Efficient full-waveform inversion with marine acquisition geometry

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GeoConvention 2012:Vision

Summary

Full-waveform inversion (FWI) is a nonlinear data fitting procedure based on seismic data to derive a accurate velocity model. With the increasing demand for high resolution images in complex geological settings, the importance of improvements in acquisition and inversion become more and more critical. However, these improvements will be obtained at high computational cost, as a typical marine survey contains thousands of shot and receiver positions, and FWI needs several passes through massive seismic data. Computational cost of FWI will grow exponentially as the size of seismic data and desired resolution increase. In this paper we present a modified Gauss-Newton (GN) method that borrows ideas from compressive sensing, where we compute the GN updates from a few randomly selected sequential shots. Each subproblem is solved by using a sparsity promoting algorithm. With this approach, we dramatically reduce the size and hence the computational costs of the problem, whilst we control information loss by redrawing a different set of sequential shots for each subproblem.

Introduction

Full-waveform inversion (FWI) can be formulated as a wave-equation based nonlinear optimization problem in which we invert the model by minimizing the two norm of the difference between observed data and estimated data. One well-known class of algorithms for FWI problem are Gauss-newton (GN) methods which involves the inverse of approximated Hessian (see e.g. Pratt et al., 1998; Erlangga and Herrmann, 2009, and the references therein). The action of the Hessian can be evaluated by combined action of the Jacobian operator (Reverse time migration) and its adjoint (Linear born scattering forward modeling), each evaluation based on one single shot and one single frequency requires at least 3 forward/adjoint simulations. This generates a huge mount of simulations for the whole FWI problem (Li et al., 2011), moreover, the computational cost grows exponentially as data size increases which is the main obstacle preventing the successful application of FWI to industrial scaled data.

As we presented in our earlier work (Li et al., 2011), to avoid multiple passes through all data when we calculate GN updates, we regularize the model updates to be sparse in the curvelet domain (Herrmann et al., 2008). We then compute the GN updates using sparsity promoting techniques derived from work in compressive sensing (CS) (Candès et al., 2006; Donoho, 2006), according to which a signal can be recovered from a severely subsampled data set. This allows us to replace a large number of conventional sequential sources by a limited number of simultaneous phase-encoded sources. As a result we can significantly lower the computational cost of computing GN updates by controlling the number of shots used in the algorithm. However, randomized source superposition relies on full data acquisition which means we should have all the receivers fixed along the seismic line. This means that we can not apply this approach to marine data. To overcome this problem, we replace the randomized superposition by randomly selecting subsets of sequential shots, Instead of solving each sparsity promoting GN subproblem precisely, we only solve each subproblem approximately until a particular residual decrease and sparsity level are achieved. These subproblems are solved by using SPGℓ_1 (a spectral projected gradient method Berg and Friedlander, 2008). We redraw a different set of shots for each GN subproblem, which allows more information to enter without extra computational costs.

Theory

Dimensionality reduction: Full-waveform inversion can be considered as an unconstrained optimization problem in which we minimize a least-squares misfit between observed data and synthetic data

$$\underset{\mathbf{m}}{\text{minimize}}\ \Phi(\mathbf{m}) := \left\{ \frac{1}{2K} \sum_{i=1}^{K} \|\mathbf{d}_i - \mathscr{F}_i[\mathbf{m},\mathbf{q}_i]\|_2^2 = \frac{1}{2}\|\mathbf{D} - \mathscr{F}[\mathbf{m},\mathbf{Q}]\|_F^2 \right\}, \tag{1}$$

with $\mathbf{d}_i$ monochromatic shot records of the Earth response to monochromatic sources $\mathbf{q}_i$, $\mathscr{F}_i[\mathbf{m},\mathbf{q}_i]$, $i = 1 \cdots K$ monochromatic nonlinear forward operators, and $K = N_f \cdot N_s$, with N_f the number of frequencies and N_s the number of sources. In the acoustic constant-density case, this operator is parameterized by the unknown velocity model $\mathbf{m}$ and involves the inversion of a large system of linear equations that represents a discretization of the time-harmonic Helmholtz equation.

In order to reduce the computational cost we use a small subset of all the shots. To achieve this we multiply the data and source function in Equation 1 from the right hand side with a tall matrix whose size is N_s (number of all source) $\times N_{s'}$ (number of subset sources), with $N_{s'} \ll N_s$

$$\underset{\mathbf{m}}{\text{minimize}}\ \underline{\Phi}(\mathbf{m}) := \left\{ \frac{1}{2K'} \sum_{i=1}^{K'} \|\mathbf{D}\mathbf{w}_i - \mathscr{F}_i[\mathbf{m},\mathbf{Q}\mathbf{w}_i]\mathbf{w}_i\|_2^2 = \frac{1}{2}\|\underline{\mathbf{D}} - \mathscr{F}[\mathbf{m},\underline{\mathbf{Q}}]\|_F^2 \right\}, \tag{2}$$

with $\{\underline{\mathbf{D}}, \underline{\mathbf{Q}}\} := \{\mathbf{DW}, \mathbf{QW}\}$ (Romero et al., 2000; Beasley, 2008; Krebs et al., 2009).
Here, we generate matrix $\mathbf{W}$ by randomly picking columns from the identity matrix, which means we randomly pick a small subset of sequential shots from all the data.

Modified Gauss-Newton subproblem with sparsity promotion: In our approach, we solve the FWI problem with a Gauss-newton method which linearizes the function inside the convex ℓ_2-norm. When all sources are used, this method yields linear overdetermined GN subproblems. We turn this overdetermined system into a underdetermined system by subsampling the sources, as described above. This gives us GN subproblems as follows:

$$\underset{\delta\mathbf{m}}{\text{minimize}}\ \frac{1}{2}\|\underline{\mathbf{D}} - \mathscr{F}[\mathbf{m},\underline{\mathbf{Q}}] - \nabla\mathscr{F}[\mathbf{m},\underline{\mathbf{Q}}]\delta\mathbf{m}\|_F^2. \tag{3}$$

where $\nabla\mathscr{F}[\mathbf{m},\underline{\mathbf{Q}}]$ is the linear born-scattering operator. Rather than solving this problem by using standard method with ℓ_2 regularization on the result, we use ℓ_1 regularization instead to promote sparsity of the model update $\delta\mathbf{m}$ in some transform-domain. We compute the update $\delta\mathbf{m}$ by solving the following constrained optimization problem (LASSO problem):

$$\underset{\mathbf{x}}{\text{minimize}}\ \frac{1}{2}\|\underline{\delta\mathbf{D}} - \nabla\mathscr{F}[\mathbf{m},\underline{\mathbf{Q}}]\mathbf{S}^H\mathbf{x}\|_F^2 \quad \text{subject to} \quad \|\mathbf{x}\|_1 \leq \tau, \tag{4}$$

where $\underline{\delta\mathbf{D}} = \underline{\mathbf{D}} - \mathscr{F}[\mathbf{m},\underline{\mathbf{Q}}]$. In this expression, $\mathbf{S}^H$ is the inverse of the sparsifying transform and $\mathbf{x}$ is a vector of coefficients in the transform domain. The constraint enforces the ℓ_1-norm of $\mathbf{x}$ to be smaller than some constant τ. In practice, LASSO problems are solved with a spectral projected gradient (SPG) algorithm implemented in SPGℓ_1 (Berg and Friedlander, 2008). The parameter τ is given by the algorithm automatically using the tradeoff curve between the optimal value of the misfit and the one norm of the solution(Herrmann et al., 2011).

Examples

To test the performance of our inversion algorithm in a realistic setting we generate data with a synthetic velocity model (Fig. 1a) constrained by well information, while the source signature is a $12\,\mathrm{Hz}$ Ricker wavelet. We use a smooth starting model without lateral information (Fig. 1b) for the inversion process. All simulations are carried out with 350 shot and 701 receiver positions sampled at a 20m and 10m intervals, with offset between $100\mathrm{m}$ and $3000\mathrm{m}$. To improve convergence we divide the whole FWI problem sequentially into 10 overlapping frequency bands, each of them has 10 frequencies of the interval $2.9-22.5\mathrm{Hz}$ (Bunks et al., 1995). 10 GN subproblems (LASSO) are solved for each frequency band. For each LASSO subproblem, we use 2 randomly selected sequential shots and roughly 20 SPGℓ_1 iterations. Hence, we are able to speed up the algorithm 87 times compared to solving FWI with GN iterations, each using 10 LSQR iterations with all the data. In this example, we carried out two experiments based on whether to use different sources for each subproblems (renewals) or not. The results for with and without renewals are included in Fig. 1c and Fig. 1d . In these results we obtained a significant improvement by using renewals which remove the crosstalk artifacts and bring more contributing information to the results.

Conclusion

We modified the Gauss-Newton (GN) method, where we replace standard GN subproblems with sparsity promoting LASSO problems. This modification can gives us high quality inversion results. Significant speedup is attainable by limiting the number of sources. Sparse recovery in combination with randomized dimensionality reduction allows us to speed up FWI significantly by iterating on small subsets of the data only. We are able to obtain good inversion results from reduced experiments based on randomized subsets of marine sequential shots, where renewals are very important for it allows more information to enter into the problem without increasing the computational costs.

Acknowledgements

We would like to thank the sponsors of the SINBAD project: BG Group, BGP, BP, Chevron, ConocoPhillips, Petrobras, PGS, Total SA, and WesternGeco. This work was partly supported by a NSERC Discovery Grant (22R81254) and CRD Grant DNOISE II (375142-08).

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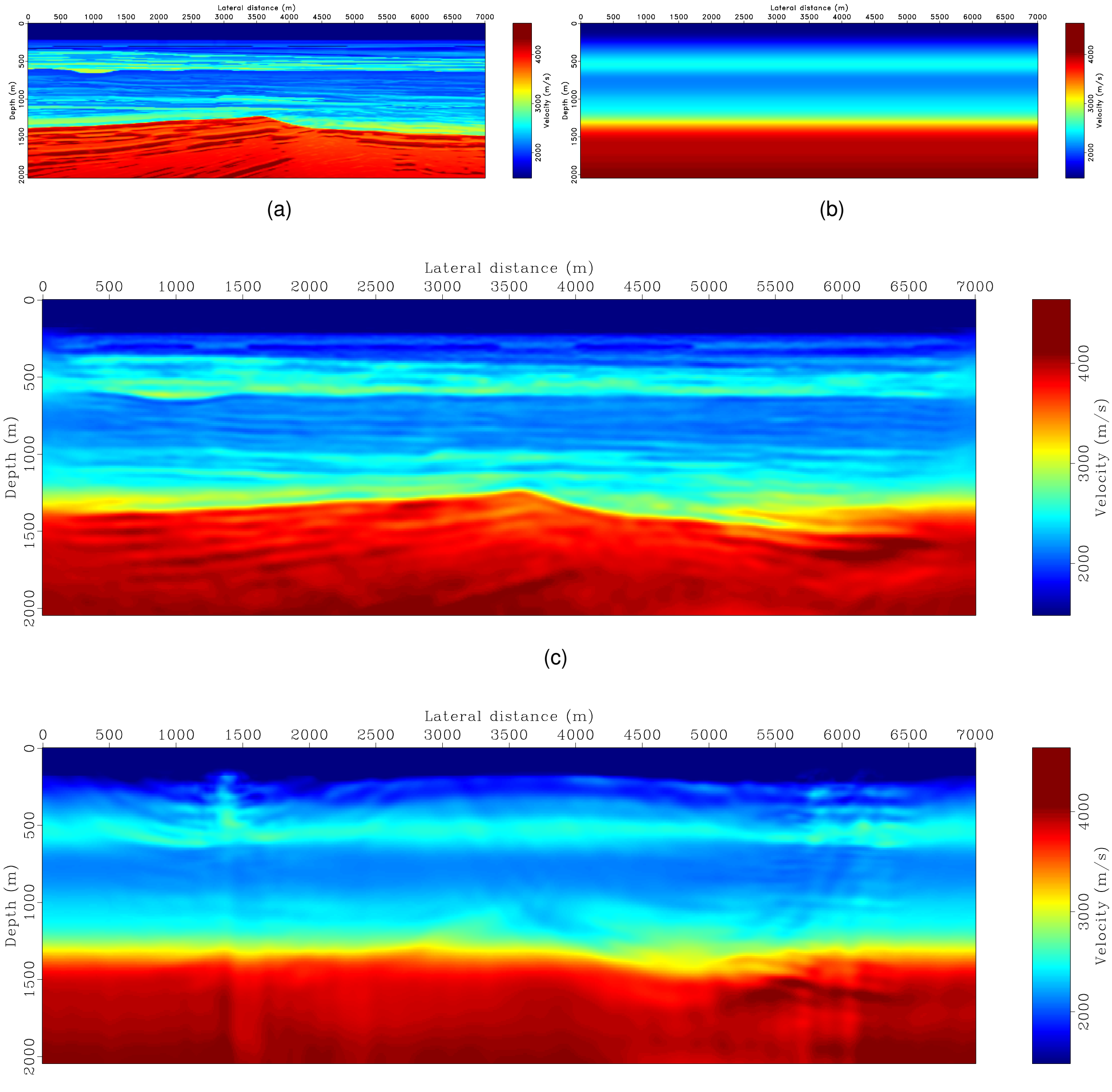

Figure 1: Inversion results with modified GN method by using 2 randomly selected sequential shots, starting from 2.9Hz over 10 frequency bands. (a) original model ($\mathbf{m}$). (b) initial model ($\mathbf{m}_0$) used to start FWI. (c) full-waveform inversion with renewals. (d) the same as (c) but without renewals