Curvelet processing and imaging: adaptive ground roll removal

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Abstract

In this paper we present examples of ground roll attenuation for synthetic and real data gathers by using Contourlet and Curvelet transforms. These non-separable wavelet transforms are locoalized both (x,t)- and (k,f)-domains and allow for adaptive seperation of signal and ground roll. Both linear and non-linear filtering are discussed using the unique properties of these basis that allow for simultaneous localization in the both domains. Eventhough, the linear filtering techniques are encouraging the true added value of these basis-function techniques becomes apparent when we use these decompositions to adaptively substract modeled ground roll from data using a non-linear thesholding procedure. We show real and synthetic examples and the results suggest that these directional-selective basis functions provide a usefull tool for the removal of coherent noise such as ground roll.

Introduction

Recently, the world of two dimensional transforms has been considerably expanded by the introduction of new varieties of two dimensional multiresolution transforms. Examples of them are ridgelets, beamlets, ridgelet packets, curvelets and contourlets. These transformations provide us with invaluable tools for seismic processing that were not avaliable before. In this abstract, we present some examples of the use of two of these transforms to eliminate ground roll in seismic gathers. Although in principle this kind of coherent noise seems to be easy to eliminate by prediction and subtraction using conventional linear Radon transforms, or simple filtering with two dimensional Fourier transforms, in practice its prediction proves to be difficult since it contains dispersive energy and usually is aliased. This is why we need to look at more sophisticated transforms that can take care of the complicated nature of this noise.

In the last years wavelet transforms have been used with success to attenuate random noise in seismic data. Most of these applications use thresholding which seperates the noise from signal. This approach is build on the premise that wavelets concentrate the signal's energy into a limited number of large coefficients. The large coefficients are preserved during thresholding and the success of this method depends on (i) how well the basis functions are able to concentrate the signal's energy and (ii) how well these functions are able to localize. Both Curvelets and Contourlets (ref. 1-4), greatly improve the directional selectivity and are able to locally and sparsely represent the signal and ground roll.

Non-separable wavelets: Contourlets & Curvelets

Curvelets, as proposed by E.J. Candès and F. Guo. (ref. 1-3), and contourlets (ref. 4) are a new family of non-seperable wavelet bases that represent seismic reflectors - that can be considered to lie on piece-wise smooth curves - very sparsely which makes them ideal for use in seismic processing. These transforms can obtain nearly optimal sparseness for seismic signals for three reasons. First, they have a rapid decay for the reconstruction error as a function of the largest coefficients. Second, they have the ability to concentrate the signal's energy on a limited number of coefficients. Lastly, they have the ability to map (coherent) noise and signal to different areas of the transformed domain. Curvelets/Contourlets achieve these goals because they are multi-scale, multi-directional, anisotropicly shaped, directionaly selective, local and almost orthogonal (they are a tight frame, see for details see referances 1-4). Because of their anisotropic shape, these basis functions align themselves with seismic events and because they are local, they are more flexible than a representation yielded by high-resolution Radon (ref. 5).

The use of these transforms to remove predicted ground roll starts off by stating a simple, generic denoising problem

$$\mathbf{d} = \mathbf{m} + \mathbf{n}. \tag{1}$$

In this case, our data is simply made up of a superposition of ground roll-free data $\mathbf{m}$ and ground roll $\mathbf{n}$. To solve this denoising problem for ground roll being white noise (which it is not), we need to construct a diagonal decision operator that minimizes the energy differences between the data with and without ground roll. It has been shown by D. L. Donoho, I. M. Johnstone and S. G. Mallat (ref. 6 and 7) and others that for a certain class of models, one can obtain near optimal signal to noise ratios for denoised data by projecting noisy data onto a basis function representation that is optimal for that particular class of models. Now most of the model's energy will reside in only a few coefficients which allows for the definition of a shrinkage estimator that separates noise from the model. For basis functions that are also local, one can show that soft thresholding on the coefficients suffices to approximatly solve the denoising problem, i.e.

$$\hat{\mathbf{m}} = \mathbf{B}^{-1}\Theta_{\mu}\left(\mathbf{B}\mathbf{d}\right) \tag{2}$$

where $\mathbf{B}^{-1}$ is the (pseudo)-inverse of **B**, and Θ_{μ} is either a linear filtering operation, removing certain frequency bands, or a non-linear soft/hard thresholding operator which mutes coefficients according to their magnitude. For orthonormal basis functions, the threshold equals (ref. 6 and 7):

$$\mu = \sigma\sqrt{2\log_e N} \quad (3)$$

Here, σ is the standard deviation of the noise and N being the number of data samples. Because of the superior localization in both position and angle, we relax the white additive noise in Eq. 1 towards colored ground roll. In this case, we simply replace the constant threshold level of Eq.3 by a level a level defined in terms of the predicted ground roll. This proocedure simply corresponds to threshold-level given by

$$\mu = \eta|\mathbf{Bn}| \quad (4)$$

We are thus, with the non-linear approach, able to adaptively decide which parts of the signal belong to our ground roll and threshold them. The μ contains the relative ground roll model and η represents an additional control parameter which sets the confidence interval (e.g. 95 % for η = 3) (de)-emphasizing the thresholding.

Model for the ground roll

Ground roll noise is highly dispersive in the time-offset domain, and therefore, very difficult to model. At the same time, it has a dispersive nature that makes it difficult to eliminate with most tools; therefore proper modeling is necessary for testing our method. To do that, we take advantage of the fact that the ground roll has a relatively simple mapping in the frequency – slope domain (ref 8.). Thus, we create the ground roll directly in the slant stack transform, following an approximated pattern of dispersive curves and overtones as described in the reference.

Two Transforms, Two Methods: linear versus non-linear

Denoising according the linear approach can be seen as a direct method. First, take the transform (we use Contourlets in this case because they have simple representation in terms of frequency bands), then remove certain bands where ground roll resides (see Fig, 1 (d)) and reconstruct with the inverse transform. In the second non-linear approach, we use a prediction of the ground roll to locally guide us which coefficient to mute. Because, these basis functions live in some wedge of the frequency plane and are localized in physical space, they average over some area and hence integrate out possible errors in the predicted ground roll. By simply defining the threshold in terms of the predicted ground roll we obtain a robust method that adaptively removes the ground roll. The noise model does not need to be exact, it only needs to give a relatively accurate predicition of the noise characteristics.

Examples

Using a synthetic example shown in Figure 1a, we predict the noise in Figure 1b by using the contourlet transform. Figure 1c is the signal obtained by subtraction. Figure 1d shows the different bands in contourlet space, which is acually a representation of a polar coordinate system in the 2-D frequency domain that seperates data in location, frequency and orientation. Figure 2 exemplifies the use of the contourlet transform to attenuate ground roll in the shot gather number 25 from Yilmaz. Although the noise is well predicted (Figure 2b), some of the signal has leaked to the noise prediction for shallow events and subtraction. This predicted noise is muted to prevent attenuation of signal in the shallow reflections. Figure 2c shows the gather after subtraction of the noise. Figure 2d shows a cartesian map of the contourlet transform for one particular window in the data. Figure 3a represents a synthetic set of hyperbolic events simulating a gather, with the additon of dispersive grond roll generated as already described. This synthetic was generated with the expressed purpose to challenge the methods of ground roll extraction that we wish to test by containing steeply dipping hyperbolas which start to take on the same slope of the ground roll. We can now predict the noise with the curvelet transform. Figure 3b shows the denoised signal and Figure 3c shows the predicted ground roll. It is worth noting that the predicted model of the ground roll and the actual ground roll applied to the synthetic example contain not only different slopes but also a different number of slopes. The difference between the ground roll applied in the synthetic example and the ground roll which was used as a model in the thresholding is shown in Figure 3d.

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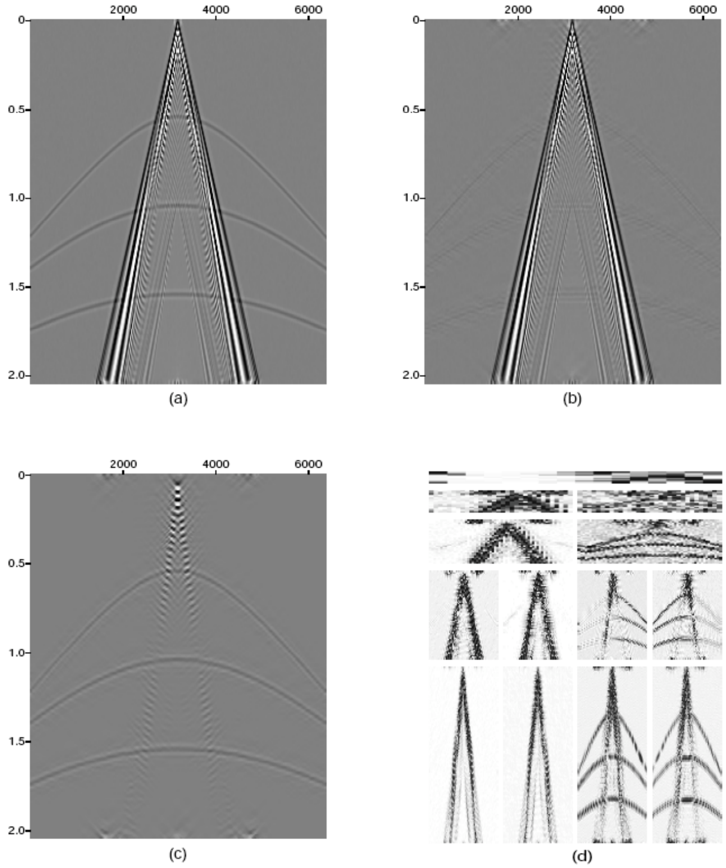

Figure1: Synthetic example of ground roll attenuation by contourlet transform (a) Data. (b) Predicted noise. (c) Data after subtracting noise. (d) Contourlet tranform.

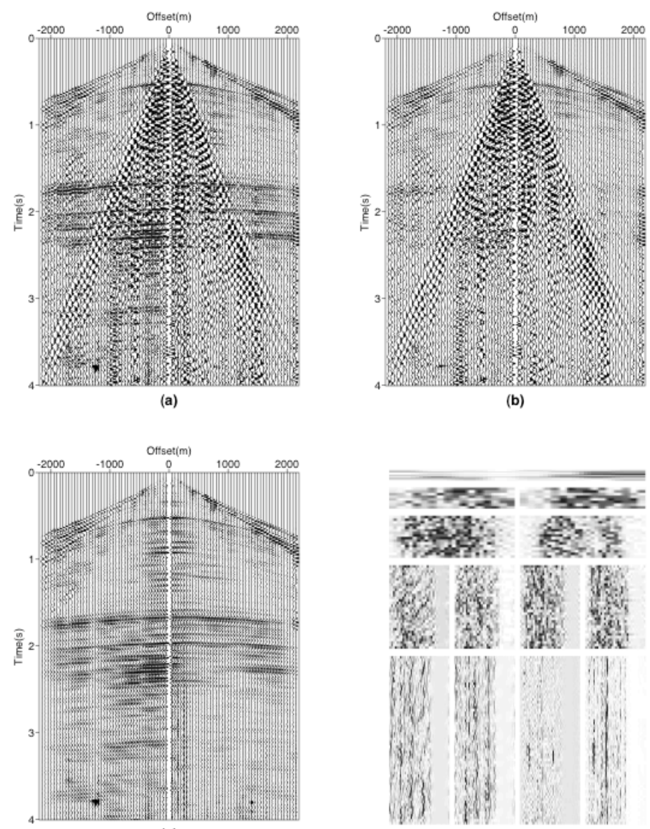

Figure 2: Ground roll attenuation by using Contourlets. (a) data. (b) Noise prediction from the Contourlet transform. (c) data after substracting predicted noise. (d) Contourlet space for one particular window on the data.

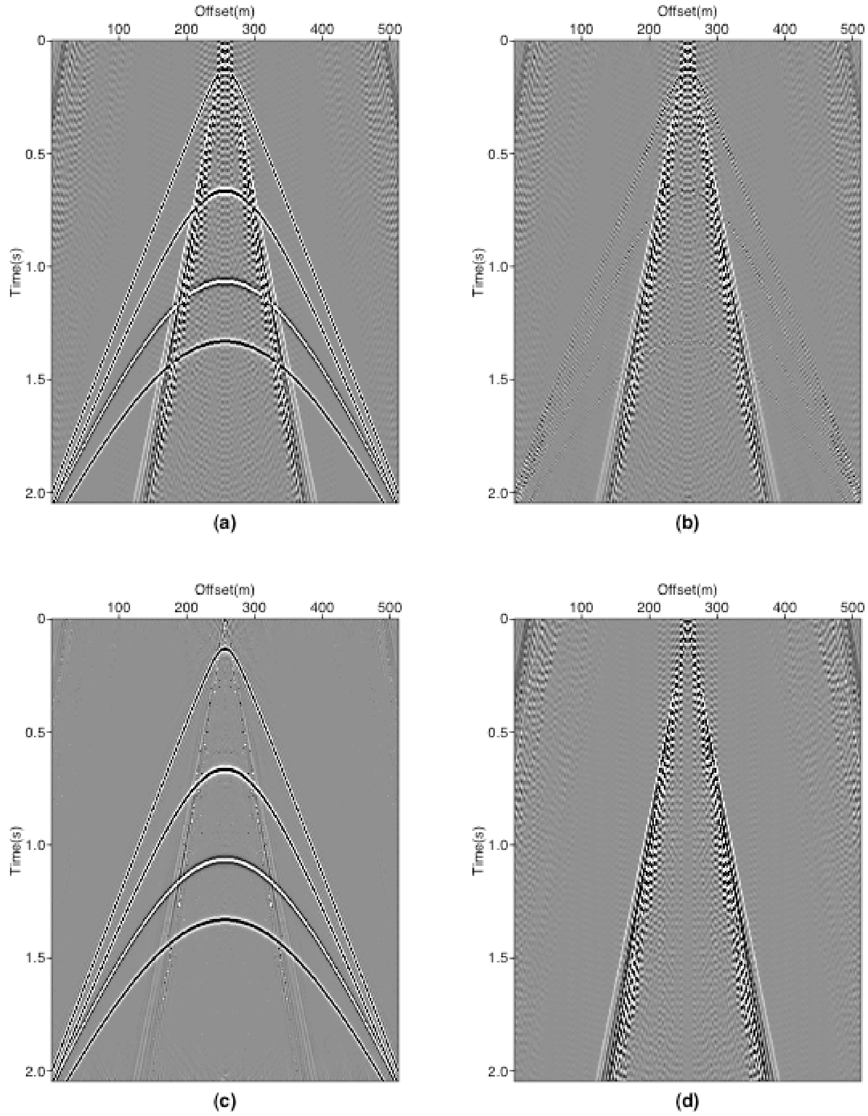

Figure 3. Synthetic example of denoising using a prediction of the ground roll. (a) Synthethetic with some events and ground roll. (b) Predicted noise by our non-linear adaptive substraction method (cf. Eq.2). (c) Denoised result. (d) Difference in the actual and the modelled ground roll used for the threshold in the denoising.

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