SLIM

Seismic Laboratory for
Imaging and Modeling

Non-Linear Data COntinuation with Redundant Frames

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CONTEXT

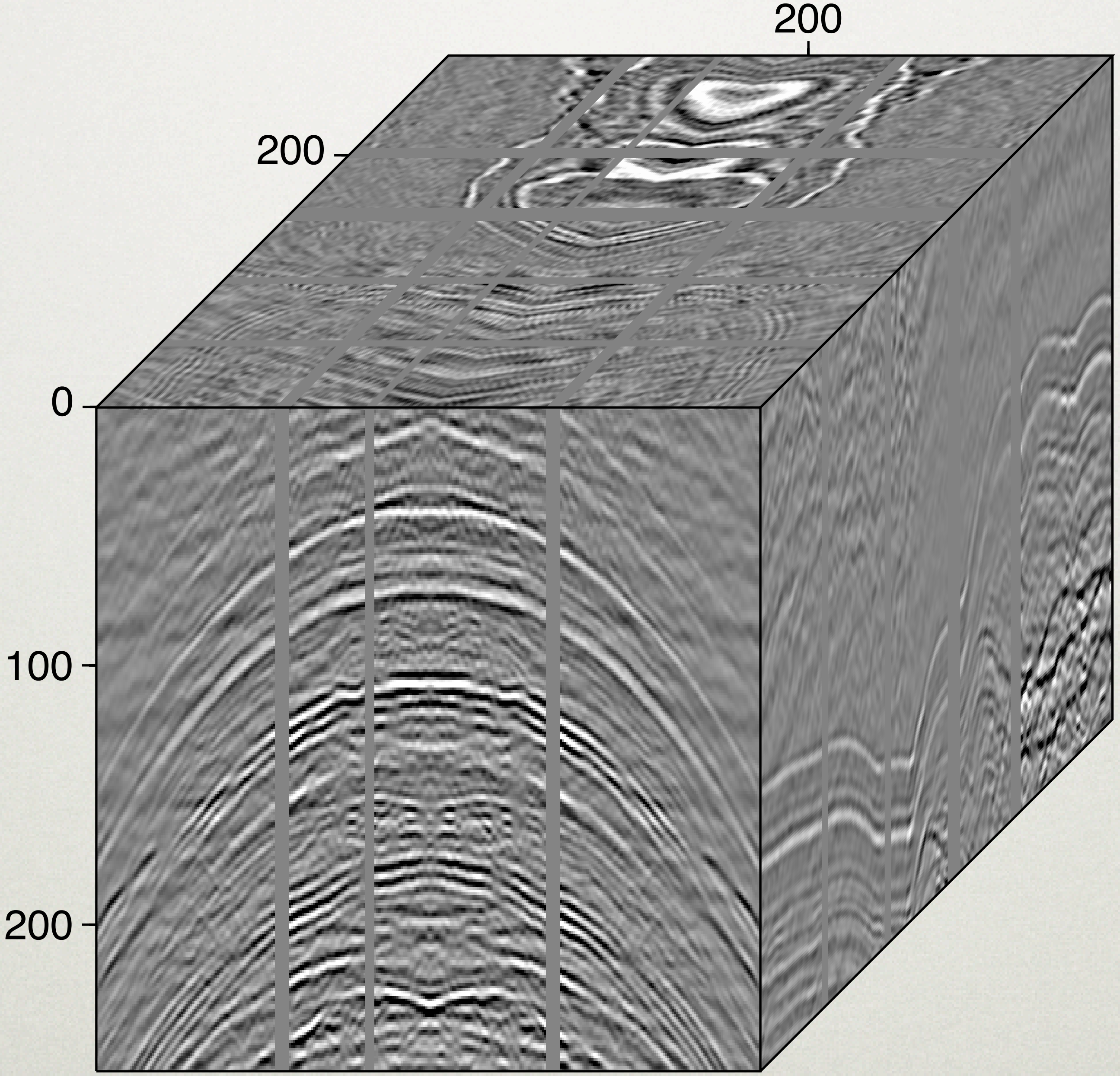
- Linear least-squares data continuation [Claerbout, 92]
- Discrete & unequally sampled Fourier Transforms [Sacchi, 96 ; Schoneville, 01; Zwartjes, 04]
- Inpainting with Morphological Component Analysis using Redundant Directional Frames such as Curvelets [Candes; Donoho; Demanet; Ying, 05; Elad, 05]
- Model- (migration or Radon) based data continuation [Trad, 03]

OUR MOTIVATION

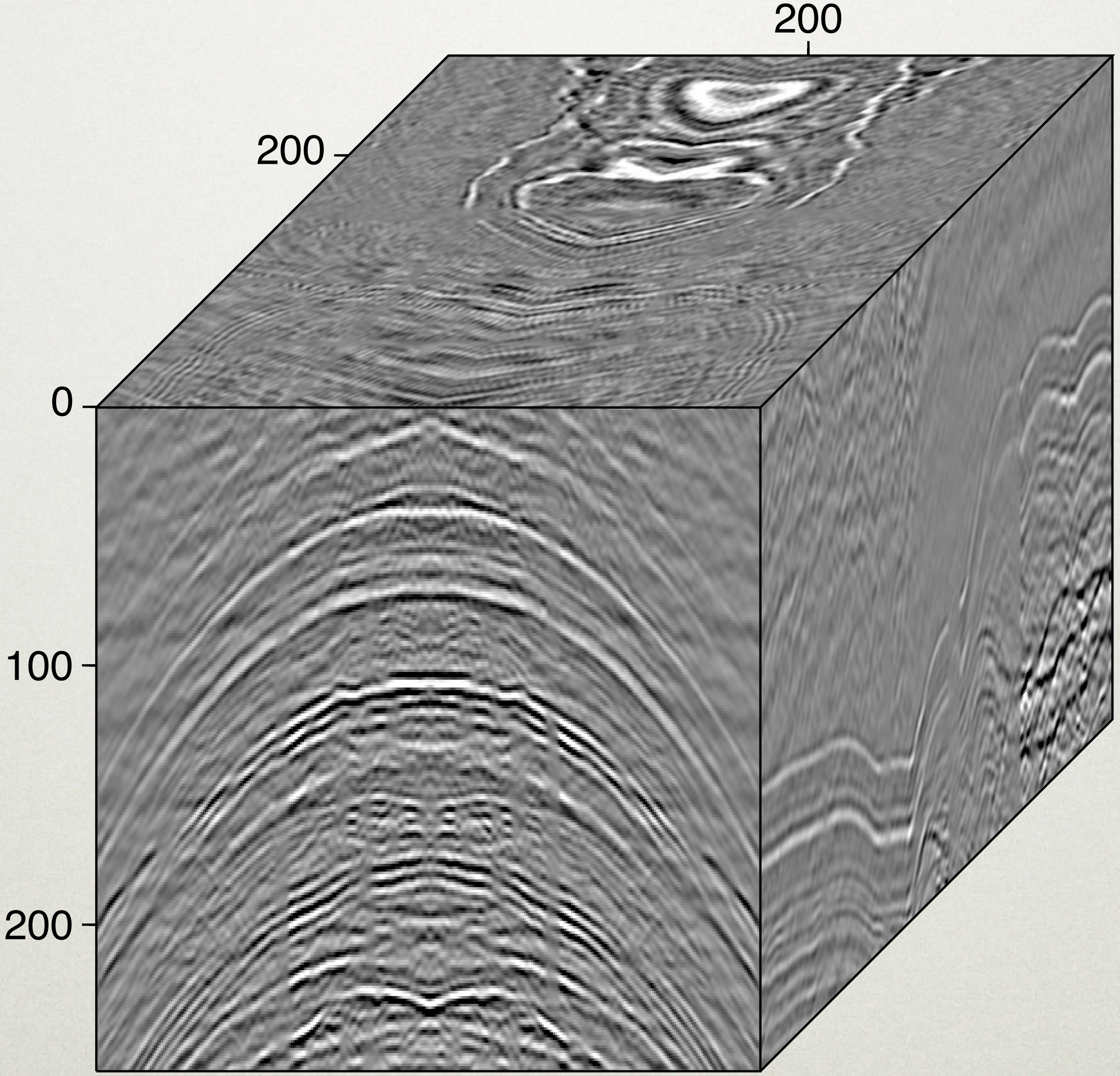
Devise a data continuation & de-aliasing scheme that

- is non-parametric/non-adaptive
- truly exploits the 3-D continuity along wavefront in $\mathbf{d}(r, s, t)$
- is noise resilient
- exploits redundant frames
- is $n \log n$

3-D REAL DATA



3-D REAL INTERPOLATED RESULT



Seismic Interpolation

Forward problem:

regular data

modeling/frame operator

$$\mathbf{d}_{acq} = \mathbf{P}\mathbf{d} \qquad \text{with} \qquad \mathbf{d} = \mathbf{F}\mathbf{x} + \mathbf{n}$$

white noise

picking operator

model

Conventional interpolation problem:

$$\hat{\mathbf{x}} = \arg\min_{\mathbf{x}} \frac{1}{2}\underbrace{\|\mathbf{P}(\mathbf{d} - \mathbf{F}\mathbf{x})\|_2^2}_{\textit{misfit data}} + \underbrace{\lambda J(\mathbf{x})}_{\textit{regularization}}$$

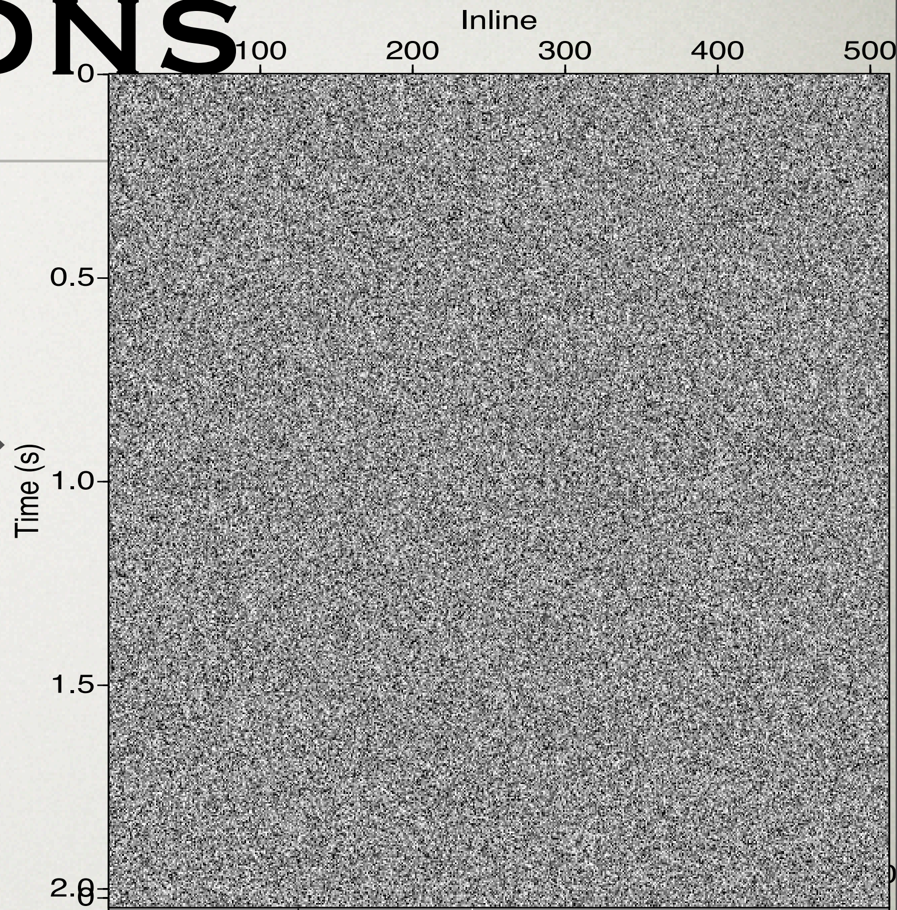
[Claerbout, 92; Sacchi, 96 ; Trad, 03; Elad, 05]

REGULARIZATIONS

Linear quadratic (lsqr migration):

$$\hat{\mathbf{x}} = \arg\min_{\mathbf{x}} \frac{1}{2}\|\mathbf{P}(\mathbf{d} - \mathbf{F}\mathbf{x})\|_2^2 + \lambda\|x\|_2^2$$

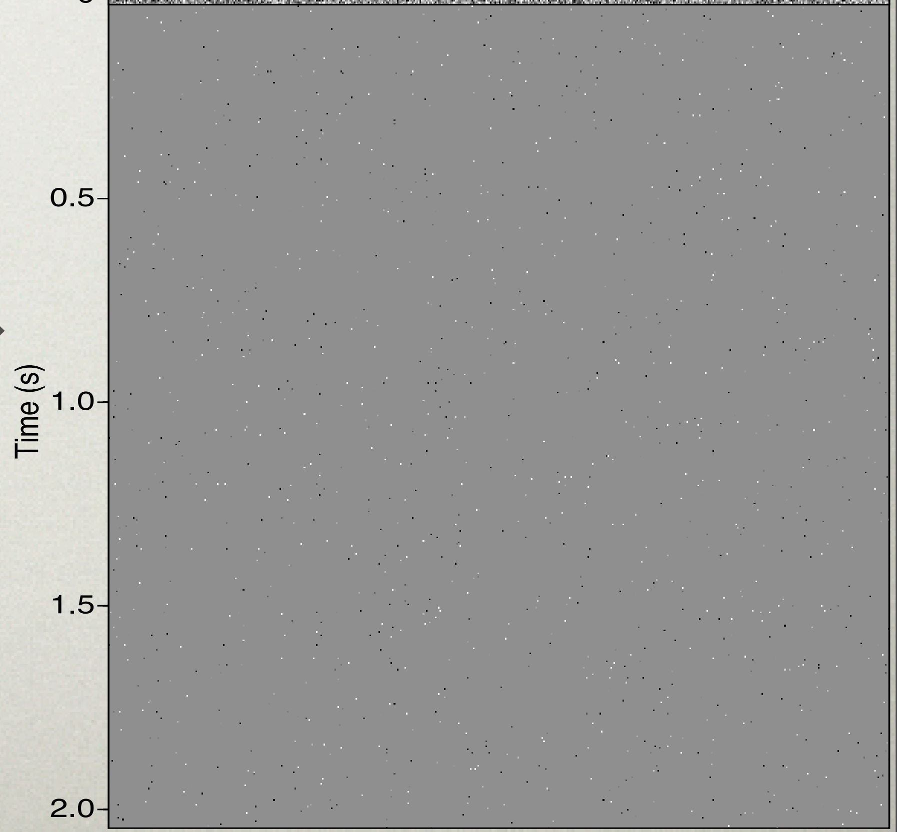
- **data white Gaussian**
- **uncorrelated**



Non-linear ℓ^1:

$$\hat{\mathbf{x}} = \arg\min_{\mathbf{x}} \frac{1}{2}\|\mathbf{P}(\mathbf{d} - \mathbf{F}\mathbf{x})\|_2^2 + \lambda\|x\|_1$$

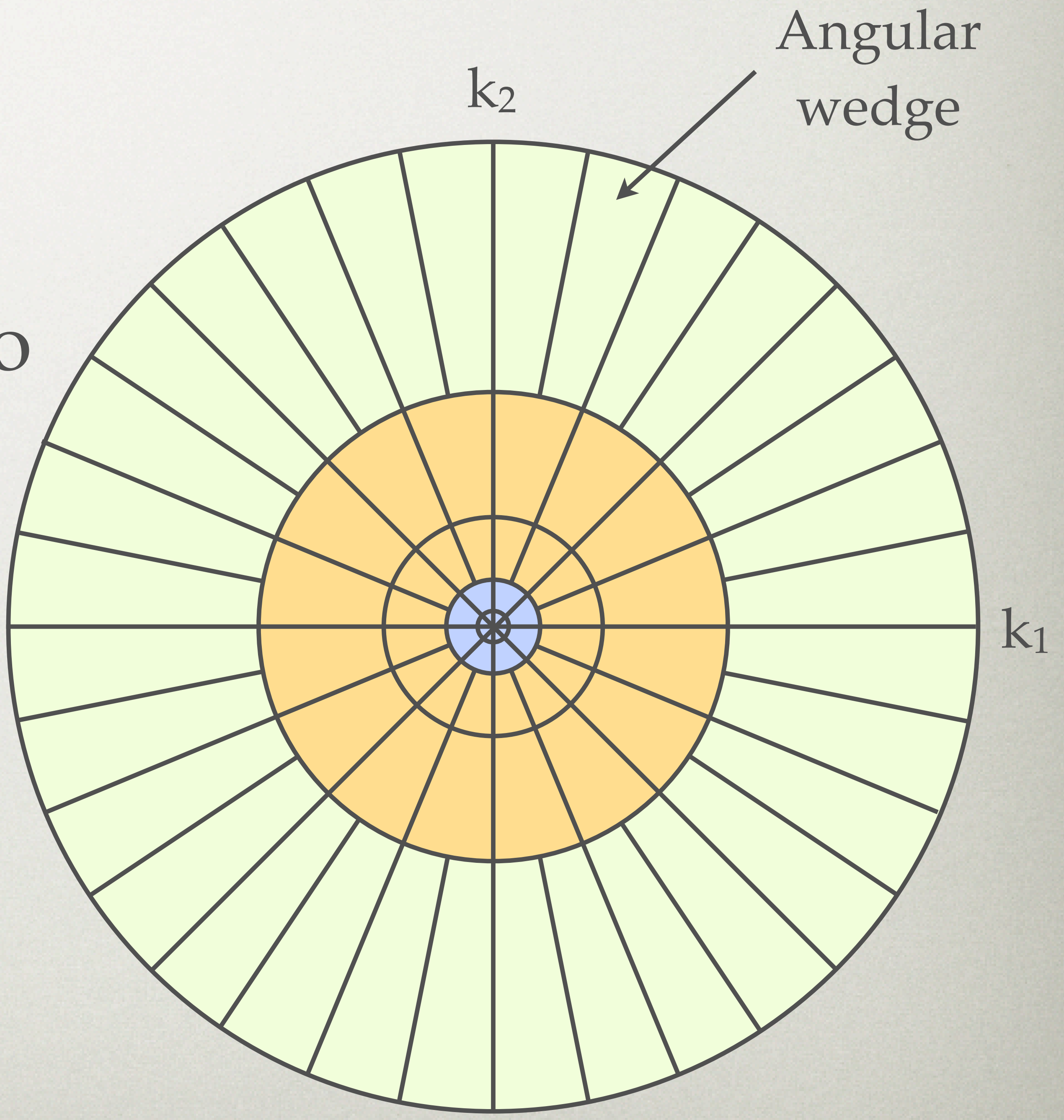
- **data white Cauchy**
- **uncorrelated**



CURVELETS

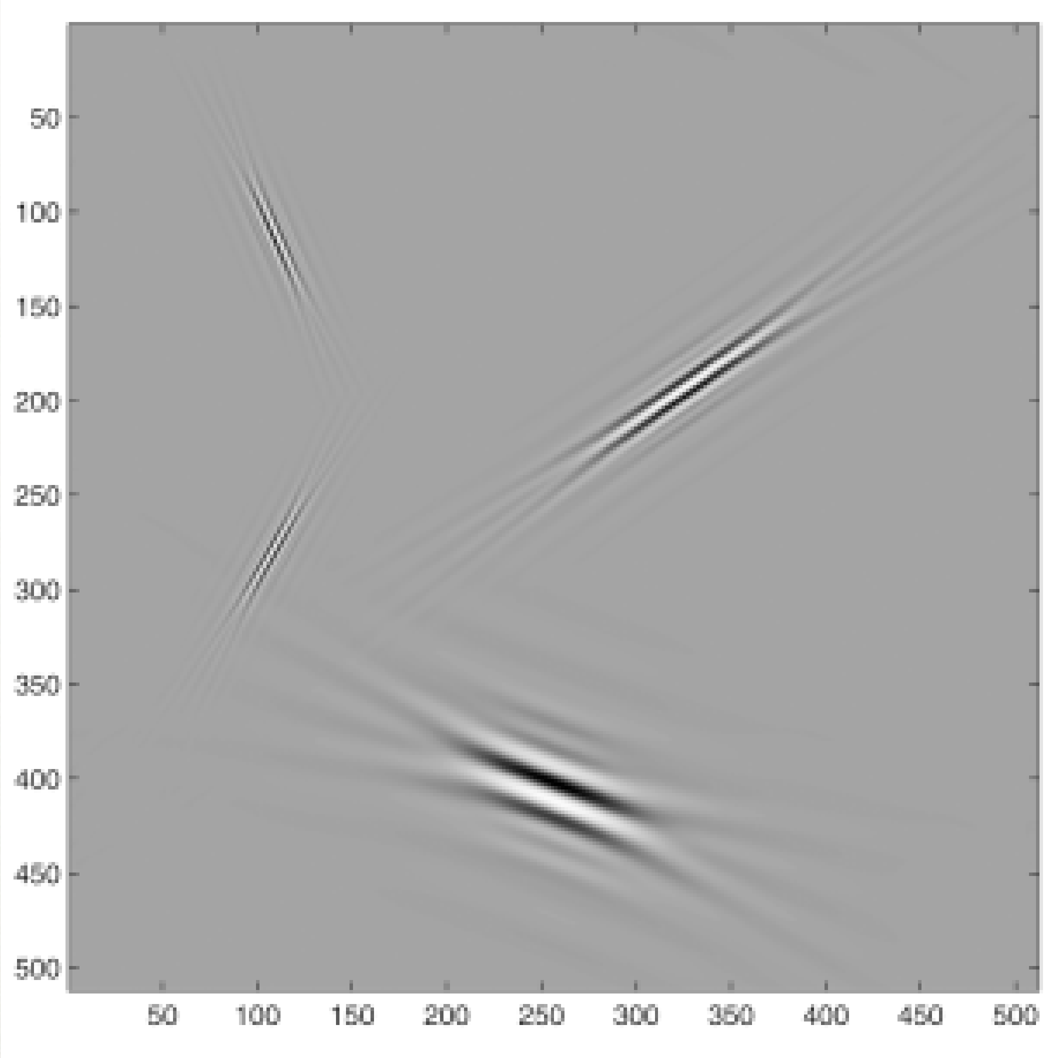
[Candes & Donoho, 99; Demanet; Ying, 05]

- Tight frames
- Partitioning of the 2-D/ 3-D Fourier domain into angular wedges of second dyadic coronae
- Parabolic scaling law



CURVELET PROPERTIES

- tight frames (n log n)
- multi-scale
- multi-directional
- highly anisotropic
- localized both in space & frequency
- moderate redundancy



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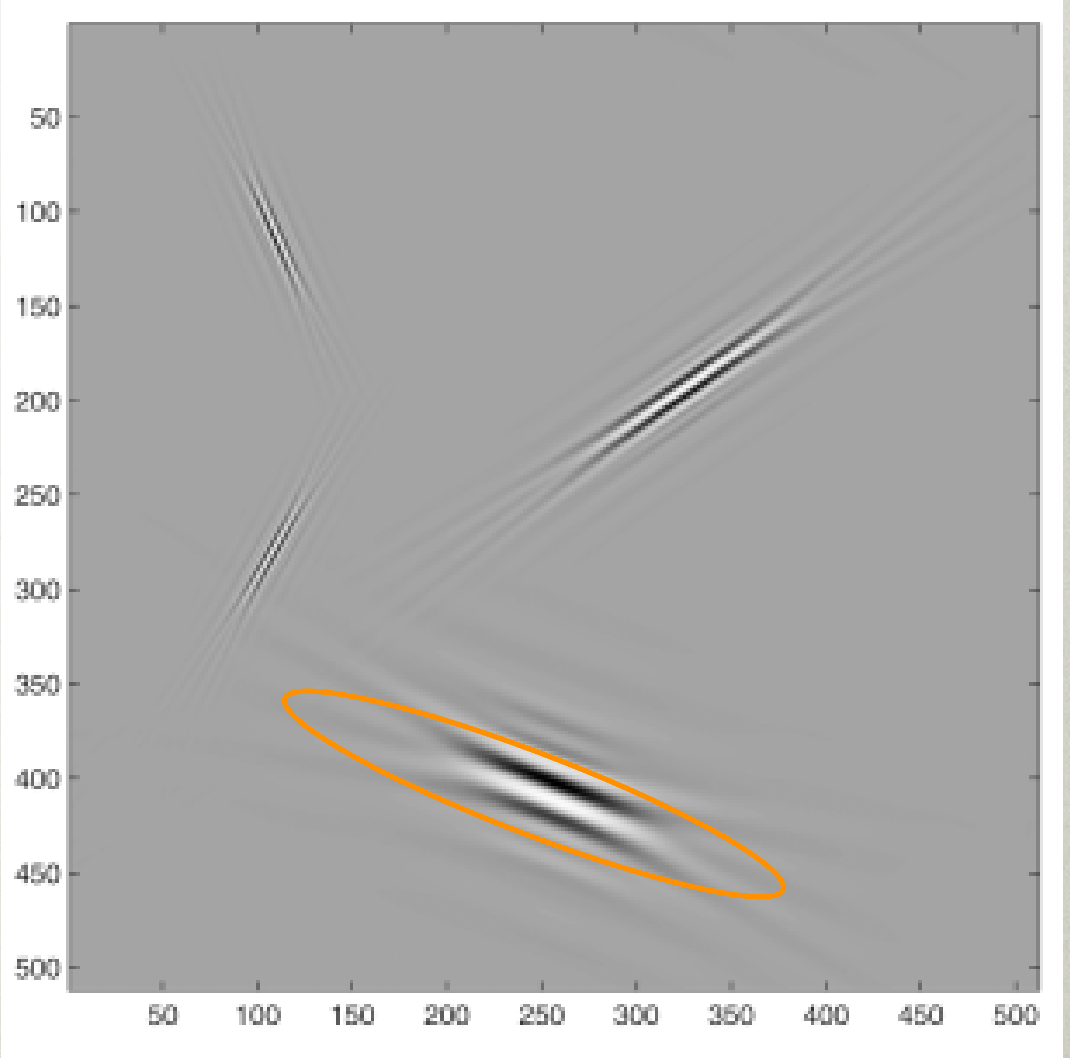

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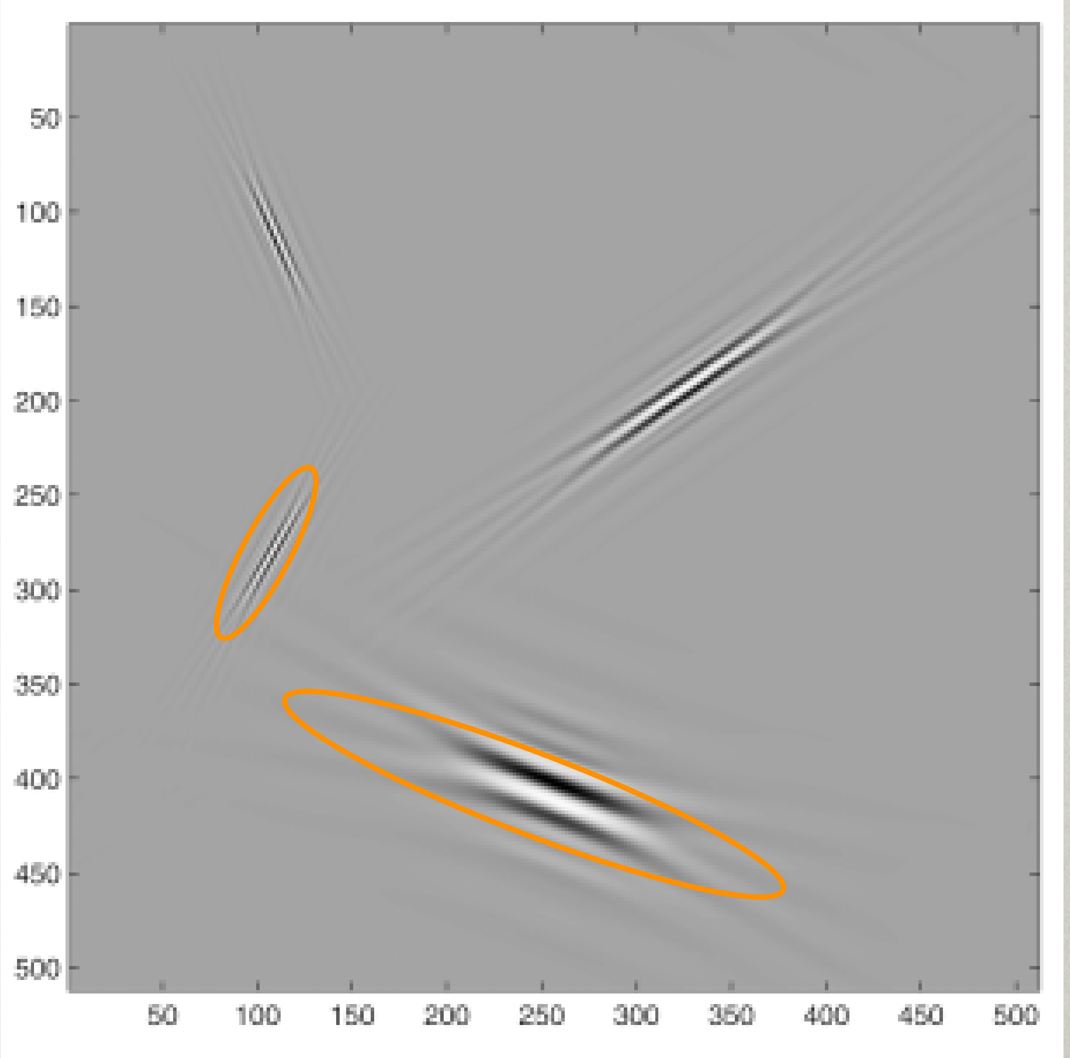

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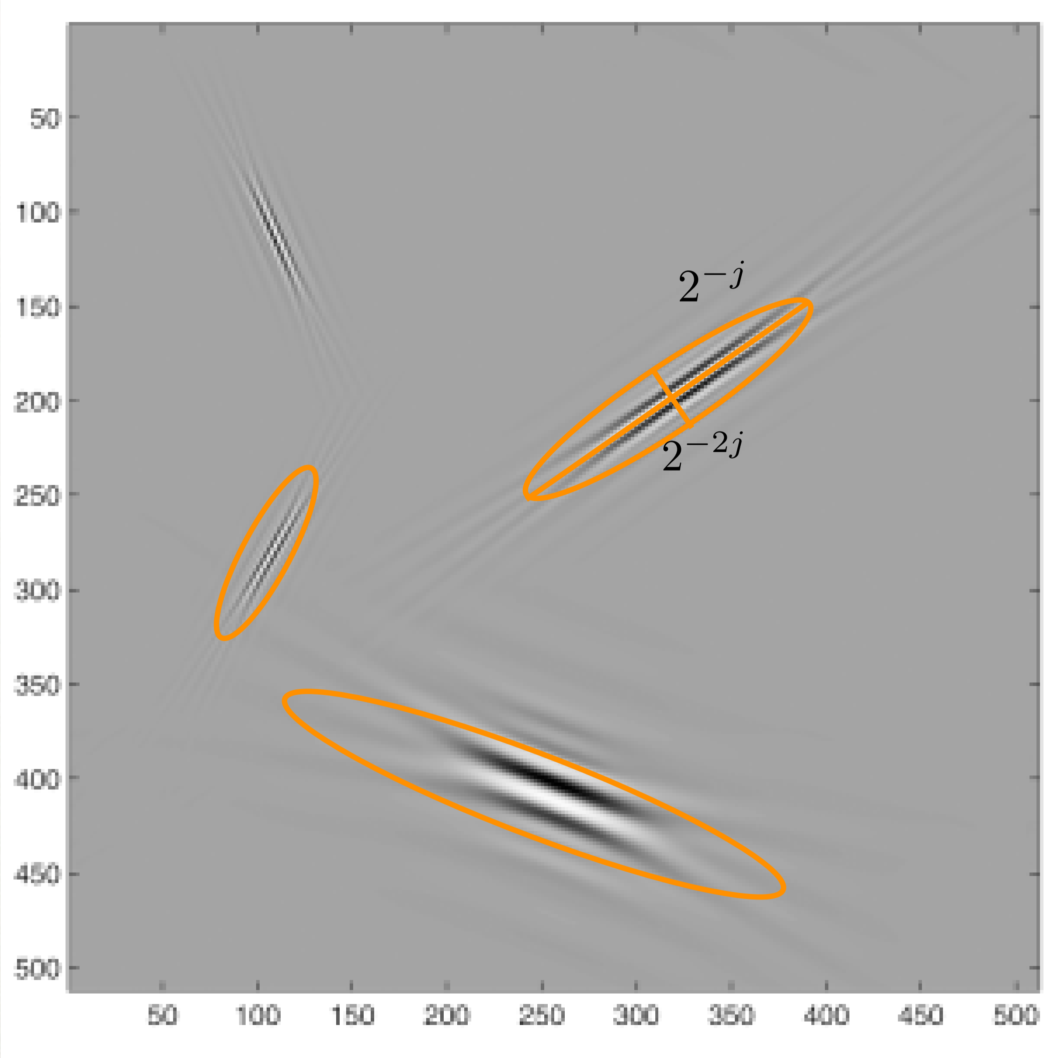
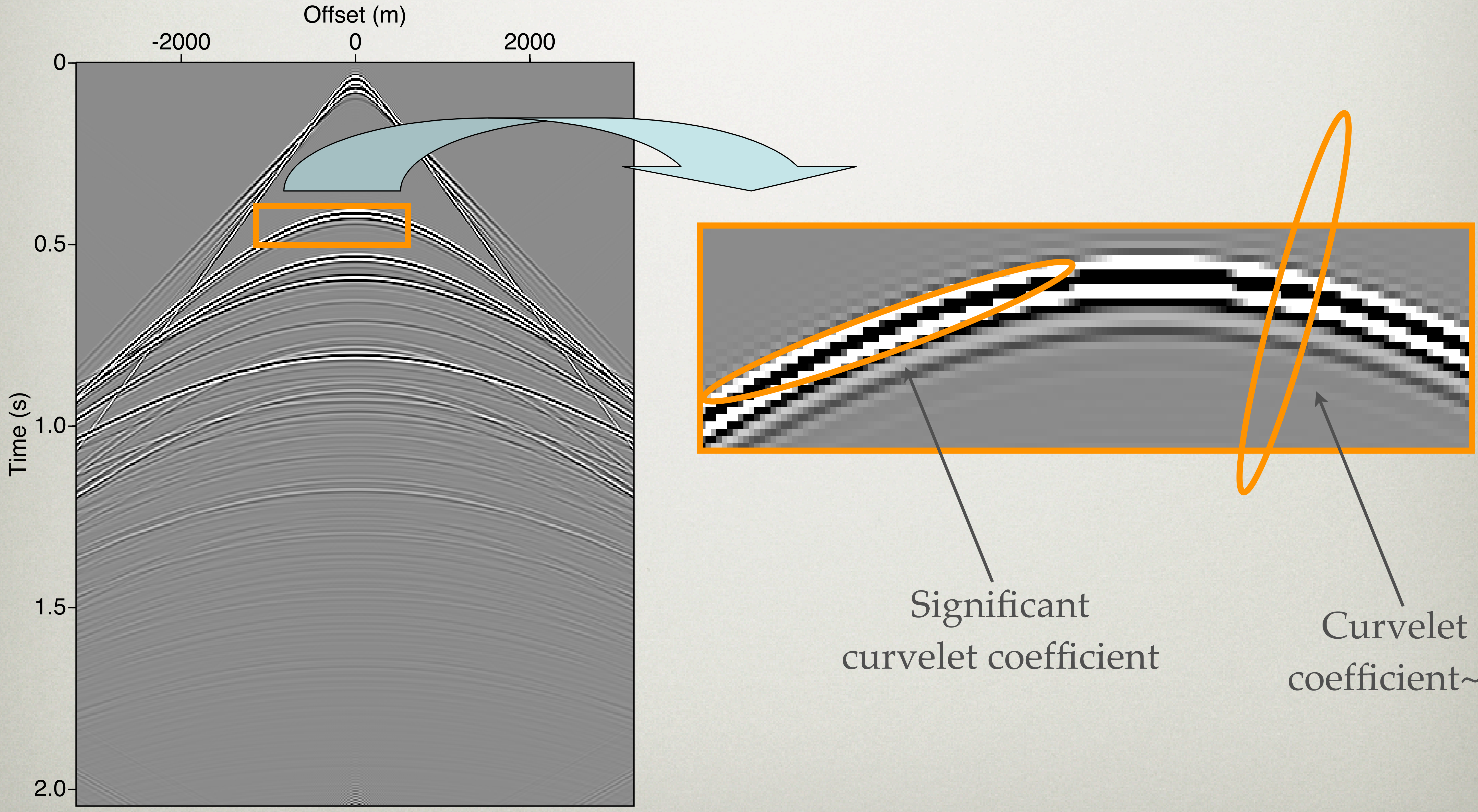


CURVELETS & SEISMIC DATA



Significant curvelet coefficient

Curvelet coefficient~0

CURVELET NON-LINEAR APPROXIMATION RATE

Optimal: [Donoho, 01]

$$\|\mathbf{f} - \mathbf{f}_p^O\|_2^2 \propto p^{-2}, \quad p \rightarrow \infty$$

Fourier:

$$\|\mathbf{f} - \mathbf{f}_p^F\|_2^2 \propto p^{-1/2}, \quad p \rightarrow \infty$$

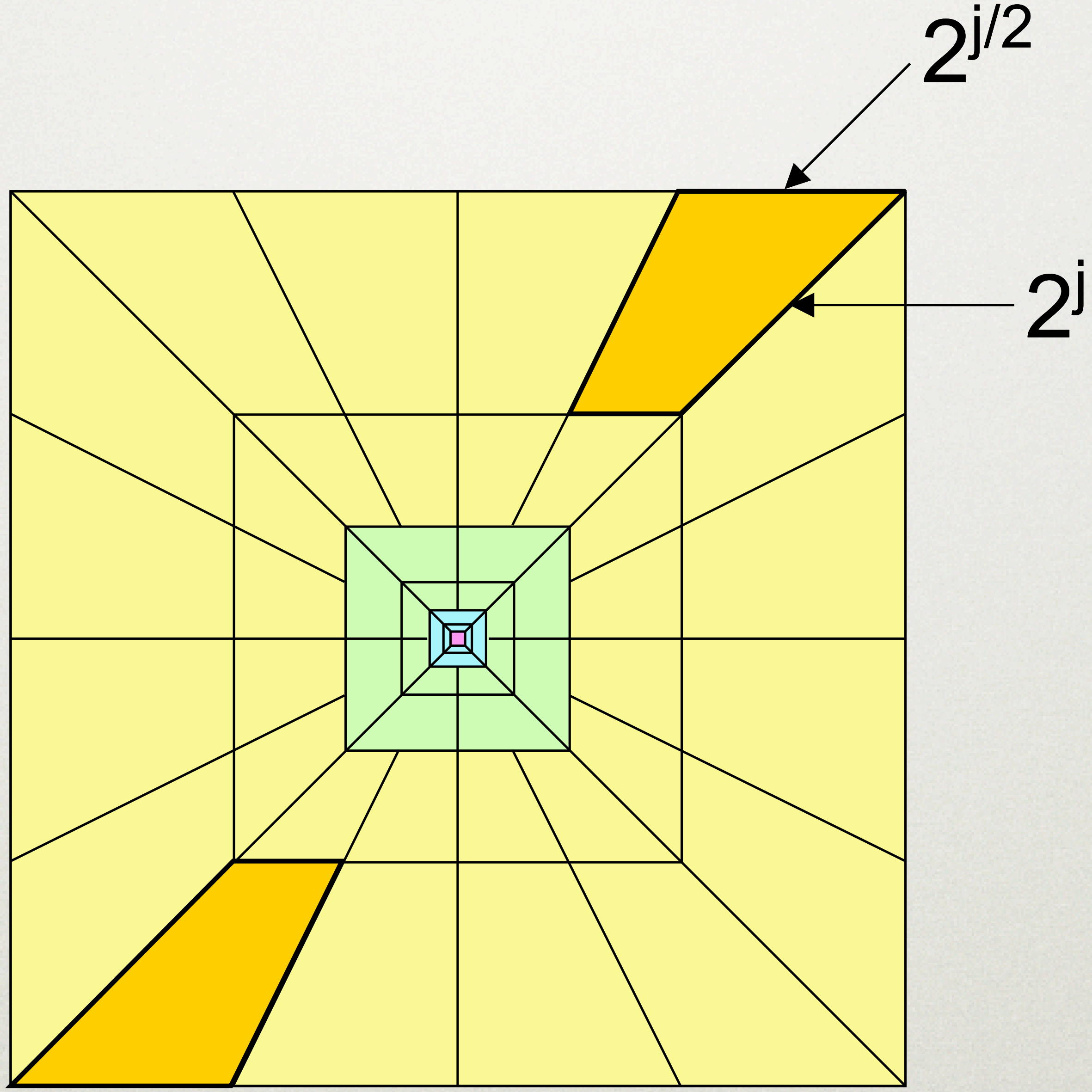
Wavelets:

$$\|\mathbf{f} - \mathbf{f}_p^W\|_2^2 \propto p^{-1}, \quad p \rightarrow \infty$$

Curvelets: [Candes & Donoho, 99]

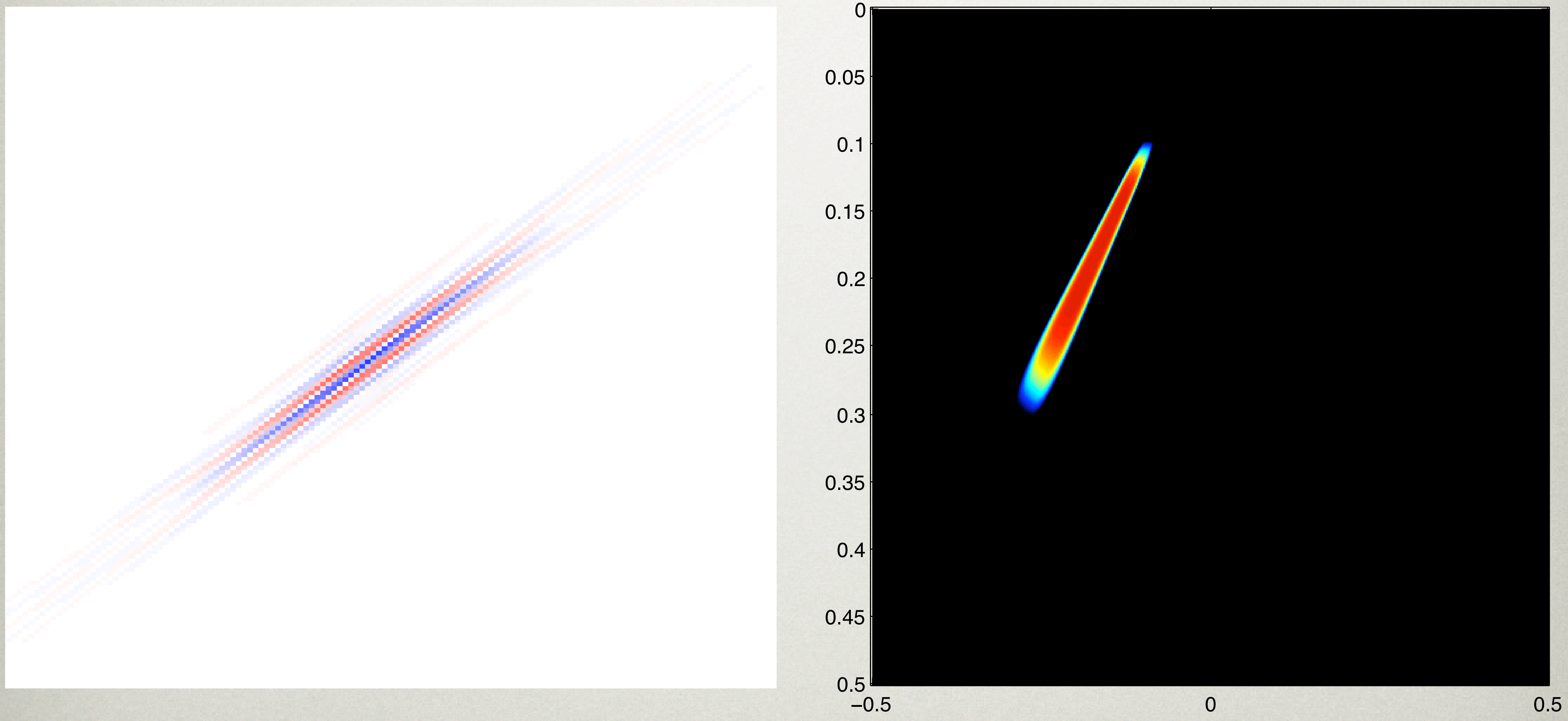
$$\|\mathbf{f} - \mathbf{f}_p^C\|_2^2 \leq C\ p^{-2}\ (\log p)^3, \quad p \rightarrow \infty$$

NUMERICAL CONSTRUCTION



Curvelets live in a wedge in the 2-3 D Fourier plane...

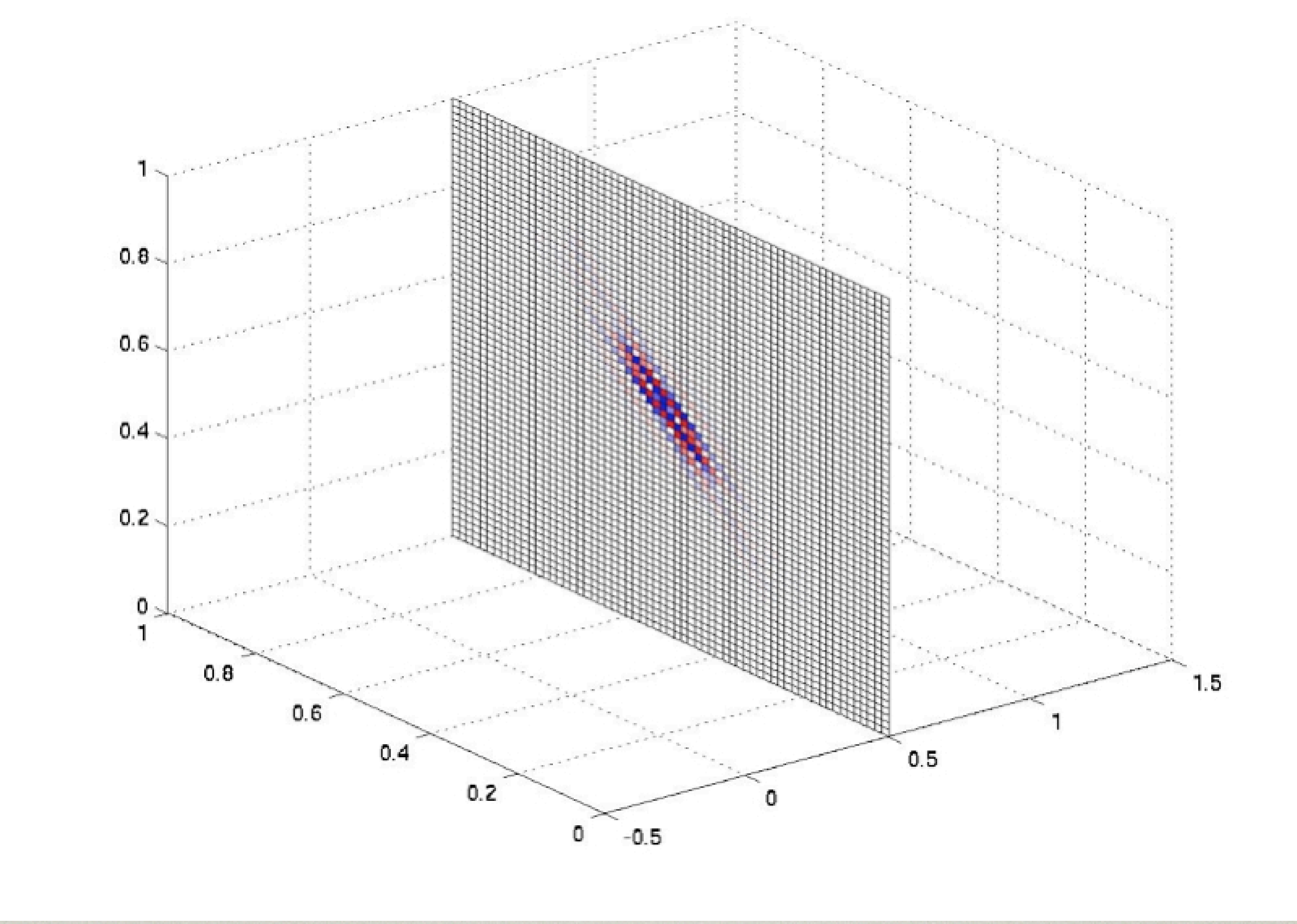
NUMERICAL CONSTRUCTION



localized in both domains

3-D CURVELETS

[Ying '05]



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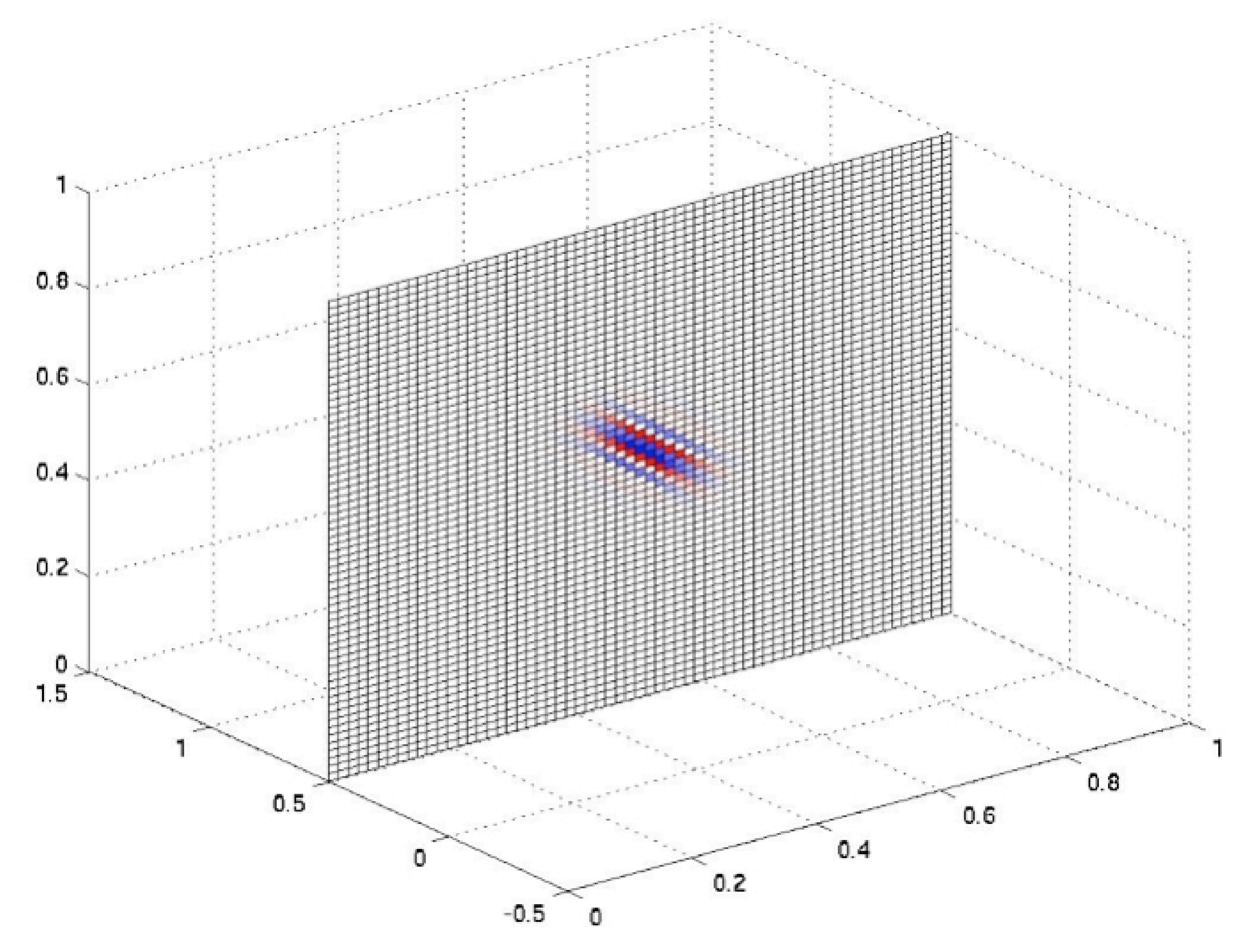

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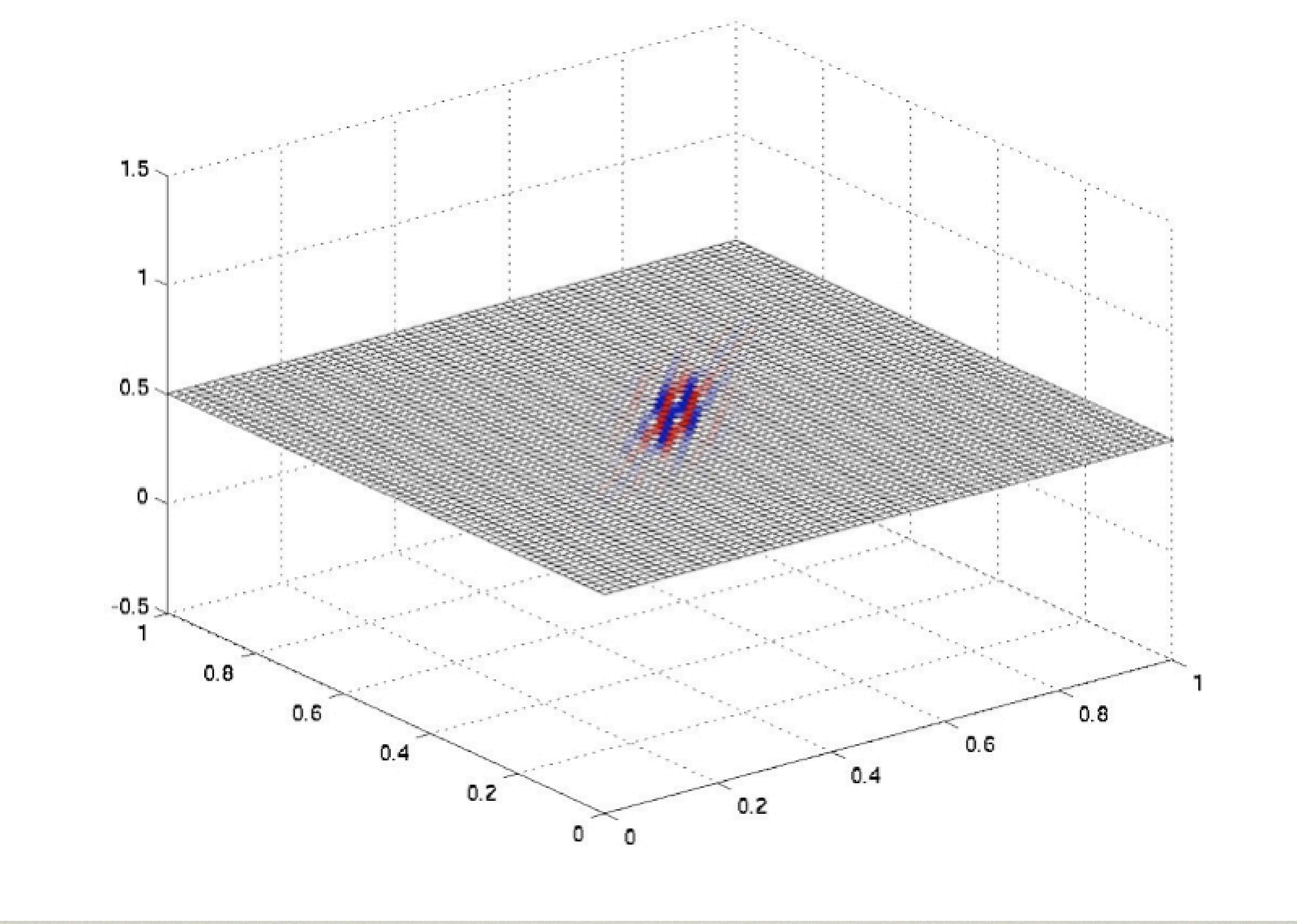


CURVELETS FOR SEISMIC DECONVOLUTION

Our forward problem:

'Modeling' operator

reg. data

$$\text{data} \longrightarrow \mathbf{d} = \mathbf{IdC}^{*}\mathbf{x} + \mathbf{n} \longleftarrow \text{noise} \qquad \& \qquad \mathbf{m} = \mathbf{C}^{*}\mathbf{x}$$

curvelet synthesis operator

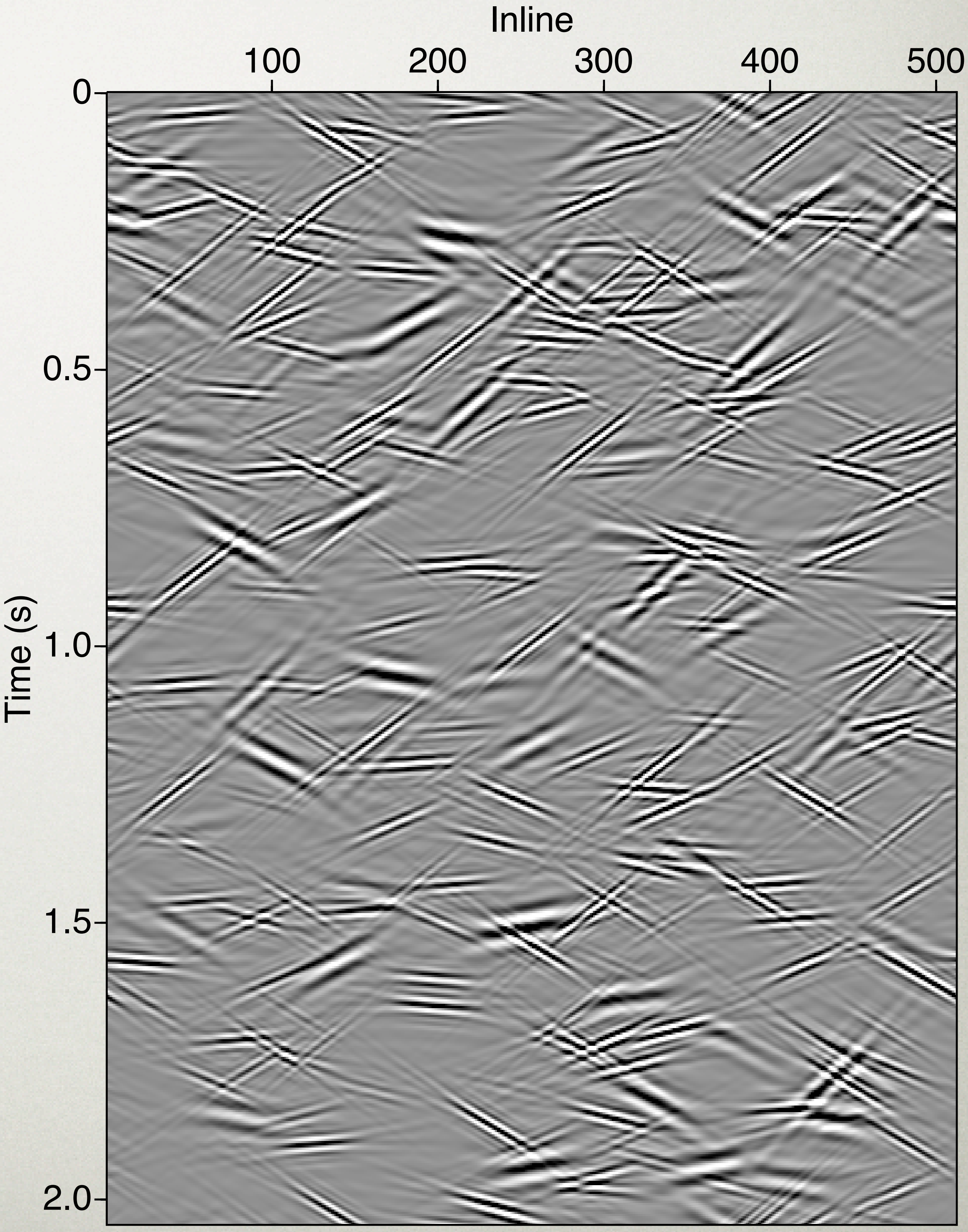
curvelet coefficients

Our interpolation problem:

$$\hat{\mathbf{x}} = \arg\min_{x} \frac{1}{2} \|\mathbf{P}\left(\mathbf{d} - \mathbf{F}\mathbf{x}\right)\|_2^2 + \|\mathbf{x}\|_1^1 \ \text{ with } \ \mathbf{F}\cdot = \mathbf{IdC}^{*}\cdot$$

$$\hat{\mathbf{x}} = \arg\min_{x} \frac{1}{2} \|\mathbf{P}\left(\mathbf{d} - \mathbf{F}\mathbf{x}\right)\|_2^2 + \|\mathbf{x}\|_1^1$$

- Seismic data is assumed as a *superposition* of curvelets
- Curvelet coefficient vector is assumed to be sparse (*prior)*

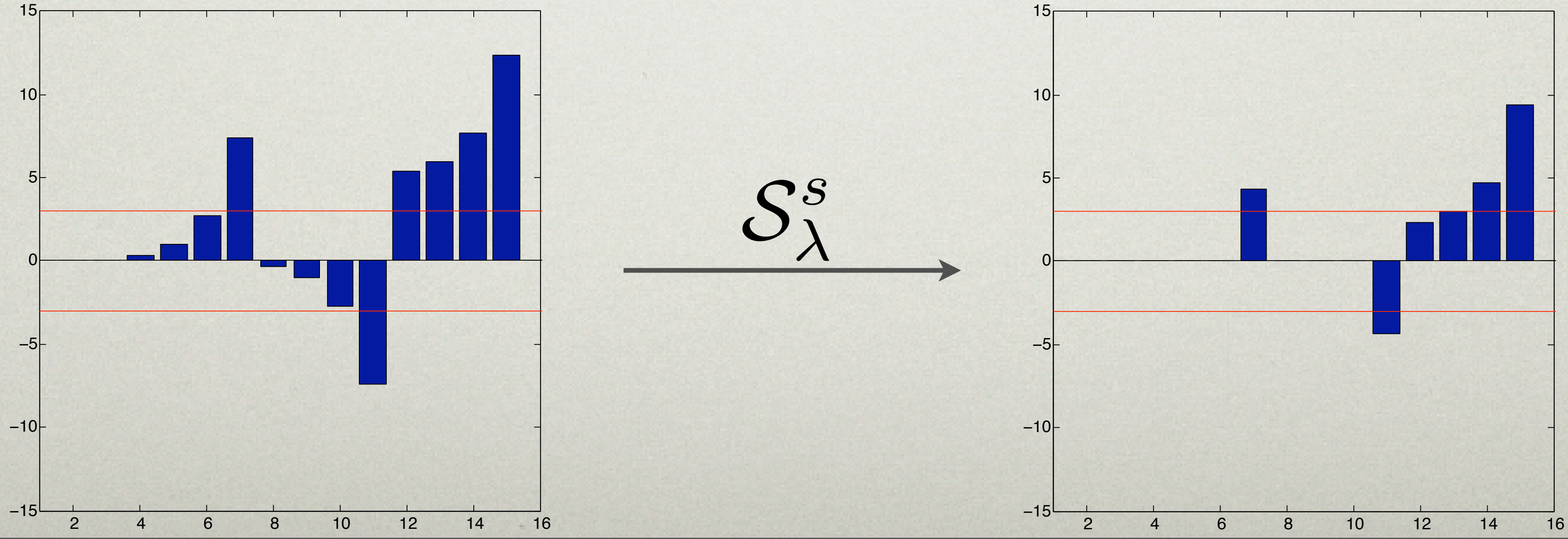


ℓ^1-norm Optimization by Iterative Thresholding

Denoising with Landweber iterations and soft-thresholding [Daubechies, 05; Elad, 05]

$$\mathbf{x}^m = S^s_{\lambda_m}\left[\mathbf{x}^{m-1} + \mathbf{F}^T\left(\mathbf{d} - \mathbf{F}\mathbf{x}^{m-1}\right)\right]$$

$$S^s_\lambda(x) = \begin{cases} x - \text{sign}(x)\lambda & |x| \geq \lambda \\ 0 & |x| < \lambda \end{cases}$$



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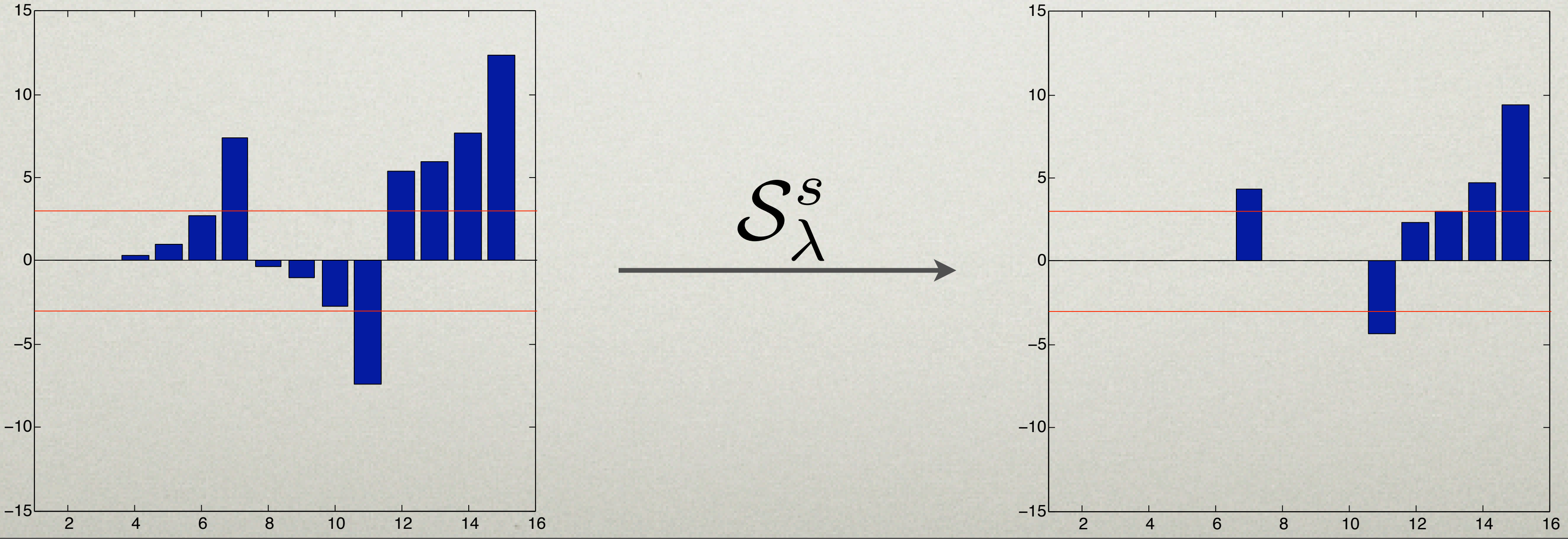

ℓ^1-NORM OPTIMIZATION BY ITERATIVE THRESHOLDING

Data-continuation with Landweber iterations and soft-thresholding [Daubechies, 05; Elad, 05]

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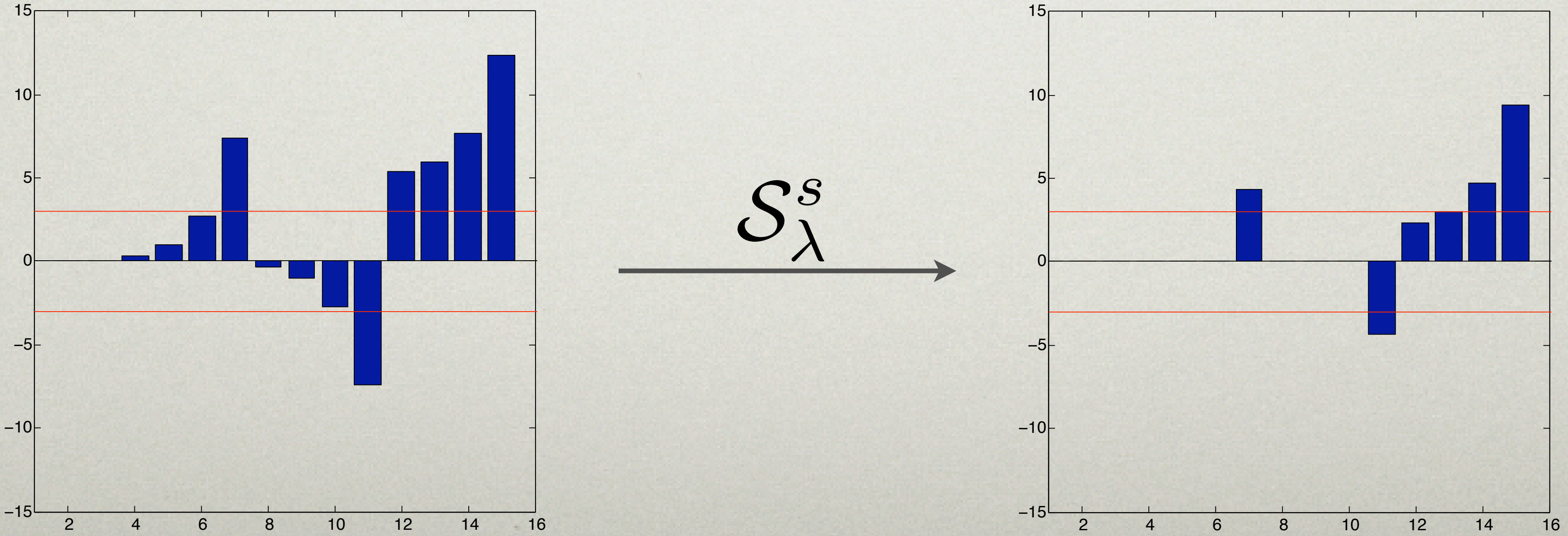

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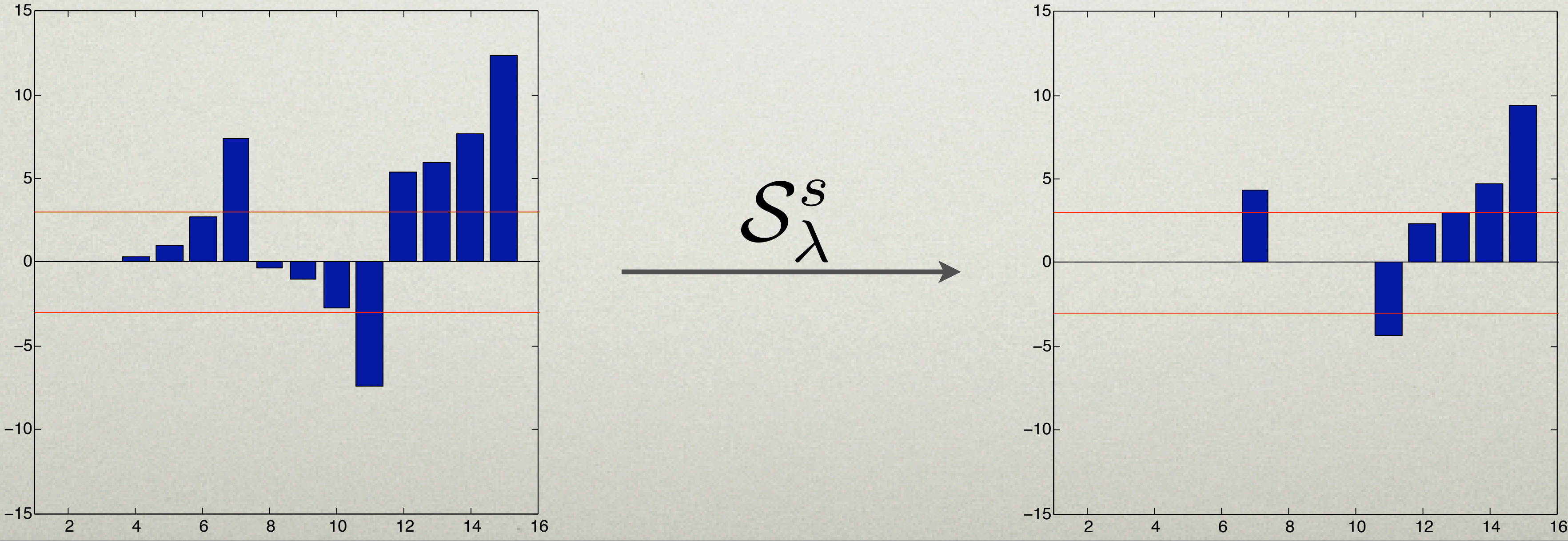

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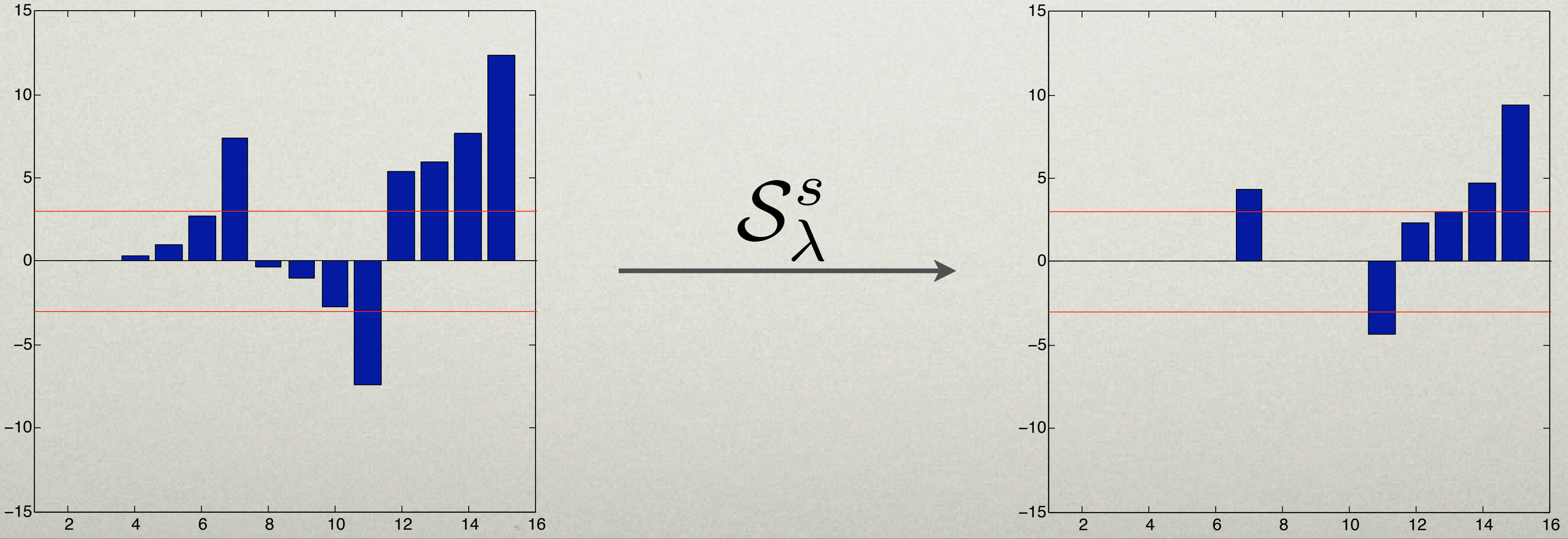
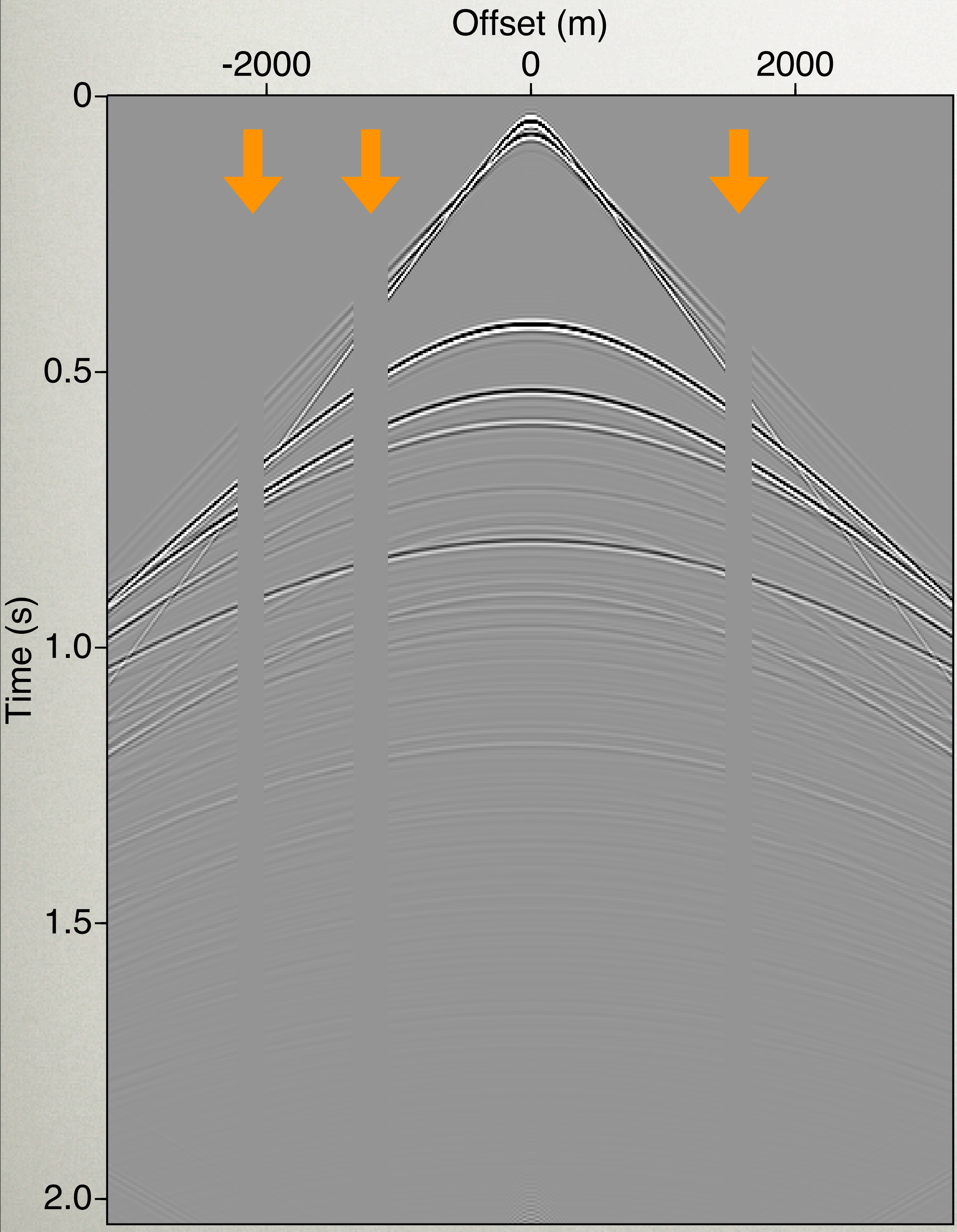


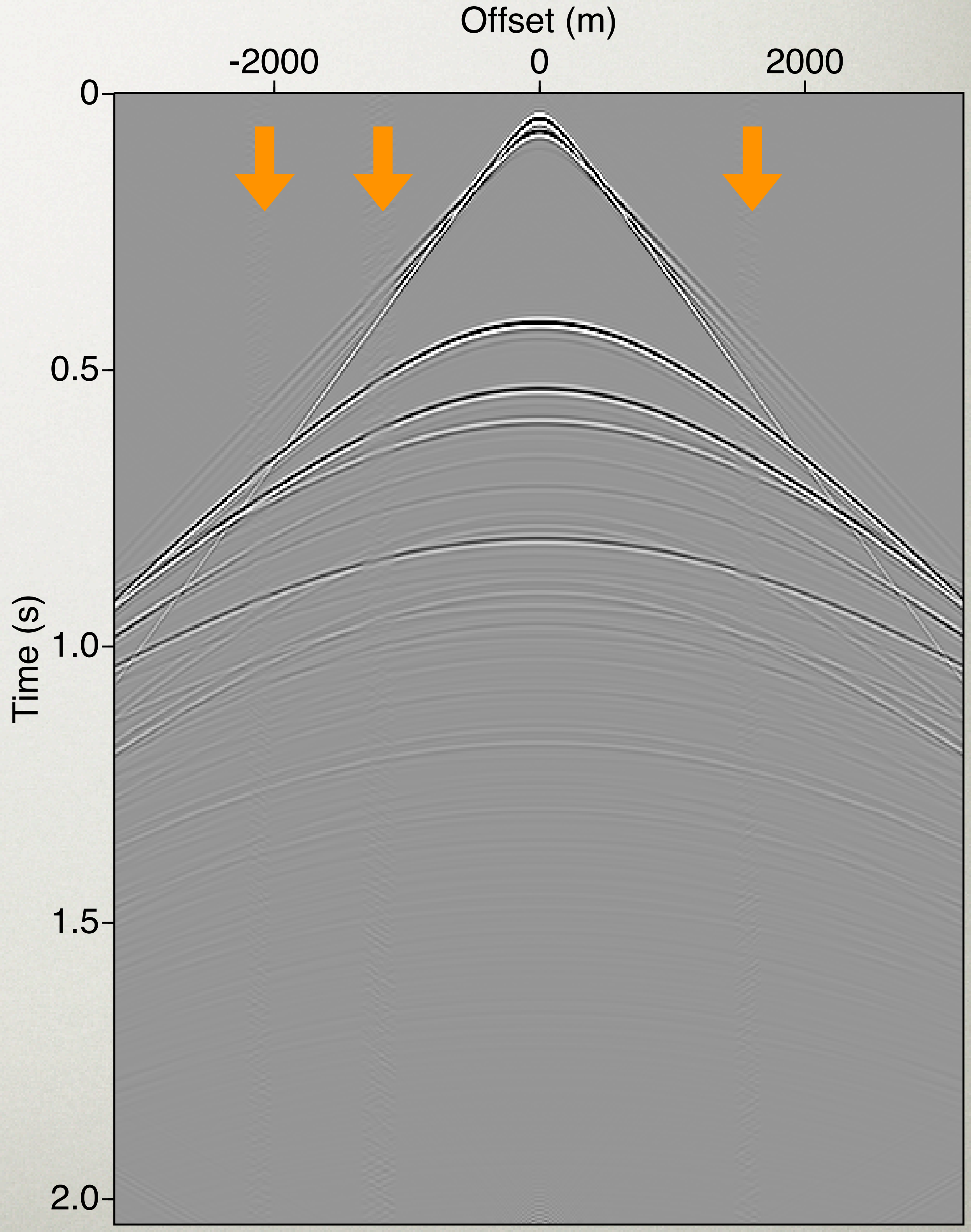
EXAMPLES

- 2-D synthetic data (512 offsets x 512 time samples)
  - Data continuation
  - De-aliasing
- 3-D real data (280 shots x 368 receivers x 256 time samples)
  - Data continuation

2-D SYNTHETIC SHOT

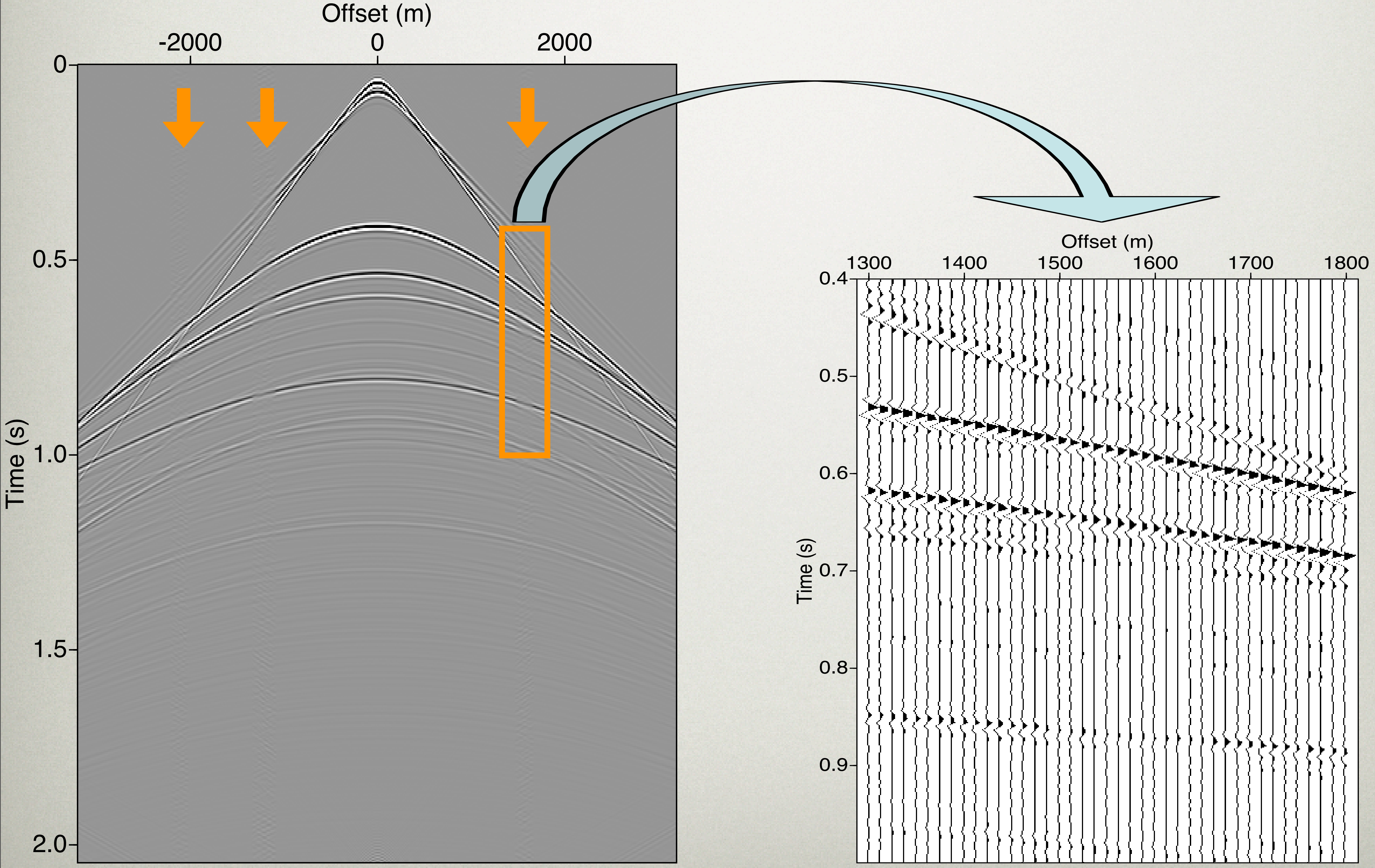
DATA CONTINUATION (NOISE-FREE)





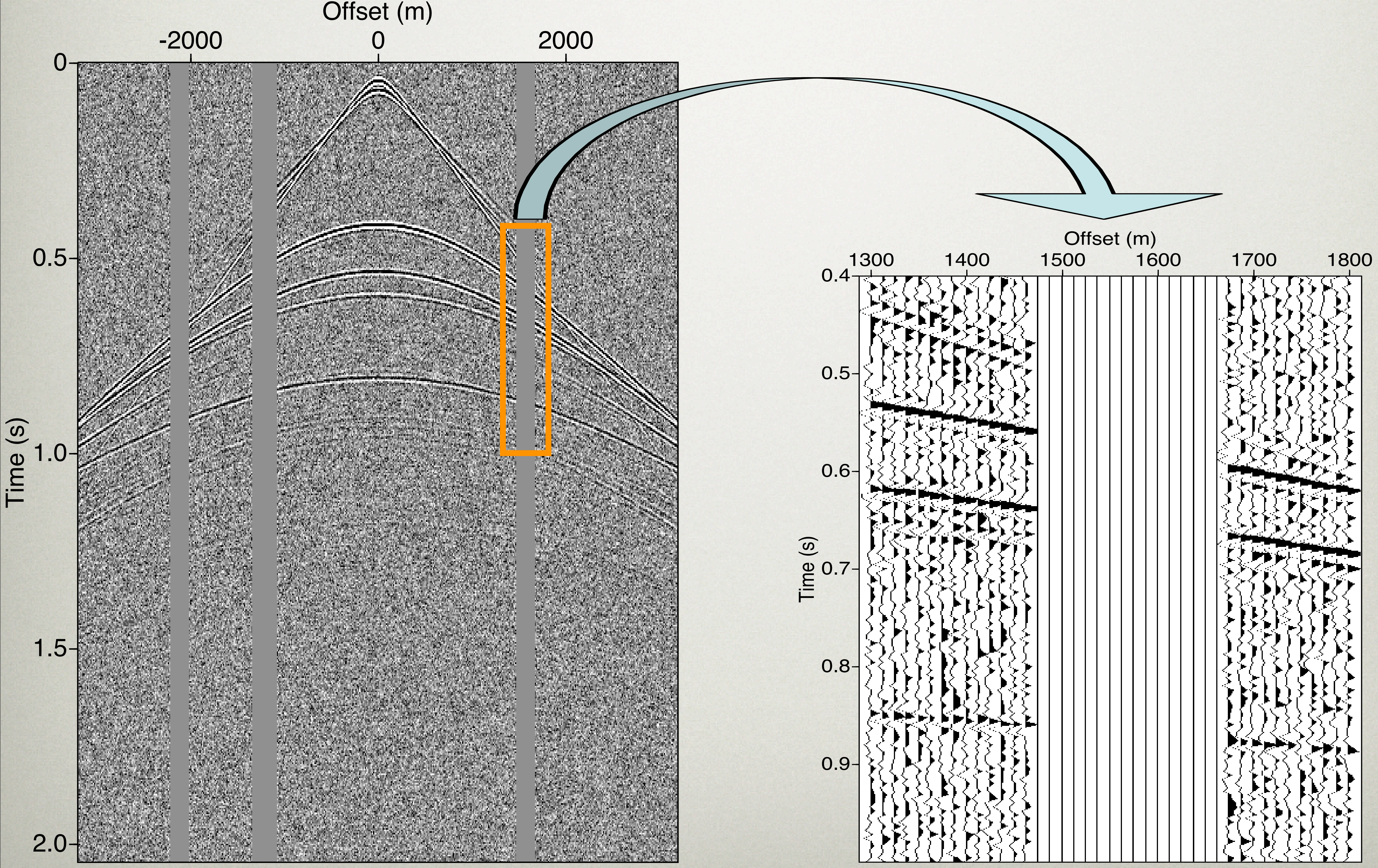
2-D SYNTHETIC SHOT

DATA CONTINUATION (NOISE-FREE)



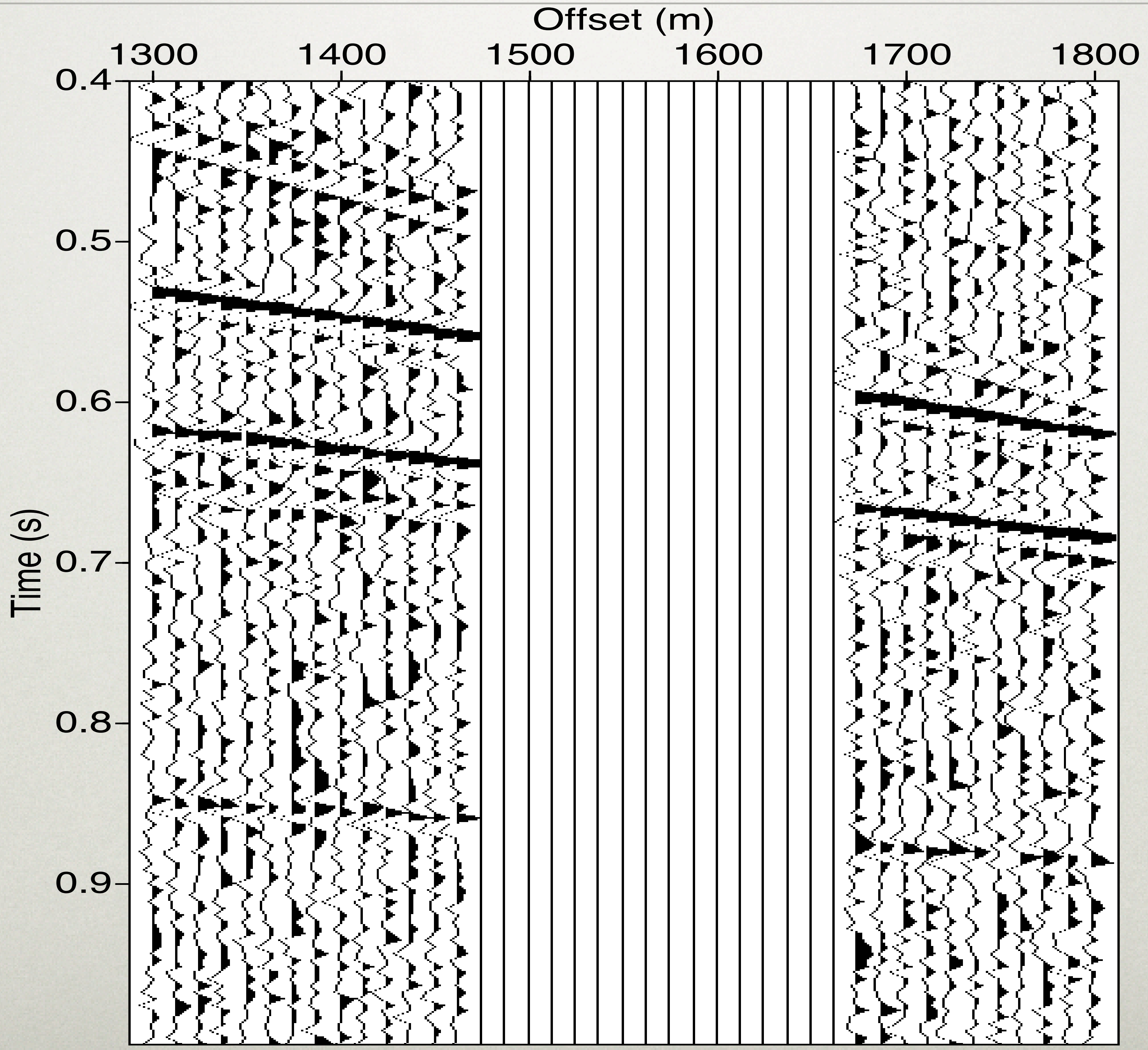
2-D SYNTHETIC SHOT

DATA CONTINUATION (NOISY - SNR = 0 DB)



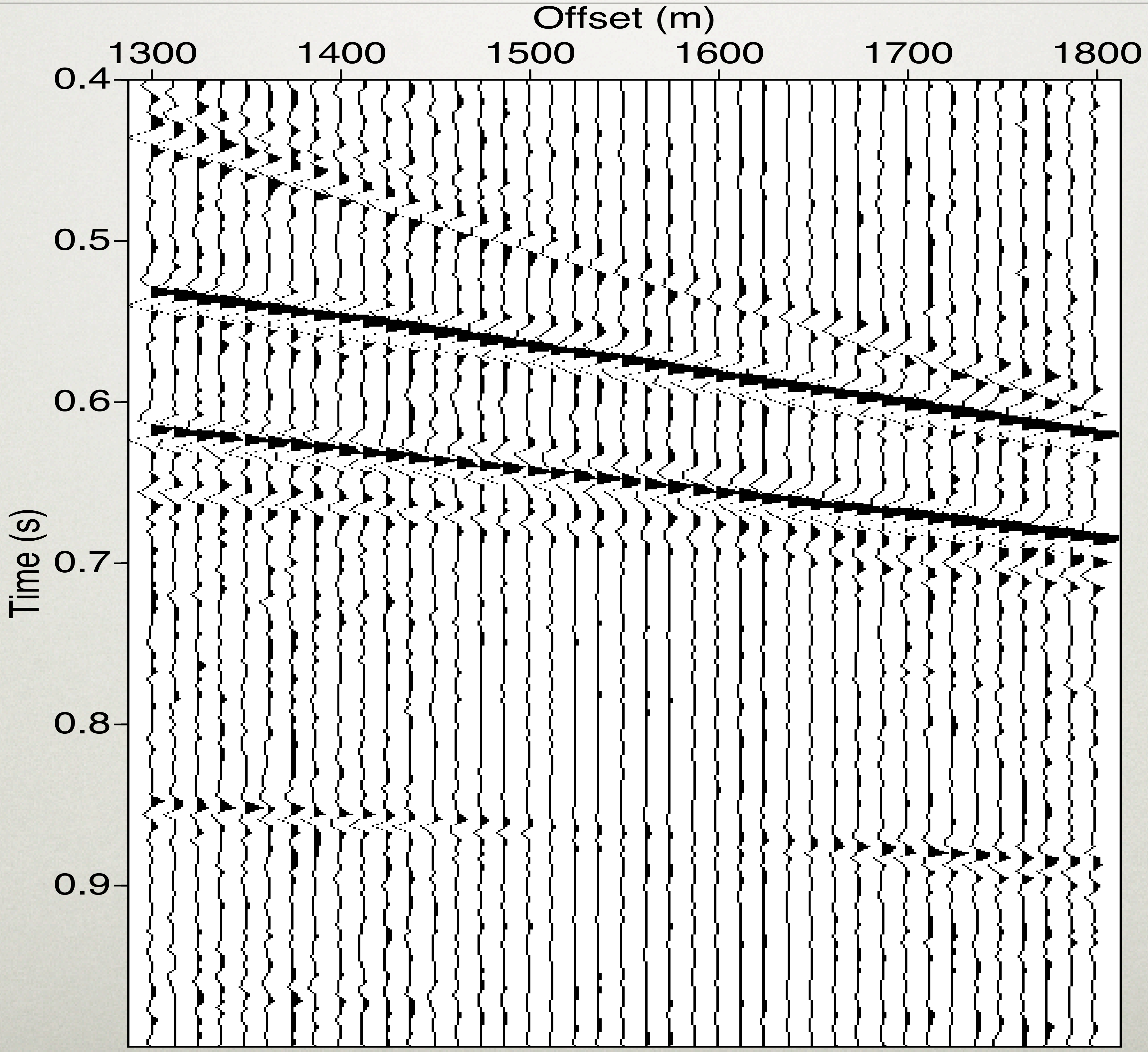
2-D SYNTHETIC SHOT

DATA CONTINUATION (NOISY)



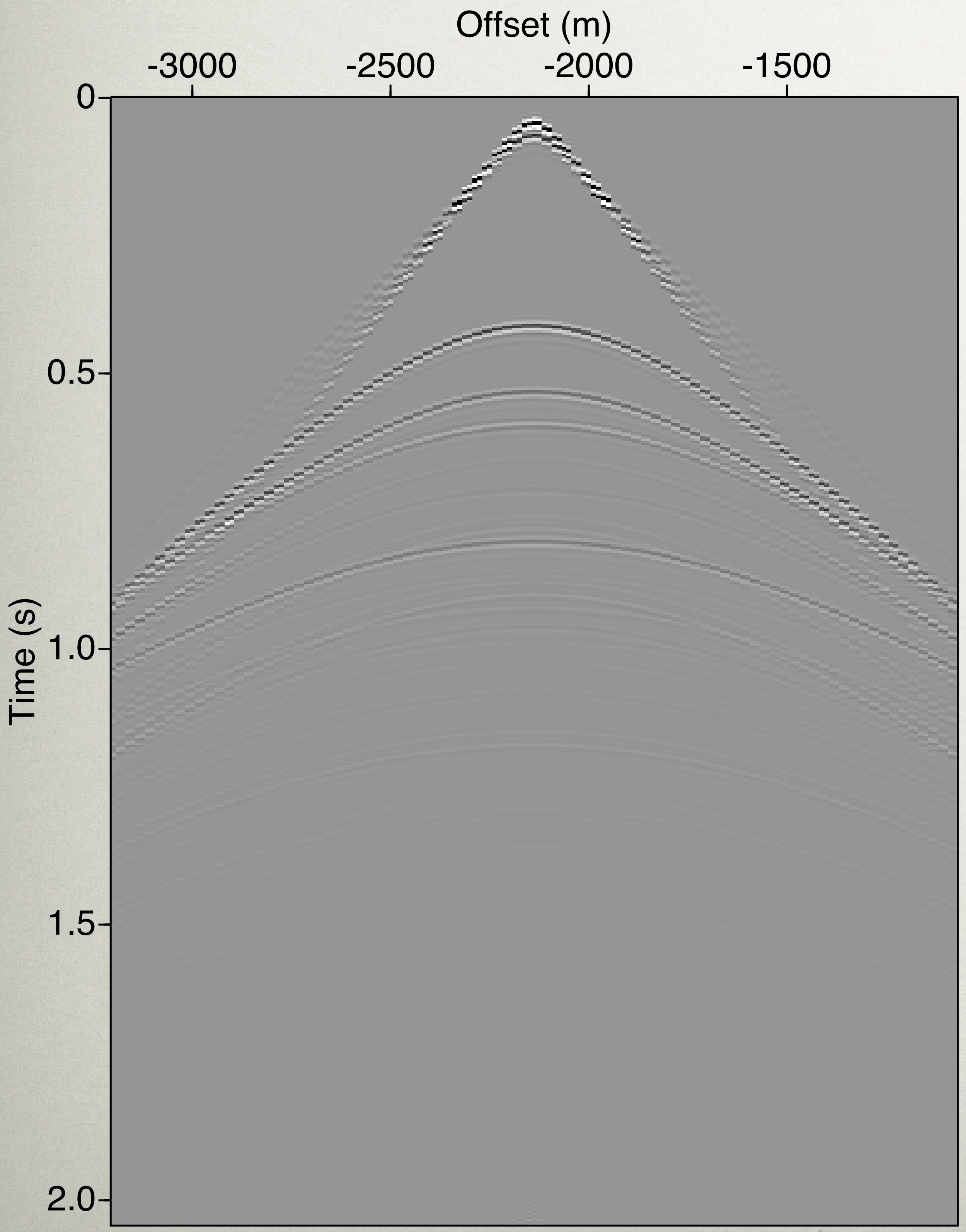
2-D SYNTHETIC SHOT

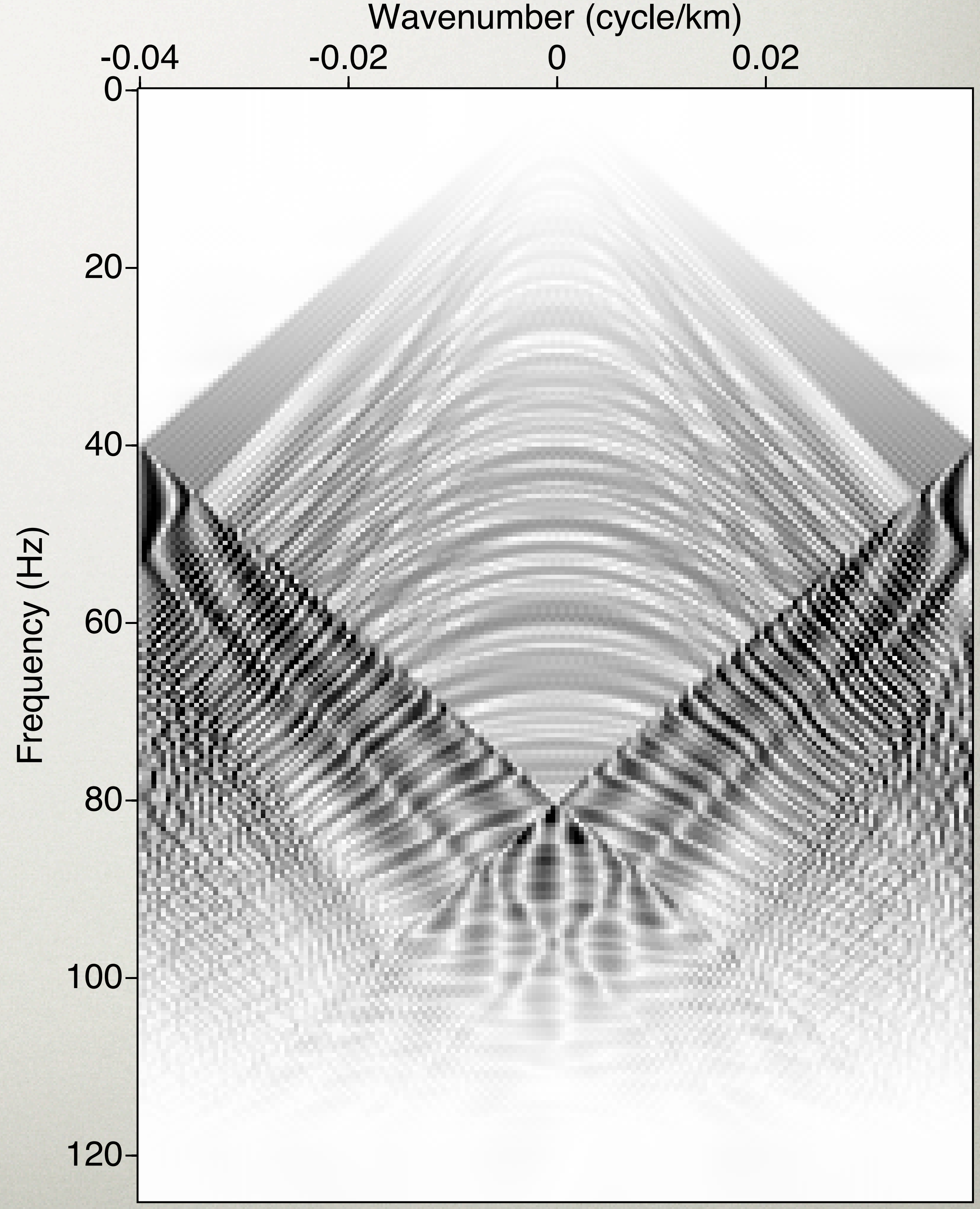
DATA CONTINUATION (NOISY)



2-D Synthetic Shot

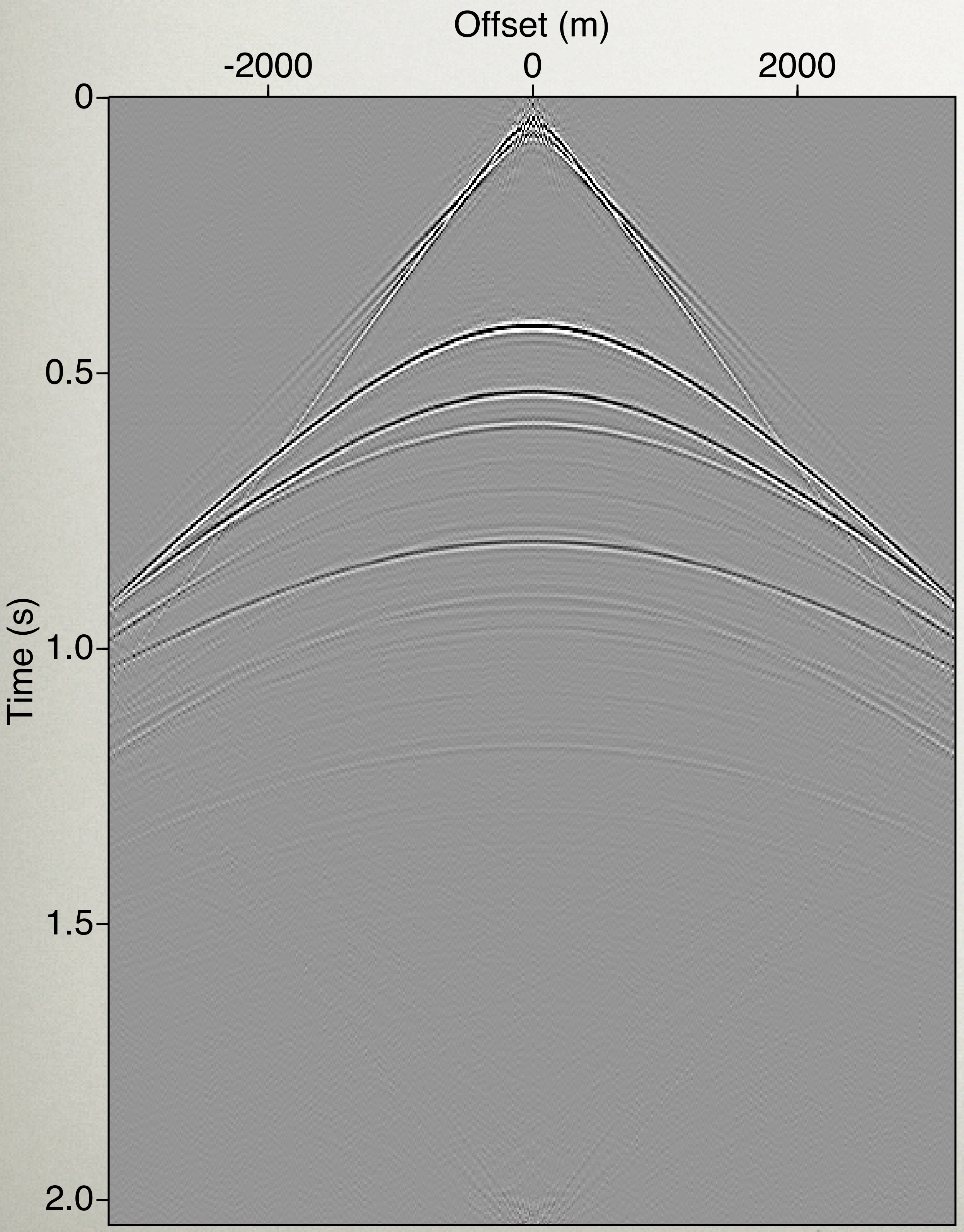
De-Aliasing

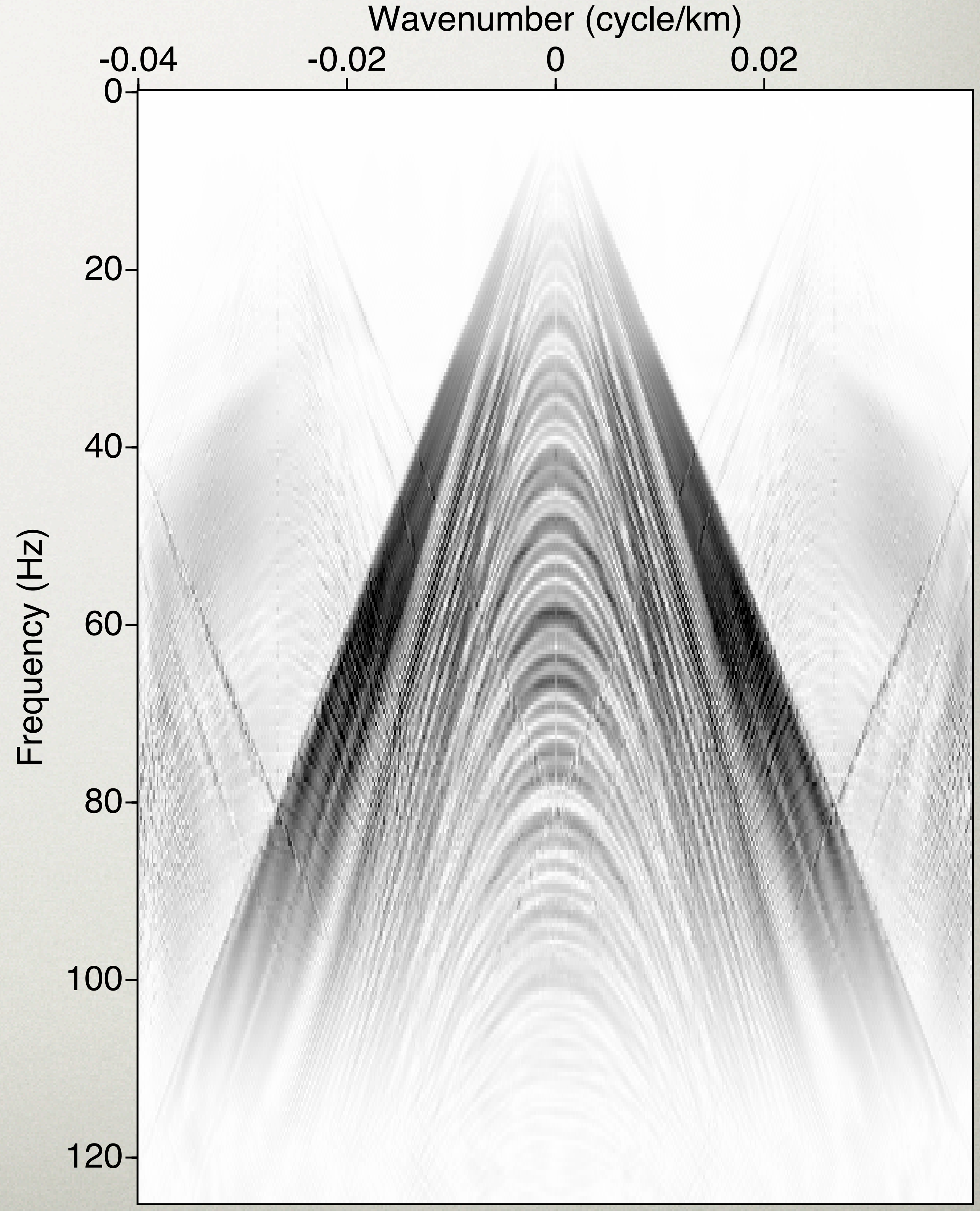




2-D SYNTHETIC SHOT

DE-ALIASING





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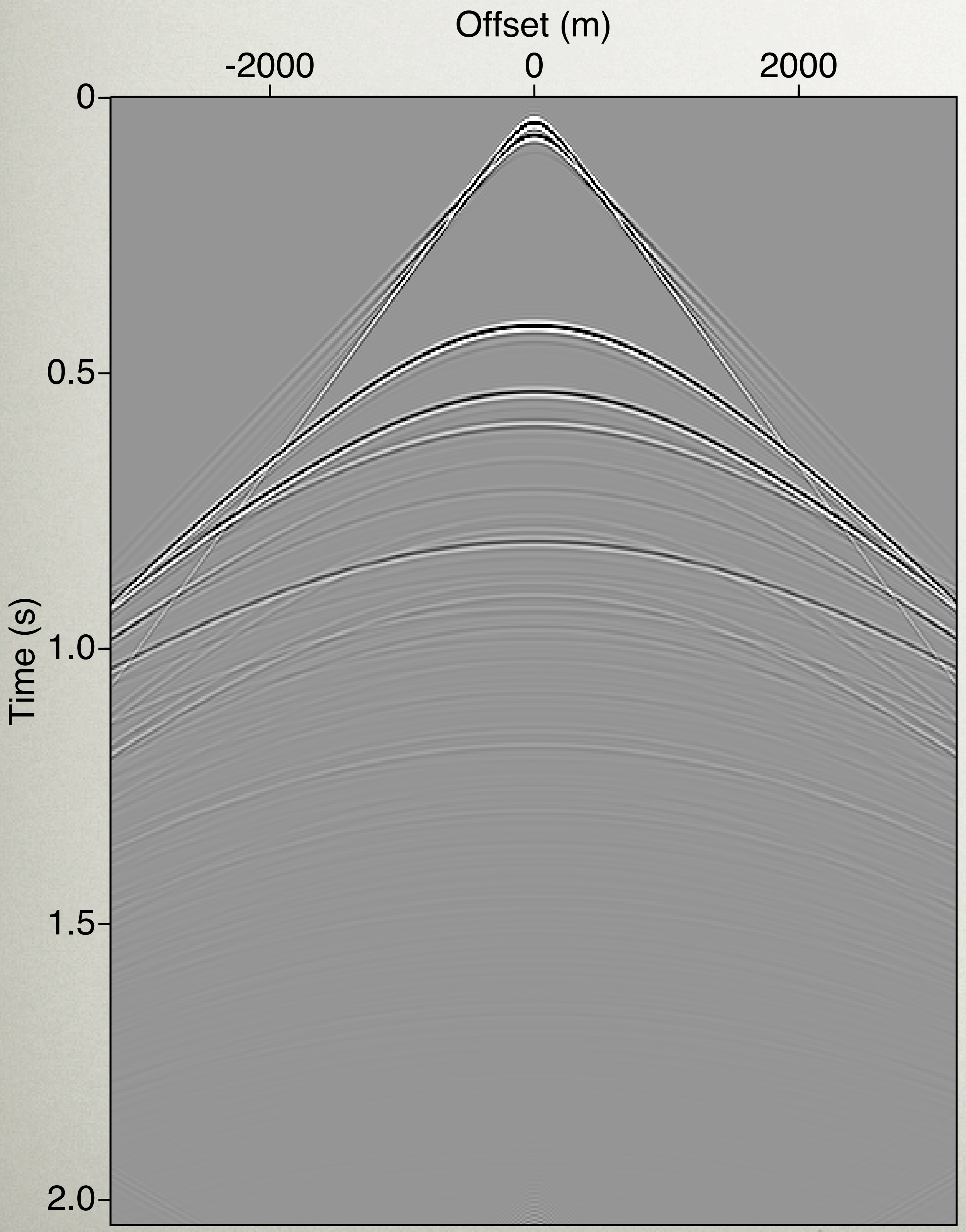
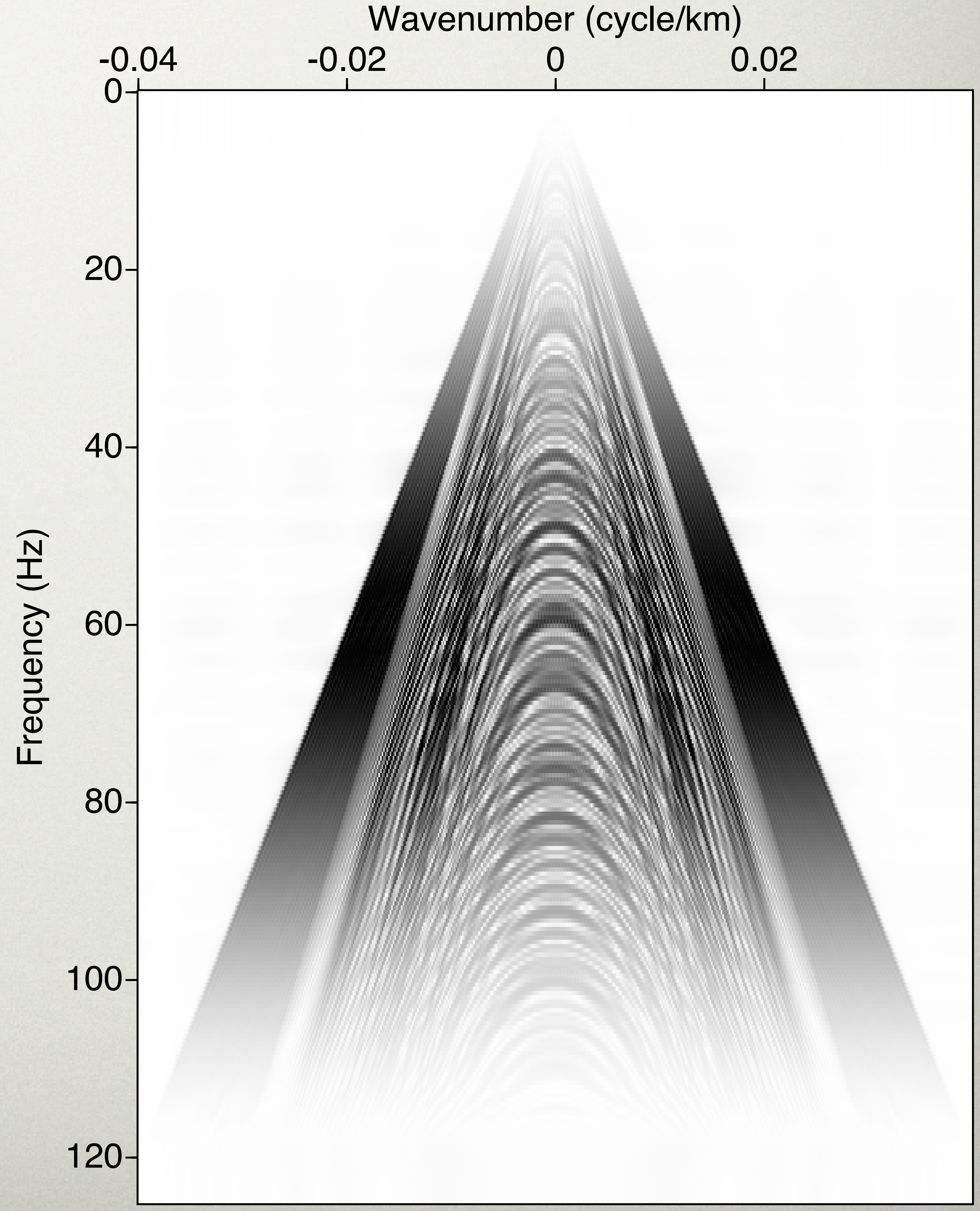
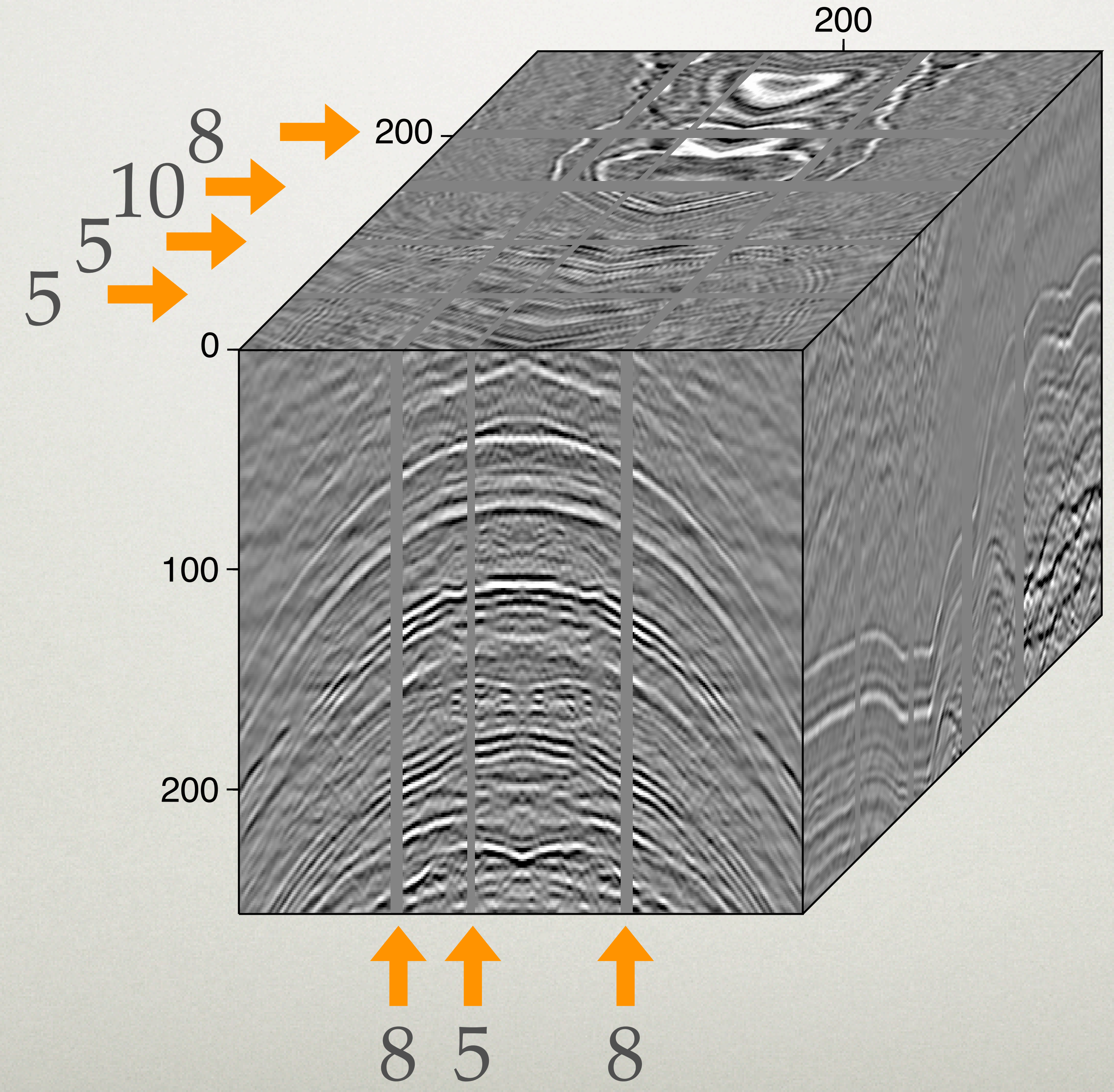
DE-ALIASING

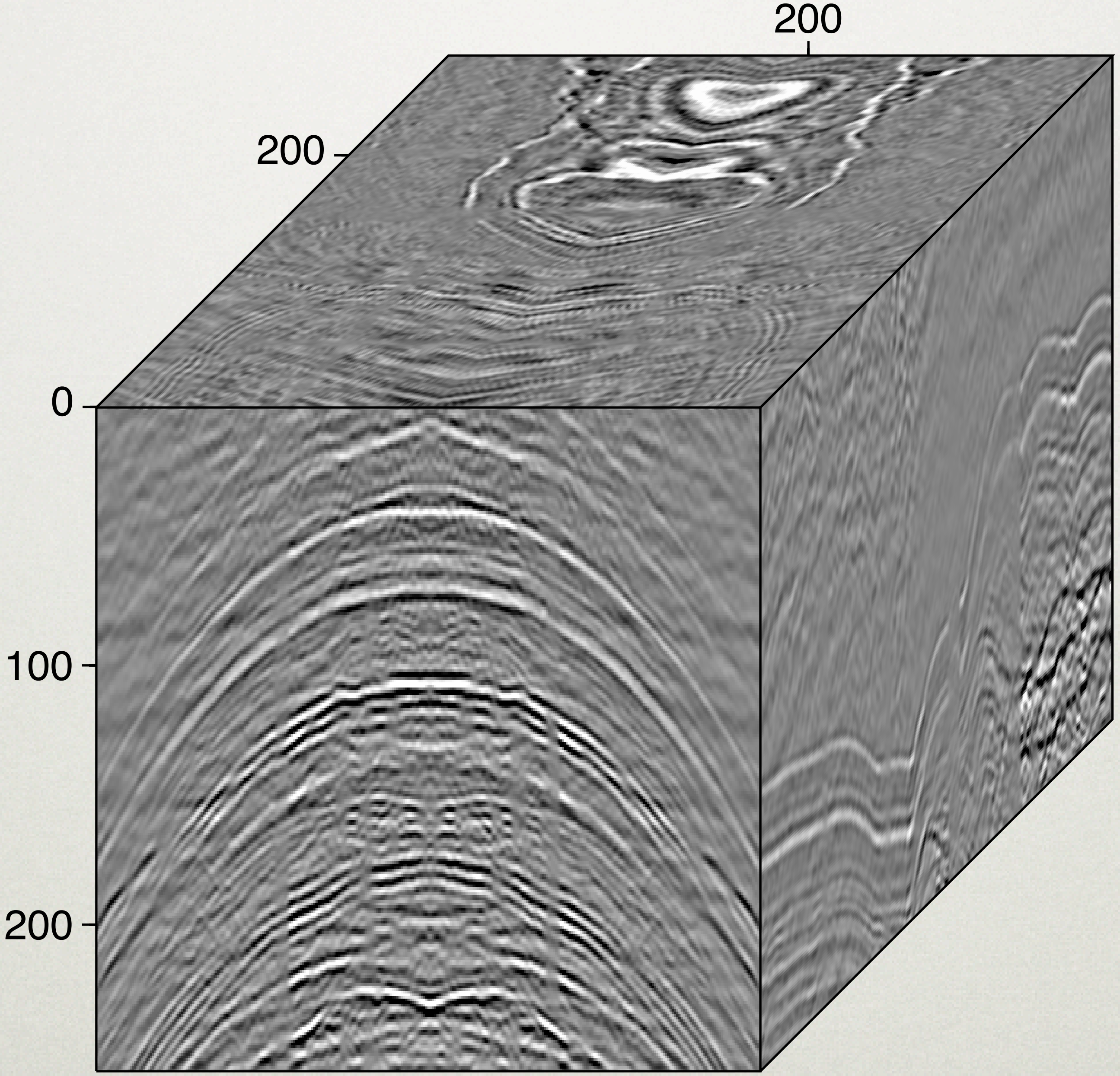




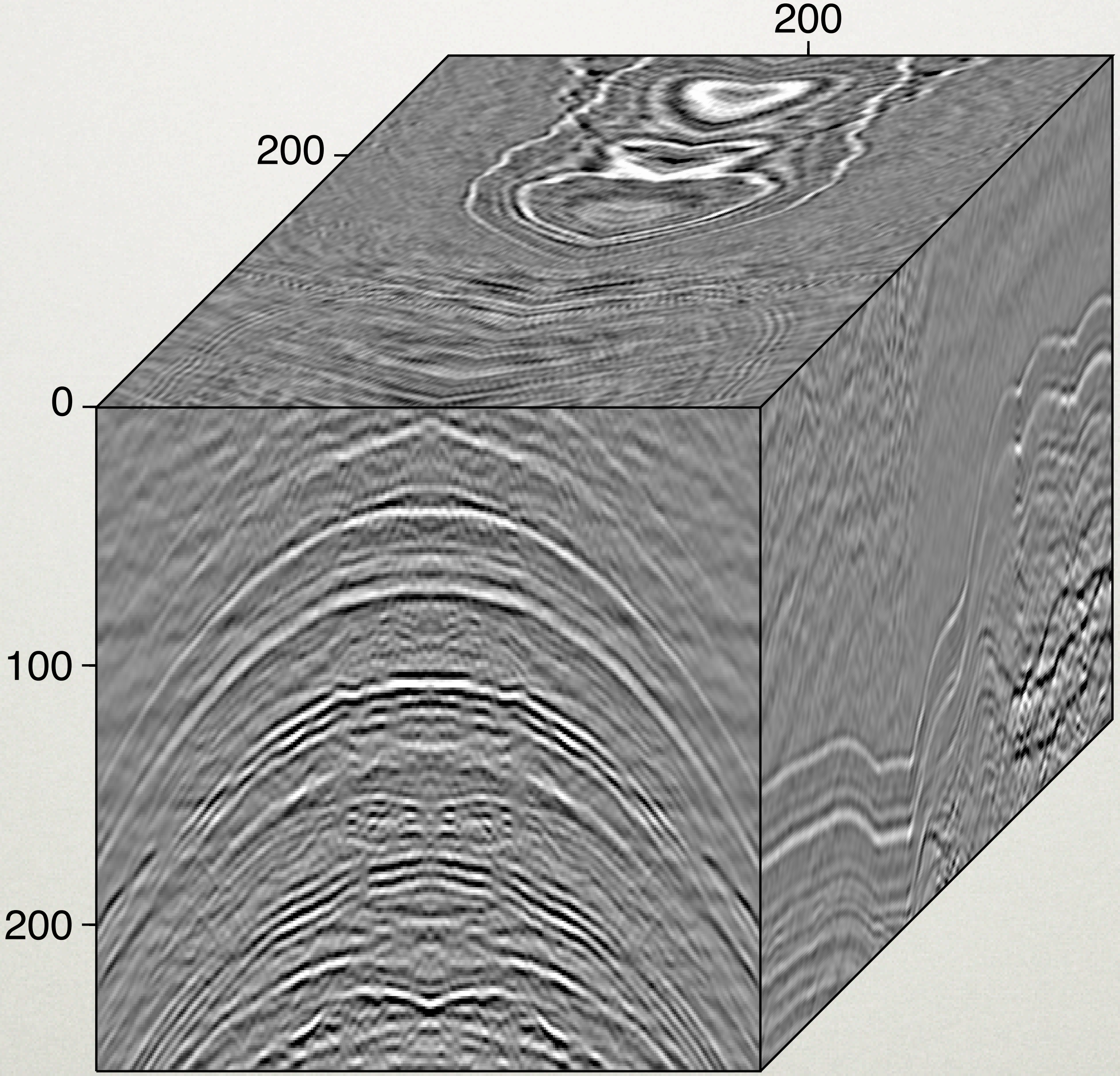
3-D REAL DATA

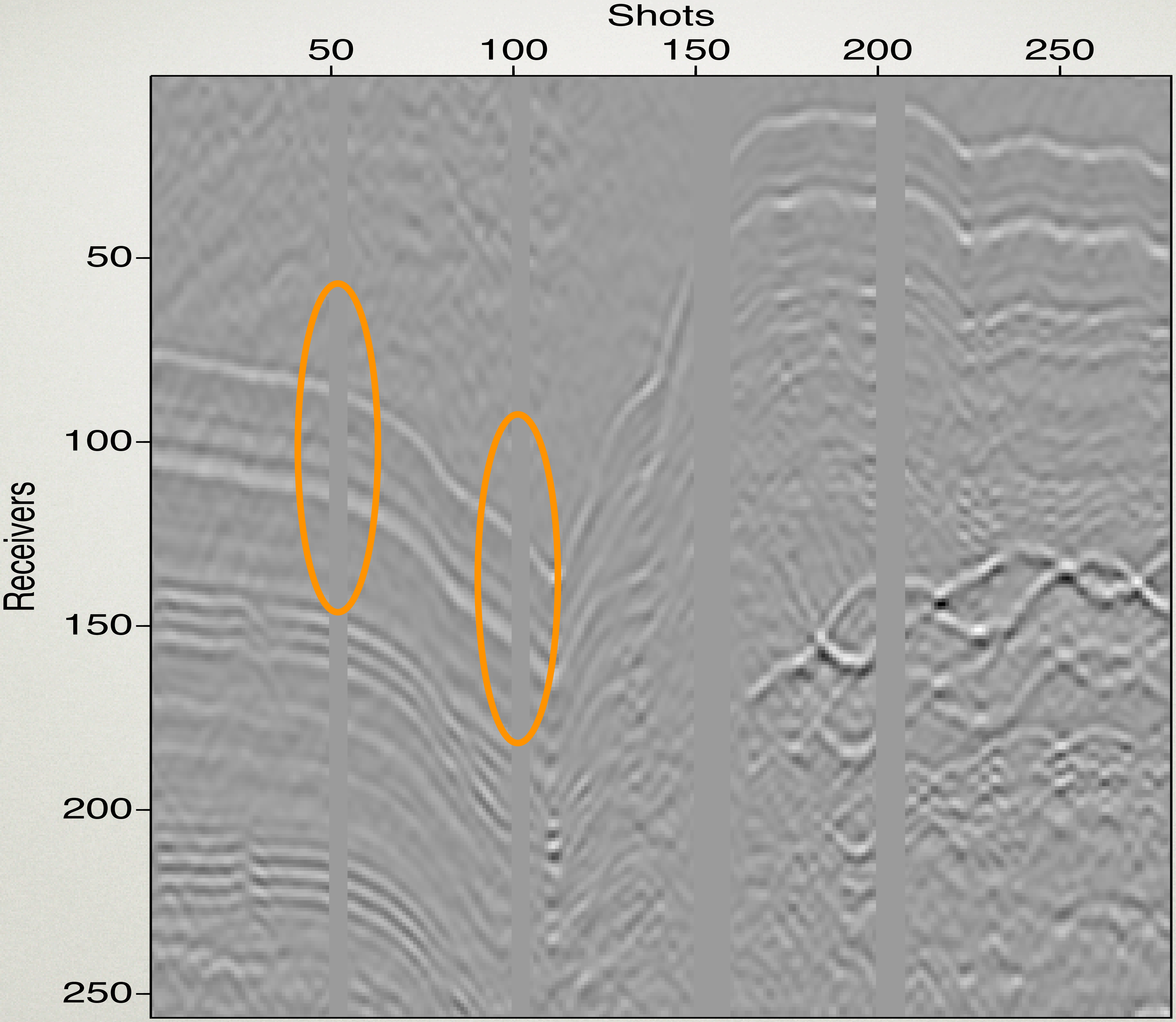


3-D REAL INTERPOLATED RESULT



3-D Real Model




Shots
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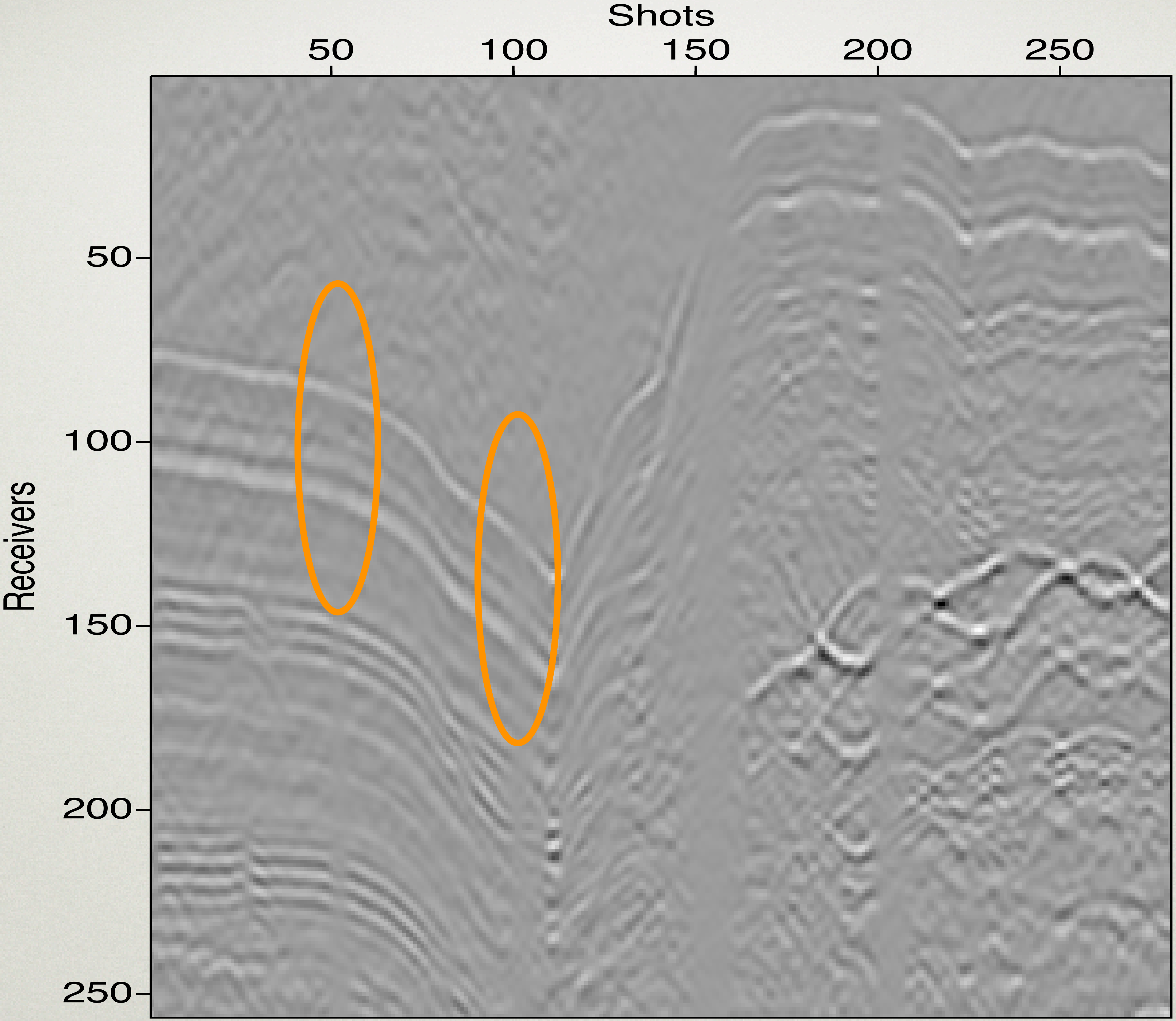
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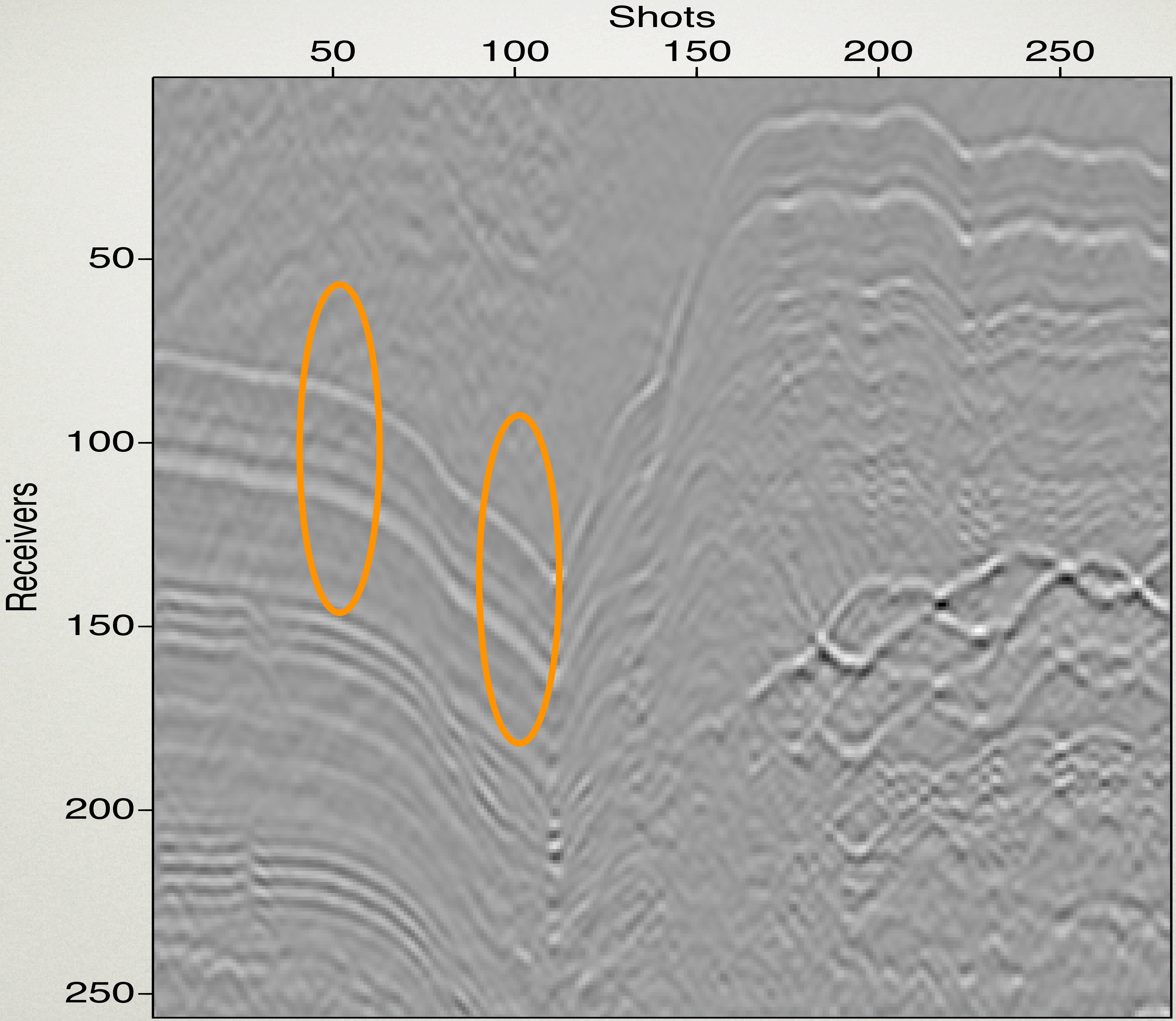
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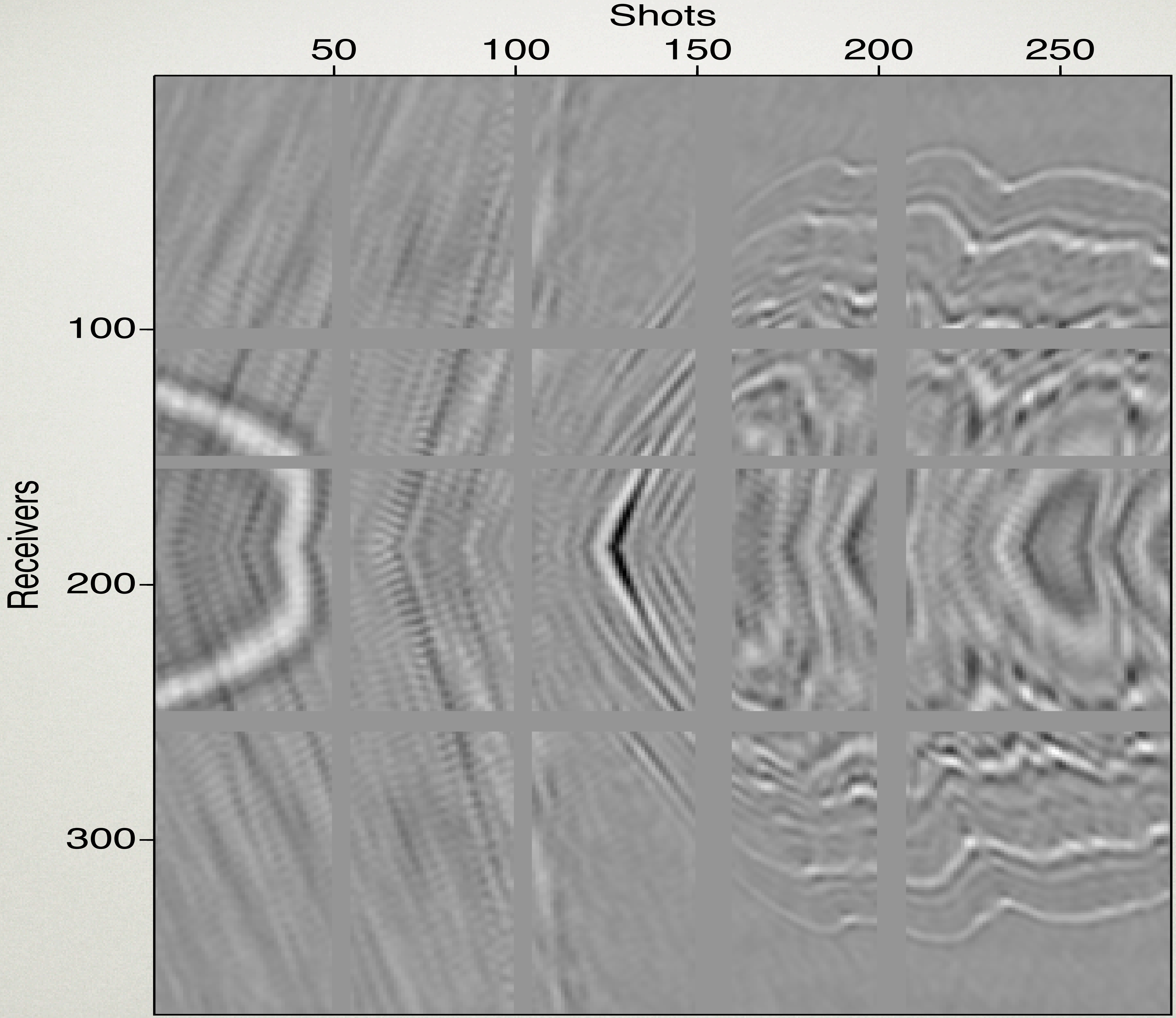
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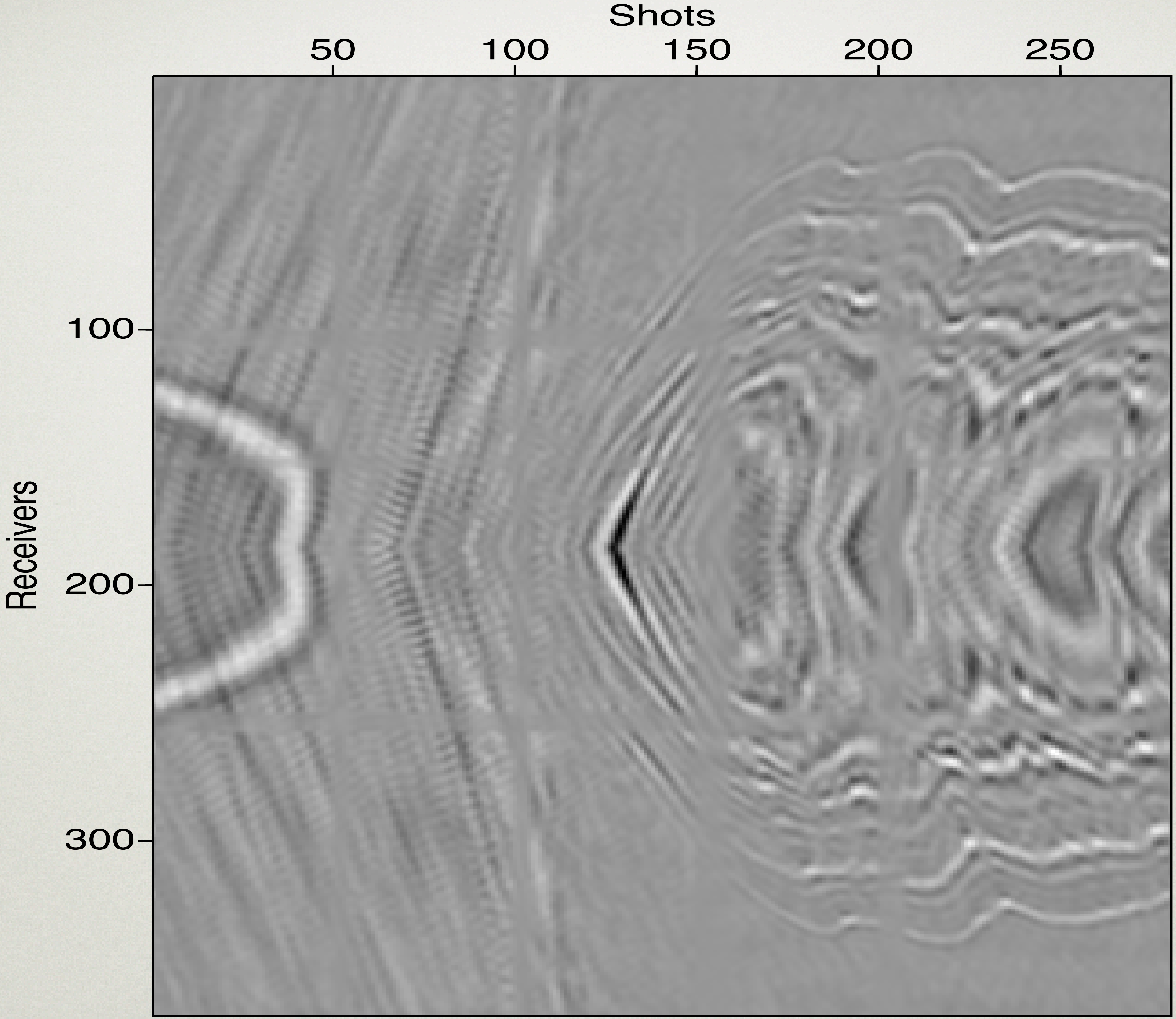
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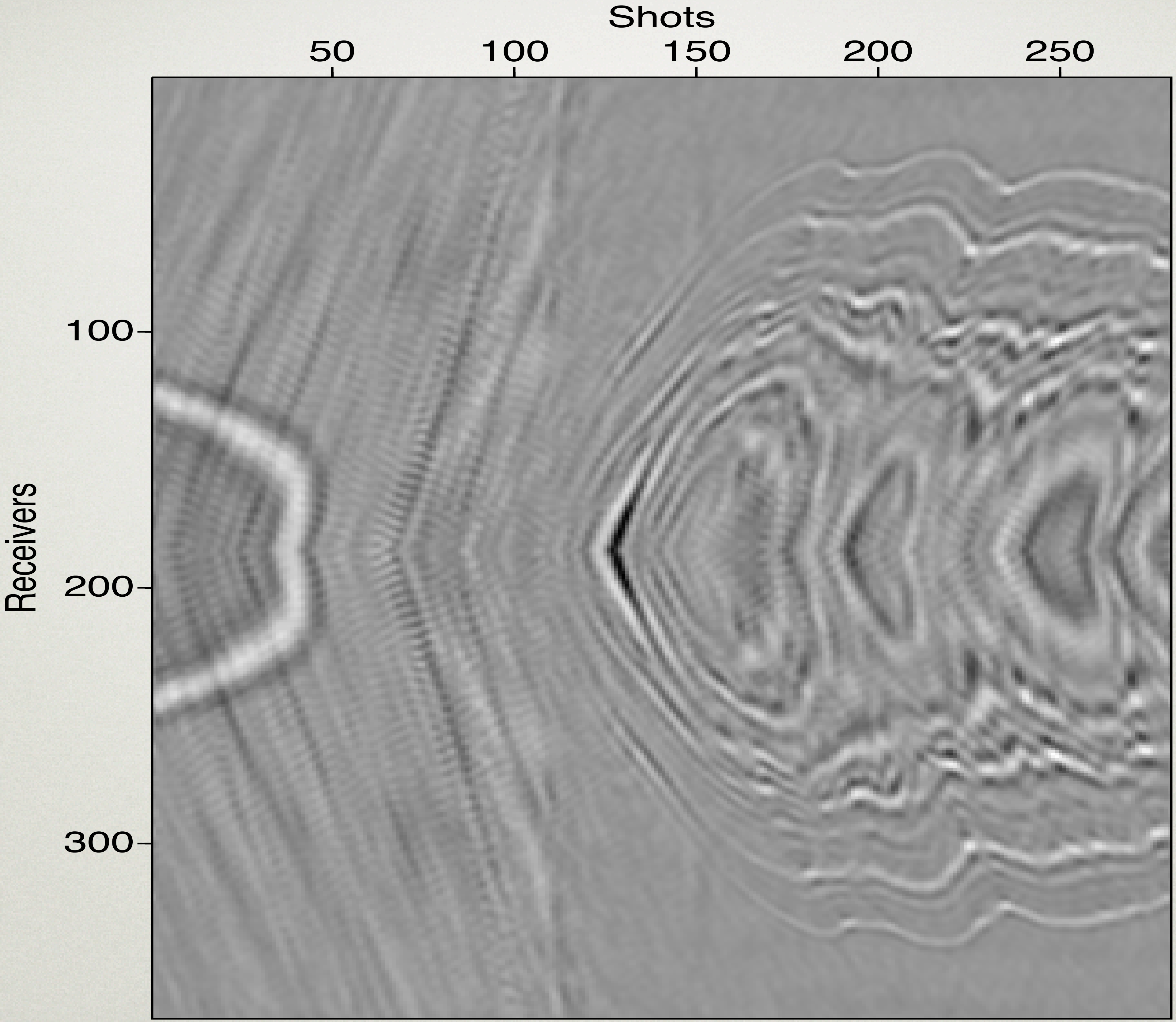
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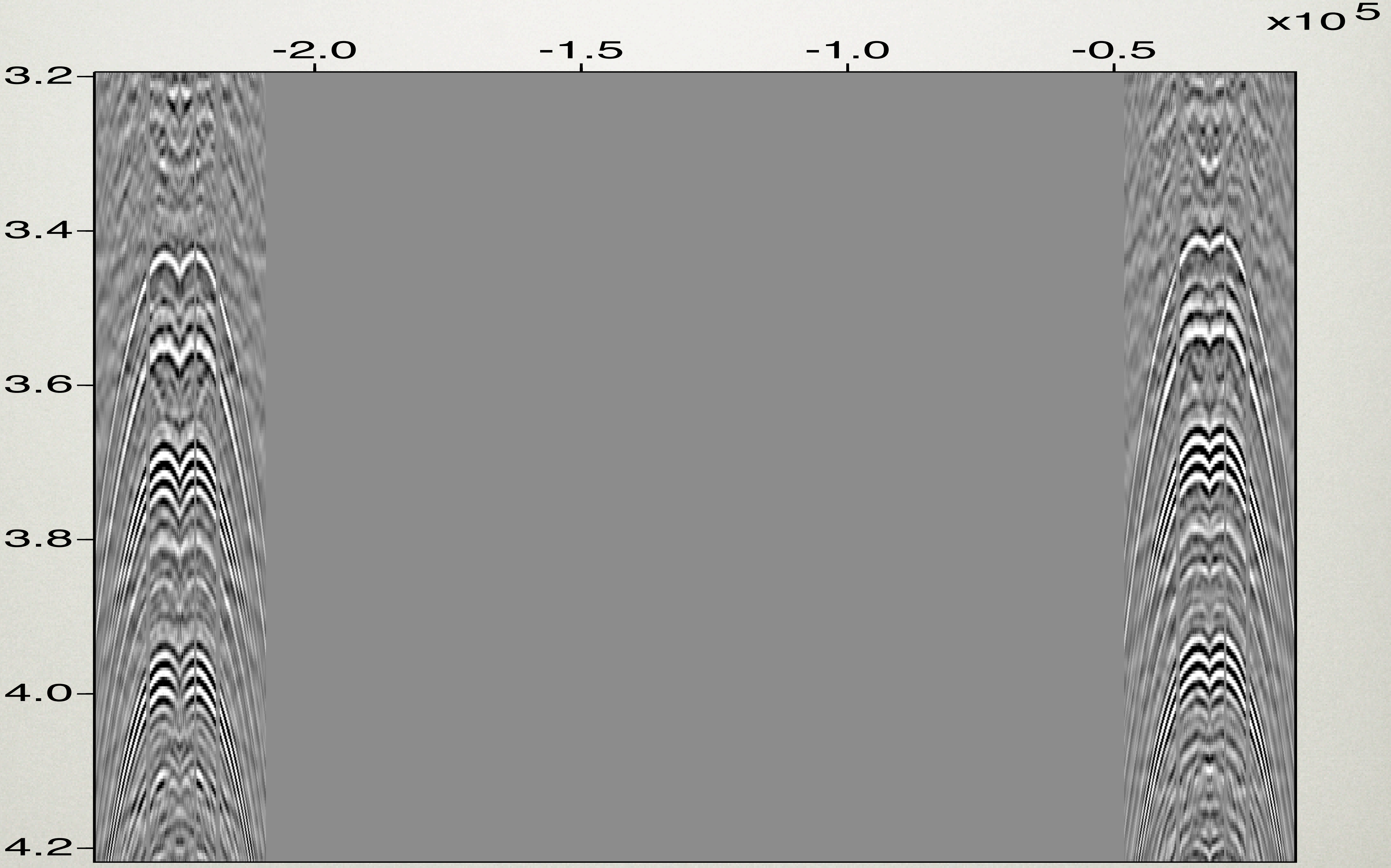
3-D Real Data



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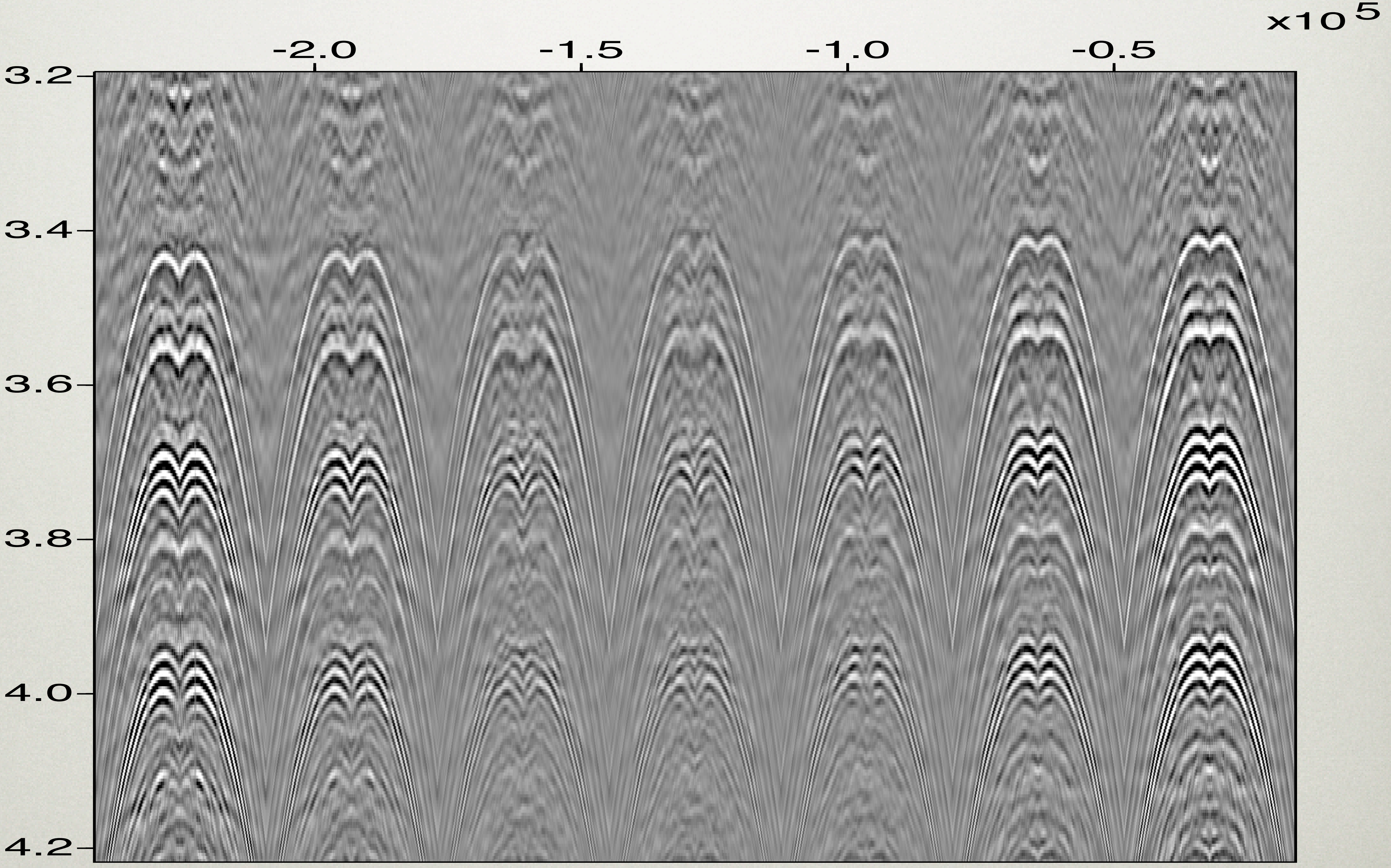

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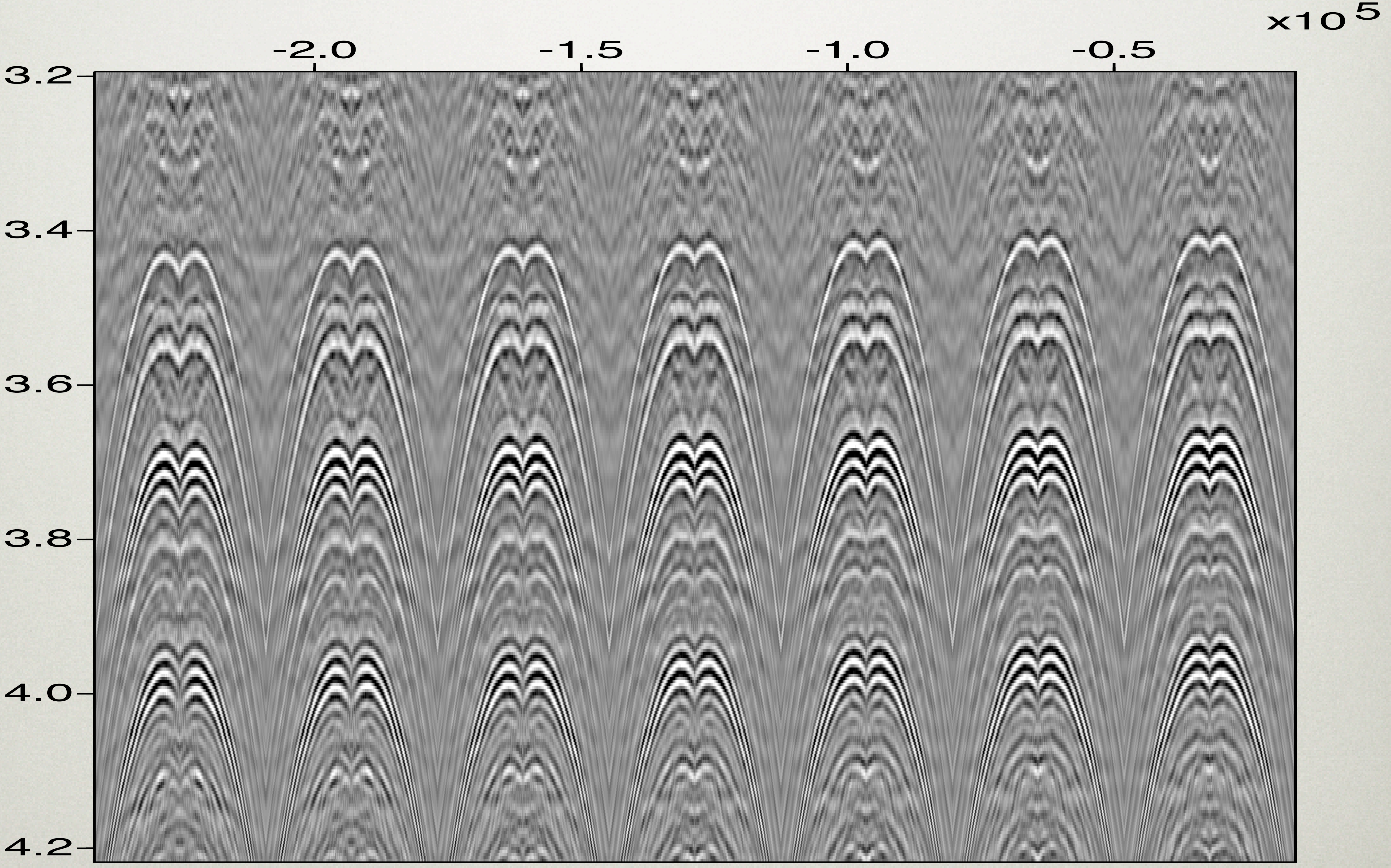

CONCLUSIONS

- Our method explores the 3-D continuity of reflection events in the data cube
- Aims for the sparsest set of Curvelet coefficients that match the data
- The iterated thresholding is resilient to (coherent) noise
- Results are encouraging
- More sophisticated solvers may improve convergence
- Curvelet Frame can be extended to include DCT

ACKNOWLEDGMENTS

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- SINBAD sponsored through ITF by British Gas, British Petroleum, ExxonMobil, Shell
- Western Geco for providing the data
- NSERC (Discovery grant 22R81254)

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