SLIM
Seismic Laboratory for Imaging and Modeling

Non-linear regularization in seismic imaging

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thanks to creators CurveLab and G. Hennenfent

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Earth and Ocean Sciences

UBC

Context

Least-squares migration & migration deconvolution [Nemeth '99, Chavent '99, de Hoop '00, Hu '01, Kuhl '03]

Sparseness/minimal structure constrained imaging [Wang '03-'05]

Continuity enhancement with anistropic diffusion [Kuhl, Fehmers, Imhof, Schertzer '03]

Curvelet frames [Stein '93, Smit '97, Candes & Donoho, Do '02, Demanet '05, Ying '05]

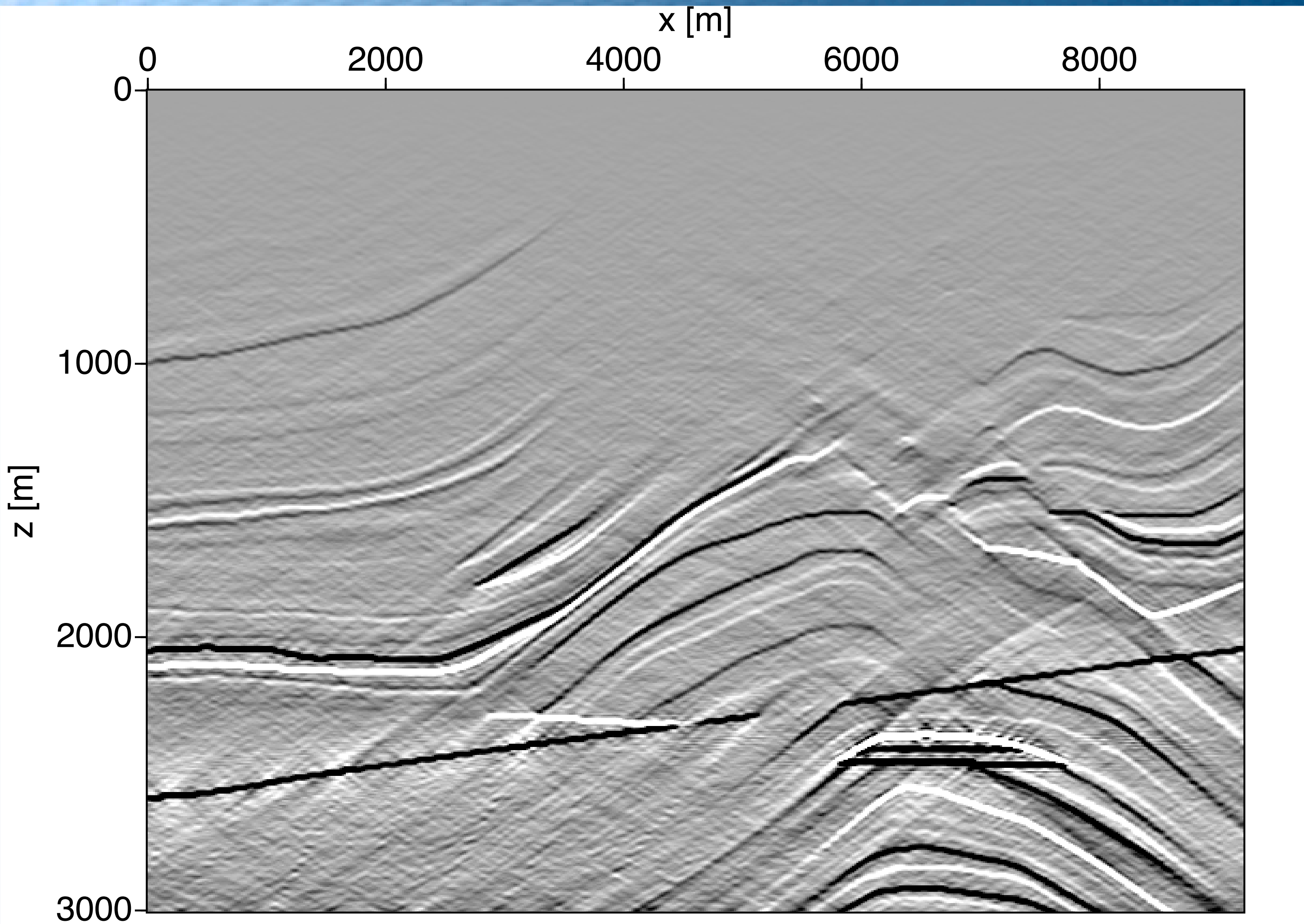
Decon. & Wavelet-Vaguelette/Q-SVD [Donoho '95, Mallat '97, Candes '01, Neelamani '03, Daubechies '05, Starck '05]

Regularizations

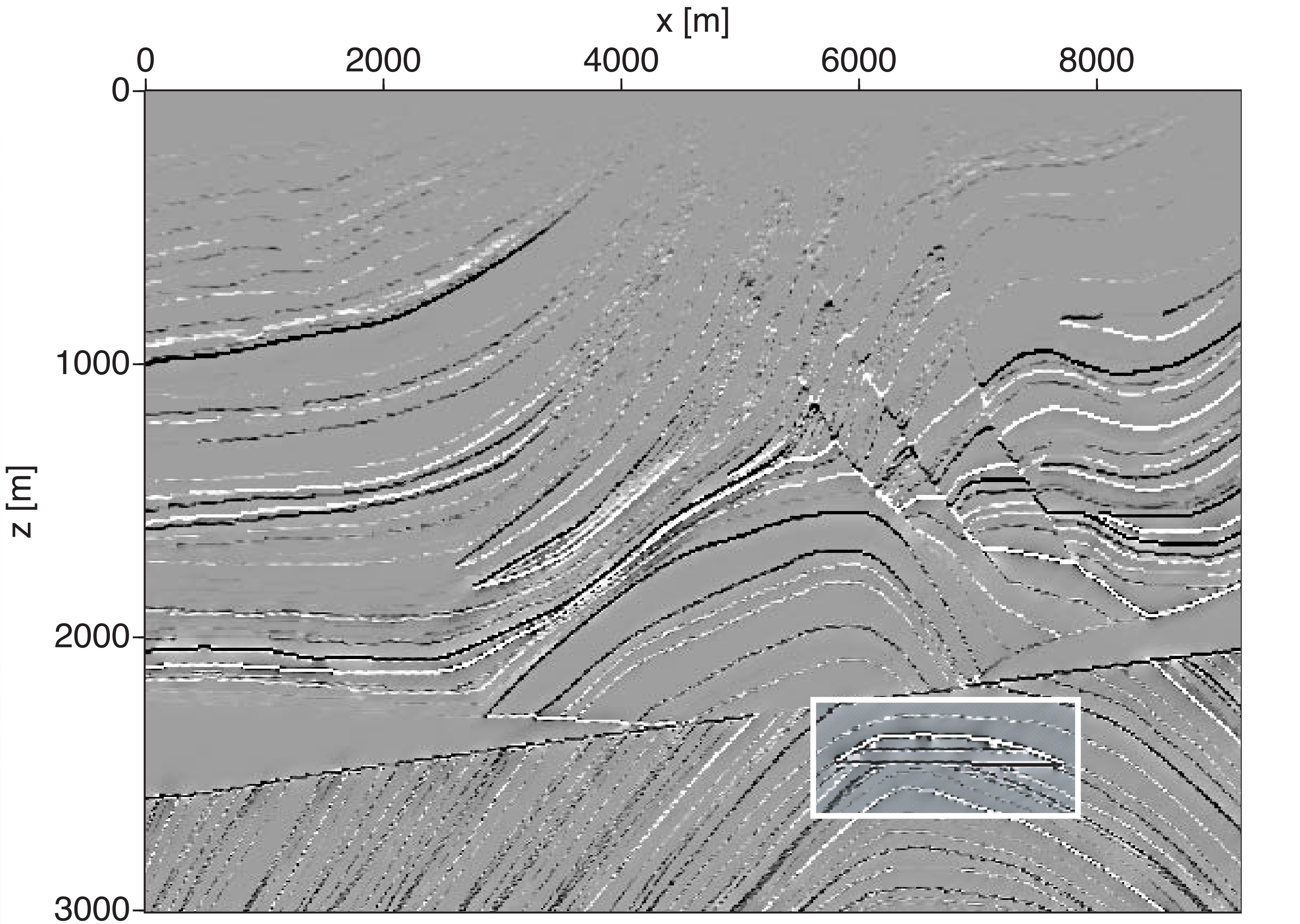
Preserve frequency content of seismic images:

- ***sparseness* of directional frames: e.g. curvelet frames** [Candes & Donoho '02, H '03-'05]
- ***invariance* of curvelet frames under imaging operators** [Douma '04, Demanet '03-'05, H & M '04]
- ***continuity* along reflectors through Curvelets & anisotropic TV/diffusion** [H & M '03-05]

Conventional imaging



Non-linear imaging



Seismic imaging

[Nemeth '99, Chavent '99, de Hoop '00, Hu '01, Kuhl '03]

The forward problem:

$$\overbrace{\mathbf{d}}^{\text{data}} = \underbrace{\mathbf{K}}_{\text{scat. oper.}} \overbrace{\mathbf{m}}^{\text{model/refl.}} + \underbrace{\mathbf{n}}_{\text{noise}}$$

Conventional inverse problem:

$$\hat{\mathbf{m}} = \arg\min_{\mathbf{m}} \underbrace{\|\mathbf{d} - \mathbf{K}\mathbf{m}\|_2^2}_{\text{data misfit}} + \underbrace{\lambda\|\mathbf{m}\|_p^p}_{\text{regularization}}$$

Regularizations

Linear quadratic (lsqr migration):

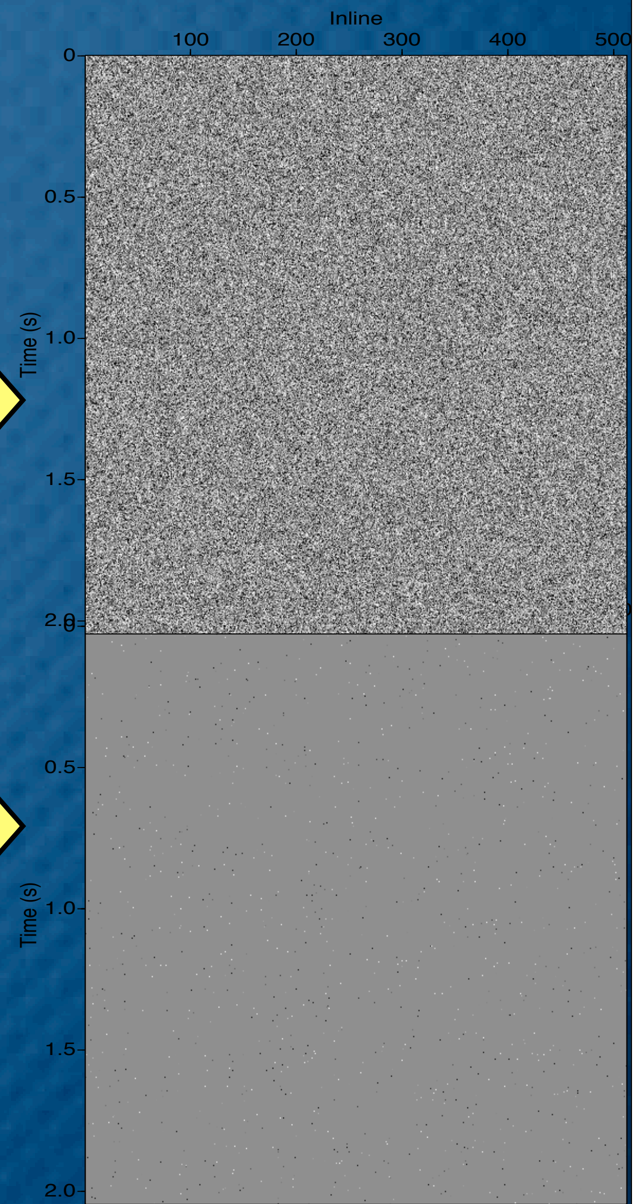
$$\hat{\mathbf{m}} = \arg\min_{\mathbf{m}} \|\mathbf{d} - \mathbf{K}\mathbf{m}\|_2^2 + \lambda \|\mathbf{m}\|_p^p$$

- **reflectivity white Gaussian**
- **uncorrelated**

Non-linear:

$$\hat{\mathbf{m}} = \arg\min_{\mathbf{m}} \|\mathbf{d} - \mathbf{K}\mathbf{m}\|_2^2 + \lambda \|\mathbf{m}\|_1^1$$

- **reflectivity white Cauchy**
- **uncorrelated**



[Nemeth '99, Chavent '99, de Hoop '00, Hu '01, Kuhl, Wang '03]

Non-linear imaging

[Donoho ‘95, Candes ‘01, H & M ‘03-’05]

Wish list: seek a *transformed domain*

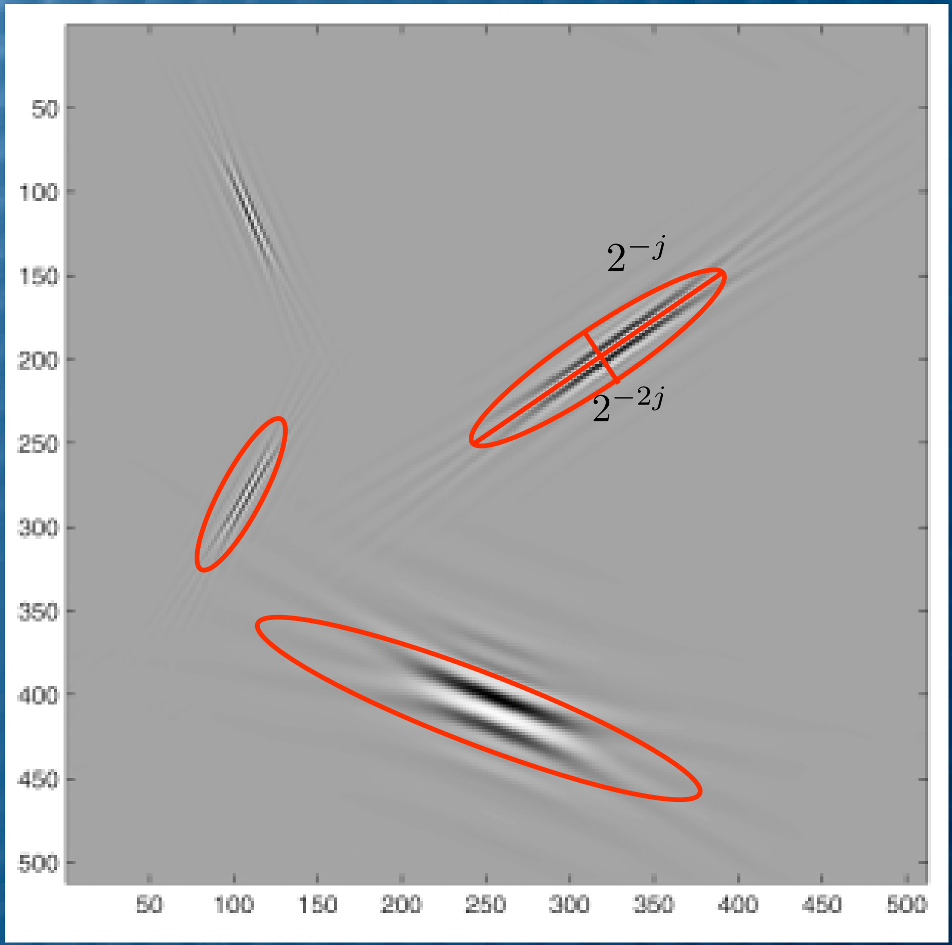
- **where atoms remain *invariant* under physics of wave propagation**
- ***that is sparse, energy concentration***
- ***optimal* for *curved* reflectors**
- ***local* both in space & spatial freq.**

Aim to exploit sparseness of curvelet frames in a non-linear estimation & optimization procedure!

Curvelets

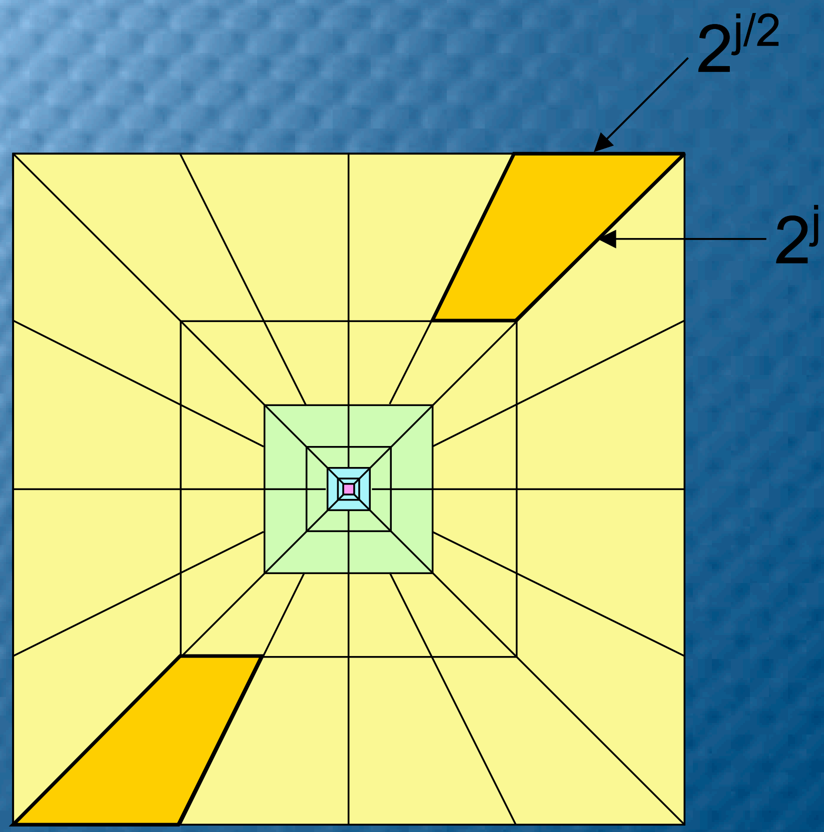
[Candes, Donoho, Demanet, Ying ‘02-’05]

- Tight frames
- Partitioning of the 2-D/3-D Fourier domain into angular wedges of second dyadic coronae
- Parabolic scaling law
- n log n



Numerical construction

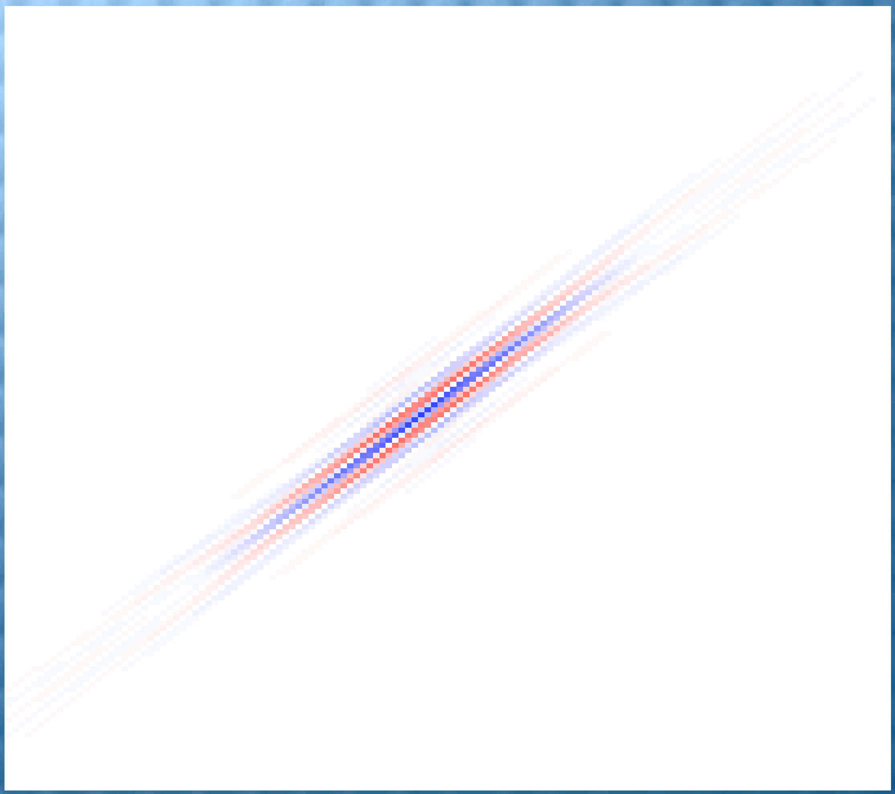
[Candes & Donoho ‘02-’05, Do ‘02, Demanet ‘05, Ying ‘05]

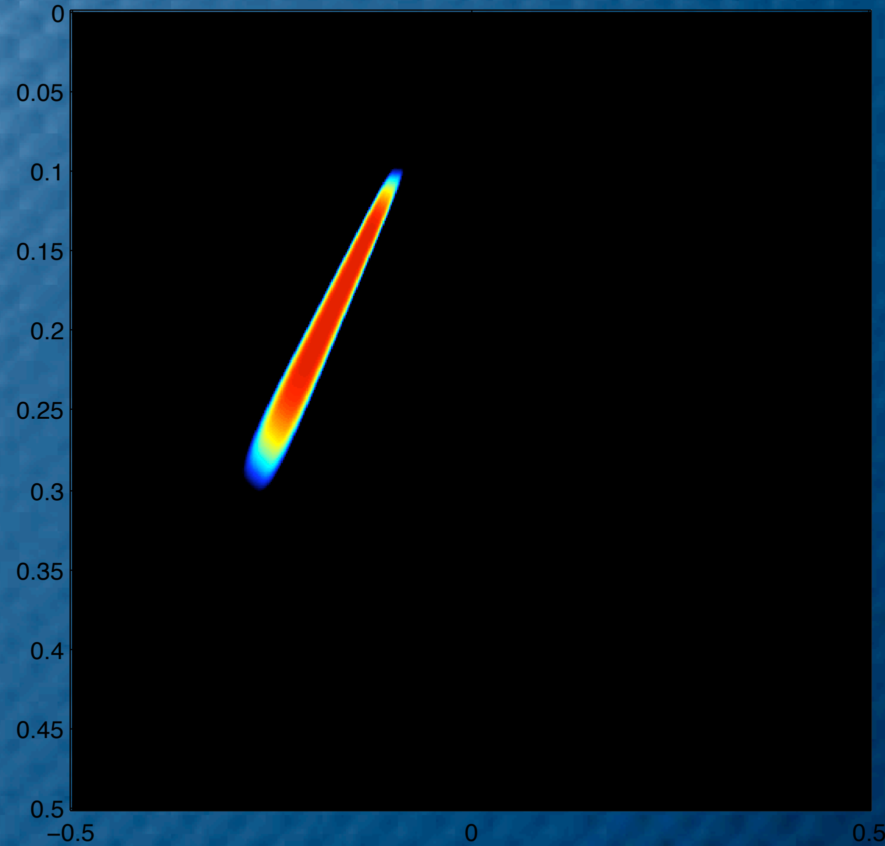


Curvelets live in a wedge in the 2-3 D Fourier plane...

Numerical construction

[Candes & Donoho '02-'05, Do '02, Demanet '05, Ying '05]

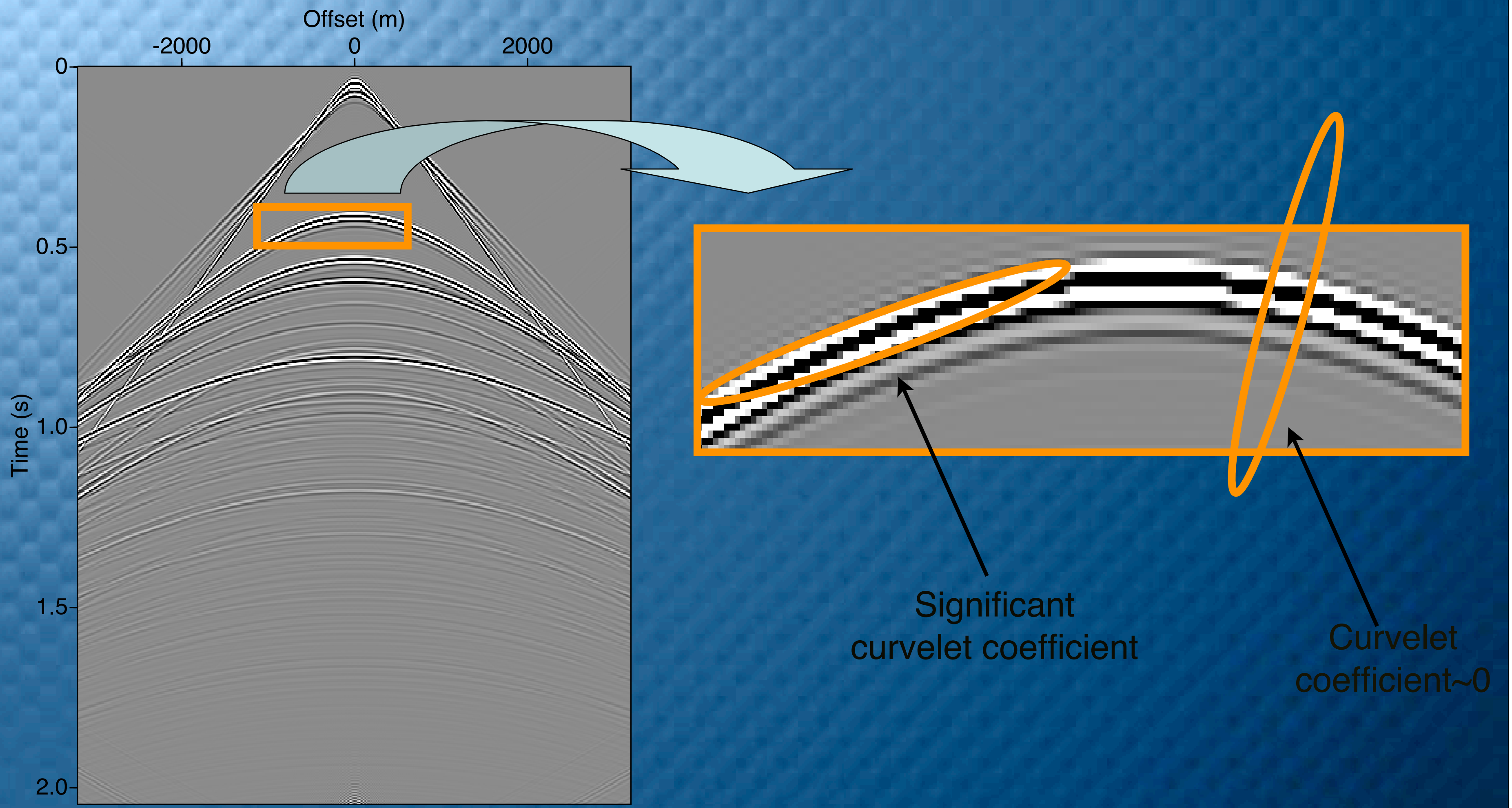




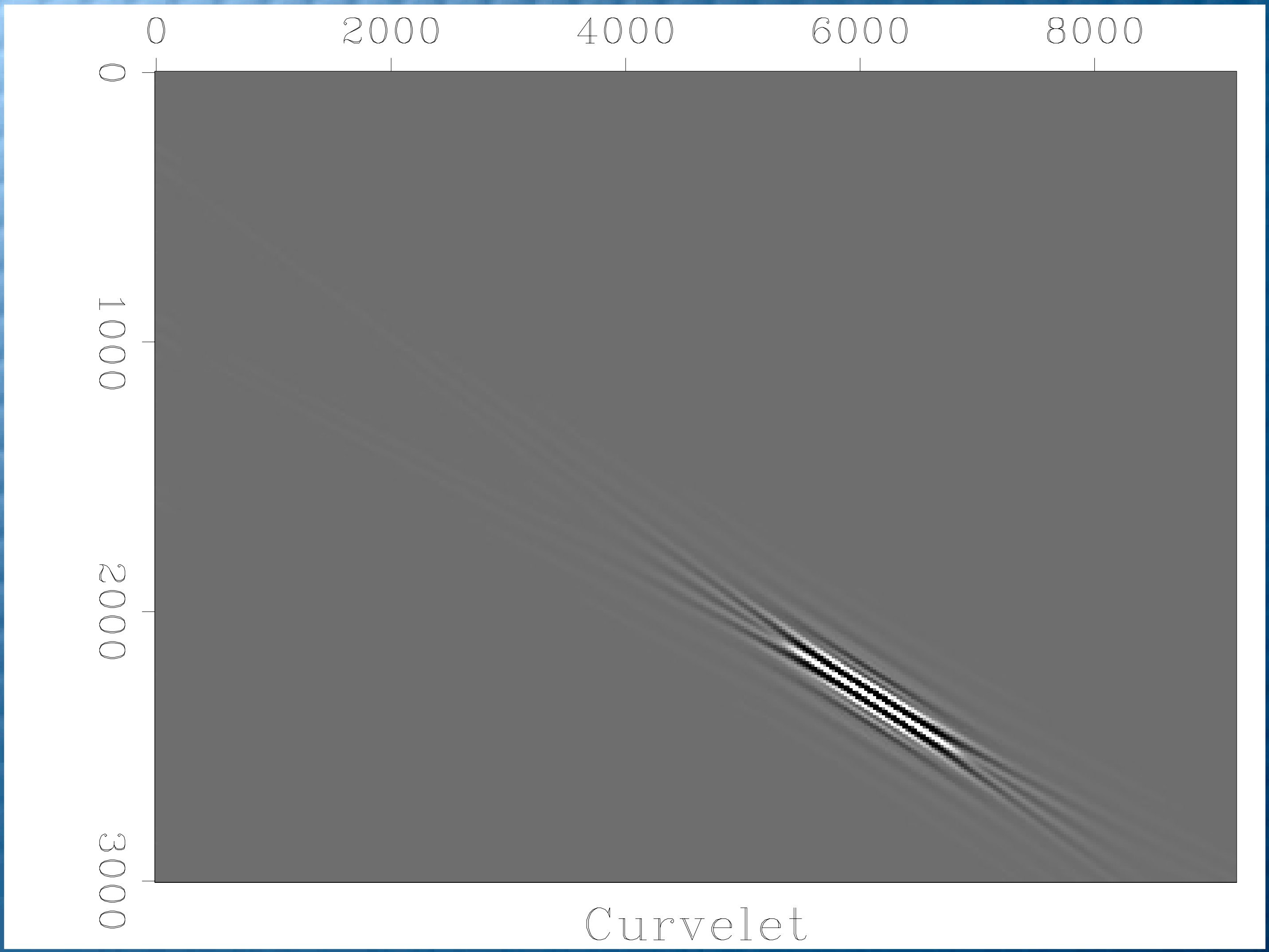
localized in both domains

Curvelets & Seismic

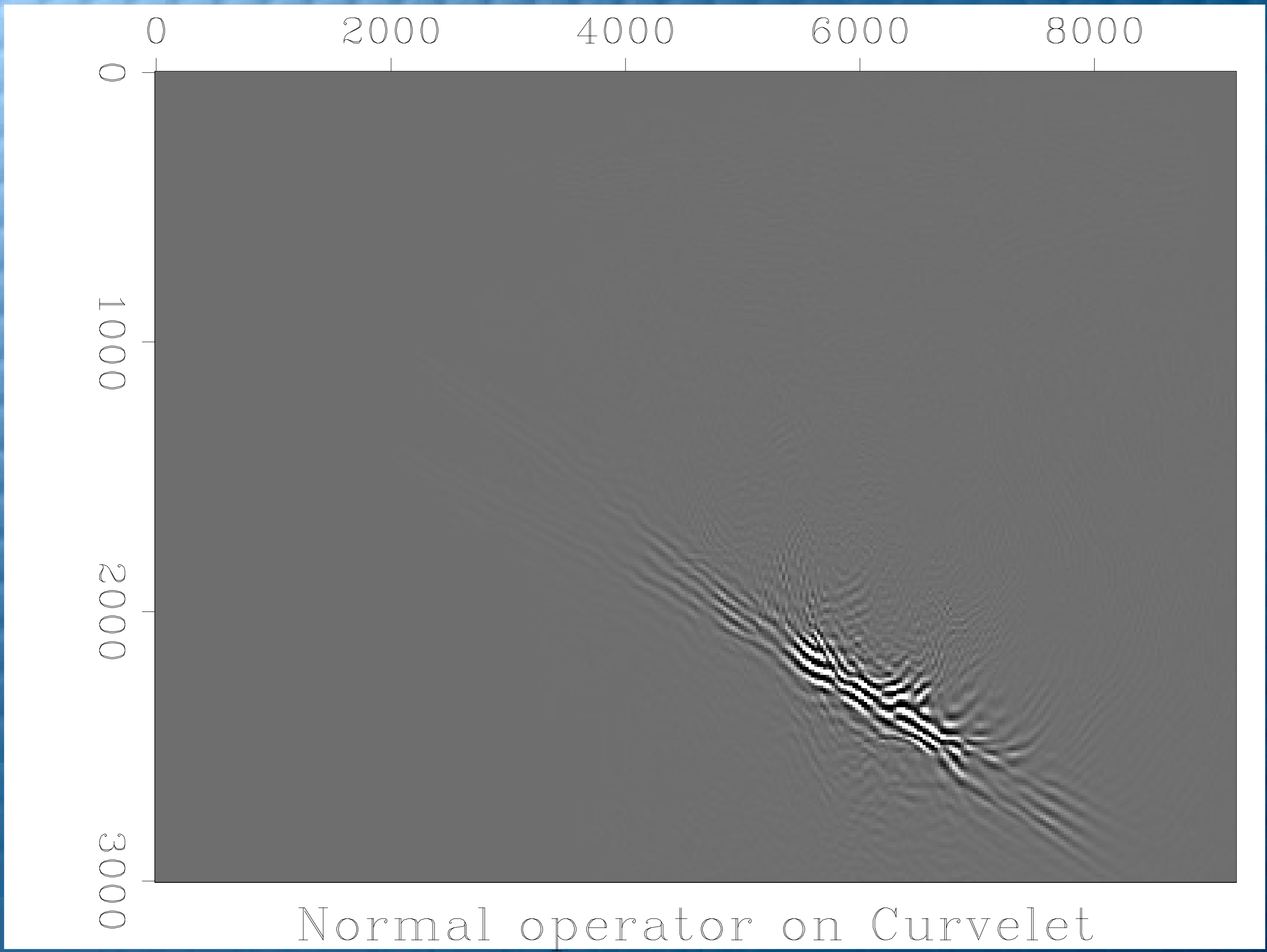
[Douma '04, H, H & M '03-'05]



Curvelets & waves



Curvelets & waves



Sparseness & continuity constrained imaging and inversion

Combine curvelet atoms

- ***sparseness*** **&** ***locality***
- ***invariance*** **under scattering-migration**

$$\mathbf{C}\mathbf{K}^T\mathbf{K}\mathbf{C}^T\cdot \approx \mathbf{\Gamma}^2\cdot$$

Quasi-singular values

with regularization functionals

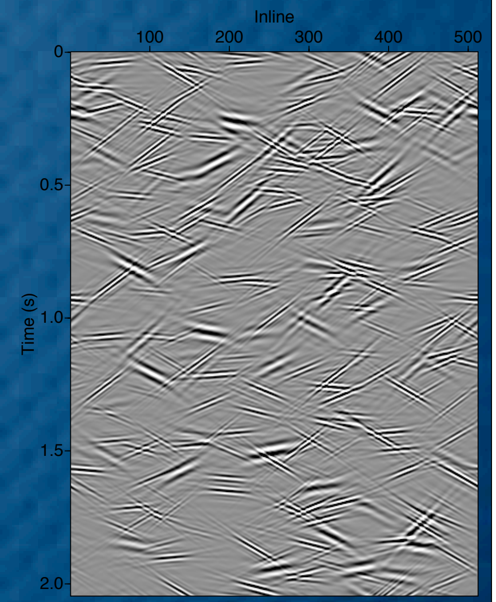
- **enhance the sparseness (ℓ^1-norm)**
- **enhance continuity**

Iterative denoising

[Donoho '98, Mallat '98 Candes '02, Starck '04, Daubechies '05]

$$\hat{\mathbf{x}} = \arg\min_{x} \frac{1}{2}\|\mathbf{d} - \mathbf{F}\mathbf{x}\|_2^2 + \|\mathbf{x}\|_1^1$$

- **with** $\mathbf{m} = \mathbf{F}\mathbf{x},\ \mathbf{F}\cdot \triangleq \mathbf{C}^T\cdot$
- **sparse superposition of curvelet atoms**
- ***multiple* Landweber iterations:**



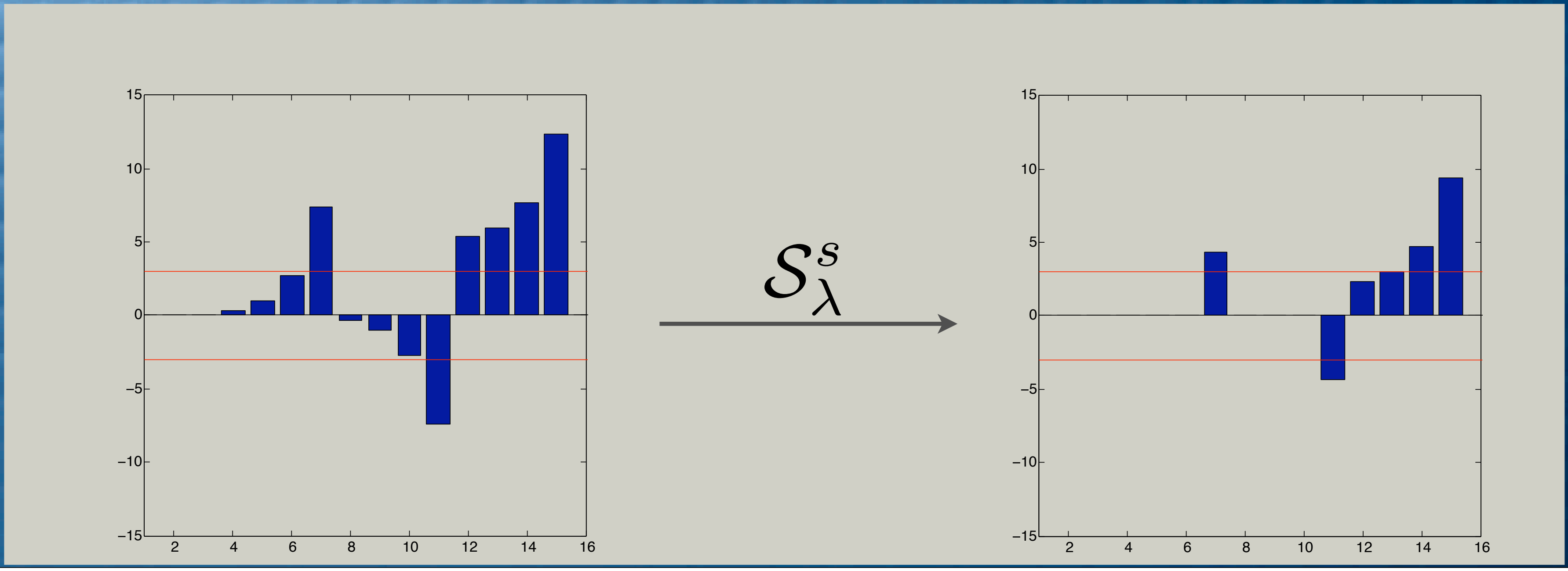
$$\mathbf{x}^m = S^s_{w(\lambda_m)}\left(\mathbf{x}^{m-1} + \mathbf{F}^*\left(\mathbf{d} - \mathbf{F}\mathbf{x}^{m-1}\right)\right)$$

Soft thresholding

[Donoho '98, Mallat '98 Candes '02, Starck '04, Daubechies '05]

$$\boxed{\mathbf{x}^m = S^s_{\lambda_m}\left(\mathbf{x}^{m-1} + \mathbf{F}^T\left(\mathbf{d} - \mathbf{F}\mathbf{x}^{m-1}\right)\right)}$$

$$S_\lambda(x) = \begin{cases} x - \mathrm{sign}(x)\lambda & |x| \geq \lambda \\ 0 & |x| < \lambda. \end{cases}$$



Preconditioning

[Donoho '95, Candes '01, Mallat '97, Neelamani '03, Daubechies '05, H & M '04-'05]

Compose modeling operator and model with

$$\mathbf{K}\cdot \mapsto \mathbf{F}\cdot \triangleq \mathbf{K}\left(-\Delta\right)^{-1/2}\mathbf{C}^T\mathbf{\Gamma}^{-1}\cdot$$

and $\mathrm{m} \mapsto \Gamma\mathrm{C}\mathrm{m}$ **such that approximately**

$$\boxed{\langle \mathbf{Ff}, \mathbf{Fg}\rangle \approx \langle \mathbf{f}, \mathbf{g}\rangle \quad \text{or} \quad \mathbf{F}^T\mathbf{F}\cdot \approx \mathbf{Id}\cdot}$$

Also use $\mathrm{E}\{\varepsilon\varepsilon^T\} \approx \mathbf{Id} \quad \text{with} \quad \boldsymbol{\varepsilon} = \mathbf{F}^T\mathbf{n},\, \mathbf{n} \in N(0,1)$

$$\boxed{\begin{array}{rcl} \mathbf{F}^T\mathbf{d} & = & \mathbf{F}^T\mathbf{F}\mathbf{x} + \mathbf{F}^T\mathbf{n} \\ \mathbf{y} & = & \underbrace{\mathbf{N}}_{\approx\mathbf{I}}\mathbf{x} + \boldsymbol{\epsilon} \end{array}}$$

Sparseness & continuity constrained imaging and inversion

Two different optimization strategies

I divide-and-conquer first threshold then

$$\hat{\mathbf{m}} = \arg\min_{m} J_a(\mathbf{m}) \quad \text{s.t.} \quad |\mathbf{\Gamma C m} - \hat{\mathbf{x}}|_\mu \leq \mathbf{e}_\mu$$

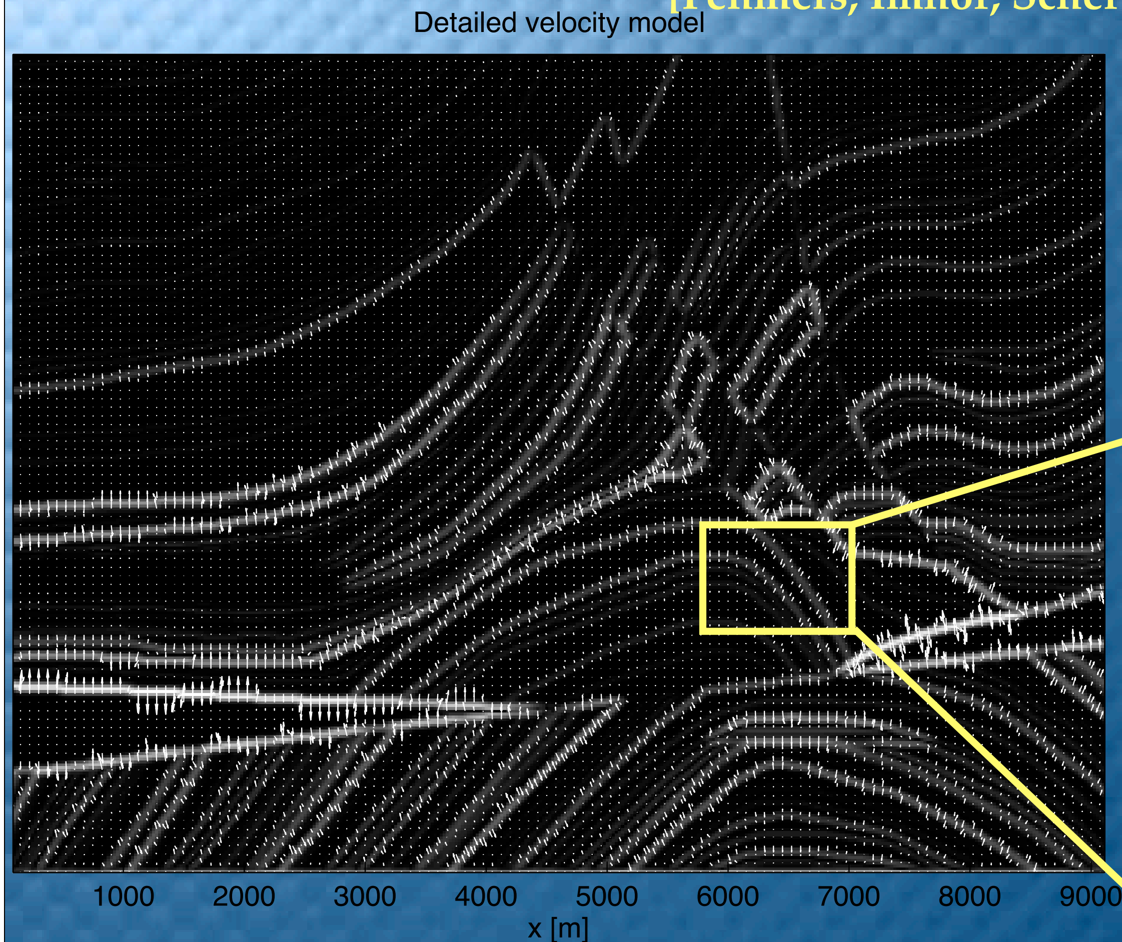
II jointly minimize sparsness & continuity

$$\hat{\mathbf{m}} = \arg\min_{m} J(\mathbf{m}) \quad \text{s.t.} \quad \|\mathbf{d} - \mathbf{K m}\|_2 \leq N\sigma_n$$

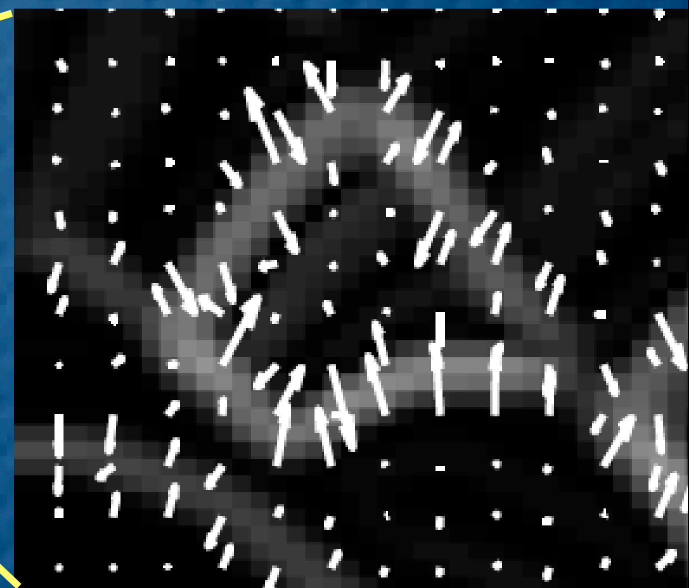
Quasi-Newton: fast convergence.

Anisotropic diffusion

[Fehmers, Imhof, Schertzer '03]

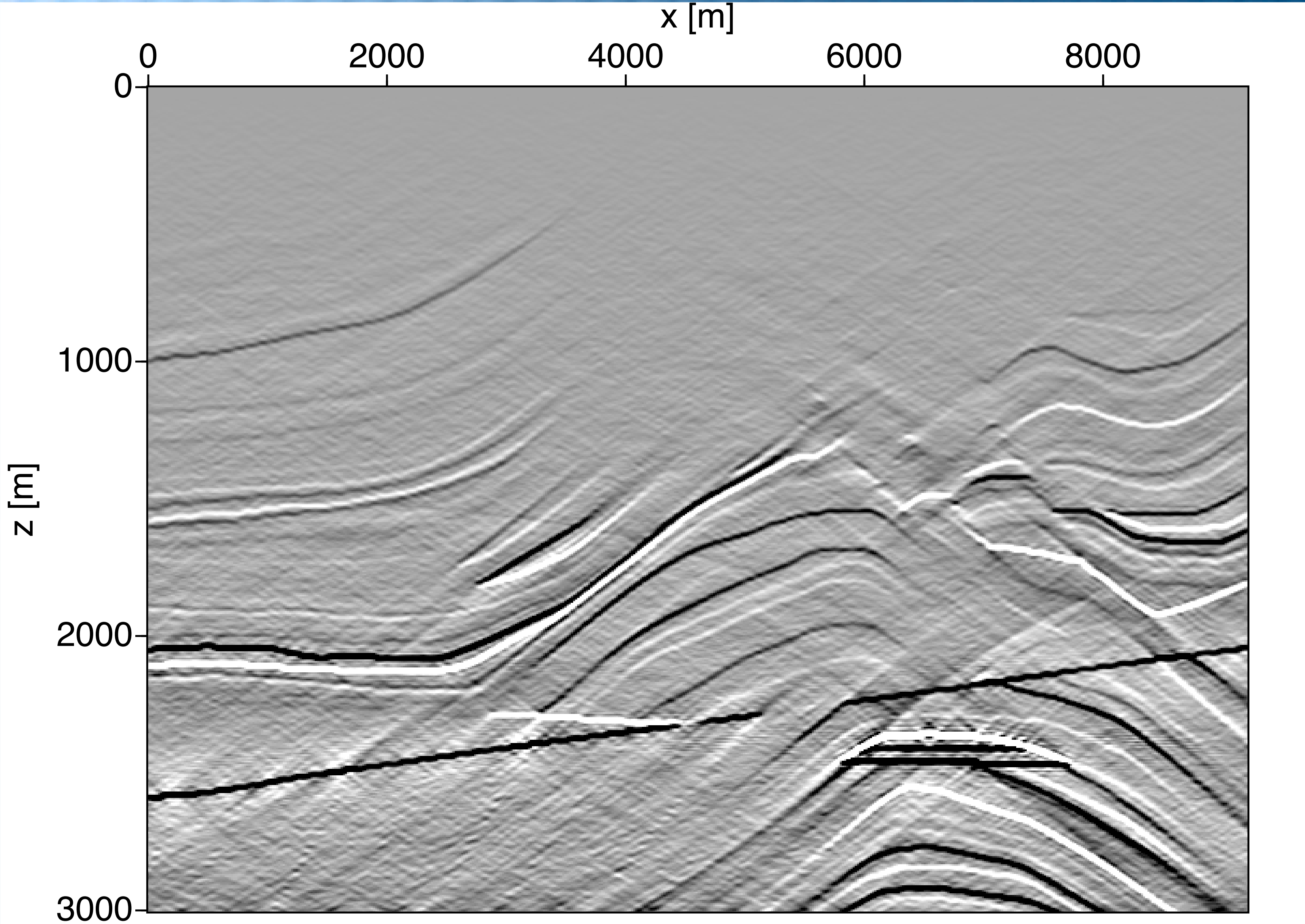


$$J_a(\mathbf{m}) = \|\mathbf{\Lambda}^{\frac{1}{2}}\nabla\mathbf{m}\|_2$$

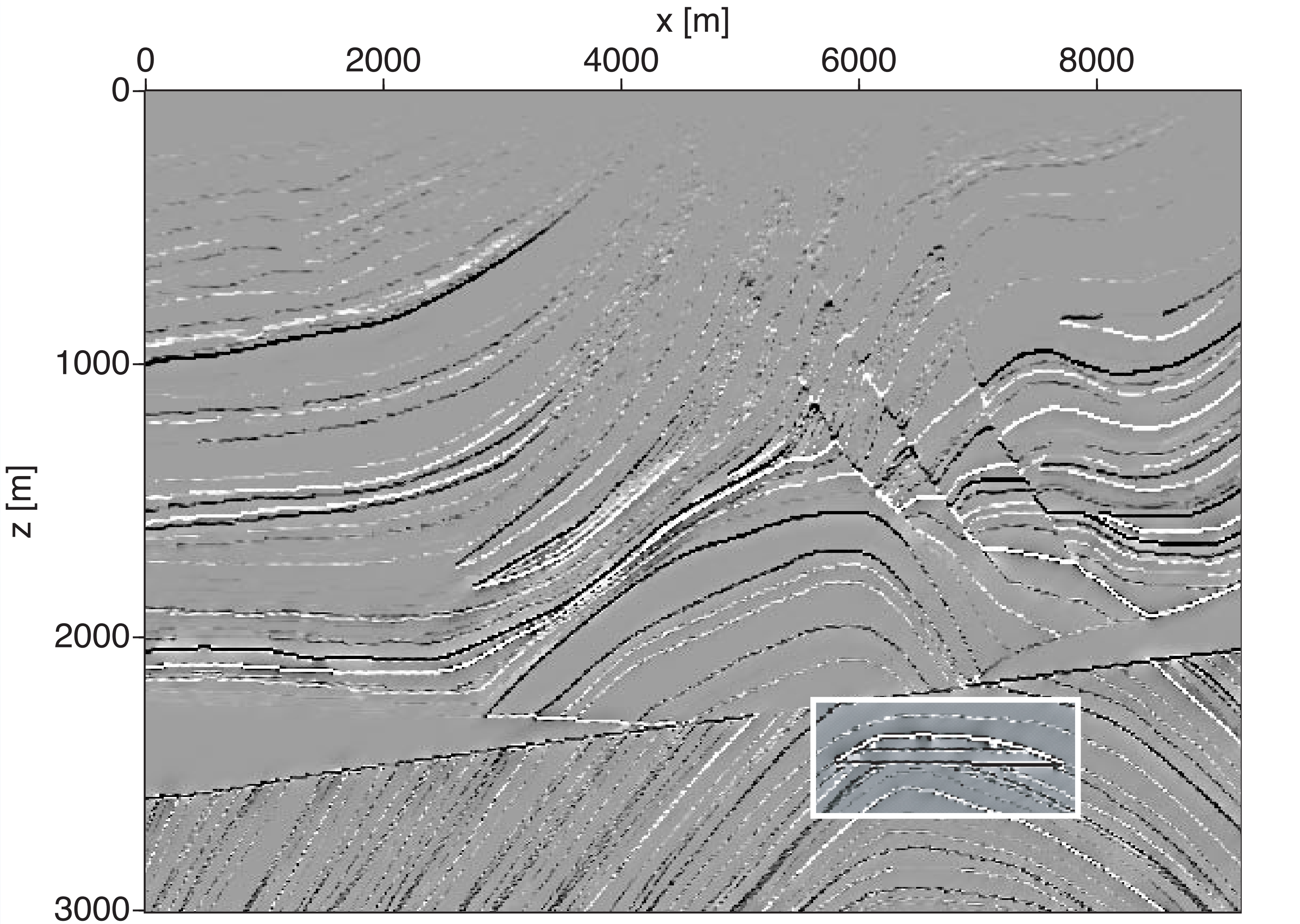


Penalize along and in between reflectors!

Non-linear imaging



Non-linear imaging



Sparsity- & continuity-enhanced imaging

Results were obtained for $SNR = 0\,\mathrm{dB}$

Sparseness & continuity constraints improved the results

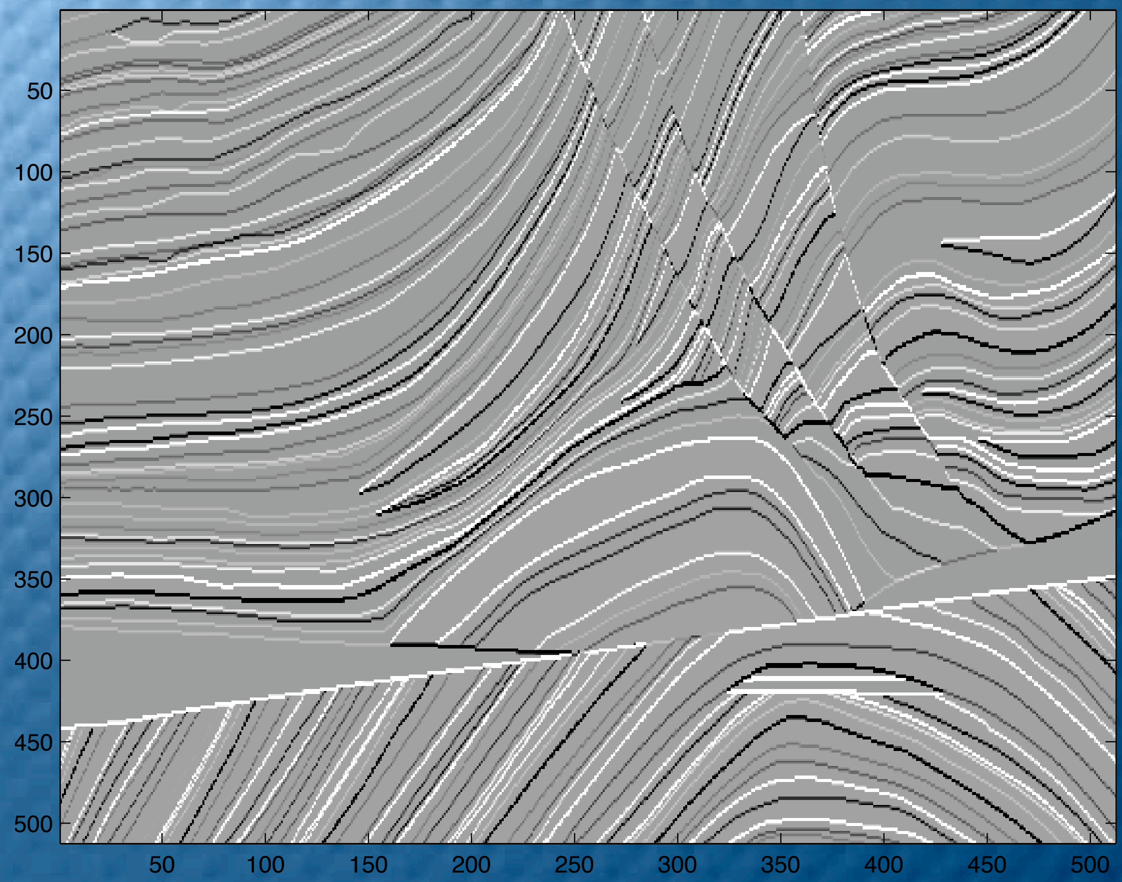
Non-trivial even for noise-free case

- **bad illumination**
- **null space modeling operator**

Apply strategy II: *joint* minimization of both constraints.

Sparsity- & continuity-enhanced imaging

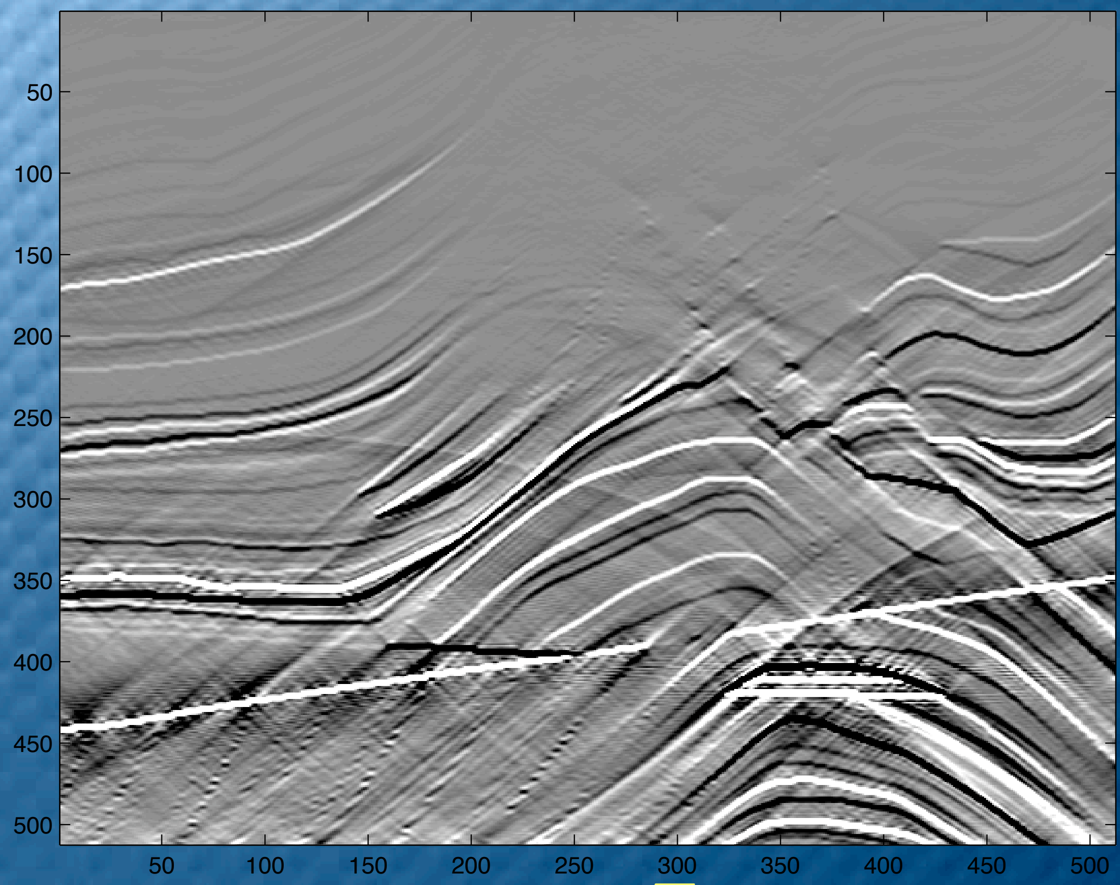
Marmoussi Model



$$\hat{\mathbf{m}} = \mathbf{m}$$

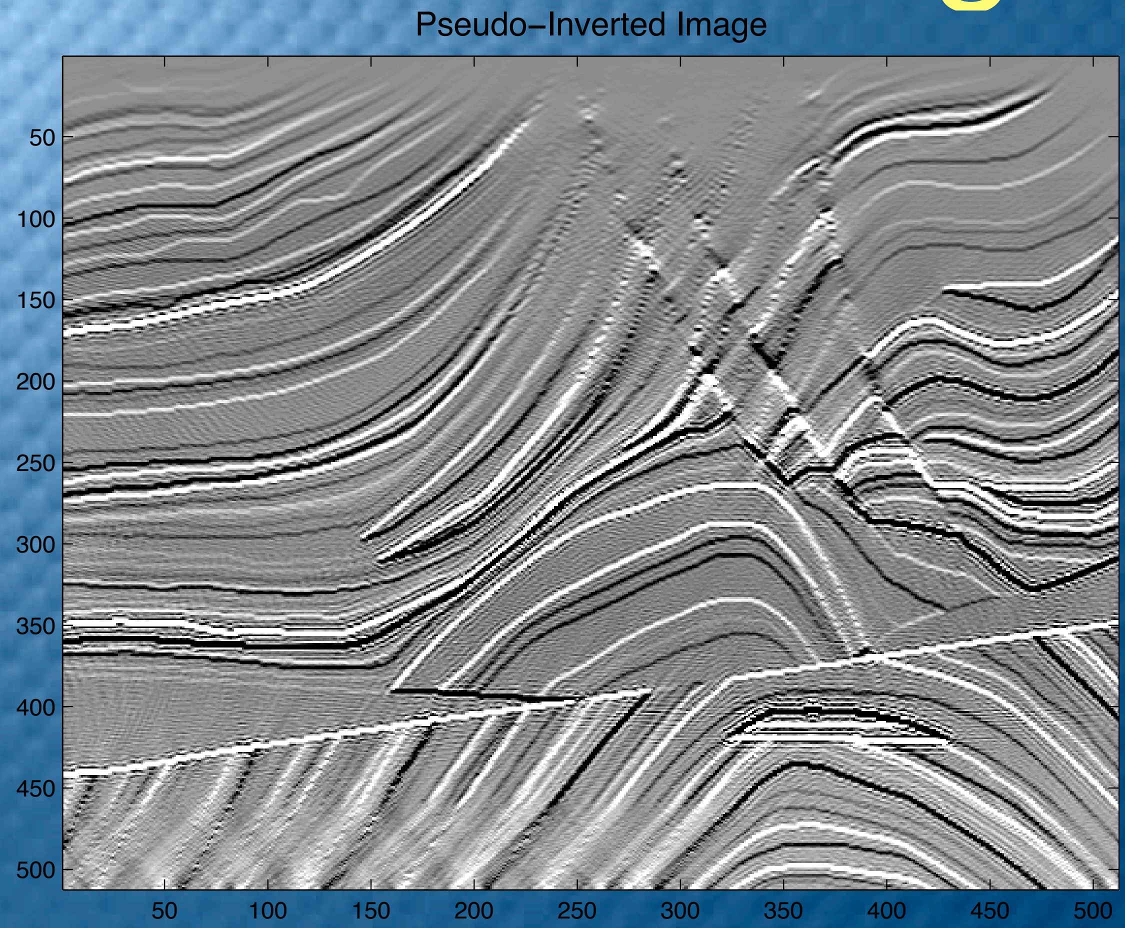
Sparsity- & continuity-enhanced imaging

Migrated Noise Free Image



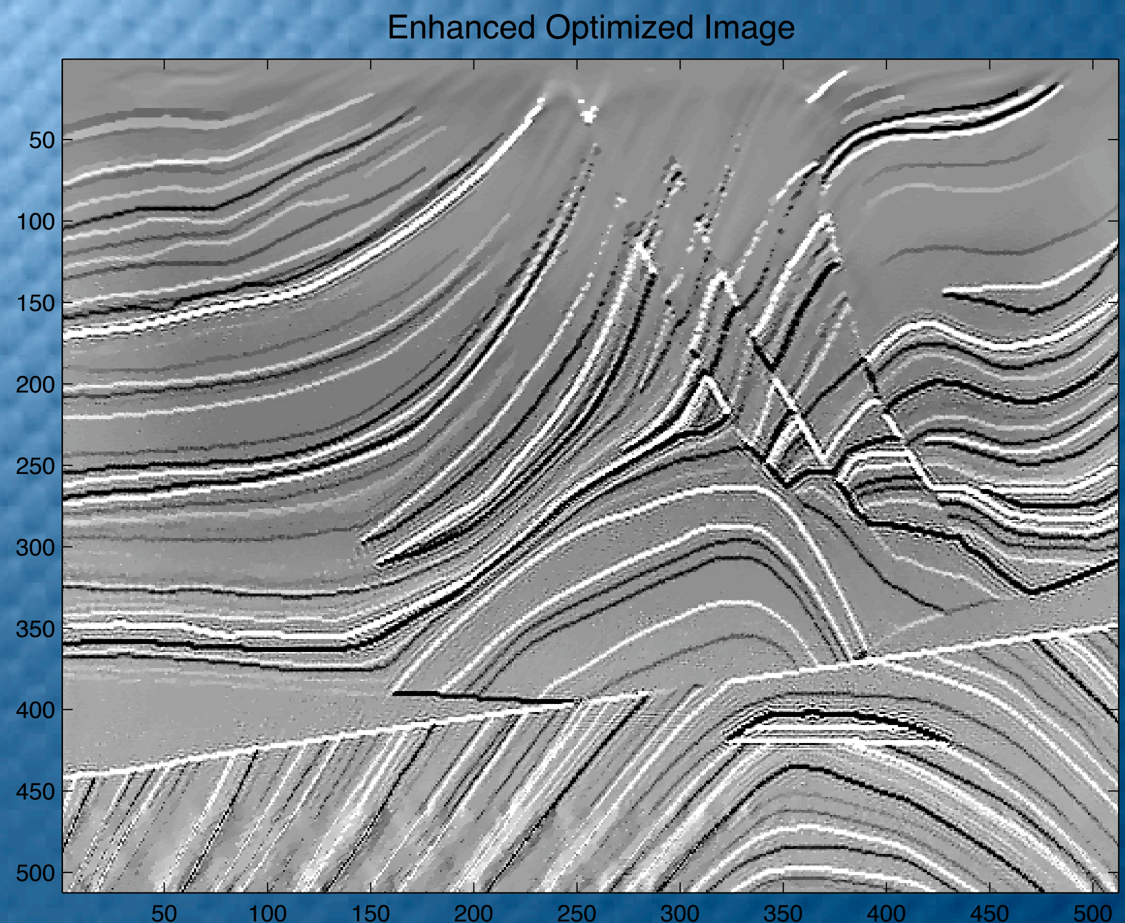
$$\hat{\mathbf{m}} = \mathbf{K}^T\mathbf{d}$$

Sparsity- & continuity-enhanced imaging



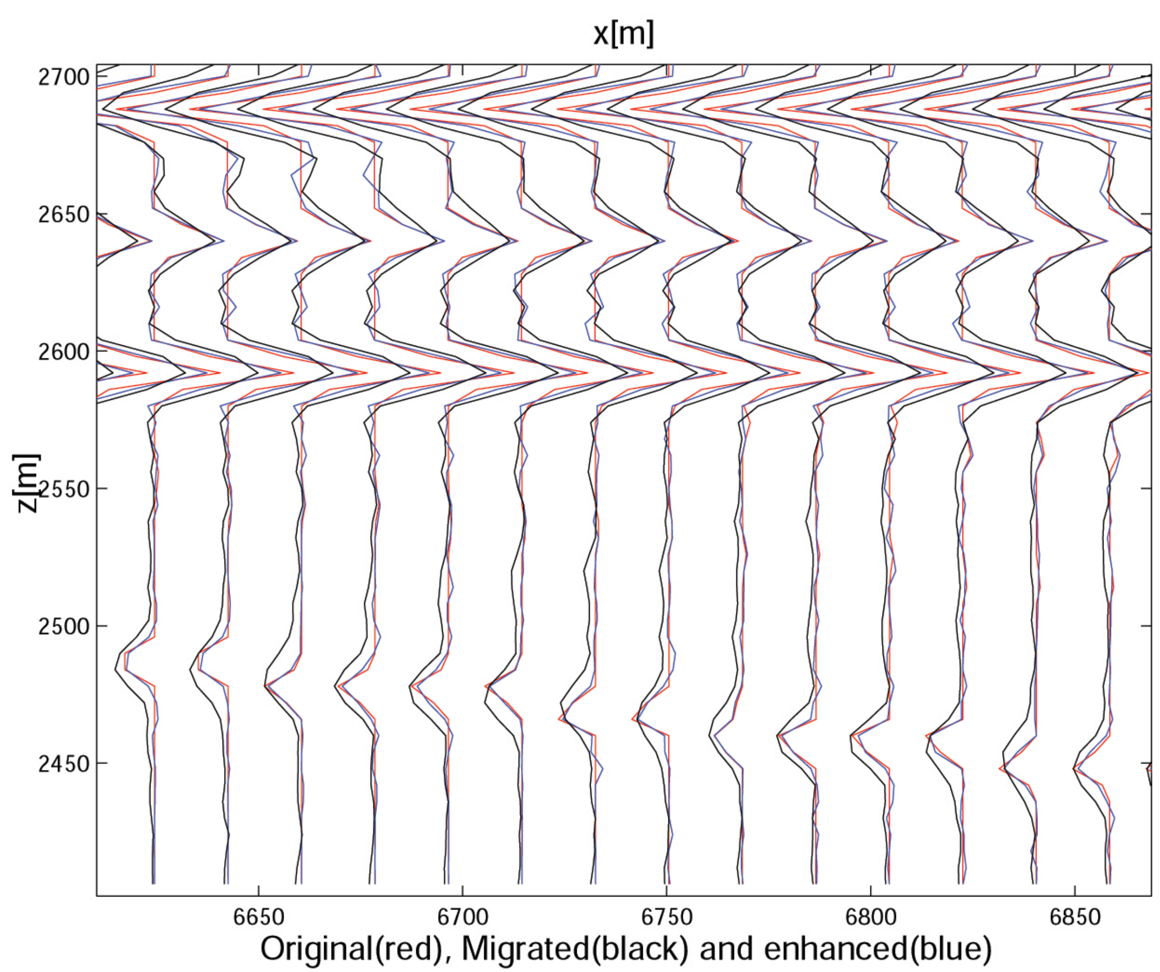
$$\hat{\mathbf{m}} = \mathbf{K}^{\dagger}\mathbf{d}$$

Sparsity- & continuity-enhanced imaging



$$\hat{\mathbf{m}} = \arg\min_{m} J(\mathbf{m}) \quad \text{s.t.} \quad \|\mathbf{d} - \mathbf{K}\mathbf{m}\|_2 \leq N\sigma_n$$

Sparsity- & continuity-enhanced imaging



Acknowledgments

Authors of CurveLab [Candes, Donoho, Demanet and Ying '05]

SINBAD sponsored through ITF by British Gas, British Petroleum, ExxonMobil and Shell

NSERC (Discovery grant 22R81254)