Compressive sensing: a paradigm shift for the imaging sciences?

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Joint work with Gilles Hennenfent, Yogi Erlangga, and Tim Lin

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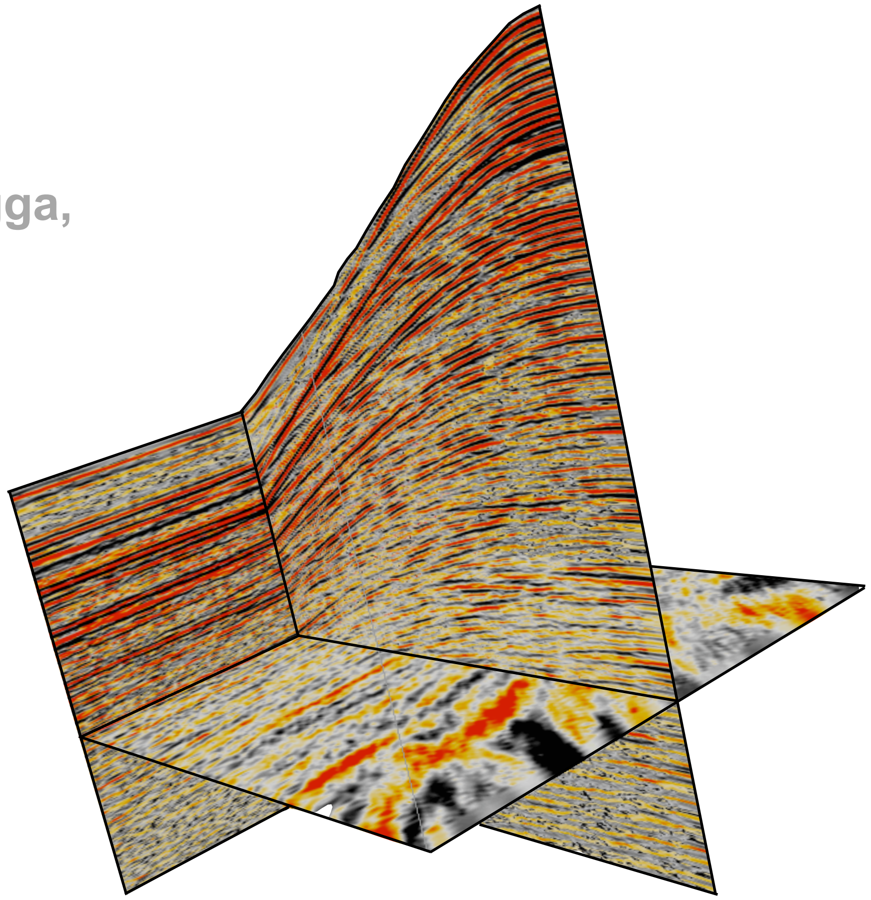

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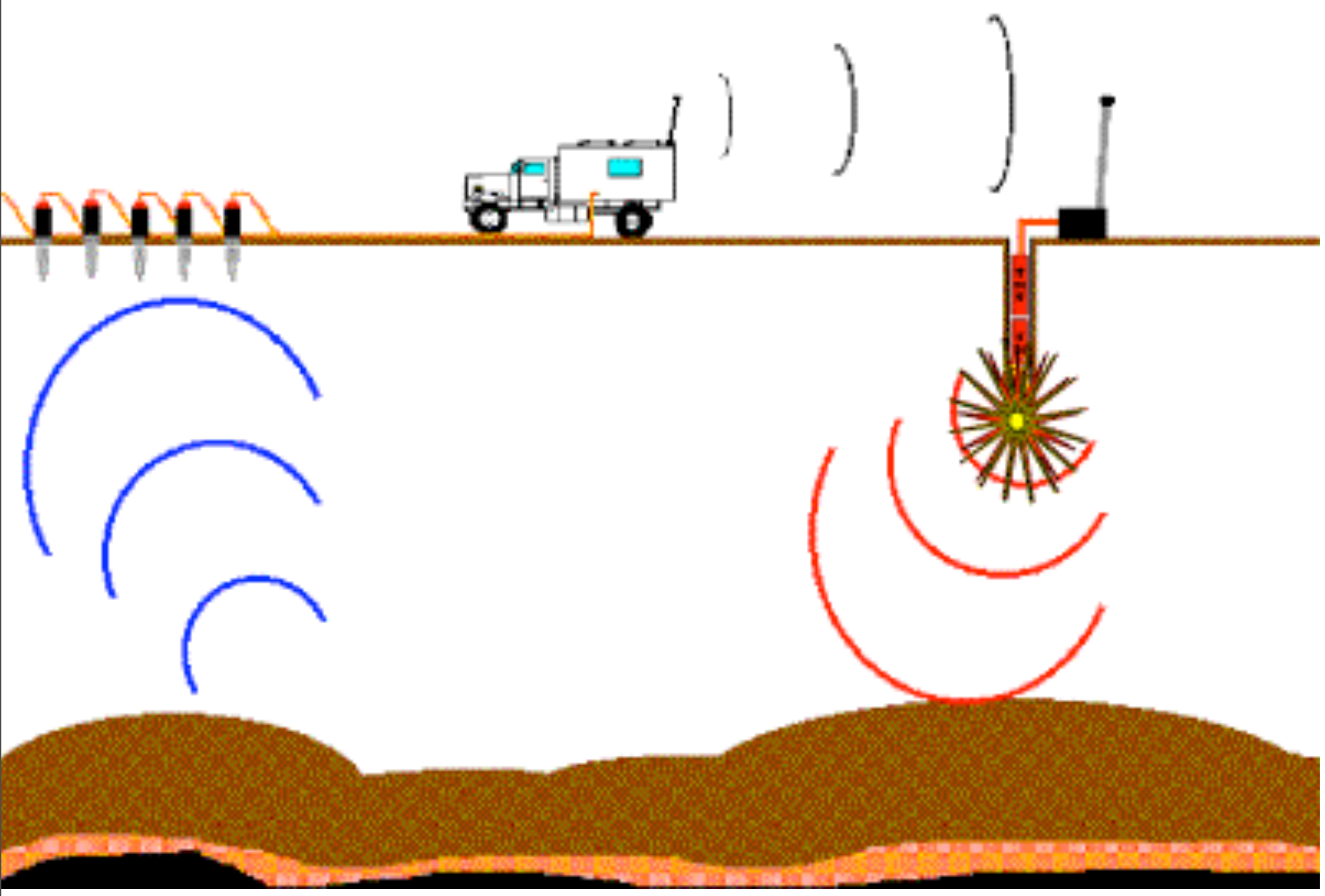
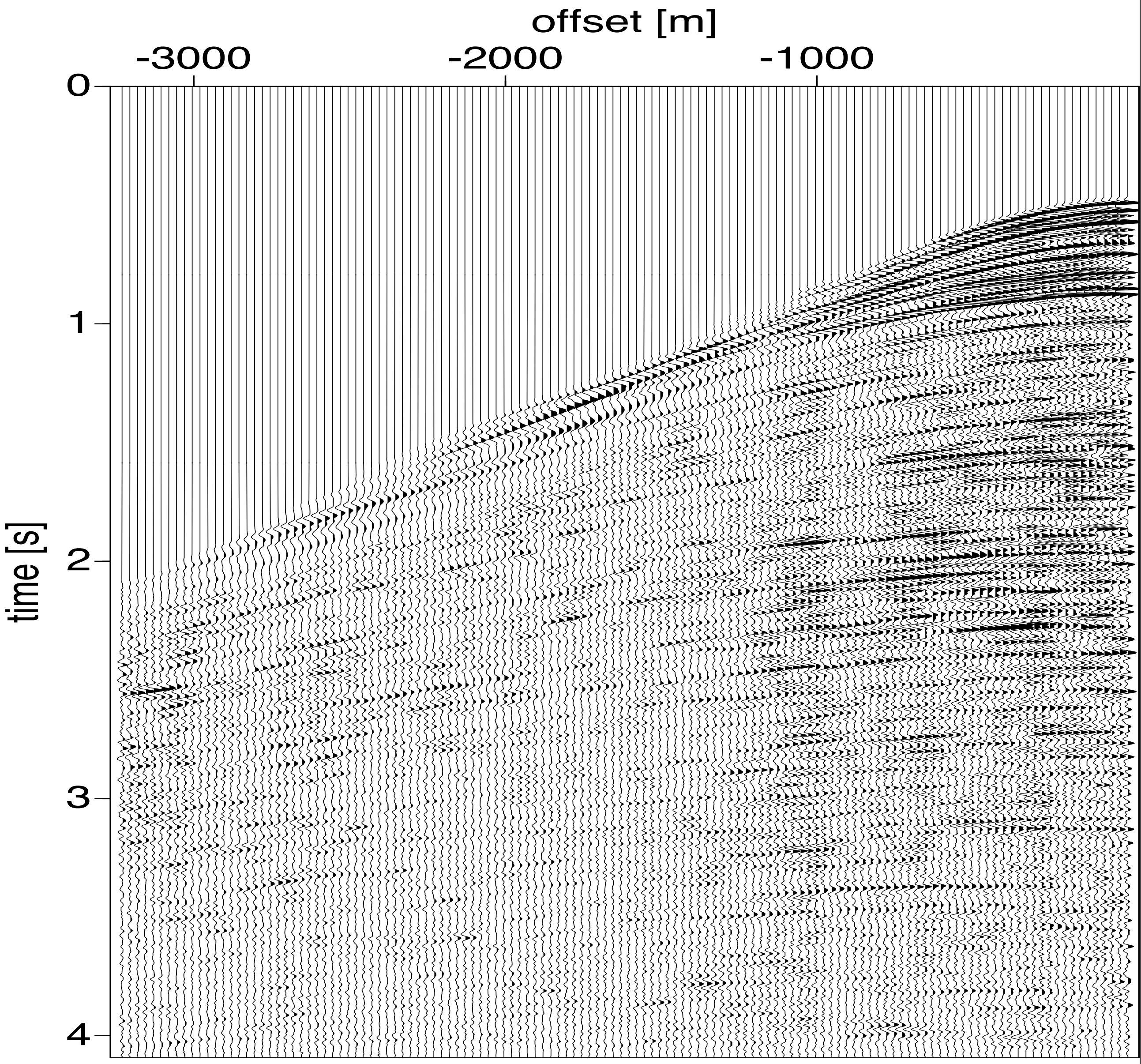
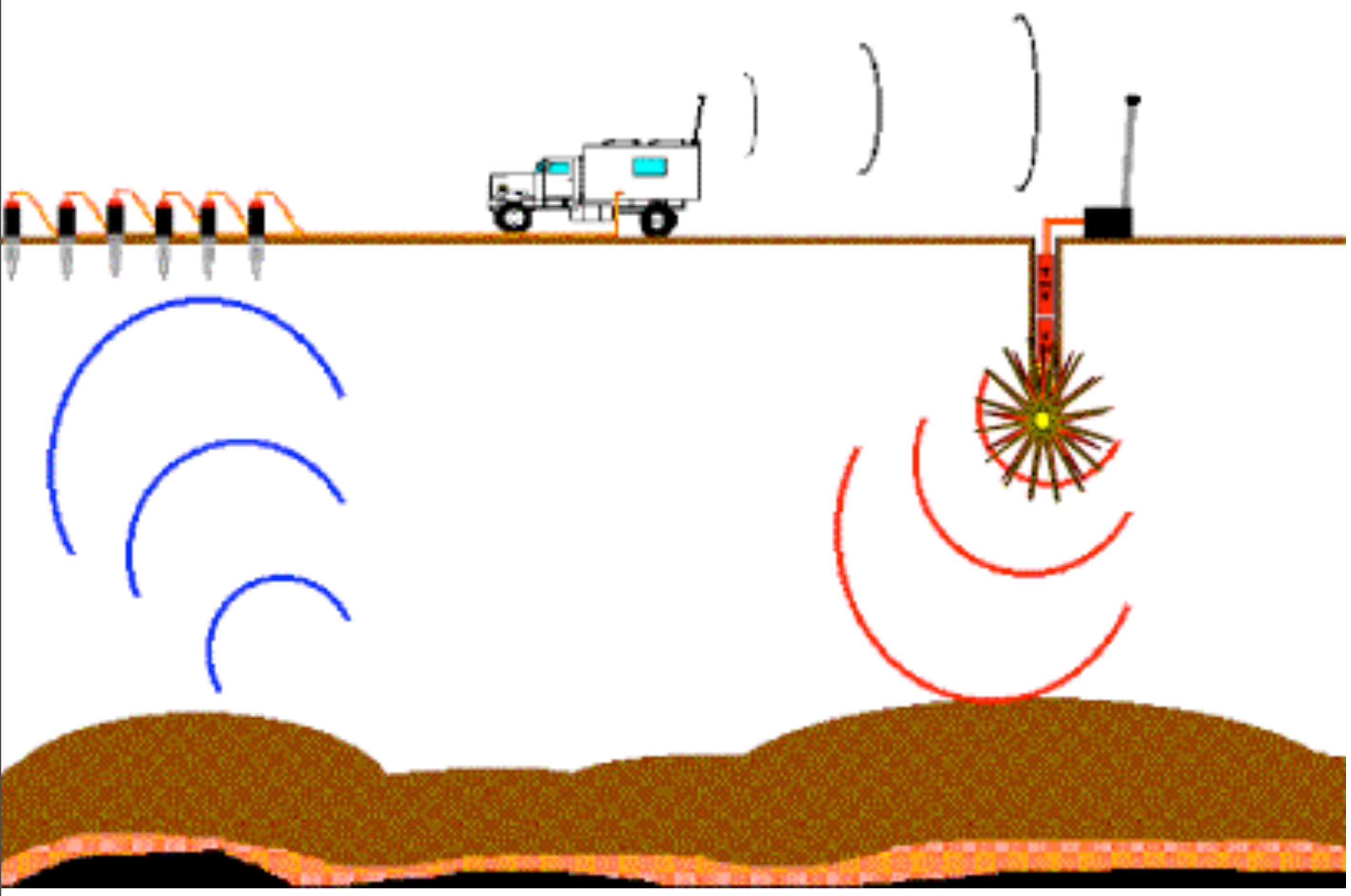
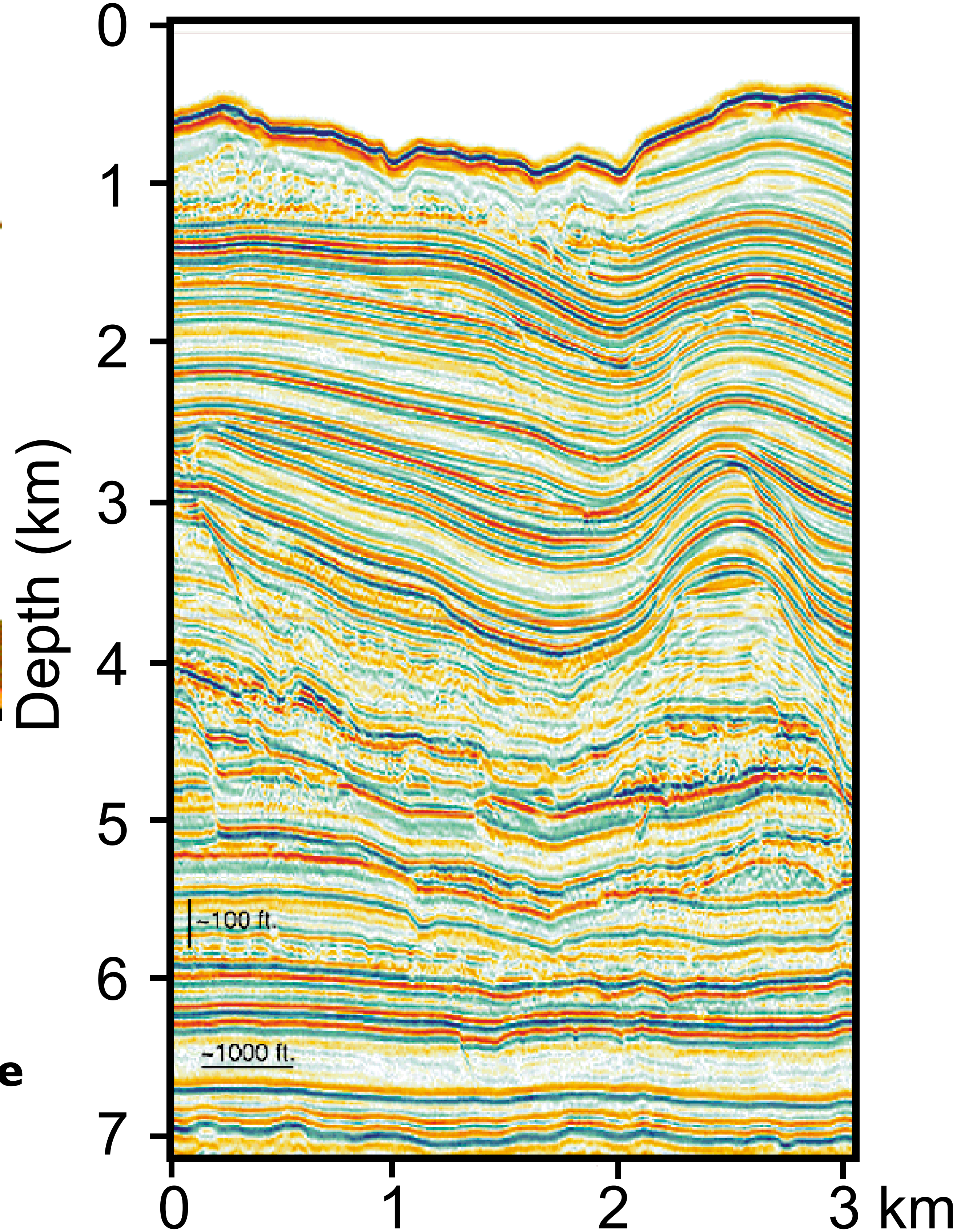
Seismic data acquisition





Exploration seismology



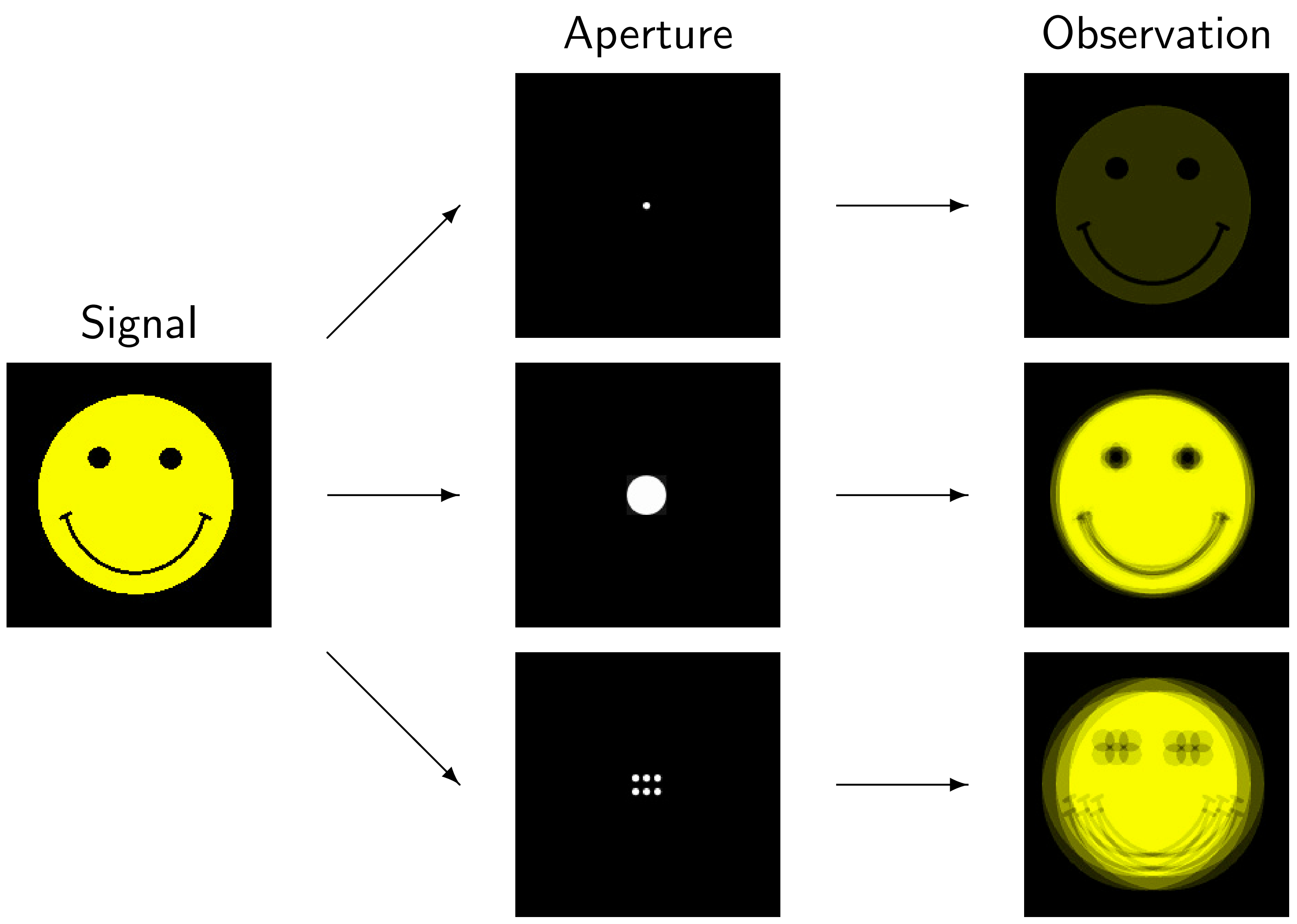


- **create images of the subsurface**
- **need for higher resolution/deeper**
- **clutter, data incompleteness, and large data size are problems**

Motivation

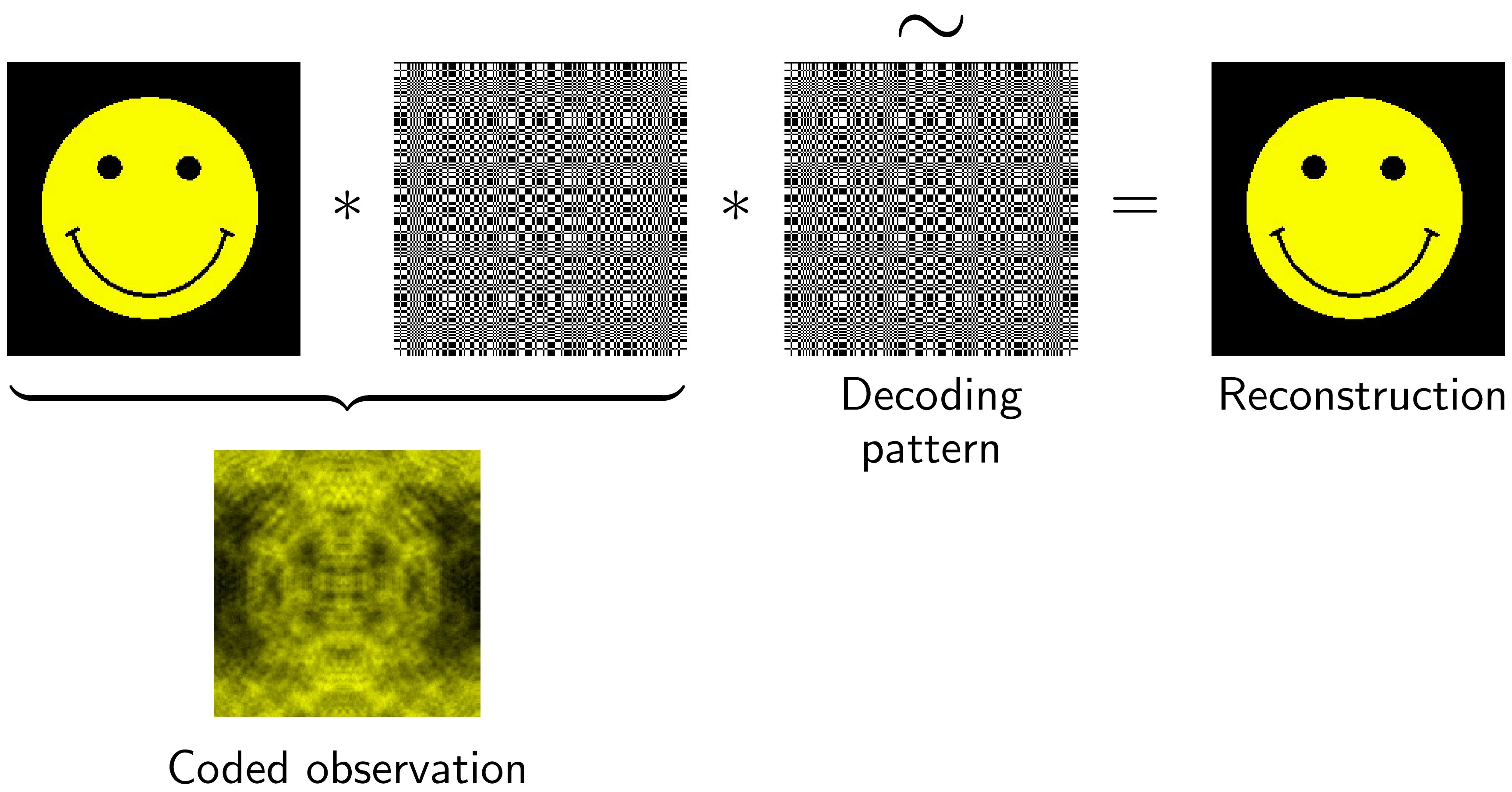
- Current state of affairs:
  - (Seismic) data *acquisition*, *processing*, *modeling*, and *imaging* are firmly rooted in the *paradigm* of *regular* **Nyquist** sampling
  - Acquisition is based on *regularly*-sampled & *source*-separated data volumes
- Major *impediment* related to the *size* of seismic data volumes
  - costs of acquisition & modeling dependent on size of the discretization
  - difficult to form *explicit* (imaging) operators
  - difficult to solve *implicit* (imaging) operators
- Recent theoretical developments
  - New *nonlinear* sampling theory--known as ***Compressive Sensing*** (CS)--that supersedes the overly *pessimistic* ***Nyquist*** *sampling* ***criterion***
  - New CS-based acquisition techniques are being developed (MRI, AD, Radar, etc)
  - New *continuous* and *simultaneous* acquisition strategies <=> instances of CS
- Success of compressive sensing hinges on
  - transform-domain sparsity (e.g. sparsity attained by multiscale/directional curvelets)
  - linear subsampling schemes that reduce interferrences
  - nonlinear recovery by sparsity promotion

An example [adapted from Roummel F. Marcia and Rebecca M. Willett, 2008]



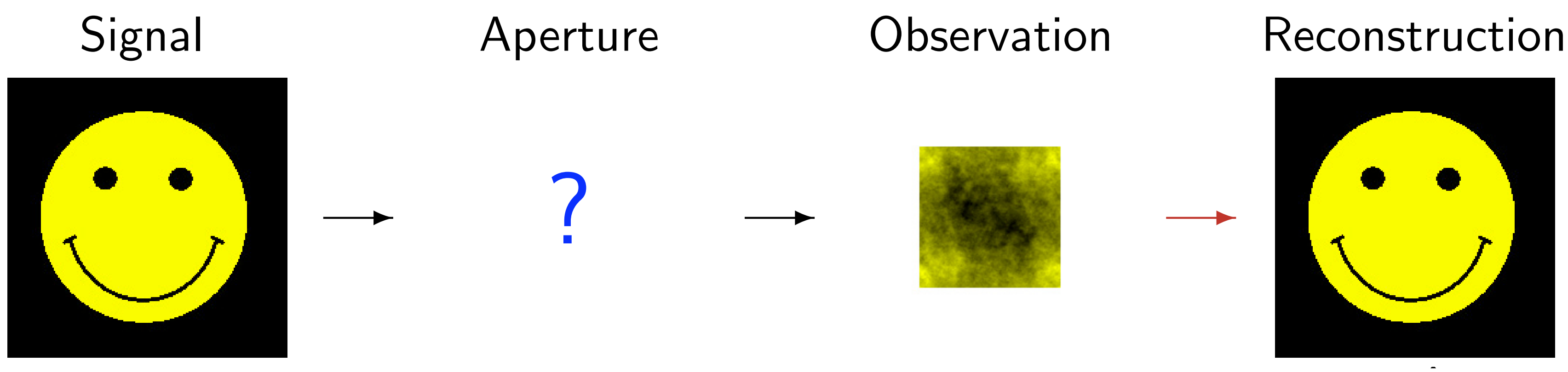
- Large pin holes (samples) blur
- Regularly-sampled multiple pinholes lead to coherent interferences

An example [adapted from Roummel F. Marcia and Rebecca M. Willett, 2008]



- Coded aperture gives brighter image
- Requires linear decoding
- ***No reduction*** in acquired data volumes

An example [adapted from Roummel F. Marcia and Rebecca M. Willett, 2008]



- Reduce the dimensionality by subsampling
  - create observations that contain the same information (encoded)
  - facilitate recovery by nonlinear transform-domain sparsity promotion
- Design physically realizable encoding schemes that break interferences
- Design nonlinear recovery techniques that exploit transform-domain sparsity/compressibility of certain structured signals
  - multiscale & directional wavelets
  - multiscale, multidirectional, and anisotropic curvelets

Today's agenda

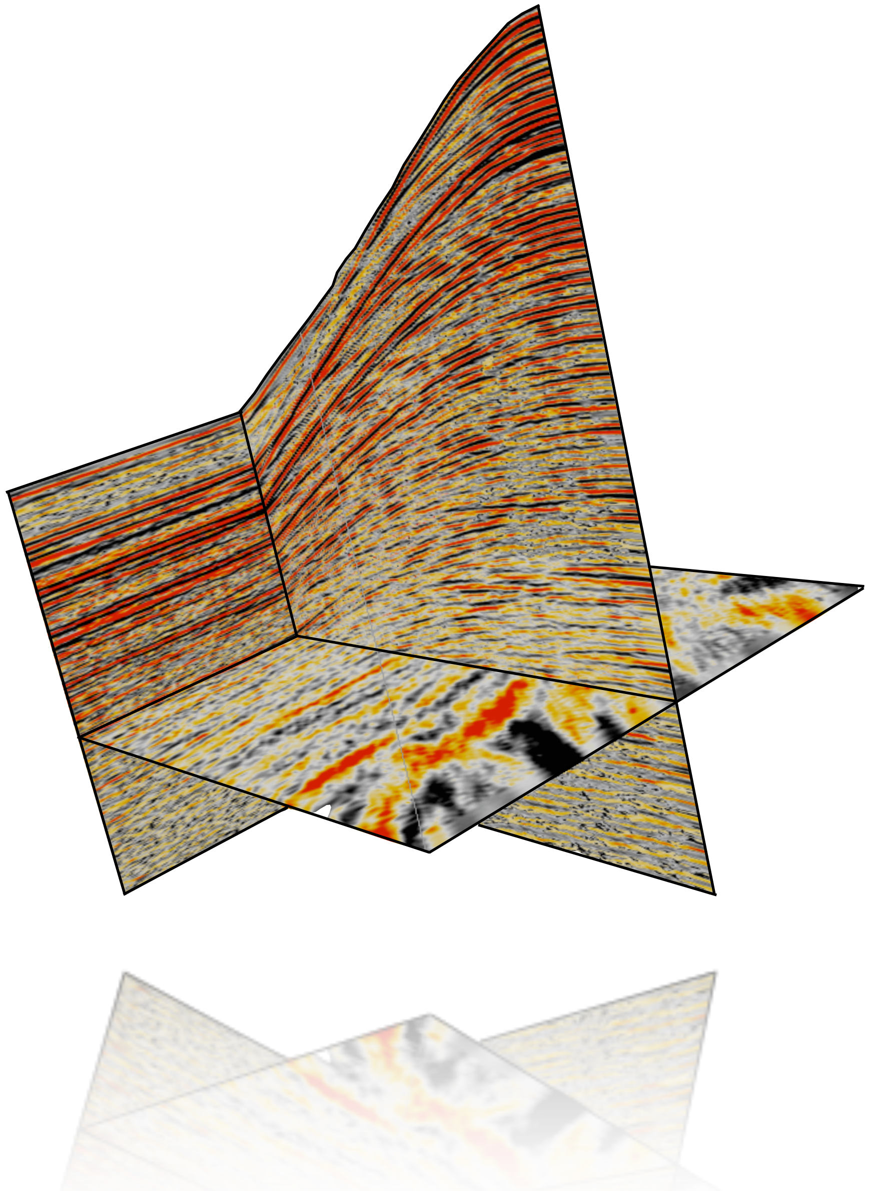
- Brief introduction to *compressive sensing*
  - sparsifying transform
  - favorable (random) acquisition (acquisition-grid and source-function design)
  - nonlinear recovery by sparsity promotion

- CS applied to wavefield reconstruction from incomplete acquisitions
  - use the "***Dirac" basis*** as the ***sampling*** domain
  - reduction of the number of sources & receivers

- CS applied to ***explicit*** one-way wavefield computations
  - use of the ***modal*** domain as the ***sampling*** domain
  - reduction of the number of eigenvectors & frequencies

- CS applied to ***implicit*** simultaneous full-waveform simulation
  - use ***simultaneous sources*** as the ***sampling*** domain
  - reduction of the number of right-hand sides & frequencies

Compressive sensing



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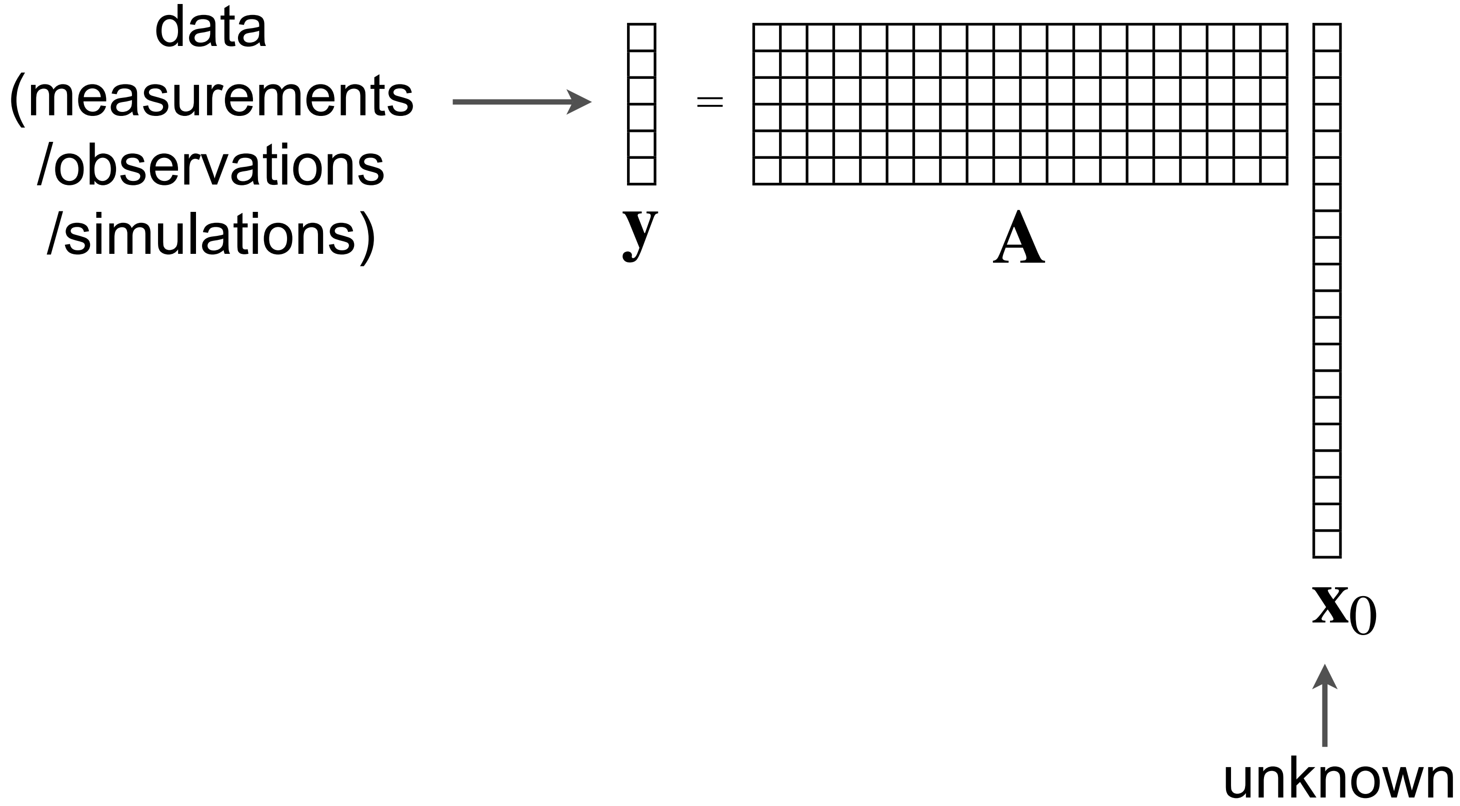
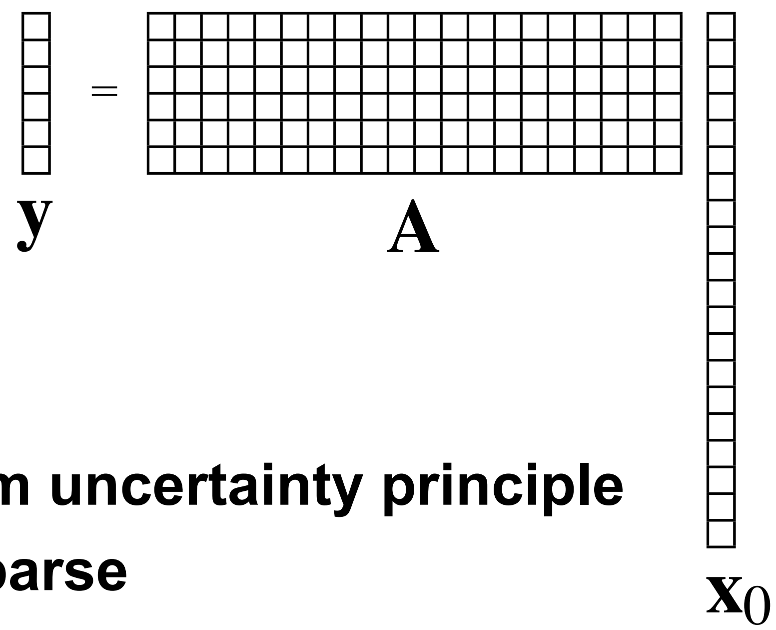
Problem statement

Consider the following (severely) underdetermined system of linear equations



Is it possible to recover $\mathbf{x}_0$ accurately from $\mathbf{y}$?

Perfect recovery



- conditions
  - **A** obeys the **uniform uncertainty principle**
  - **x**$_0$ is **sufficiently sparse**

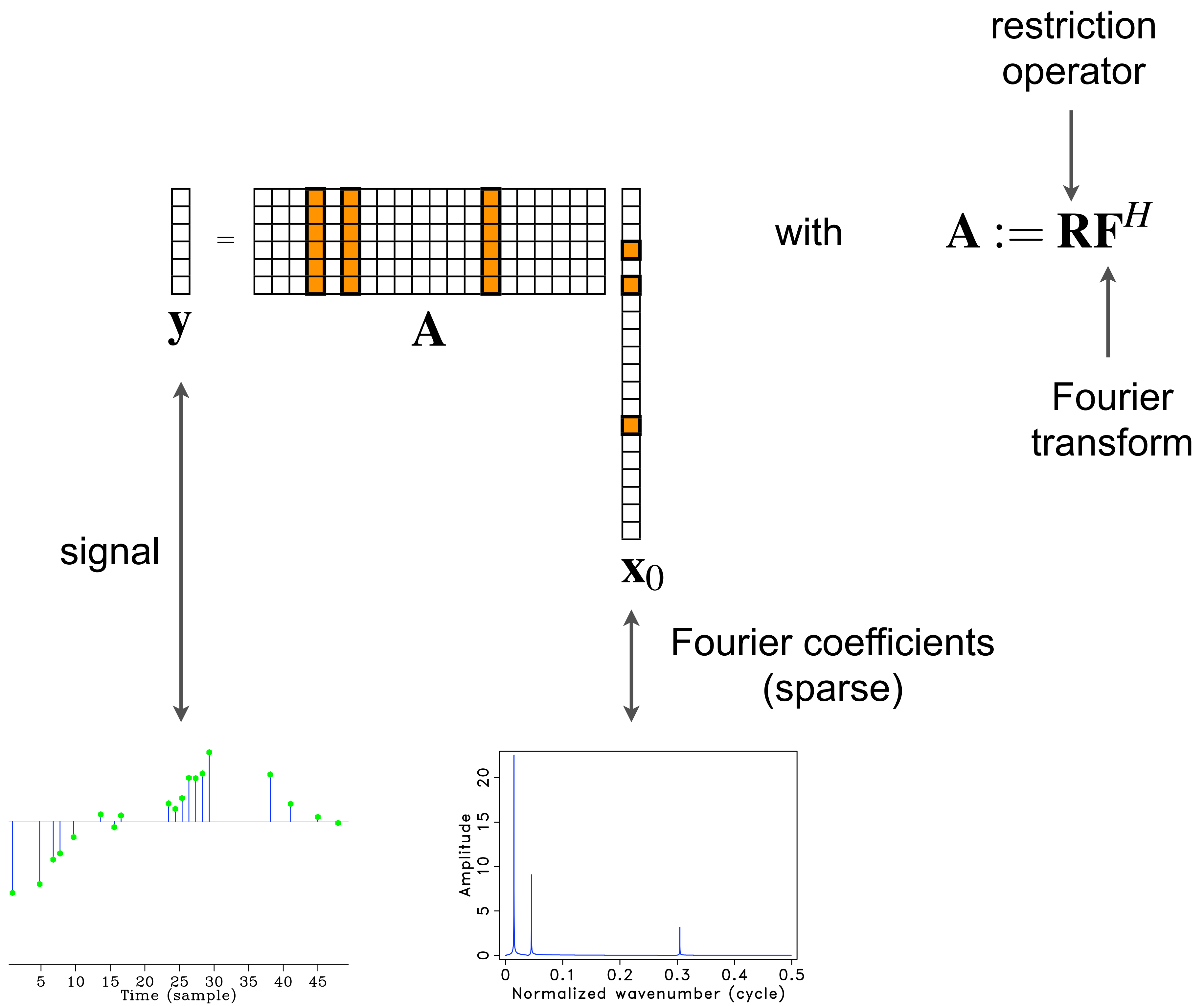
- procedure

$$\underbrace{\min_{\mathbf{x}} \|\mathbf{x}\|_1}_{\text{sparsity}} \quad \text{s.t.} \quad \underbrace{\mathbf{A}\mathbf{x} = \mathbf{y}}_{\text{perfect reconstruction}}$$

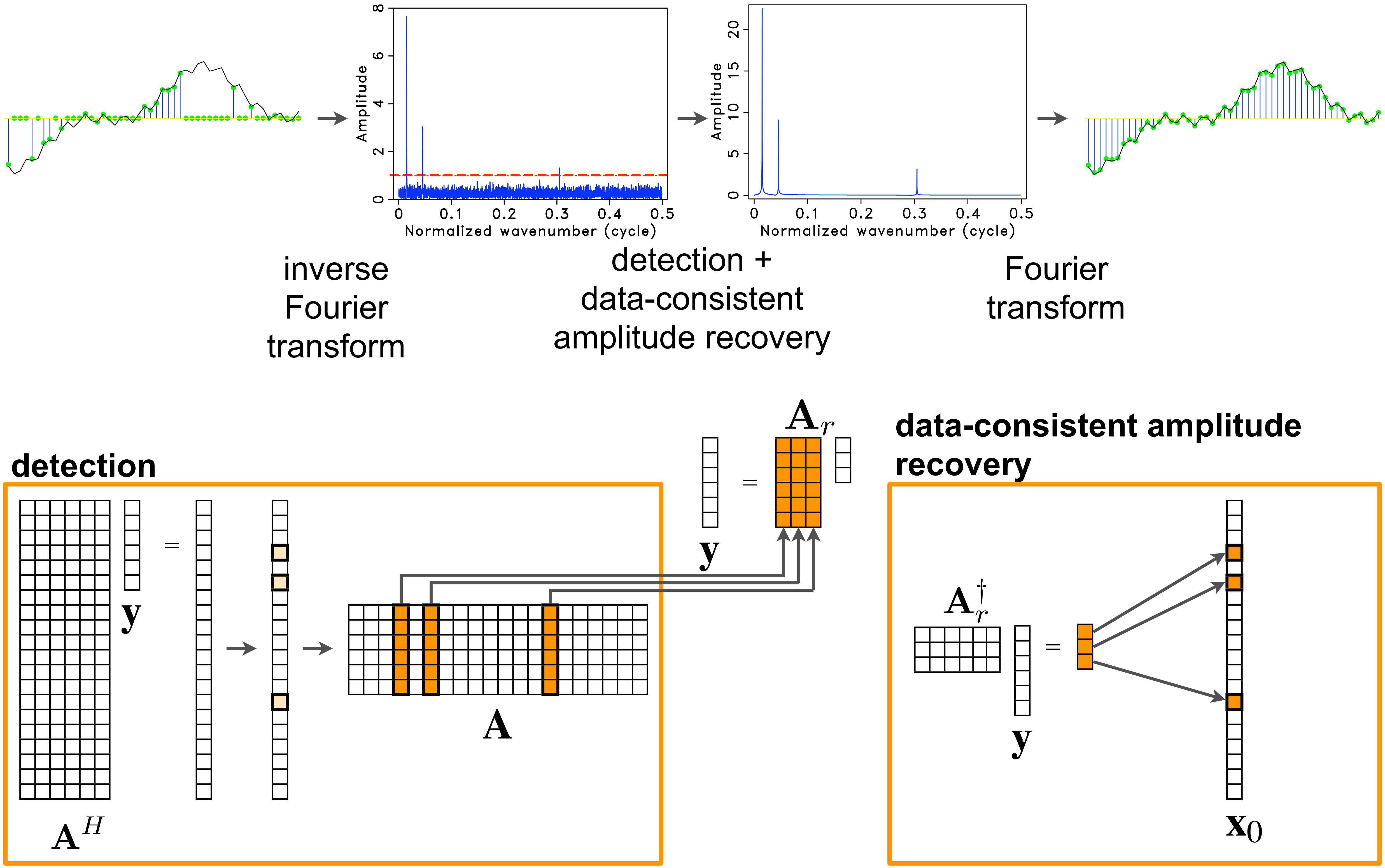
- performance
  - *S*-sparse vectors recovered from roughly on the order of *S* measurements (to within constant and *log* factors)

[Candès et al.'06]
[Donoho'06]

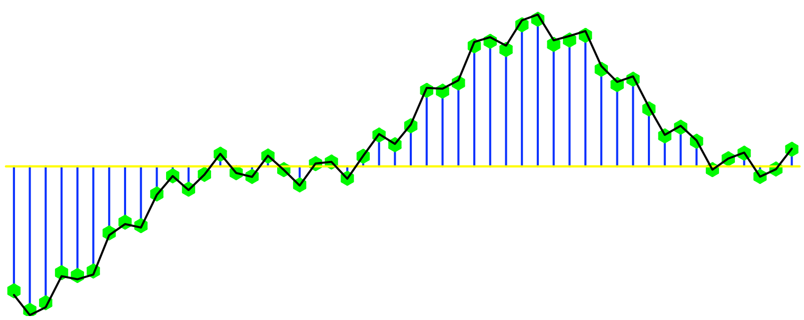
Simple example



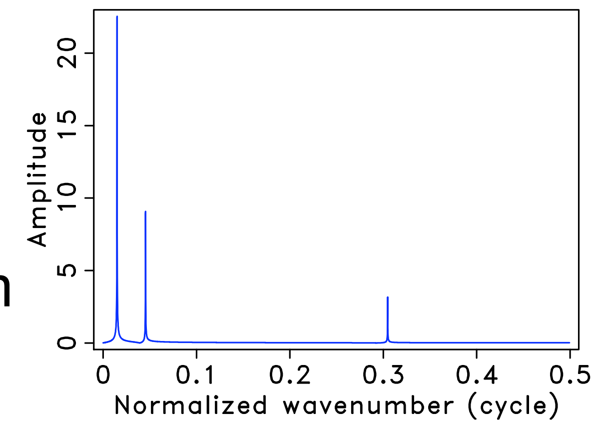
NAIVE sparsity-promoting recovery



Coarse sampling schemes

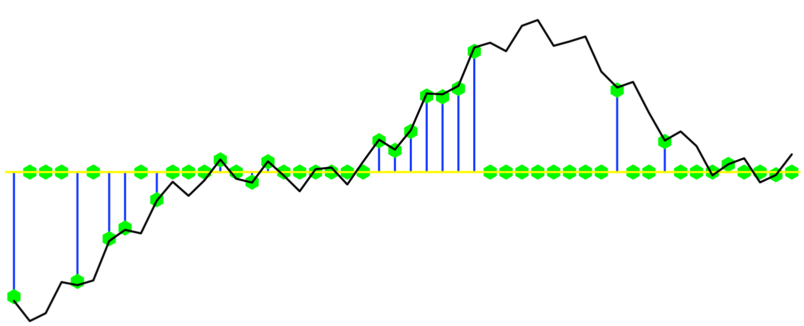


Fourier transform

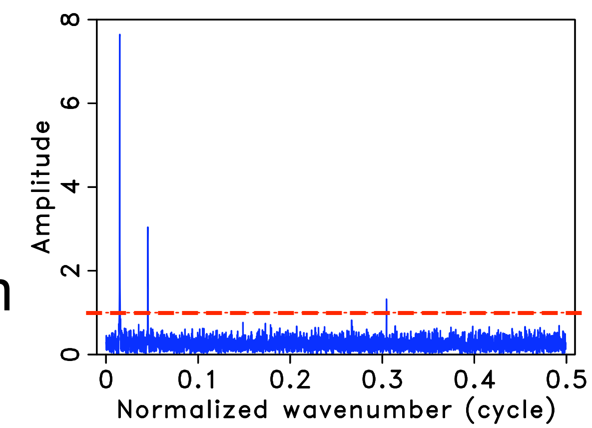


few significant coefficients

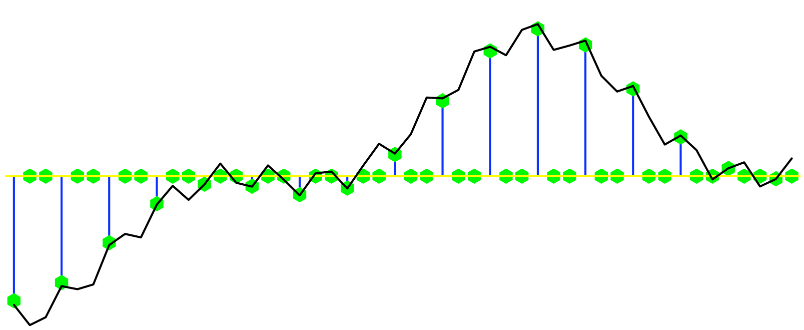
3-fold under-sampling



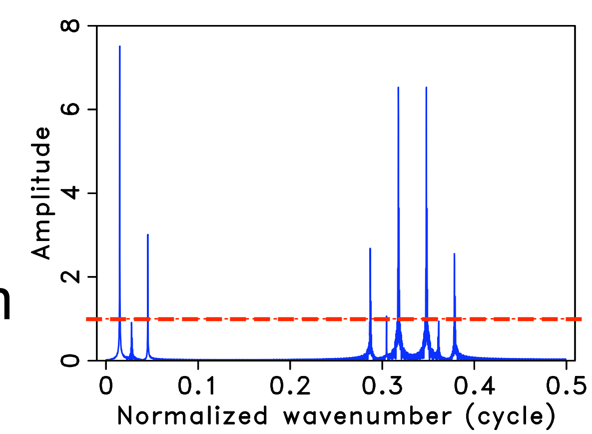
Fourier transform



✓ significant coefficients detected



Fourier transform



✗ ambiguity

[Hennenfent and Herrmann ‘08]

Extensions

Incomplete and noisy measurements:

$$\mathbf{y} = \overbrace{\mathbf{R}}^{\text{Restriction}} \underbrace{\mathbf{M}}_{\text{Measurement}} \mathbf{m} + \mathbf{n}$$

y *incomplete* (compressively sampled) and noisy data

m the *unknown* model

M “**arbitrary*”*** *measurement* matrix

Fourier, eigenfunctions

simultaneous sources

R *restriction* matrix

n Gaussian noise

Extensions

- Use CS principles to select appropriate
  - measurement basis **M**
  - sparsifying transform **S**
  - driven by signal type, physics, and type of acquisition (e.g. fMRI vs seismic)

- Sparse signal representation:

$$\mathbf{y} = \mathbf{A}\mathbf{x}_0$$

with

$$\mathbf{A} = \mathbf{R}\mathbf{M}\mathbf{S}^H$$

restriction matrix (R) **measurement matrix** (M) **sparsity matrix** (S^H)

Selection is aimed at turning *aliases/coherent subsampling artifacts* into harmless ***noise*** ...

Extensions

- According to CS theory (valid for orthonormal bases for **M** & **S**) recovery depends on ***restriction***, ***mutual coherence***, and ***sparsity***

- ***Mutual coherence*** between **M** & **S** (off-diagonals of Gramm matrix),

$$\mu(\mathbf{M}, \mathbf{S}) = \sqrt{m} \max_{(k,l)\in[1\cdots m]\times[1\cdots m]} |\langle m_k,\ s_l\rangle|$$

  and appropriate subsampling
  - controls leakage
  - maps interferences into noise
  - importance sample in the band

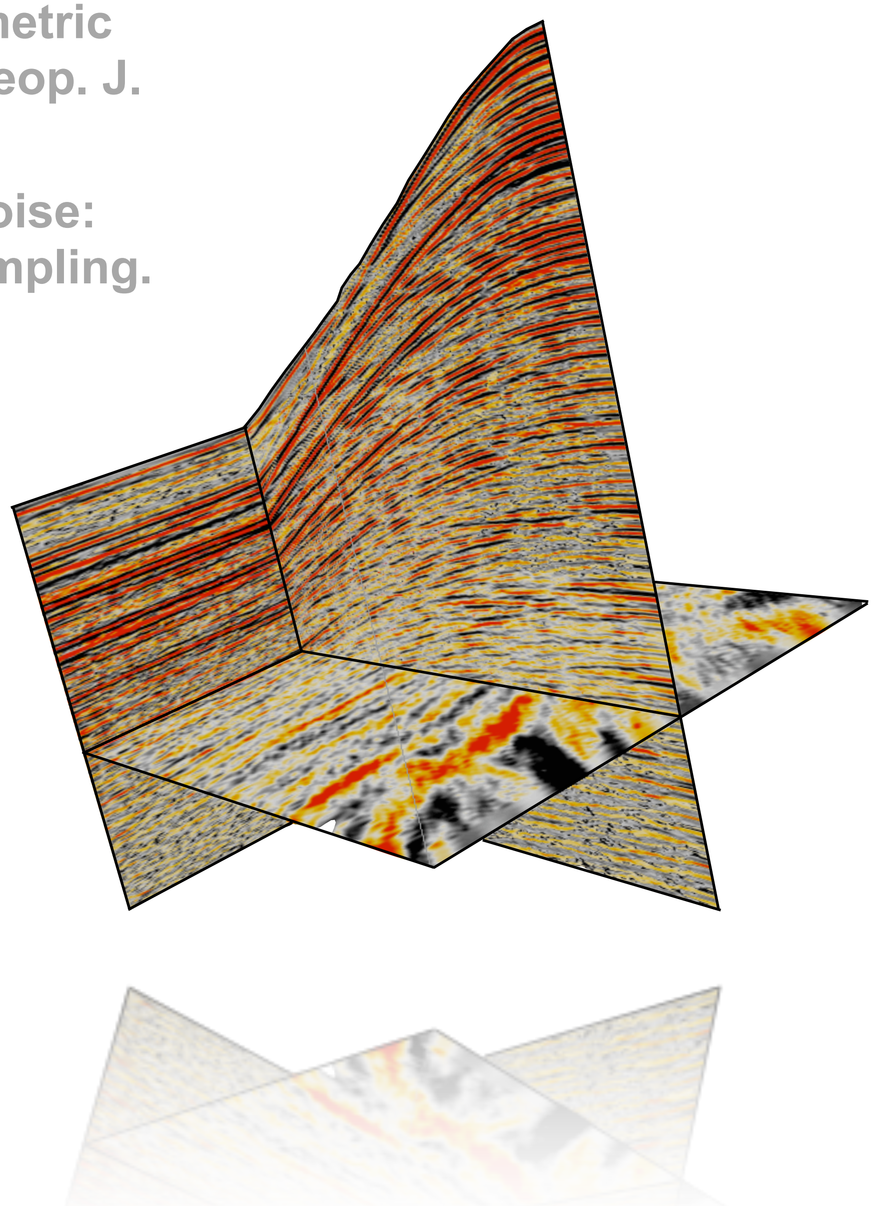
- ***Compressibility***, i.e.,

$$|\mathbf{x}_{i\in I}| \leq Ci^{-r},\ r \geq 1 \quad \text{and} \quad x_{I(1)} \geq x_{I(2)} \geq \cdots \geq x_{I(m)}$$

Wavefield reconstruction

Herrmann, F. J. and Hennenfent, G. Non-parametric seismic data recovery with curvelet frames, Geop. J. Int., Vol. 173, No. 1, pp. 233-248, 2008

Hennefent, G. and Herrmann, F. J. Simply denoise: wavefield reconstruction via jittered under-sampling. Geophysics, Vol. 73, No. 3, pp. V19–V28, 2008.



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Key elements

- *sparsifying transform*
  – typically **localized** in the time-space domain to handle the complexity of seismic data

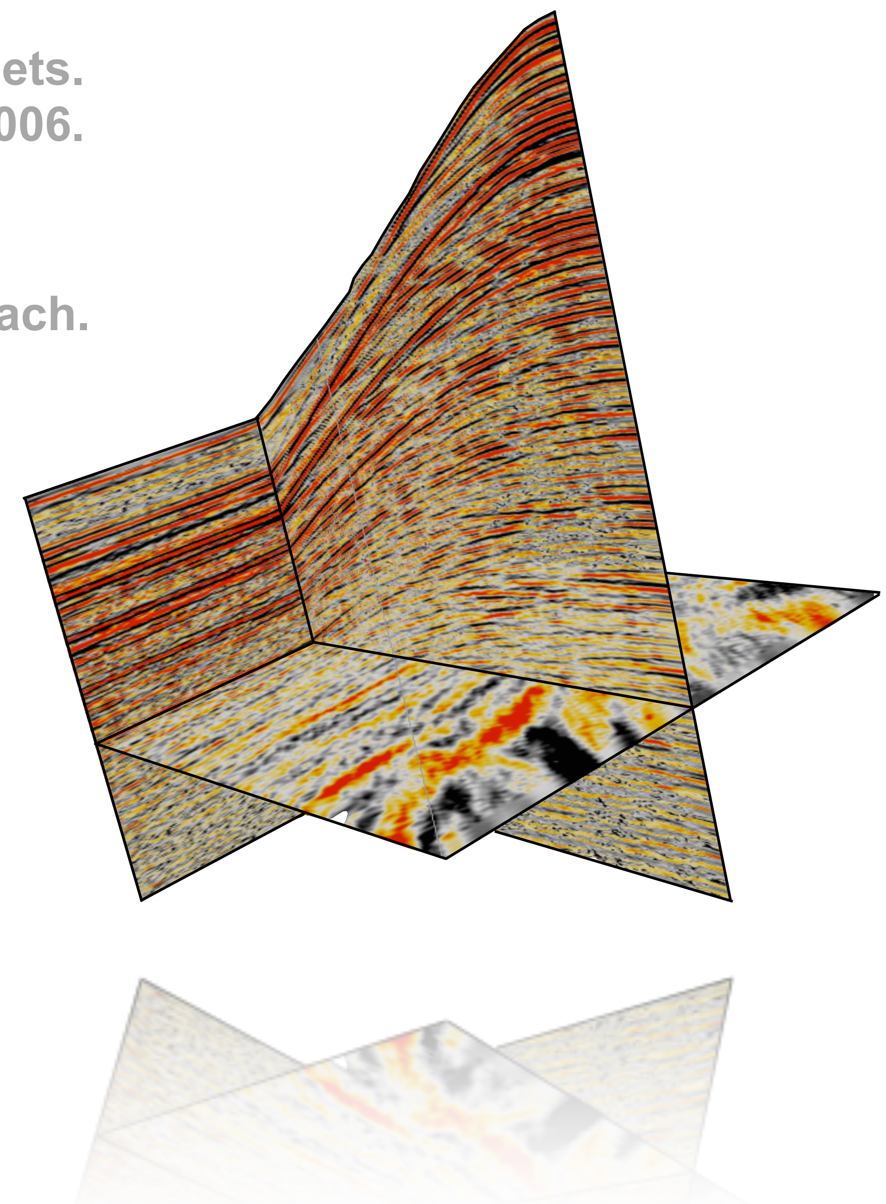
- *advantageous coarse sampling*
  – generates incoherent random undersampling “noise” in the sparsifying domain
  – does not create large gaps
    - because of the limited spatiotemporal extent of transform elements used for the reconstruction

- *sparsity-promoting solver*
  – requires few matrix-vector multiplications

Curvelet transform

Hennenfent, G and Herrmann, F. J., Seismic Denoising with Nonuniformly Sampled Curvelets. Comp. in Sc. & Eng., vol. 8, no. 3, pp. 16-25, 2006.

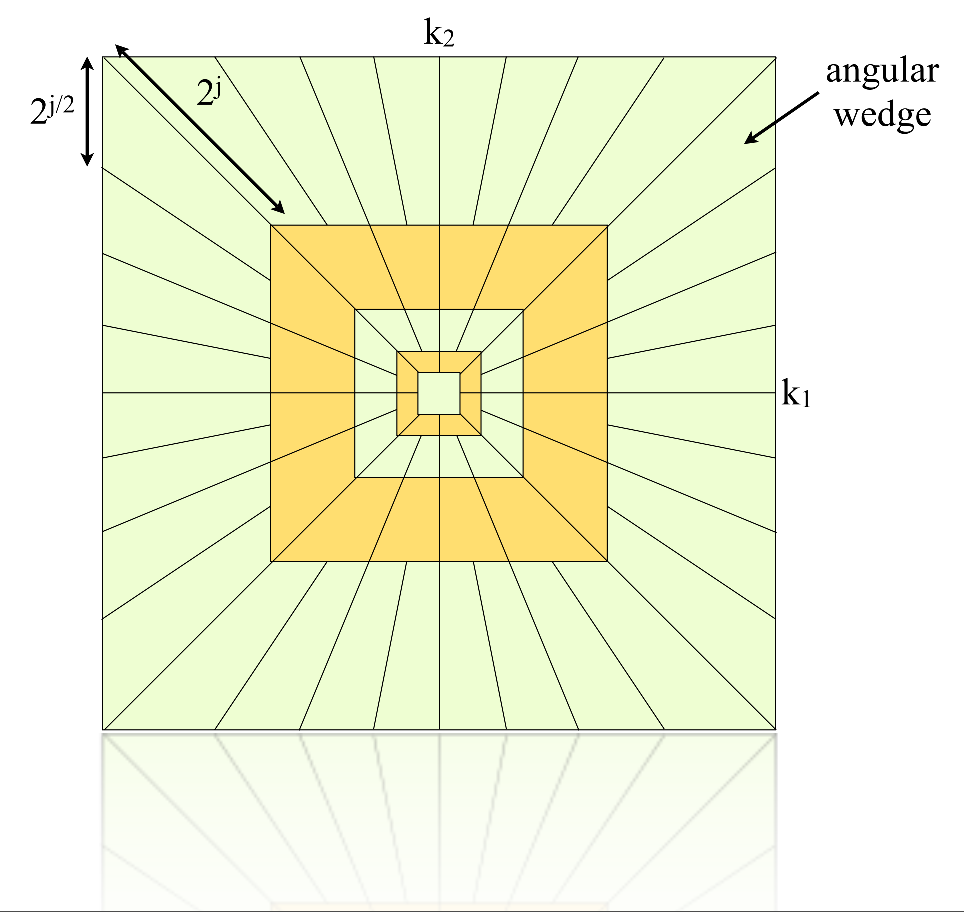
Herrmann, F. J., Wang, D., Hennenfent, G. and Moghaddam, P. Curvelet-based seismic data processing: a multiscale and nonlinear approach. Geophysics, Vol. 73, No. 1, pp. A1–A5, 2008.



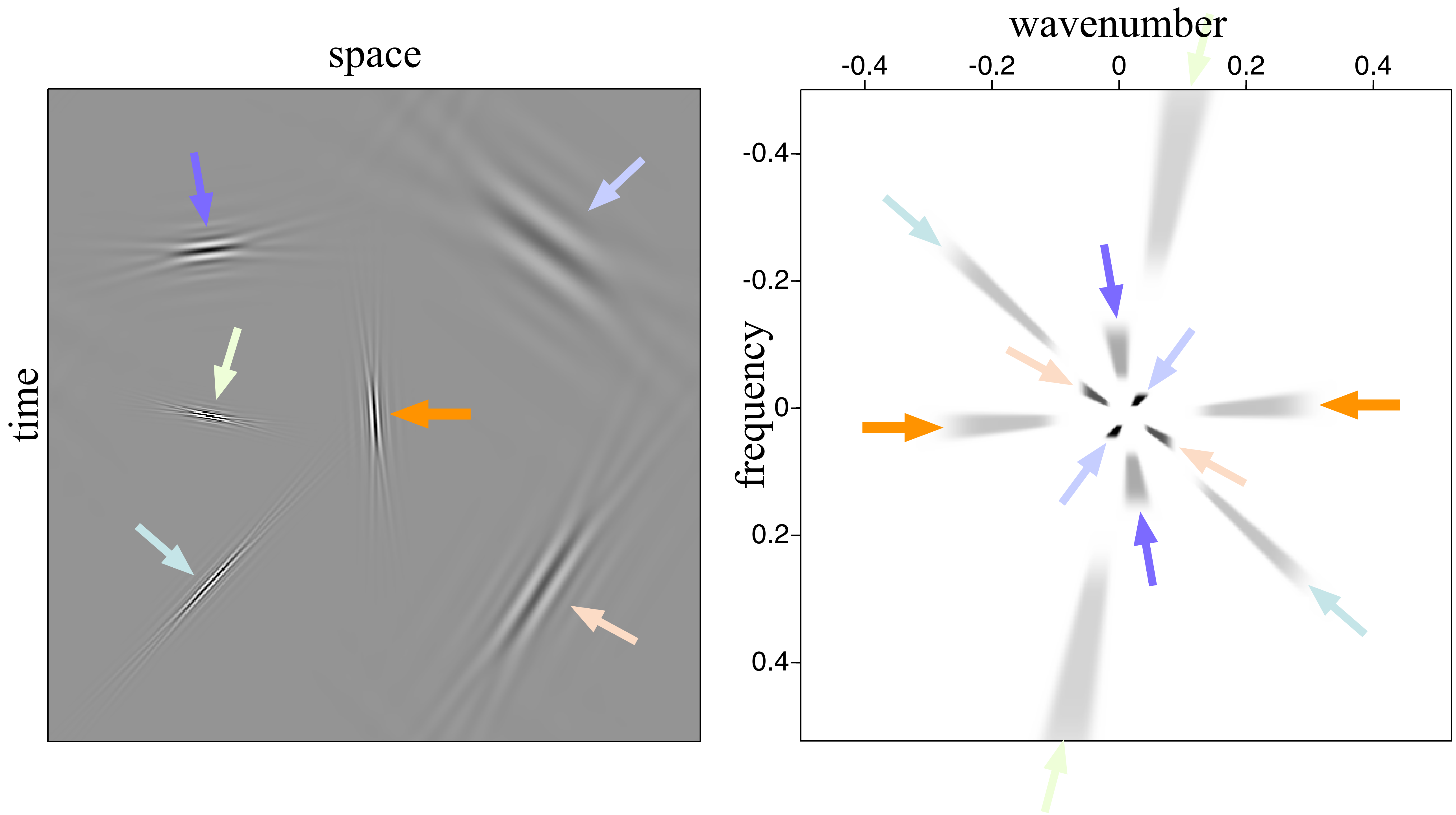
Representations for seismic data

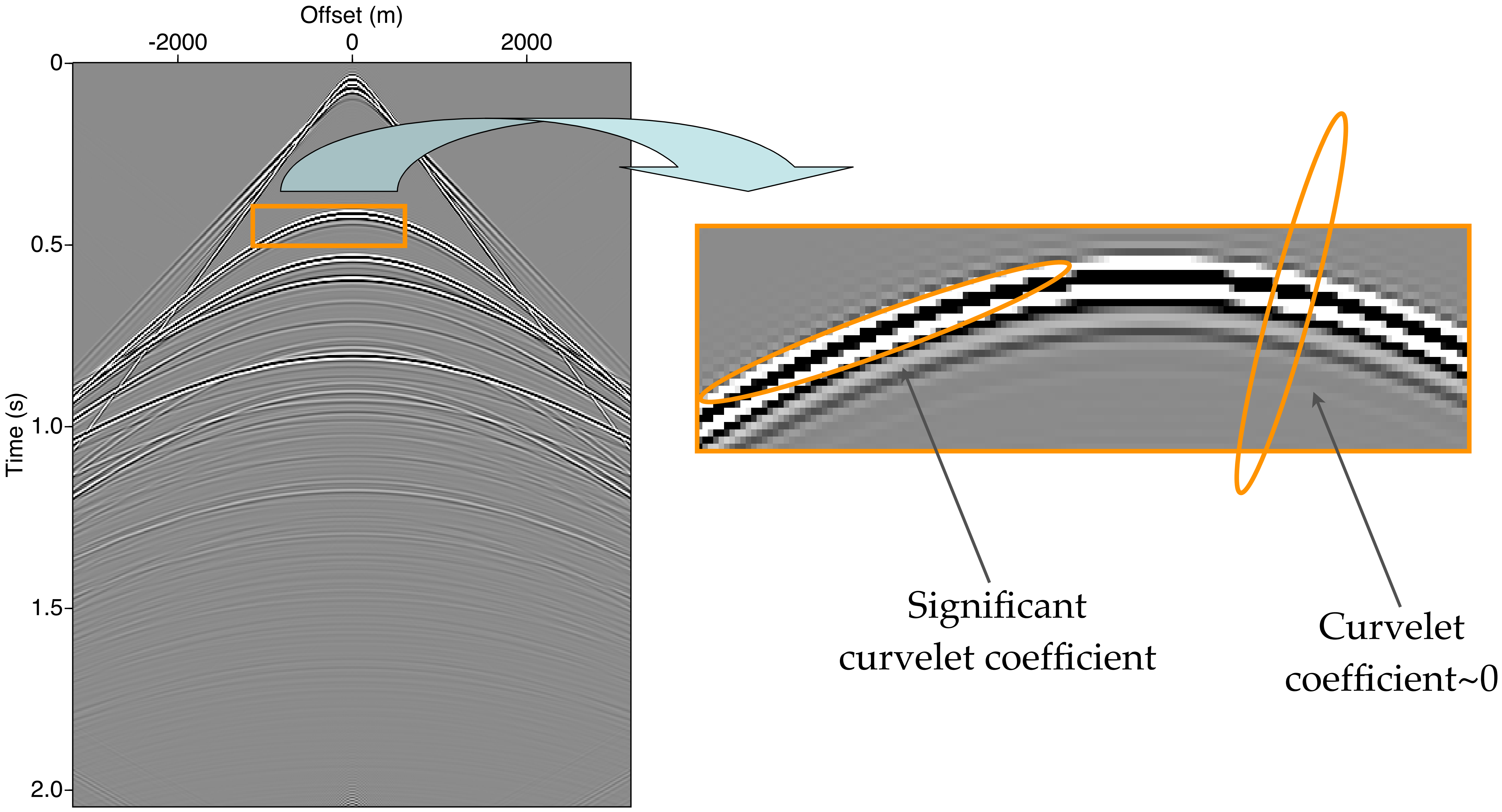
Transform	Underlying assumption
FK	plane waves
linear/parabolic Radon transform	linear/parabolic events
wavelet transform	point-like events (1D singularities)
curvelet transform	**curve-like events (2D singularities)**

- curvelet transform
  - **multiscale**: tiling of the FK domain into dyadic coronae
  - **multidirectional**: coronae sub-partitioned into angular wedges, # of angles doubles every other scale
  - **anisotropic**: parabolic scaling principle
  - **local**



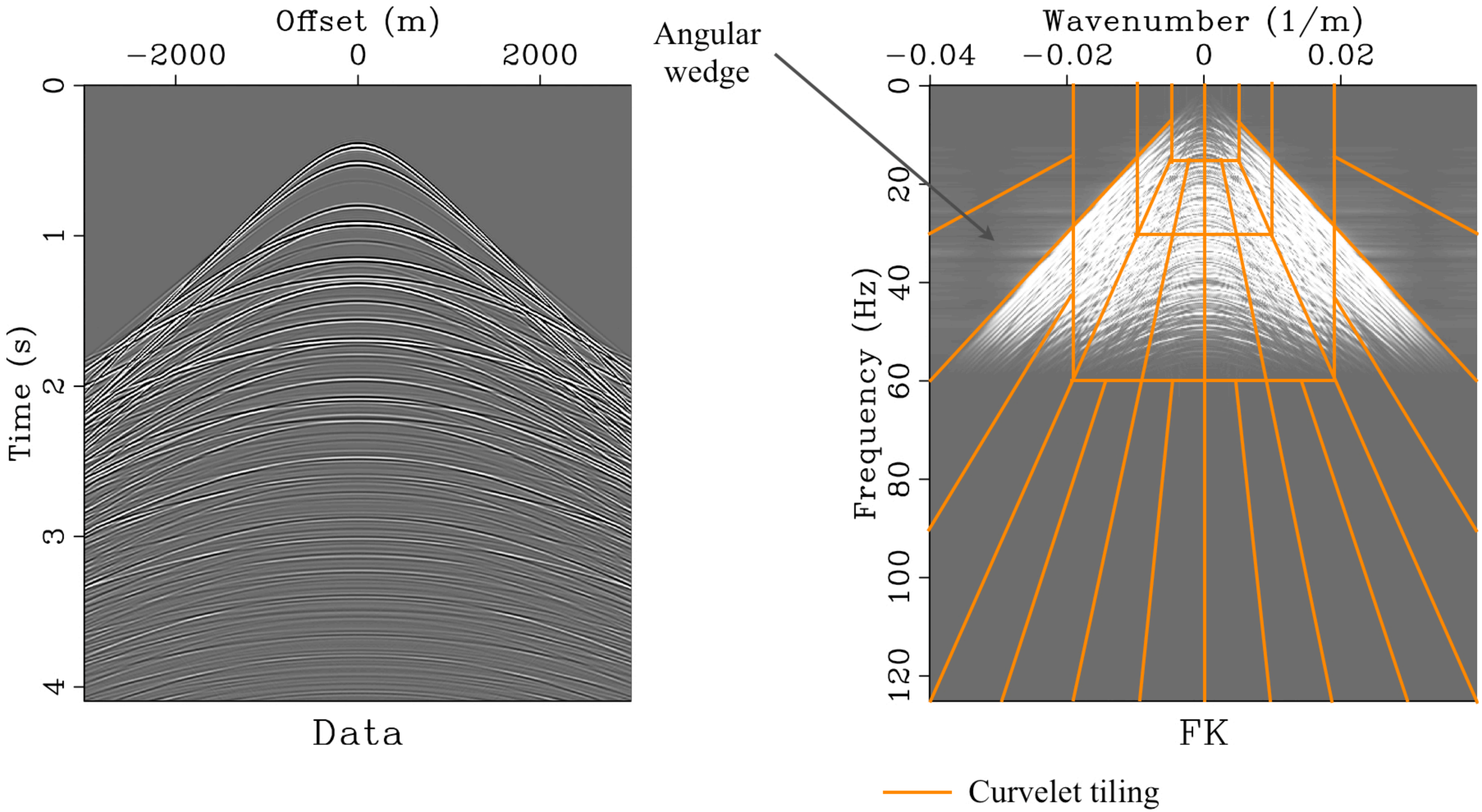
2D discrete curvelets





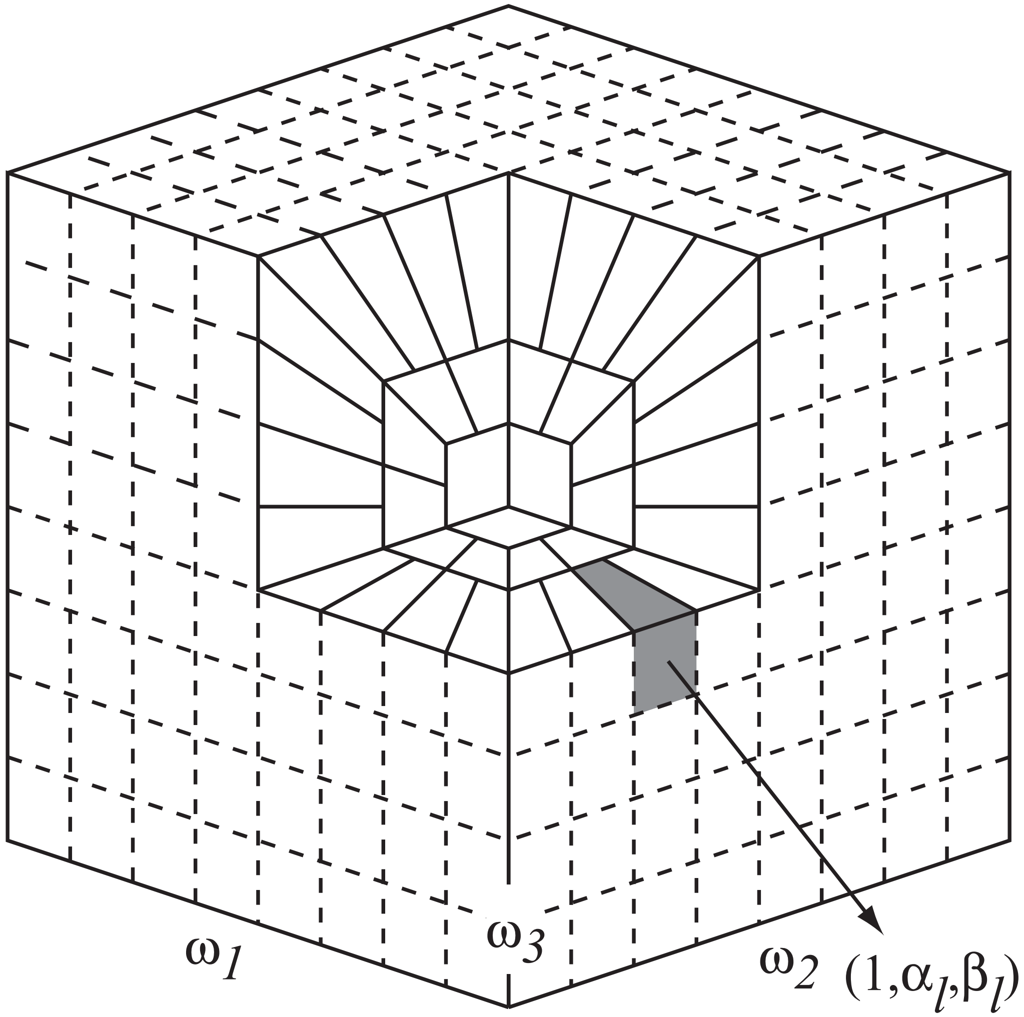
Offset (m)
-2000
0
2000
0
0.5
1.0
1.5
2.0
Time (s)
Significant
curvelet coefficient
Curvelet
coefficient~0

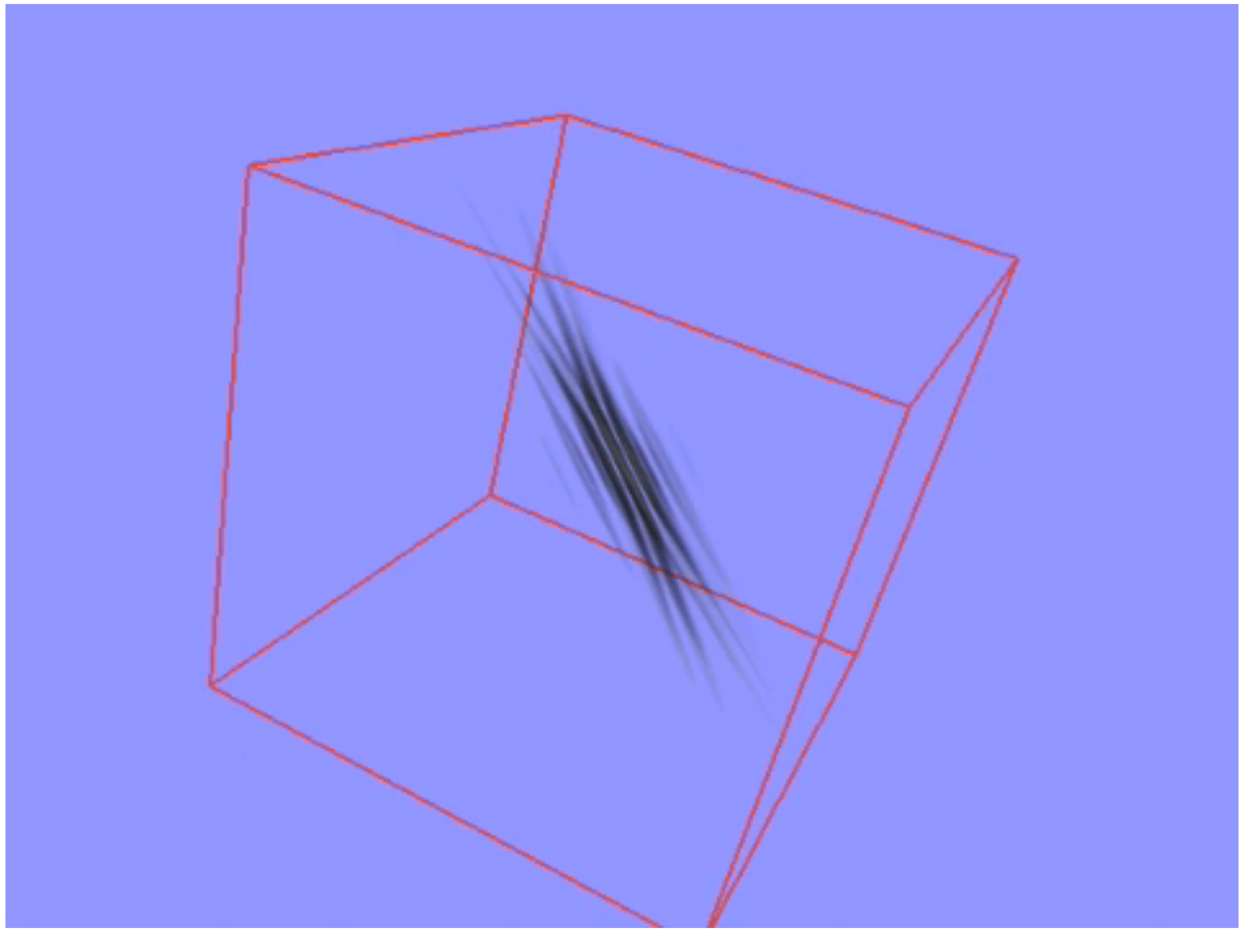
Curvelet tiling & seismic data



Curvelet tiling superimposed on the FK domain

3D discrete curvelets





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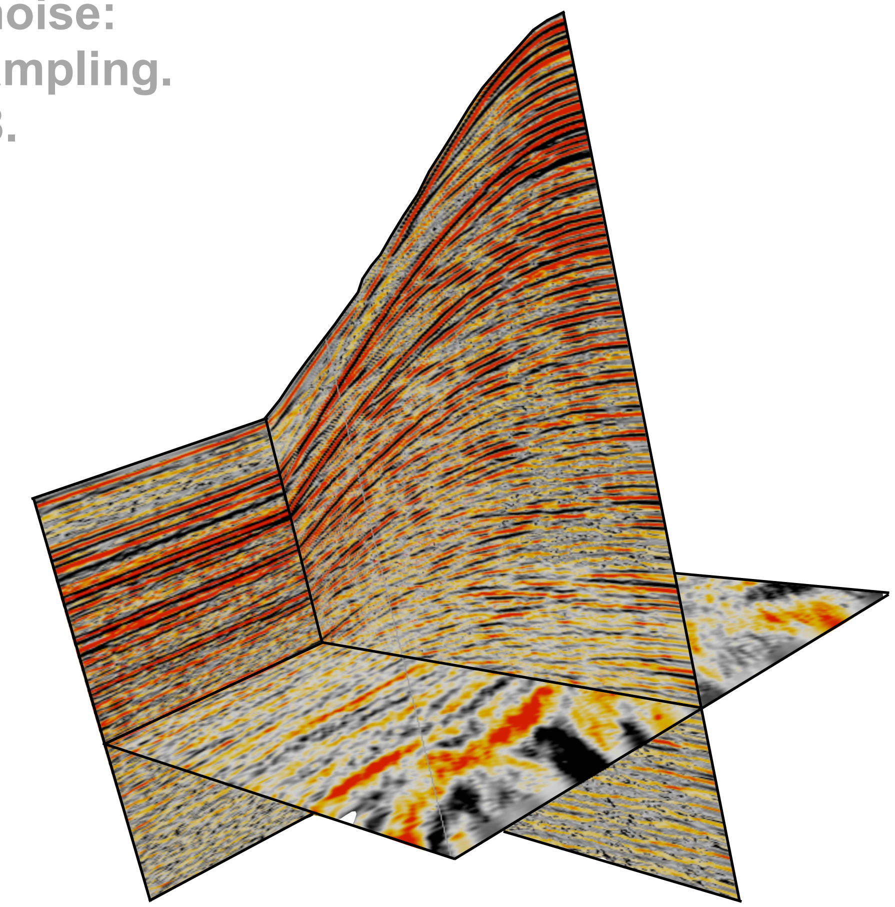
Jittered undersampling

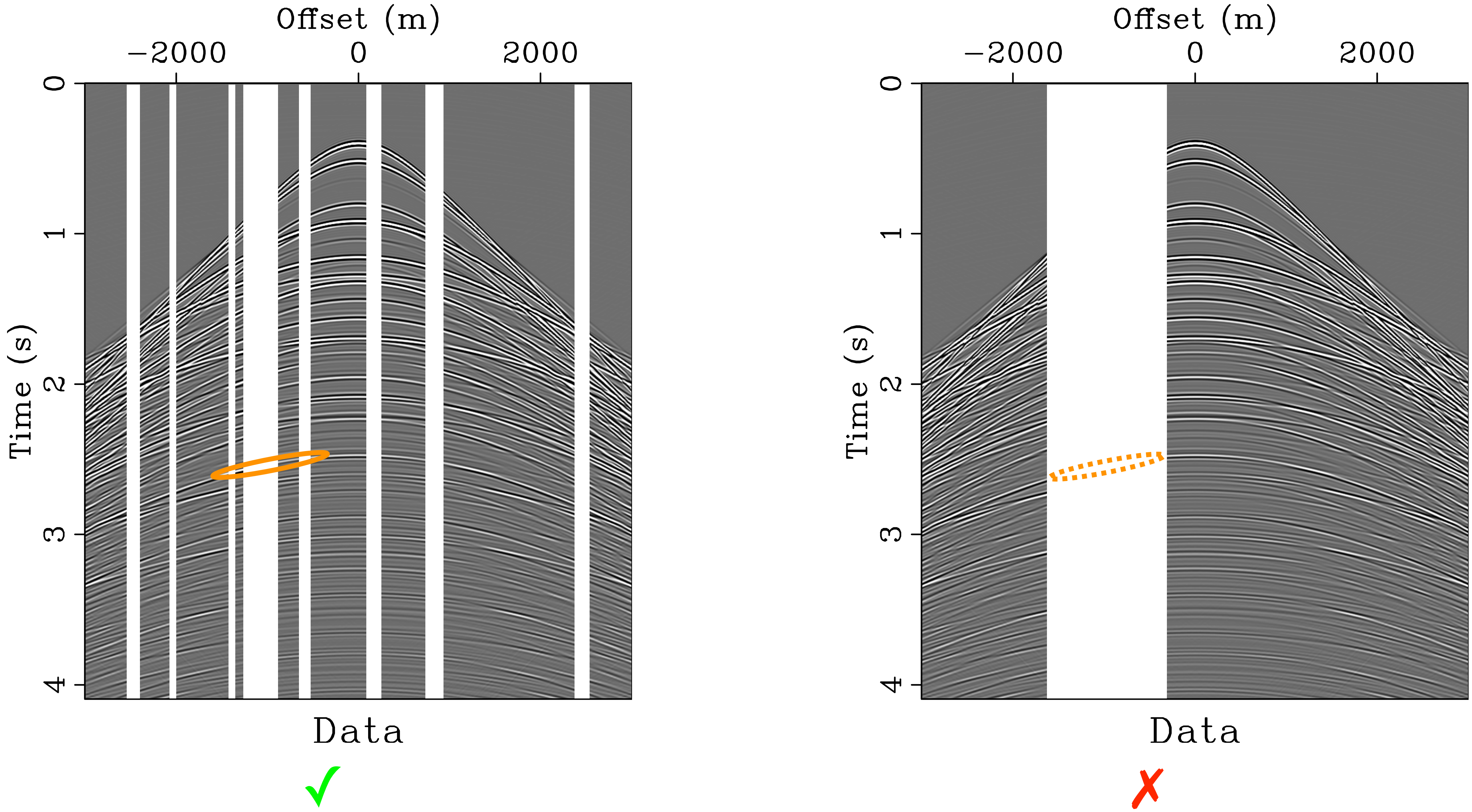
Hennefent, G. and Herrmann, F. J. Simply denoise: wavefield reconstruction via jittered under-sampling. Geophysics, Vol. 73, No. 3, pp. V19–V28, 2008.



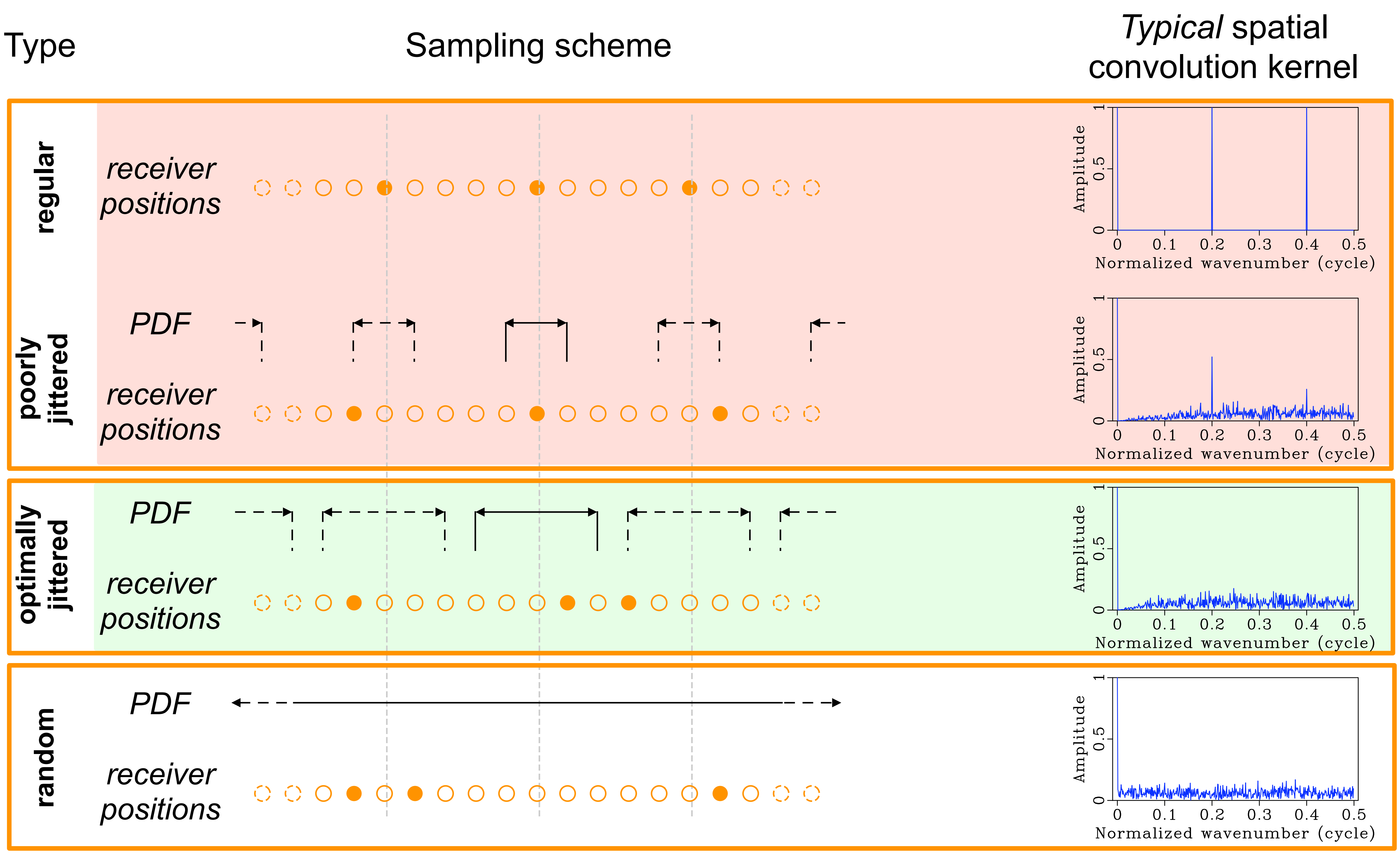
SINBAD 2008 Project Overview
Houston, October 13-17

Localized transform elements & gap size

$$\tilde{\mathbf{x}} = \arg\min_{\mathbf{x}} ||\mathbf{x}||_1 \quad \text{s.t.} \quad \mathbf{y} = \mathbf{A}\mathbf{x}$$



Discrete random jittered undersampling



[Hennenfent and Herrmann‘08]

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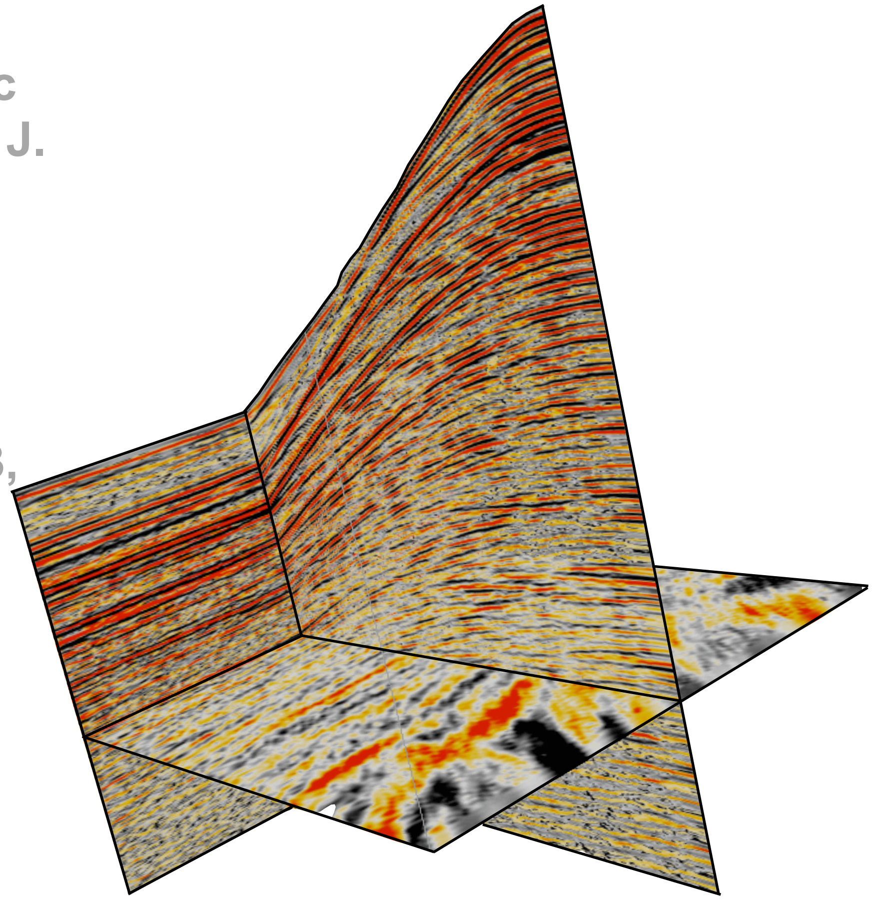
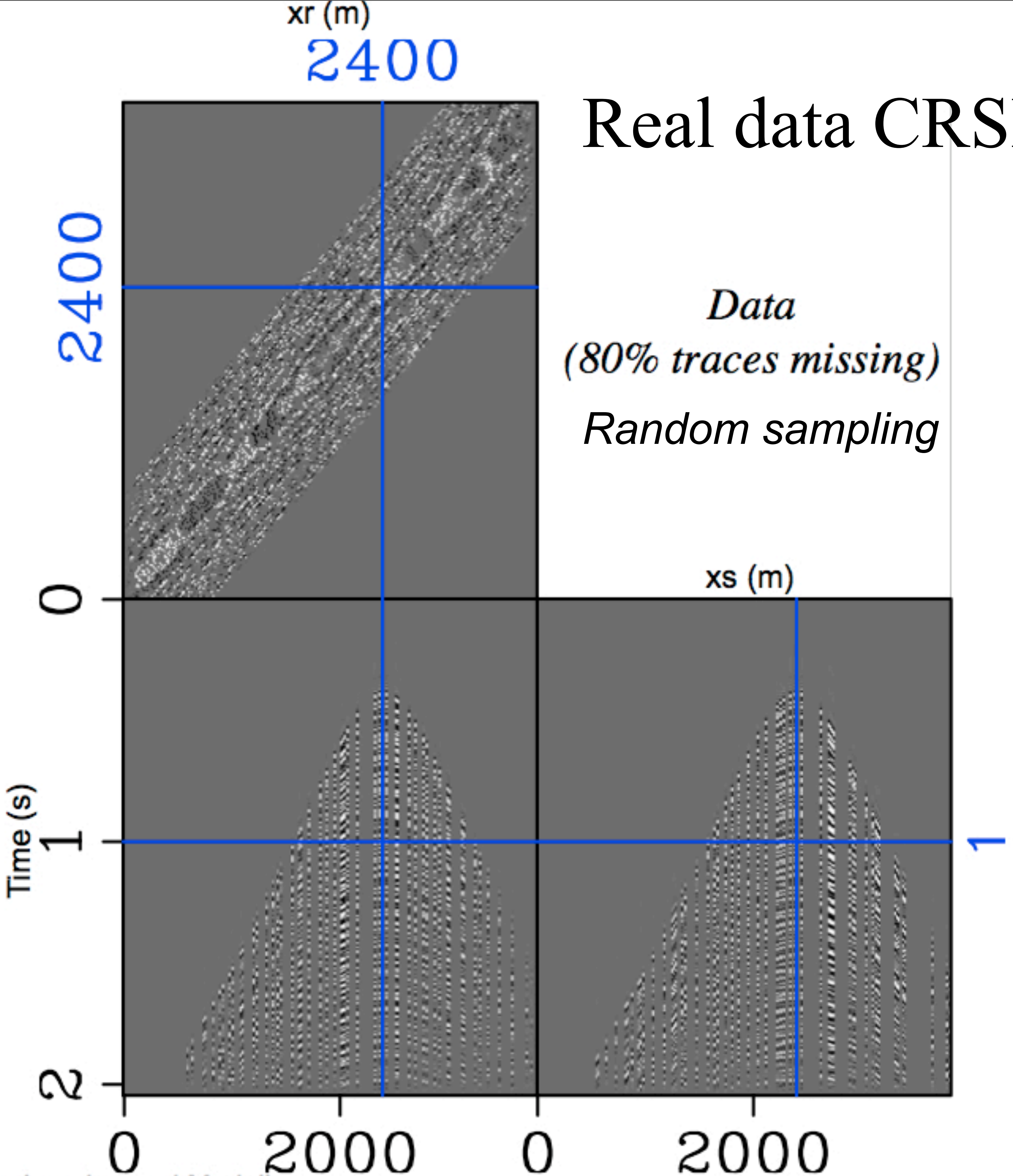
Sparsity-promoting solvers

Herrmann, F. J. and Hennenfent, G. Non-parametric seismic data recovery with curvelet frames, Geop. J. Int., Vol. 173, No. 1, pp. 233-248, 2008.

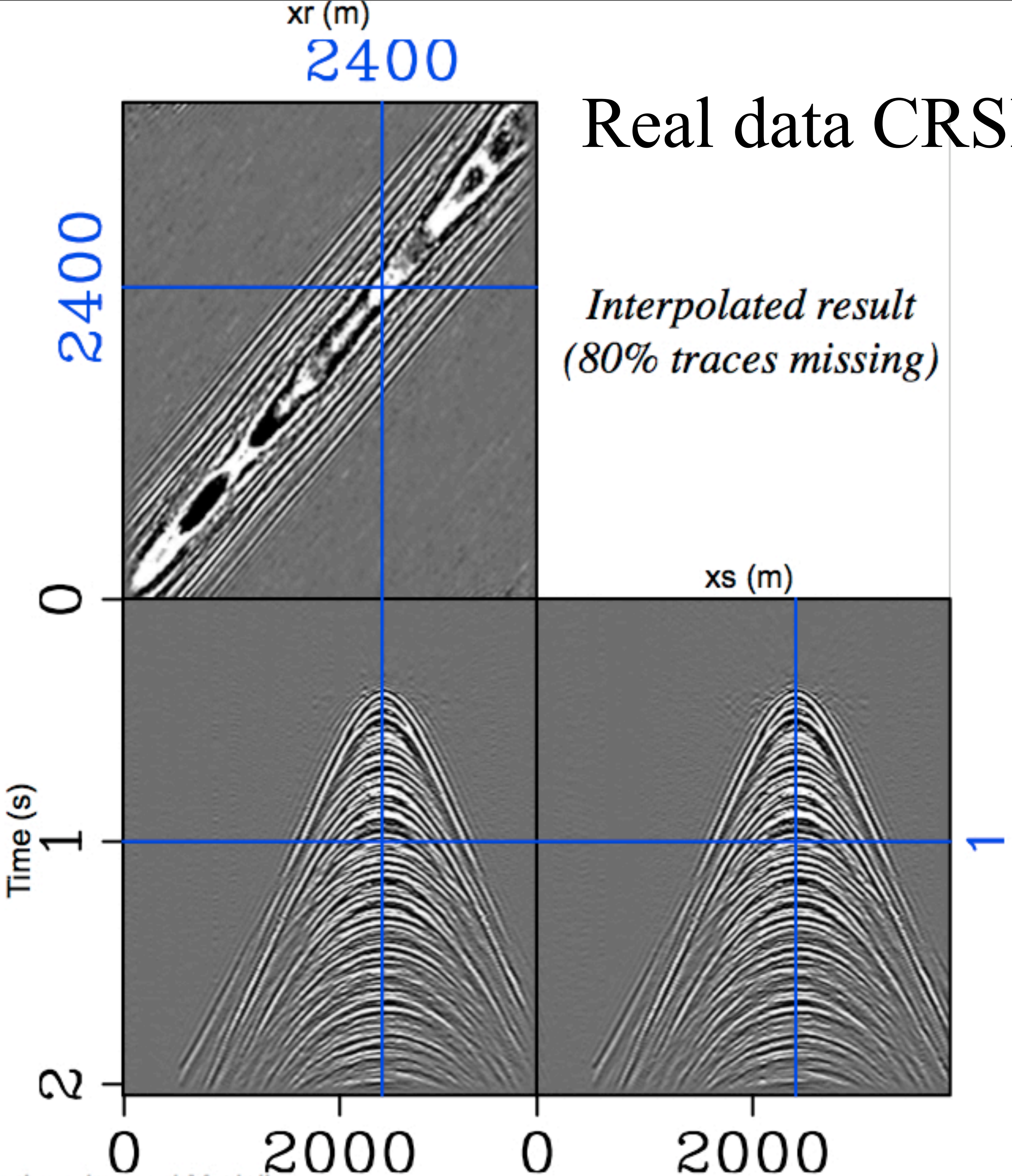
G. Hennenfent, E. van den Berg, M. P. Friedlander, and F. J. Herrmann. New insights into one-norm solvers from the Pareto curve. Geophysics, Vol. 73, No. 4, 2008.



Real data CRSI example



Real data CRSI example



Interpolated result (80% traces missing)

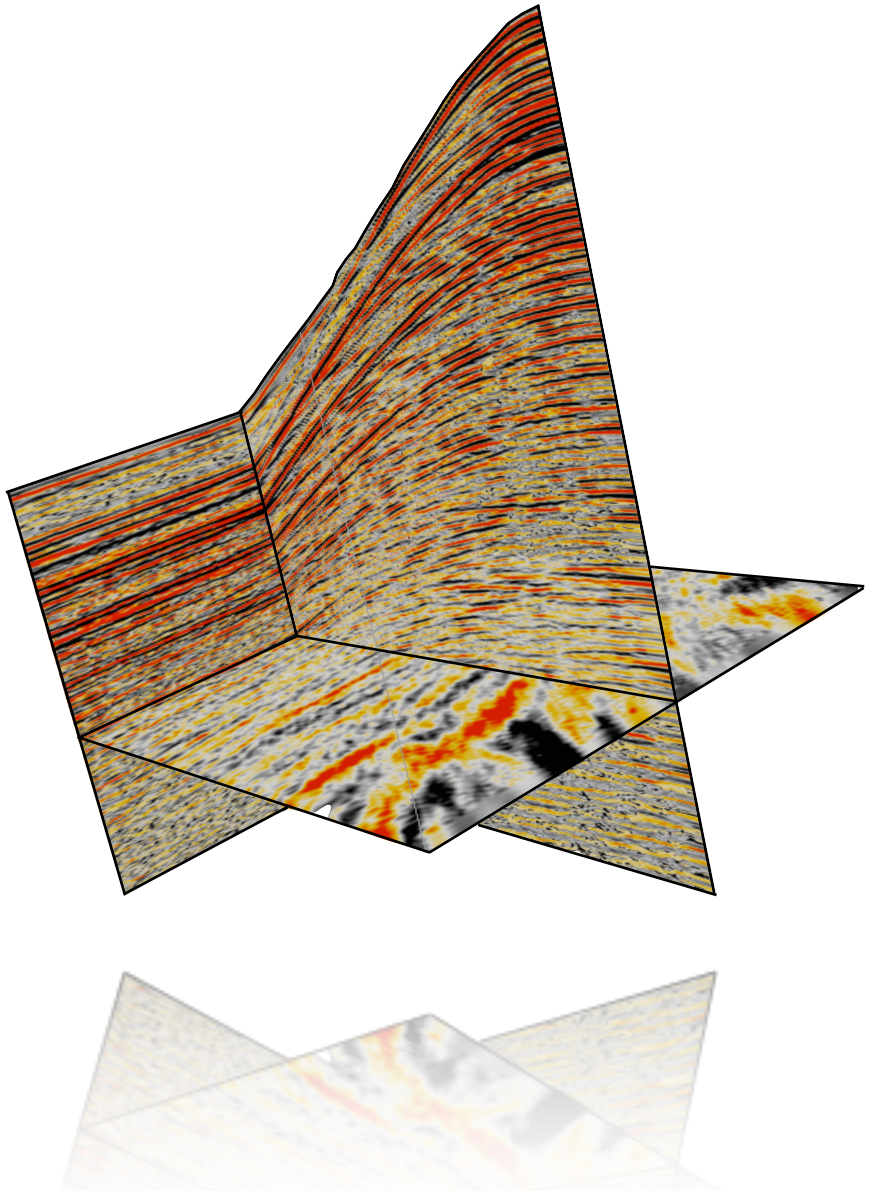
Observations

- Random subsampling breaks *constructive* interferences, i.e. *aliases*
- Turns *aliases* into incoherent *noise*
- Works by virtue of
  - *incoherence* (correlations) between the *rows* of the *Dirac* measurement basis and the columns of the *Fourier synthesis* basis
  - maximum *spreading* of Diracs in Fourier domain
  - maximum *leakage*
  - *independence* amongst columns of **A**, i.e., there exists a subset of columns of **A** that forms an orthonormal basis
- According to theory of compressive sensing
  - recovery stable w.r.t. noise
  - measurement & sparsity bases can be more general
- Apply these principles to compressive wavefield computations
  - reduce memory imprint
  - reduce the number of sources = number of right-hand sides of linear systems

Compressive one-way wavefield extrapolation

T. T. Y. Lin and F. J. Herrmann. Compressed wavefield extrapolation. Geophysics, 72(5):SM77–SM93, 2007.

L. Demanet and G. Peyre. Compressive wave computation, 2008. Stanford. Submitted for publication.



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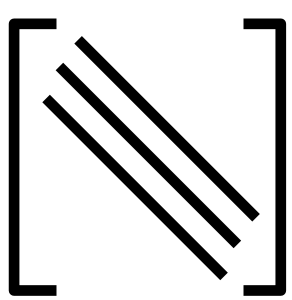
Introduction

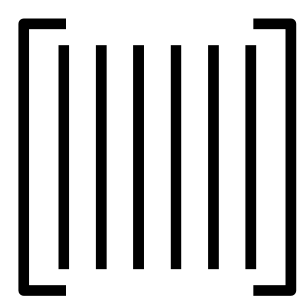
- Goal: employ the complete one-Way Helmholtz operator for W

Grimbergen, J., F. Dessing, and C. Wapenaar, 1998, Modal expansion of one-way operator on laterally varying media: Geophysics, **63**, 995–1005.

$$\mathbf{W}^{\pm} = e^{\mp j \Delta x \mathbf{H}_1} \qquad \mathbf{H}_2 = \mathbf{H}_1 \mathbf{H}_1$$

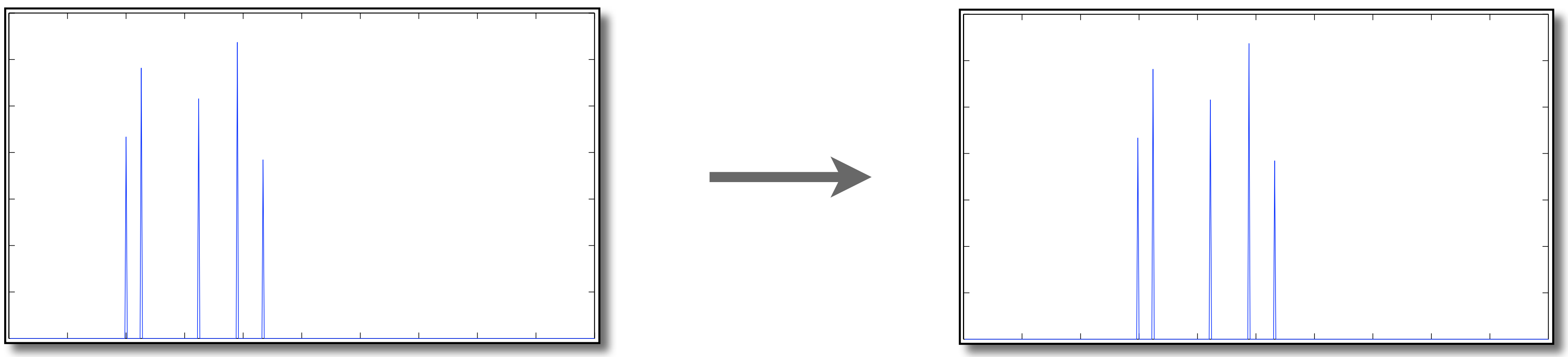
- Problem: computation & storage complexity
  - creating and storing $\mathbf{H}_2$ is trivial
  - however $\mathbf{H}_1$ is *not* trivial to compute and store

$\mathbf{H}_2 =$ 

$\mathbf{H}_1 =$ 

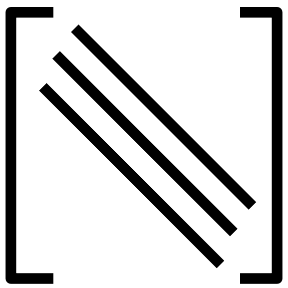
Our approach

- Consider a related, but simpler problem: shifting (or translating) a signal



- operator is $\mathbf{T}_{\Delta x} = e^{-j\frac{\Delta x}{2\pi}\mathbf{D}}$
- $\mathbf{D}$ is differential operator

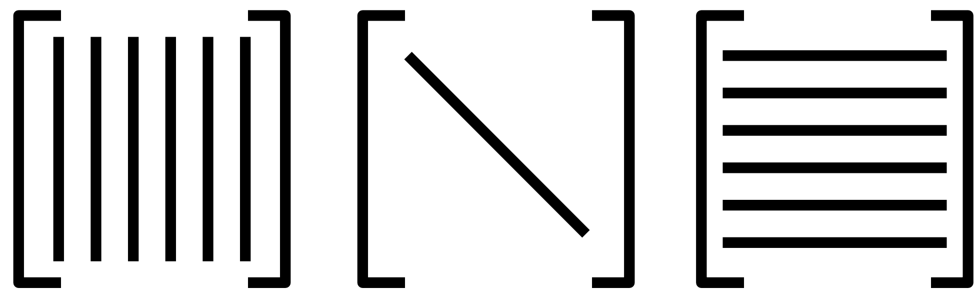
$\mathbf{D} =$



Our approach

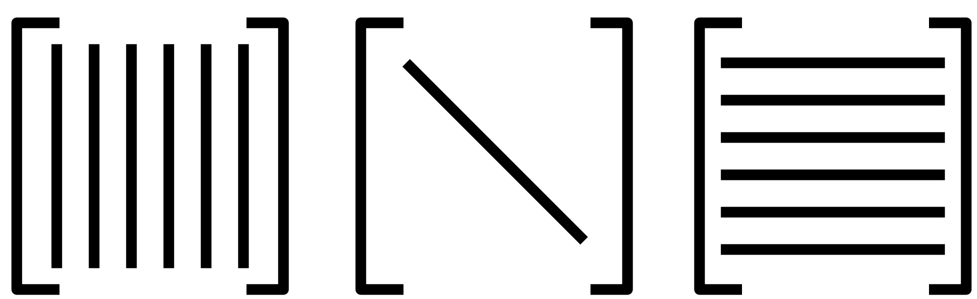
- Computation requires similar approach to $\mathbf{W}^{\pm}$

$$\mathbf{D} = \mathbf{L}\boldsymbol{\Lambda}\mathbf{L}^{\mathrm{T}} =$$



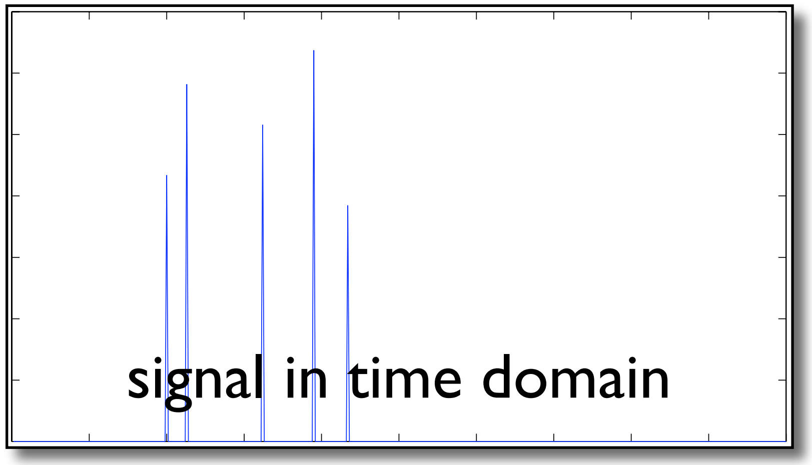
$\mathbf{L}$ $\boldsymbol{\Lambda}$ $\mathbf{L}^{\mathrm{T}}$

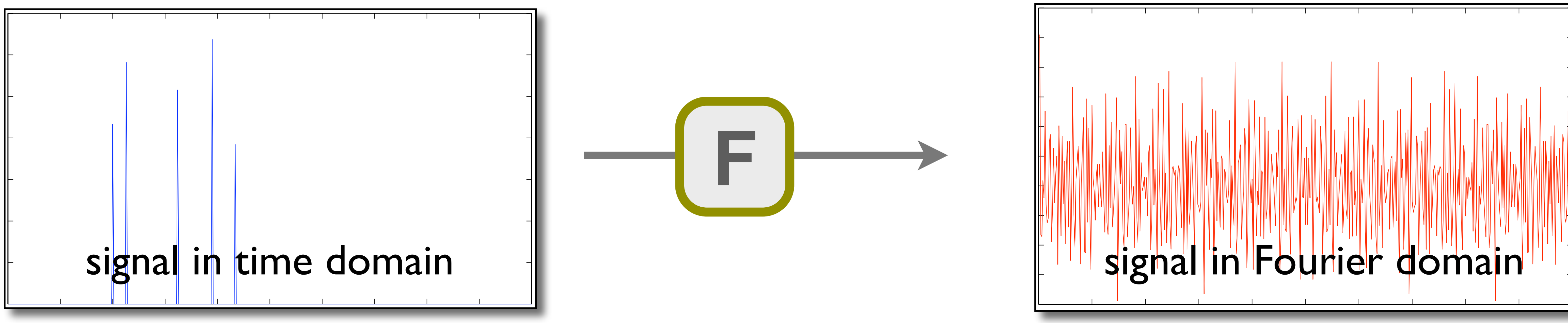
$$\mathbf{T}_{\Delta x} =$$

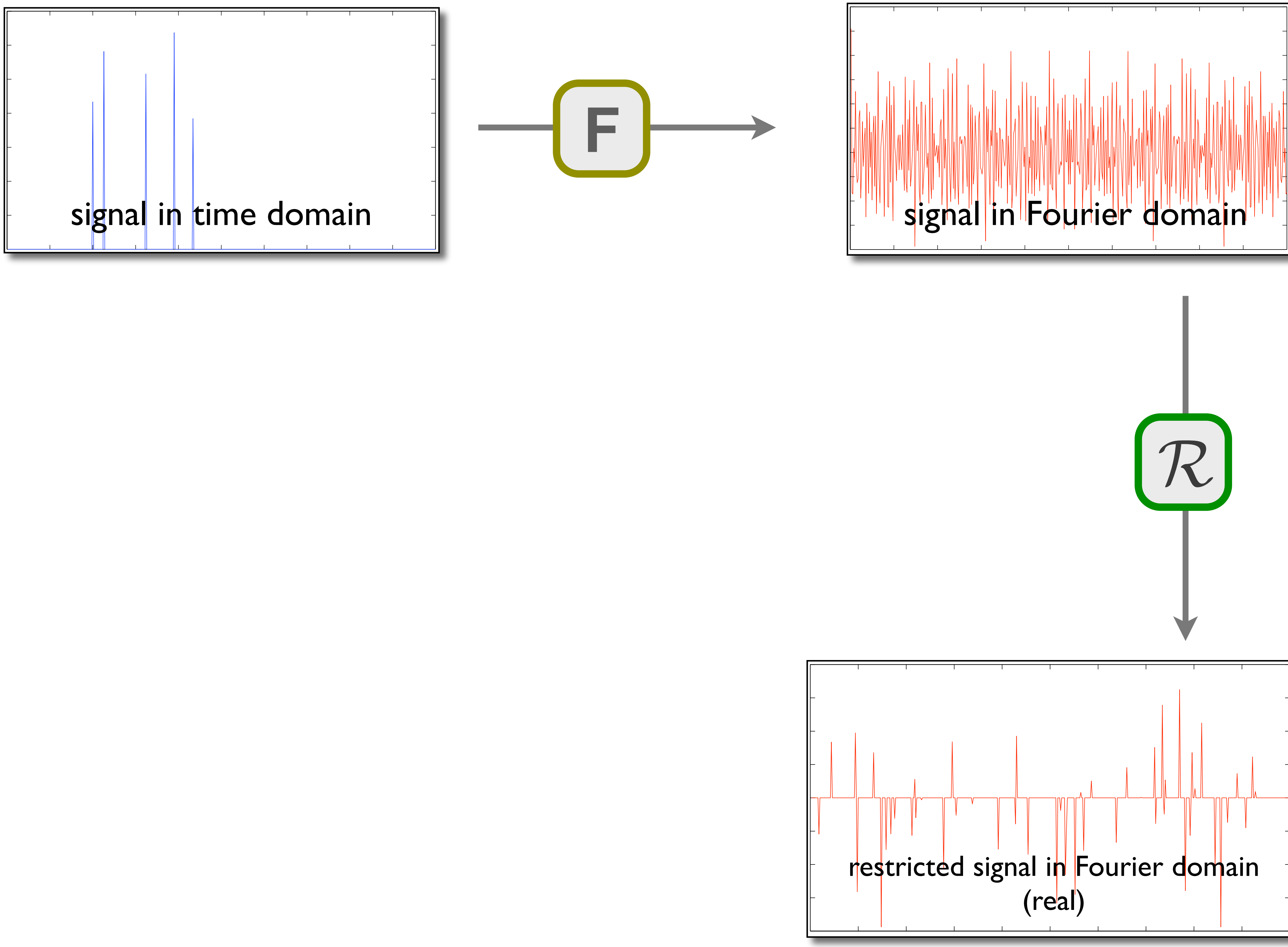


$\mathbf{L}$ $e^{-j\frac{\Delta x}{2\pi}}\boldsymbol{\Lambda}$ $\mathbf{L}^{\mathrm{T}}$

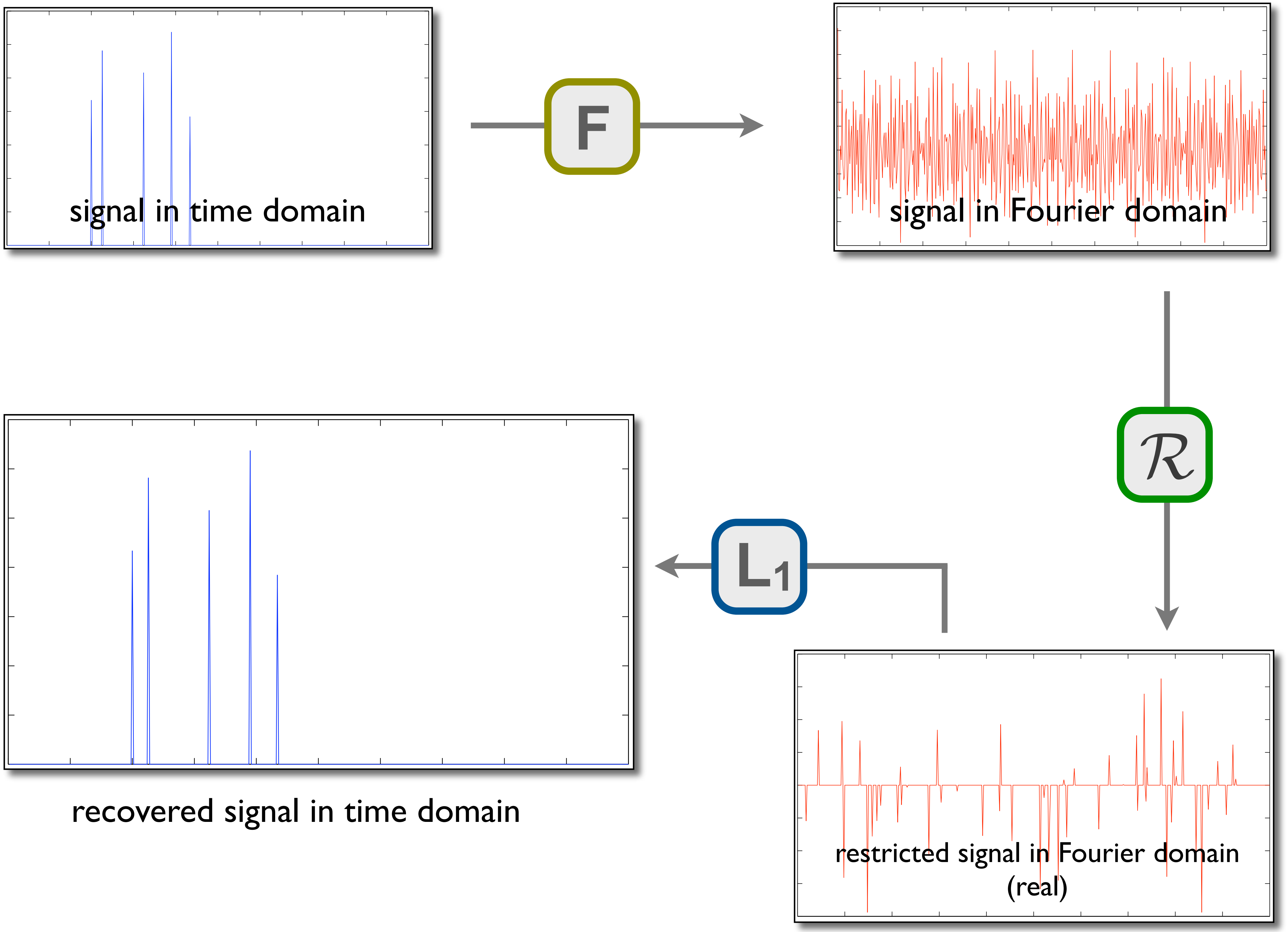
- However, for $\mathbf{D}$, $\mathbf{L} = \mathrm{DFT}$, so computation trivial with FFT


signal in time domain


signal in time domain
F
signal in Fourier domain



SLIM
Seismic Laboratory for Imaging and Modeling



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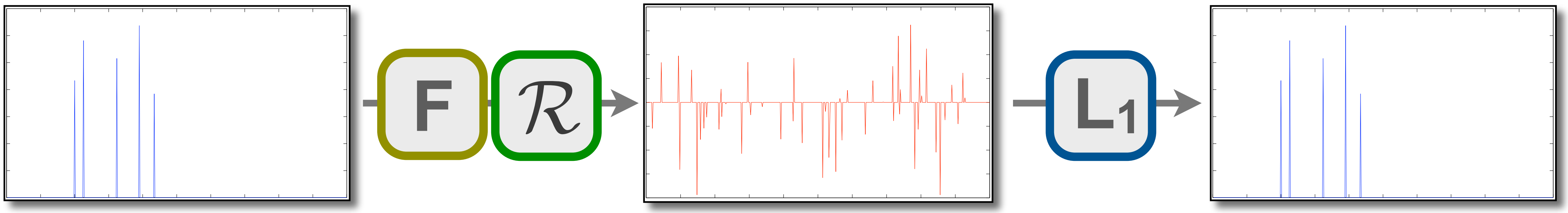
Seismic Laboratory for Imaging and Modeling

Compressed Sensing

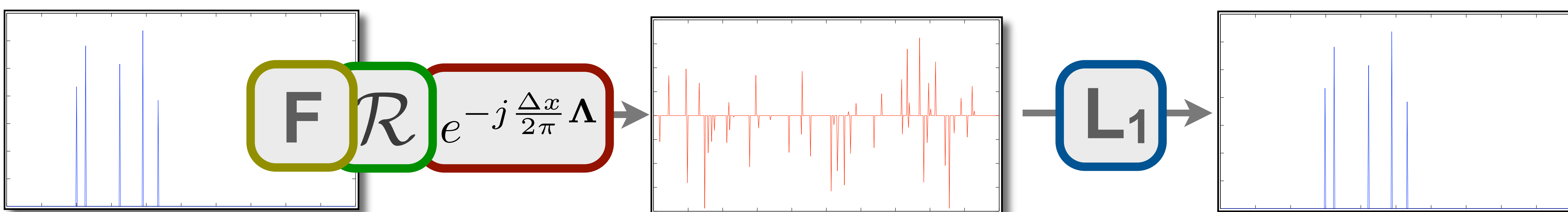


signal in space domain

incomplete signal in Fourier domain

signal in space domain

Compressed Computations



signal in space domain

incomplete and *shifted* signal in Fourier domain

shifted signal in space domain

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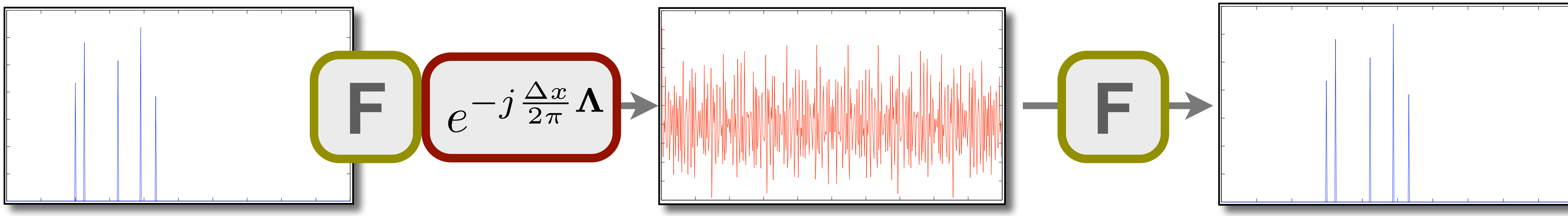
Straightforward Computation

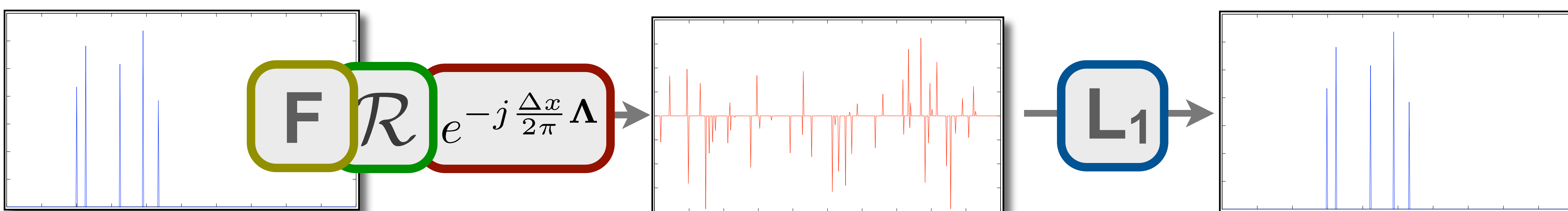


signal in space domain

shifted signal in Fourier domain

shifted signal in space domain

Compressed Computation

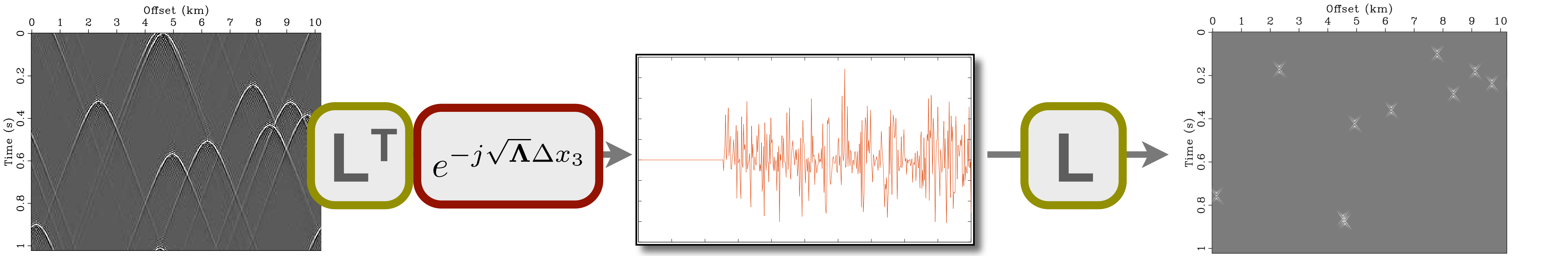


signal in space domain

incomplete and *shifted* signal in Fourier domain

shifted signal in space domain

Straightforward 1-Way inverse Wavefield Extrapolation

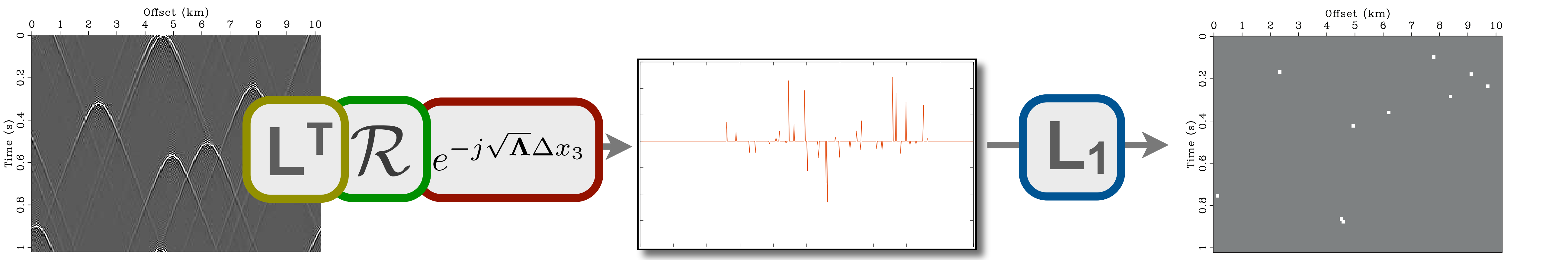


wavefield in space-time domain

back-extrapolated wavefield in H2 domain

back-extrapolated to impulse source in space-time domain

Compressed 1-Way Wavefield Extrapolation



wavefield in space-time domain

incomplete back-extrapolated wavefield in H2 domain

back-extrapolated to impulse source in space-time domain

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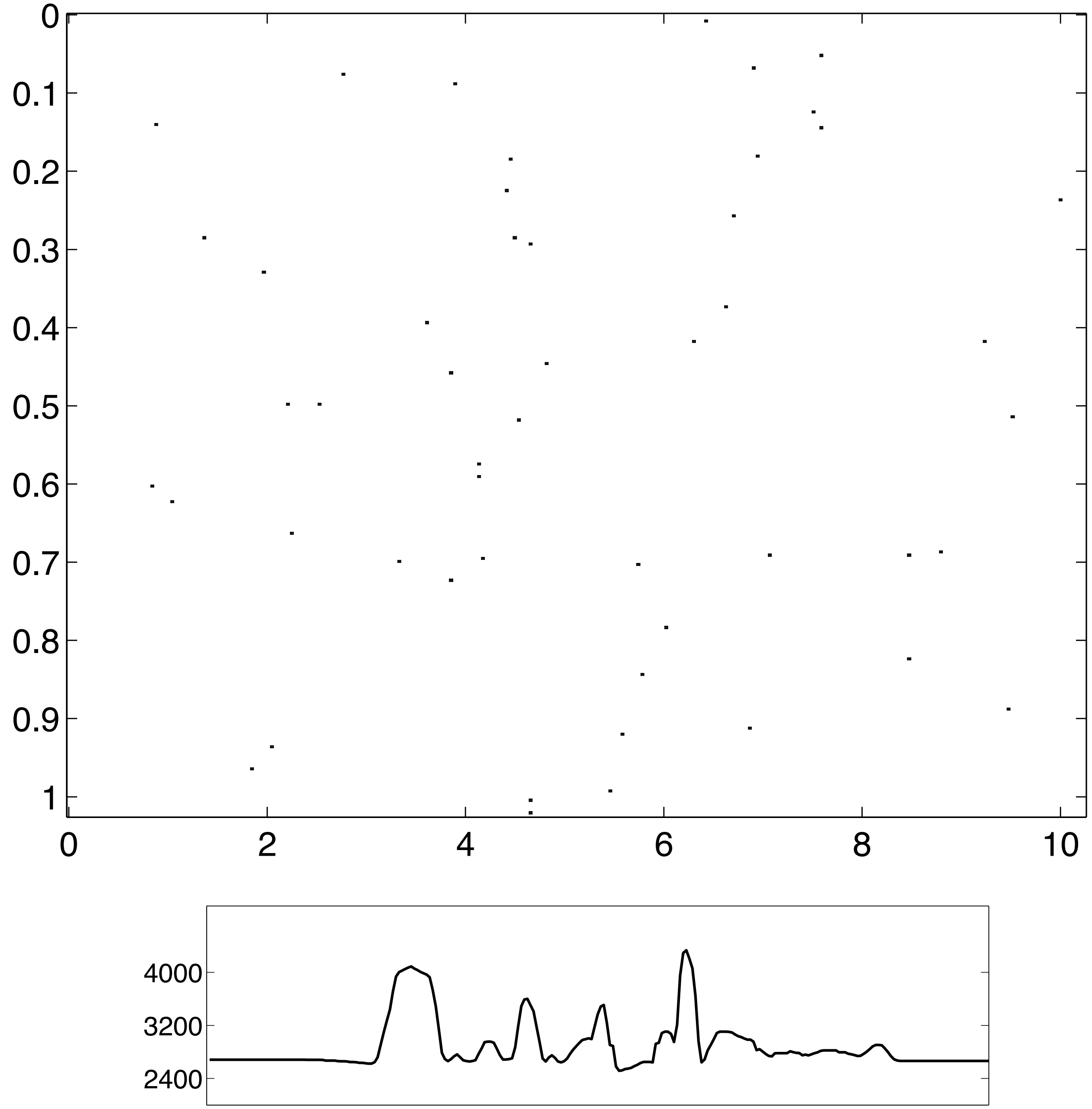
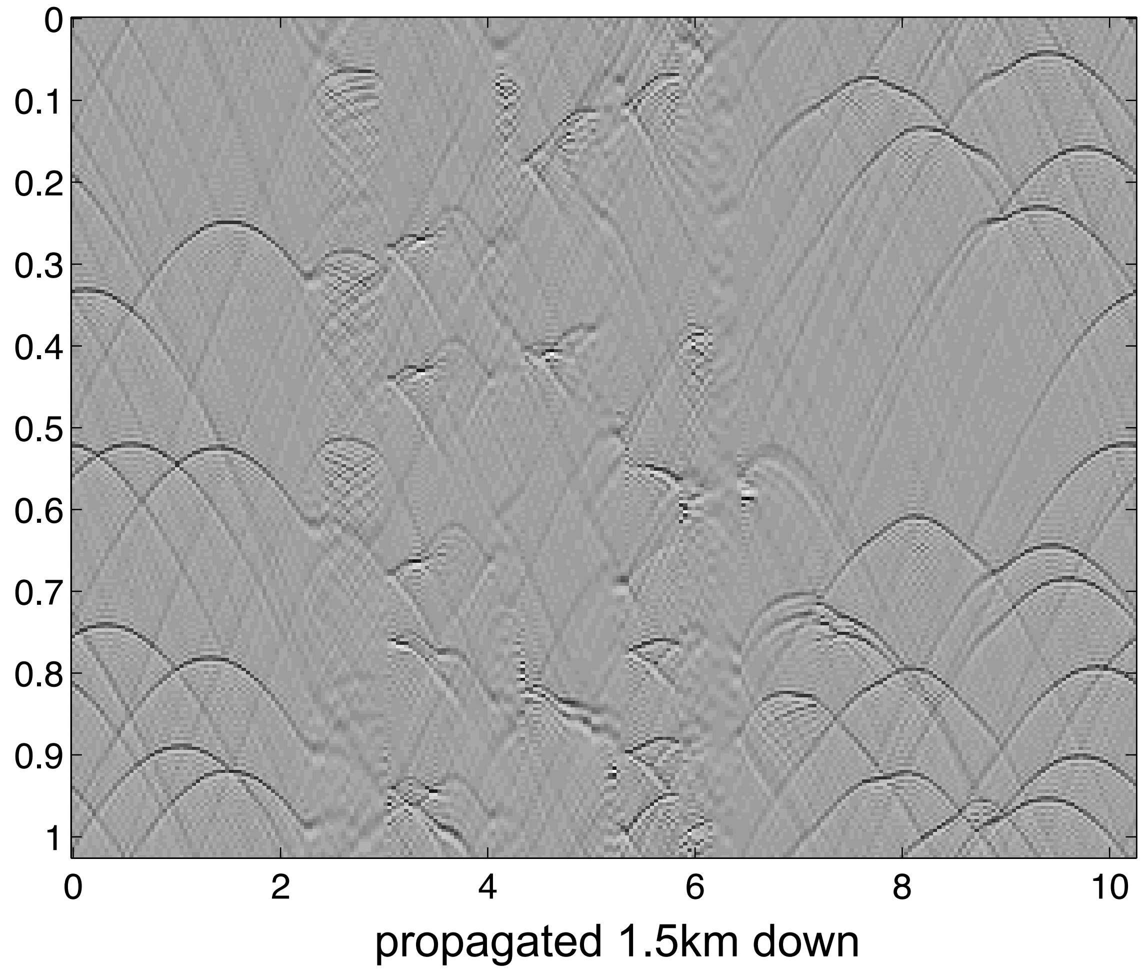
Compressed wavefield extrapolation

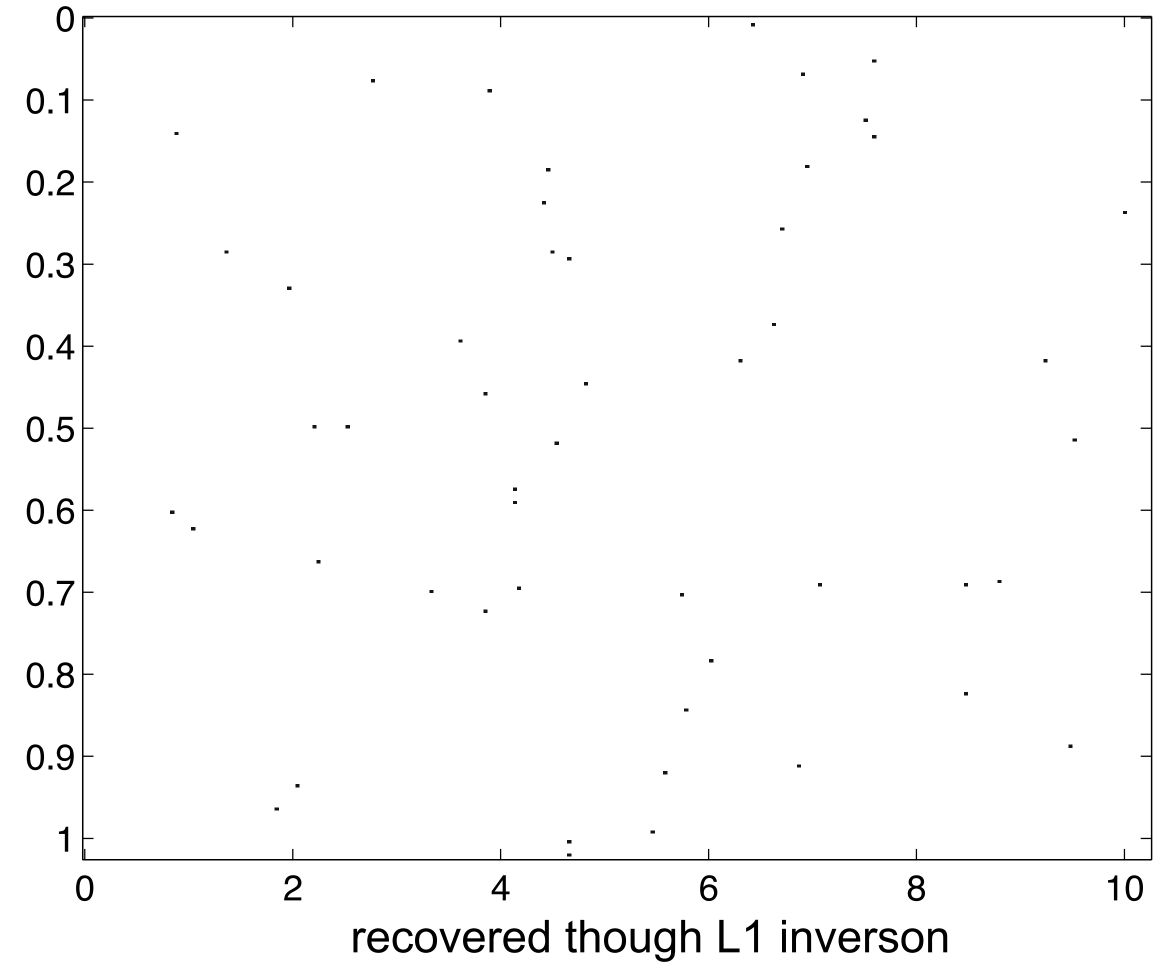
simple 1-D space/time propagation example with point scatters



Compressed wavefield extrapolation

simple 1-D space/time propagation example with point scatters

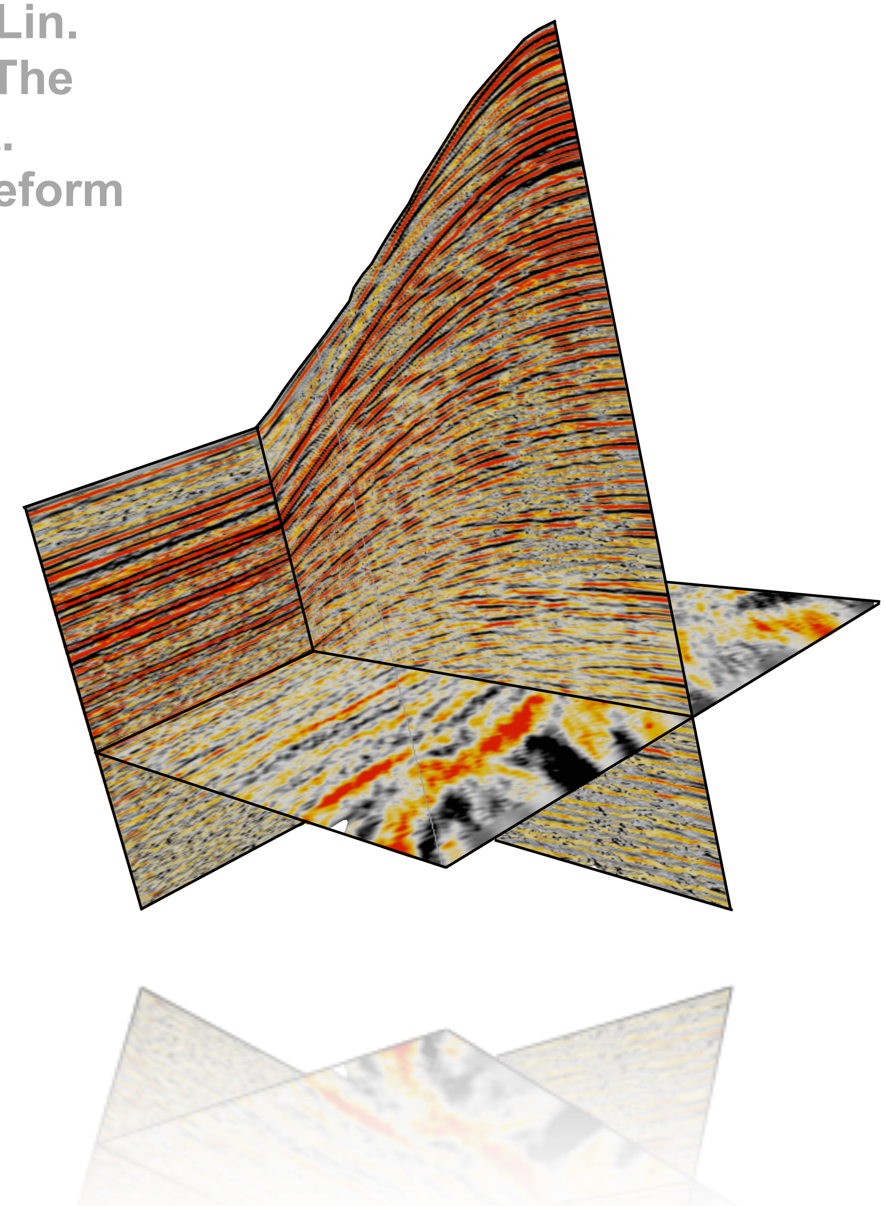




Restricted L transform to ~0.01 of original coefficients

Simultaneous simulation & recovery

Felix J. Herrmann, Yogi Erlangga, and Tim T. Y. Lin.
Seismic Laboratory for Imaging and Modeling. The university of British Columbia Technical Report.
TR-2008-3. Compressive simultaneous full-waveform simulation.



Delft University of Technology
Delft, February 2, 2009

Wavefield computations

- For n_s shots and n_f frequencies, the linear (Helmholtz) systems are independent
- The multi-shot and multi-frequency problem is embarrassingly parallel

$$
\begin{bmatrix}
\mathcal{H}_{\omega_1} & 0 & & \\
0 & \mathcal{H}_{\omega_2} & \ddots & \\
 & \ddots & \ddots & 0 \\
 & & 0 & \mathcal{H}_{\omega_{n_f}}
\end{bmatrix}
\begin{bmatrix}
\overbrace{[\mathbf{u}_1 \; \mathbf{u}_2 \; \cdots \; \mathbf{u}_{n_s}]_{\omega_1}}^{\mathbf{U}_{\omega_1}} \\
\vdots \\
\vdots \\
\underbrace{[\mathbf{u}_1 \; \mathbf{u}_2 \; \cdots \; \mathbf{u}_{n_s}]_{\omega_{n_f}}}_{\mathbf{U}_{n_f}}
\end{bmatrix}
=
\begin{bmatrix}
\overbrace{[\mathbf{b}_1 \; \mathbf{b}_2 \; \cdots \; \mathbf{b}_{n_s}]_{\omega_1}}^{\mathbf{B}_{\omega_1}} \\
\vdots \\
\vdots \\
\underbrace{[\mathbf{b}_1 \; \mathbf{b}_2 \; \cdots \; \mathbf{b}_{n_s}]_{\omega_{n_f}}}_{\mathbf{B}_{n_f}}
\end{bmatrix}
$$

$$\mathcal{H}_{\omega_j} := \mathcal{H}(\omega_j), \quad \omega_j = 2\pi j \Delta f, \quad j = 1, \ldots, n_f$$

Δf frequency sample interval

Wavefield computations

$$
\overbrace{\begin{bmatrix} \mathcal{H}_{\omega_1} & 0 & & \\ 0 & \mathcal{H}_{\omega_2} & \ddots & \\ & \ddots & \ddots & 0 \\ & & 0 & \mathcal{H}_{\omega_{n_f}} \end{bmatrix}}^{\mathbf{H}} \overbrace{\begin{bmatrix} \mathbf{U}_{\omega_1} \\ \mathbf{U}_{\omega_1} \\ \vdots \\ \mathbf{U}_{n_f} \end{bmatrix}}^{\mathbf{U}} = \overbrace{\begin{bmatrix} \mathbf{B}_{\omega_1} \\ \mathbf{B}_{\omega_2} \\ \vdots \\ \mathbf{B}_{n_f} \end{bmatrix}}^{\mathbf{B}}
$$

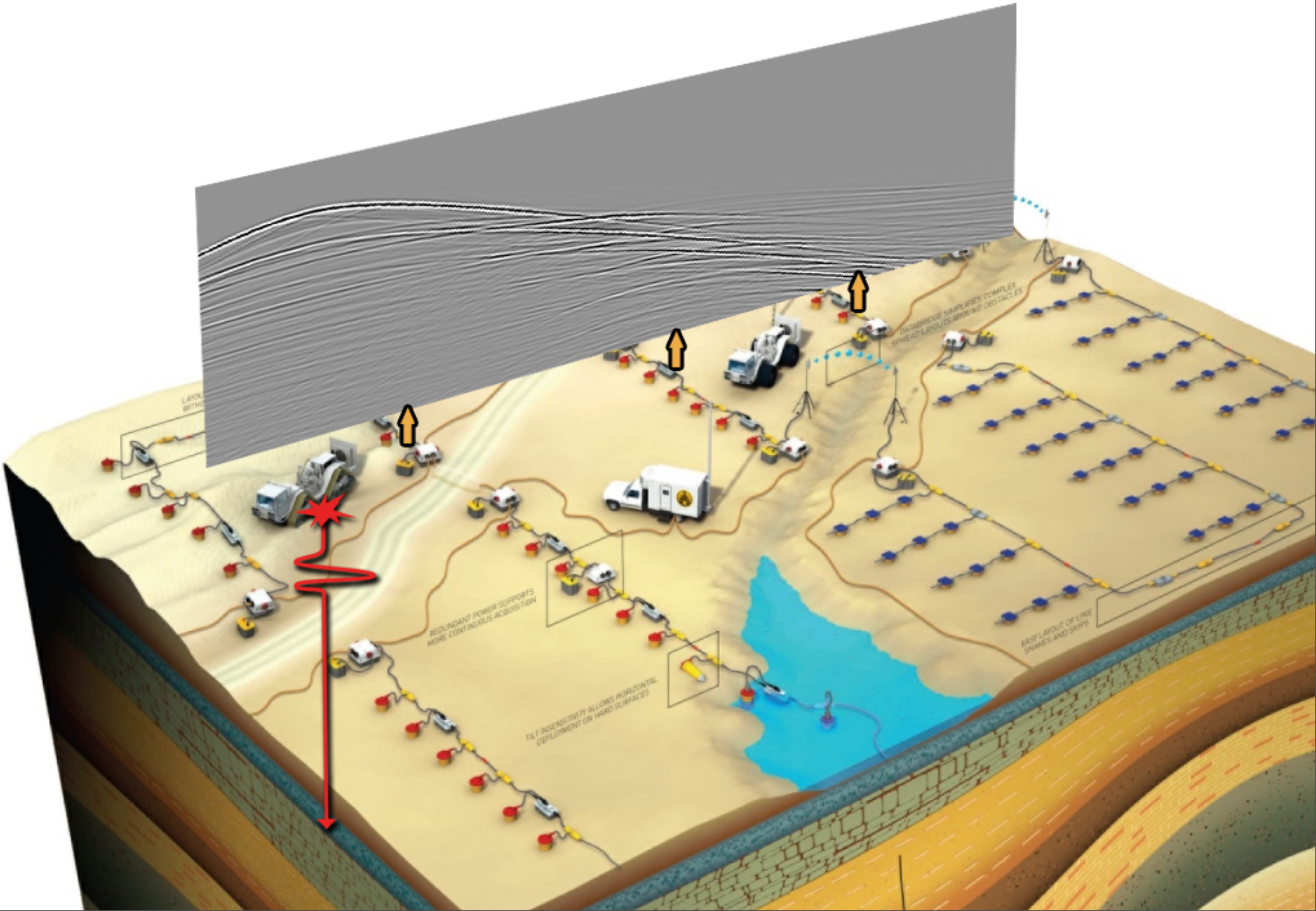
$$\Downarrow$$

$$\mathbf{HU} = \mathbf{B}$$

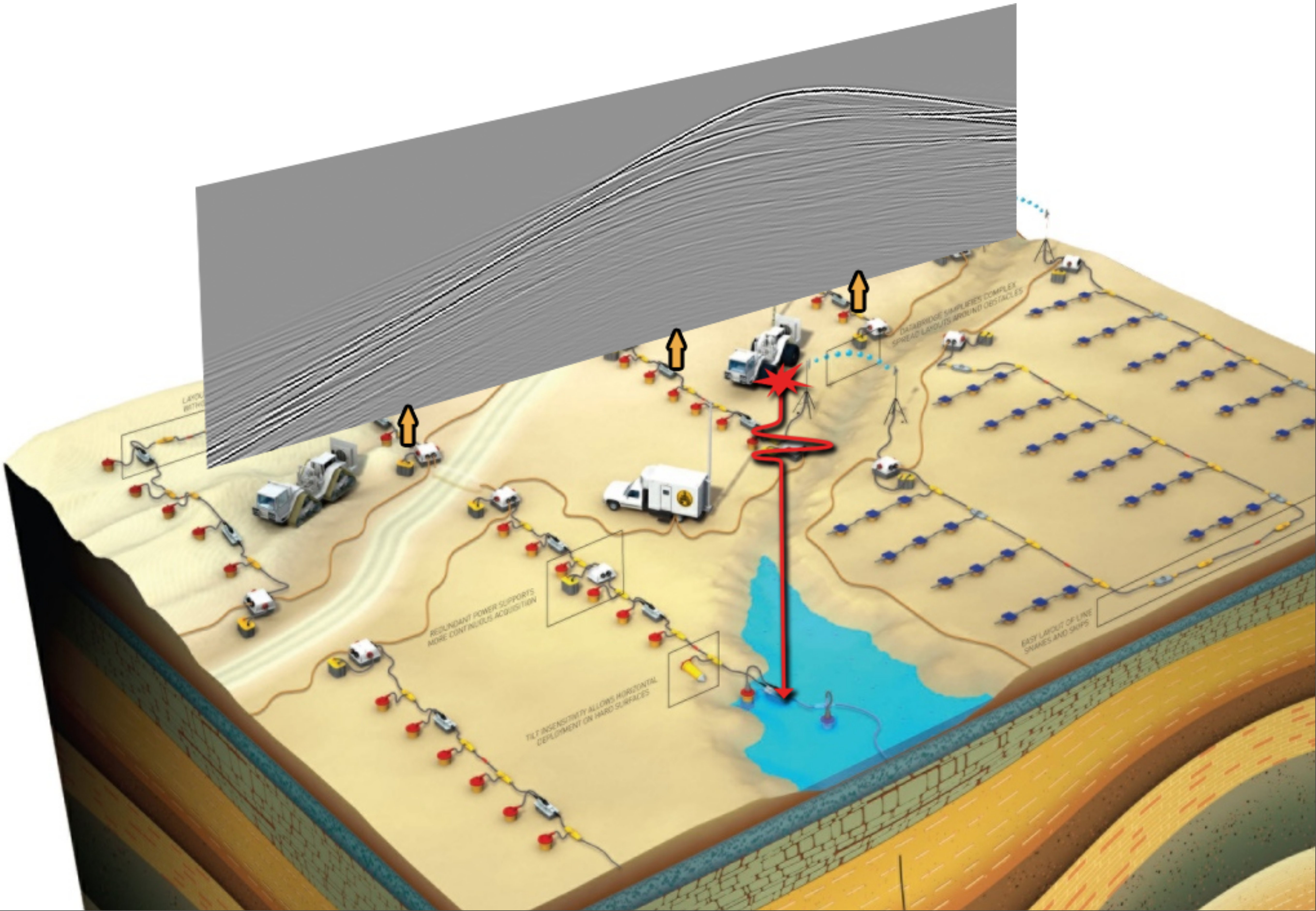
- Natural for CS setting!

Acquisition image courtesy ION (www.iongeo.com)

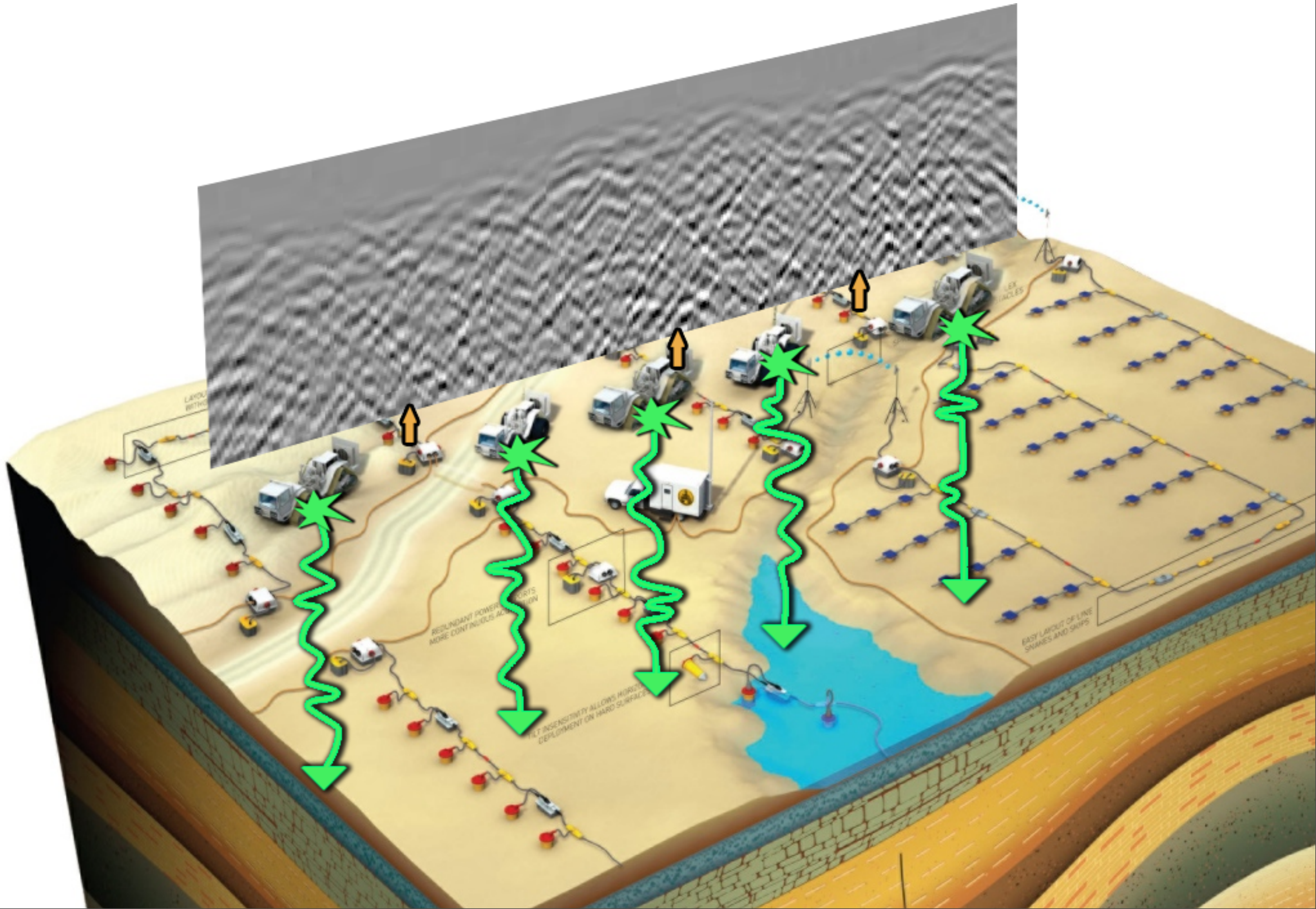
Individual shots



Individual shots



Simultaneous & continuous shots



Simultaneous modeling & acquisition

- **Current paradigm:**
  - ***separate*** single-source experiments in the field
  - ***separate*** single-shot simulations in the computer
  - **Con:** expensive

- **New paradigm:**
  - ***simultaneous*** *&* ***continuous*** source experiments in the field
  - ***simultaneous*** (continuous) simulations in the computer
  - **continuous** simultaneous simulations are equivalent to **multiple** simultaneous experiments
  - **Con:** *postprocessing necessary to separate into individual shots*

- **Key observation: this is *really* another instance of CS ...**
  - **design new simultaneous acquisitions & recovery schemes!**

CS sampling of frequencies and shots (rhs)

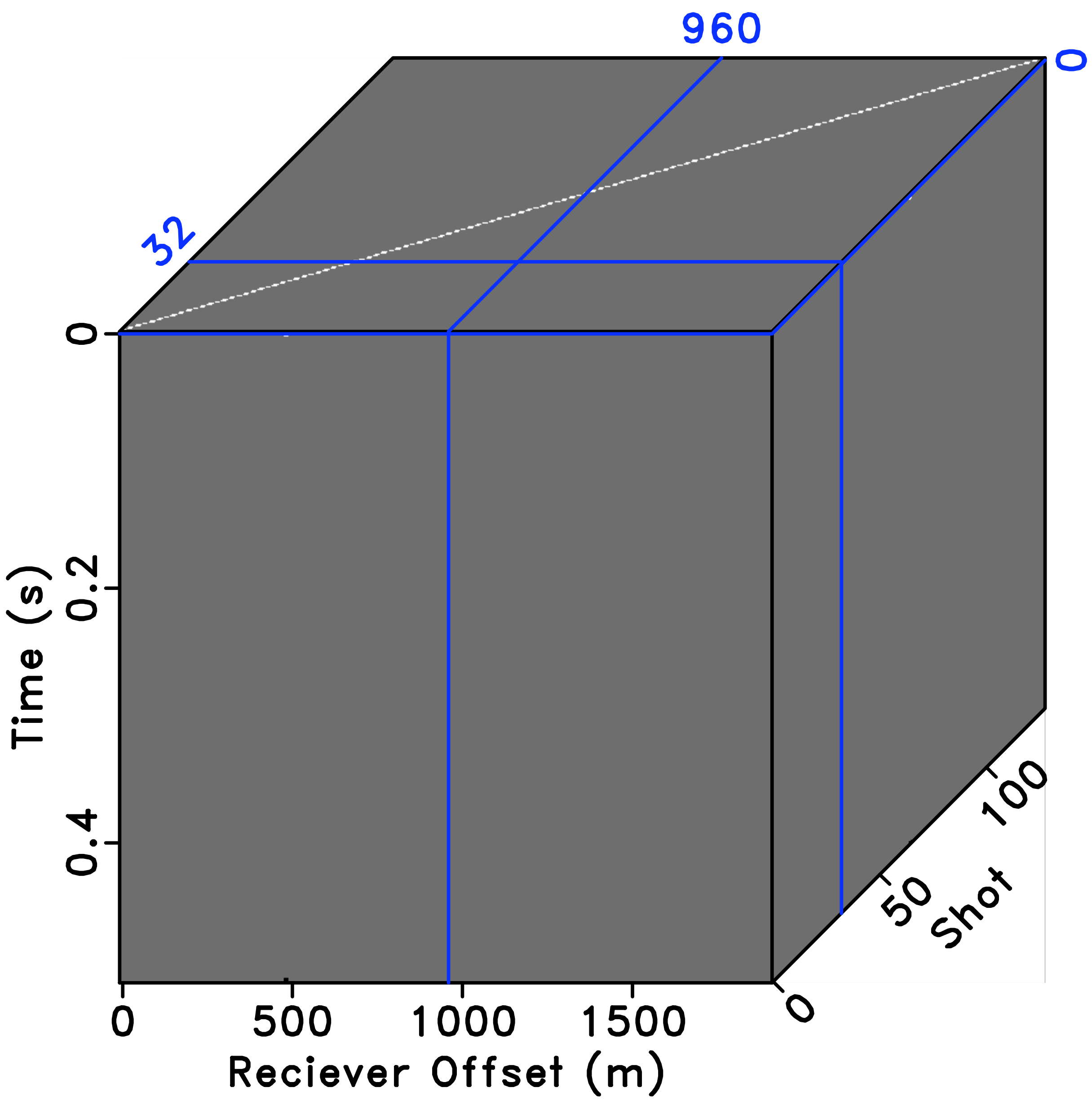
- **CS with Random Convolution (Romberg '08)**
  - Replace Gaussian matrix along shots with restricted **random convolution** over the whole seismic data
  - CS with 3D Fourier Transform $\mathbf{F}_3$ and multiply each coefficient with a unit-norm complex number of randomly determined phase, followed by inverse 2D Fourier on shot-receiver plane, then restrict in both temporal-frequency and shot coordinates

$$\mathbf{RM} = \overbrace{\begin{bmatrix} \mathbf{R}_1^{\Sigma} \otimes \mathbf{I} \otimes \mathbf{R}_1^{\Omega} \\ \vdots \\ \mathbf{R}_{n_{s'}}^{\Sigma} \otimes \mathbf{I} \otimes \mathbf{R}_{n_{s'}}^{\Omega} \end{bmatrix}}^{\text{sub sampler}} \overbrace{\left(\mathbf{F}_2^* \operatorname{diag}\left(e^{\hat{i}\boldsymbol{\theta}}\right) \otimes \mathbf{I}\right)}^{\text{random phase encoder}} \mathbf{F}_3,$$

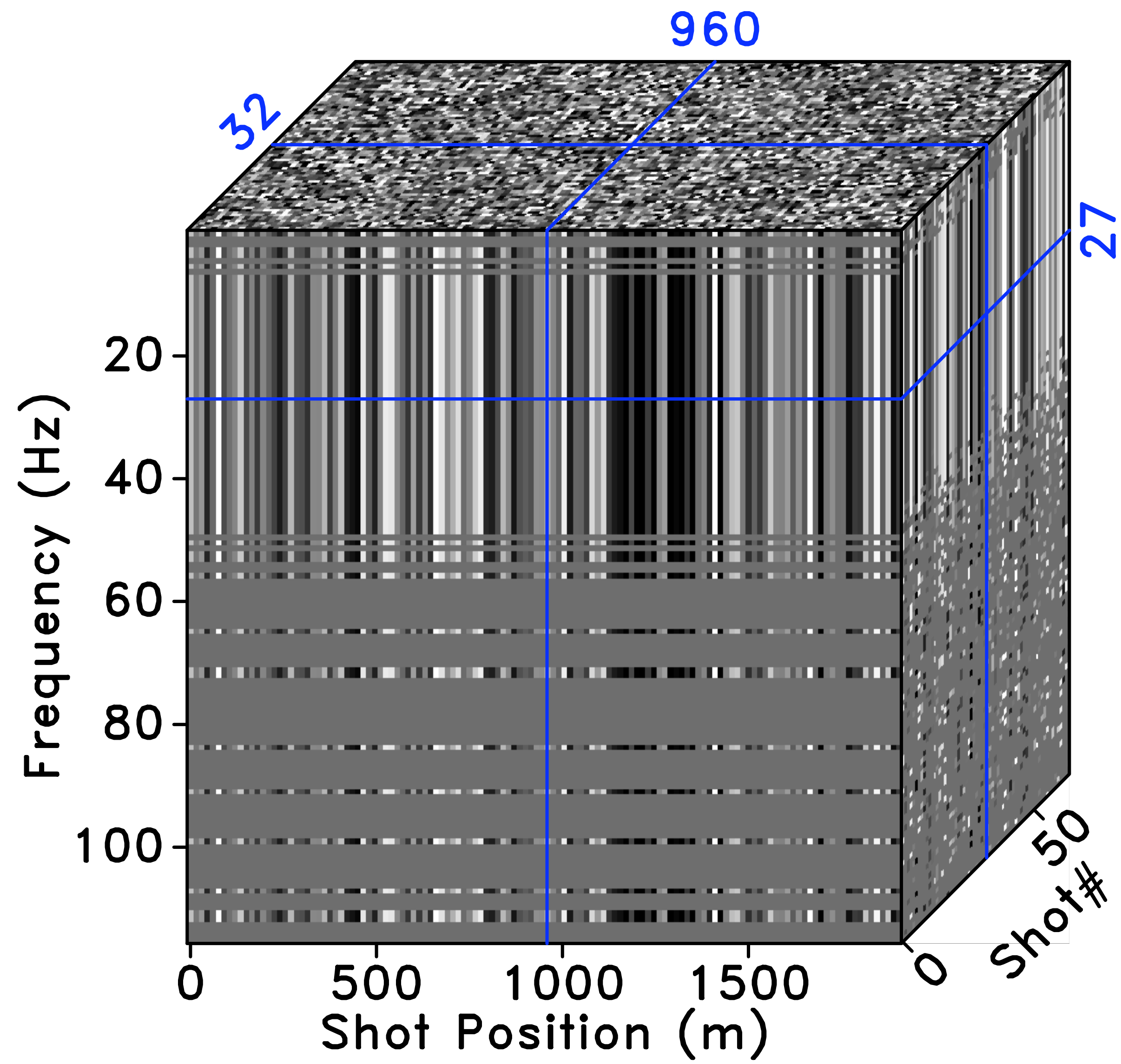
$$\theta_w = \text{Uniform}([0, 2\pi])$$

Applying to Shot Sources

separated source



compressed source



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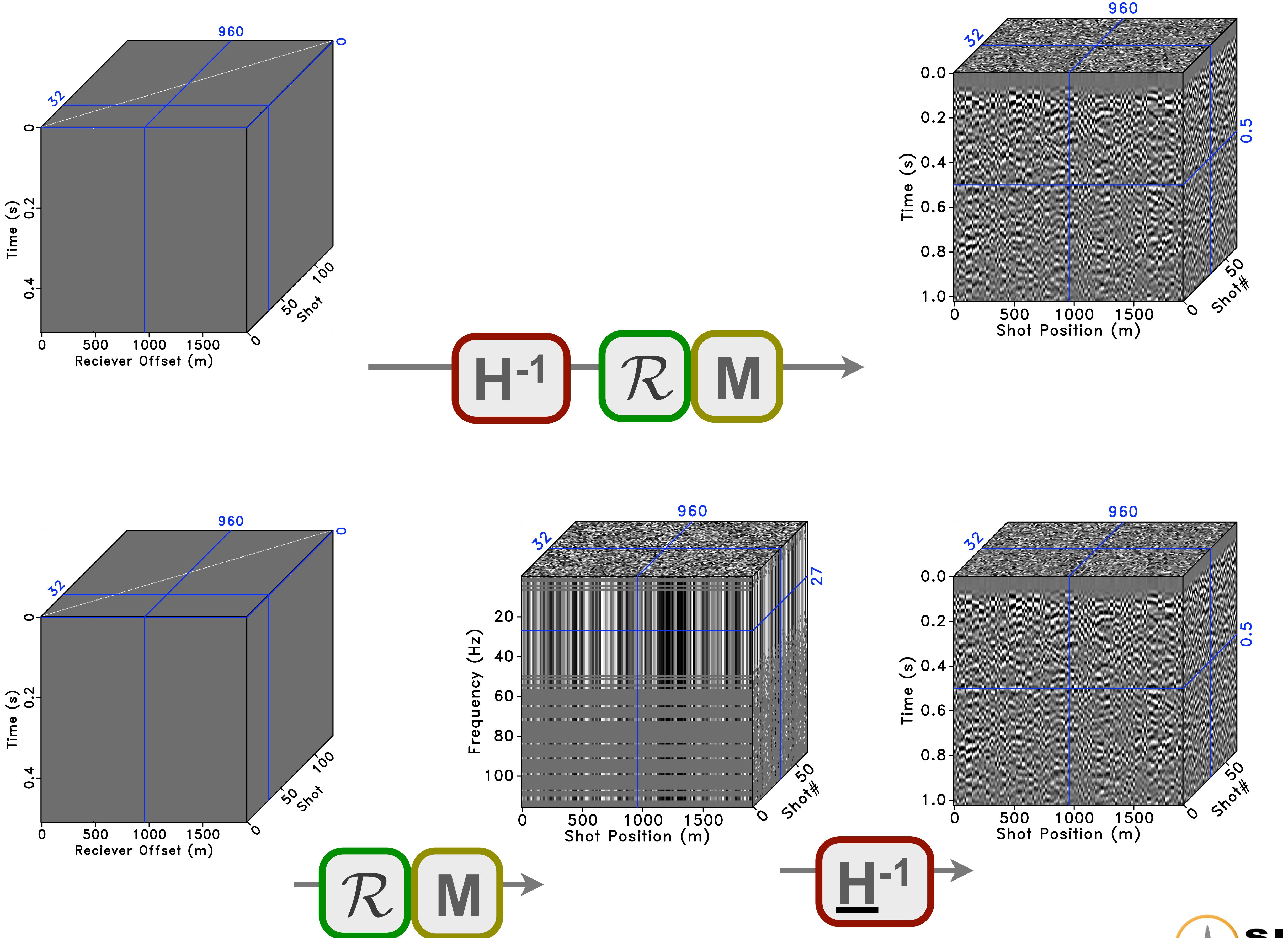
Equivalence

Show equivalence between

- CS sampling of ***full*** solution for separate single-source (sweep) experiments
- Solution of ***reduced*** *system* after CS sampling the collective single-shot source wavefield => **simultaneous source experiments**

$$\begin{cases} \mathbf{B} = \mathbf{D}^* \underbrace{\mathbf{s}}_{\text{single shots}} \\ \mathbf{H}\mathbf{U} = \mathbf{B} \\ \mathbf{y} = \mathbf{RMDU} \end{cases} \iff \begin{cases} \underline{\mathbf{B}} = \underline{\mathbf{D}}^* \underbrace{\mathbf{RMs}}_{\text{simul. shots}} \\ \underline{\mathbf{H}\mathbf{U}} = \underline{\mathbf{B}} \\ \underline{\mathbf{y}} = \underline{\mathbf{DU}} \end{cases}$$

Show that $\mathbf{y} = \underline{\mathbf{y}}$.


960
32
0
Time (s)
0.2
0.4
0
500
1000
1500
Reciever Offset (m)
50
100
Shot
H-1
ℛ
M
Time (s)
0.0
0.2
0.4
0.6
0.8
1.0
Shot Position (m)
Shot#
0.5
Frequency (Hz)
20
40
60
80
100
27

CS

$$\mathbf{P_1}: \quad \begin{cases} \mathbf{y} & = \mathbf{RMd} \\ \tilde{\mathbf{x}} & = \arg\min_{\mathbf{x}} \|\mathbf{x}\|_1 \quad \text{subject to} \quad \mathbf{Ax} = \mathbf{y} \\ \mathbf{A} & = \mathbf{RMS}^* \\ \tilde{\mathbf{d}} & = \mathbf{S}^* \tilde{\mathbf{x}} \end{cases}$$

CS provides conditions under which P1 recovers **d**:

- selection of CS-matrix (Measurement & Restriction matrices)
- ***selection of sparsifying transform***

Additional complications

- large-to-extremely large problem size
- projected gradient with root finding method ($\mathrm{SPG}\ell_1$, Friedlander & van den Berg, ‘07-’08)
- CS matrix has to lead to *physically* ***realizable*** source wavefield for modeling & acquisition

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Composite sparsity transform

Using Curvelet transform for shot and receiver coordinates

- **Frequency-domain** restrictions perform well under **Wavelet** transform for seismic data (Lin et. al. ’08)
- **Spatial-domain** restrictions perform well under **Curvelet** transform for seismic data (Hennefent et. al. ’07)

Combine both transforms in the coordinate they are most suited for

- Wavelet sparsity on temporal-frequency coordinate
- 2D Curvelet sparsity on shot and receiver plane

$$\mathbf{S} = \mathbf{C}_{2d} \otimes \mathbf{W}$$

Complexity $\mathcal{O}(n^3 \log n)$

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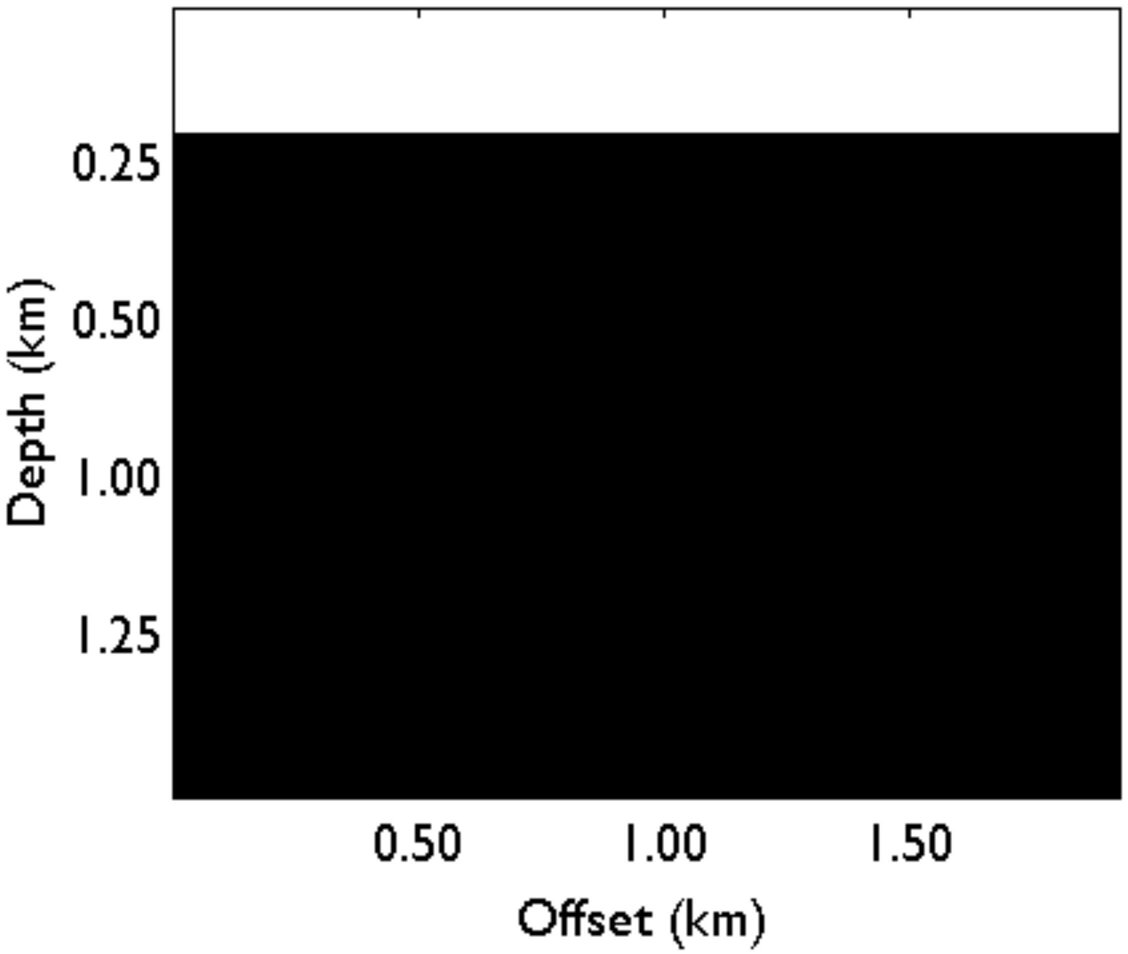
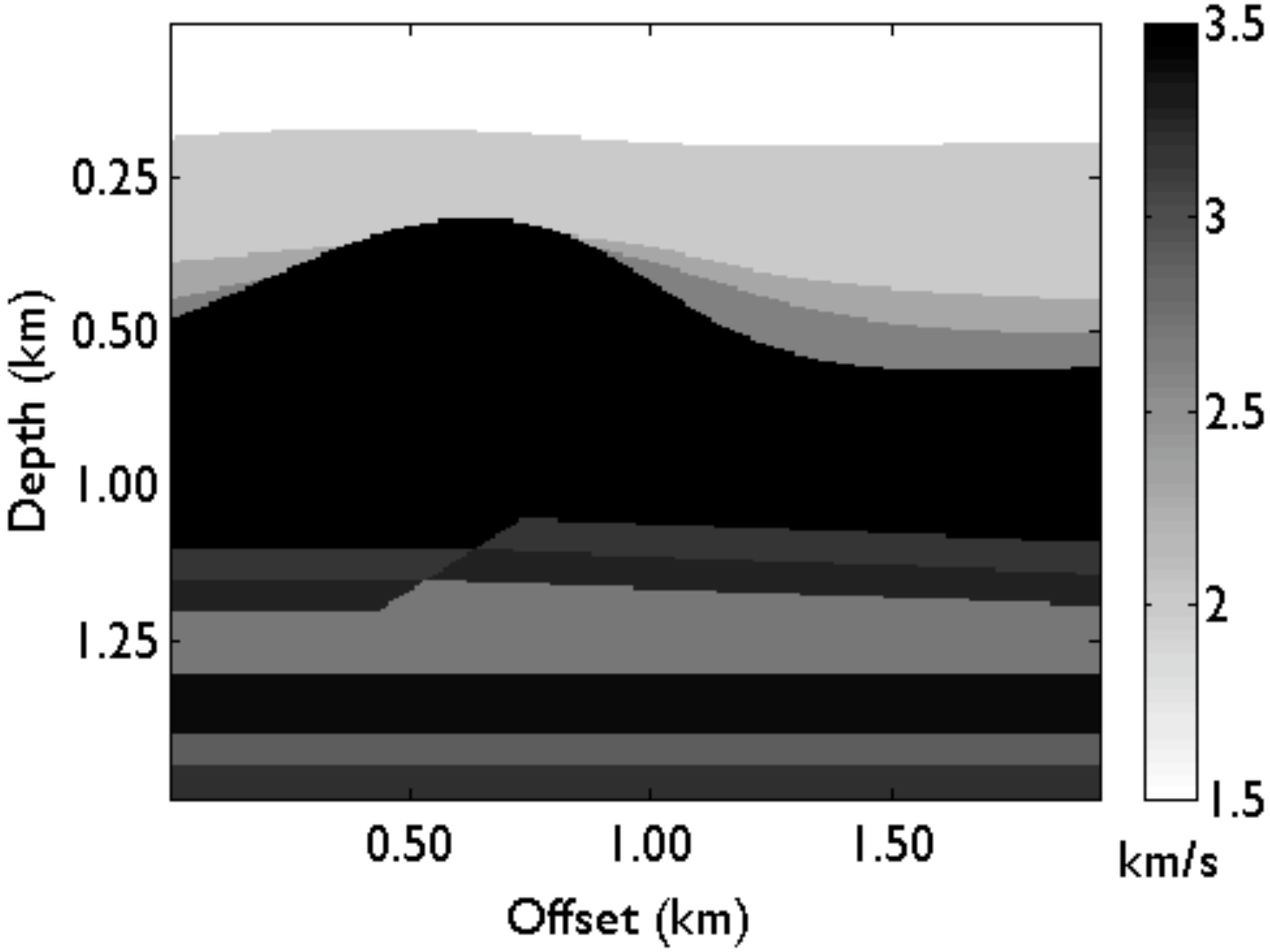
Velocity models

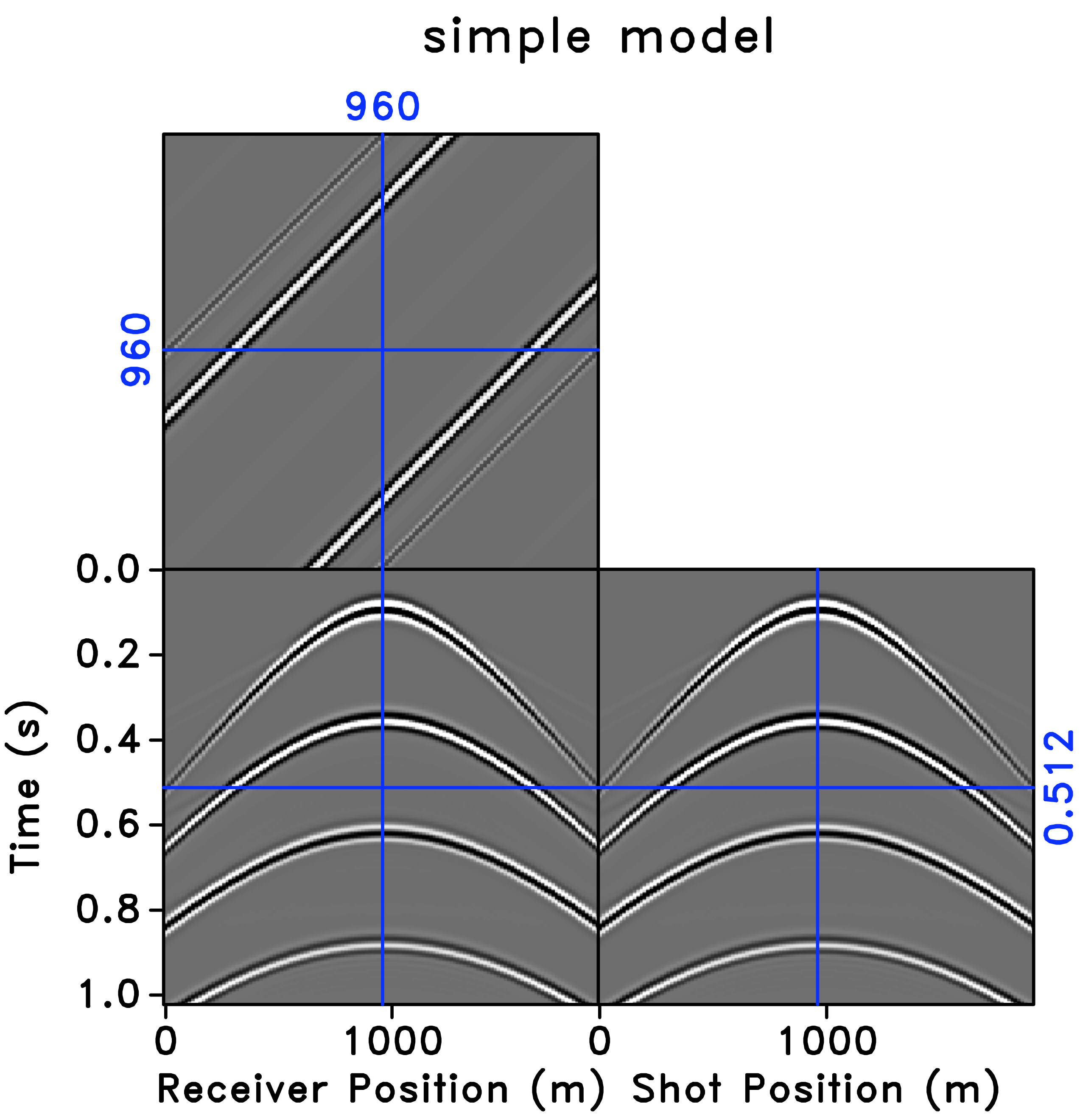
simple model

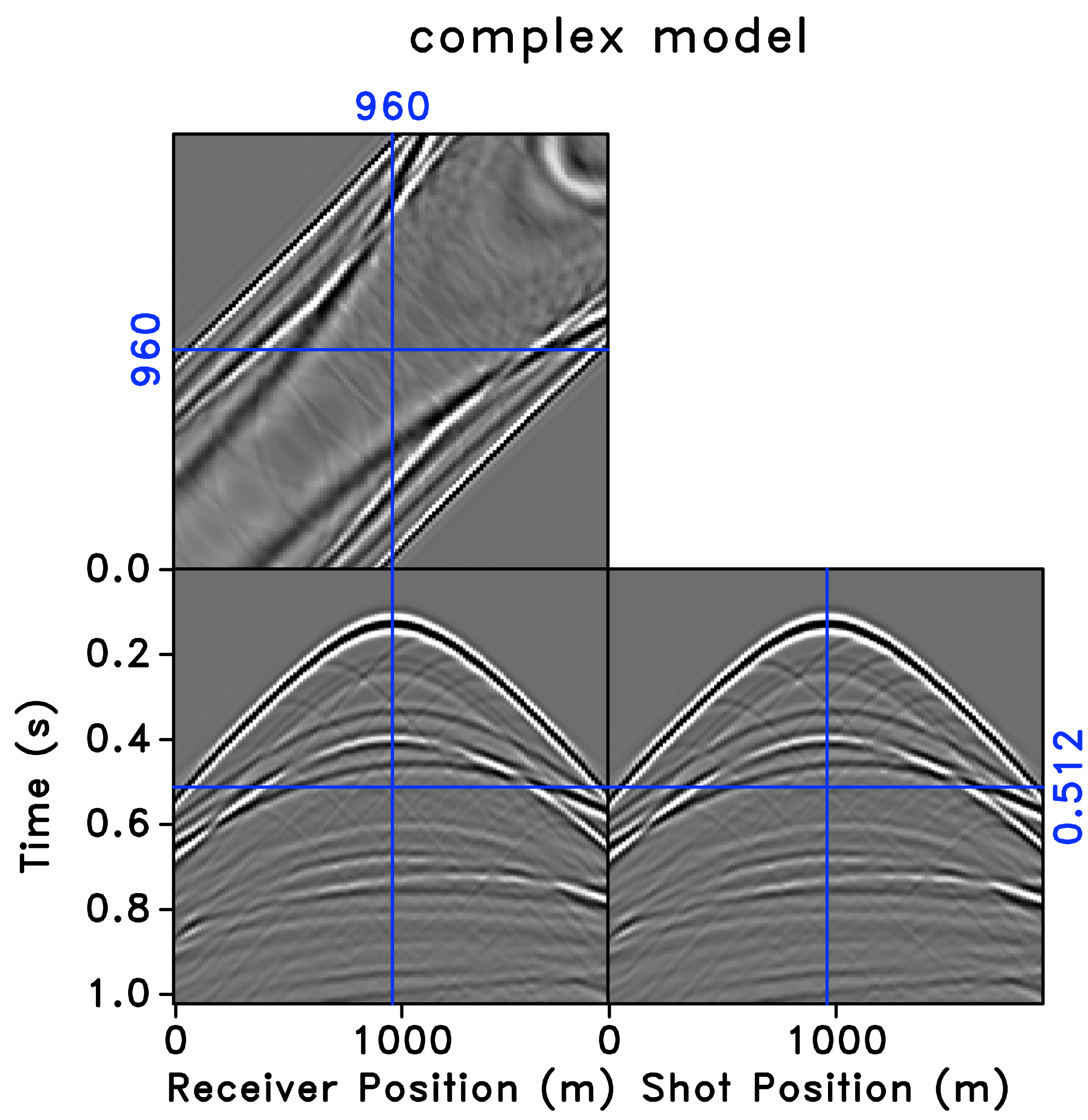


complex model



Green's functions





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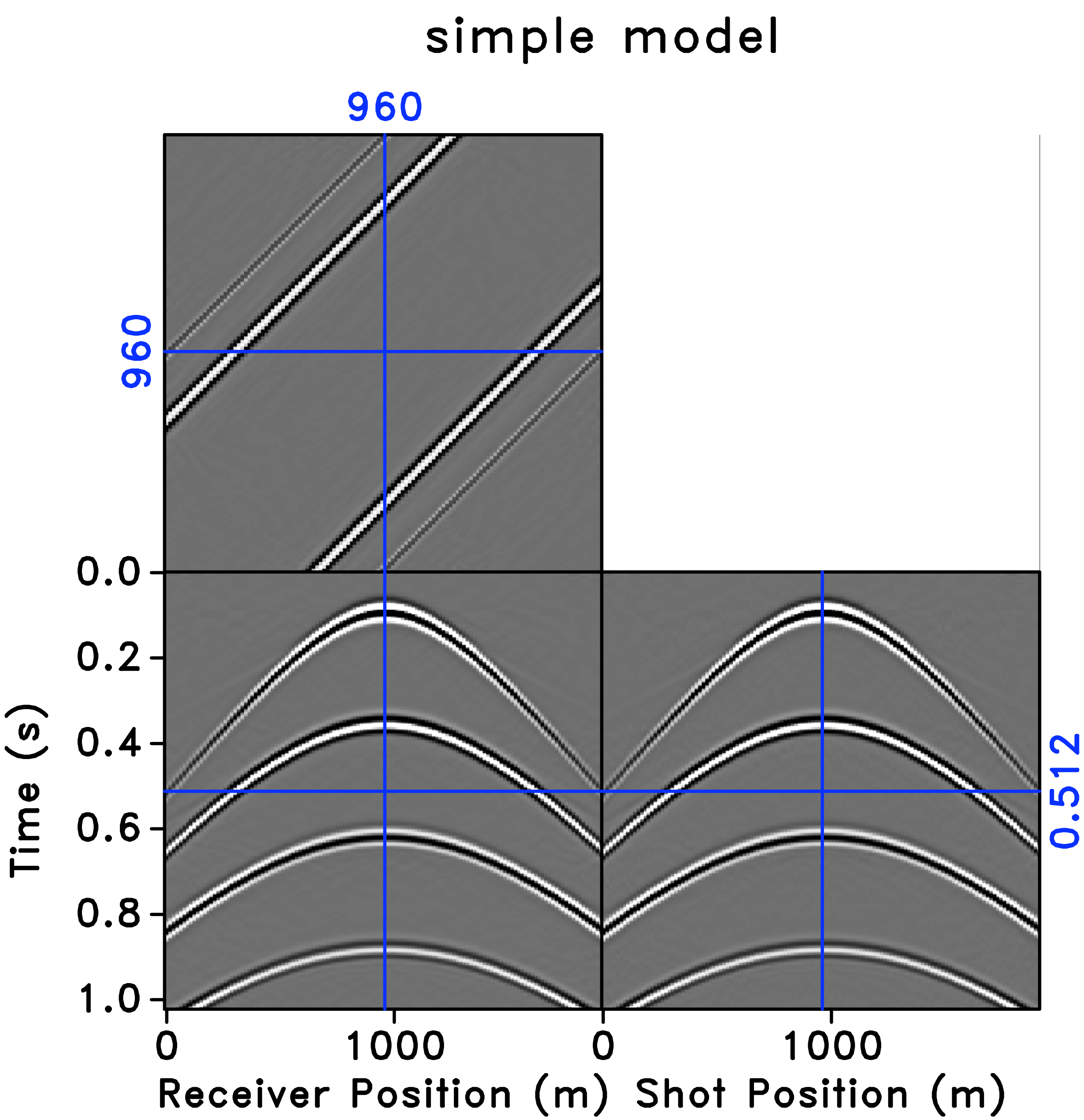
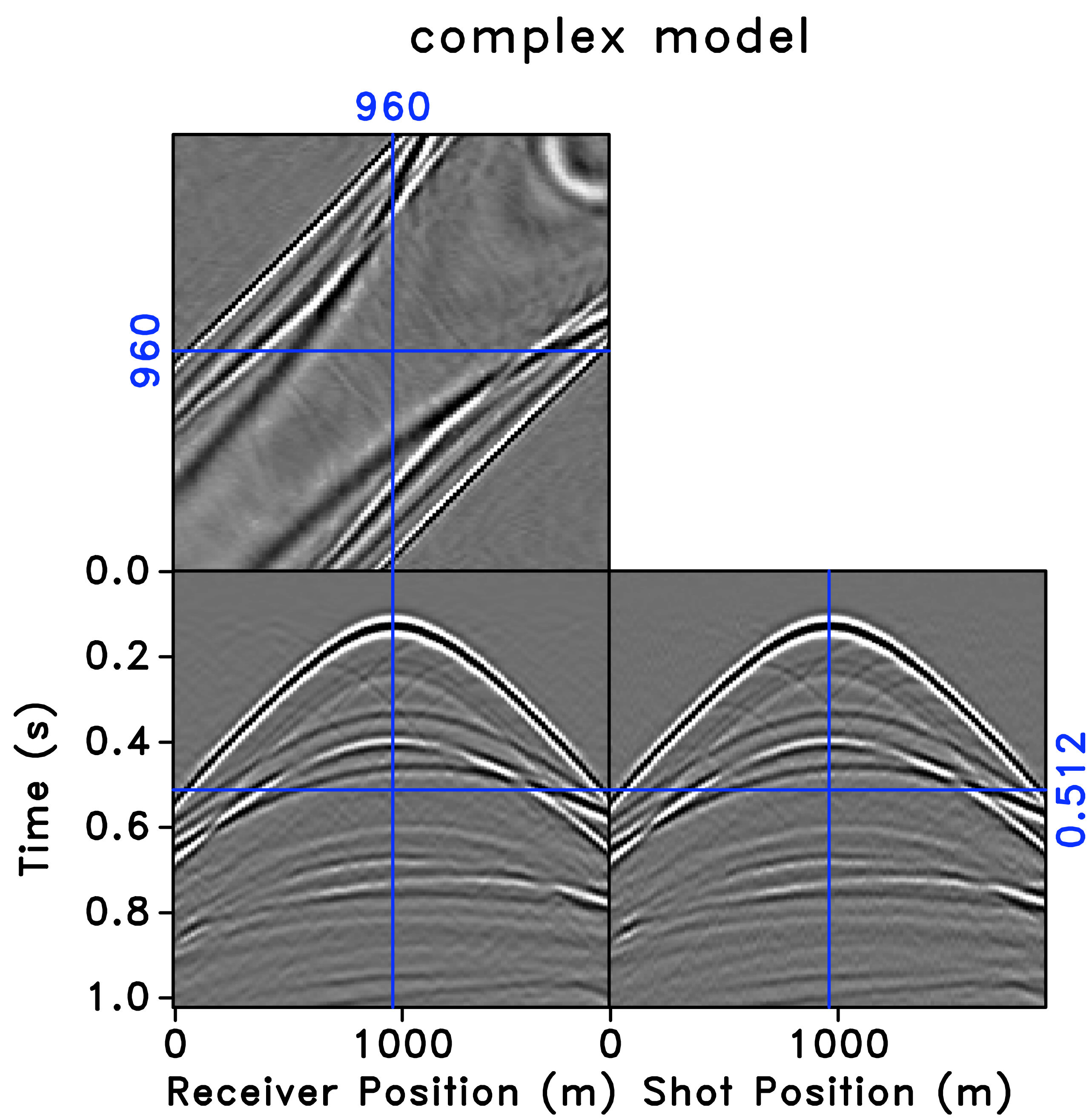
Recovered data



28.1dB



18.2dB

300 SPGL1 iteration

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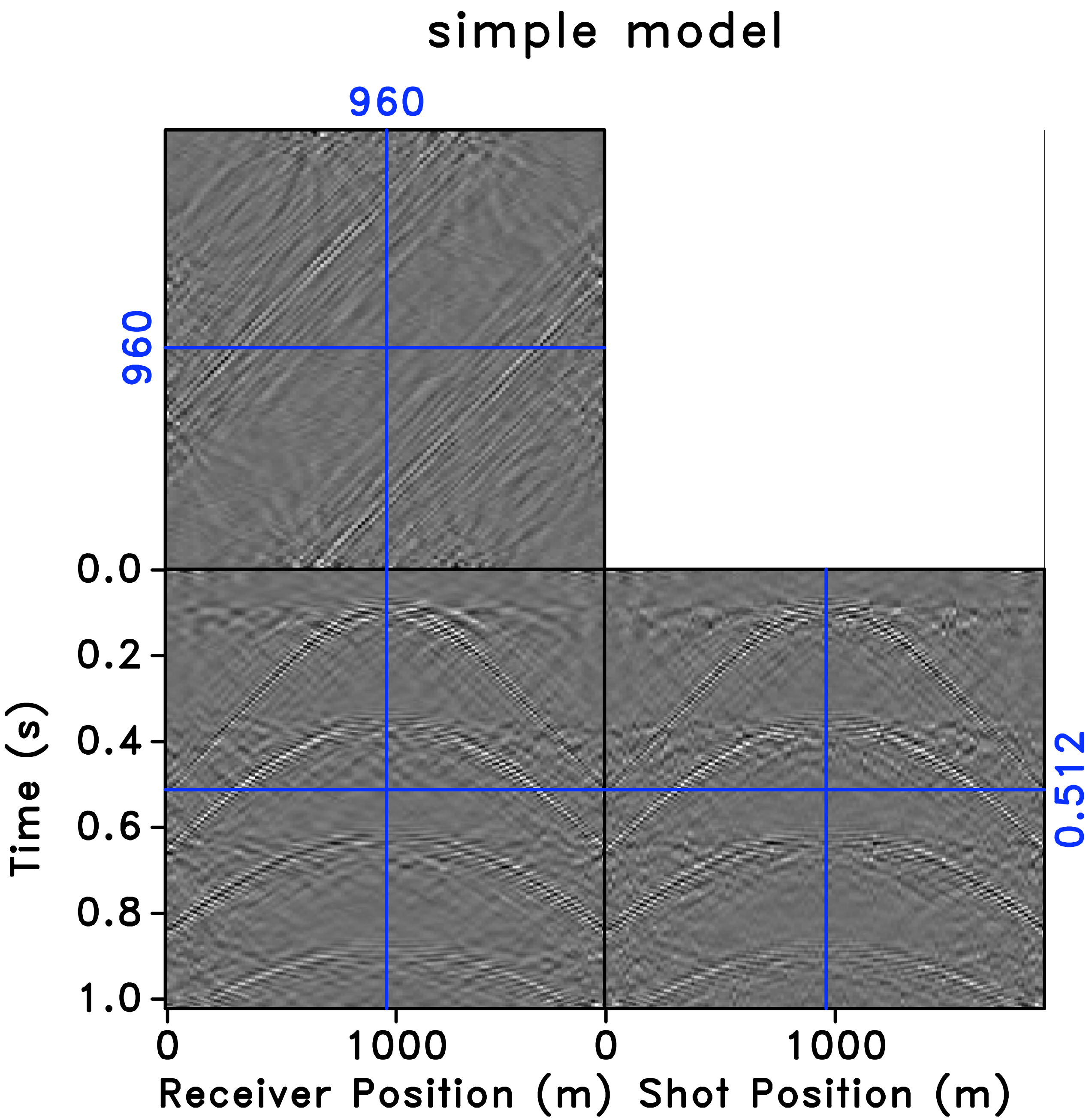
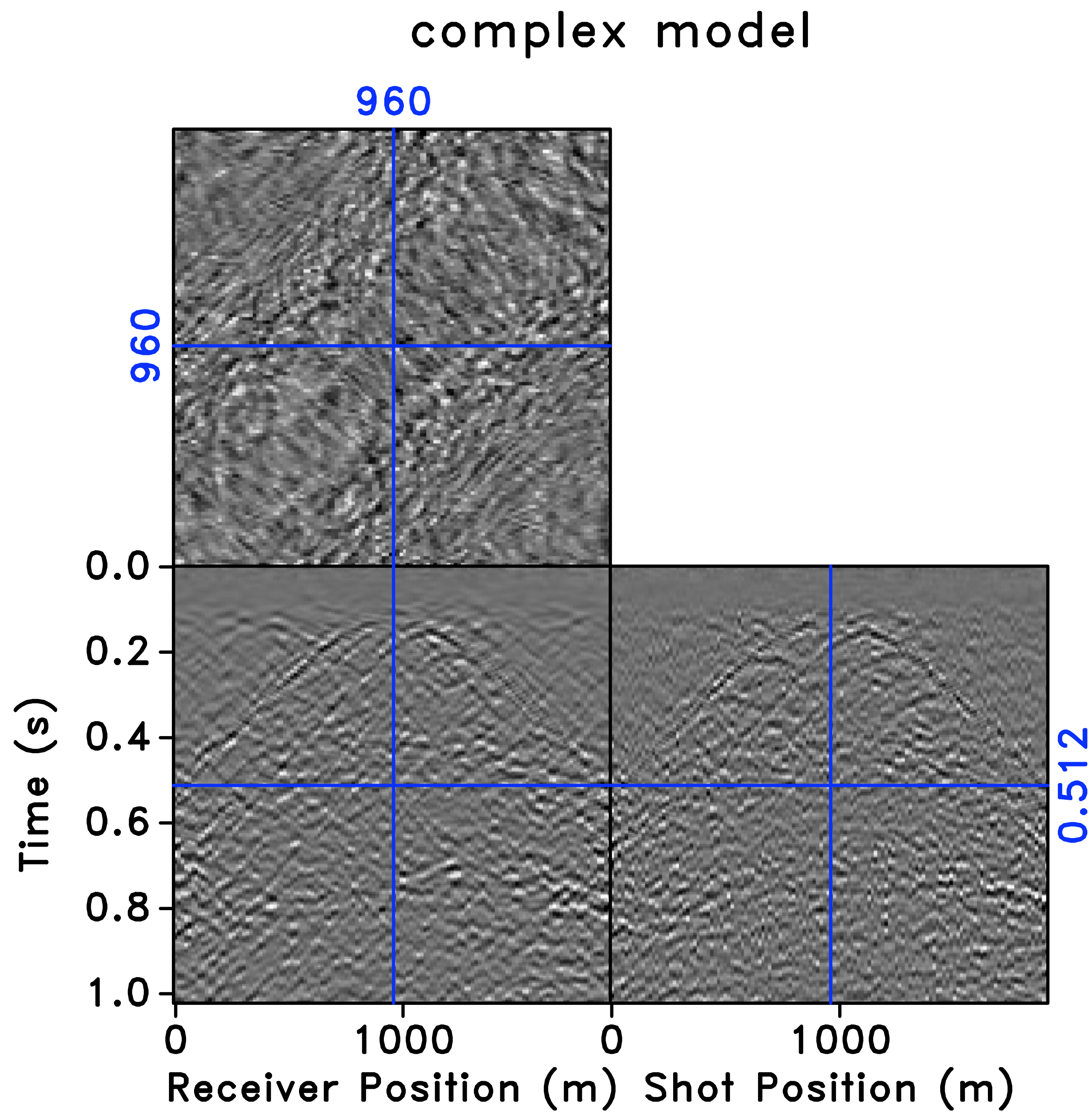
Difference



28.1dB



18.2dB

300 SPGL1 iteration

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Conclusions & outlook

- **CS** provides a **new** *linear sampling paradigm*
  - ***degree*** of *subsampling commensurate* with transform-domain ***sparsity***
  - subsampling of seismic data volumes
    - missing source-receiver locations
    - simultaneous acquisition
  - *subsampling* of solutions to PDEs

- **CS** leads to
  - acquisition of *smaller* data volumes that carry the ***same*** *information* or
  - to ***improved*** *inferences* from data using the *same* resources

- Bottom line: **acquisition & numerical modeling costs** are **no** longer determined by the ***size*** of the ***discretization*** but by the ***transform-domain compressibility*** of the **solution ...**

Acknowledgments

- E. van den Berg and M. P. Friedlander for *SPGL1* (www.cs.ubc.ca/labs/scl/spgl1) & *Sparco* (www.cs.ubc.ca/labs/scl/sparco)
- Sergey Fomel and Yang Liu for Madagascar (rsf.sf.net)
- E. Candes and the Curvelab team

This work was carried out as part of the SINBAD project with financial support from the collaborative research & development (CRD) grant DNOISE (334810-05) funded by the Natural Science and Engineering Research Council (NSERC) and matching contributions from BG, BP, Chevron, ExxonMobil and Shell.

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and... Thank you!