SLIM

Seismic Laboratory for Imaging and Modeling

Sparseness-Constrained Seismic Deconvolution with Curvelets

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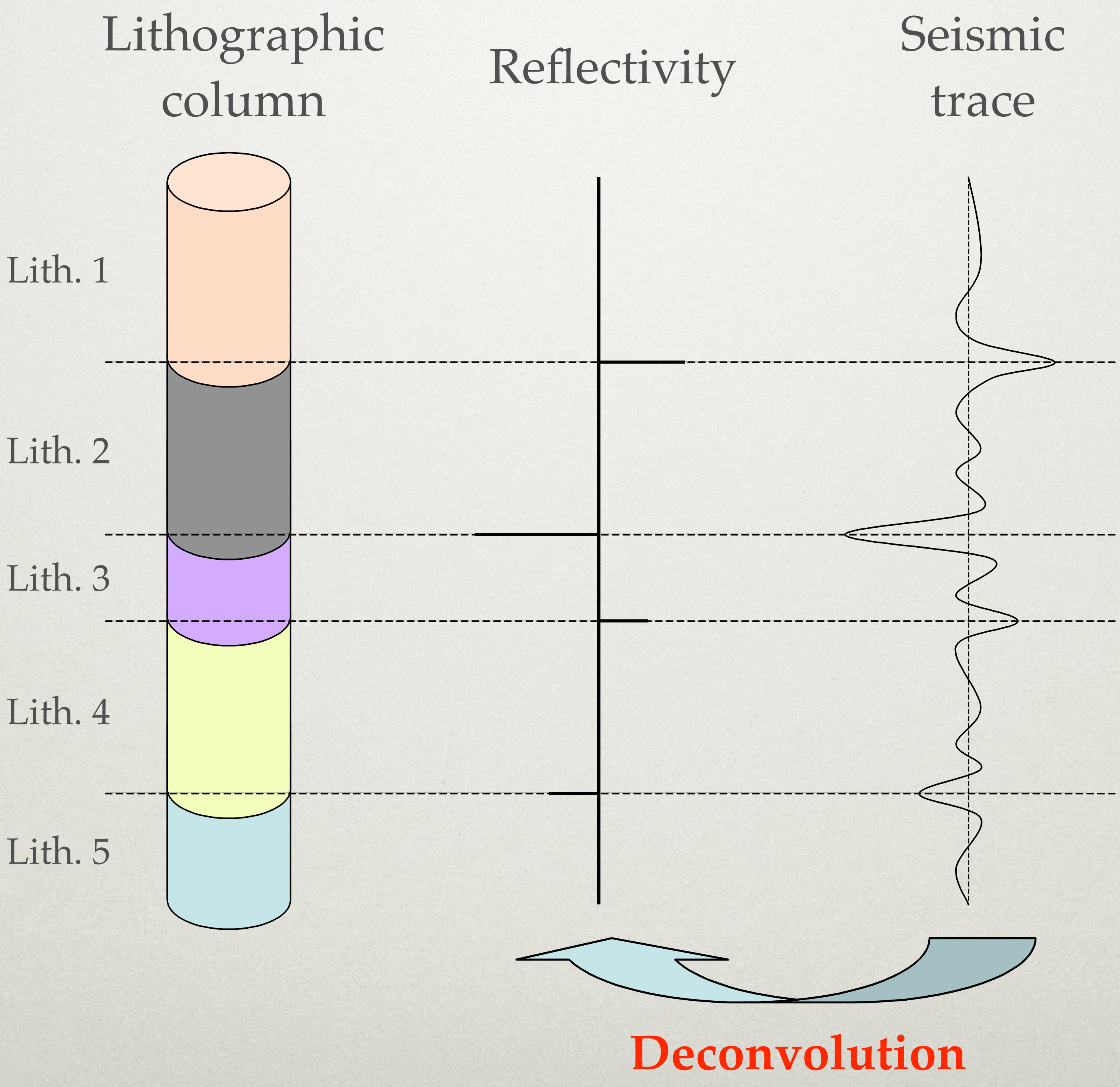
Deconvolution Problem



Context

- Least-squares [Robinson, 57]
- Homomorphic [Buttkus, 75]
- Maximum-likelihood [Chi et al., 84]

Deconvolution operator estimated (statistical)

- Minimum structure/l_1-norm
  [Oldenburg,81; Ulrych & Walker, 82; Sacchi et al., 94]
- ForWaRD [Neelamani, 04]

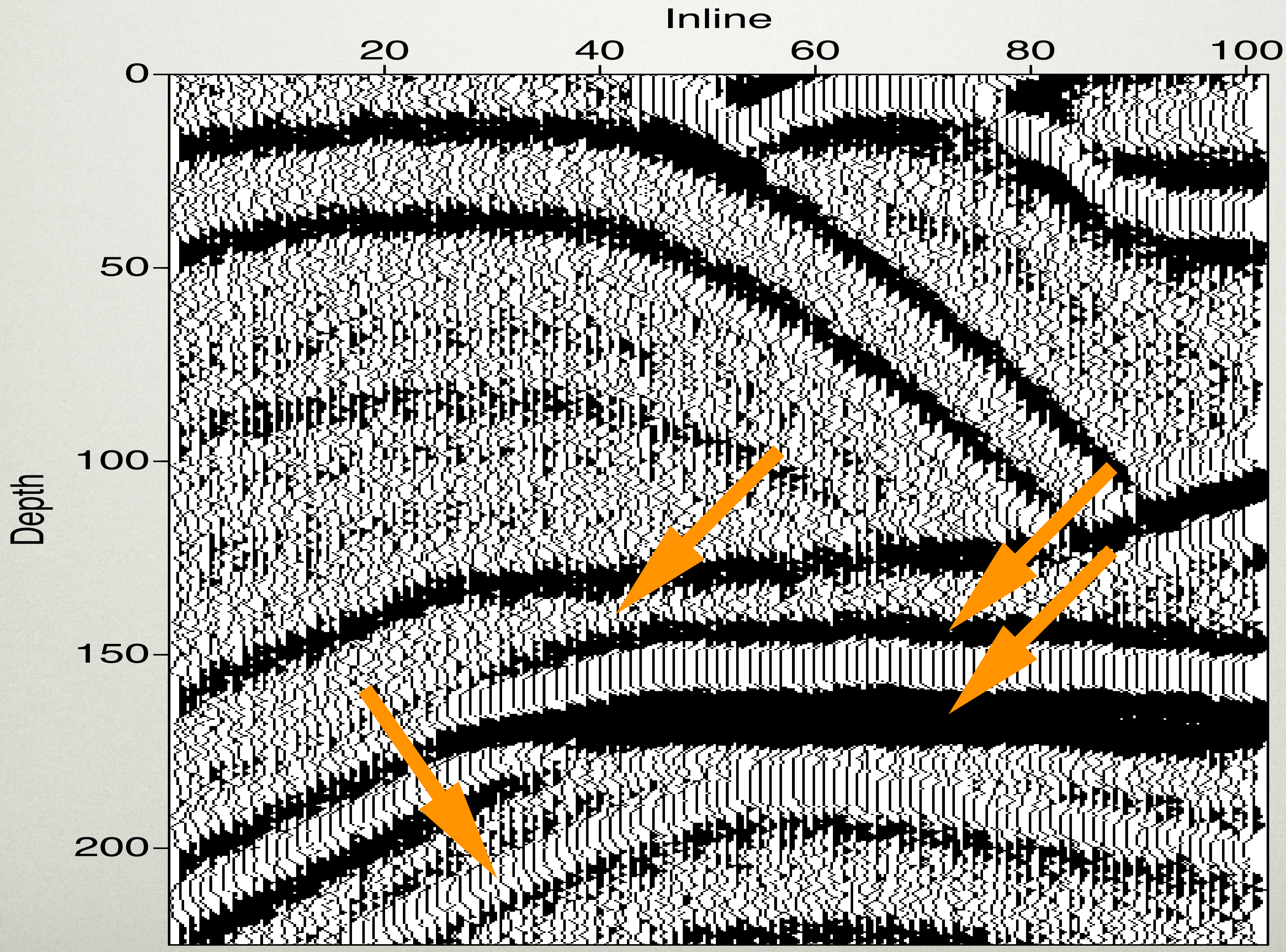
Deconvolution operator "known" (deterministic)

Our Motivation

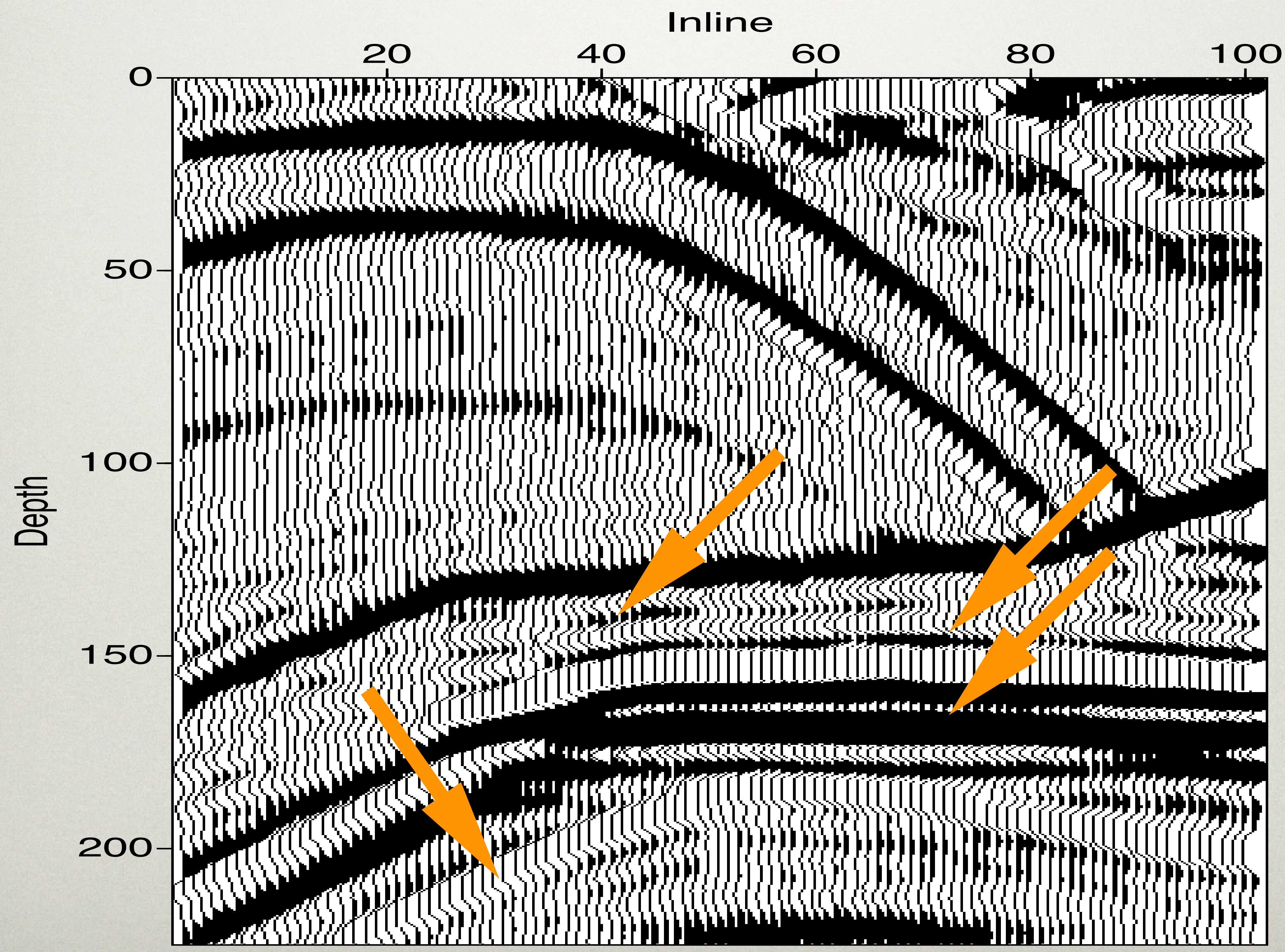
Given an estimate of the source signature:

- Boost high frequencies
- Be robust under noise
- Exploit:
  - continuity along reflectors
  - sparseness in transformed domain

Data

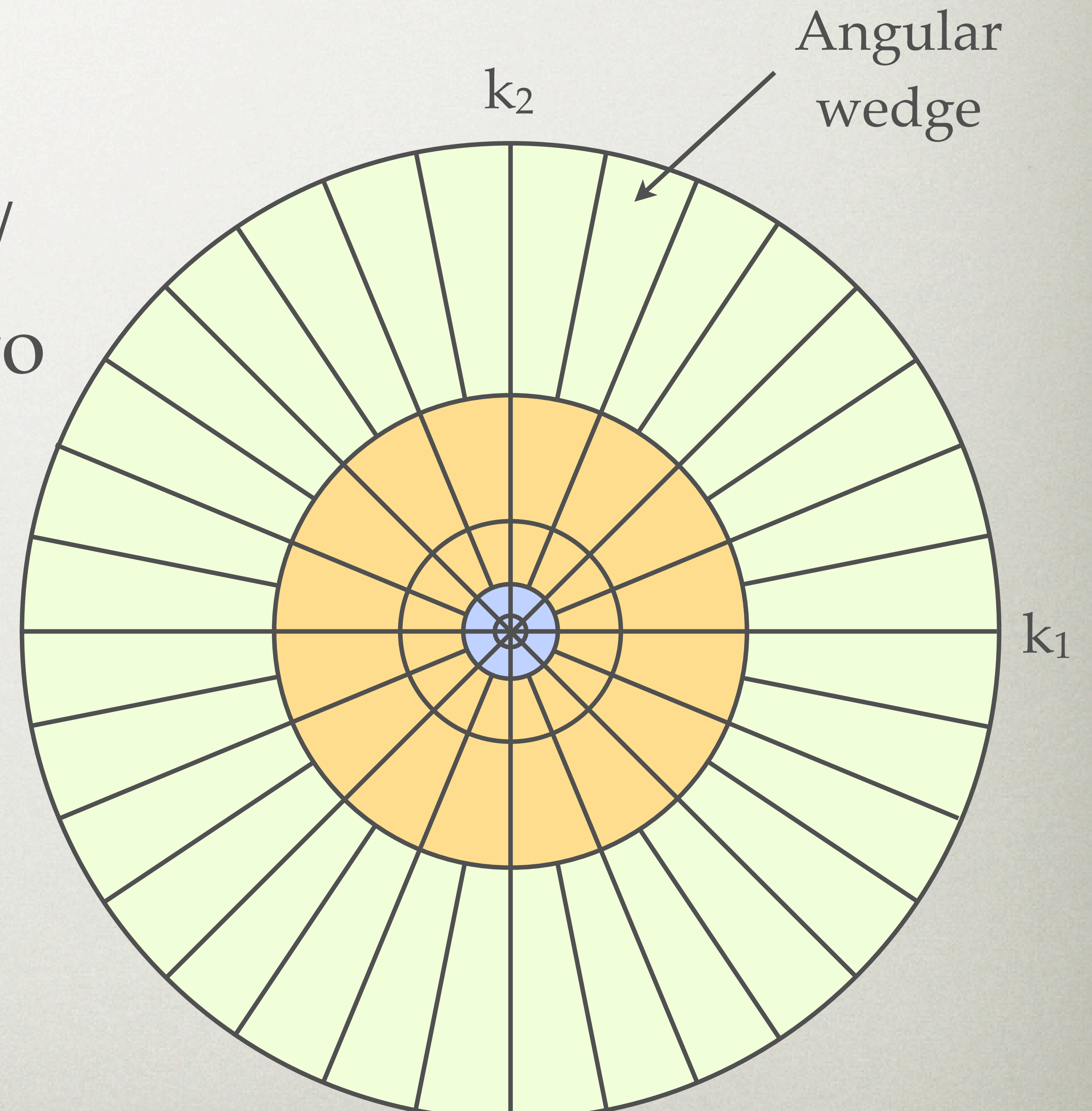


Deconvolution with Curvelet Frames



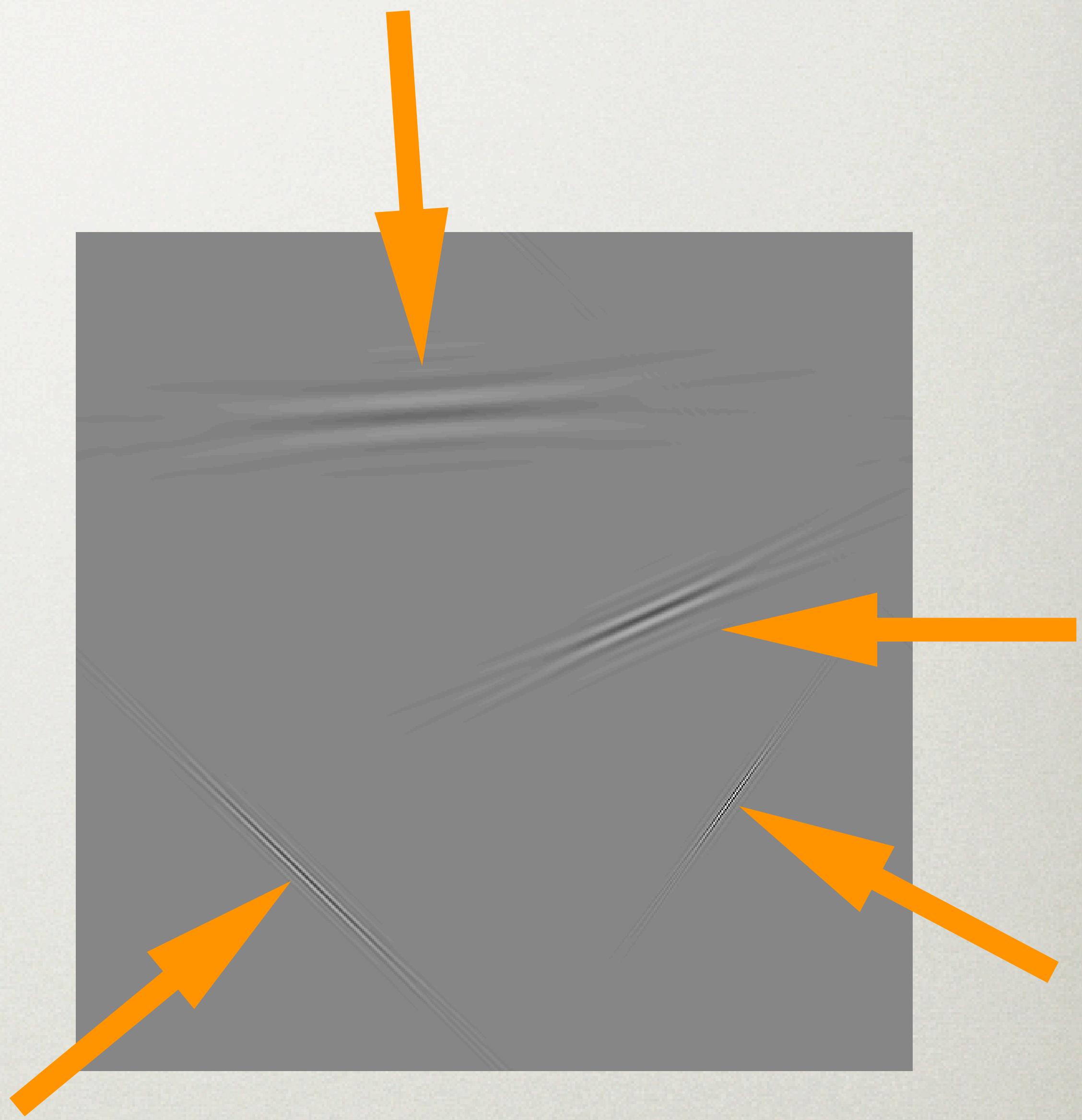
Curvelets [Candes & Donoho, 99]

- Tight frames
- Partitioning of the 2-D/ 3-D Fourier domain into angular wedges of second dyadic coronae
- Parabolic scaling law

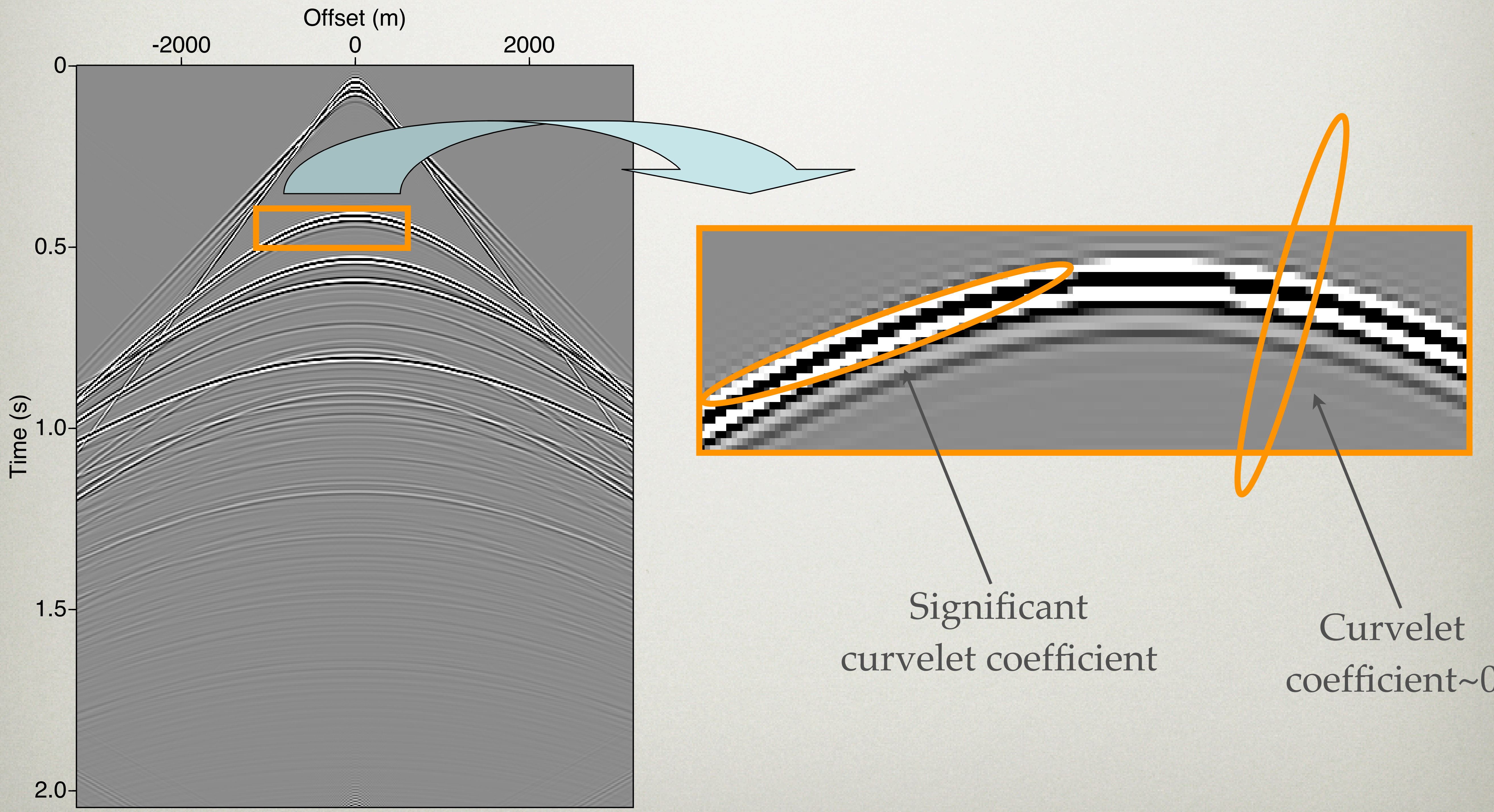


Curvelet Properties

- multi-scale
- multi-directional
- highly anisotropic
- localized both in space & frequency



Curvelets & Seismic Data



Curvelet Non-Linear Approximation Rate

Optimal: [Donoho, 01]

$$\|\mathbf{f} - \mathbf{f}_p^O\|_2^2 \propto p^{-2}, \quad p \to \infty$$

Fourier:

$$\|\mathbf{f} - \mathbf{f}_p^F\|_2^2 \propto p^{-1/2}, \quad p \to \infty$$

Wavelets:

$$\|\mathbf{f} - \mathbf{f}_p^W\|_2^2 \propto p^{-1}, \quad p \to \infty$$

Curvelets: [Candes & Donoho, 99]

$$\|\mathbf{f} - \mathbf{f}_p^C\|_2^2 \leq C \ p^{-2} \ (\log p)^3, \quad p \to \infty$$

Seismic Deconvolution

Forward problem:

$$\textit{data} \longrightarrow \underbrace{\mathbf{d}}_{} = \overset{\textit{convolution operator}}{\mathbf{K}}\underset{\textit{reflectivity}}{\mathbf{m}} + \mathbf{n} \longleftarrow \textit{white noise}$$

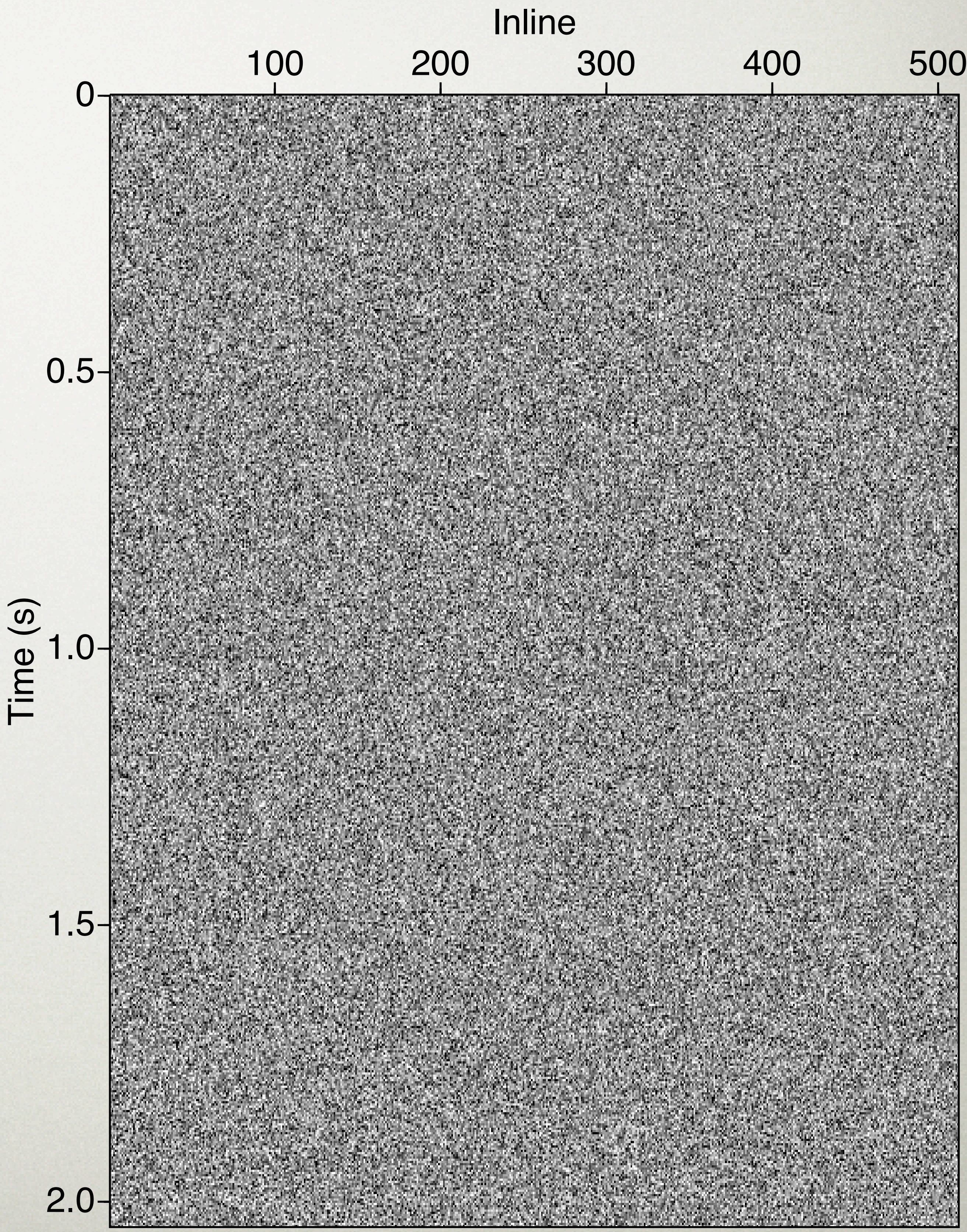
Conventional inverse problem:

$$\hat{\mathbf{m}} = \arg\min_{\mathbf{m}} \frac{1}{2} \underbrace{\|\mathbf{d} - \mathbf{K}\mathbf{m}\|_2^2}_{\textit{misfit data}} + \lambda \underbrace{\|\mathbf{m}\|_p^p}_{\textit{regularization}}$$

$$\hat{\mathbf{m}} = \arg\min_{\mathbf{m}} \frac{1}{2}\|\mathbf{d} - \mathbf{K}\mathbf{m}\|_2^2 + \lambda\|\mathbf{m}\|_2^2$$

- Assumption:

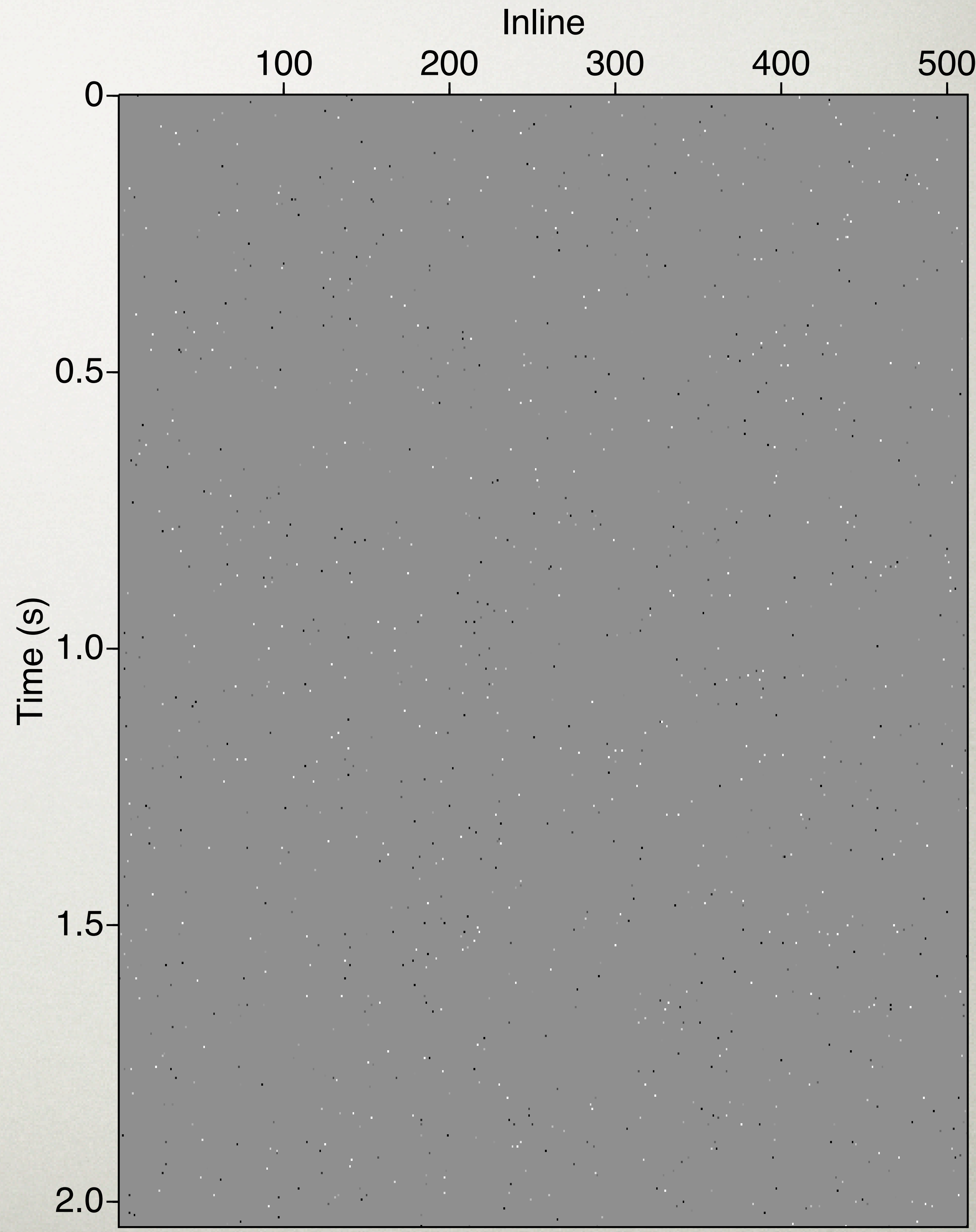
  White spectrum reflectivity



$$\hat{\mathbf{m}} = \arg\min_{\mathbf{m}} \frac{1}{2}\|\mathbf{d} - \mathbf{K}\mathbf{m}\|_2^2 + \lambda\|\mathbf{m}\|_1$$

- Assumption:

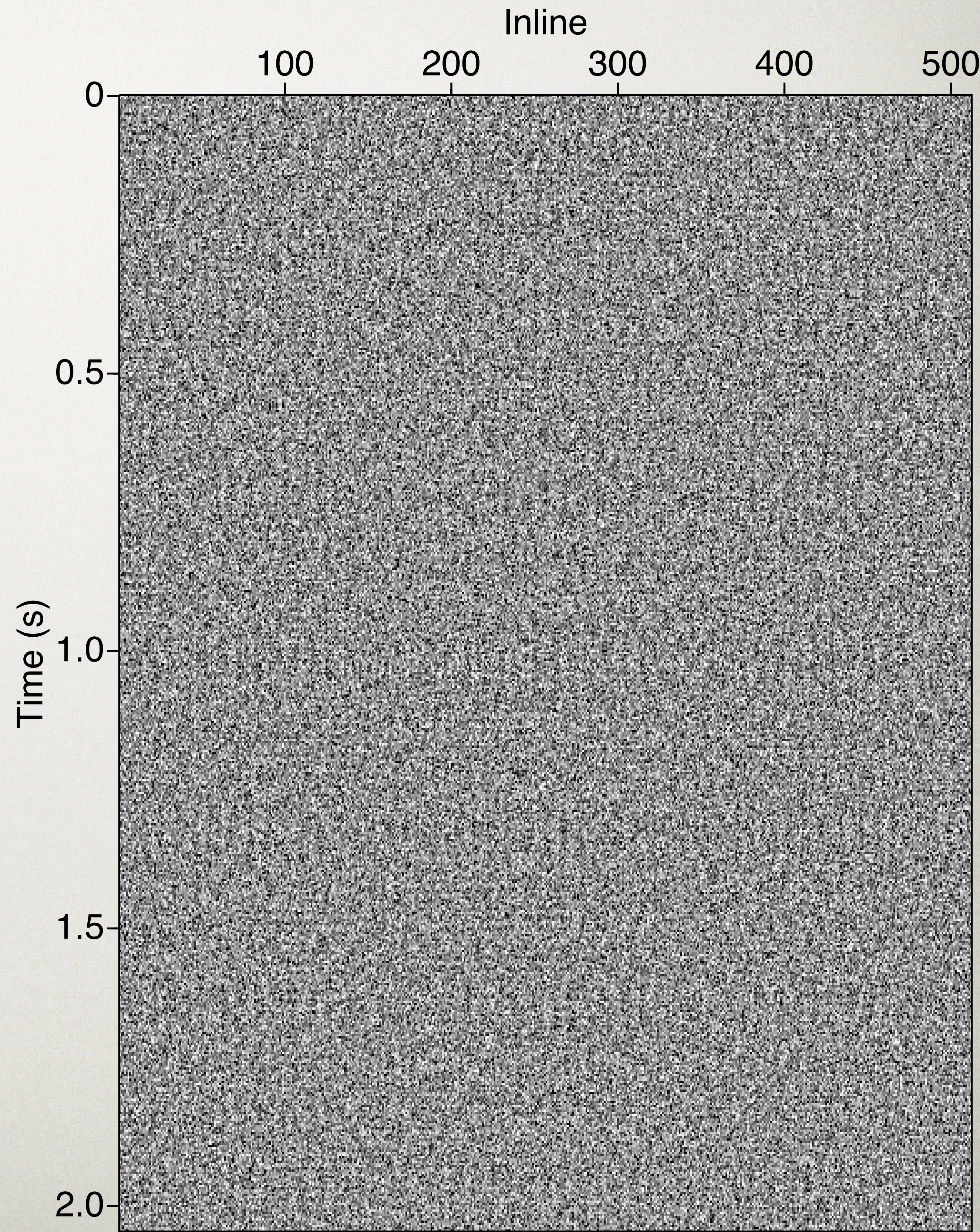
  Spiky reflectivity



$$\hat{\mathbf{m}} = \arg\min_{\mathbf{m}} \frac{1}{2}\|\mathbf{d} - \mathbf{K}\mathbf{m}\|_2^2 + \lambda\|\mathbf{B}_{\mathbf{F}}\mathbf{m}\|_1$$

- <u>Assumption</u>:

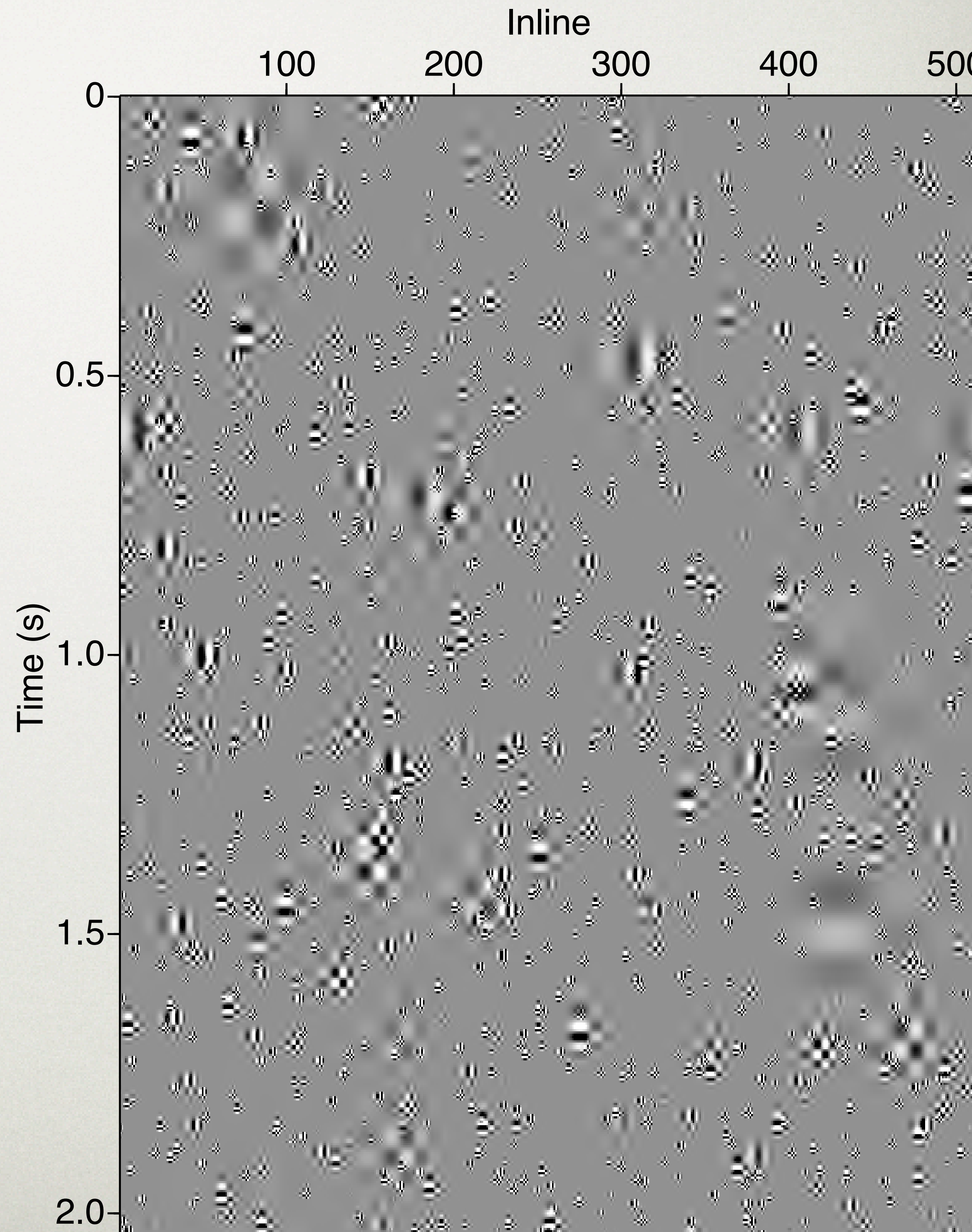
Fourier-like reflectivity



$$\hat{\mathbf{m}} = \arg\min_{\mathbf{m}} \frac{1}{2}\|\mathbf{d} - \mathbf{K}\mathbf{m}\|_2^2 + \lambda\|\mathbf{B_w}\mathbf{m}\|_1$$

- <u>Assumption</u>:

  Wavelet-like reflectivity



Curvelets for Seismic Deconvolution

Our forward problem:

$$\underbrace{\mathbf{d}}_{\textit{data}} = \underbrace{\mathbf{K}}_{\textit{convolution operator}}\underbrace{\mathbf{C}^*}_{\textit{curvelet composition operator}}\underbrace{\mathbf{x}}_{\textit{curvelet coefficients}} + \underbrace{\mathbf{n}}_{\textit{noise}} \quad \& \quad \underbrace{\mathbf{m}}_{\textit{reflectivity}} = \mathbf{C}^*\mathbf{x}$$

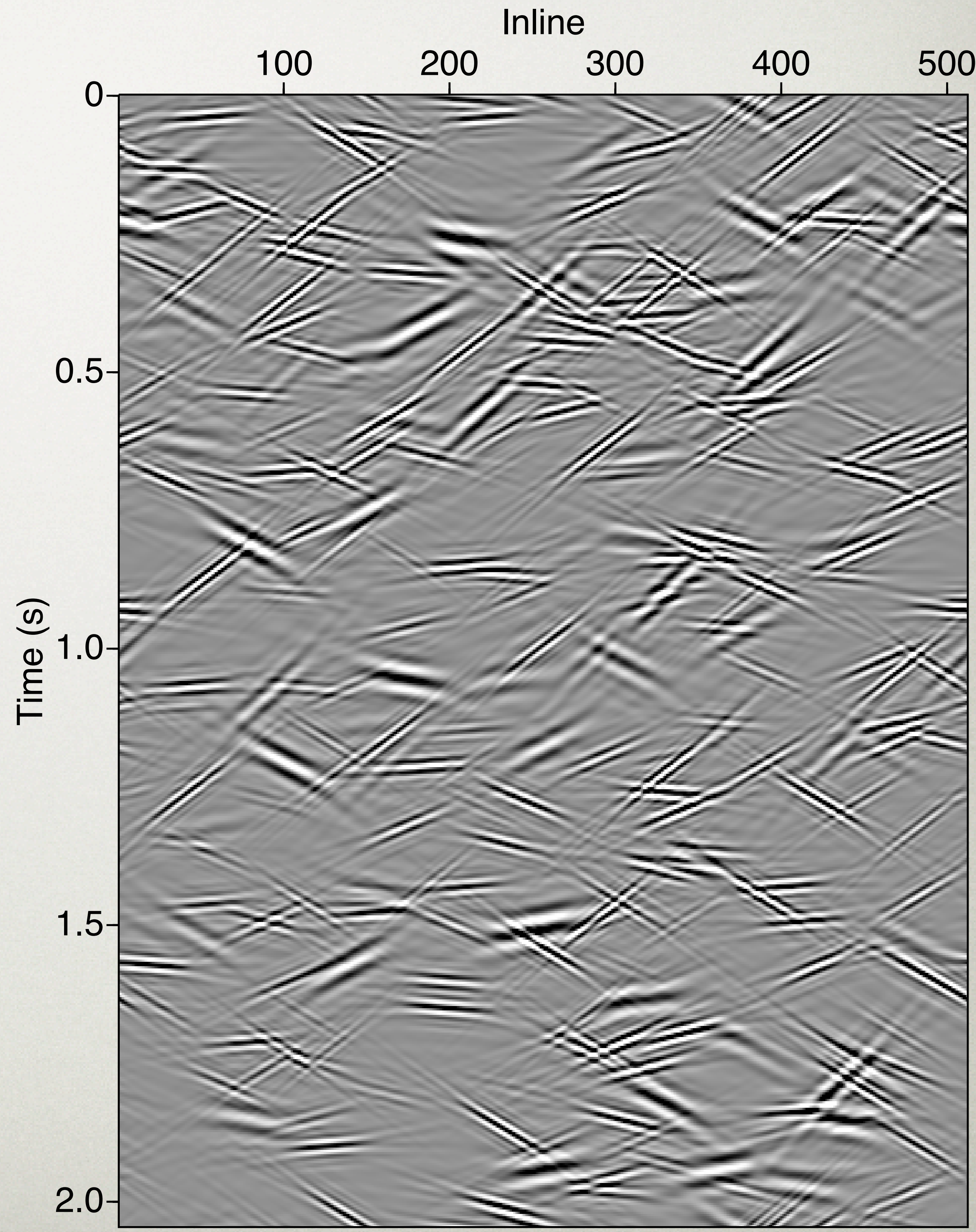
Our inverse problem:

$$\boxed{\hat{\mathbf{x}} = \arg\min_{\mathbf{x}} \frac{1}{2}\|\mathbf{d} - \mathbf{F}\mathbf{x}\|_2^2 + \lambda\|\mathbf{x}\|_1 \quad \text{with} \quad \mathbf{F} = \mathbf{KC}^*}$$

$$\hat{\mathbf{x}} = \arg\min_{\mathbf{x}} \frac{1}{2}\|\mathbf{d} - \mathbf{F}\mathbf{x}\|_2^2 + \lambda\|\mathbf{x}\|_1$$

- <u>Assumption</u>:

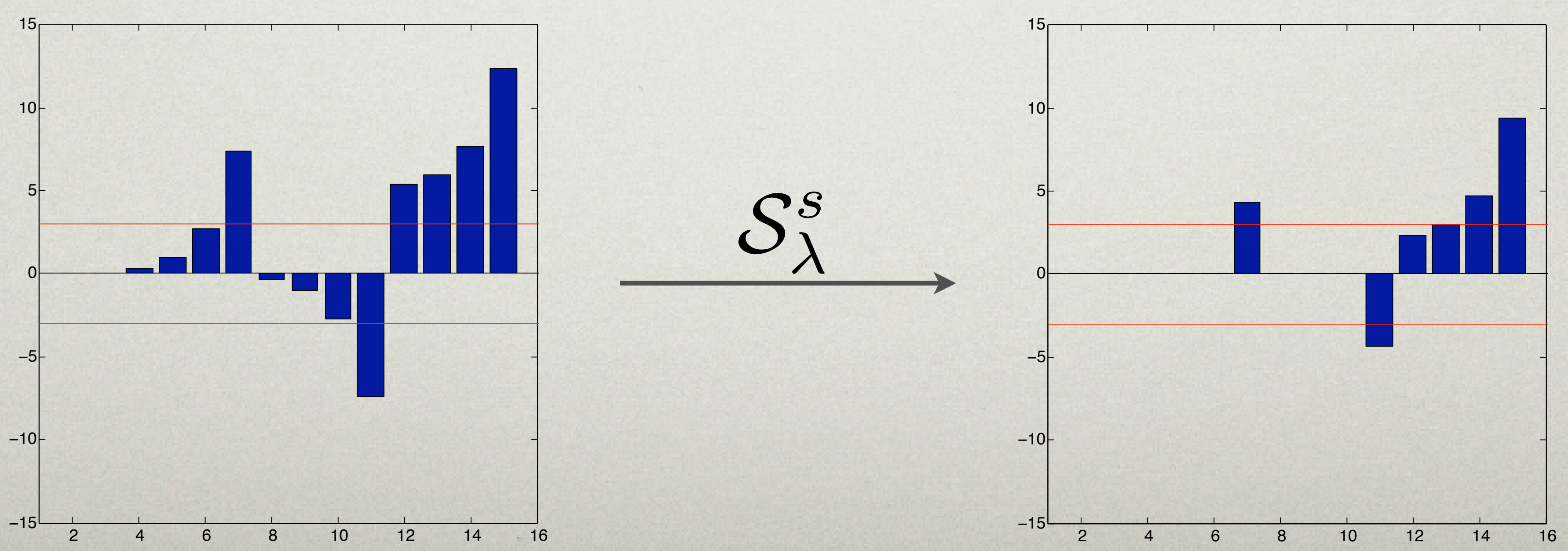
Curvelet-like reflectivity



Existing Methods for L1-norm Optimization

- Iterative Re-weighted Least-Squares (IRLS) [Gersztenkorn et al., 86]
- Landweber iterations and soft-thresholding [Daubechies, 05]

$$\mathbf{x}^m = \mathcal{S}^s_{\lambda_m}\left[\mathbf{x}^{m-1} + \mathbf{F}^*(\mathbf{d} - \mathbf{F}\mathbf{x}^{m-1})\right]$$



Our Method

1. Initialization (initial & final threshold, number of inner & outer iterations, initial guess)
2. Inner loop
   - CG iterations: estimate updated
   - Estimate soft-thresholded
3. Threshold updated
4. Return to 2 until end

Restarted CG

Noise regularization (sparseness-constraint)

Example

- Reflectivity:

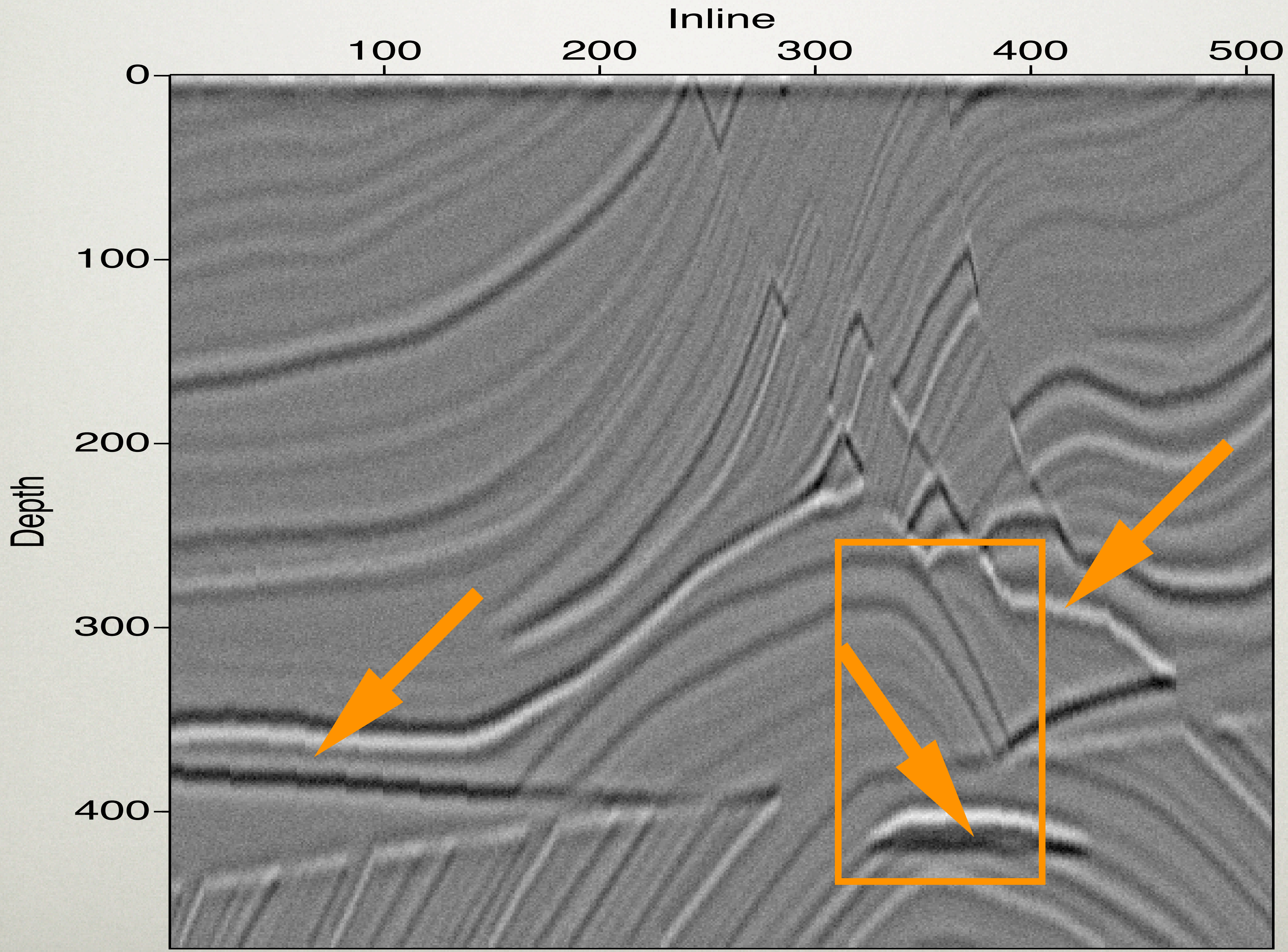
  Marmousi model

- Source:

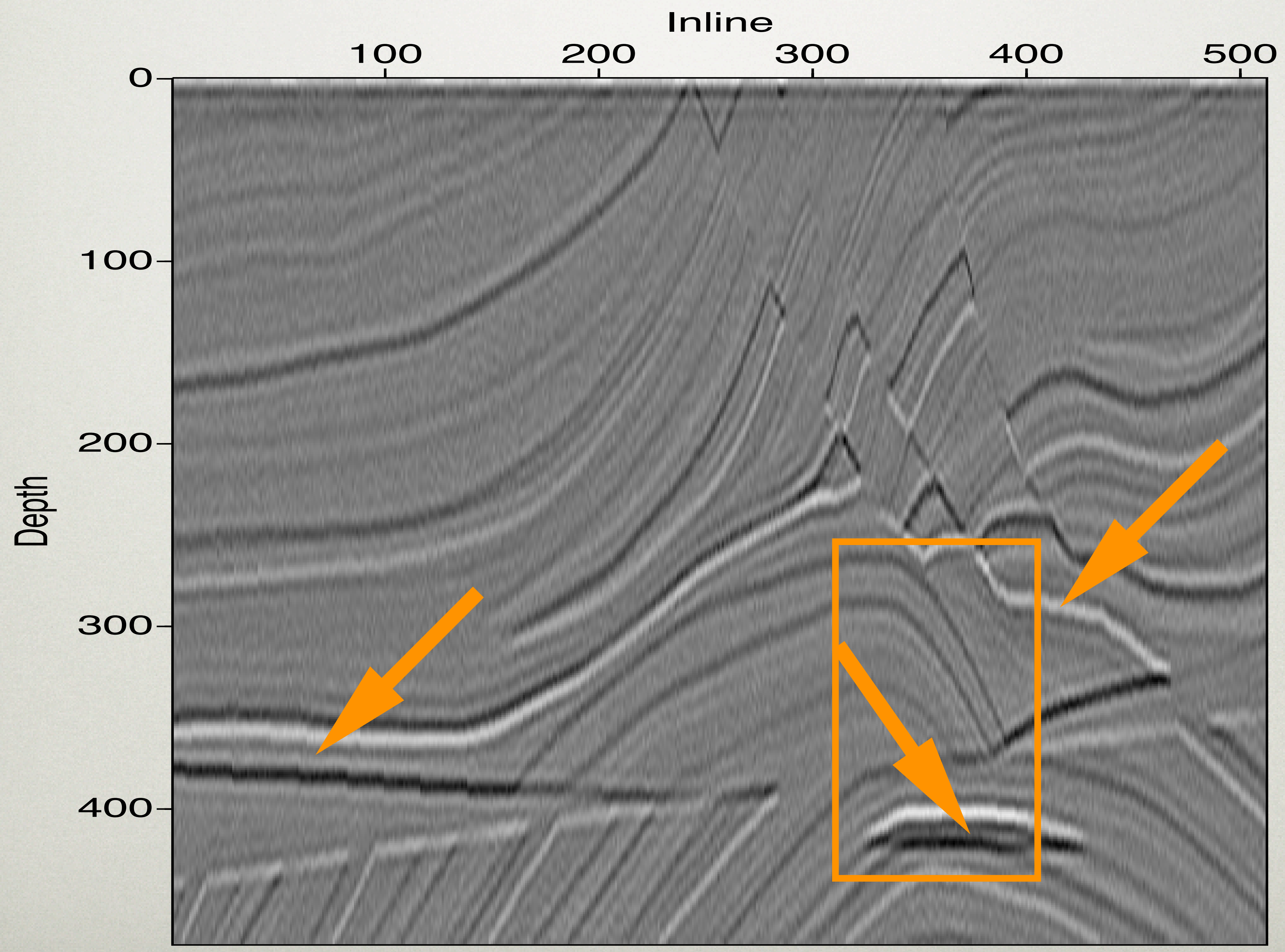
  Ricker wavelet

- Signal-to-Noise Ratio ~ 8 dB

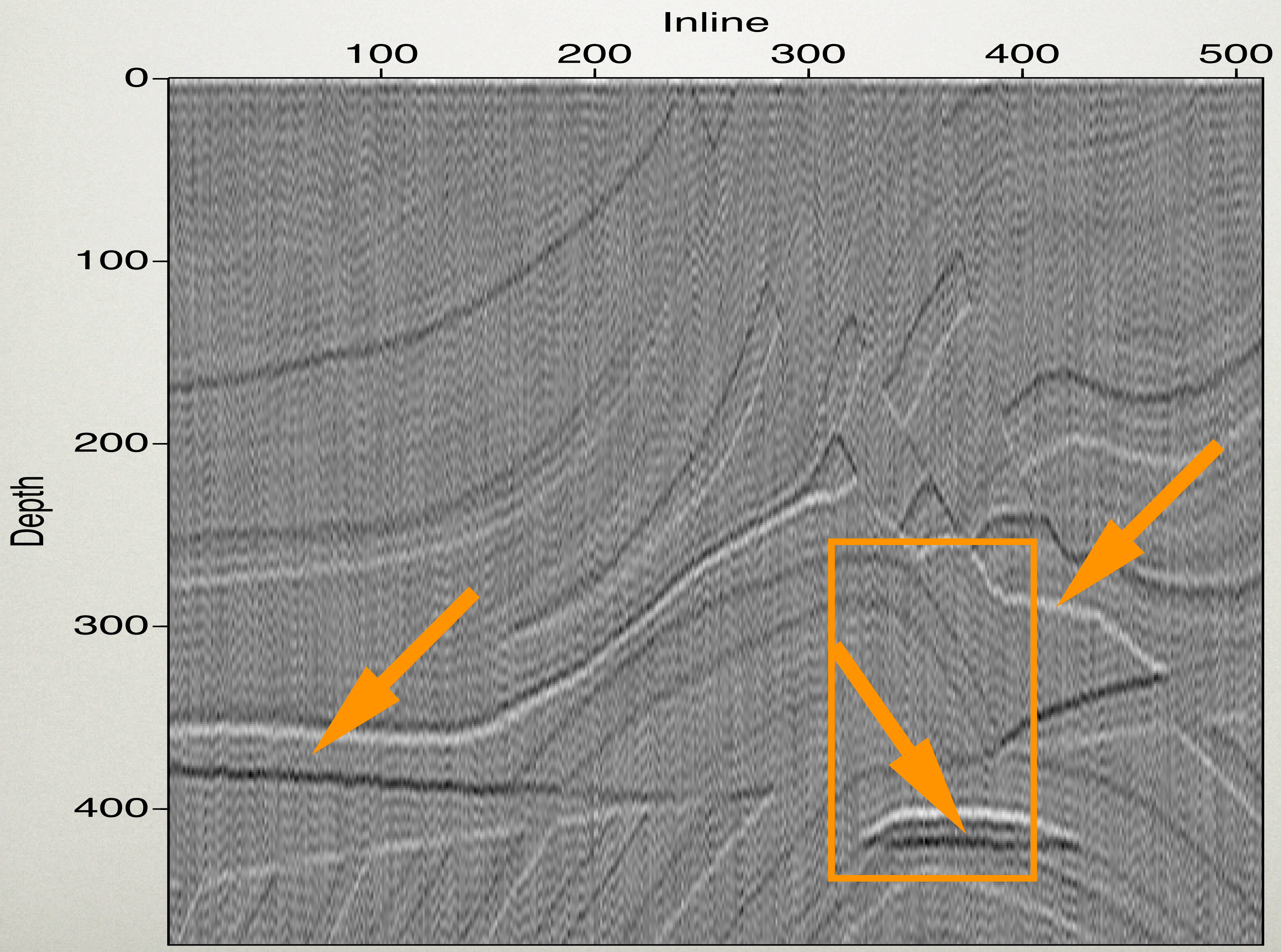
Data



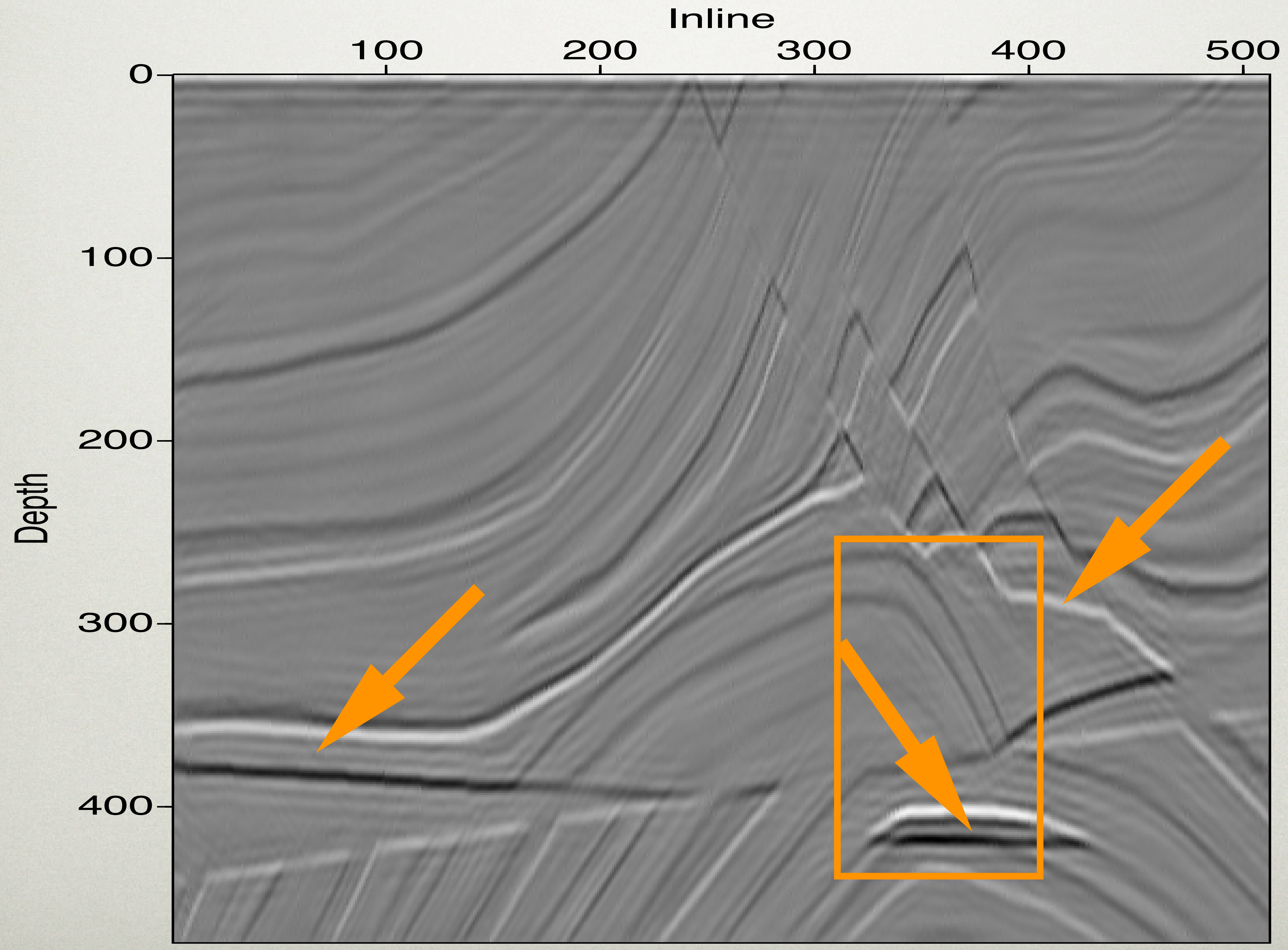
Wiener Deconvolution



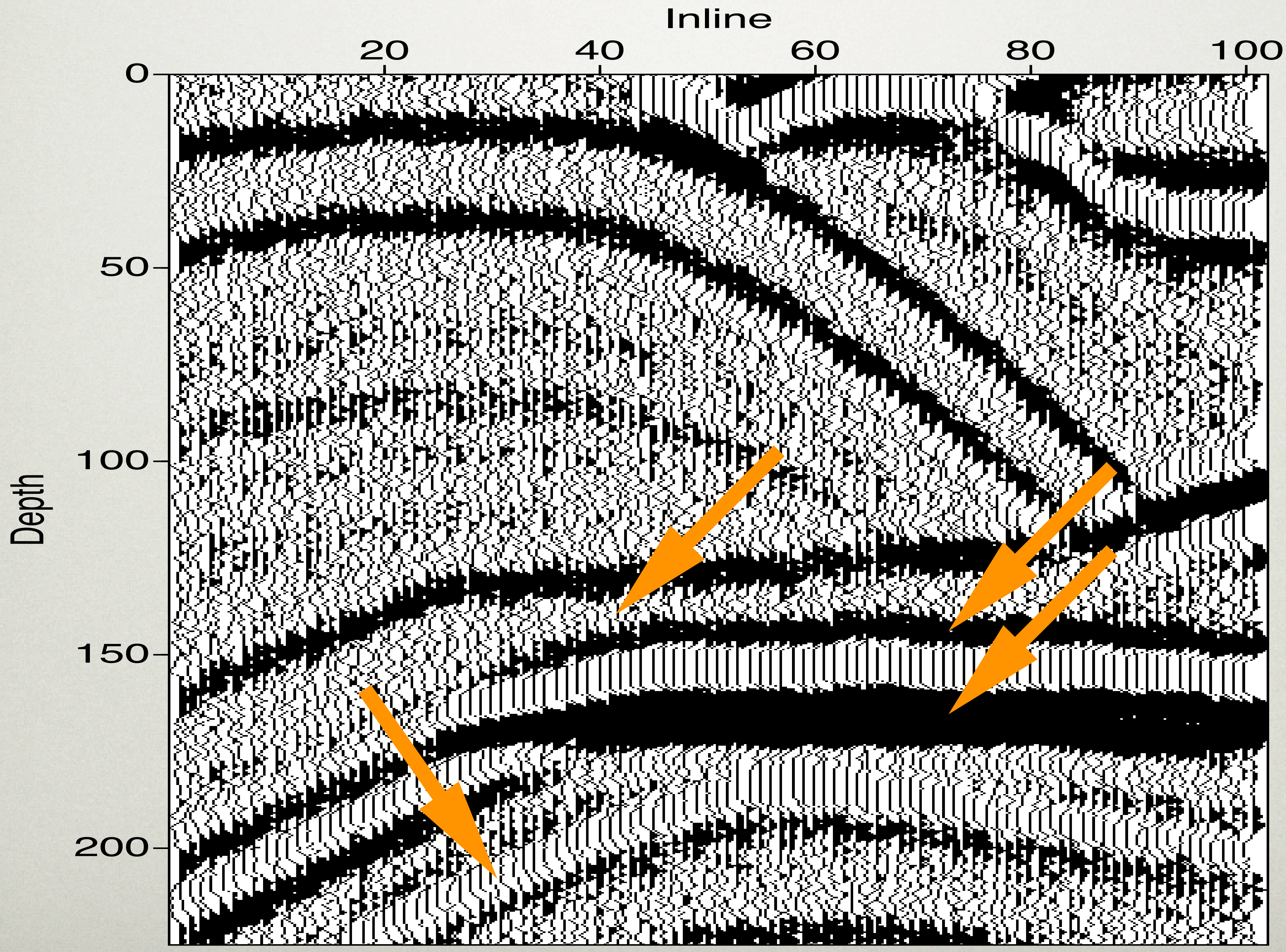
Deconvolution (CG only)



Deconvolution with Curvelet Frames



Data



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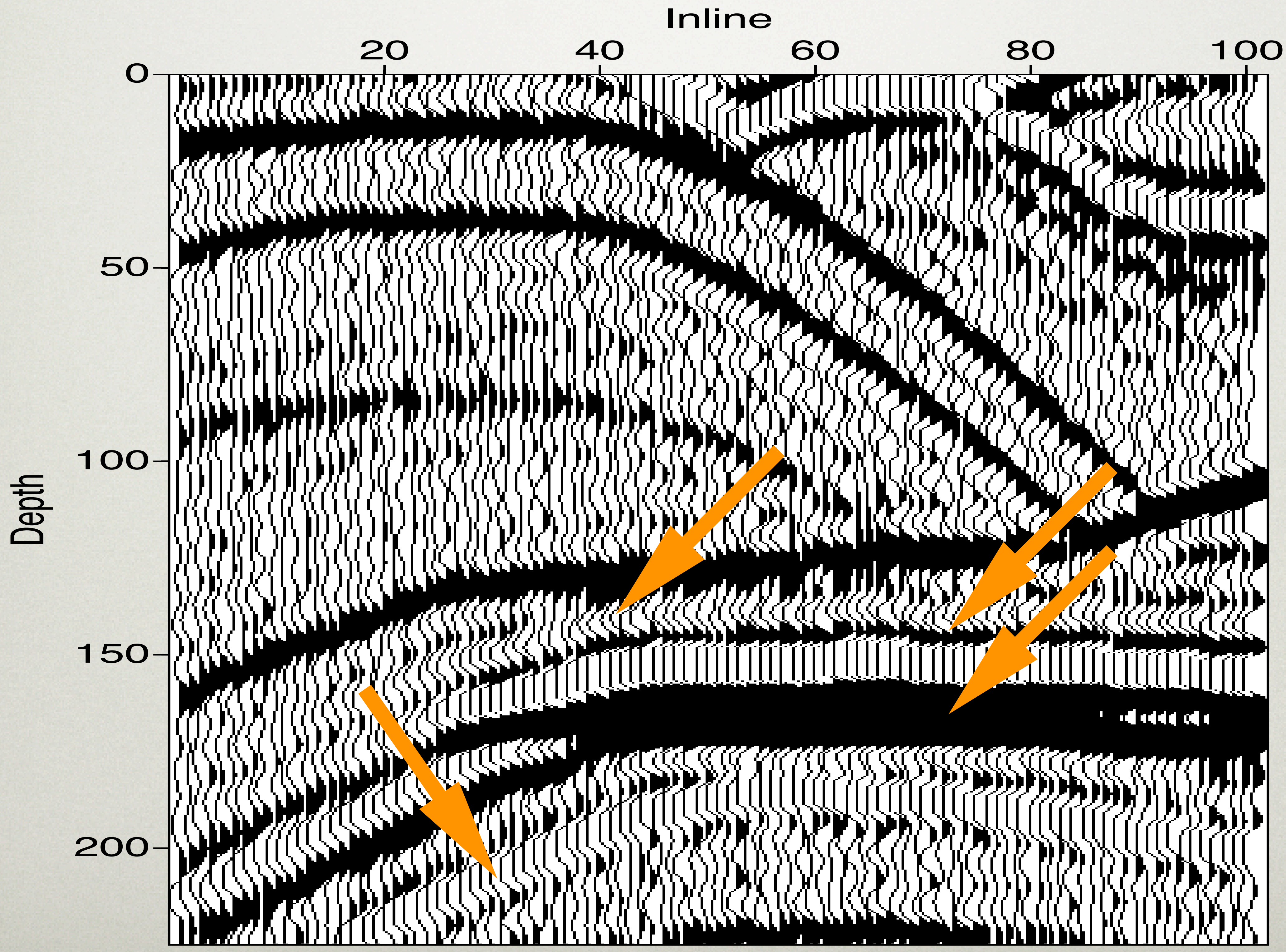

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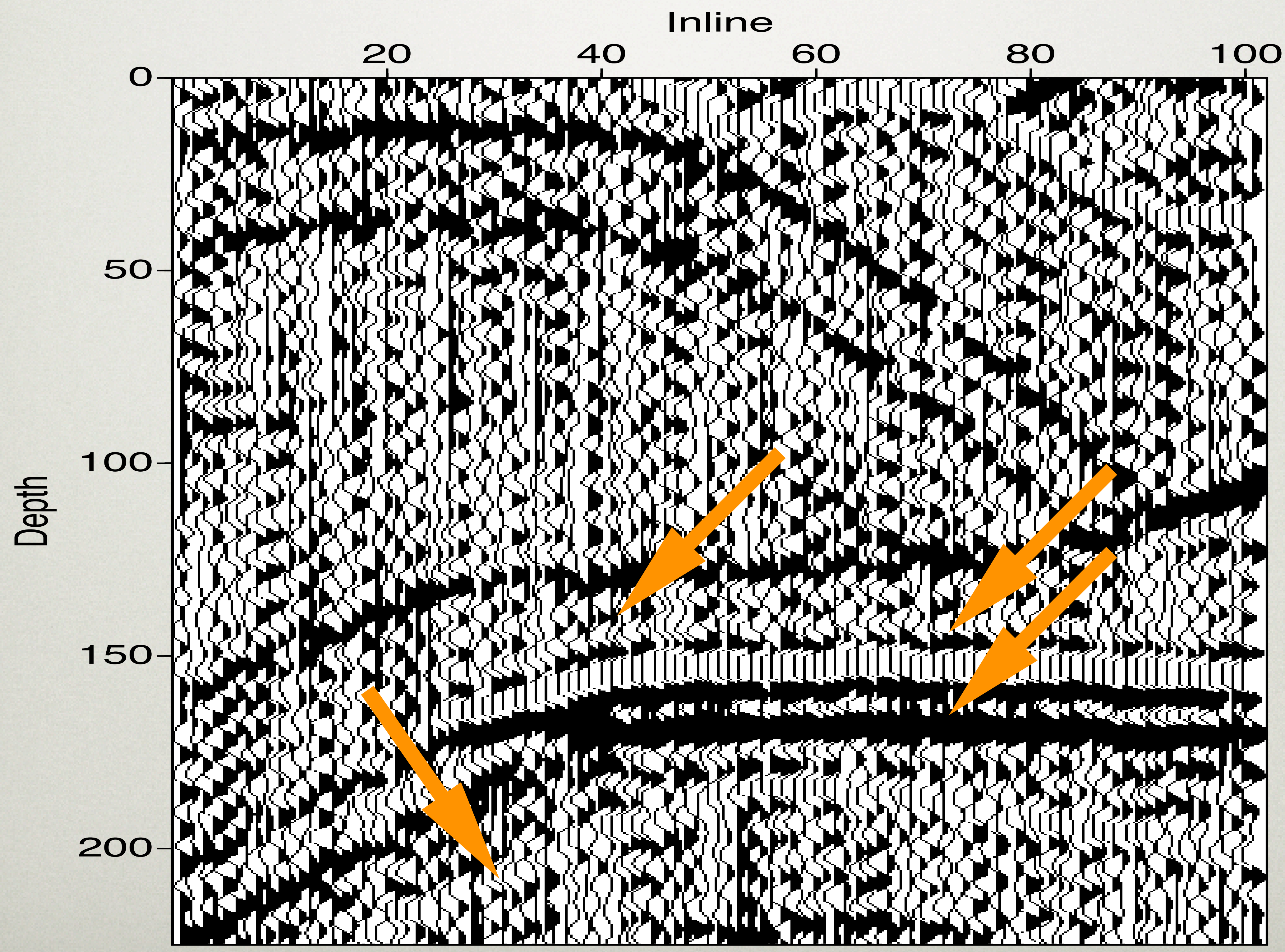
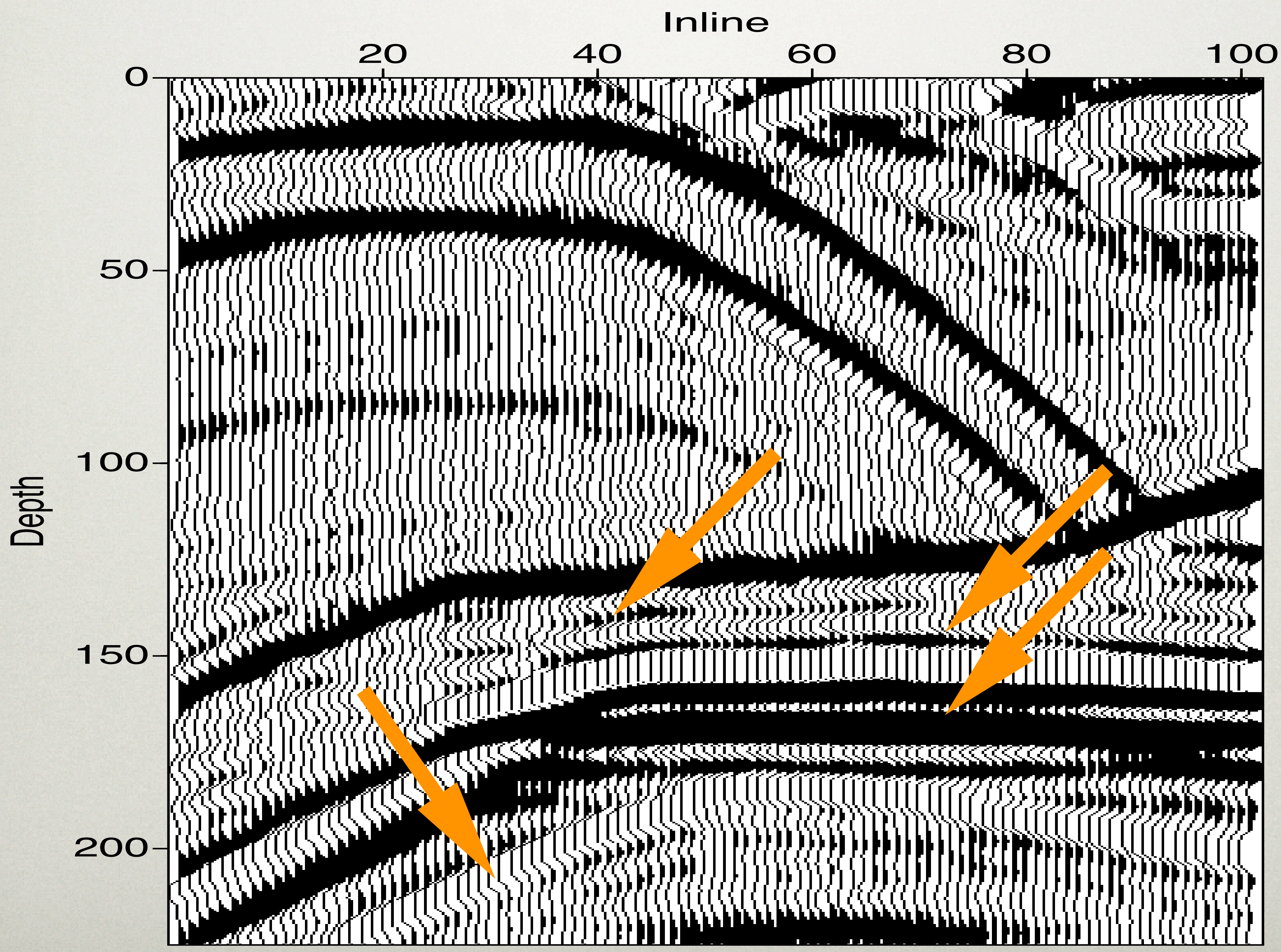


Deconvolution with Curvelet Frames



Conclusions

- 2-D deconvolution as opposed to trace-by-trace (3-D extension straightforward)
- Fast method (CG + FDCT)
- Improved continuity along reflectors
- Robust under noise

Acknowledgments

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