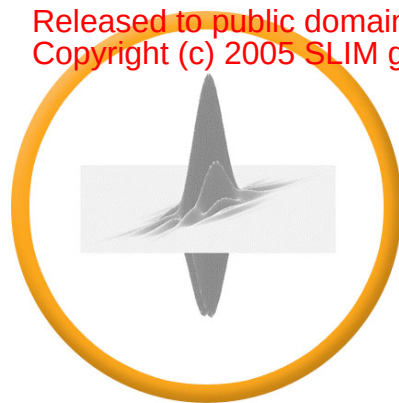




SLIM

Seismic Laboratory for
Imaging and Modeling

ROBUST CURVELET-DOMAIN DATA CONTINUATION WITH SPARSENESS CONSTRAINTS

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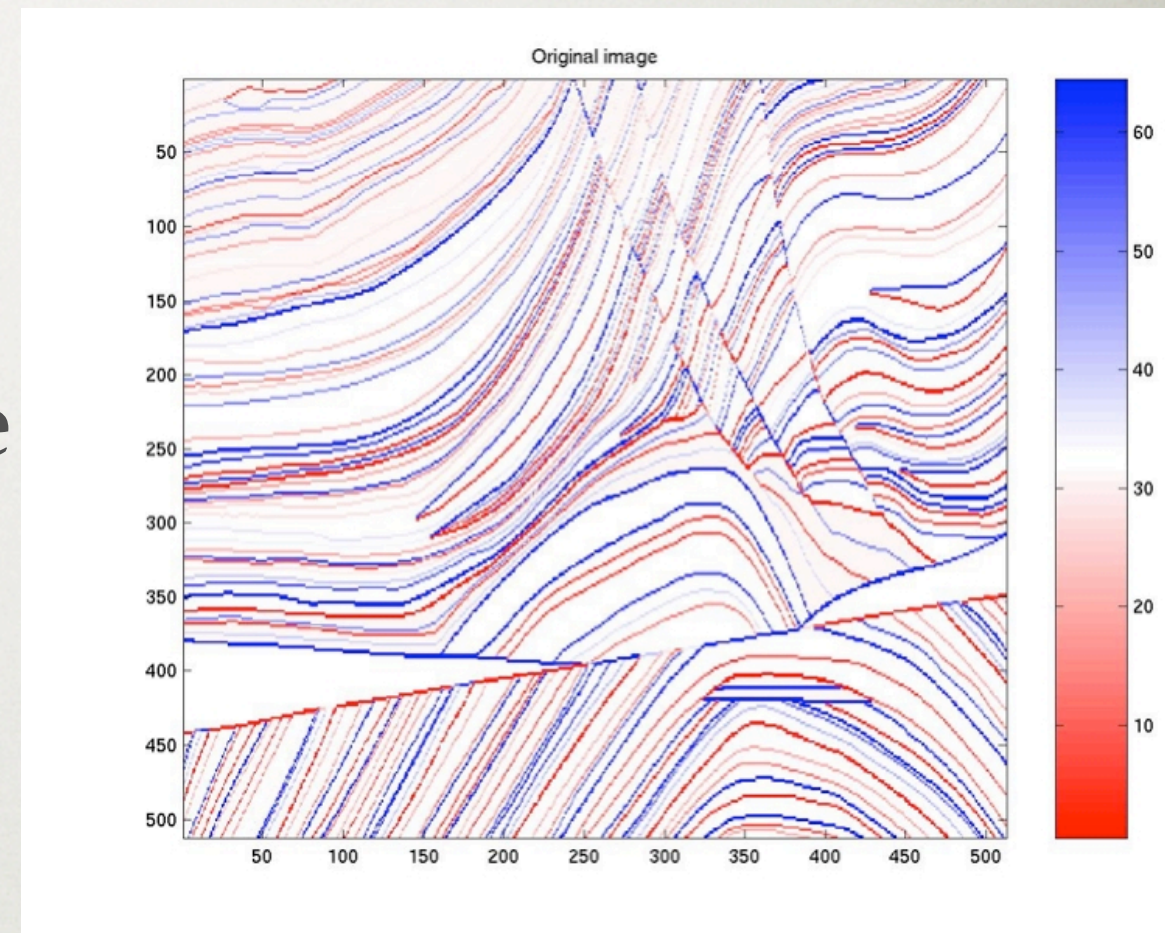
CONTEXT

- Linear least-squares data continuation [Claerbout, 92]
- Discrete & unequally sampled Fourier Transforms [Sacchi, 96 ; Schoneville, 01; Zwartjes, 04]
- Inpainting with Morphological Component Analysis using Redundant Directional Frames such as Curvelets [Candes; Donoho; Demanet; Ying, 05; Elad, 05]
- Model- (migration, NMO or Radon) based data continuation [Trad, 03]

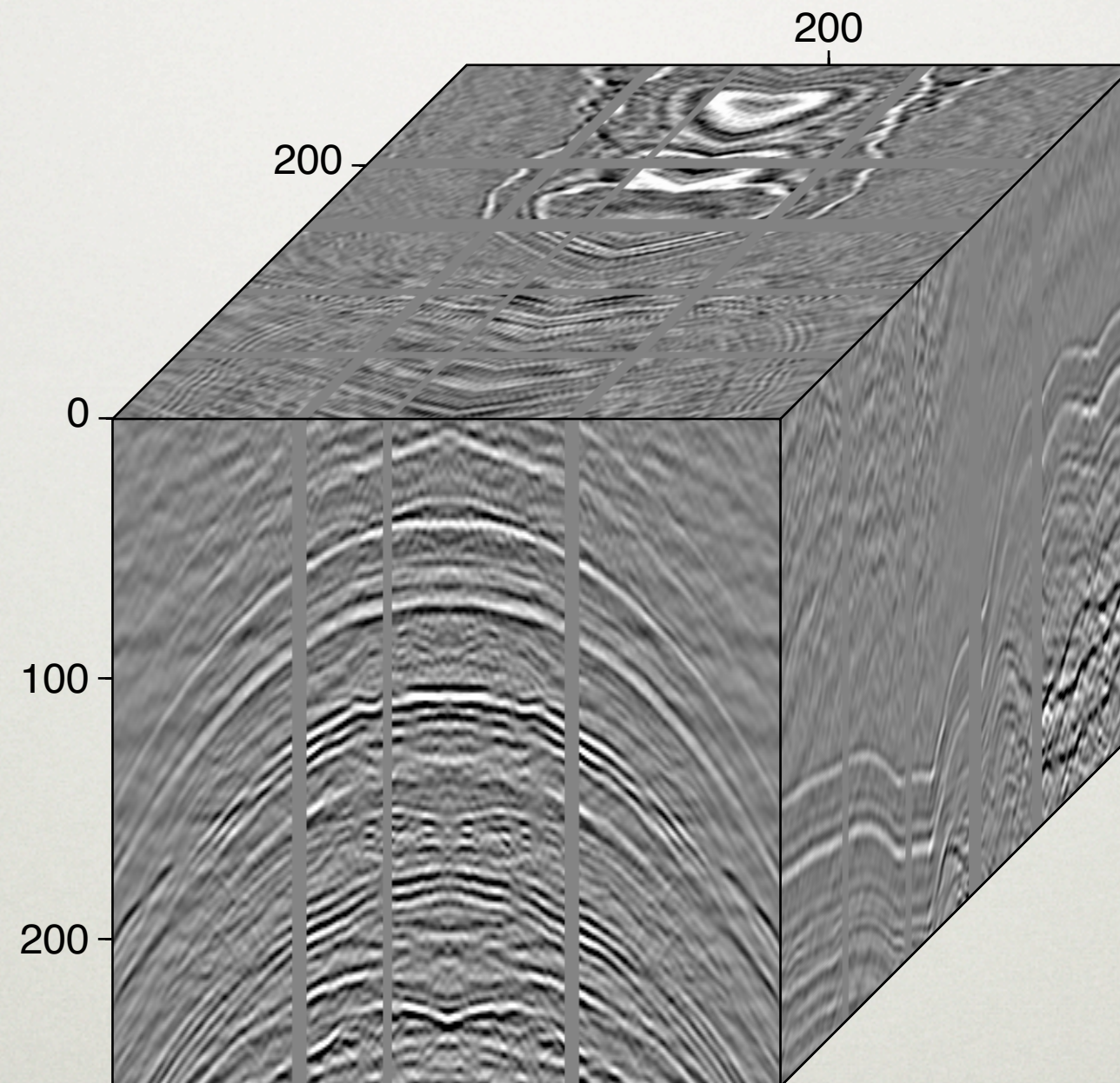
OUR MOTIVATION

Devise a data continuation & de-aliasing scheme that

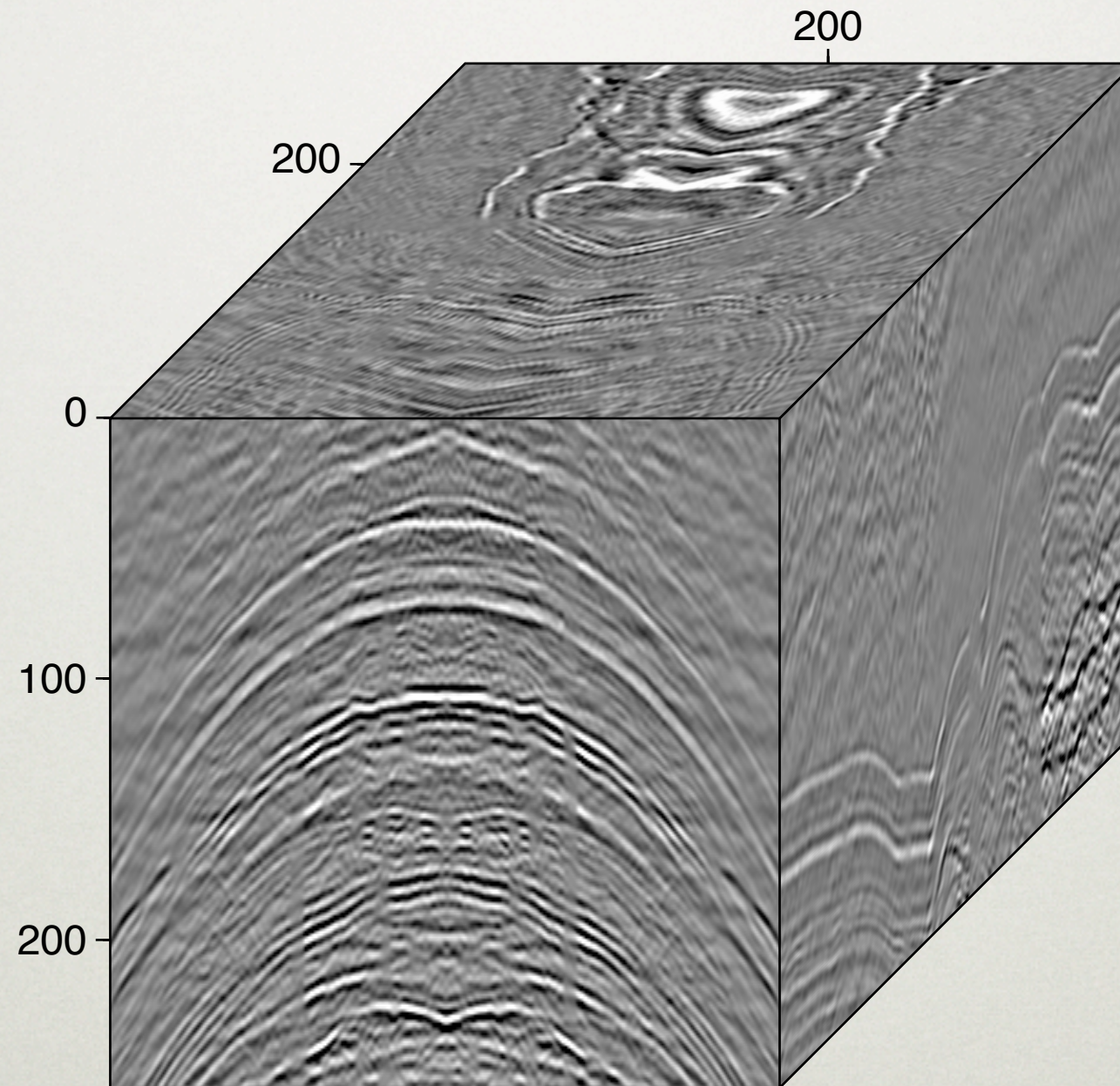
- is non-parametric / non-adaptive
- truly exploits the 3-D continuity along wavefronts in $\mathbf{d}(r, s, t)$
- is noise resilient
- exploits redundant frames
- is $n \log n$
- deals with *intermittent* regularity!



3-D REAL DATA



3-D REAL INTERPOLATED RESULT



SEISMIC INTERPOLATION

Forward problem:

$\mathbf{d}_{acq} = \mathbf{P}\mathbf{d}$ with $\mathbf{d} = \mathbf{F}\mathbf{x} + \mathbf{n}$

picking operator *regular data* *modeling/frame operator* *white noise* *model*

Conventional interpolation problem:

$$\hat{\mathbf{x}} = \arg \min_{\mathbf{x}} \underbrace{\frac{1}{2} \|\mathbf{P}(\mathbf{d} - \mathbf{F}\mathbf{x})\|_2^2}_{\text{misfit data}} + \underbrace{\lambda J(\mathbf{x})}_{\text{regularization}}$$

REGULARIZATIONS

Linear quadratic (lsqr migration):

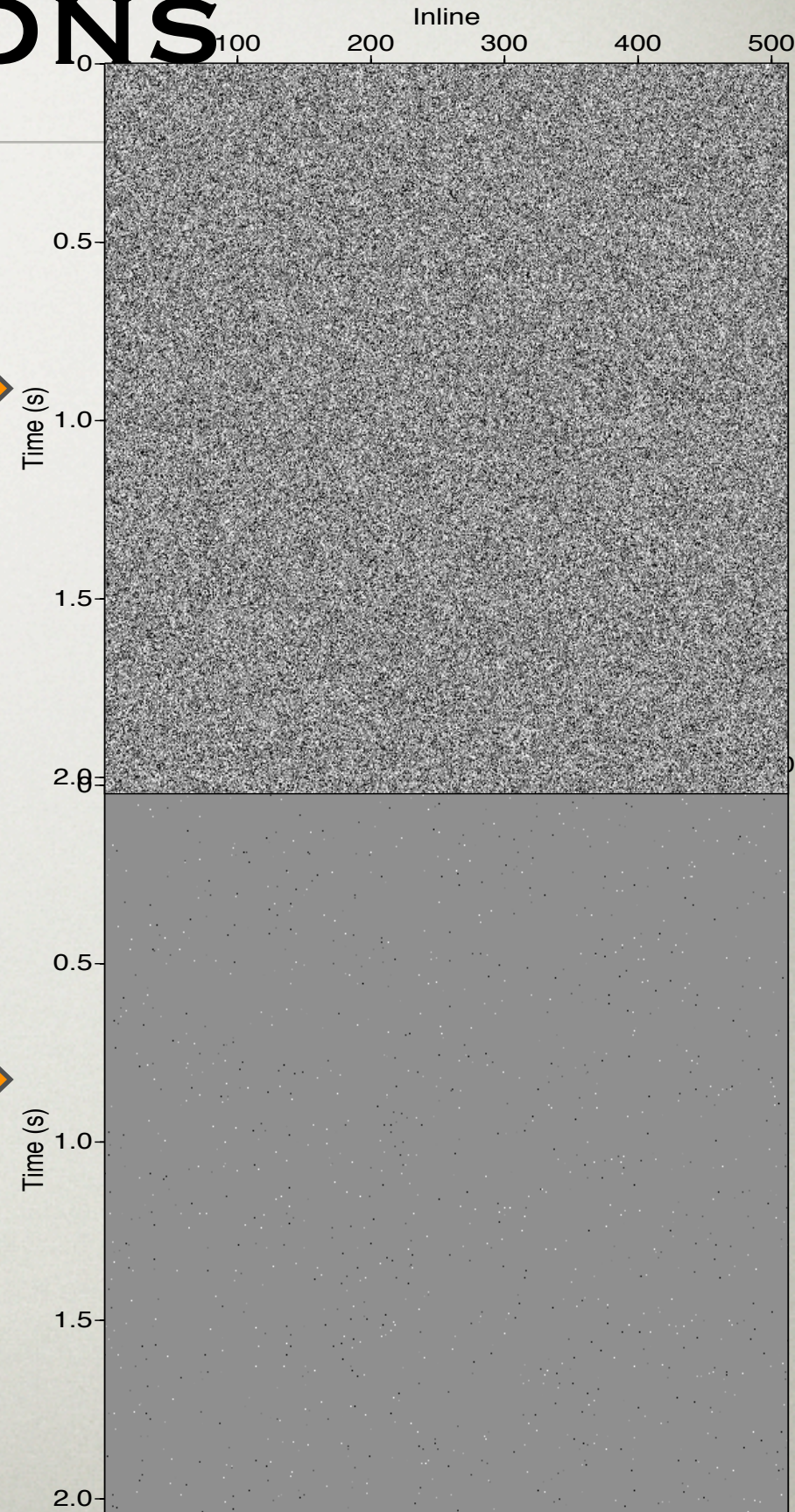
$$\hat{\mathbf{x}} = \arg \min_{\mathbf{x}} \frac{1}{2} \|\mathbf{P}(\mathbf{d} - \mathbf{F}\mathbf{x})\|_2^2 + \lambda \|\mathbf{x}\|_2^2$$

- data white Gaussian
- uncorrelated

Non-linear ℓ^1 :

$$\hat{\mathbf{x}} = \arg \min_{\mathbf{x}} \frac{1}{2} \|\mathbf{P}(\mathbf{d} - \mathbf{F}\mathbf{x})\|_2^2 + \lambda \|\mathbf{x}\|_1$$

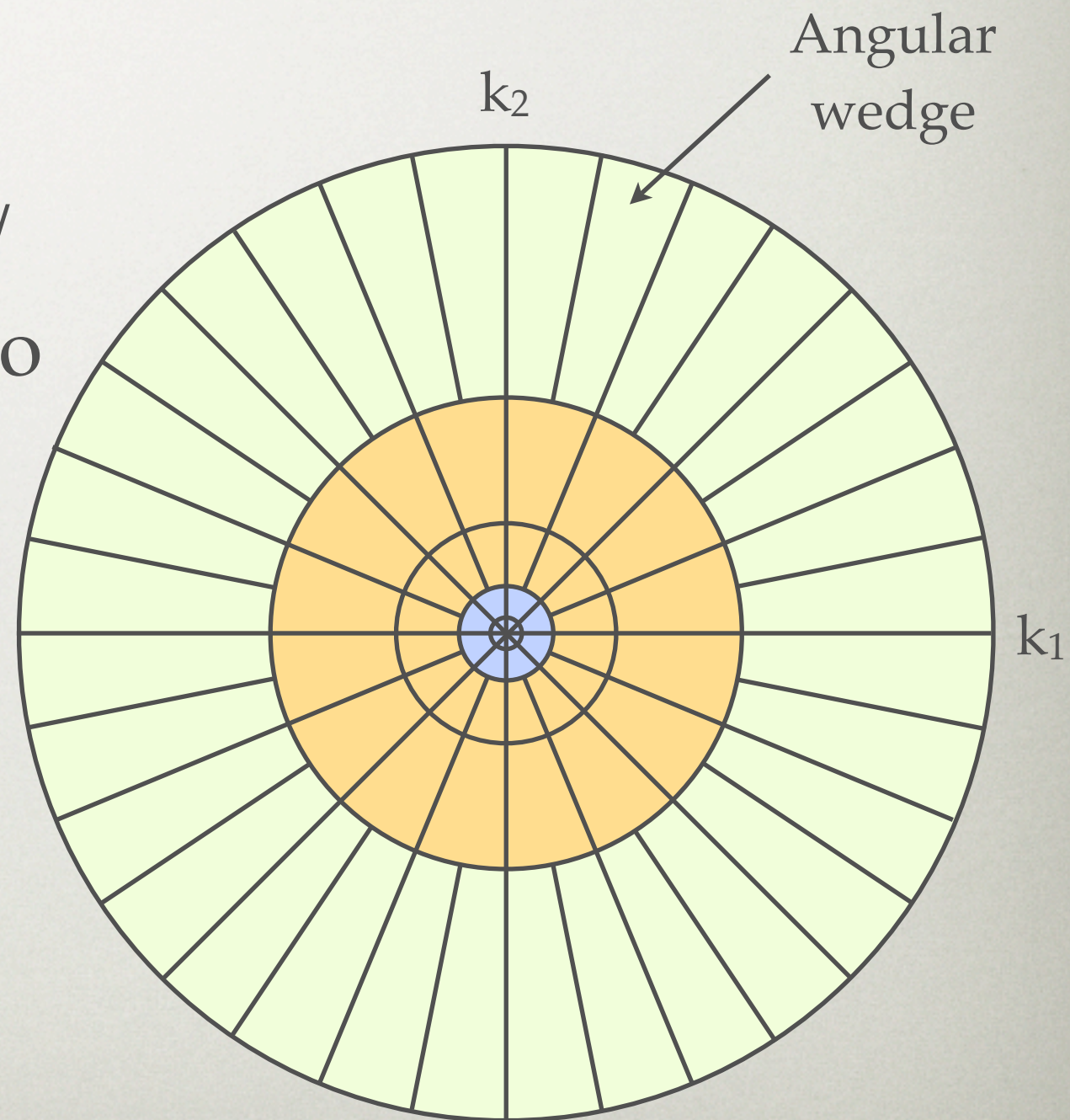
- data white Cauchy
- uncorrelated



CURVELETS

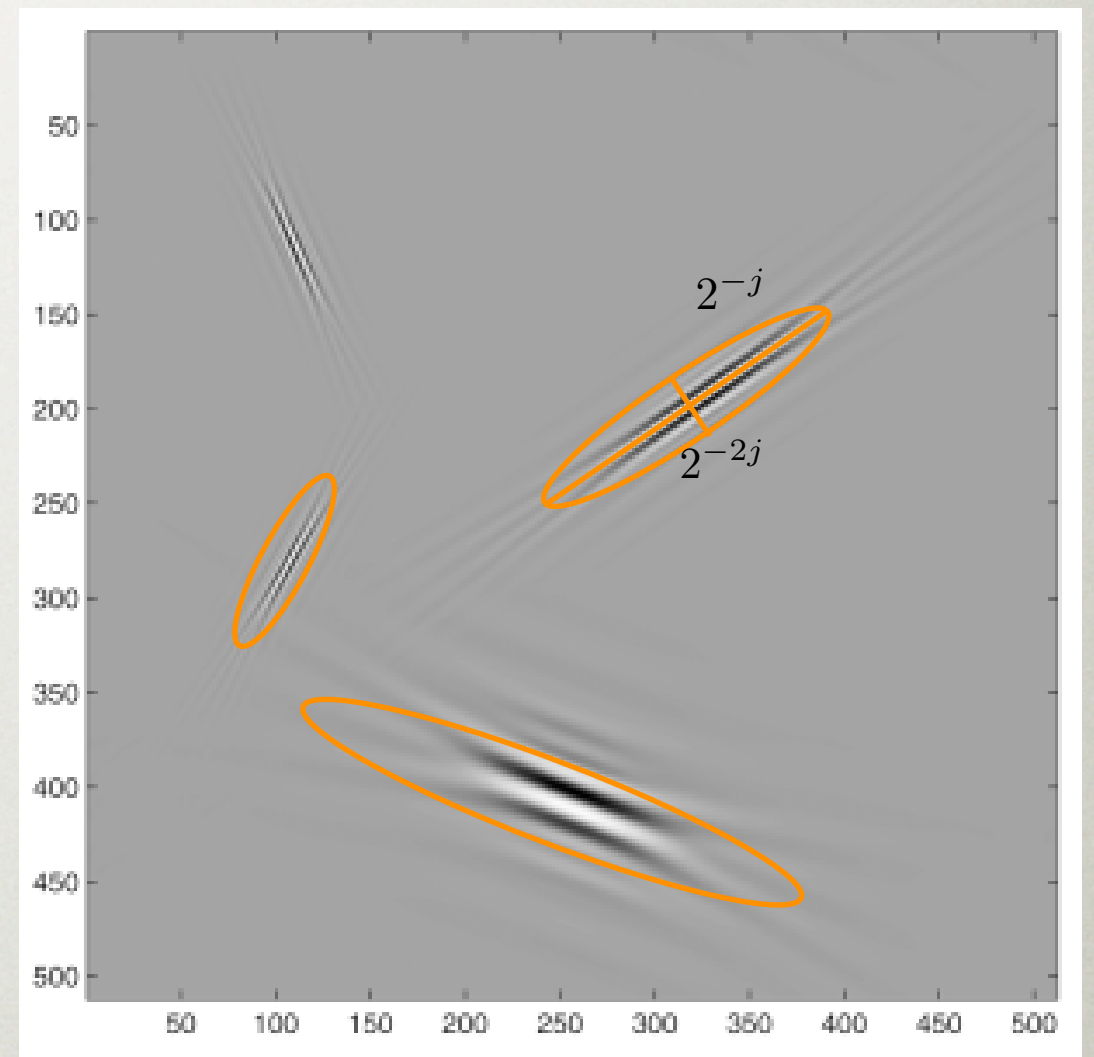
[Candes & Donoho, 99; Demanet; Ying, 05]

- Tight frames
- Partitioning of the 2-D / 3-D Fourier domain into angular wedges of second dyadic coronae
- Parabolic scaling law

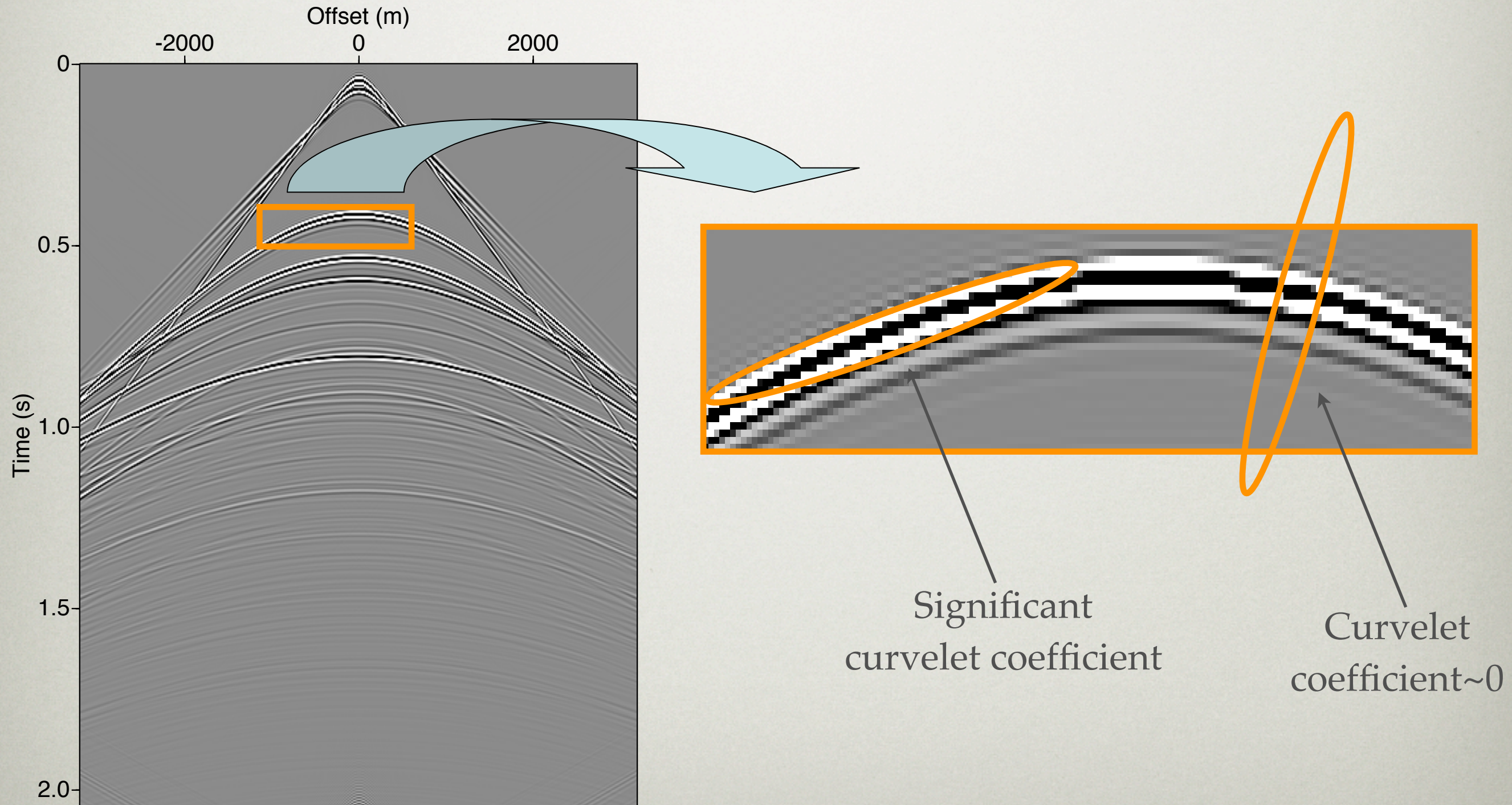


CURVELET PROPERTIES

- tight frames ($n \log n$)
- multi-scale
- multi-directional
- highly anisotropic
- localized both in space & frequency
- moderate redundancy



CURVELETS & SEISMIC DATA



CURVELET NON-LINEAR APPROXIMATION RATE

Optimal: [Donoho, 01]

$$\|\mathbf{f} - \mathbf{f}_p^O\|_2^2 \propto p^{-2}, \quad p \rightarrow \infty$$

Fourier:

$$\|\mathbf{f} - \mathbf{f}_p^F\|_2^2 \propto p^{-1/2}, \quad p \rightarrow \infty$$

Wavelets:

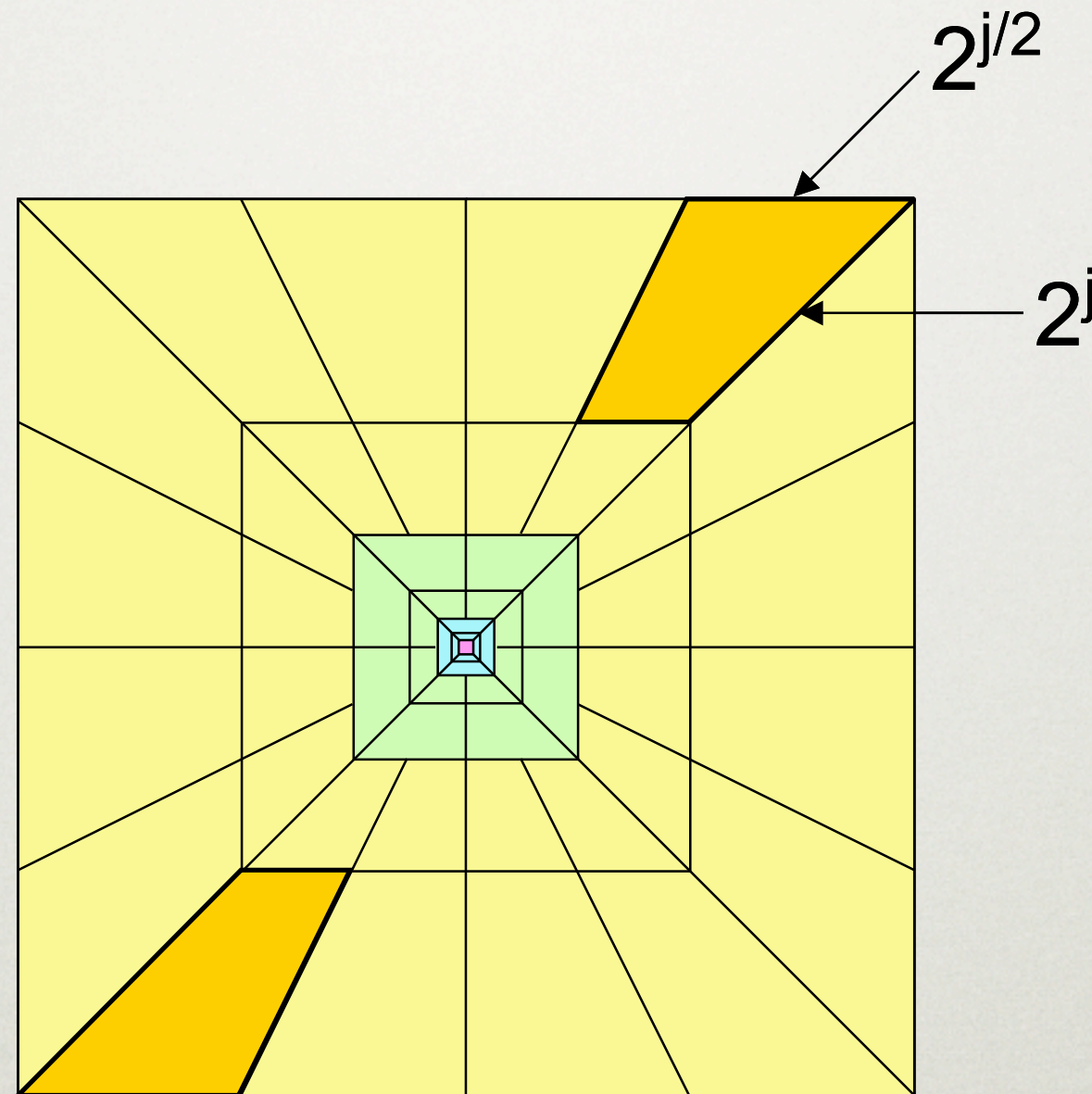
$$\|\mathbf{f} - \mathbf{f}_p^W\|_2^2 \propto p^{-1}, \quad p \rightarrow \infty$$

Curvelets: [Candes & Donoho, 99]

$$\|\mathbf{f} - \mathbf{f}_p^C\|_2^2 \leq C p^{-2} (\log p)^3, \quad p \rightarrow \infty$$

NUMERICAL CONSTRUCTION

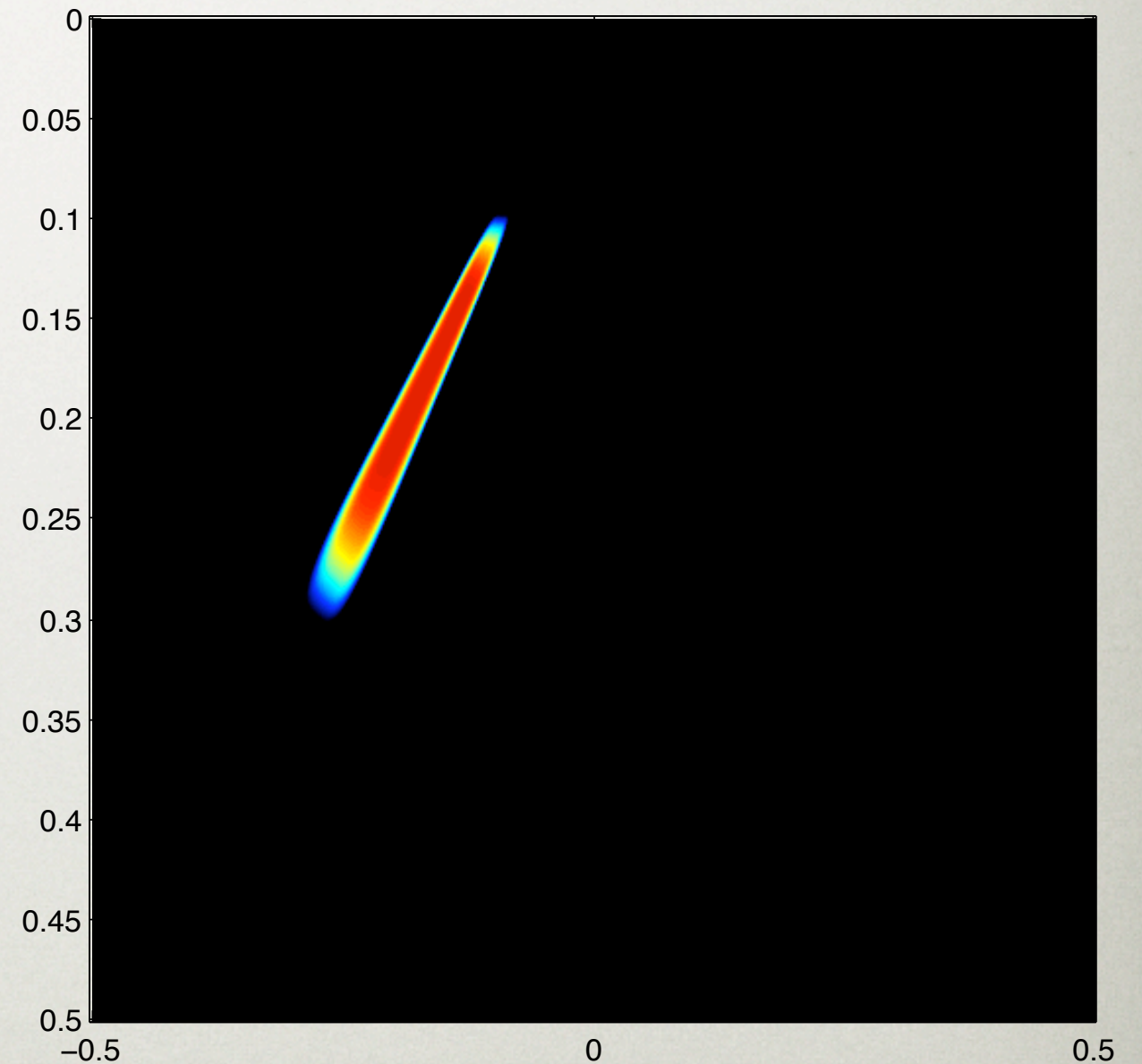
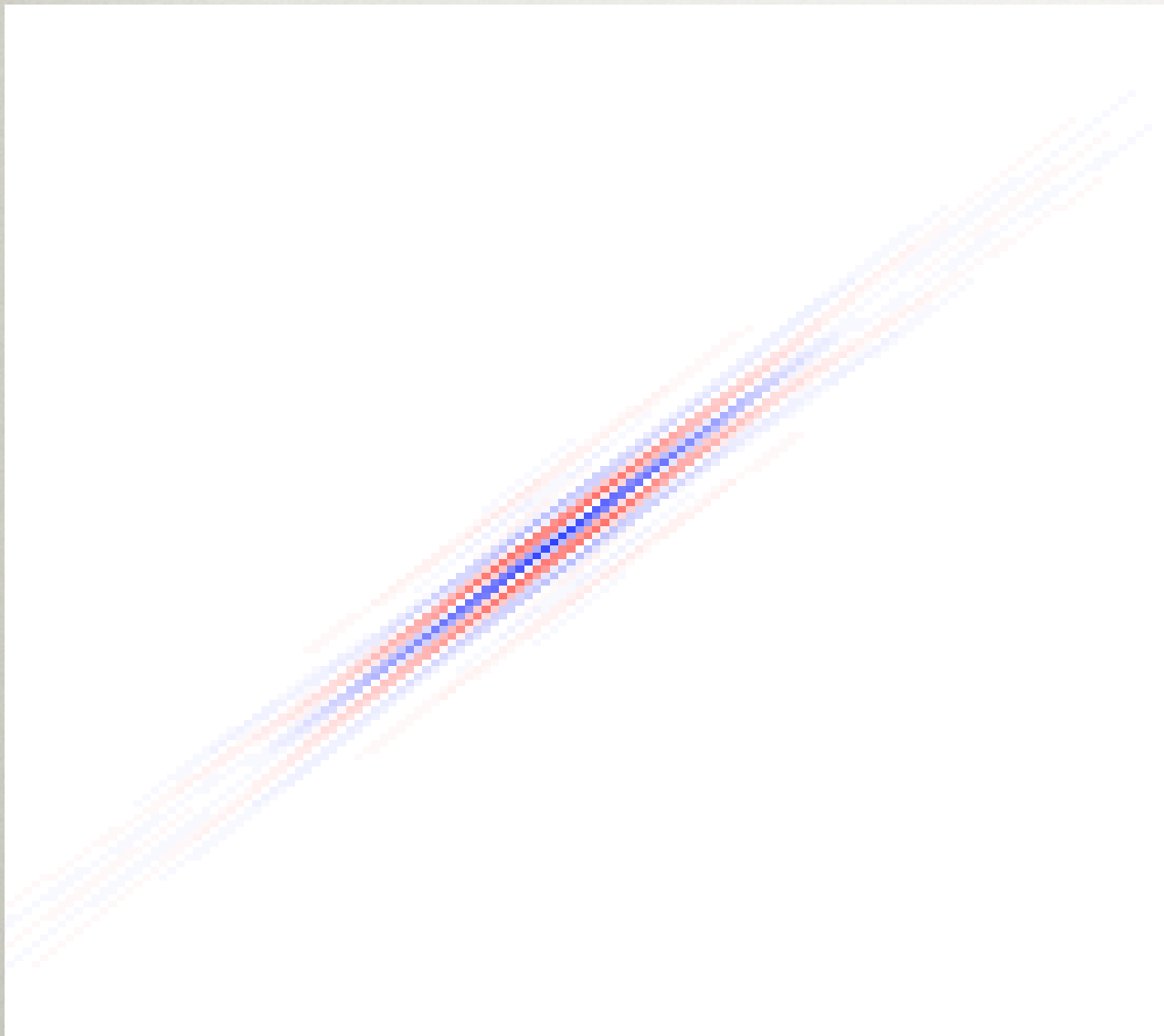
[Candes & Donoho '02-'05, Do '02, Demanet '05, Ying '05]



Curvelets live in a wedge in the 2-3 D Fourier plane...

NUMERICAL CONSTRUCTION

[Candes & Donoho '02-'05, Do '02, Demanet '05, Ying '05]



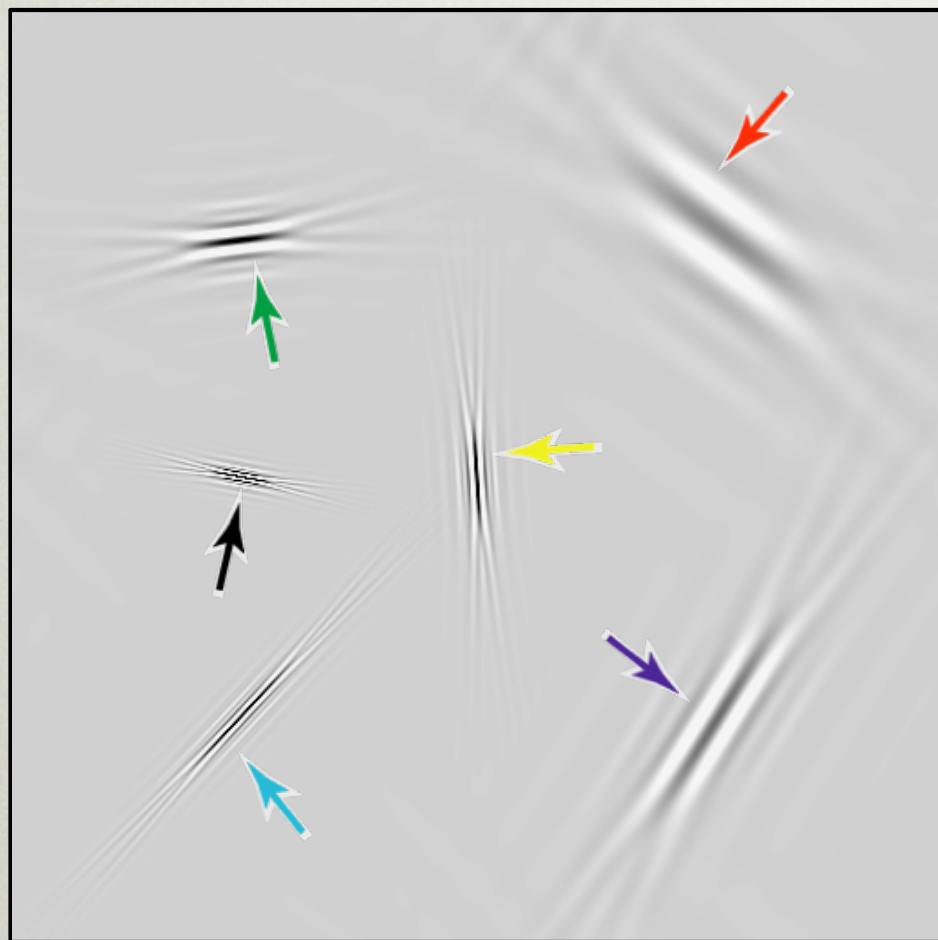
localized in both domains

CONSTRUCTION

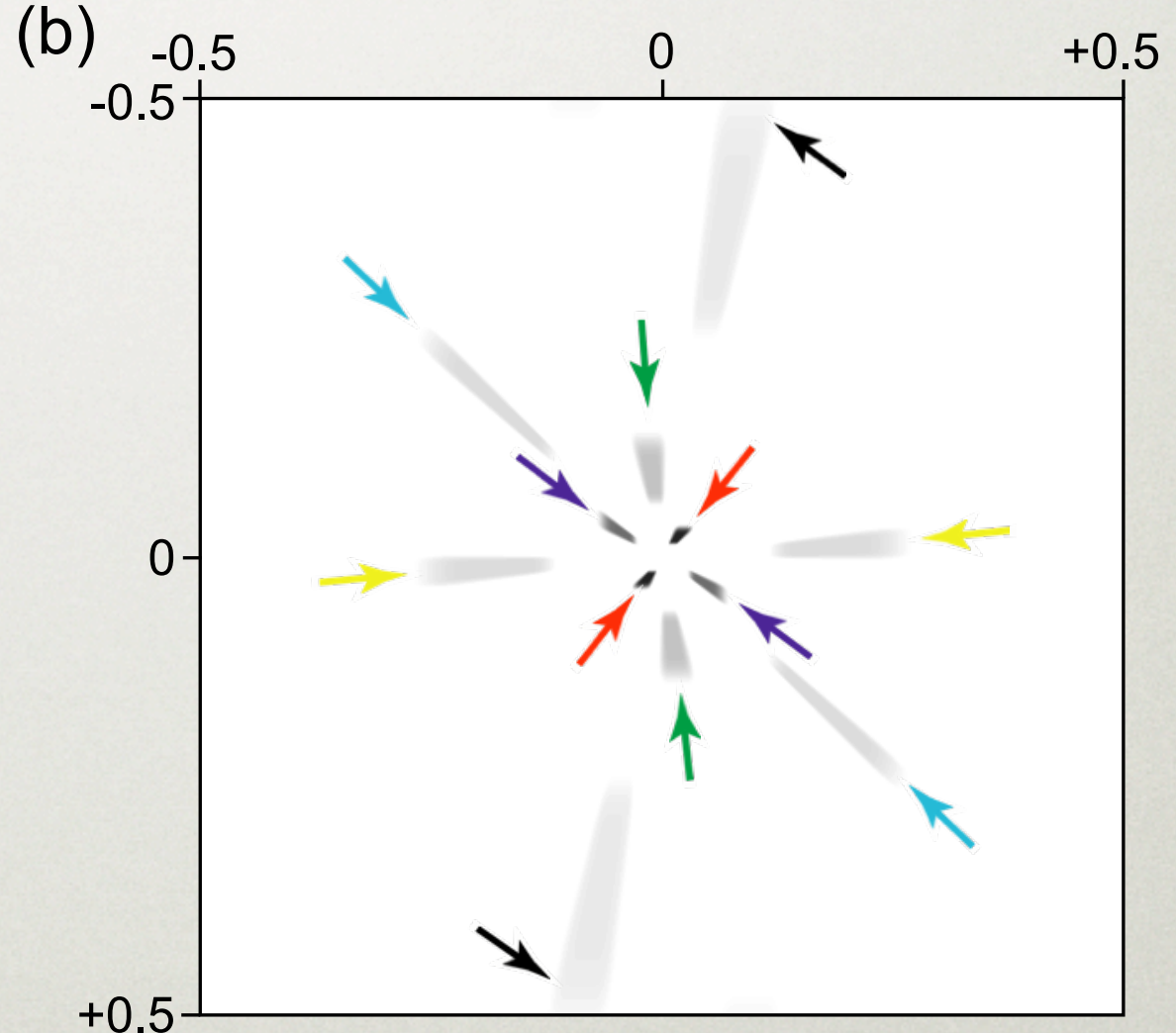
[Candes & Donoho '02-'05, Do '02, Demanet '05, Ying '05]

Partitioning example

(a) Curvelet in the space domain



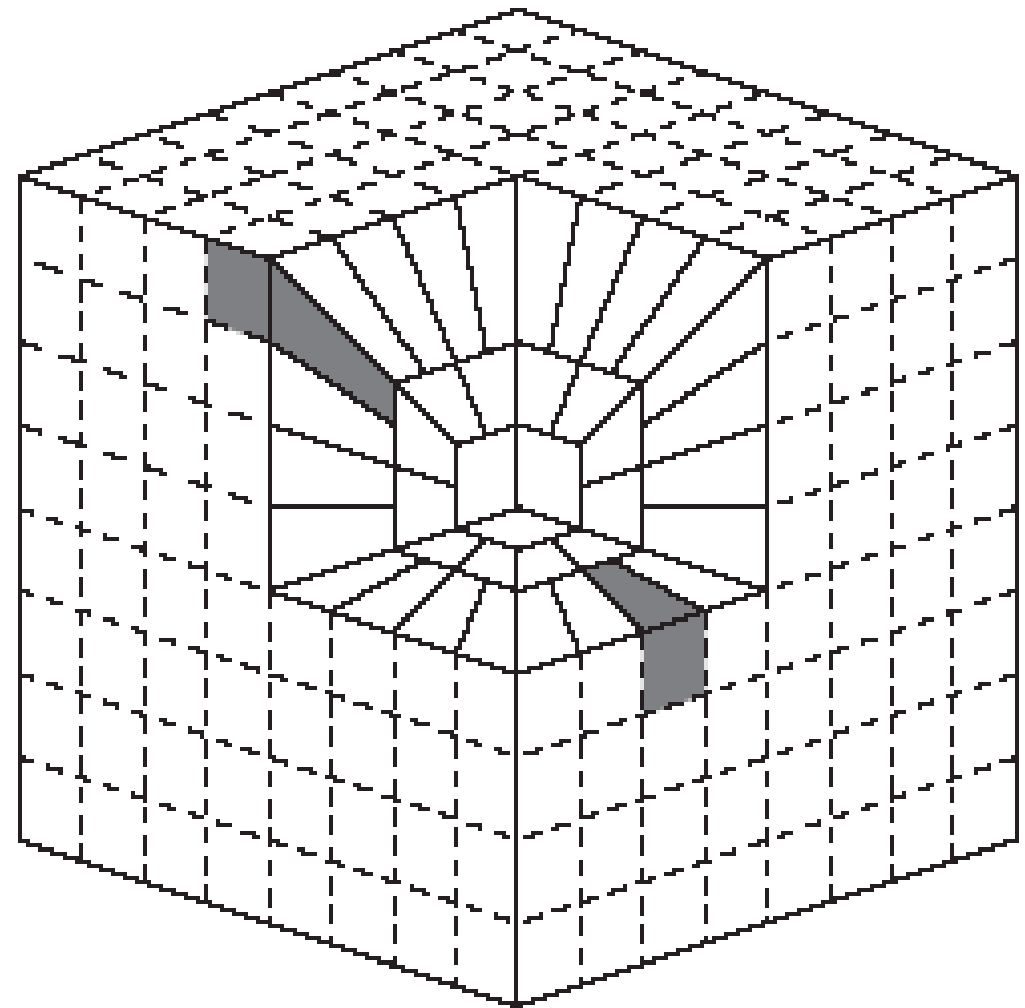
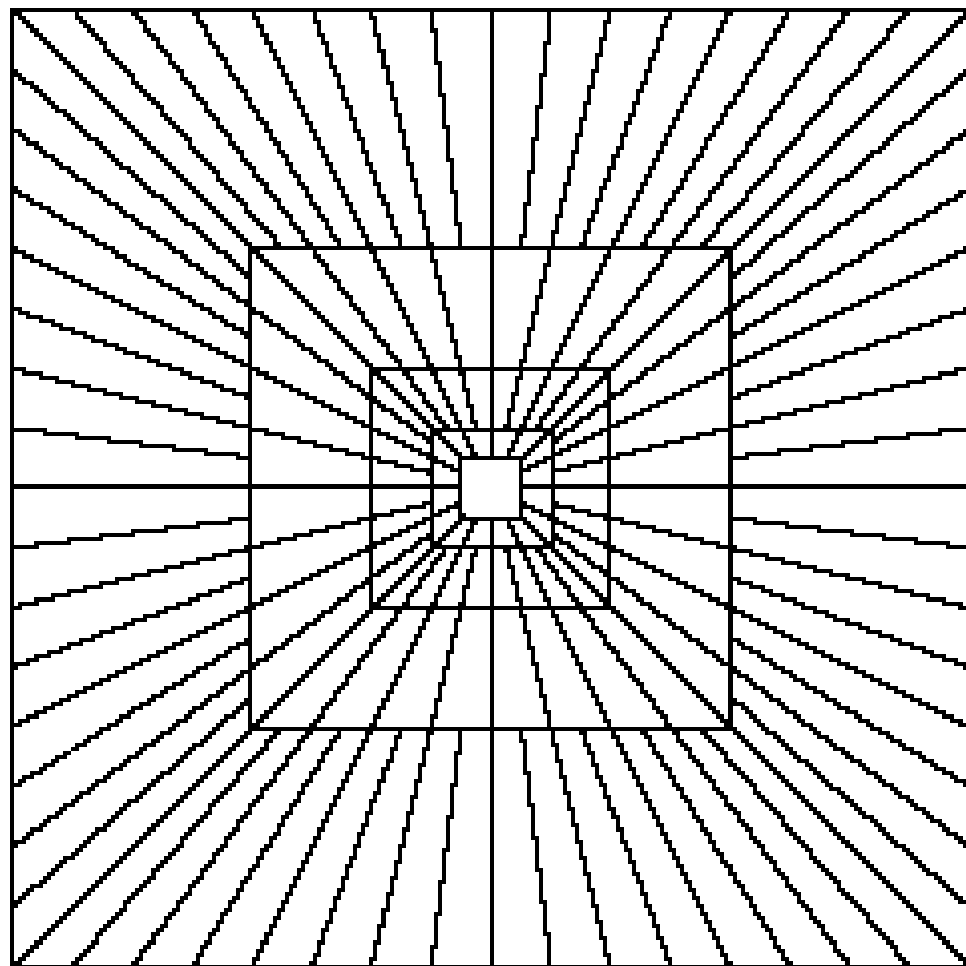
(b) Curvelet in the Fourier domain



Micro-local correspondence

EXTENSION TO 3-D

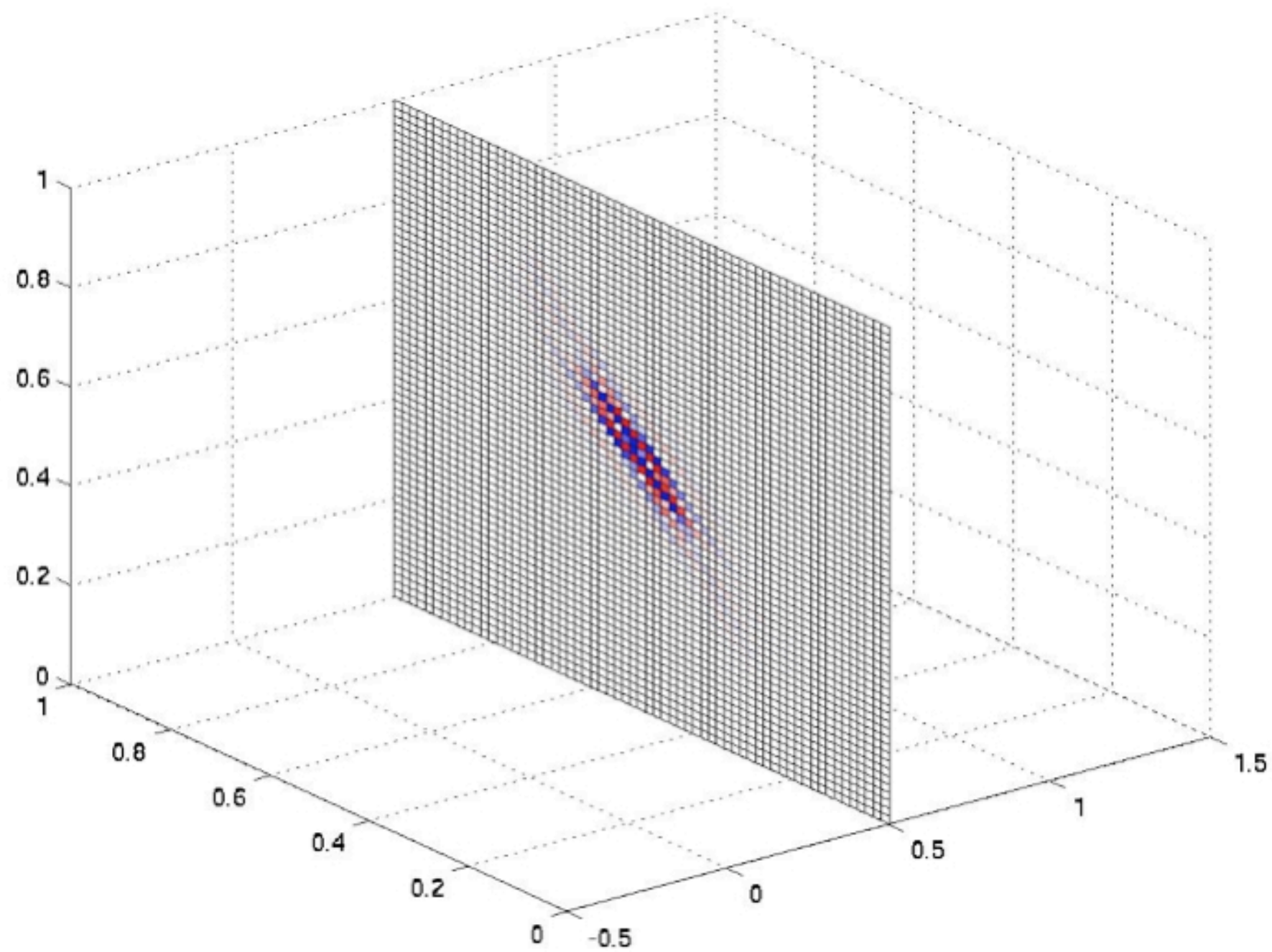
[courtesy Demanet '05, Ying '05]



Curvelets live in a wedge in the 3 D Fourier plane...

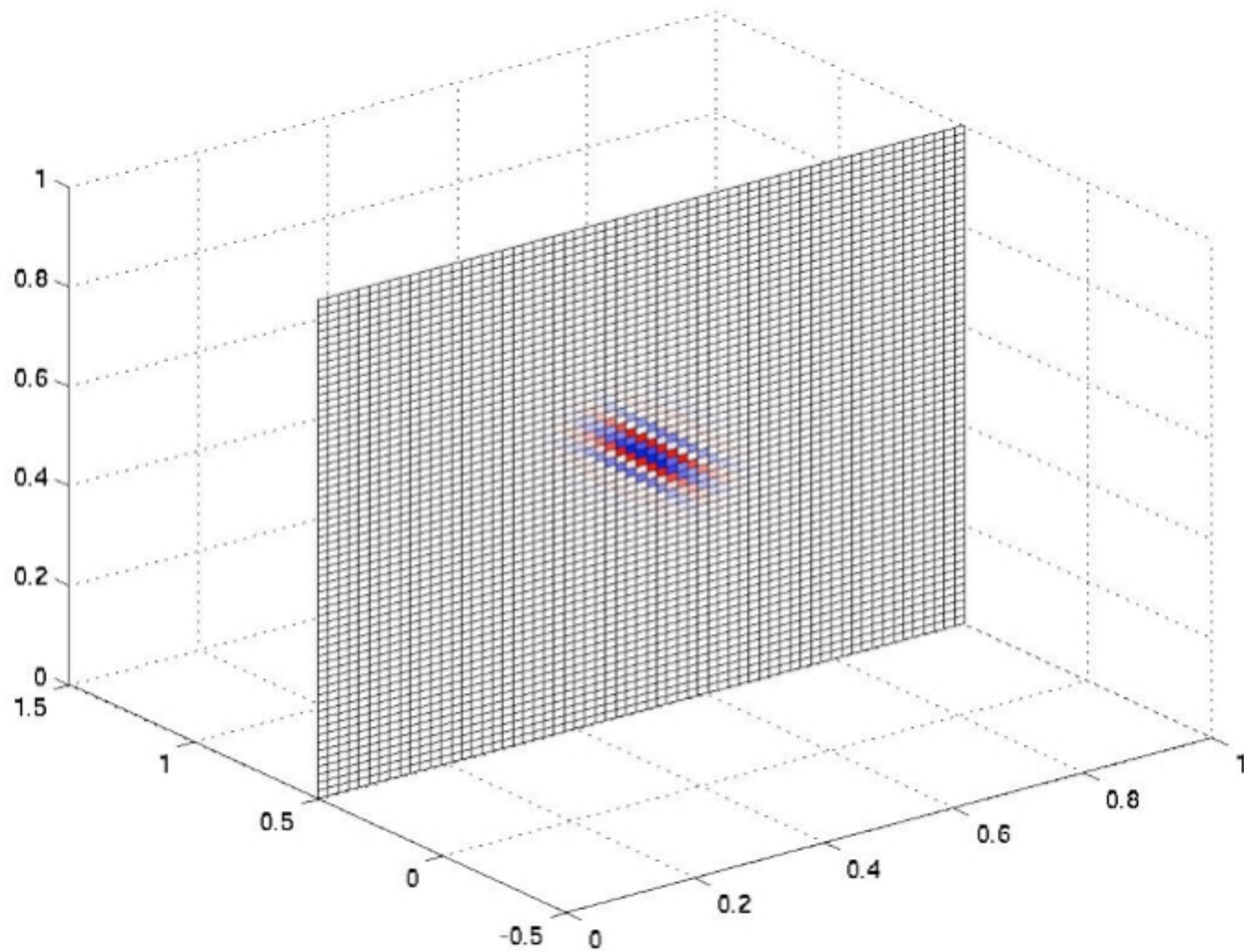
3-D CURVELETS

[Ying '05]



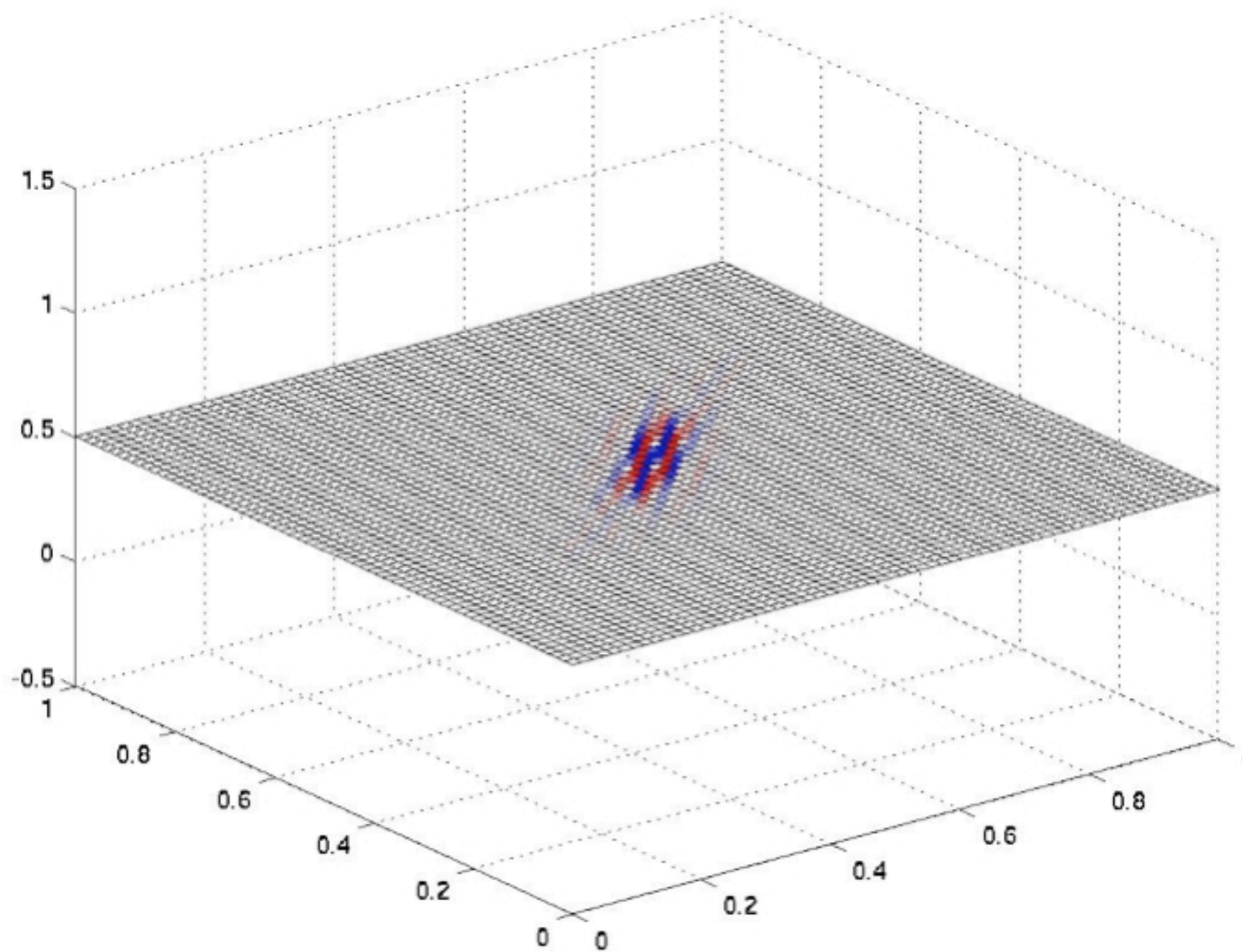
3-D CURVELETS

[Ying '05]



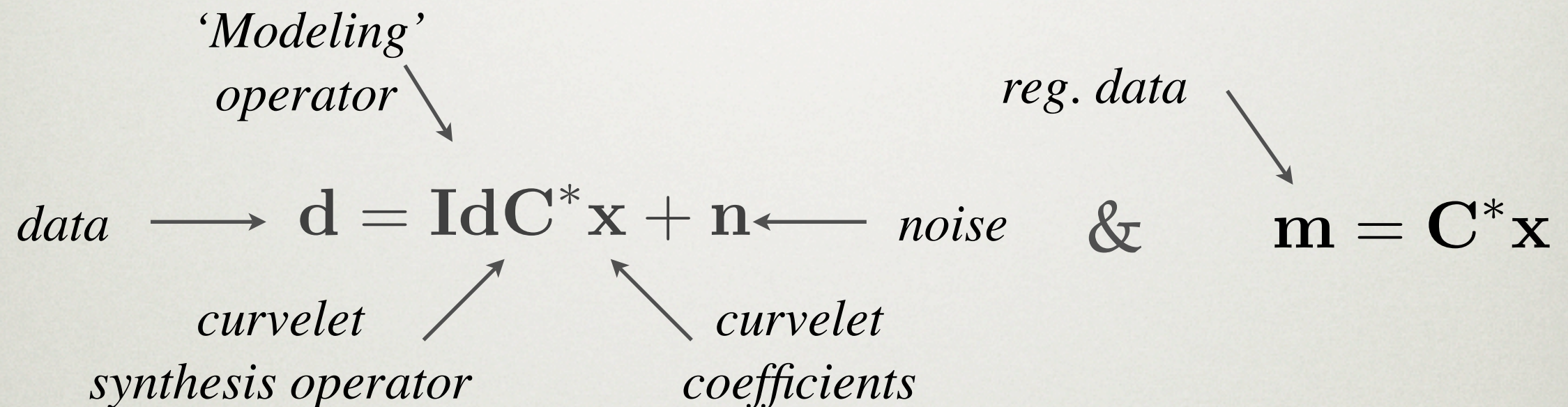
3-D CURVELETS

[Ying '05]



CURVELETS FOR SEISMIC DECONVOLUTION

Our forward problem:

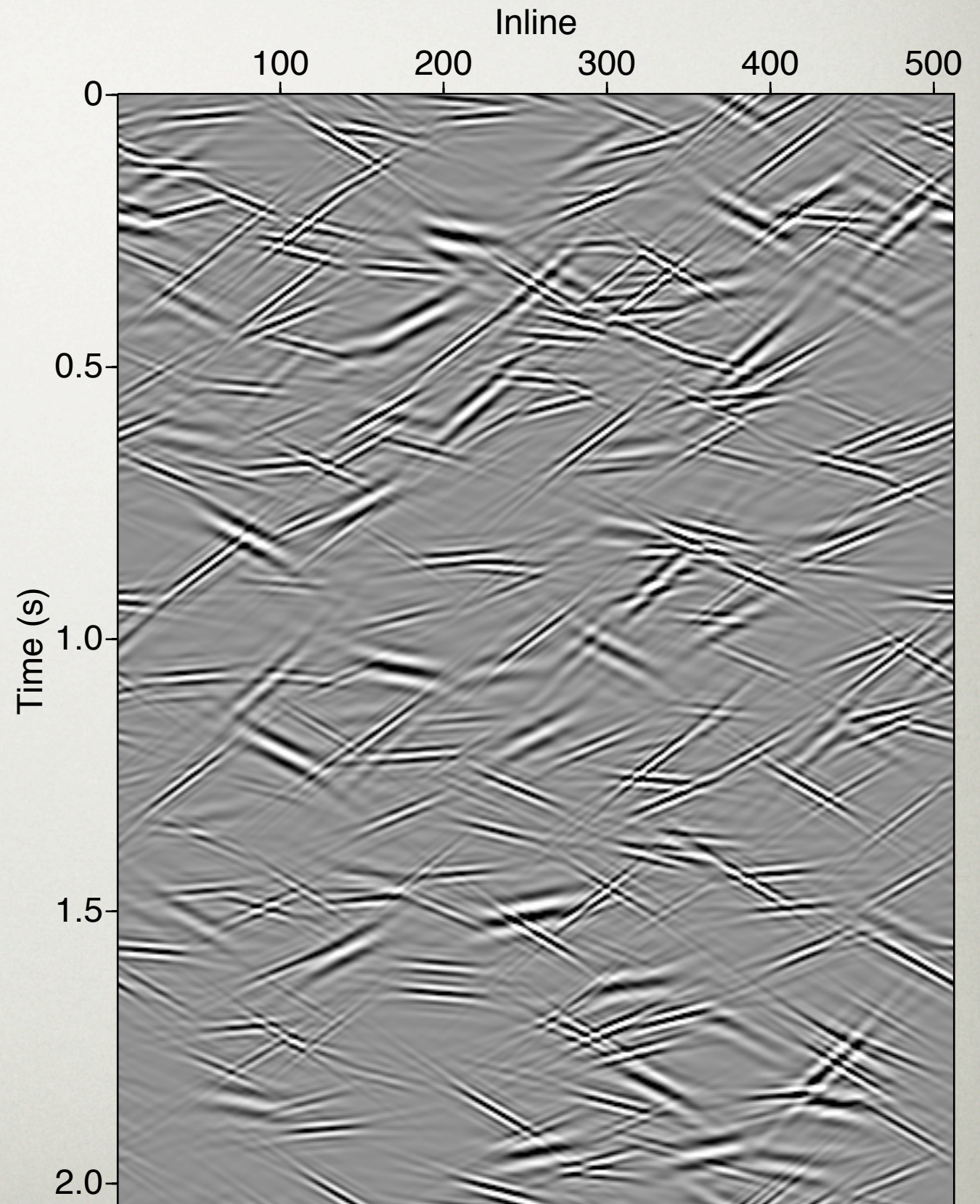


Our interpolation problem:

$$\hat{\mathbf{x}} = \arg \min_x \frac{1}{2} \|\mathbf{P} (\mathbf{d} - \mathbf{F}\mathbf{x})\|_2^2 + \|\mathbf{x}\|_1 \text{ with } \mathbf{F}\cdot = \mathbf{IdC}^*.$$

$$\hat{\mathbf{x}} = \arg \min_x \frac{1}{2} \|\mathbf{P}(\mathbf{d} - \mathbf{F}\mathbf{x})\|_2^2 + \|\mathbf{x}\|_1$$

- Seismic data is assumed as a *superposition* of curvelets
- Curvelet coefficient vector is assumed to be sparse (*prior*)

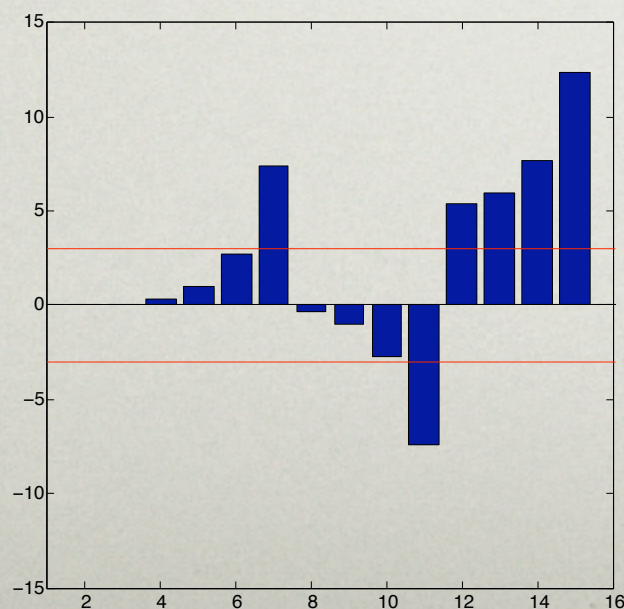


ℓ^1 -NORM OPTIMIZATION BY ITERATIVE THRESHOLDING

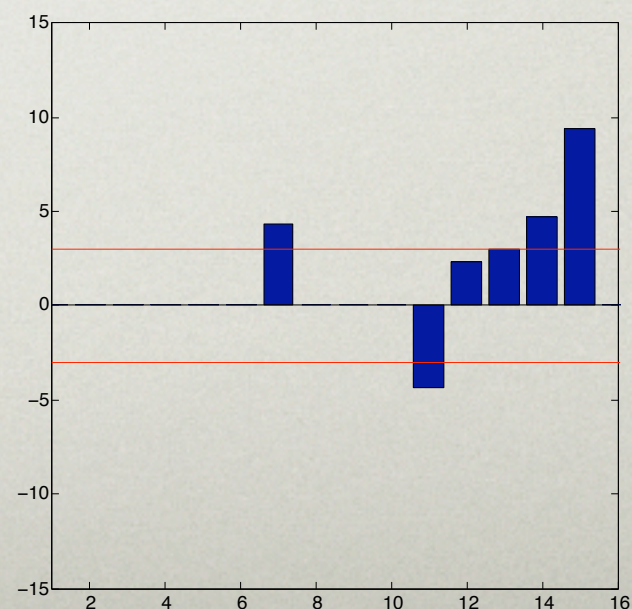
Denoising with Landweber iterations and soft-thresholding [Daubechies, 05; Elad, 05]

$$\mathbf{x}^m = S_{\lambda_m}^s \left[\mathbf{x}^{m-1} + \mathbf{F}^T (\mathbf{d} - \mathbf{F}\mathbf{x}^{m-1}) \right]$$

$$S_{\lambda}^s(x) = \begin{cases} x - \text{sign}(x)\lambda & |x| \geq \lambda \\ 0 & |x| < \lambda \end{cases}$$



$$\xrightarrow{S_{\lambda}^s}$$

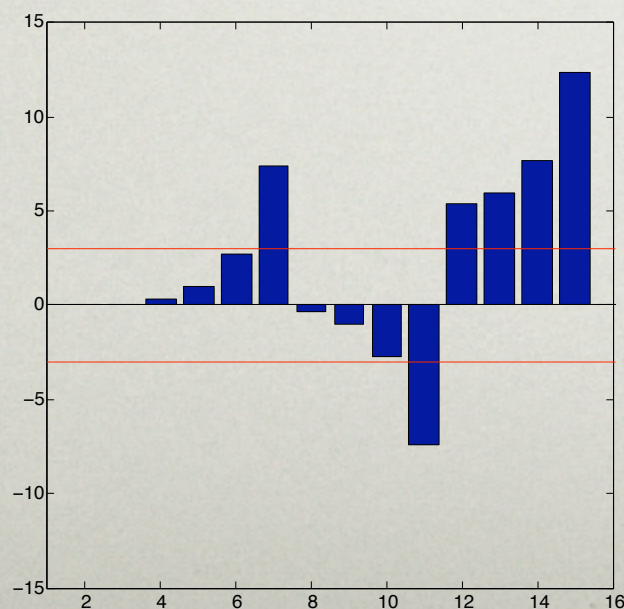


ℓ^1 -NORM OPTIMIZATION BY ITERATIVE THRESHOLDING

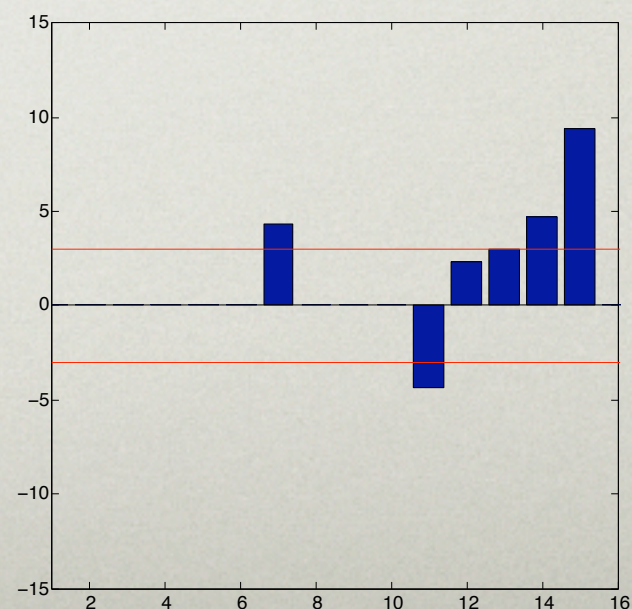
Data-continuation with Landweber iterations and soft-thresholding [Daubechies, 05; Elad, 05]

$$\mathbf{x}^m = \mathcal{S}_{\lambda_m}^s \left[\mathbf{x}^{m-1} + \mathbf{F}^* \mathbf{P}(\mathbf{d} - \mathbf{F} \mathbf{x}^{m-1}) \right]$$

$$\mathcal{S}_{\lambda}^s(x) = \begin{cases} x - \text{sign}(x)\lambda & |x| \geq \lambda \\ 0 & |x| < \lambda \end{cases}$$



$$\xrightarrow{\mathcal{S}_{\lambda}^s}$$

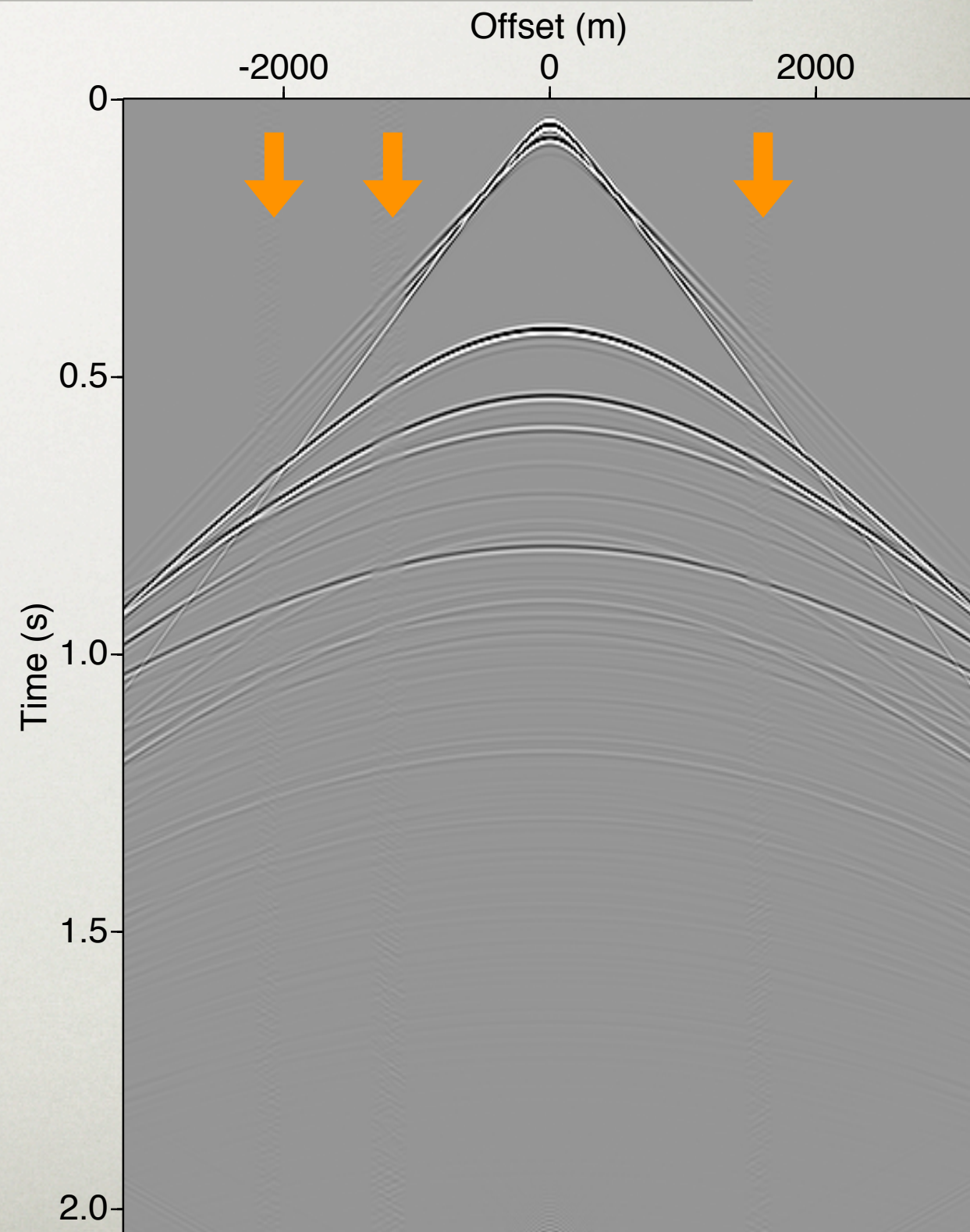
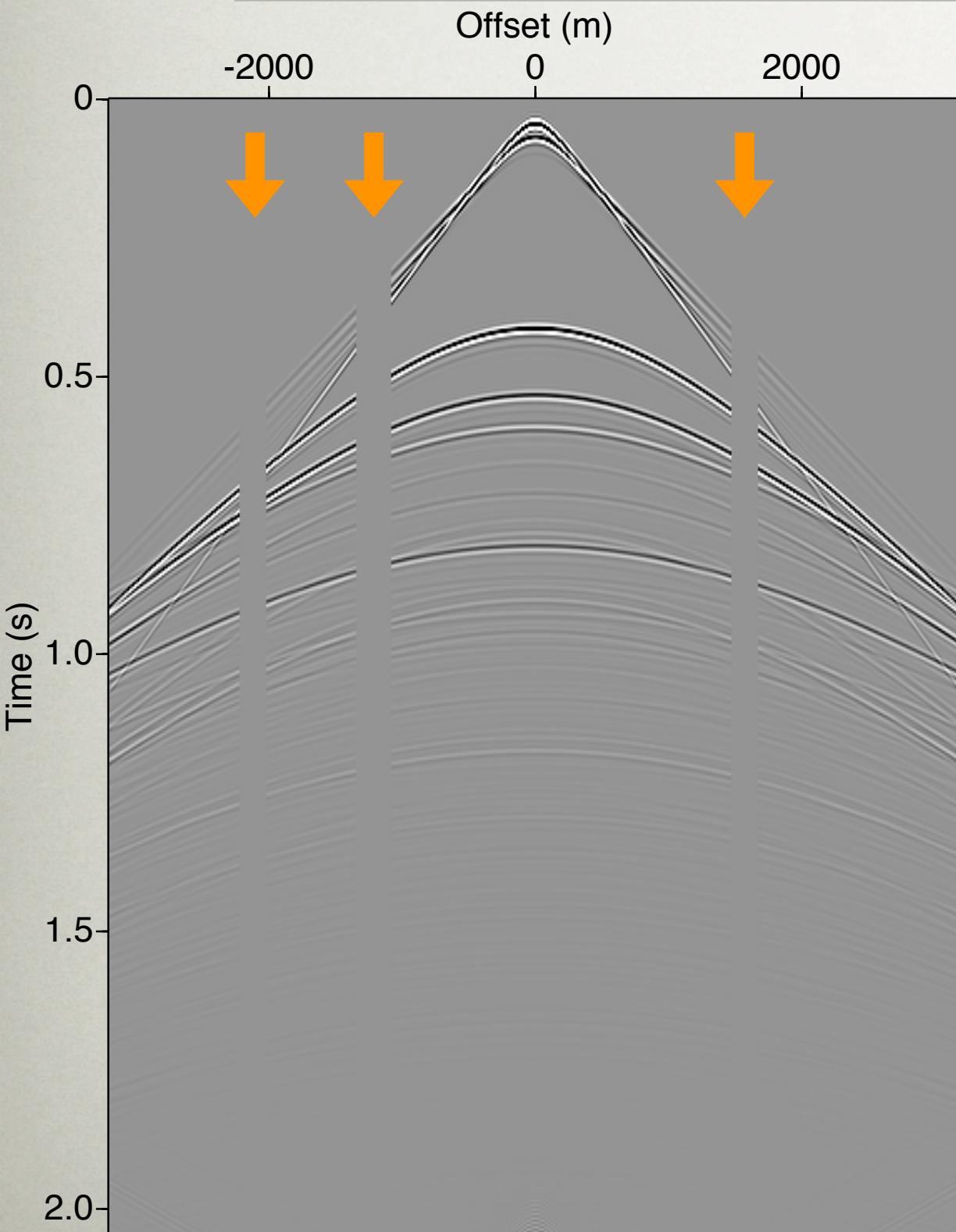


EXAMPLES

- 2-D synthetic data (512 offsets x 512 time samples)
 - Data continuation
 - De-aliasing
- 3-D real data (280 shots x 368 receivers x 256 time samples)
 - Data continuation

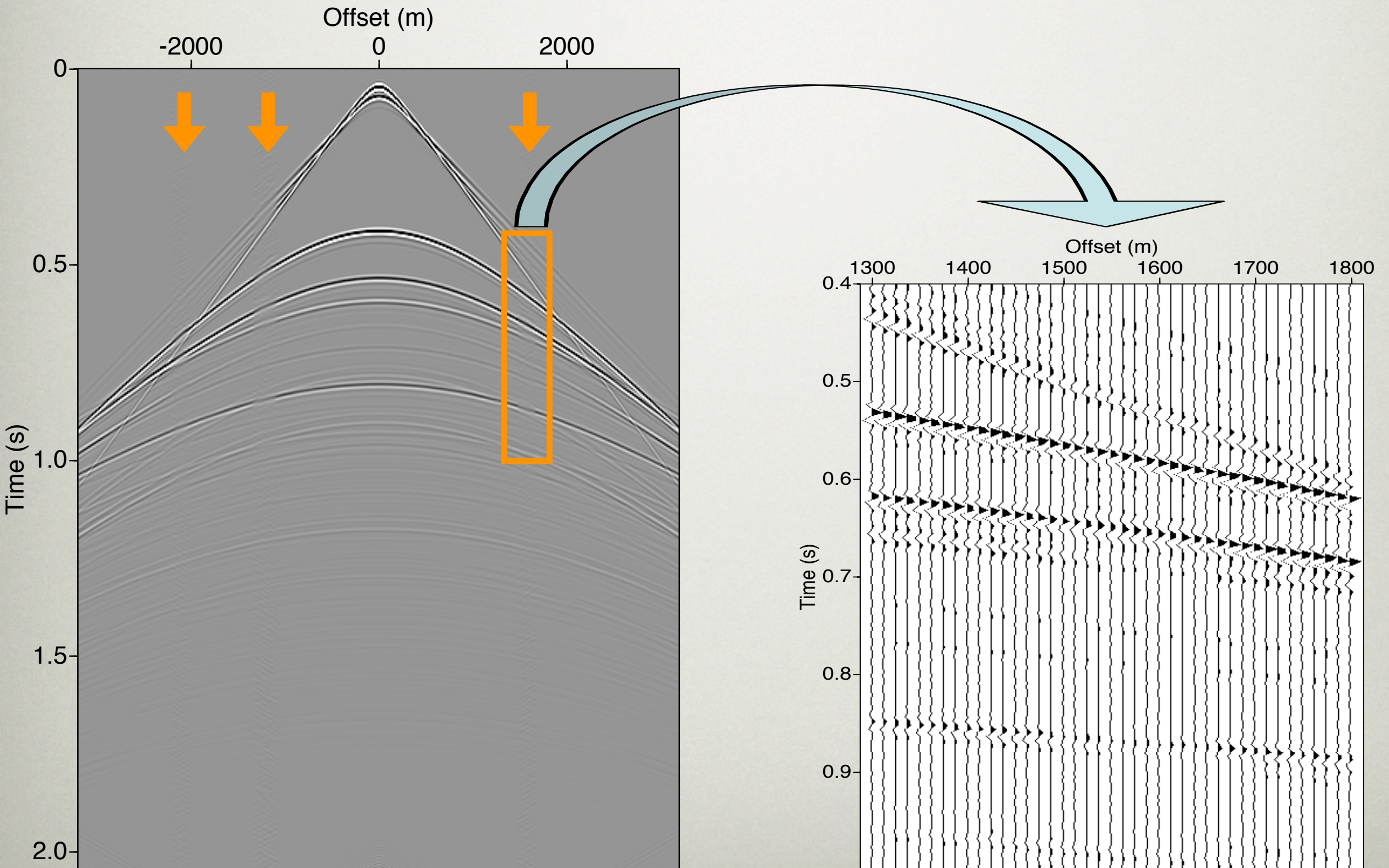
2-D SYNTHETIC SHOT

DATA CONTINUATION (NOISE-FREE)



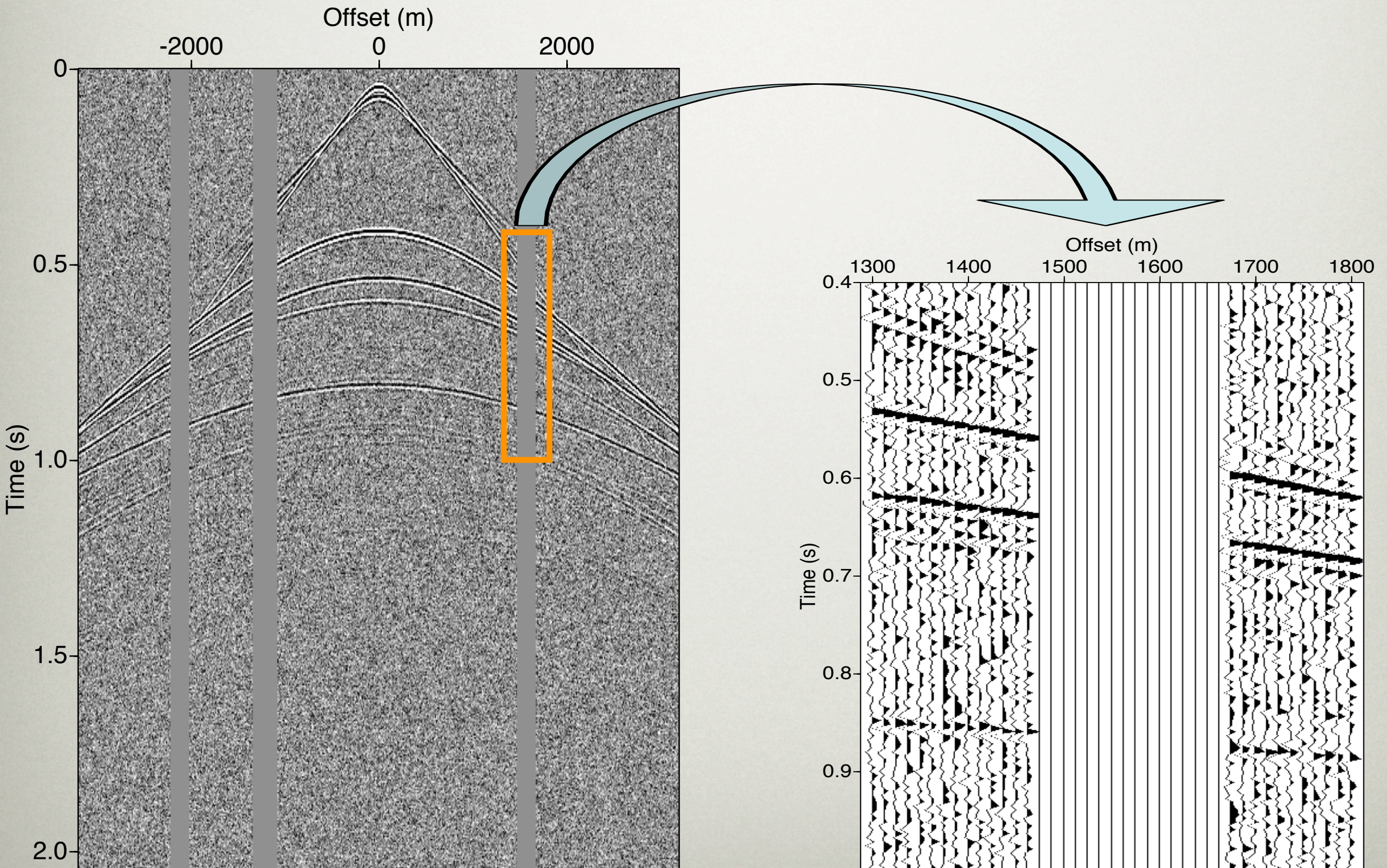
2-D SYNTHETIC SHOT

DATA CONTINUATION (NOISE-FREE)



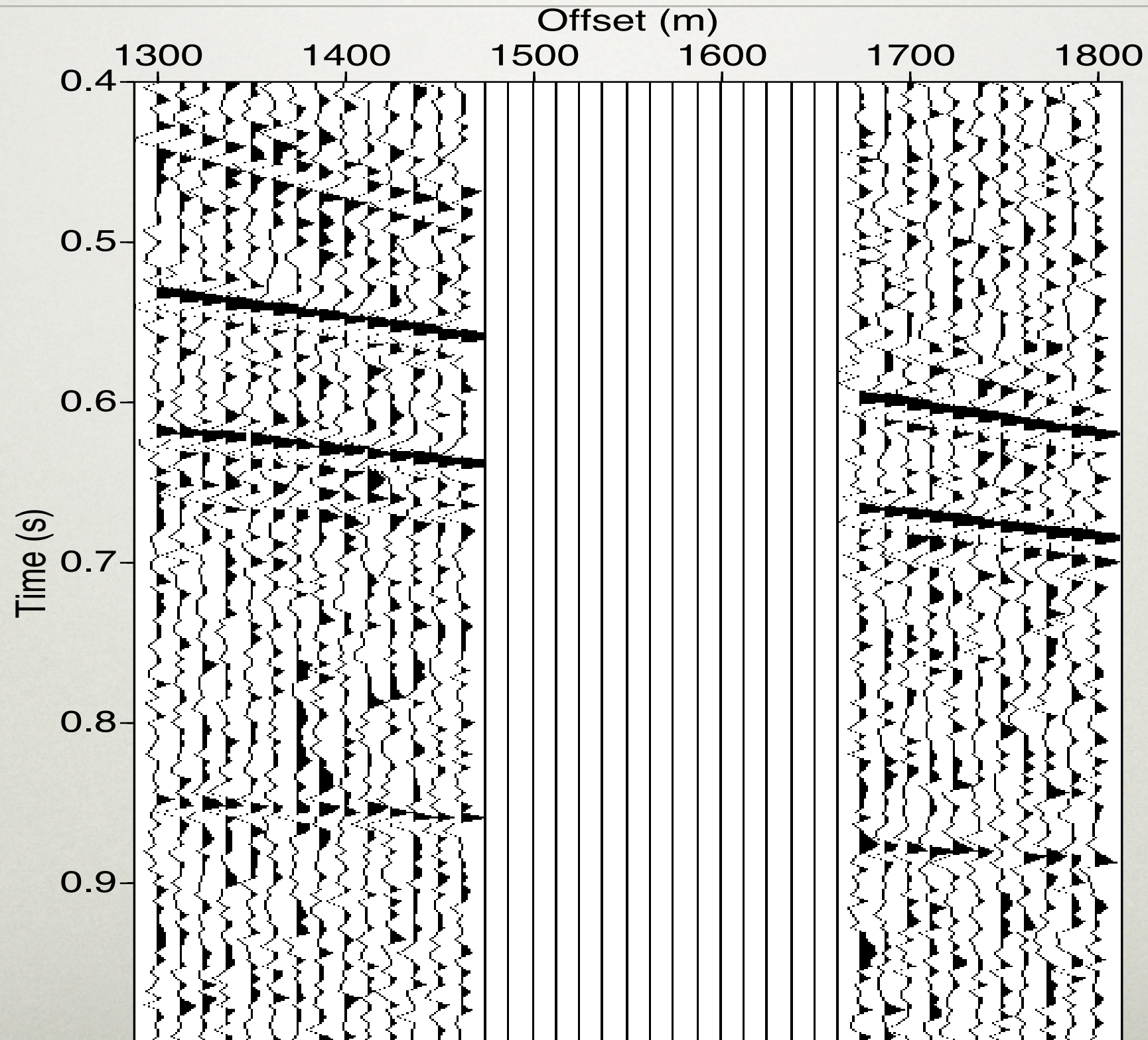
2-D SYNTHETIC SHOT

DATA CONTINUATION (NOISY - SNR = 0 DB)



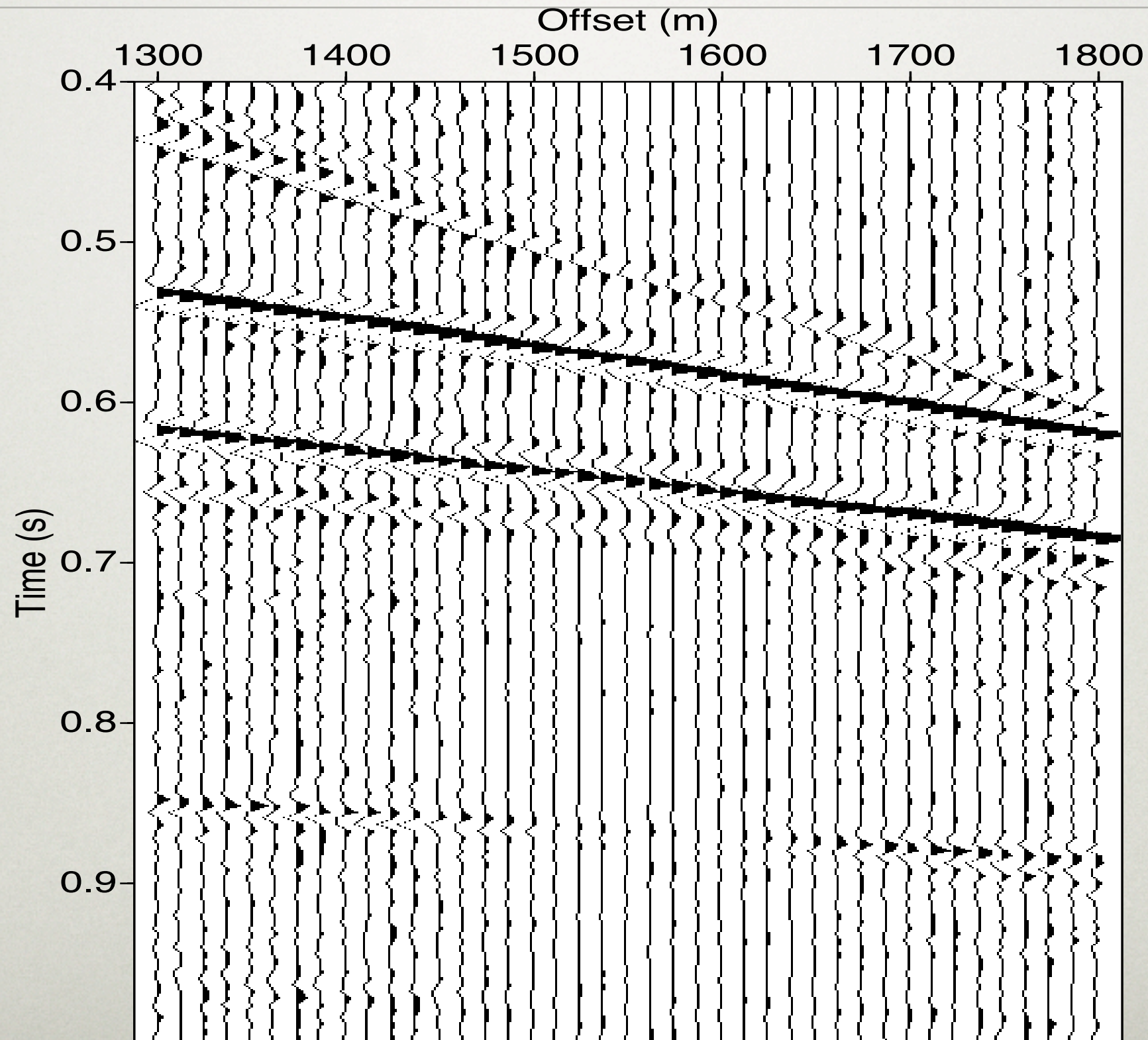
2-D SYNTHETIC SHOT

DATA CONTINUATION (NOISY)



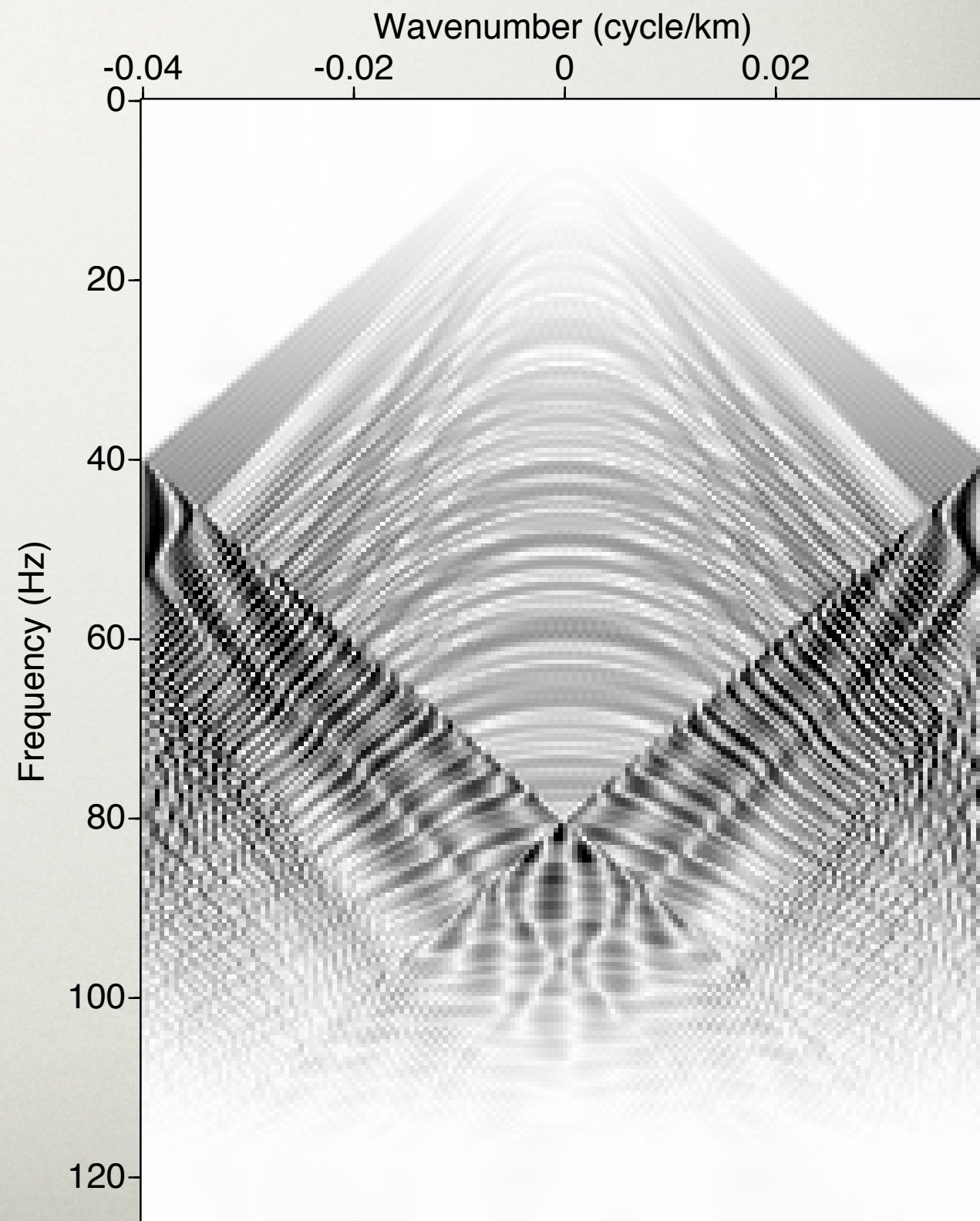
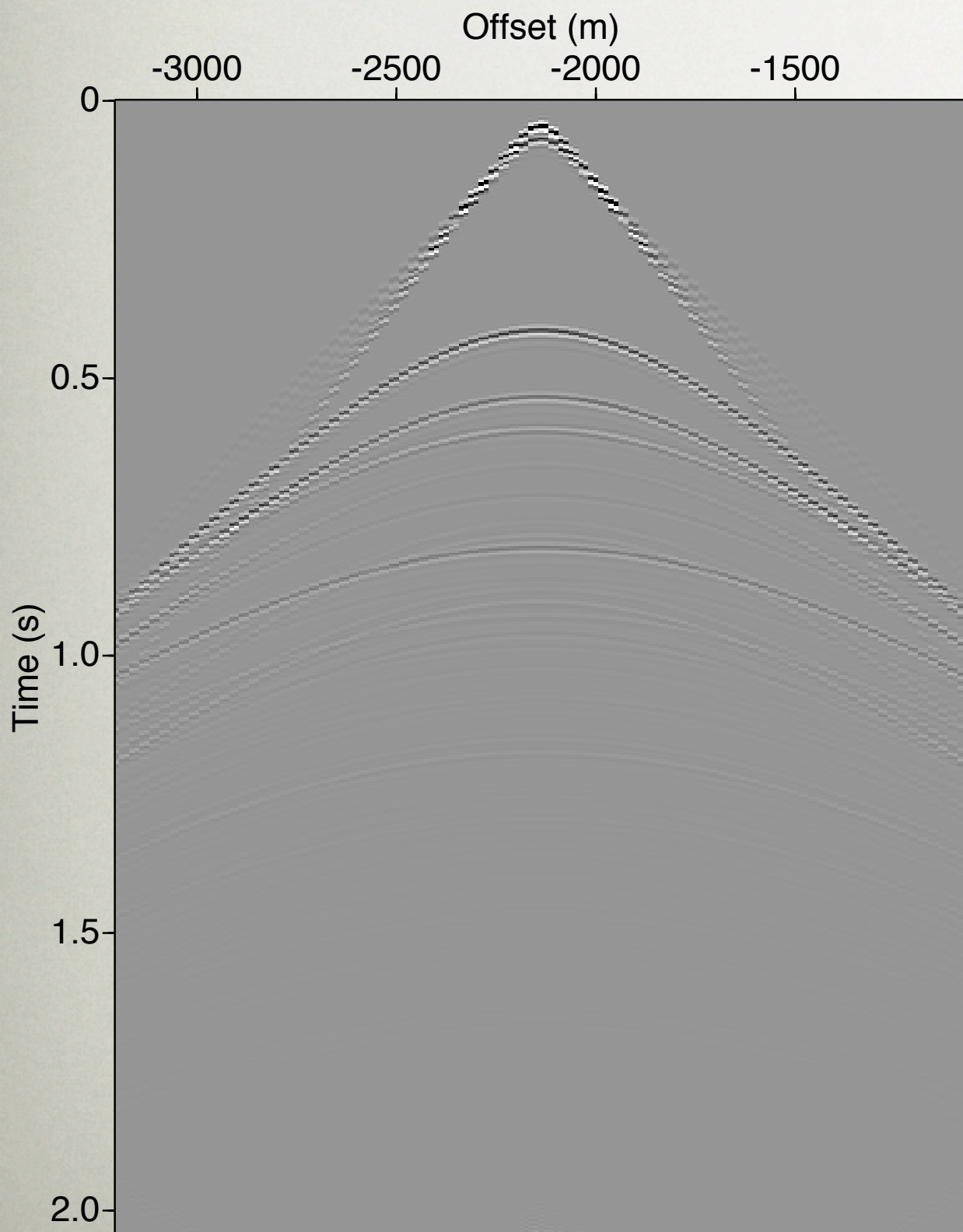
2-D SYNTHETIC SHOT

DATA CONTINUATION (NOISY)



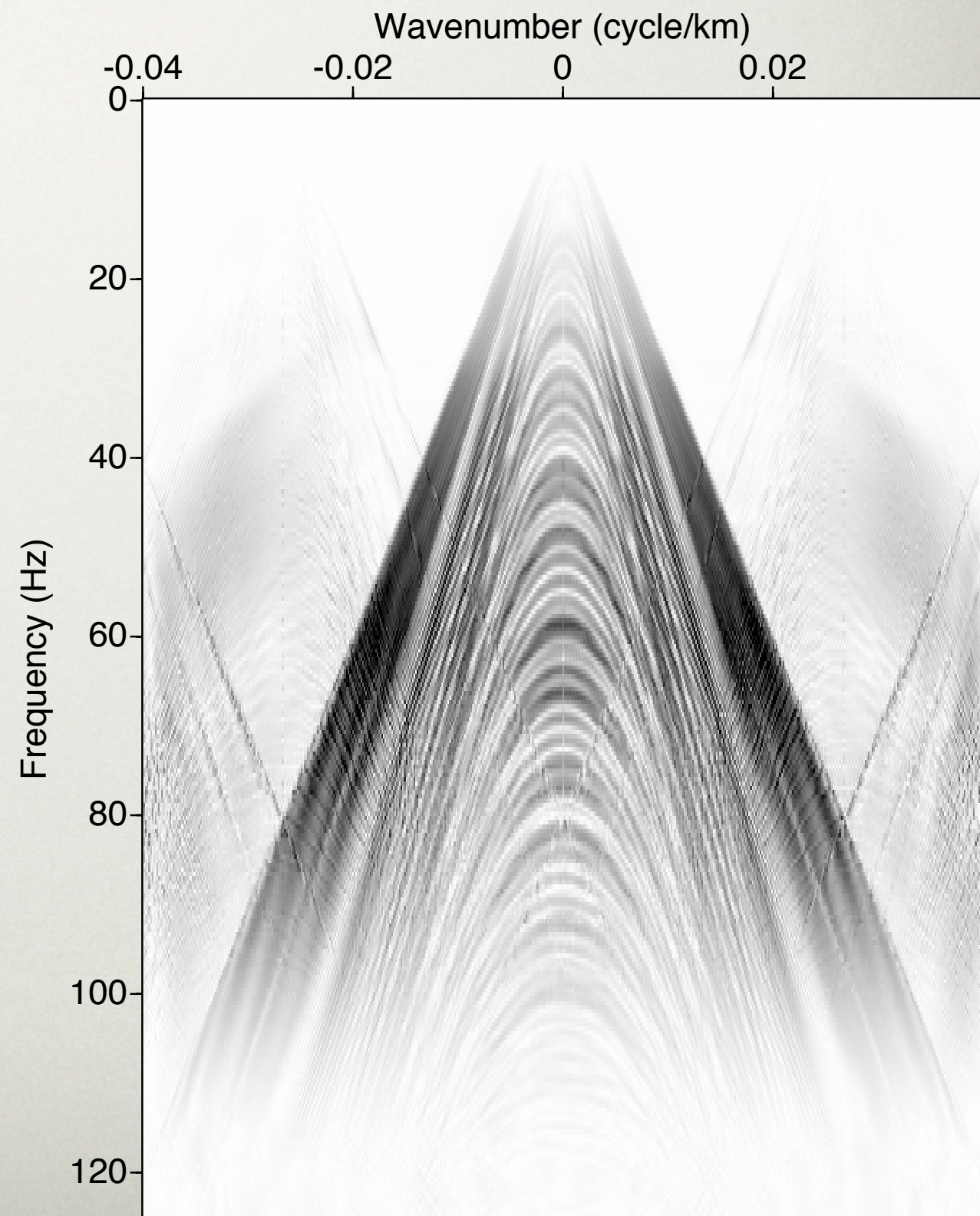
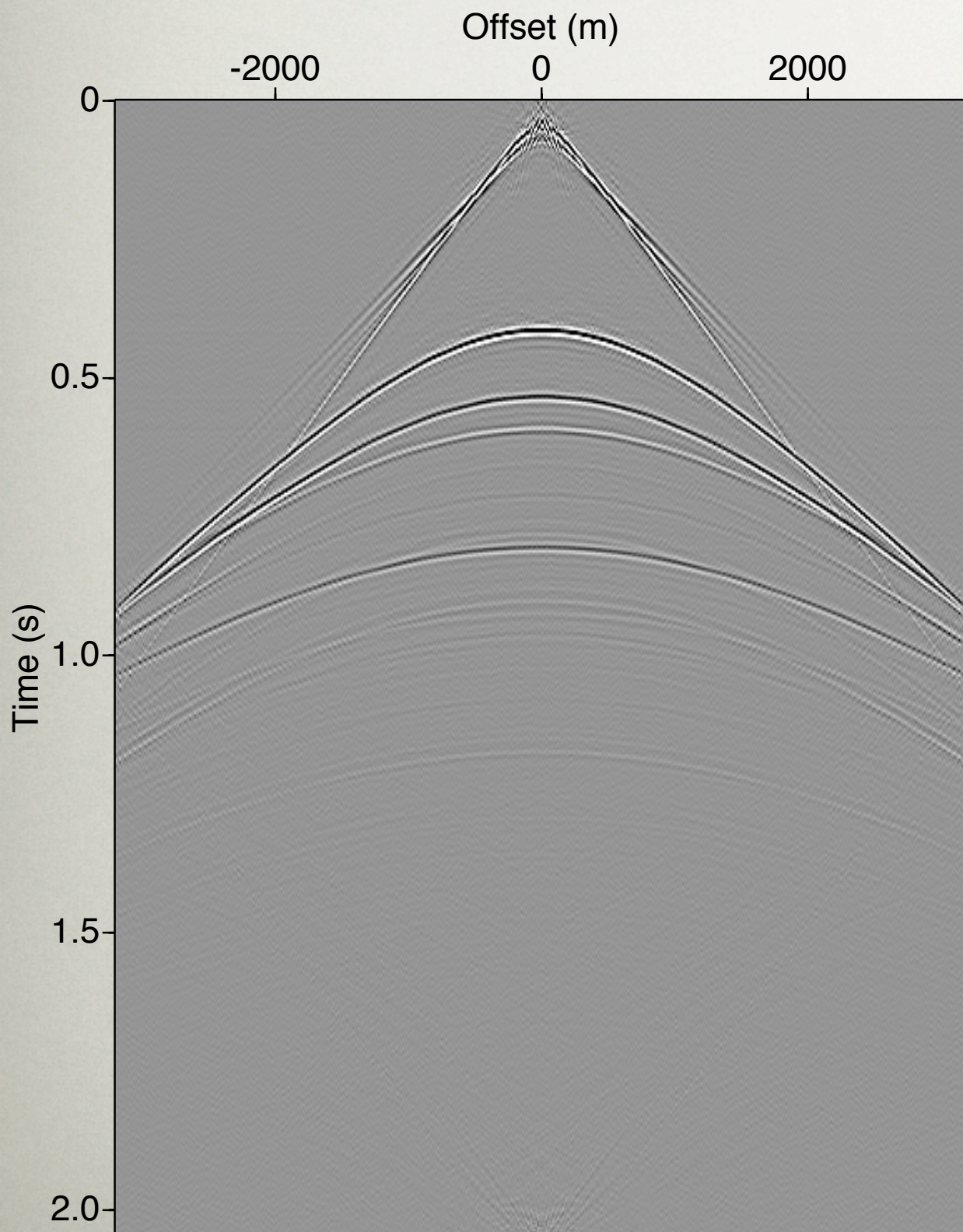
2-D SYNTHETIC SHOT

DE-ALIASING



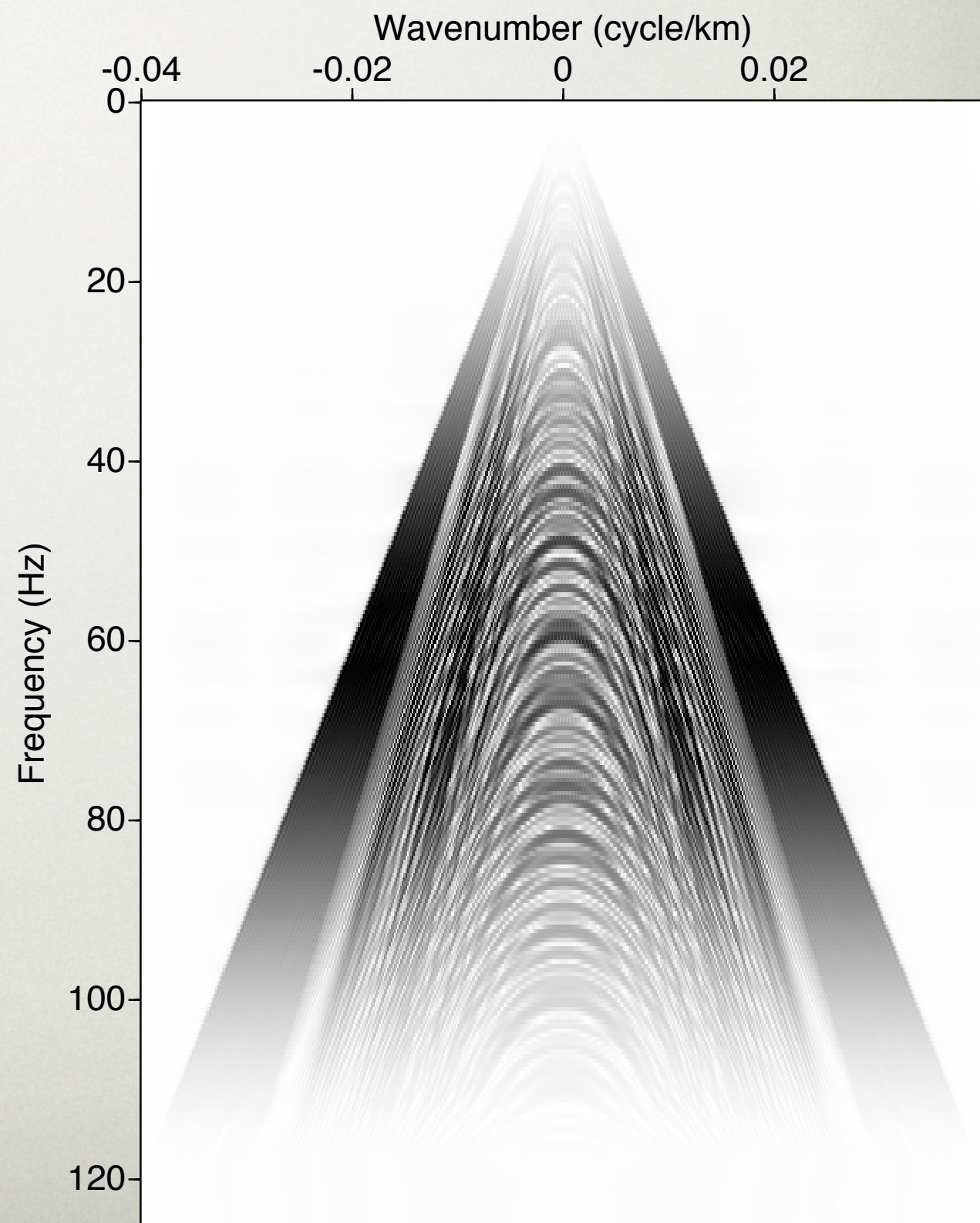
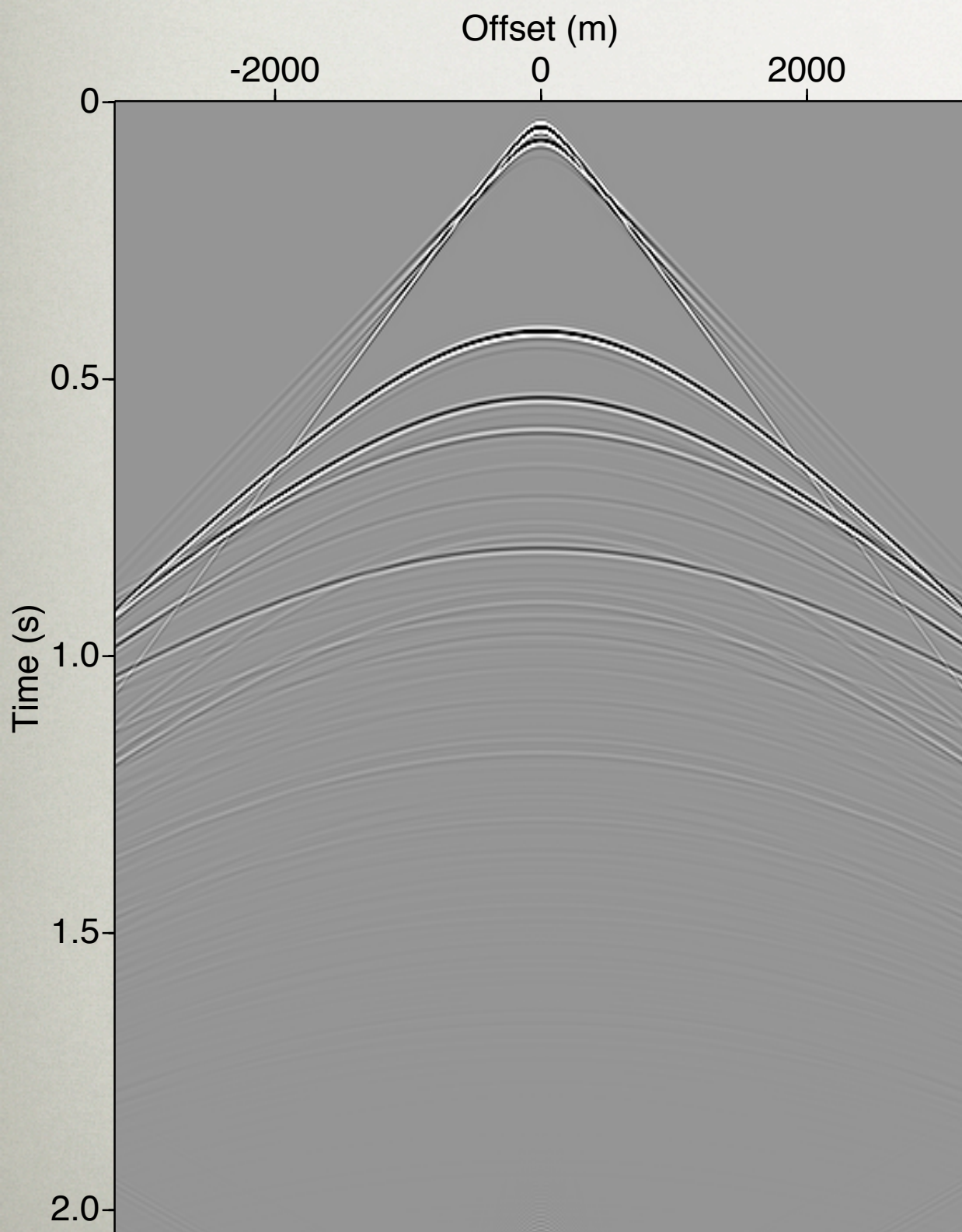
2-D SYNTHETIC SHOT

DE-ALIASING

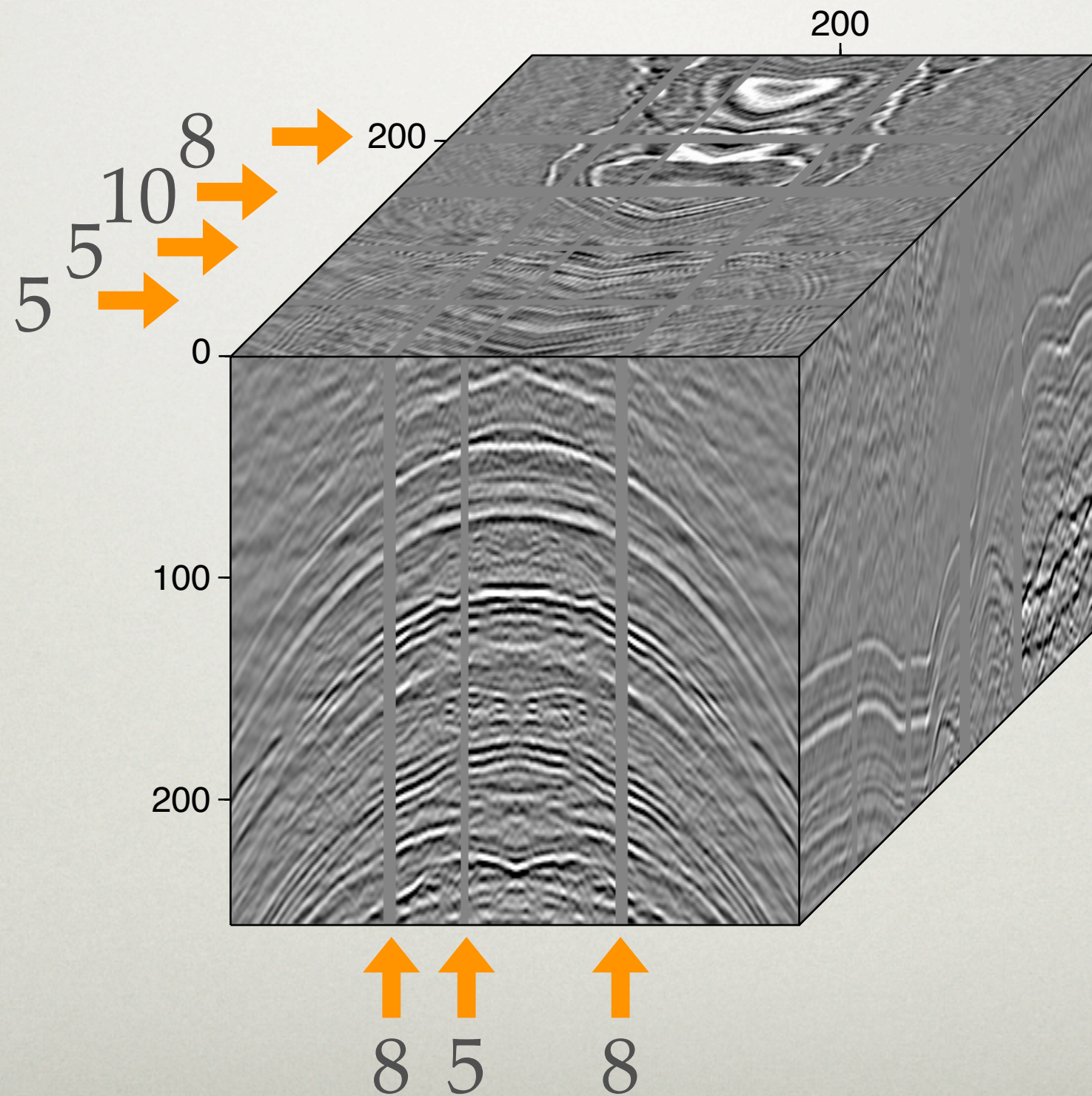


2-D SYNTHETIC SHOT

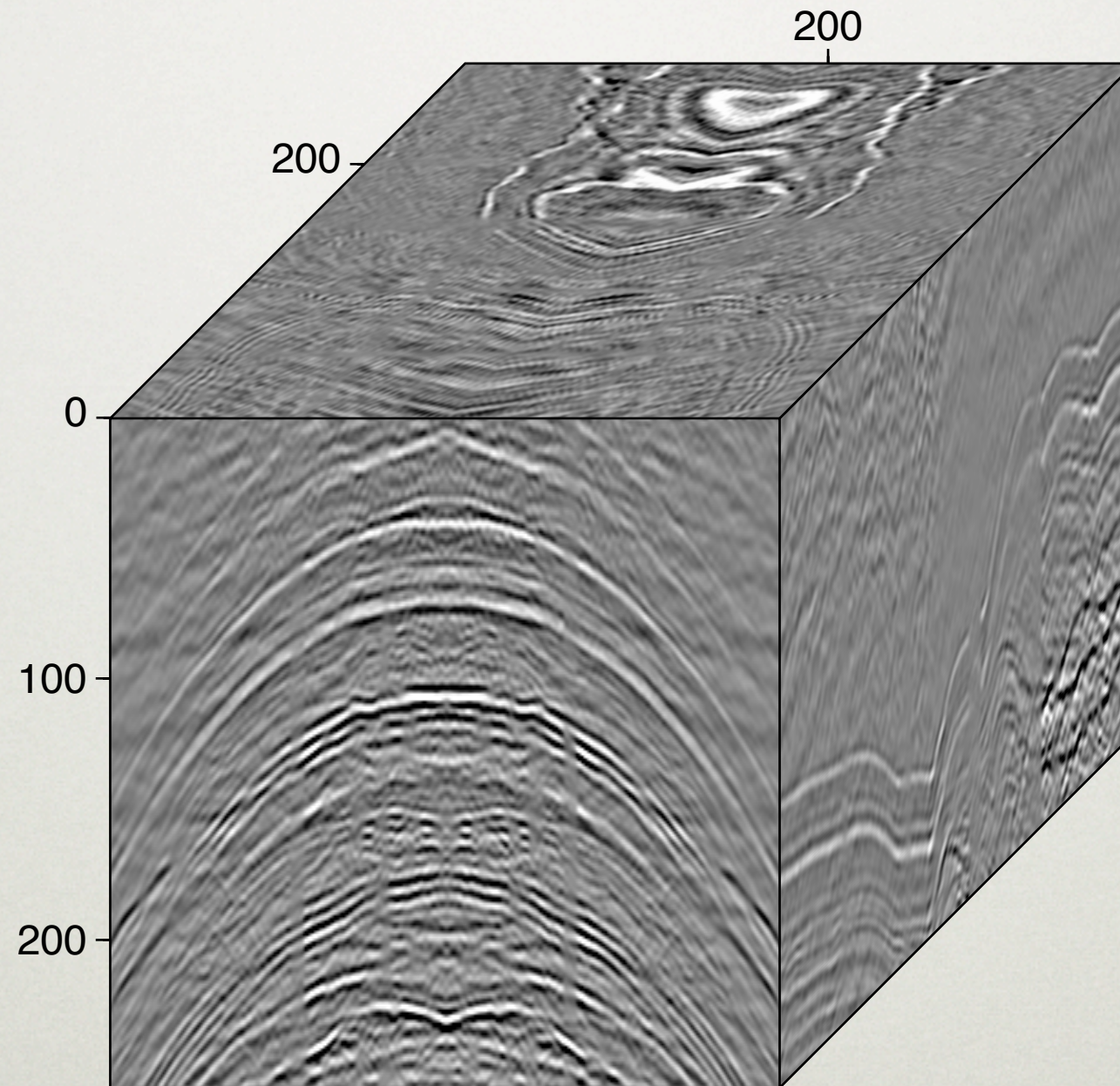
DE-ALIASING



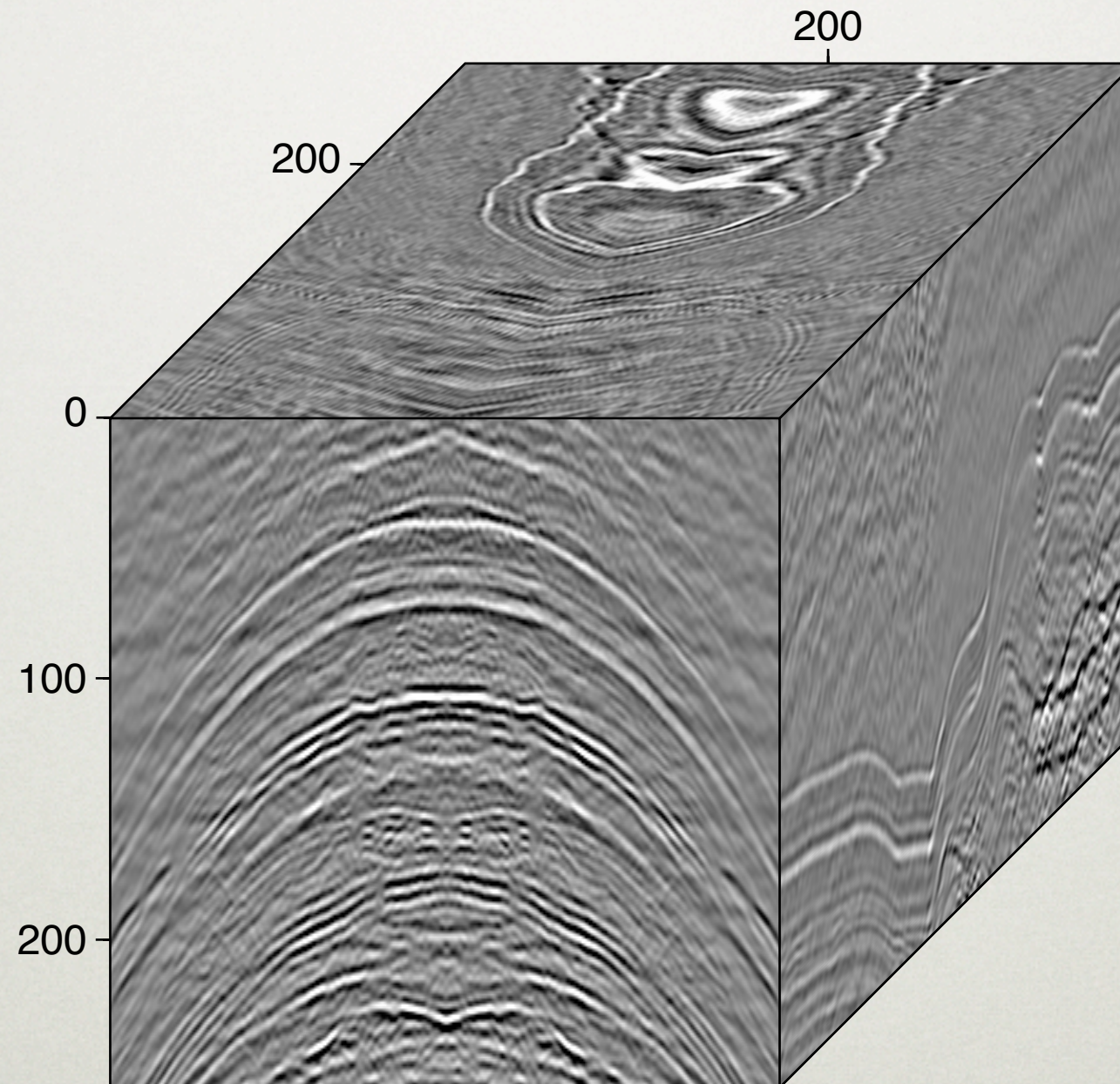
3-D REAL DATA

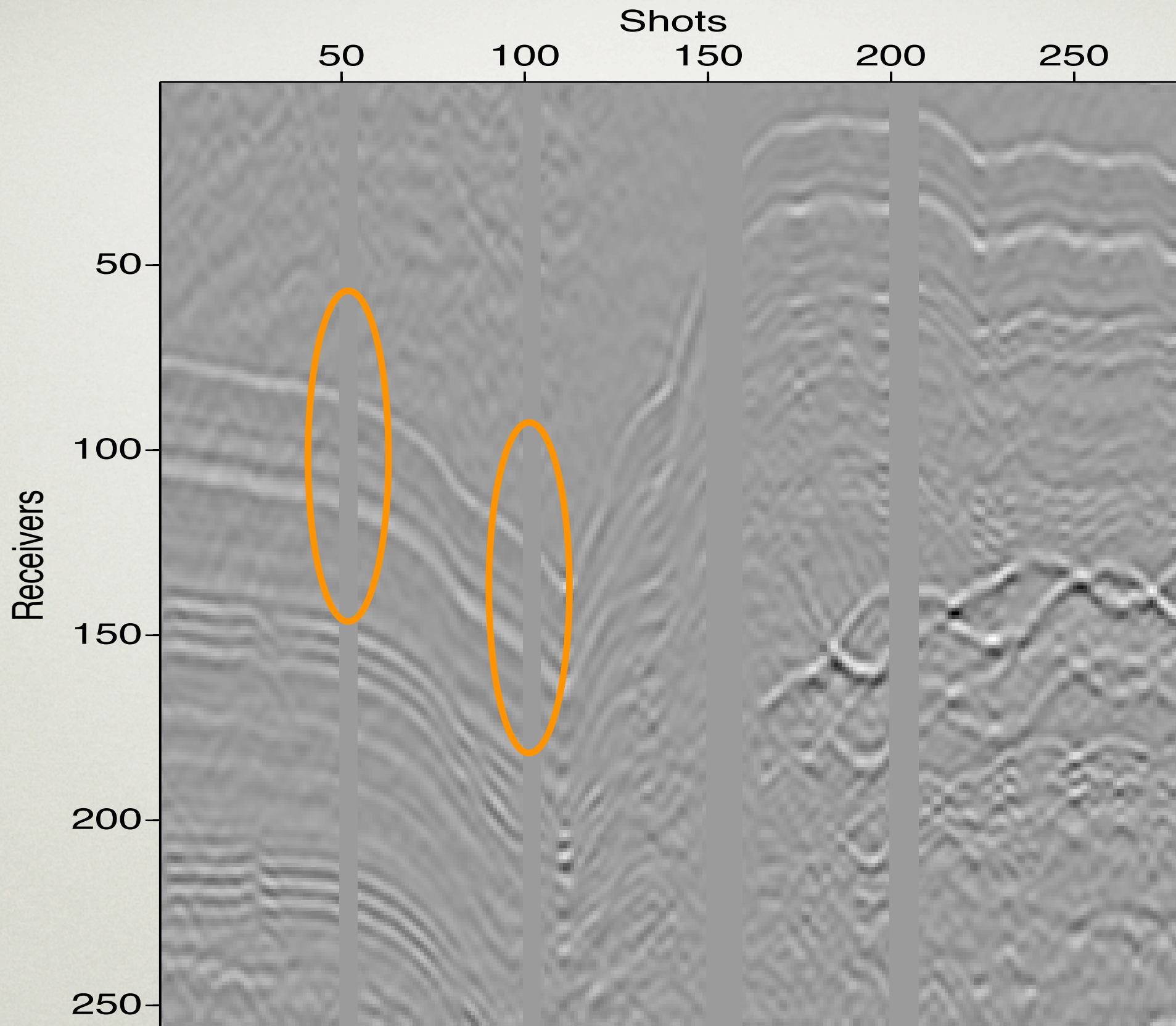


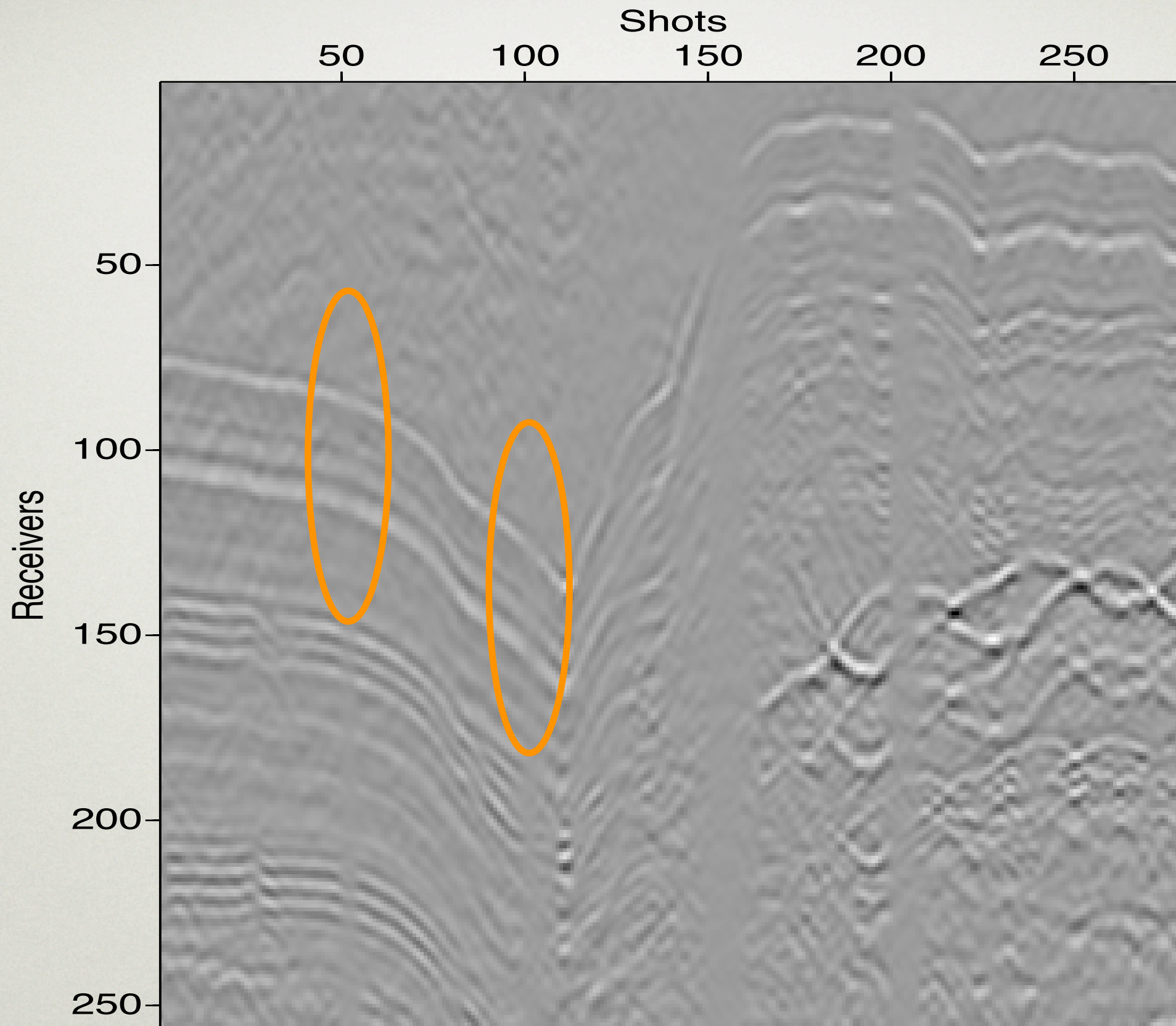
3-D REAL INTERPOLATED RESULT

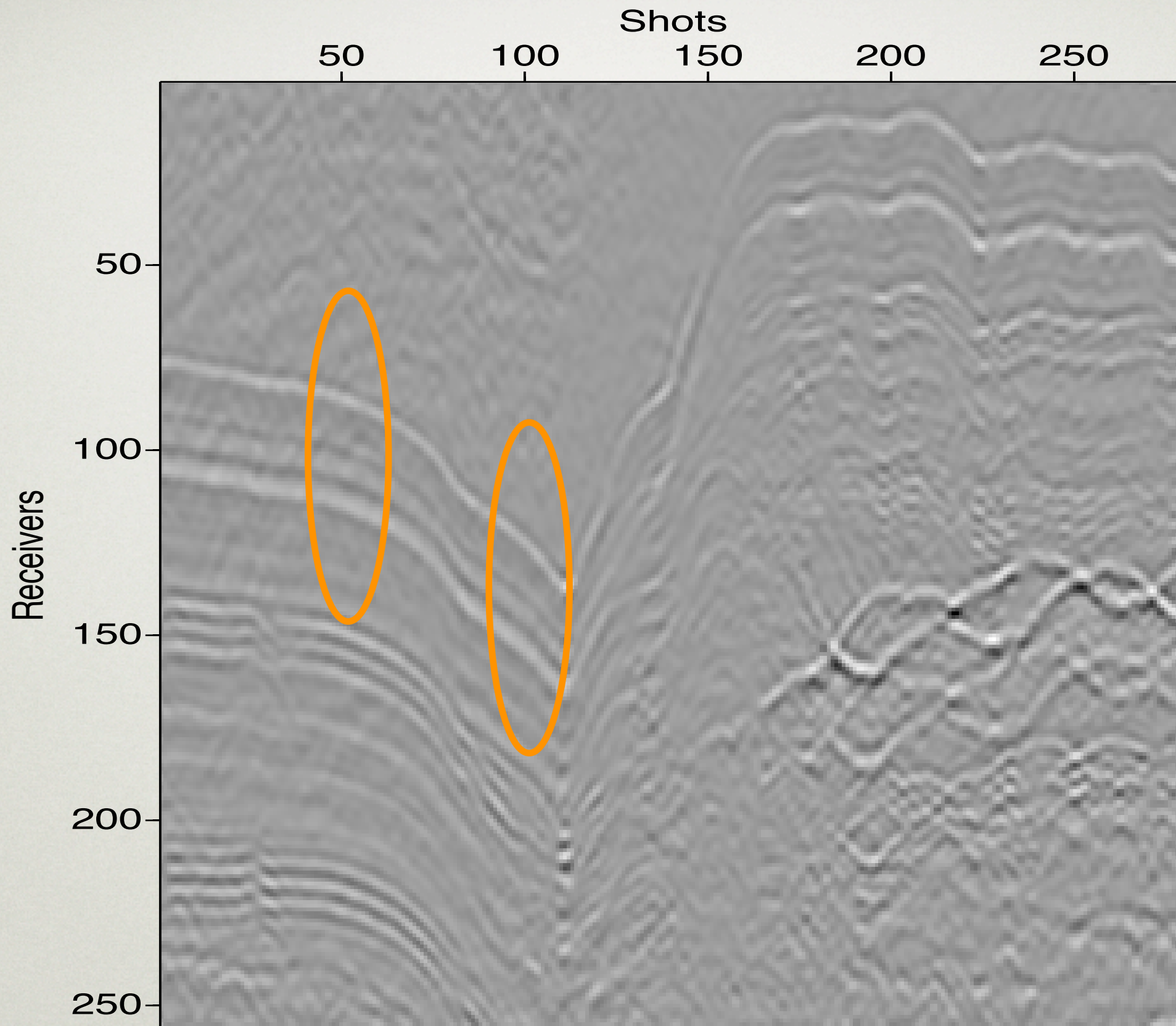


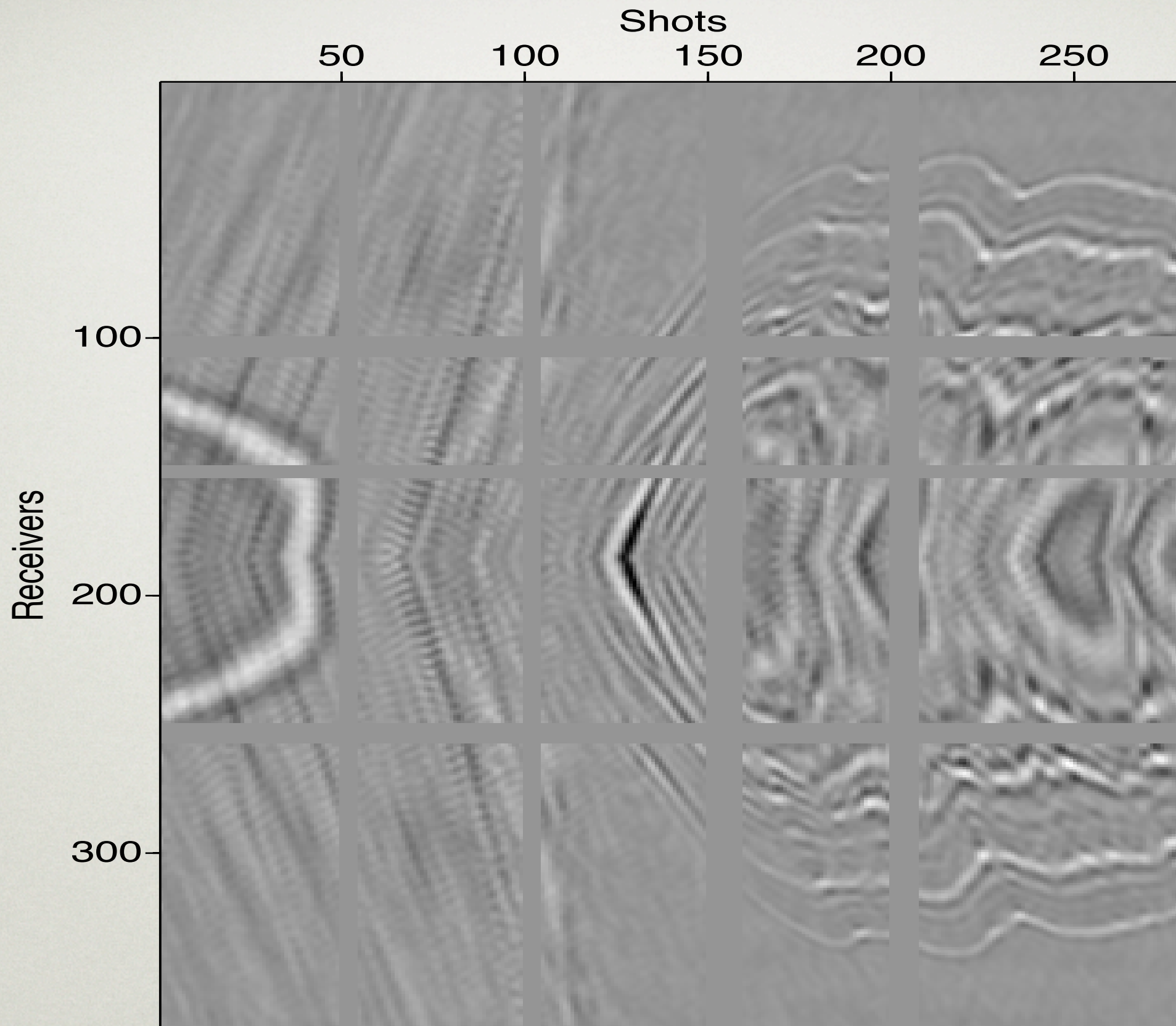
3-D REAL MODEL

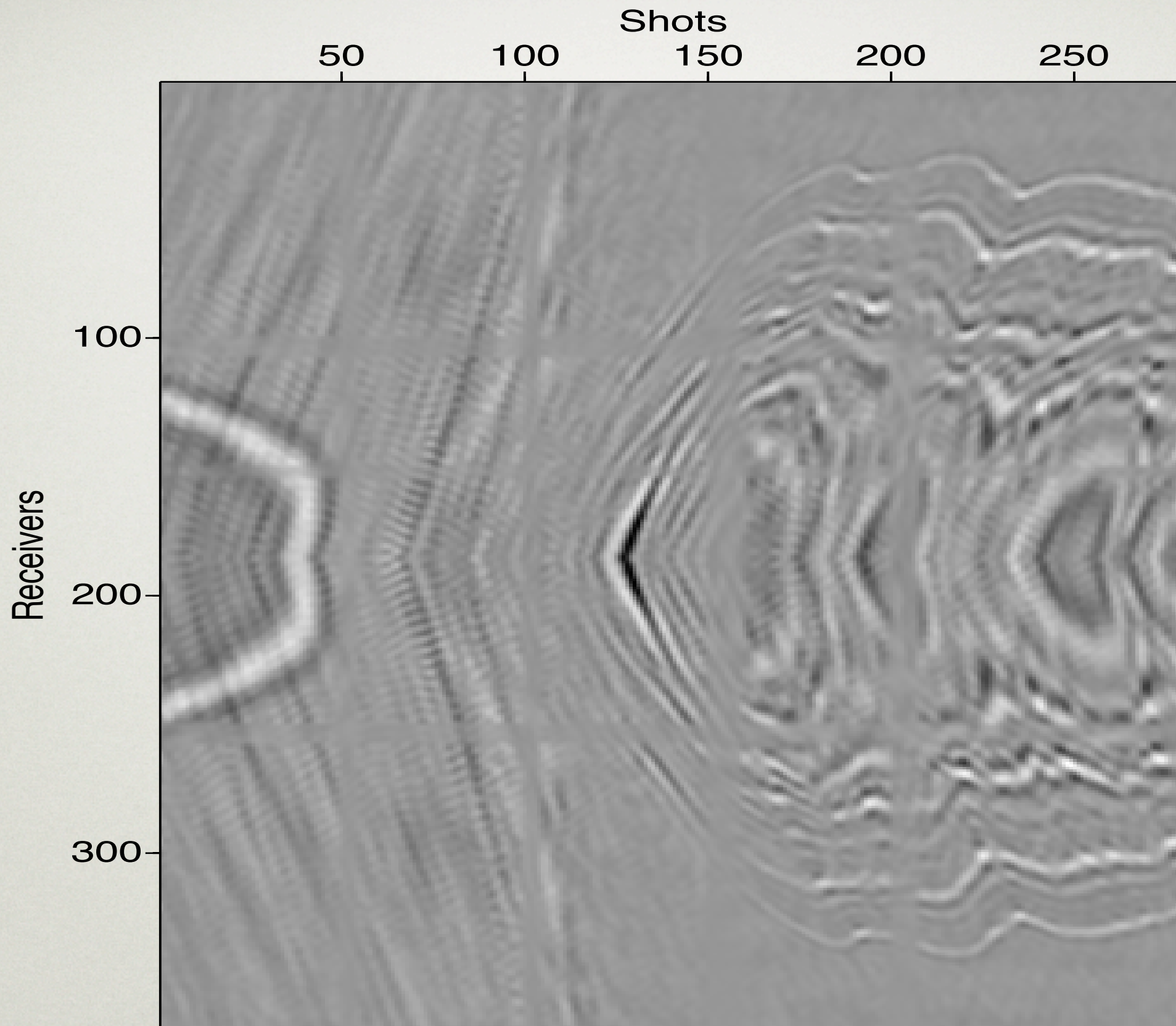


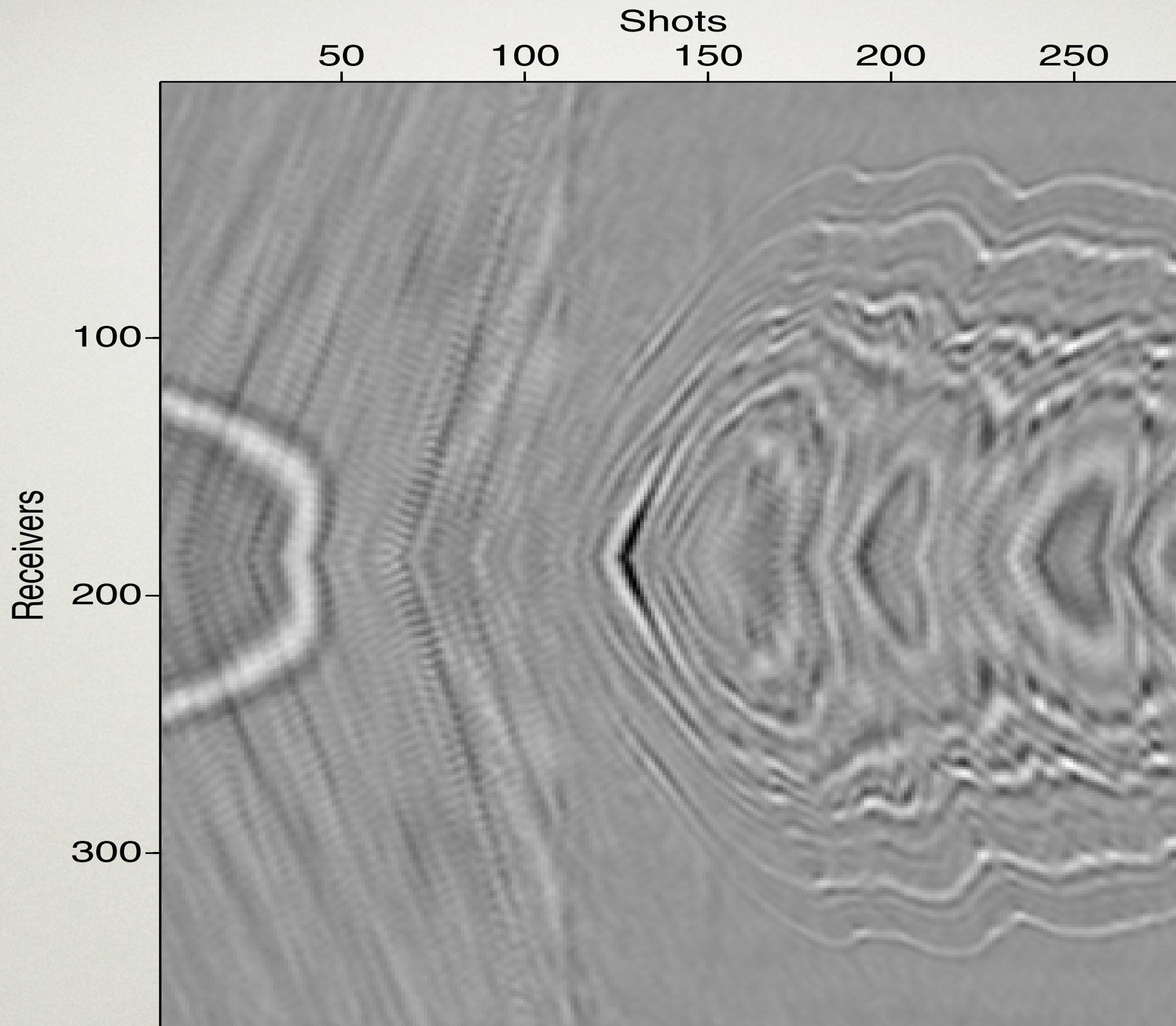




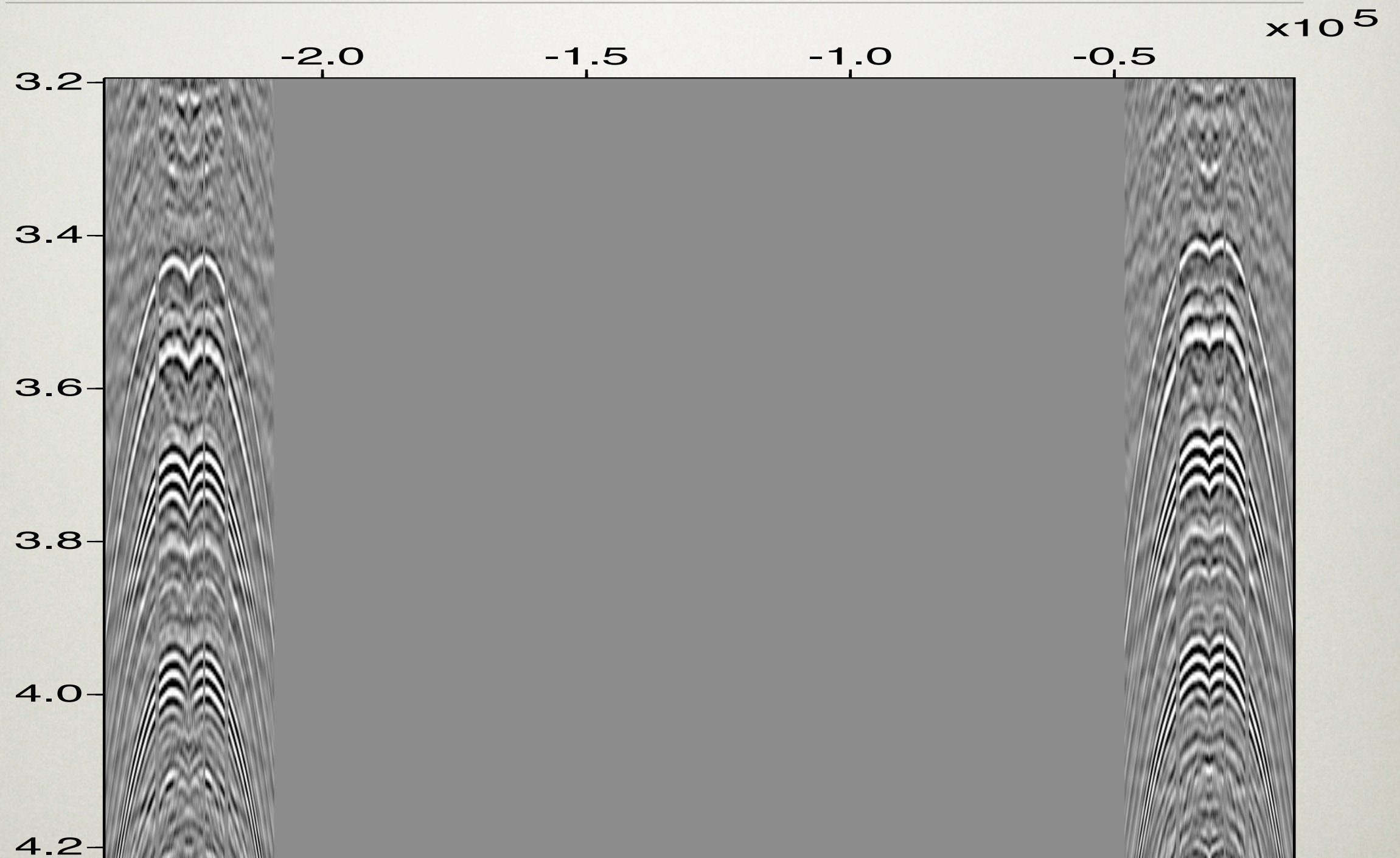




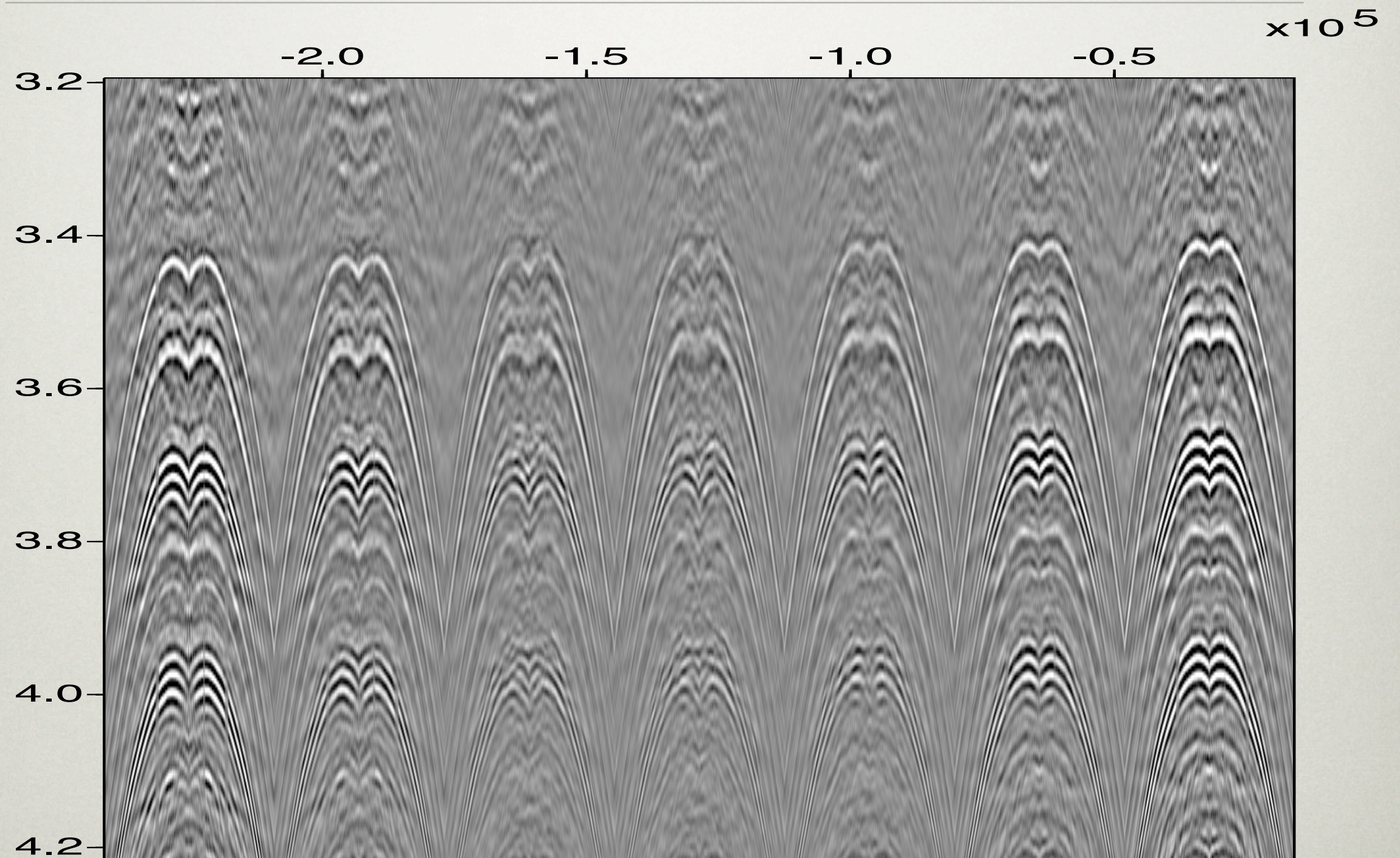




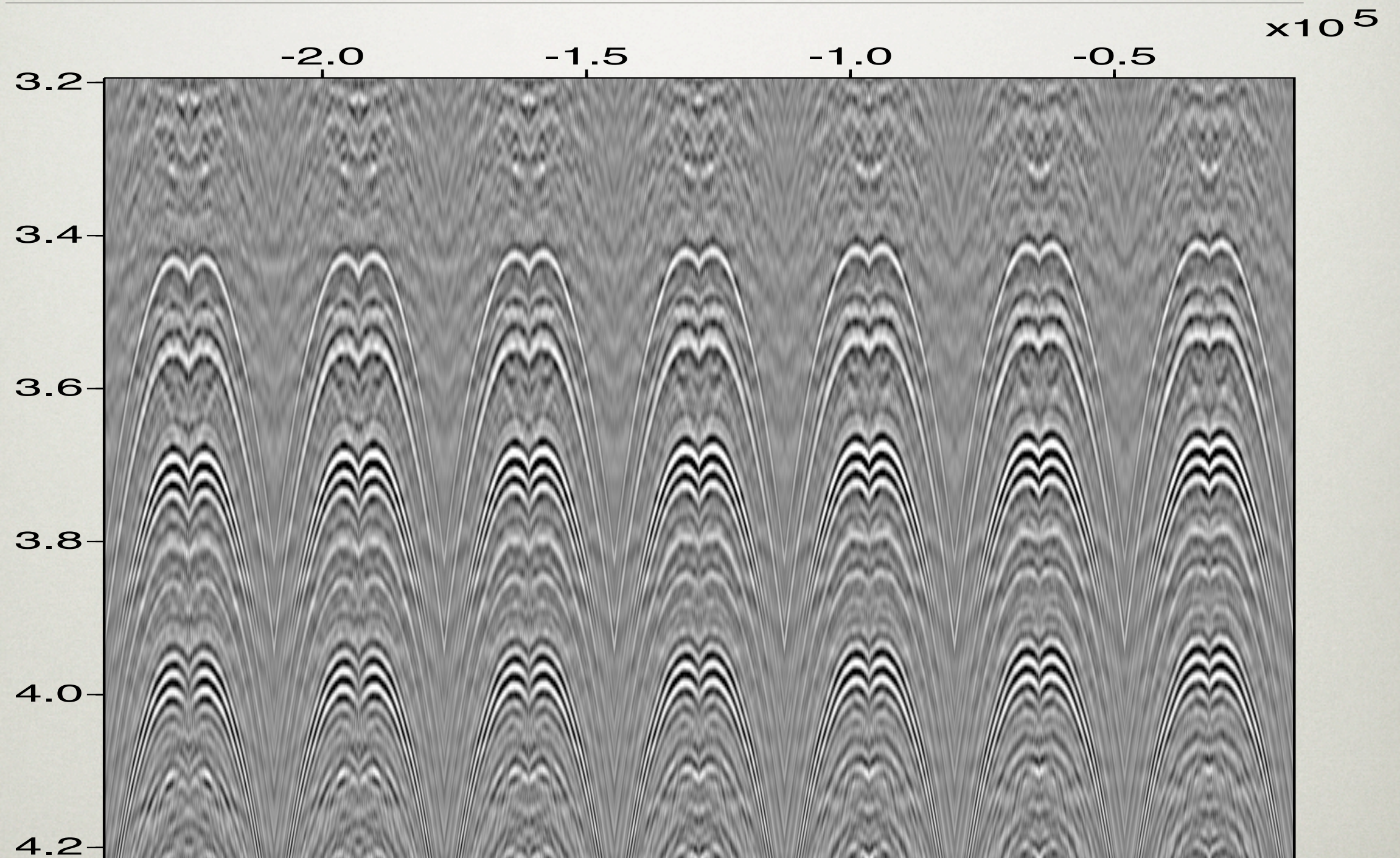
3-D REAL DATA



3-D REAL DATA



3-D REAL DATA



CONCLUSIONS

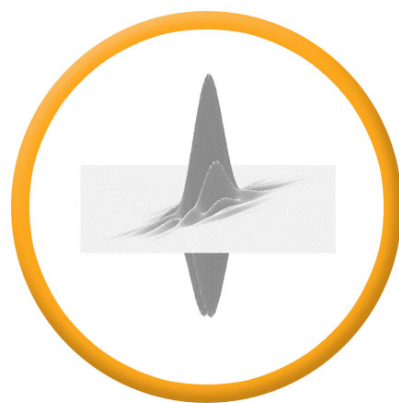
- Our method explores the 3-D continuity of reflection events in the data cube
- Sparsest set of coefficients that match the data
- The method is resilient to (coherent) noise
- Results are encouraging

CONCLUSIONS CONT'D

- More sophisticated solvers may improve convergence
- No parametric assumptions except *intermittent* regularity
- Curvelet Frame can be extended to include DCT

ACKNOWLEDGMENTS

- Authors of CurveLab (Candes, Donoho, Demanet and Ying)
- SINBAD sponsored through ITF by British Gas, British Petroleum, ExxonMobil, Shell
- Western Geco for providing the data
- NSERC (Discovery grant 22R81254)



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