’s functions are nearly diagonalized by curvelets. Consequently, seismic data can be represented by a sparse coefficient vector rendering our signal-separation method effective for seismic data with events on arbitrary piece-wise twice differentiable curves. Moreover, our approach is robust under additive possibly colored noise and can relatively easily be generalized to 3-D; speeded by using more intelligent ℓ^p -solvers such as IRLS.

Acknowledgments

The author would like to thank the authors of the Digital Curvelet Transform (Candes, Donoho, Demanet and Ying). This work was carried out as part of the SINBAD project with financial support, secured through ITF (the Industry Technology Facilitator), from the following organizations: BG Group, BP, ExxonMobil and SHELL. Additional funding came from the NSERC Discovery Grant 22R81254.

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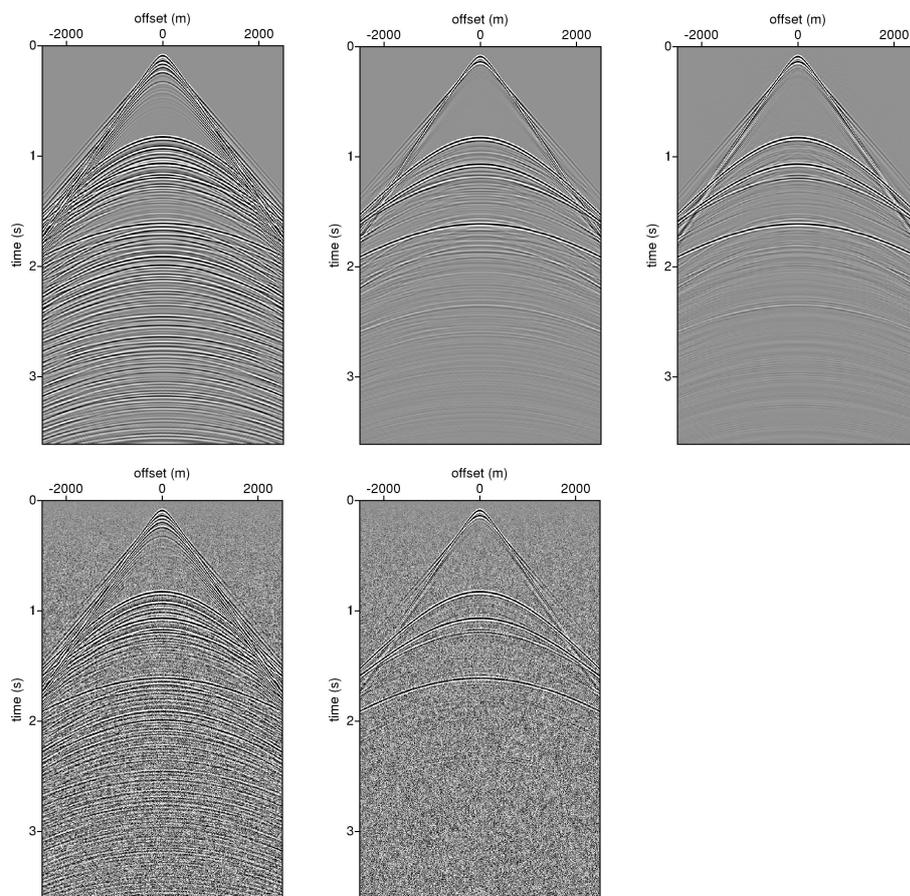


Figure 2: Synthetic example. **Top row:** Data with primaries and multiples; Matched-filter estimated primaries; Curvelet predicted primaries. **Second row:** Noisy data; Curvelet-estimated primaries.

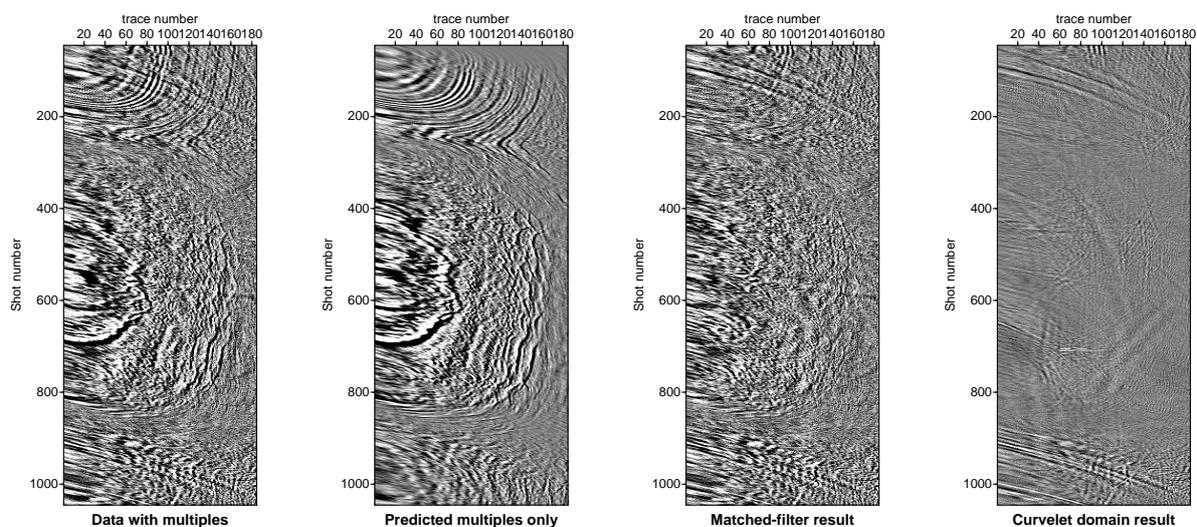


Figure 3: Real example: a time slice from a 2D Gulf of Mexico line. Data with primaries and multiples; Predicted multiples; Matched-filter estimated primaries; Curvelet predicted primaries.