Randomized wavefield inversion

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Joint work with Yogi Erlangga, and Tim Lin

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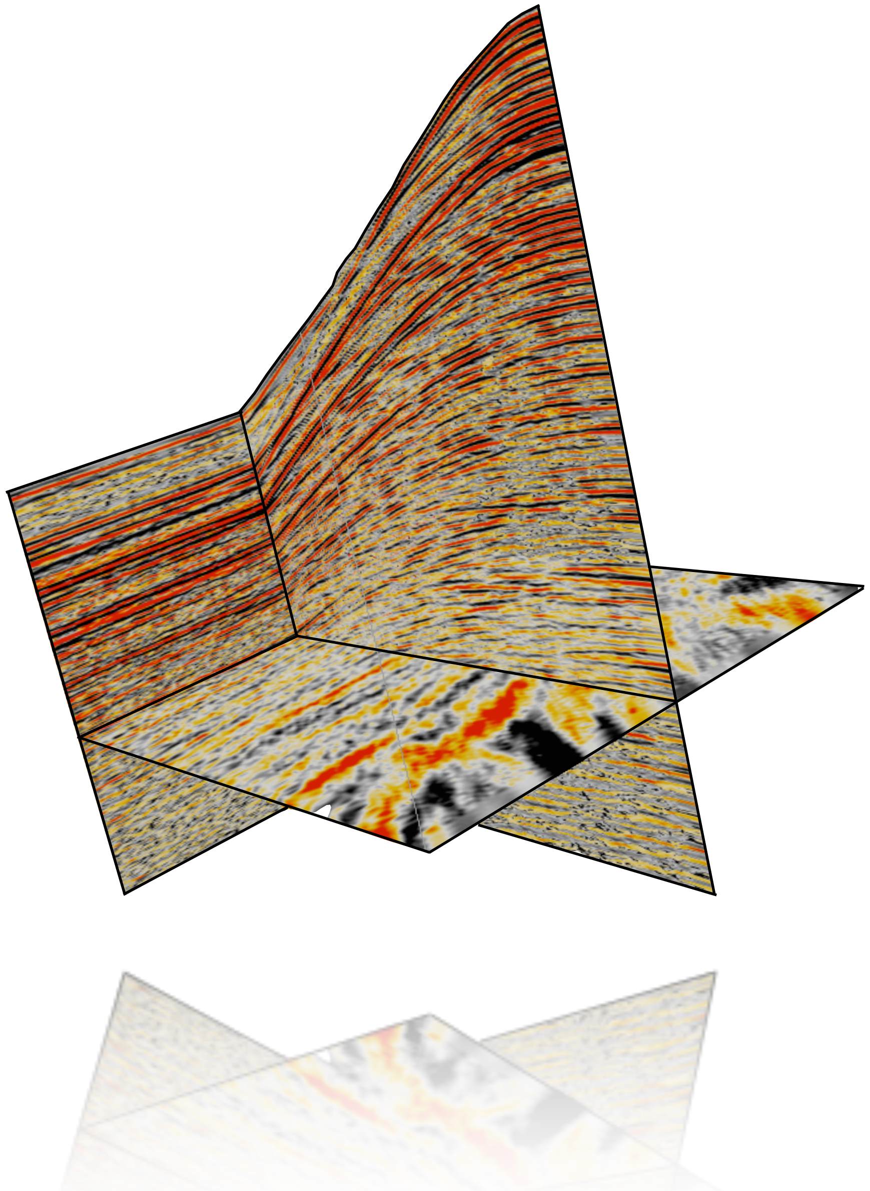


Delphi, June 4th, 2009

Motivation

- **Seismic data processing, modeling & inversion:**
  - firmly rooted in Nyquist's sampling paradigm for (modeled) wavefields
  - too *pessimistic* for signals with *structure*
  - existence of sparsifying transforms (e.g. curvelets)

- **Major impediment: "*curse of dimensionality*"**
  - *acquisition* **>>** *processing* & *inversion* **>>** *modeling* **costs** are proportional to the **size** of *data* and *image* space

- **Solution strategy:**
  - *leverage new* paradigm of *compressive sensing* (CS)
    - identify simultaneous acquisition as CS
    - reduce acquisition, simulation, and inversion costs by ***randomization*** and deliberate ***subsampling***
  - recovery from sample **rates ≈ computational cost** *proportional* to **transform-domain sparsity** of *data* or *model*

Today's agenda

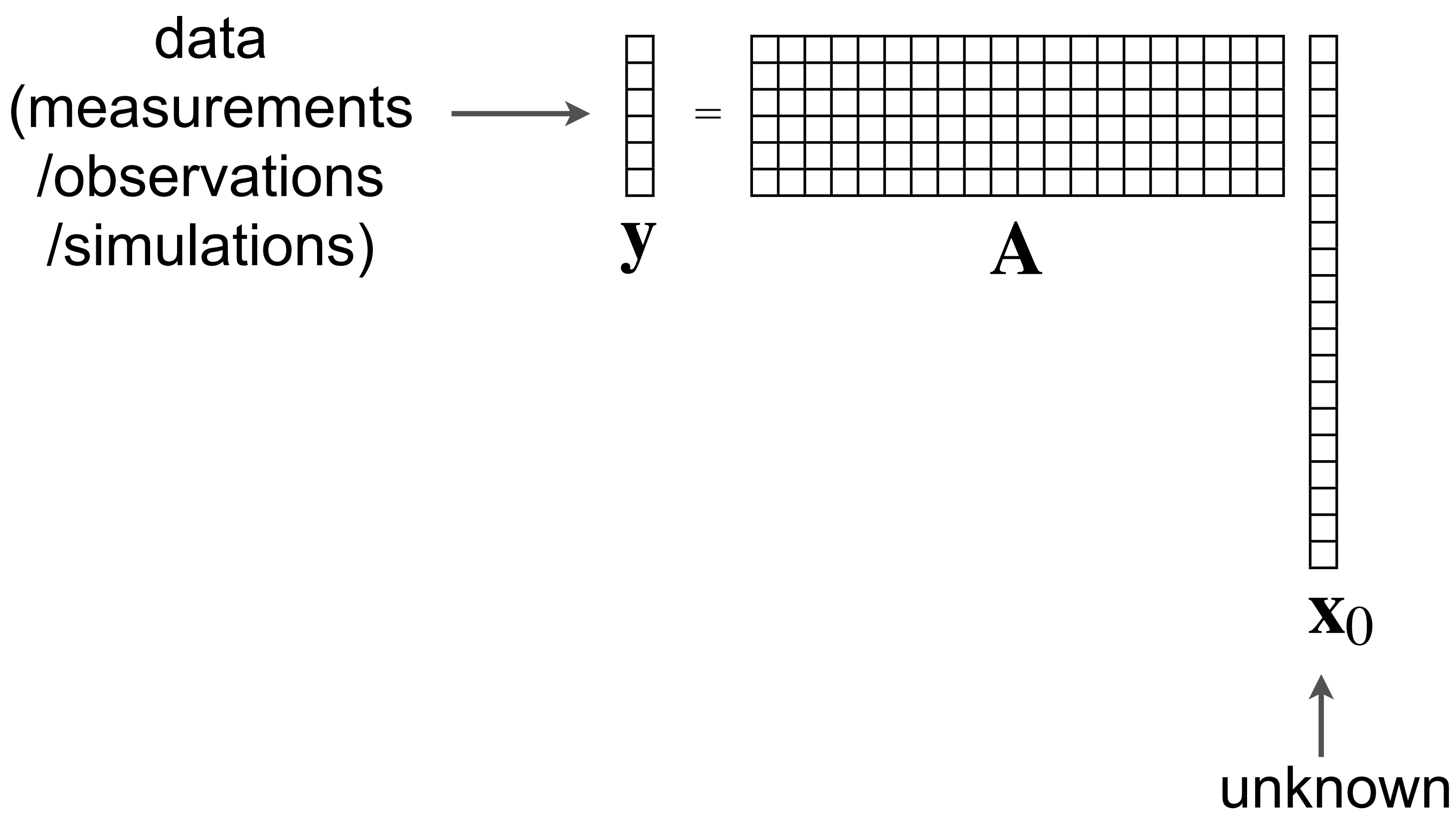
- Brief introduction to *compressive sensing*
  - ***sparsifying*** transforms
  - ***randomized* = *incoherent* downsampling**
  - ***nonlinear*** recovery by ***sparsity*** promotion

- *Sparsity-promoting recovery* from ***randomized* simultaneous measurements**
  - missing *separated* shots versus missing *simultaneous* shots
  - recovery from simultaneous data *with* and *without* primary prediction (CSed EPSI)

- J*oint sparsity-promoting recovery* from ***randomized* image volumes**
  - leverage *focusing*
  - *reduction* of model-space wavefields

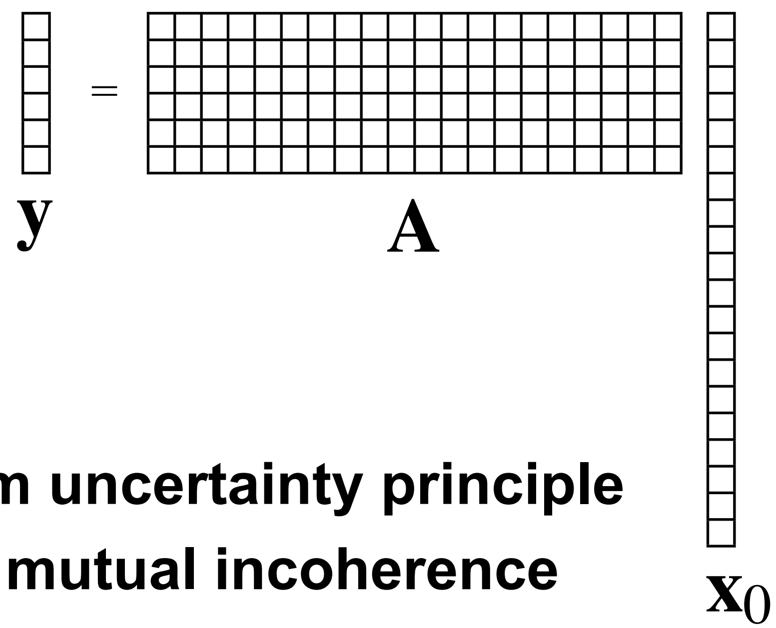
Problem statement

Consider the following (severely) underdetermined system of linear equations



Is it possible to recover $\mathbf{x}_0$ accurately from $\mathbf{y}$?

Perfect recovery



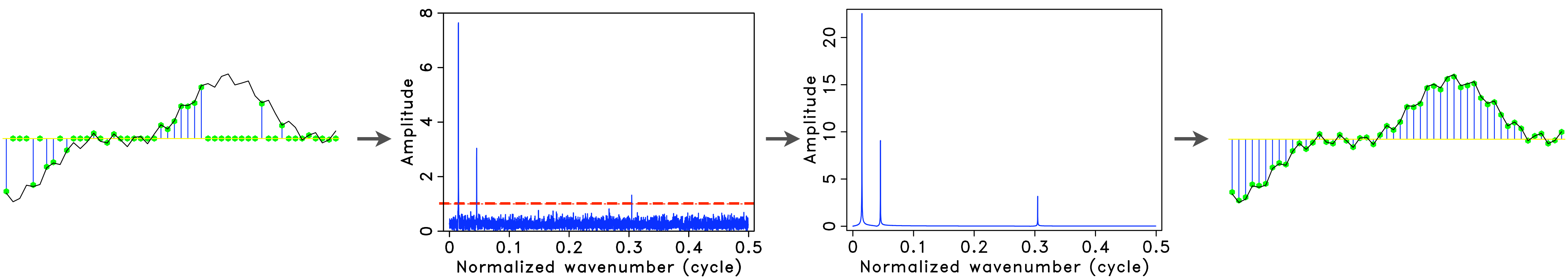
- conditions
  - **A** obeys the **uniform uncertainty principle**
  - ***randomized* A <=> mutual incoherence**
  - **x_0** is **sufficiently sparse**
- ***nonlinear*** recovery procedure:

$$\underbrace{\min_{\mathbf{x}} \|\mathbf{x}\|_1}_{\text{sparsity}} \quad \text{s.t.} \quad \underbrace{\mathbf{A}\mathbf{x} = \mathbf{y}}_{\text{perfect reconstruction}}$$

- performance
  - *S*-sparse vectors recovered from roughly on the order of *S* measurements (to within constant and *log* factors)

[Candès et al.‘06]
[Donoho‘06]

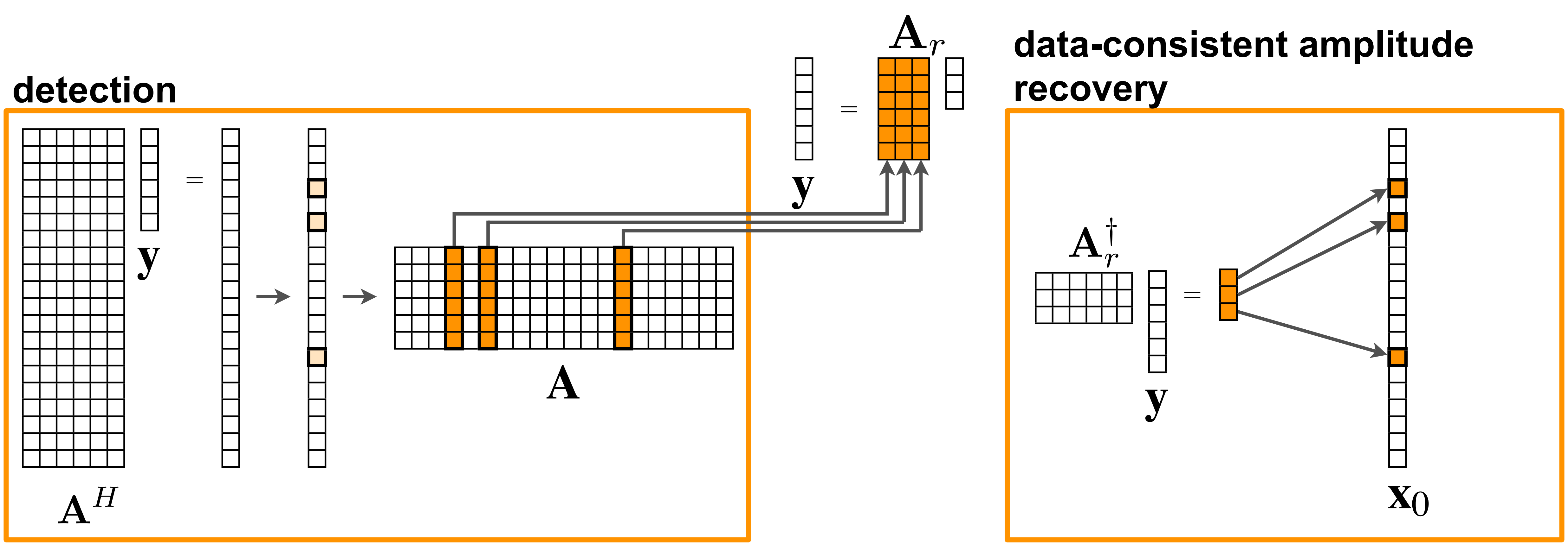
NAIVE sparsity-promoting recovery



inverse Fourier transform

detection + data-consistent amplitude recovery

Fourier transform



Extensions

- Use CS principles to select *physically* appropriate
  - *measurement* basis **M = *random* phase encoder** } **"blending"**
  - ***randomized*** *restriction* matrix **R = downsampler**
  - sparsifying transform **S** (e.g. curvelets)
  - driven by signal type, physics, and type of acquisition (e.g. fMRI vs seismic)
- Sparse signal representation:

$$\mathbf{y} = \mathbf{A}\mathbf{x}_0$$

with

$$\mathbf{A} = \mathbf{R}\mathbf{M}\mathbf{S}^H$$

restriction matrix — **measurement matrix** — **sparsity matrix**

Selection is aimed at turning *aliases/coherent subsampling artifacts* into harmless ***noise*** ...

Recovery from *randomized* simultaneous measurements

Tim T.Y. Lin and Felix J. Herrmann, Designing simultaneous acquisitions with compressive sensing. Submitted Abstract, Amsterdam, 2009, EAG

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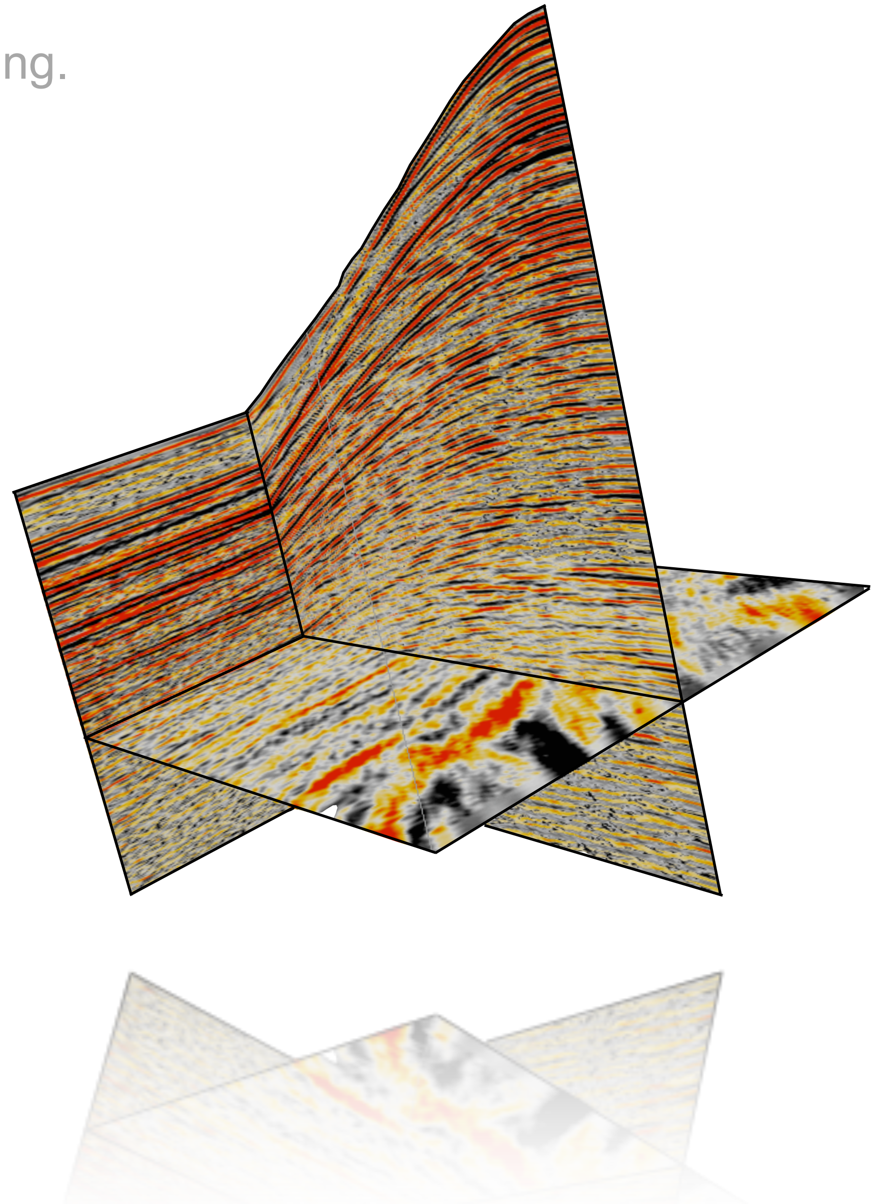


Delphi, June 4th, 2009

Relation to existing work

- **Simultaneous & continuous acquisition:**
  - *A new look at marine simultaneous sources* by C. Beasley, ‘08
  - *Simultaneous Sourcing without Compromise* by R. Neelamani & C.E. Krohn, ’08.
  - *Changing the mindset in seismic data acquisition* by A. Berkout, ’08
  - *Independent simultaneous sweeping - A method to increase the productivity of land seismic crews* by D. Howe, M. Foster, T. Allen, B. Taylor, and I. Jack, ‘08

- **Primary prediction through wavefield inversion:**
  - *Elimination of free-surface related multiples without need of the source wavelet* by L. Amundsen, ‘01
  - *Primary estimation by sparse inversion and its application to near offset reconstruction* by G. van Groenenstijn and D. Verschuur, ’09

Two questions

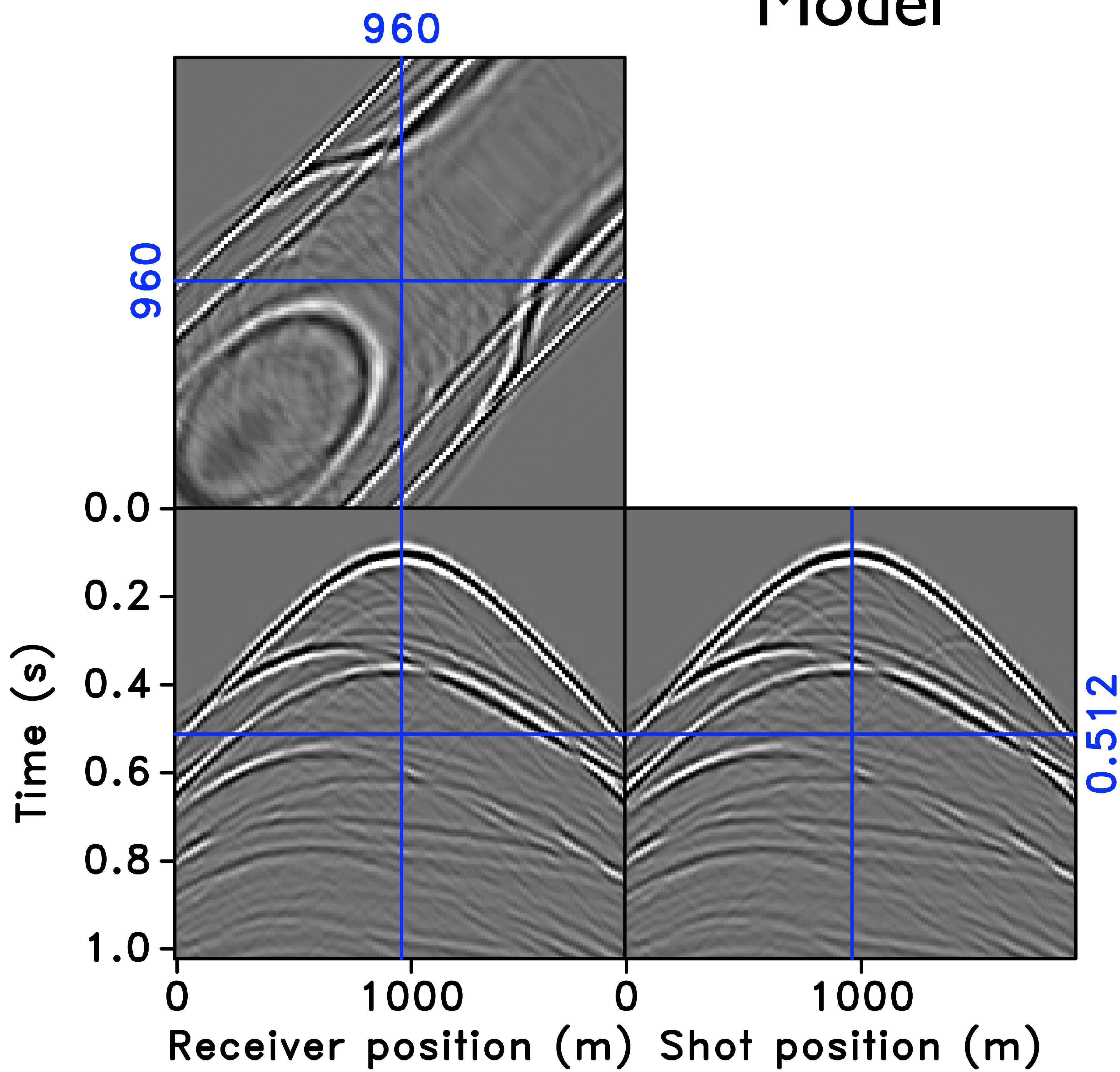
- **Question I: What is better? Having missing *single-source* or *missing randomized simultaneous* experiments?**

- Comparison between different undersampling strategies for source experiments:
  - ***Deterministic*** missing shot positions
  - ***Randomized*** jittered shot positions
  - ***Randomized*** simultaneous shots

- **Question II: What is better? First recover and then process or process directly in the compressed domain?**

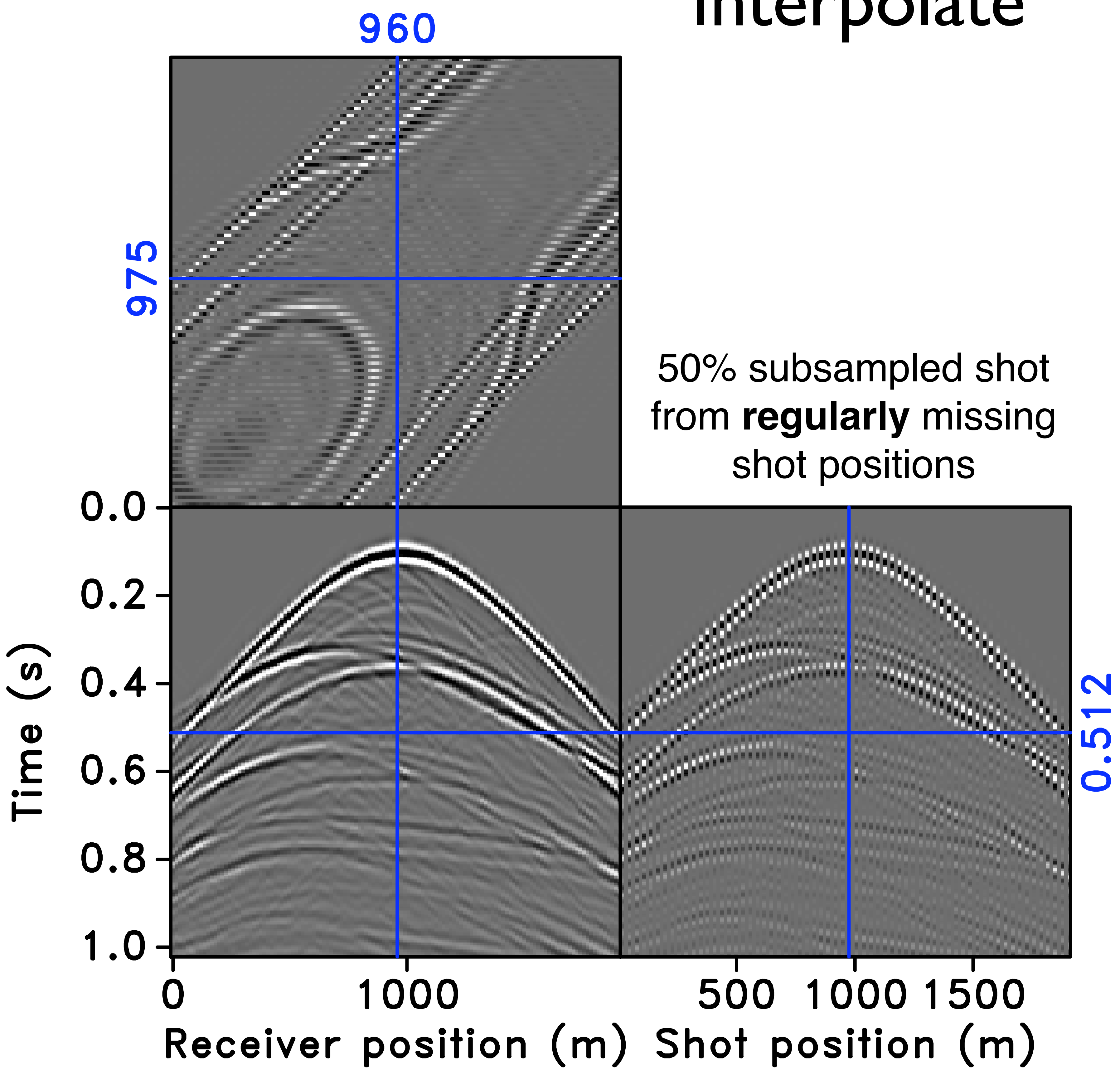
- Example: ***randomized*** primary prediction with EPSI

Model



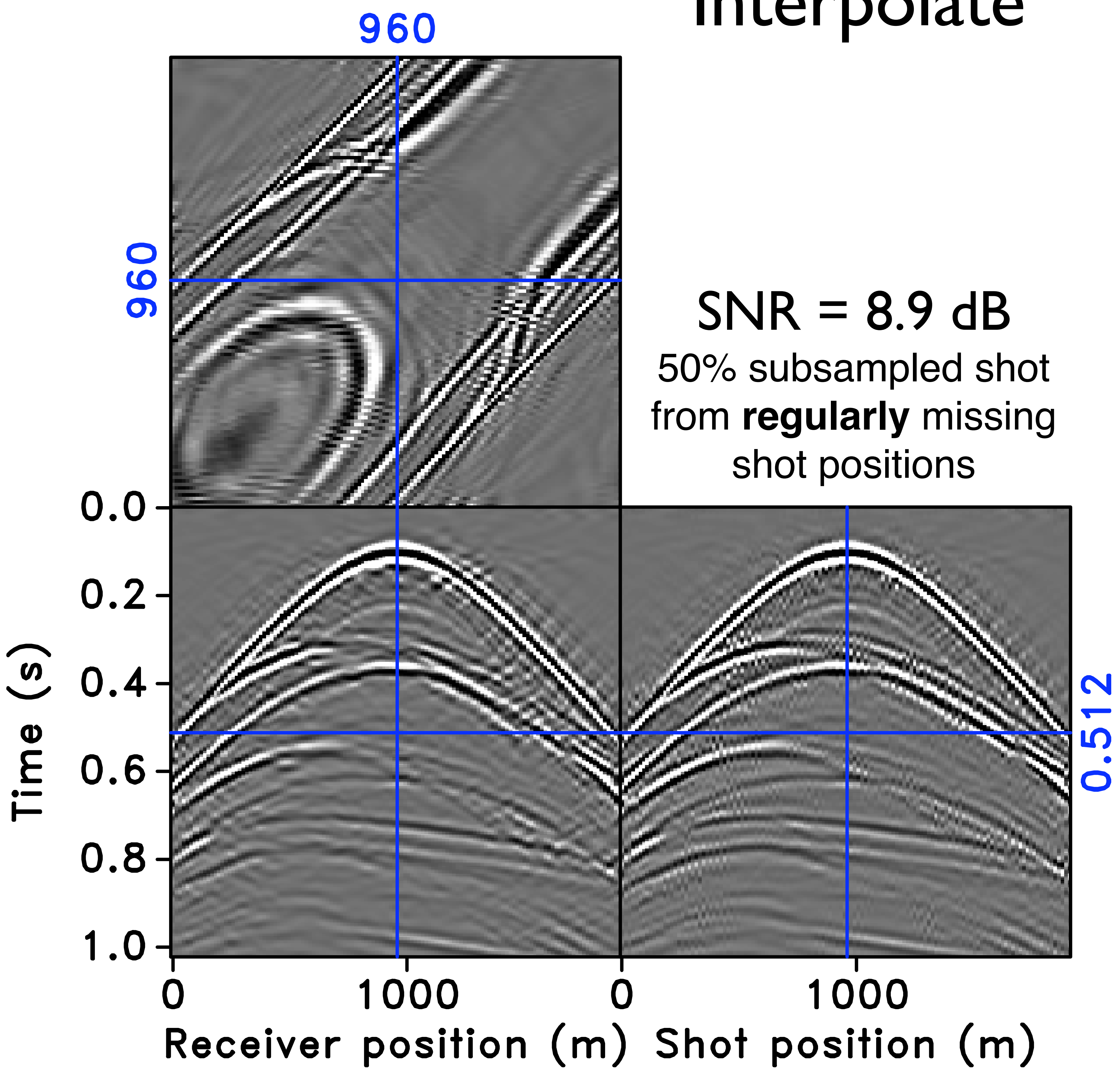
960
960
0.512
Time (s)
0.0
0.2
0.4
0.6
0.8
1.0
0
1000
0
1000
Receiver position (m)
Shot position (m)

Interpolate



50% subsampled shot from **regularly** missing shot positions

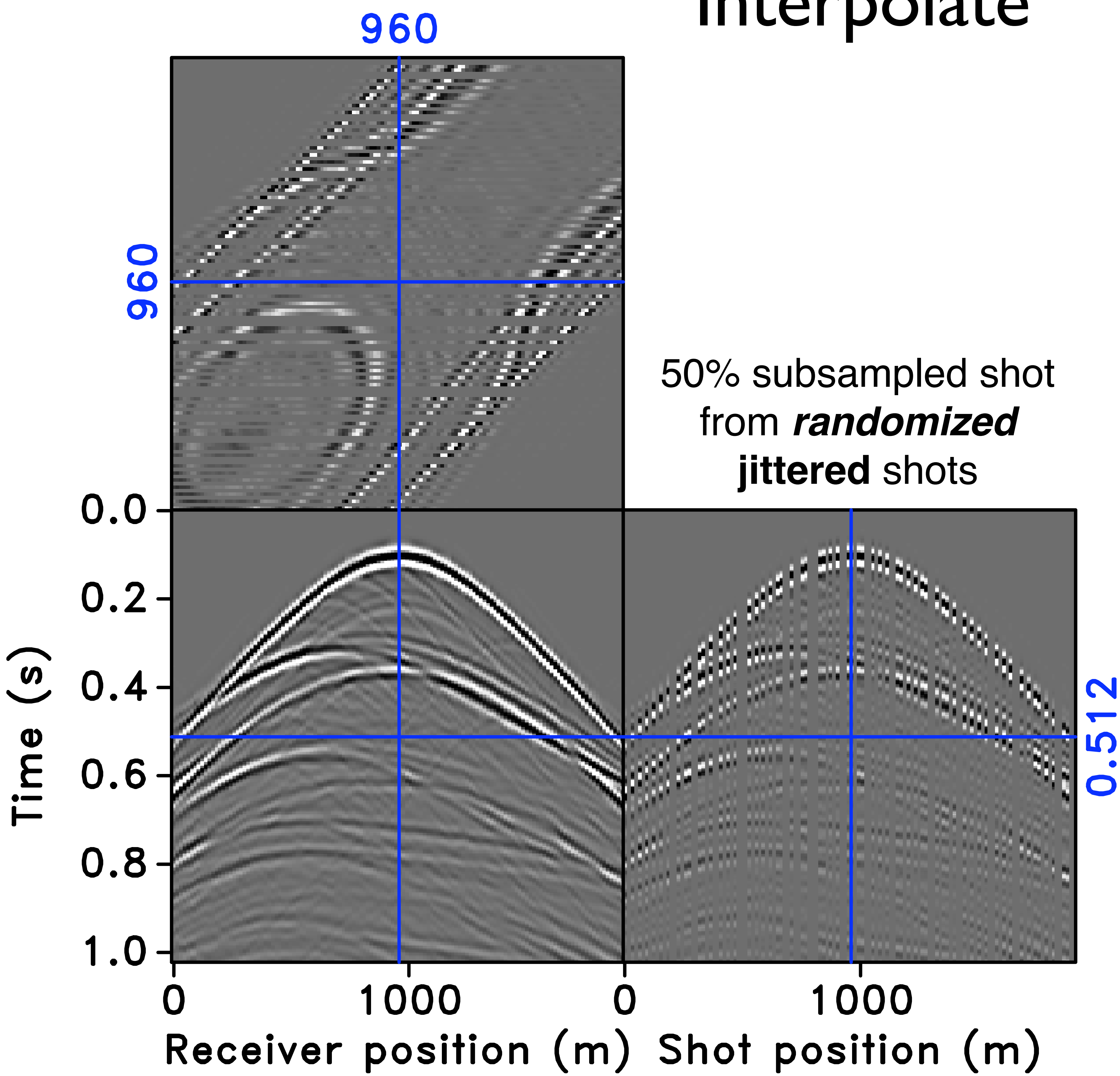
Interpolate



SNR = 8.9 dB

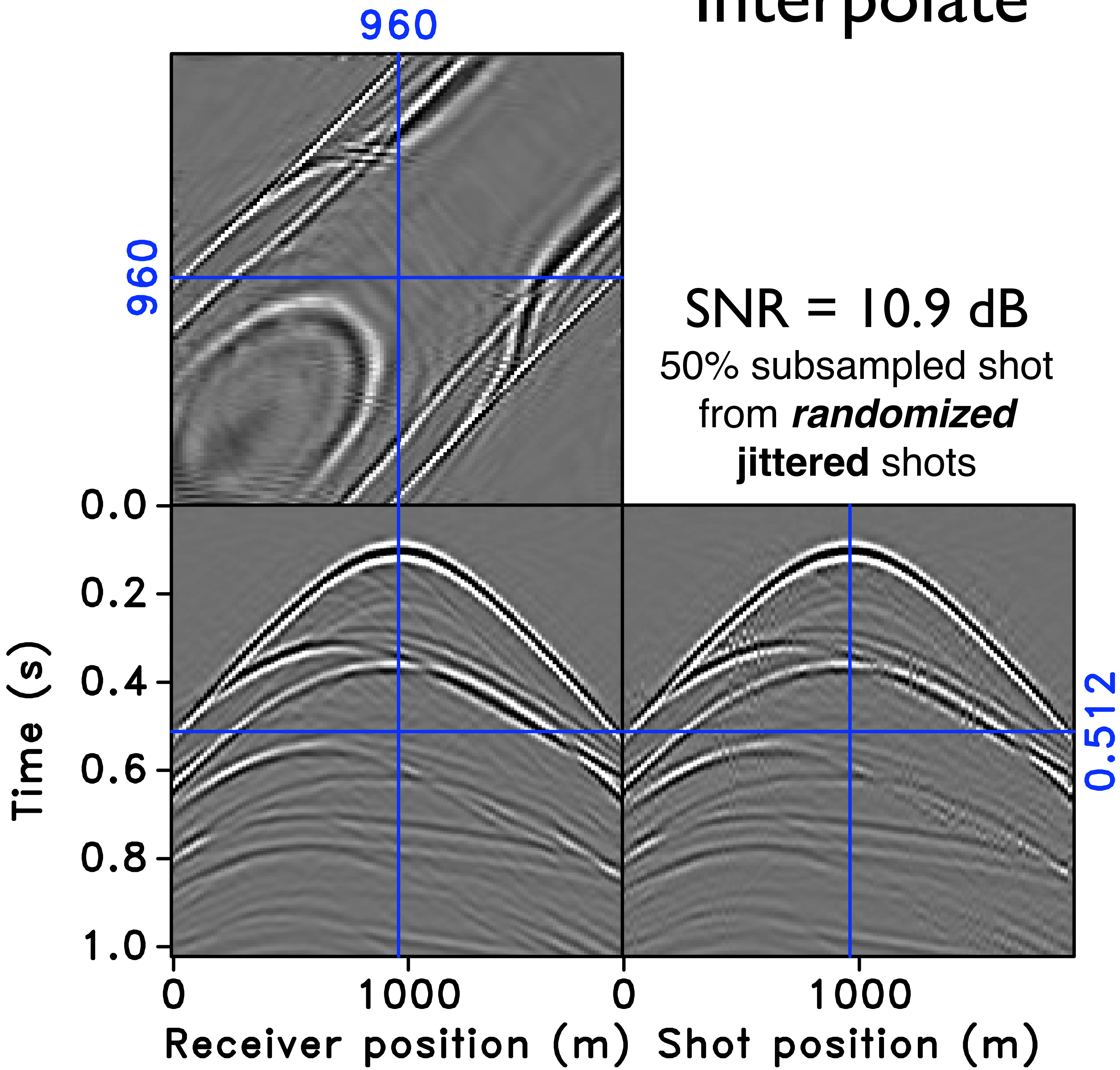
50% subsampled shot from **regularly** missing shot positions

Interpolate



50% subsampled shot from ***randomized*** **jittered** shots

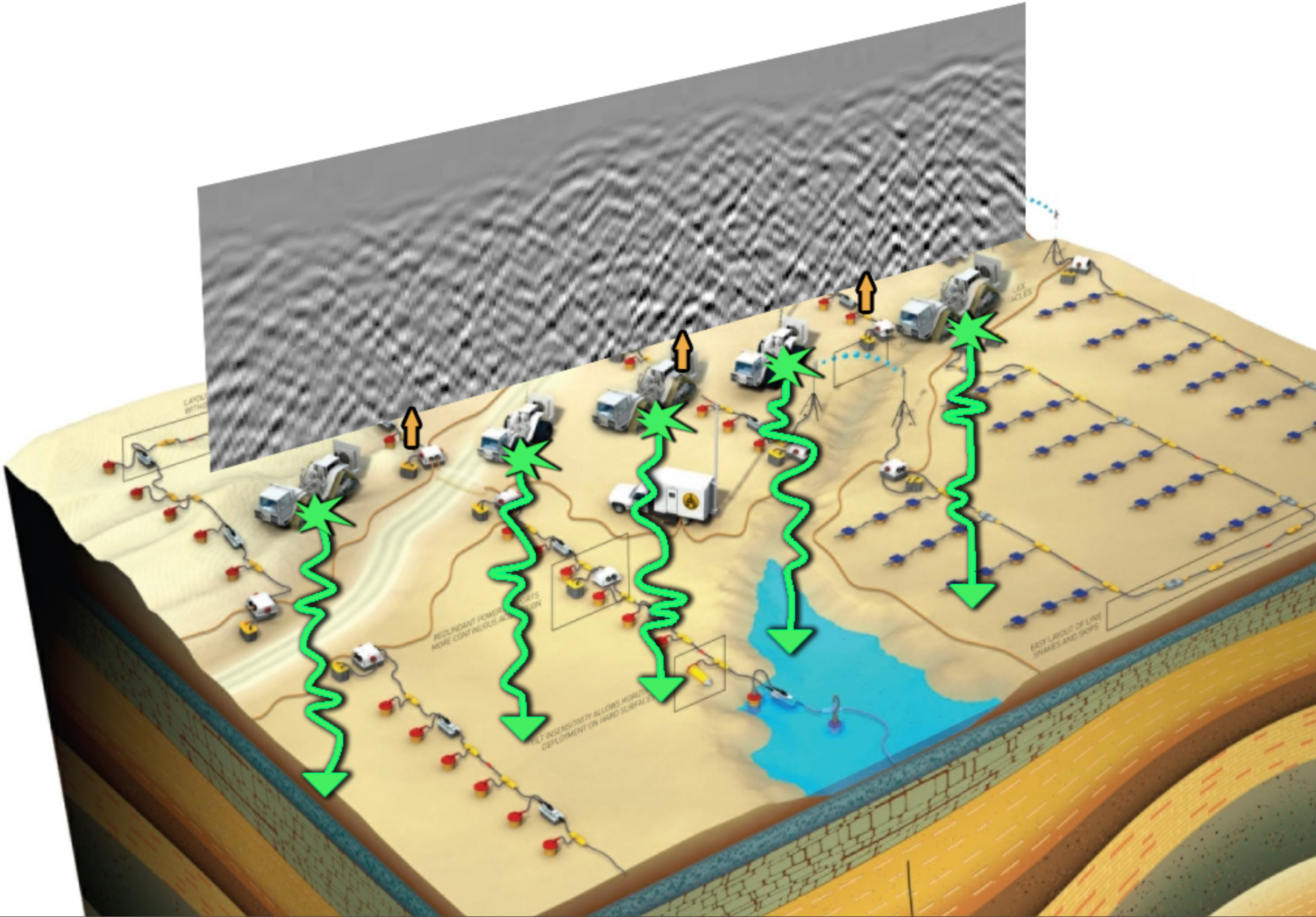
Interpolate



SNR = 10.9 dB

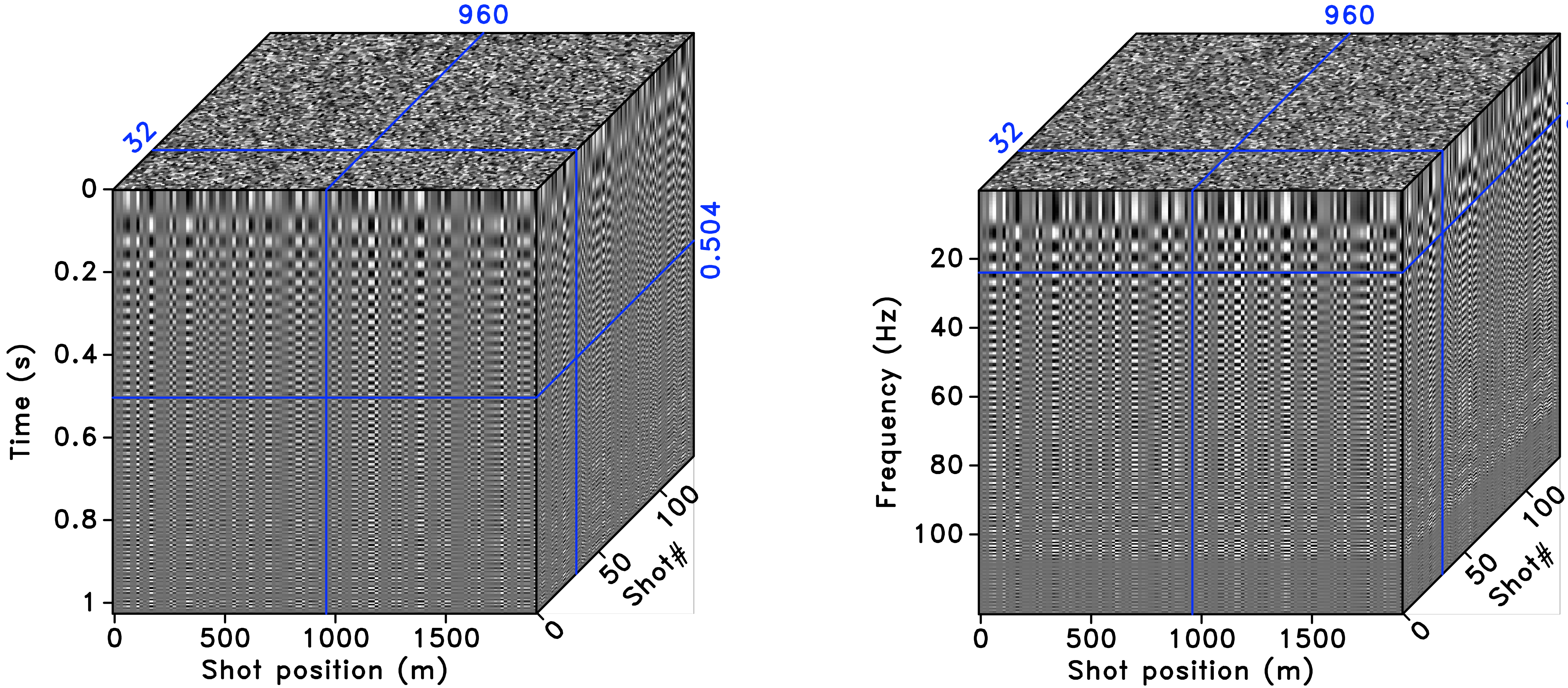
50% subsampled shot from ***randomized*** **jittered** shots

Simultaneous & continuous sources

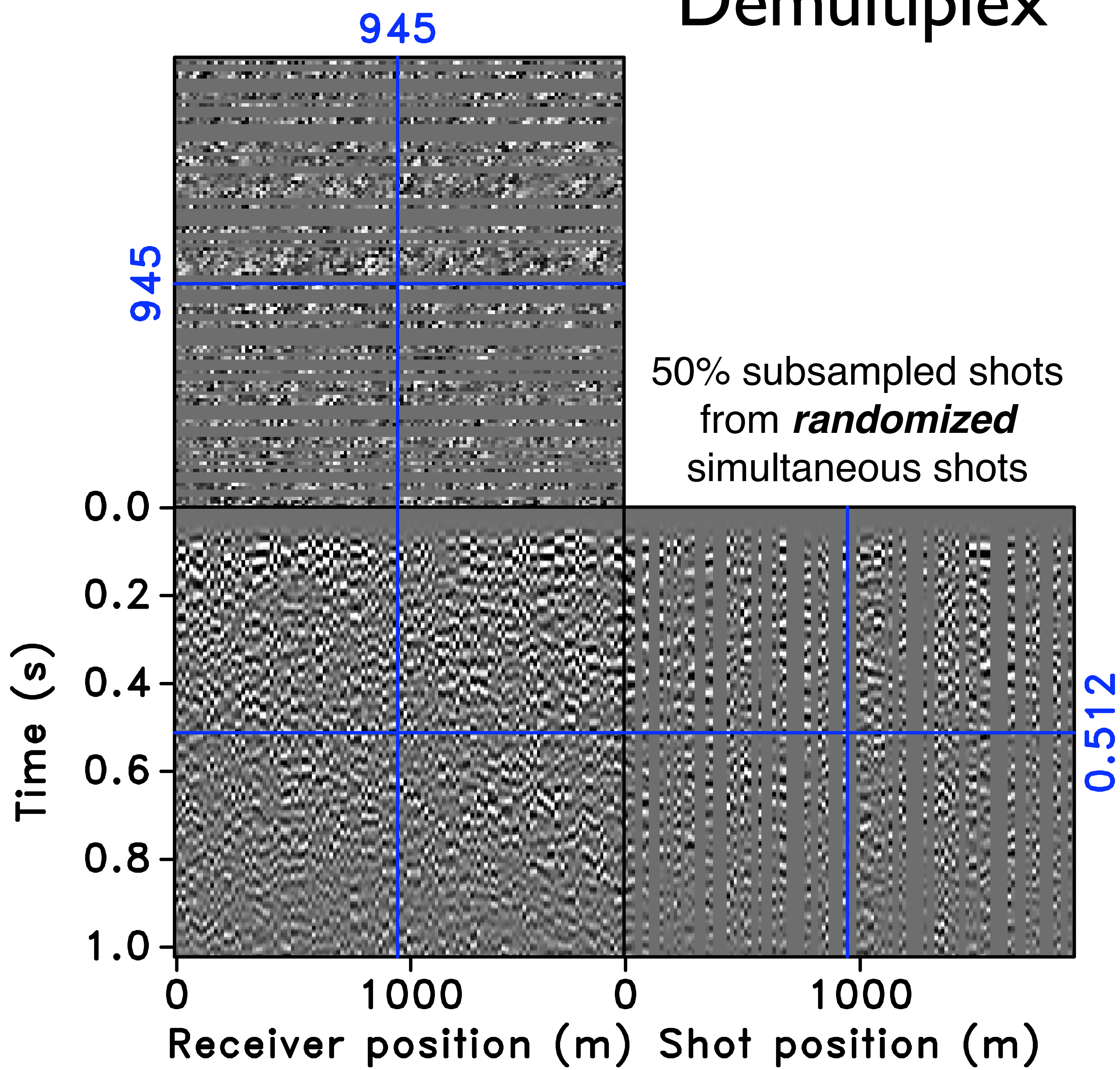


Randomized simultaneous sweep signals

- Linearly ramping seismic sweep, 5 to 110 Hz
- Simultaneous source at all positions, each randomly phase encoded

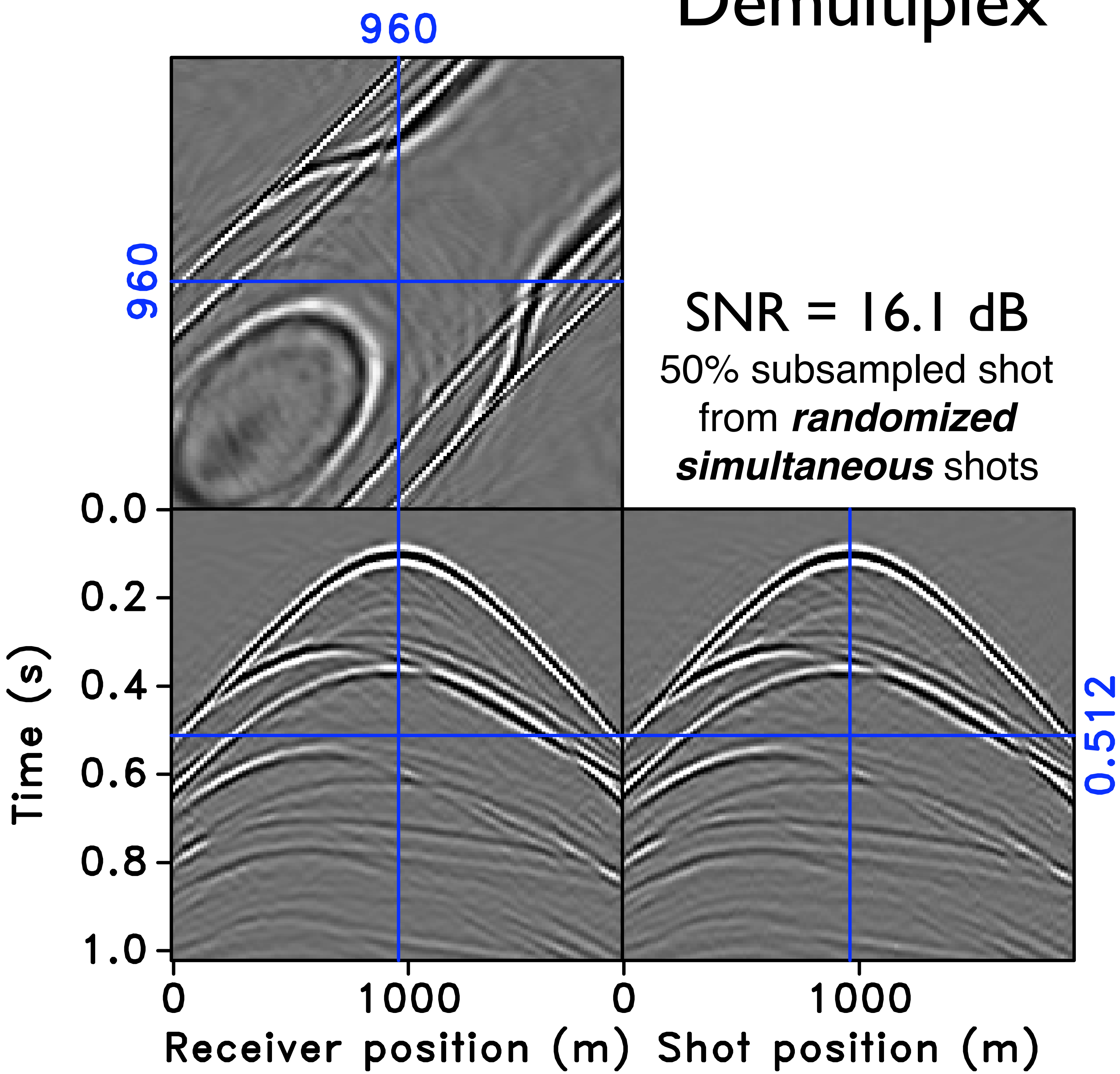


Demultiplex



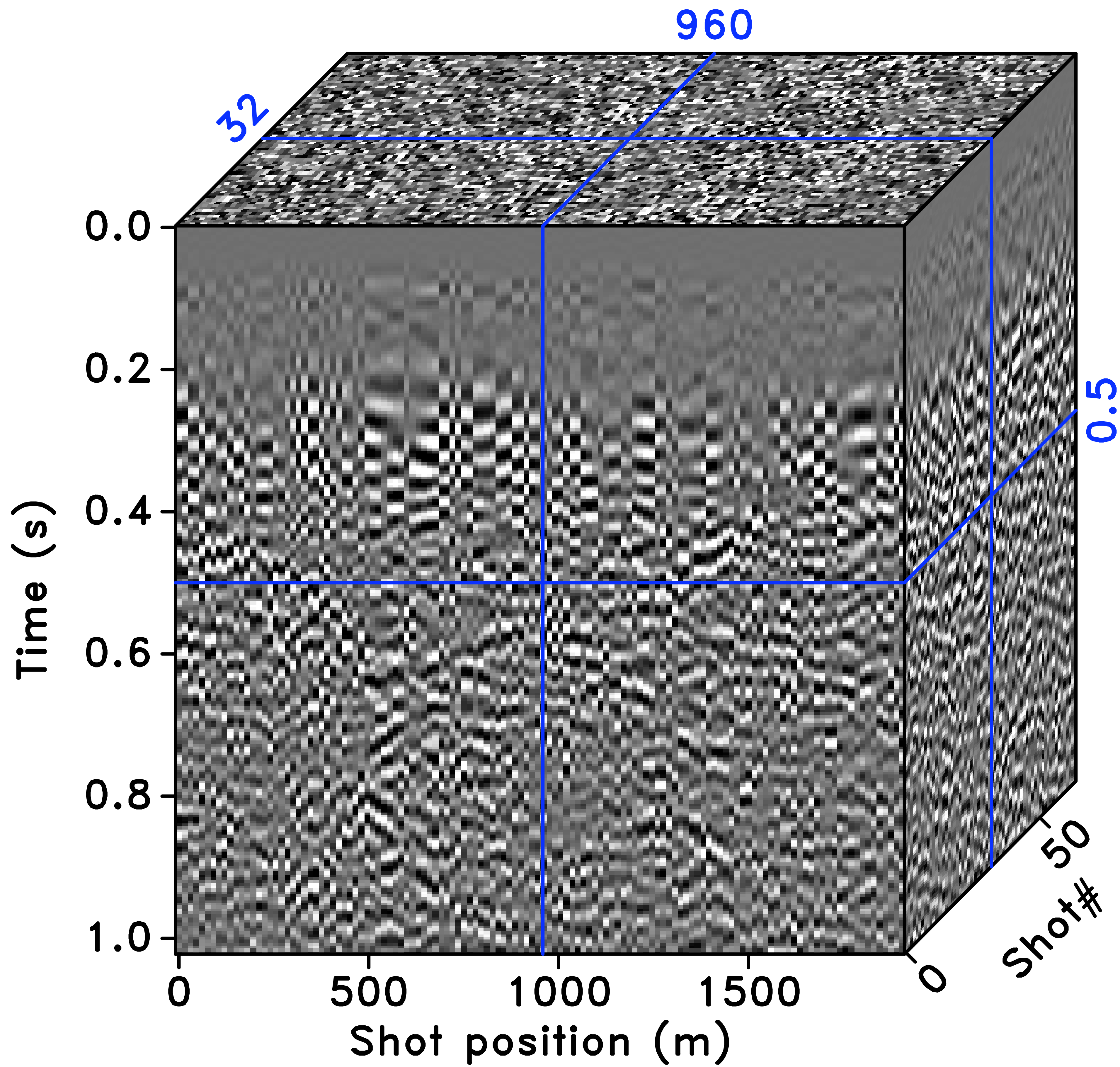
50% subsampled shots from ***randomized*** simultaneous shots

Demultiplex

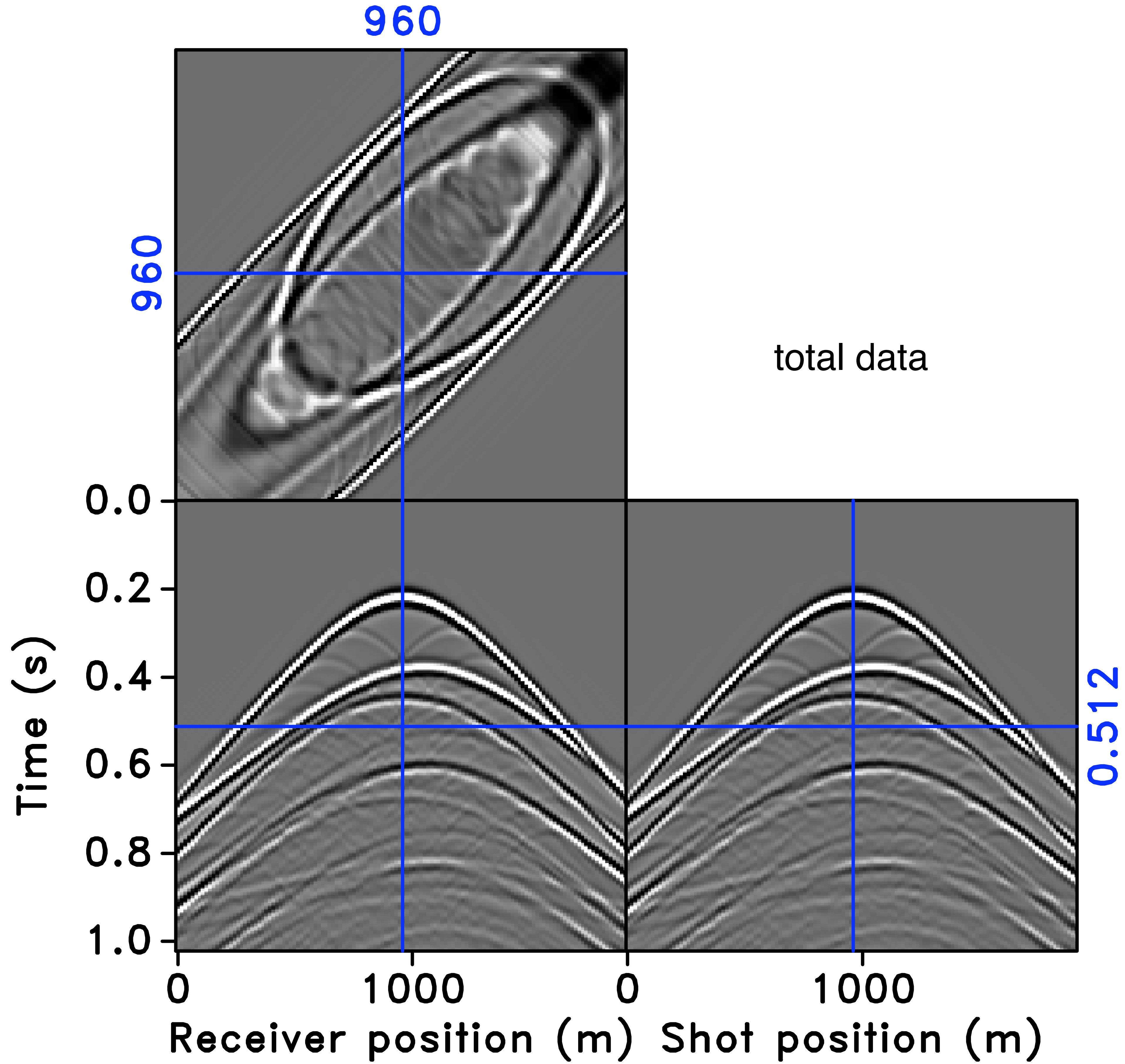


SNR = 16.1 dB

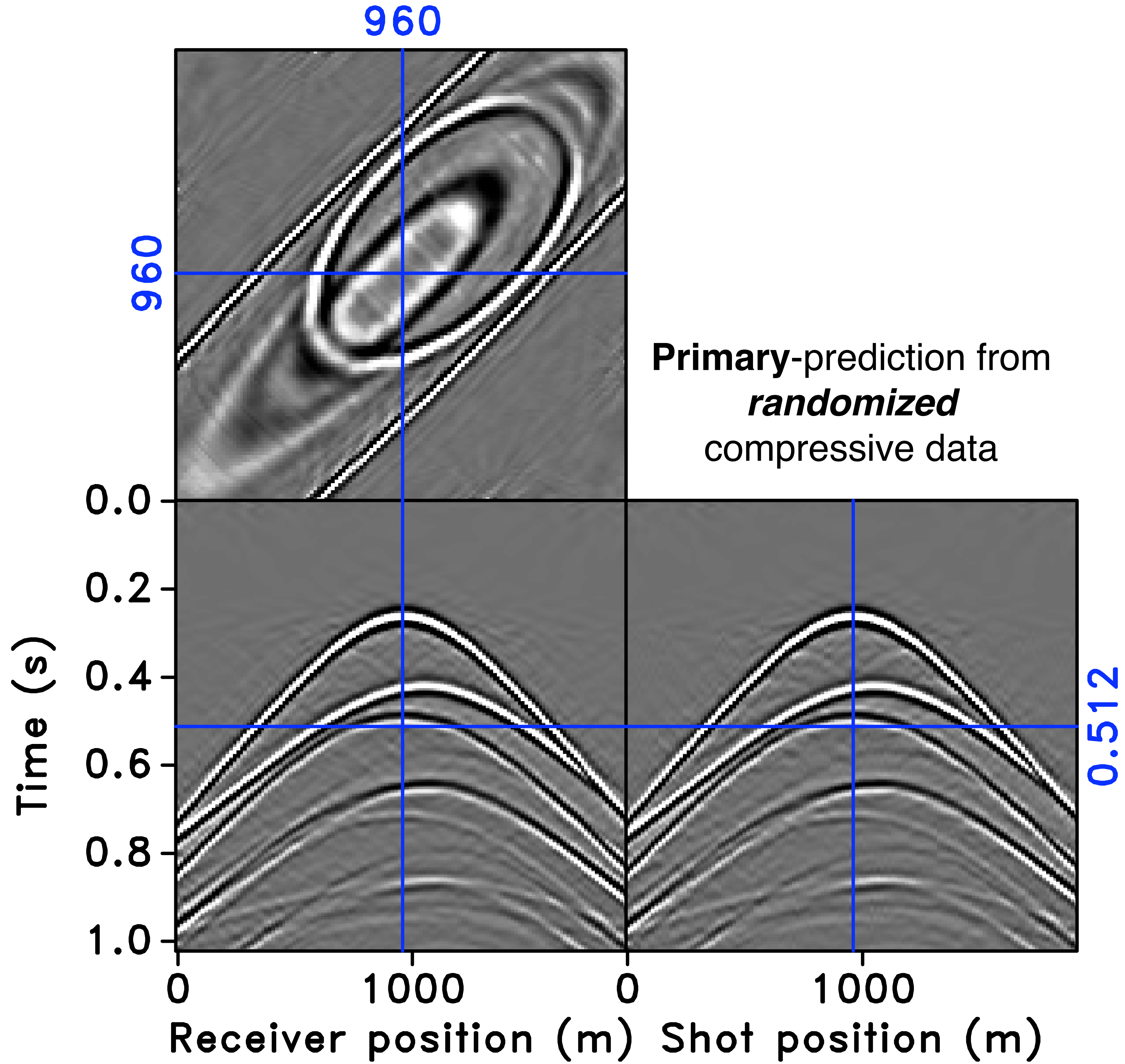
50% subsampled shot from ***randomized simultaneous*** shots



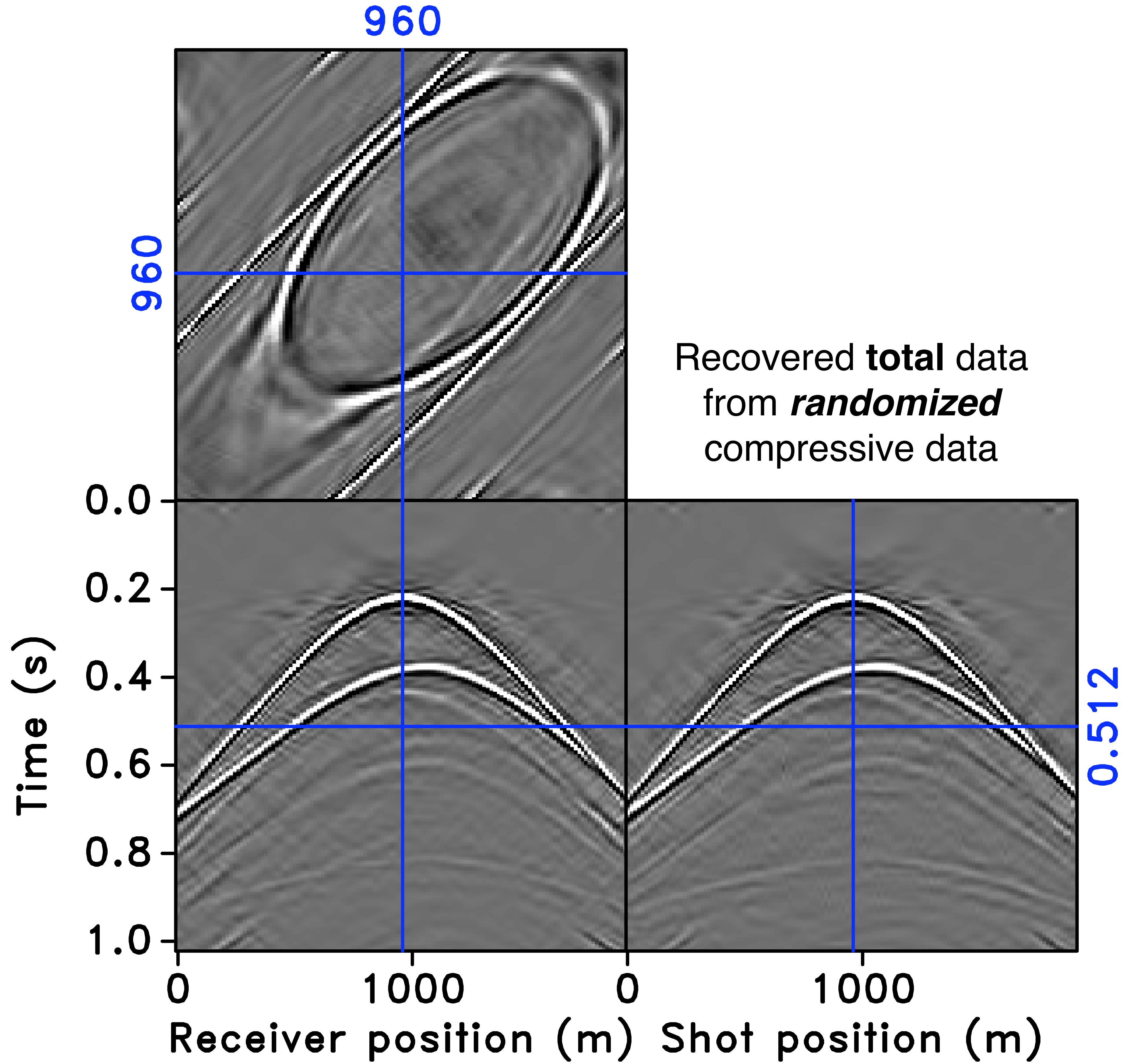
960
32
0.5
0.0
0.2
0.4
0.6
0.8
1.0
Time (s)
0
500
1000
1500
Shot position (m)
0
50
Shot#

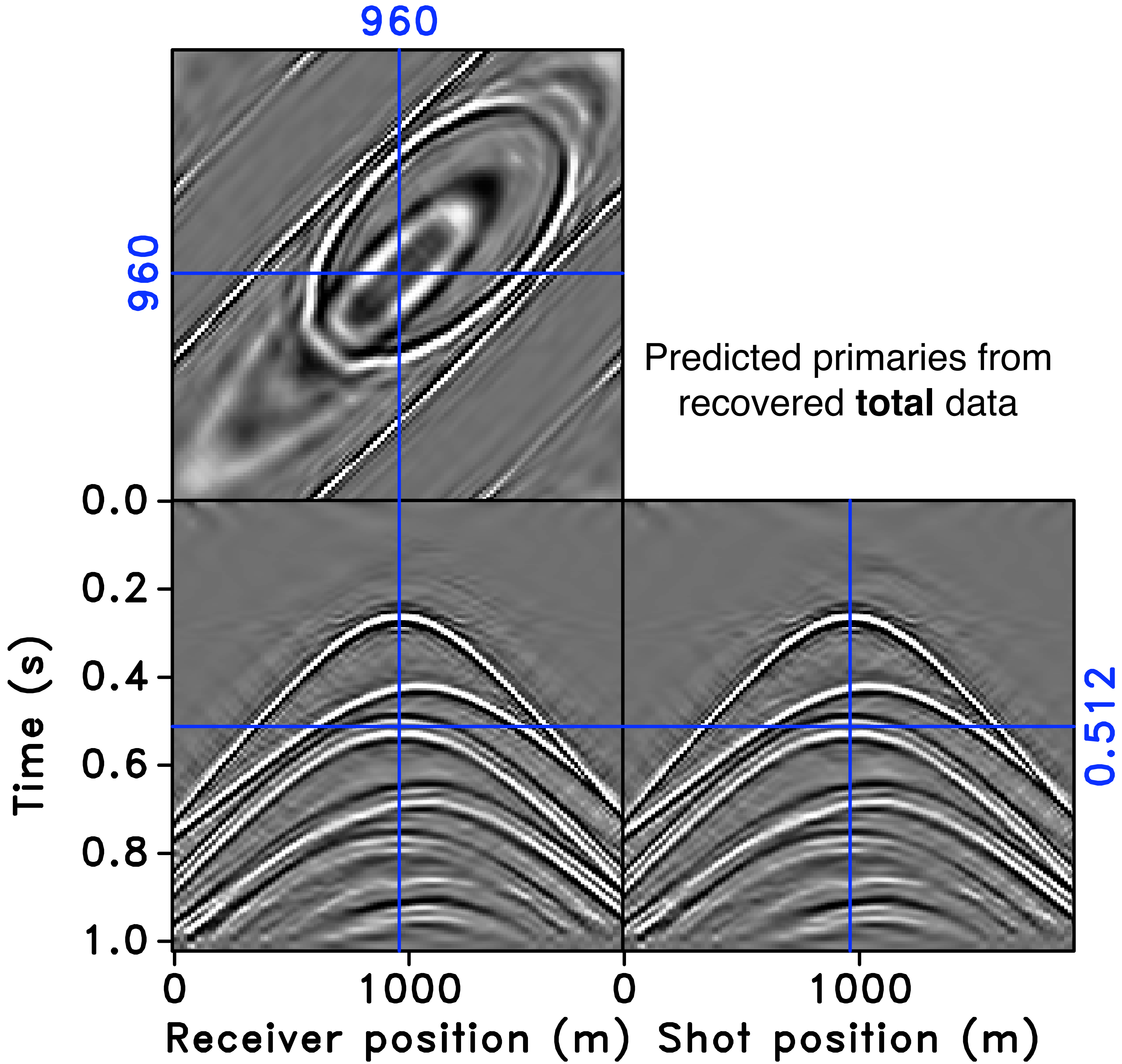


960
960
total data
0.512
Time (s)
0.0
0.2
0.4
0.6
0.8
1.0
0
1000
0
1000
Receiver position (m)
Shot position (m)



Primary-prediction from ***randomized*** compressive data


960
960
Recovered **total** data
from ***randomized***
compressive data
0.0
0.2
0.4
0.6
0.8
1.0
Time (s)
0.512
0
1000
0
1000
Receiver position (m)
Shot position (m)



Predicted primaries from recovered **total** data

Observations

- **Incoherent *randomized*** sampling *crucial* for creating *favorable* recovery conditions for ***sparsity-promoting recovery*** from "**incomplete**" data
  - depends on the choice of ***downsampled randomization*** **RM**
  - simultaneous acquisition is better for reconstruction

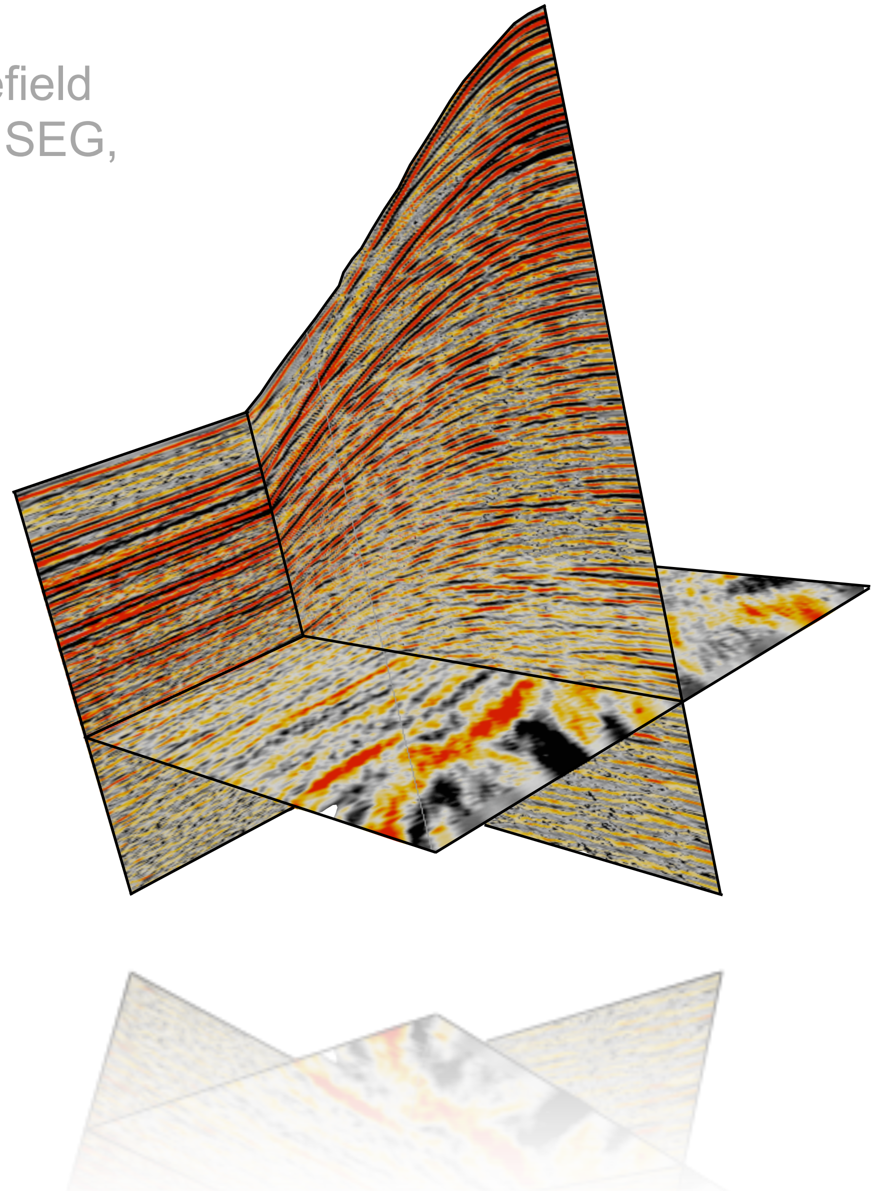
- Recovery greatly improves when **estimating primaries**
  - *deconvolved* **primaries** are **sparser** than **multiples**
  - **multiples** are mapped to **primaries**
  - example of ***randomized wavefield inversion*** with ***reduced sizes***

- Push recovery down into processing flow, i.e., compressive processing & imaging

- Extend these ideas to imaging = model-space compressive sampling

Recovery from *randomized* image volumes

Felix J. Herrmann, Compressive imaging by wavefield inversion with group sparsity. Submitted abstract, SEG, 2009, Houston. Technical Report TR-2009-01



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Strategy

- Leverage CS towards solutions of wave simulation & imaging problems
- Subsample solution deliberately, followed by CS recovery
- Speedup if recovery costs < gain in reduced system size
  - computation
  - storage
- Examples:
  - compressed imaging by CS sampling in the model space

Relation to existing work

- **Simultaneous & continuous acquisition:**
  - *Efficient Seismic Forward Modeling using Simultaneous Random Sources and Sparsity* by N. Neelamani and C. Krohn and J. Krebs and M. Deffenbaugh and J. Romberg, ‘08
- **Simultaneous simulations & migration:**
  - *Faster shot-record depth migrations using phase encoding* by Morton & Ober, ’98.
  - *Phase encoding of shot records in prestack migration* by Romero et. al., ’00.
- **Imaging:**
  - *How to choose a subset of frequencies in frequency-domain finite-difference migration* by Mulder & Plessix, ’04.
  - *Efficient waveform inversion and imaging: A strategy for selecting temporal frequencies* by Sirque and Pratt, ’04.
- **Full-waveform inversion:**
  - *3D prestack plane-wave, full-waveform inversion* by Vigh and Starr, ‘08
- **Wavefield extrapolation:**
  - *Compressed wavefield extrapolation* by T. Lin and F.J.H, ’07
  - *Compressive wave computations* by L. Demanet (SIA ’08 MS79 & Preprint)

Essentials of seismic inversion

Simulation:

$$\mathbf{H}[\mathbf{m}]\mathbf{U} = \mathbf{Q} \quad \text{and} \quad \mathbf{H}^*[\mathbf{m}]\mathbf{V} = \mathbf{\Delta R}$$

- $\mathbf{Q}$: sources
- $\mathbf{\Delta R}$: residue (data)
- $\mathbf{H}$: discretized PDE (Helmholtz)
- $\mathbf{m}$: variables (Earth)
- $\mathbf{U}$: solution (seismic wavefield)
- $\mathbf{V}$: adjoint solution (seismic wavefield)

Imaging:

$$\begin{aligned} \widehat{\boldsymbol{\delta}\mathbf{I}}(x_s, x_r, \omega) &= (\mathbf{U} \circ \mathbf{V}^*) \\ \delta\mathbf{m}(x_s = x_r, t = 0) &= \sum_{\omega} \omega^2 \mathrm{diag}\{\widehat{\boldsymbol{\delta}\mathbf{I}}\} \end{aligned}$$

- $\widehat{\boldsymbol{\delta}\mathbf{I}}$: image volume
- $(\mathbf{U} \circ \mathbf{V}^*)$: multi-D 'cross-correlation'

Essentials of seismic inversion

Simulation:

$$\mathbf{H}[\mathbf{m}]\mathbf{U} = \mathbf{Q} \quad \text{and} \quad \mathbf{H}^*[\mathbf{m}]\mathbf{V} = \mathbf{\Delta R}$$

- **discretized PDE (Helmholtz)**: $\mathbf{H}$
- **variables (Earth)**: $\mathbf{m}$
- **solution (seismic wavefield)**: $\mathbf{U}$
- **sources**: $\mathbf{Q}$
- **adjoint solution (seismic wavefield)**: $\mathbf{V}$
- **residue (data)**: $\mathbf{\Delta R}$

– High-dimensional solutions are ***extremely*** expensive to compute

– Inversion (e.g. via Gauss-Newton) requires multiple solves

– Number of blocks in **H** and number of **rhs** determine simulation & acquisition costs

Imaging:

$$\widehat{\boldsymbol{\delta}\mathbf{I}}(x_s, x_r, \omega) = (\mathbf{U} \circ \mathbf{V}^*)$$

$$\delta\mathbf{m}(x_s = x_r, t = 0) = \sum_{\omega} \omega^2 \mathrm{diag}\{\widehat{\boldsymbol{\delta}\mathbf{I}}\}$$

- **image volume**: $\widehat{\boldsymbol{\delta}\mathbf{I}}$
- **multi-D 'cross-correlation'**: $\circ$

Essentials of seismic inversion

Simulation:

sources; residue (data)

$$\mathbf{H}[\mathbf{m}]\mathbf{U} = \mathbf{Q} \quad \text{and} \quad \mathbf{H}^*[\mathbf{m}]\mathbf{V} = \mathbf{\Delta R}$$

discretized PDE (Helmholtz); variables (Earth); solution (seismic wavefield); adjoint solution (seismic wavefield)

- High-dimensional solutions are ***extremely*** expensive to compute
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- Number of blocks in **H** and number of **rhs** determine simulation & acquisition costs

Imaging:

image volume; multi-D 'cross-correlation'

$$\widehat{\boldsymbol{\delta}\mathbf{I}}(x_s, x_r, \omega) = (\mathbf{U} \circ \mathbf{V}^*)$$

$$\delta\mathbf{m}(x_s = x_r, t = 0) = \sum_{\omega} \omega^2 \mathrm{diag}\{\widehat{\boldsymbol{\delta}\mathbf{I}}\}$$

- Explicit matrix evaluations part of prestack migration are expensive, require lots of memory
- Improve recovery by formulating imaging as a CSed inversion problem where
  - *off diagonals* are *penalized* (impose focusing)
  - image recovered by wavefield inversion by ***joint*** *sparsity* promotion

Imaging by wavefield correlations

Creation of image volumes involves

$$\boldsymbol{\delta}\mathbf{I}(x_s, x_r, t) = \mathbf{F}_t^* \sum_{\omega} \omega^2 \left(\mathbf{U} \circ \mathbf{V}^*\right)$$

with

$$(\mathbf{U} \circ \mathbf{V}^*) = \begin{bmatrix} \bar{\mathbf{U}}_1 & & \\ & \ddots & \\ & & \bar{\mathbf{U}}_{n_f} \end{bmatrix} \begin{bmatrix} \mathbf{V}_1^T \\ \vdots \\ \mathbf{V}_{n_f}^T \end{bmatrix}$$

and

$$\mathbf{U}_f = \left[\mathbf{u}_1 \cdots \mathbf{u}_{n_f}\right] \text{ and } \mathbf{V}_f = \left[\mathbf{v}_1 \cdots \mathbf{v}_{n_f}\right]$$

- Extremely *large* problem size
- Gradient updates do not account for the *Hessian*
- Recast imaging into a multi-D *deconvolution* problem supplemented by *focussing*
- Penalize *off-diagonals* as part of this *focussing* procedure

Wavefield focusing

Define linear mid-point/offset coordinate transformation

$$\boldsymbol{\delta}\mathbf{I}'(m,h,t) = \mathbf{T}^{\Delta h}_{(x_s,x_r)\mapsto(m,h)}\boldsymbol{\delta}\mathbf{I}(x_s,x_r,t),$$

with $m = \frac{1}{2}(x_s + x_r) \quad \text{and} \quad h = \frac{1}{2}(x_s - x_r)$

Penalize ***defocusing*** via minimizing [Symes, ‘09]

$$\|\mathsf{P}_h \mathbf{I}'(\cdot,\, h)\|_2 \text{ with } \mathsf{P}_h \cdot = \mathbf{h}\cdot$$

an *annihilator* that increasingly *penalizes* non-zero offsets.

Remark: conventional imaging principle

$$\delta\mathbf{m} = \boldsymbol{\delta}\mathbf{I}'(\cdot, h=0, t=0)$$

Wavefield inversion with focusing

Form augmented linear system

$$\begin{aligned}(\mathbf{U}^* \circ \mathbf{S}^* \mathbf{X}) &\approx \mathbf{V}^* \\ \mathsf{P}_h \mathbf{X} &\approx \mathbf{0} \qquad \text{focuses}\end{aligned}$$

with the sparsifying transform (curvelets/wavelets along depth-midpoint slices)

$$\mathbf{S} \cdot := \mathrm{vec}^{-1}\left(\left(\mathbf{Id} \otimes \mathbf{C}\right) \mathbf{T_0}\right) \mathrm{vec}\left(\cdot\right) \cdot$$

and $\mathbf{T_0}$ source/receiver-midpoint offset mapping supplemented with the imaging condition for t=0.

Formulation by *wavefield inversion* is a two-edged sword:

- **Correct for amplitudes by wavefield inversion**
- **Reduce system size by compressive sampling ...**

System-size reduction by CS

For each angular frequency, randomly subsample with CS matrix

$$\mathbf{RM} := \overbrace{\begin{bmatrix} \mathbf{R}_1^{\sigma} \otimes \mathbf{R}_1^{\rho} \otimes \mathbf{R}_1^{\zeta} \\ \vdots \\ \mathbf{R}_{n'_f}^{\sigma} \otimes \mathbf{R}_{n'_f}^{\rho} \otimes \mathbf{R}_{n'_f}^{\zeta} \end{bmatrix}}^{\text{sub sampler}} \overbrace{\left(\mathbf{F}_3^* \left(e^{\hat{i}\boldsymbol{\theta}}\right)\right)\mathbf{F}_3}^{\text{random phase encoder}} ,$$

$$\theta_w = \text{Uniform}([0, 2\pi])$$

with

$$n'_f \times n'_\sigma \times n'_\rho \times n'_\zeta \ll n_f \times n_s \times n_r \times n_z$$

Model-space CS *subsampling* along *source*, *receiver*, and *depth* coordinates.

Compressive wavefield inversion with focussing

Compressively sample *augmented* system

$$\begin{aligned}\mathbf{RM}\left(\mathbf{U}^* \circ \mathbf{S}^*\mathbf{X}\right) &\approx \mathbf{RMV}^T \\ \mathsf{P}_h\mathbf{X} &\approx \mathbf{0}\end{aligned} \quad \text{or} \quad \mathbf{AX} \approx \mathbf{B}$$

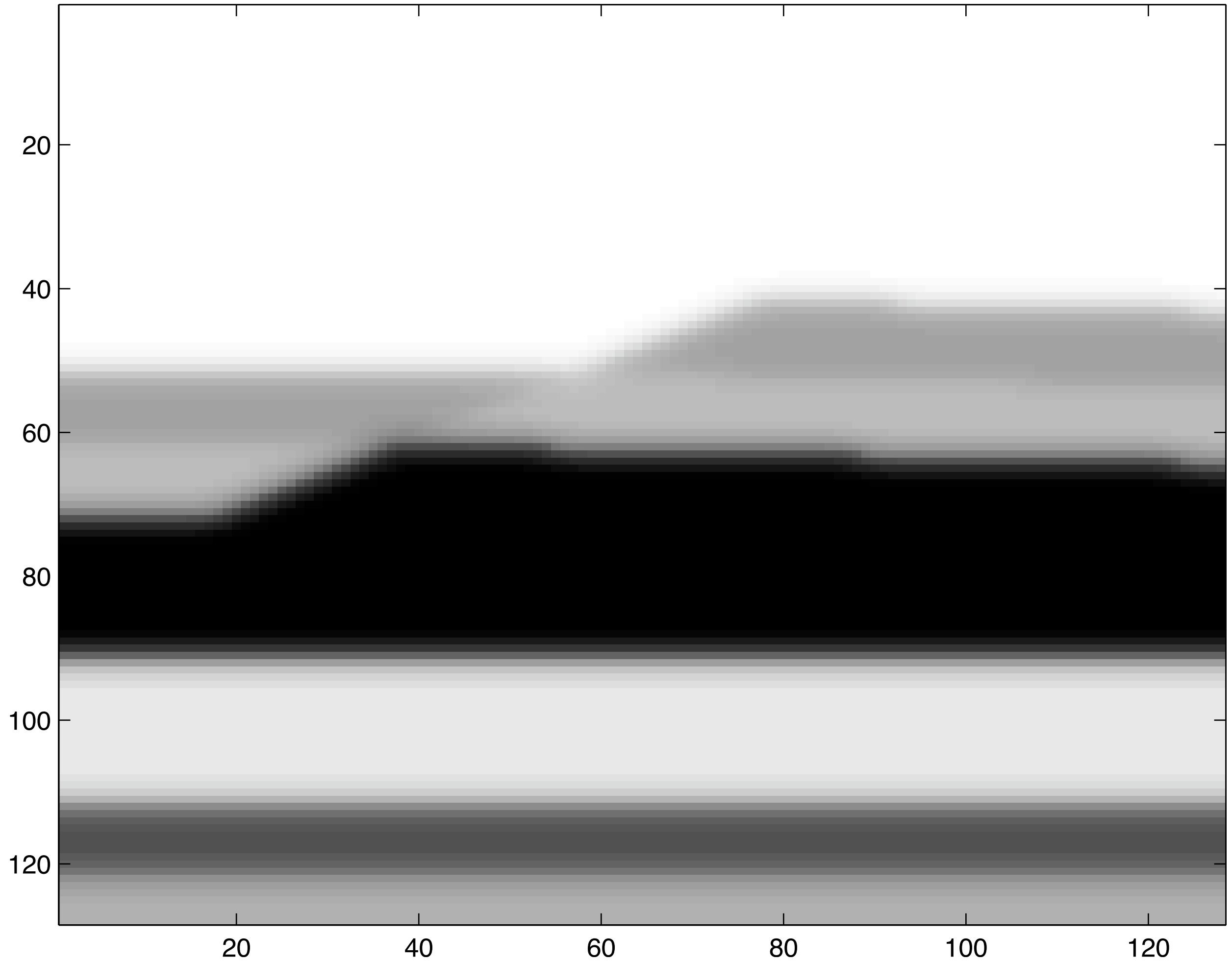
Recover focused solution by mixed (1,2)-norm minimization

$$\tilde{\mathbf{X}} = \arg\min_{\mathbf{X}} \|\mathbf{X}\|_{1,2} \quad \text{subject to} \quad \|\mathbf{AX} - \mathbf{B}\|_{2,2} \leq \sigma,$$

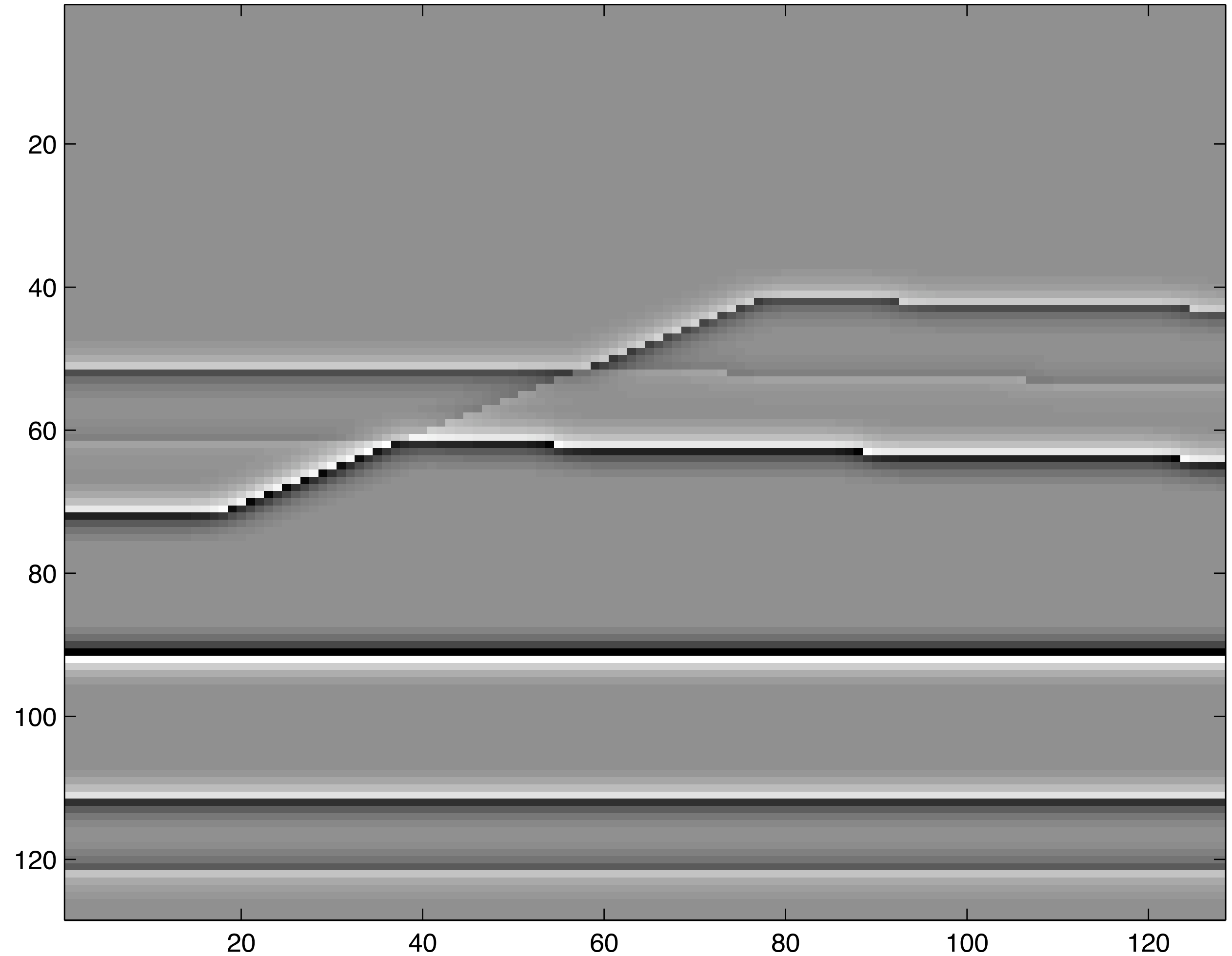
with $$\|\mathbf{X}\|_{1,2} := \sum_{i \in \mathrm{rows}(\mathbf{X})} \|\mathrm{row}_i(\mathbf{X})^*\|_2$$

and $$\|\mathbf{X}\|_{2,2} := \left(\sum_{i \in \mathrm{rows}(\mathbf{X})} \|\mathrm{row}_i(\mathbf{X})^*\|_2^2 \right)^{\frac{1}{2}}.$$

Stylized example

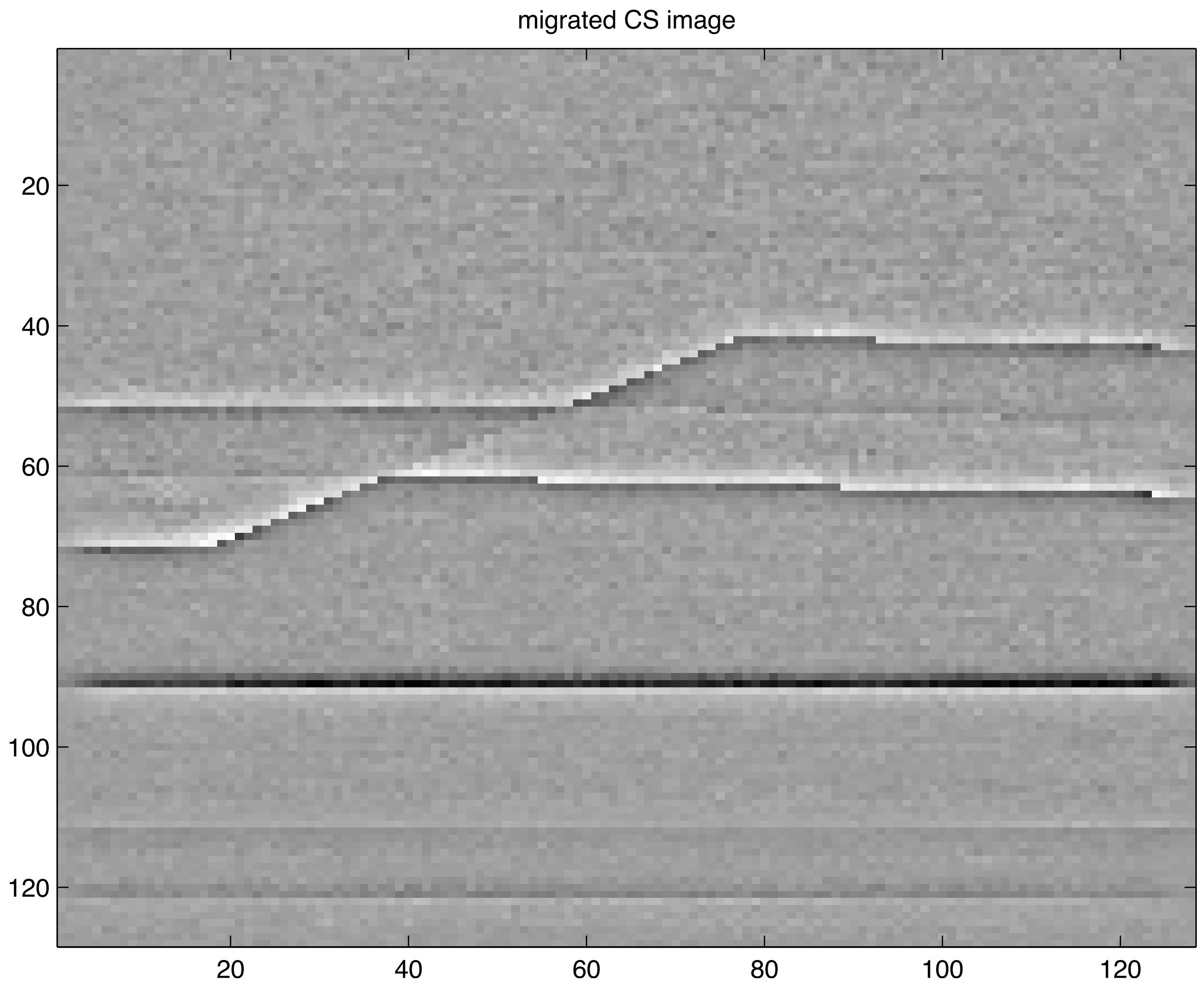


background velocity model

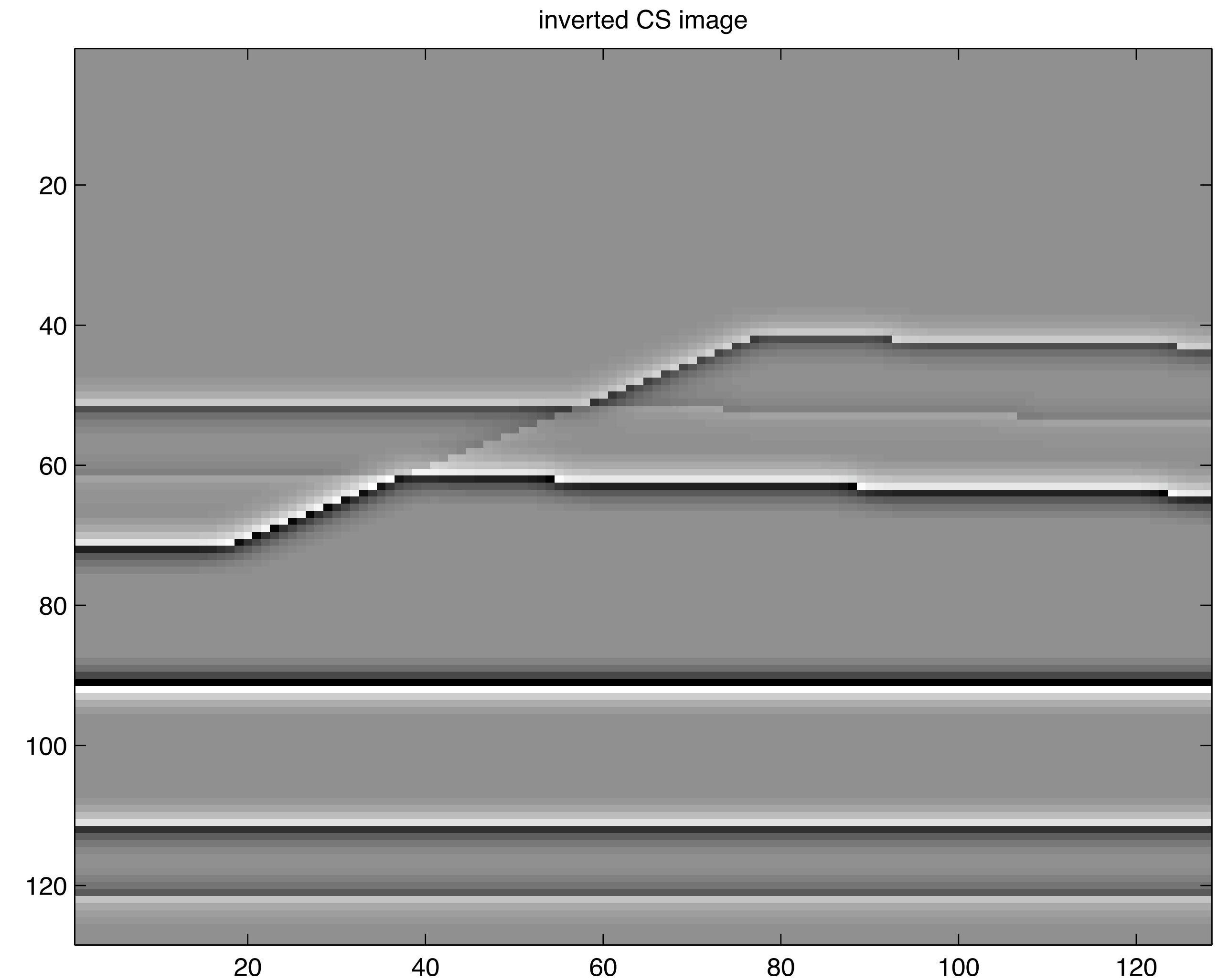


perturbation

Stylized example

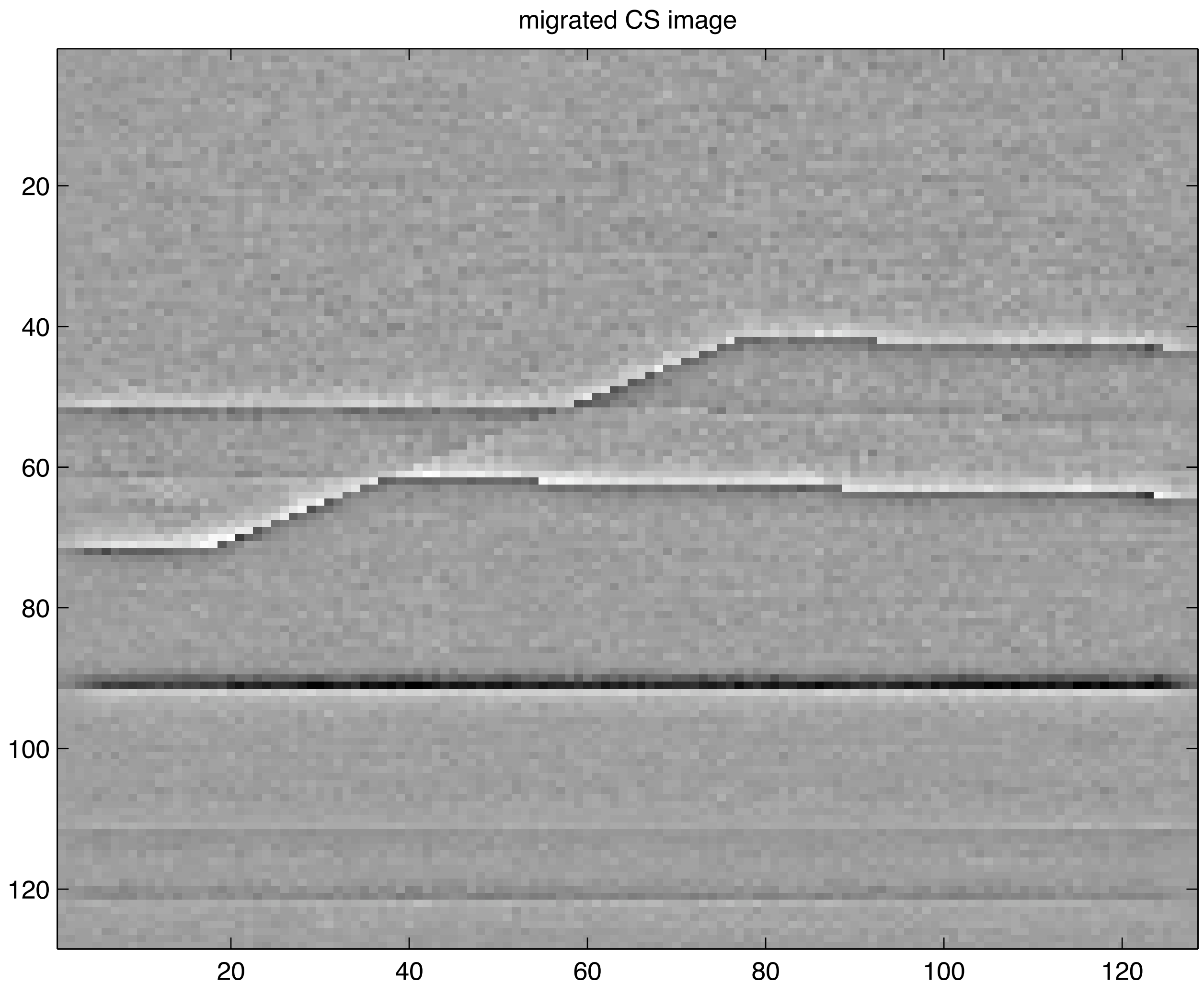


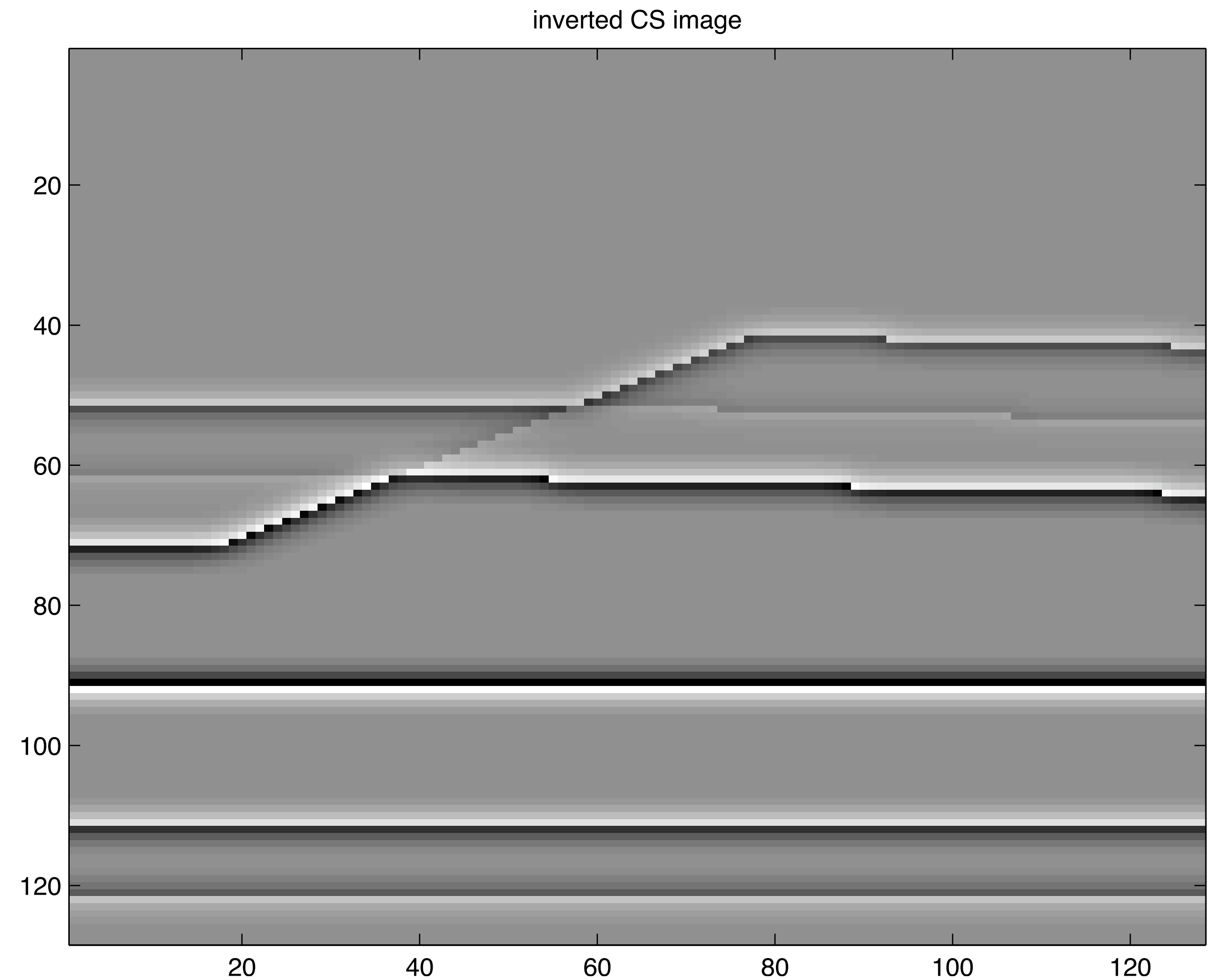
plain migration



wavefield inversion

Stylized example

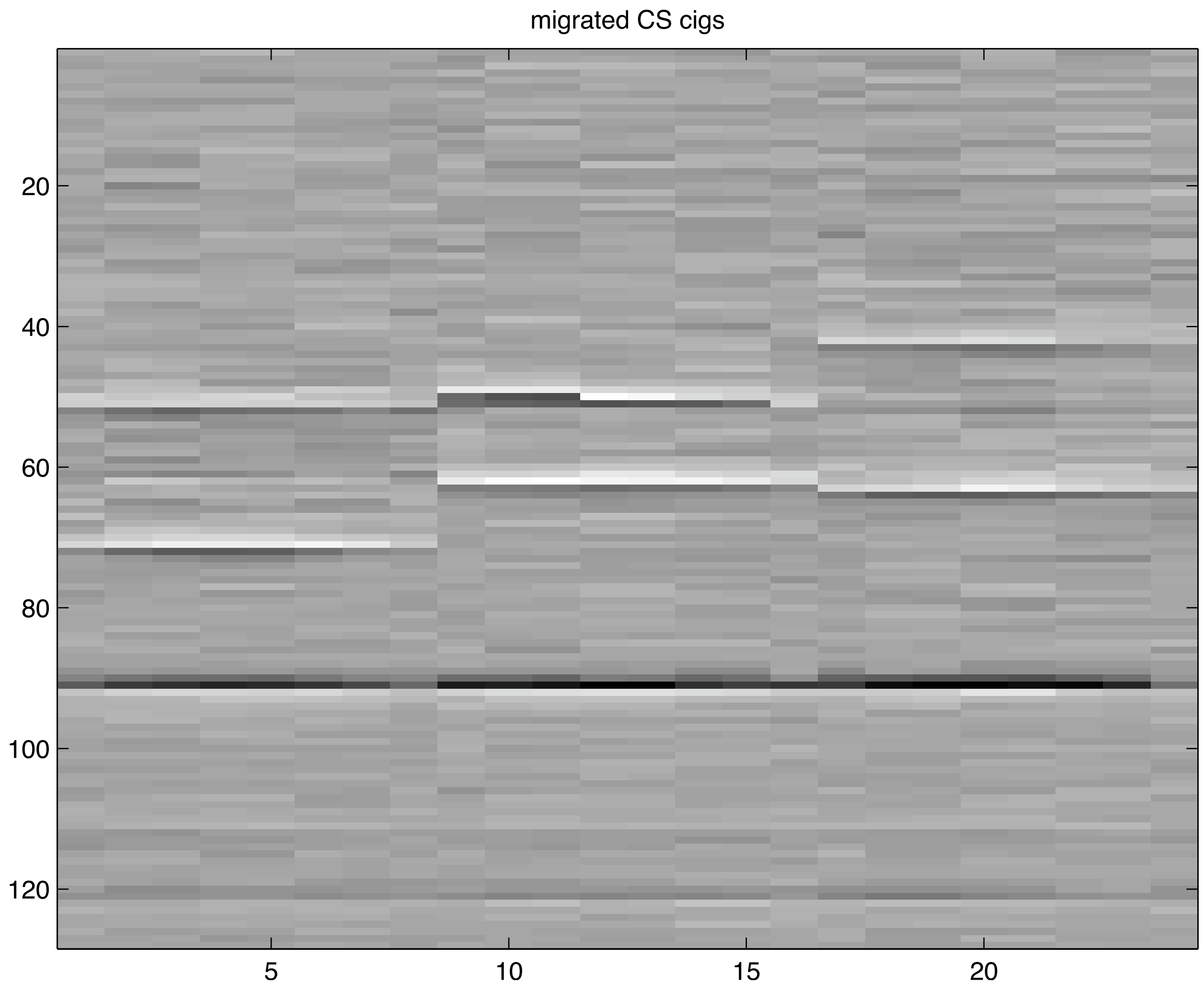




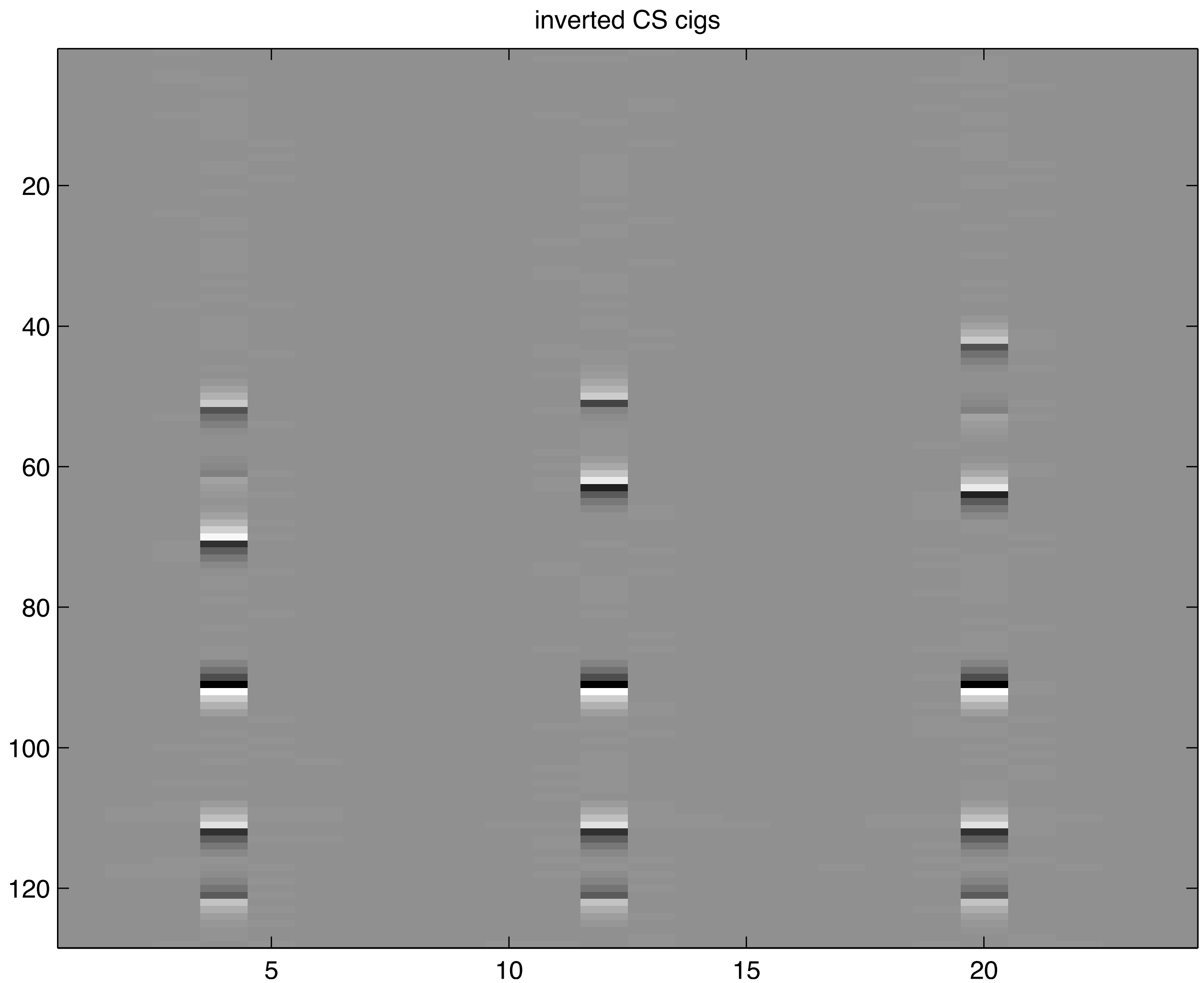
plain migration **wavefield inversion**

Recovery from 64-fold subsampling ...

Stylized example

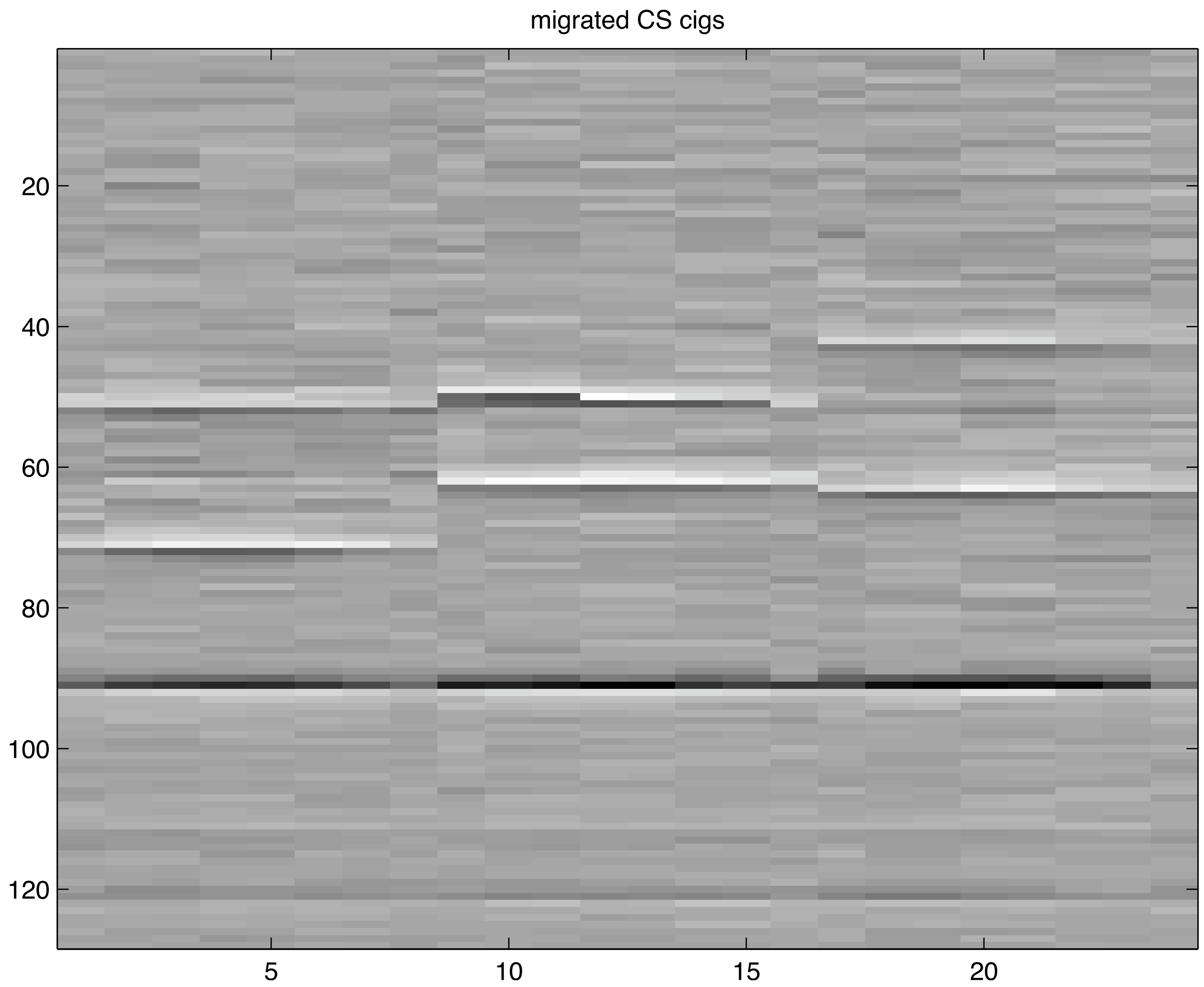


correlation based

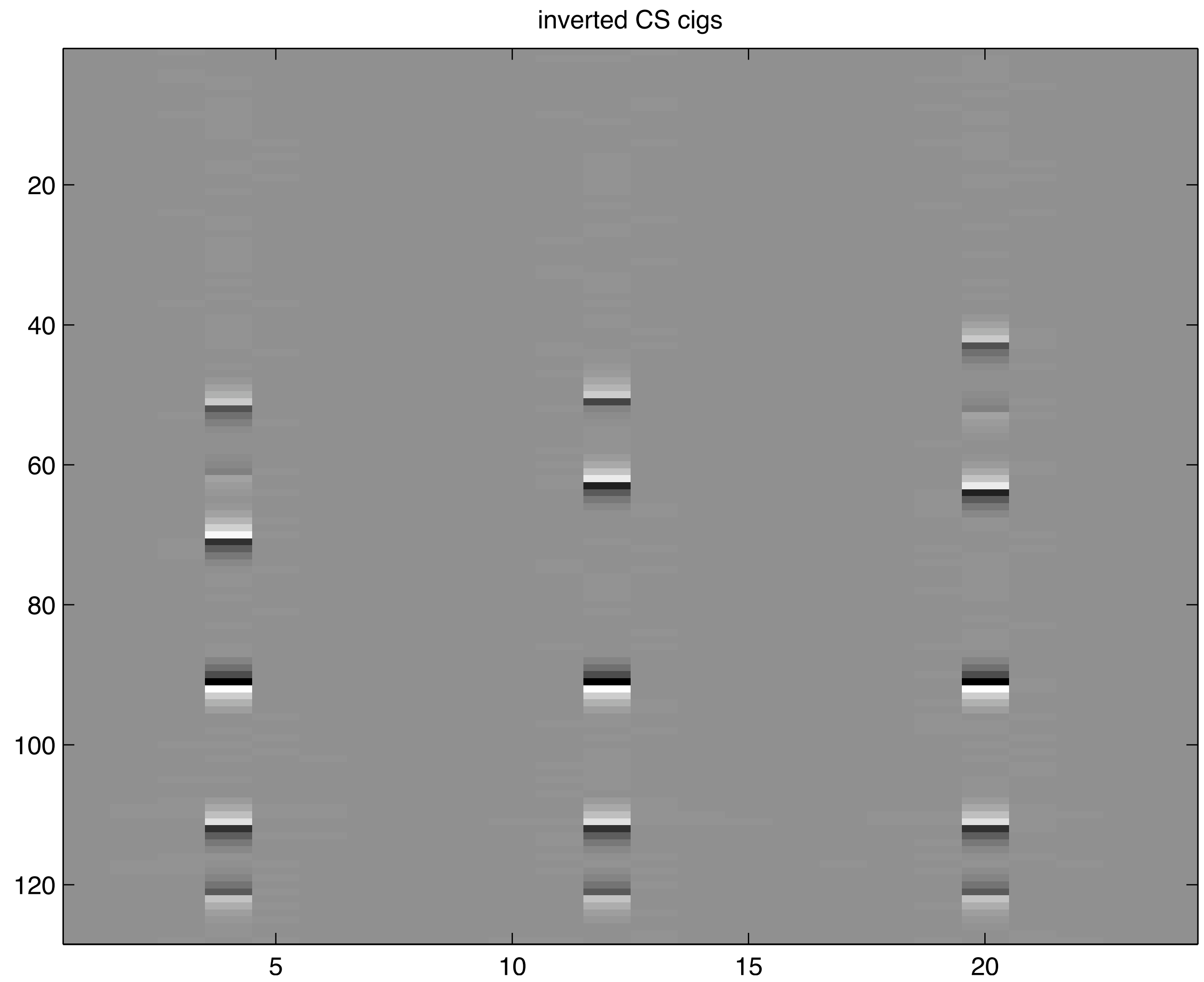


wavefield inversion

Stylized example



correlation based



wavefield inversion

Common-image gathers are focussed.

Observations

- **CS** provides a **new** *linear sampling paradigm* based on ***randomization***
  - ***reduces*** *data* volumes and hence *acquisition*, *processing* & *inversion* **costs**
  - **linearity** allows for compressive processing & inversion

- **CS** leads to
  - "acquisition" of *smaller* data volumes that carry the ***same*** *information* or
  - to ***improved*** *inferences* from data using the *same* resources
  - ***concrete implementations***

- **CS** combined with physics improved recovery by using
  - compressively-sampled multiples
  - focusing in the image space

- **Bottom line: acquisition & processing & inversion costs** are **no** longer determined by the ***size*** of the ***discretization*** but by ***transform-domain sparsity*** of the **solution ...**

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and... Thank you!