Curvelet imaging and processing: adaptive multiple elimination

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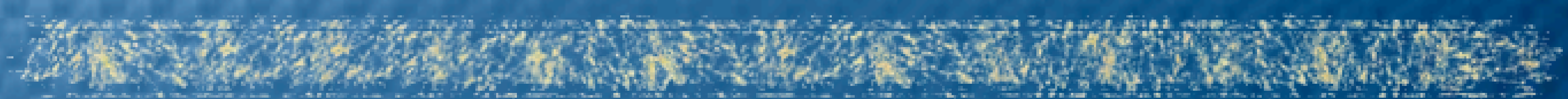


Thanks to Emmanuel Candes

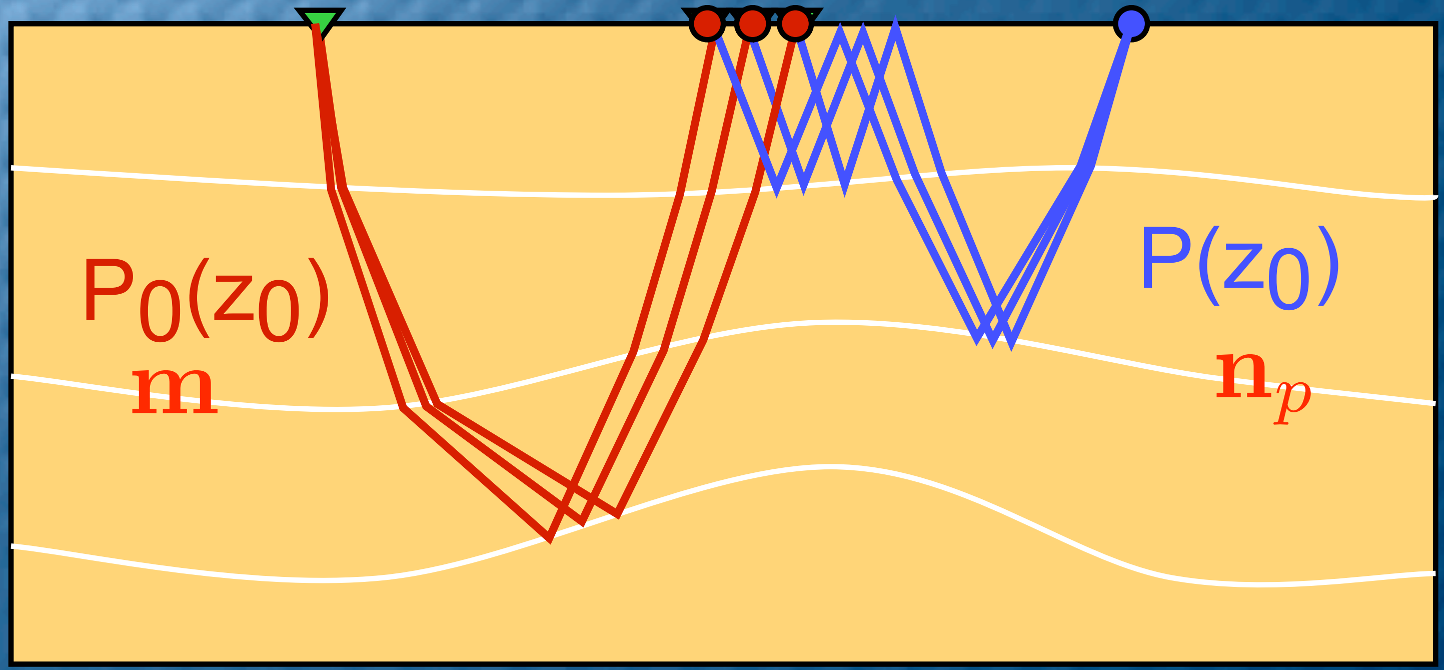
Motivation

Multiple removal:

- ★ **Geometry effects in 3D surface multiple prediction**
- ★ **New approach to adaptive subtraction robust under**
  - **phase rotations**
  - **misalignments**

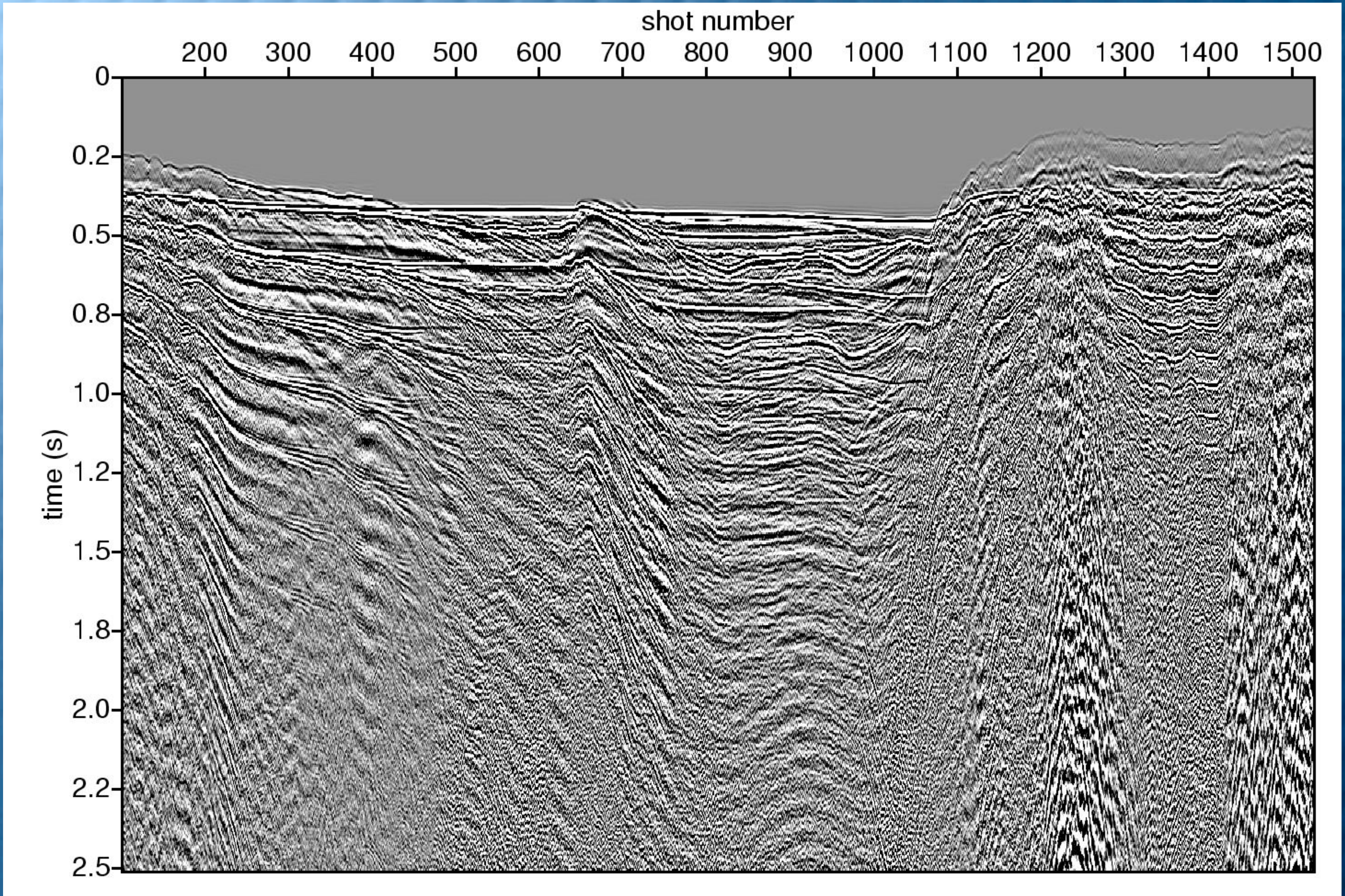
Surface multiple elimination

Multiple prediction: data convolution along surface



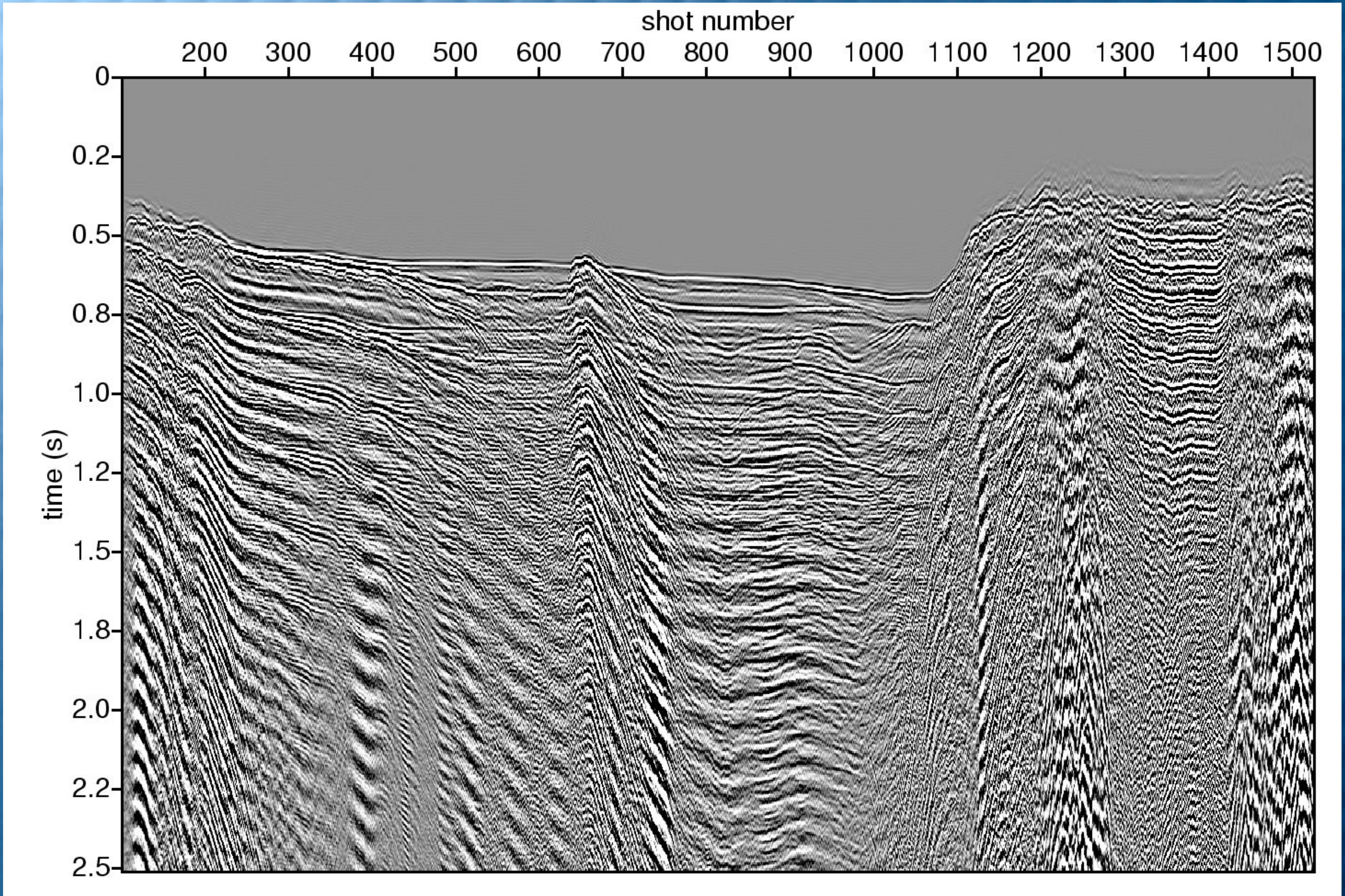
Offshore Scotland example

Common offset (500 m) with multiples



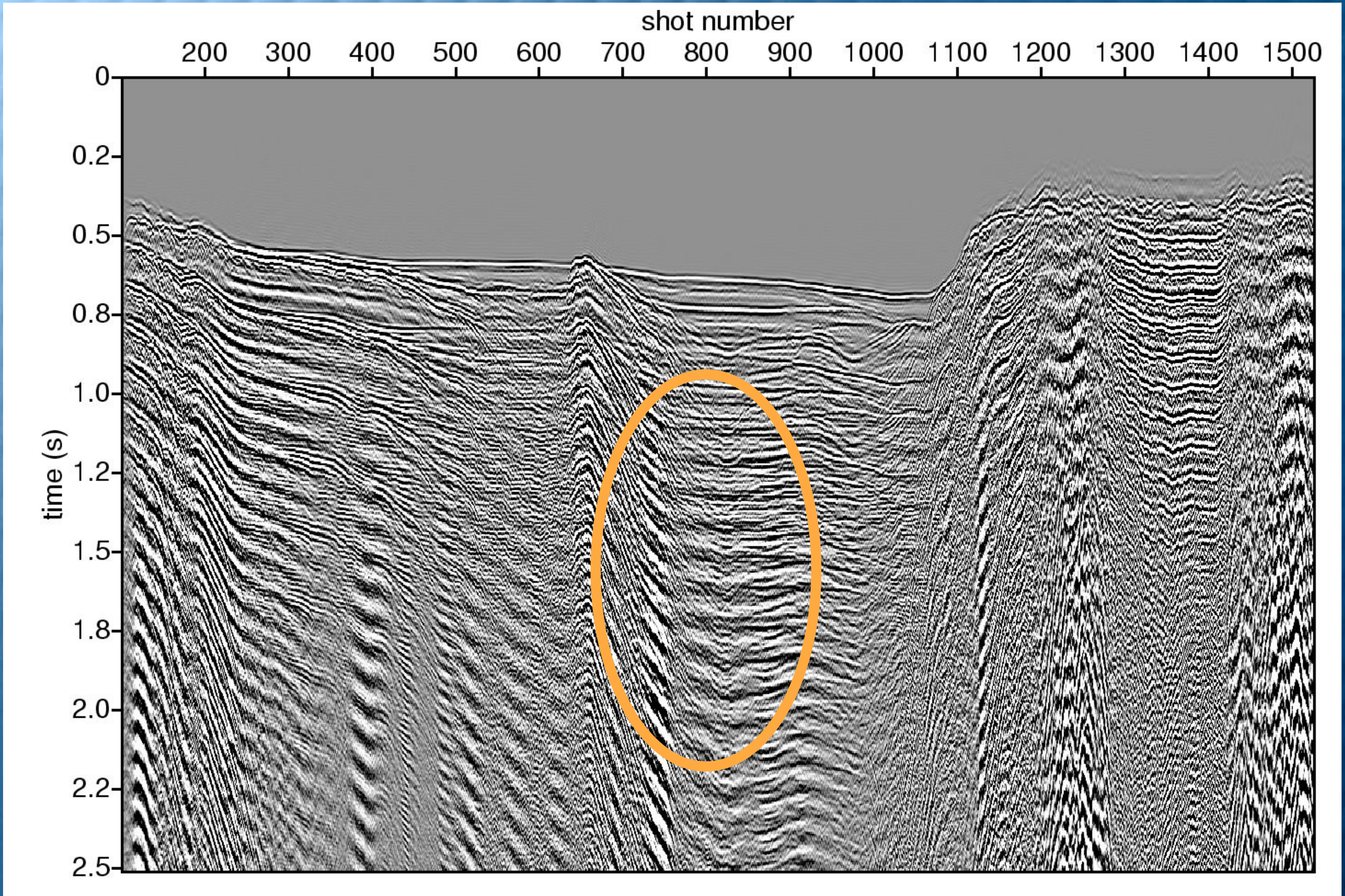
Offshore Scotland example

Predicted multiples (no adaptation)



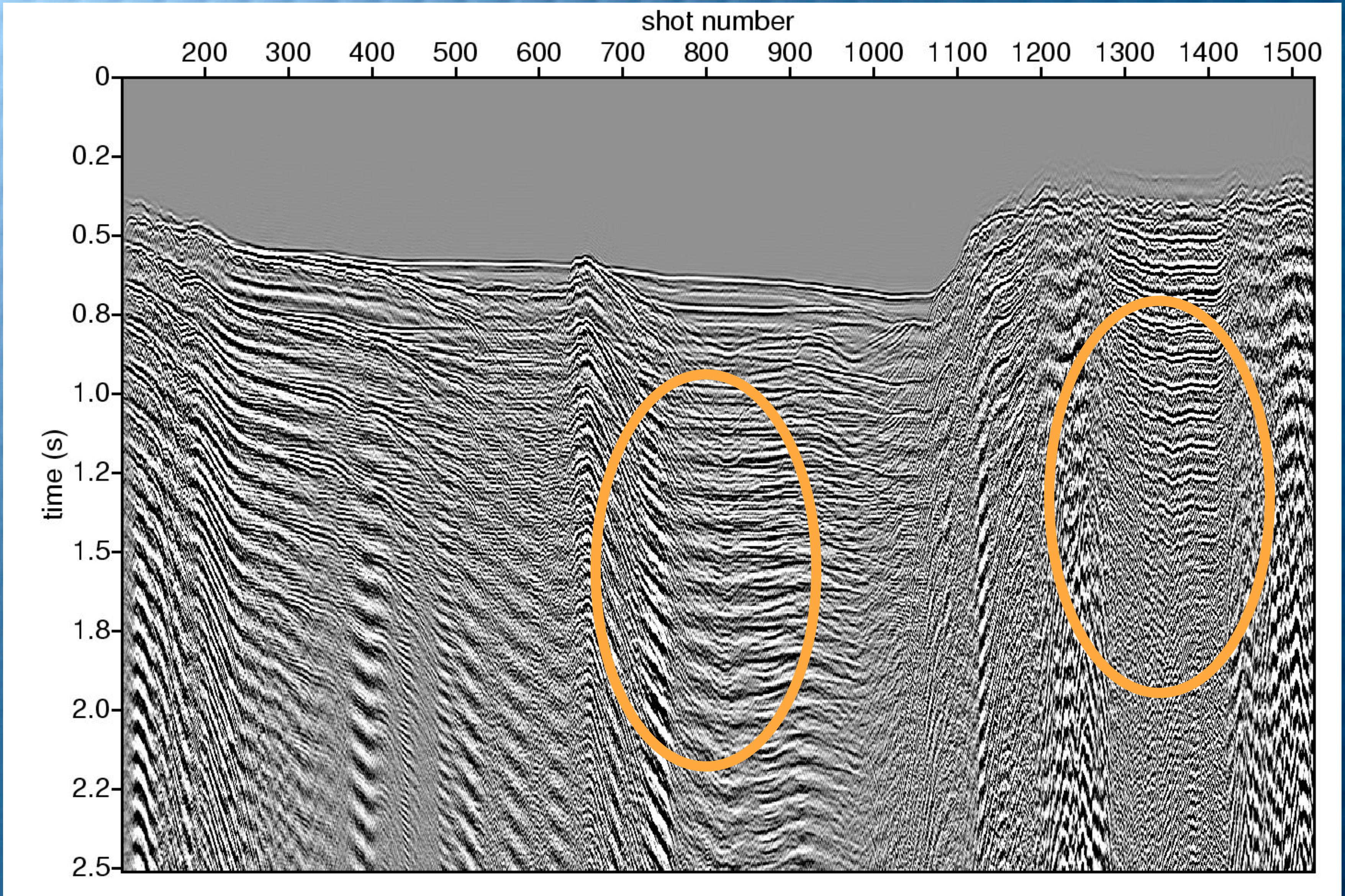
Offshore Scotland example

Predicted multiples (no adaptation)



Offshore Scotland example

Predicted multiples (no adaptation)



Offshore Scotland example

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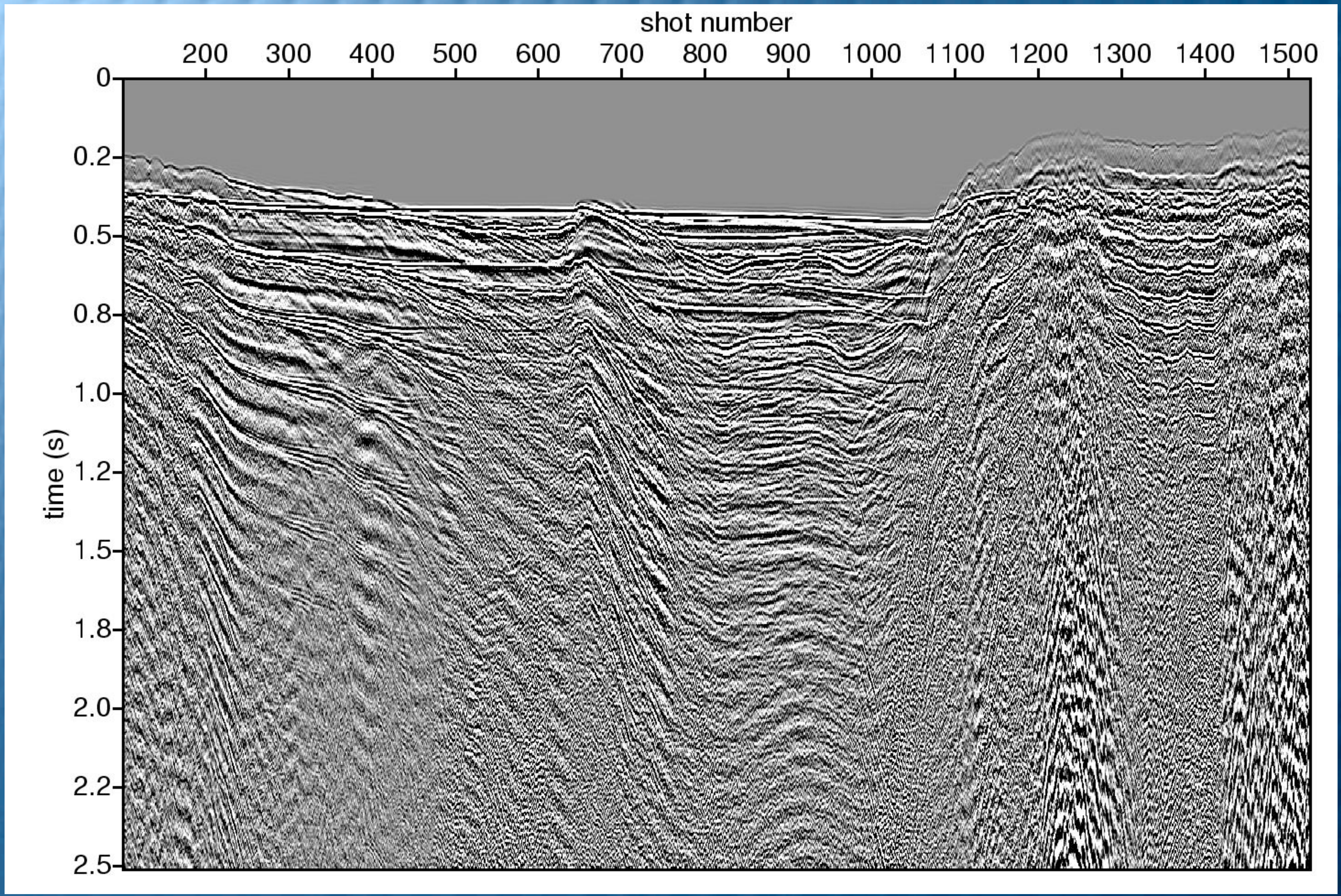
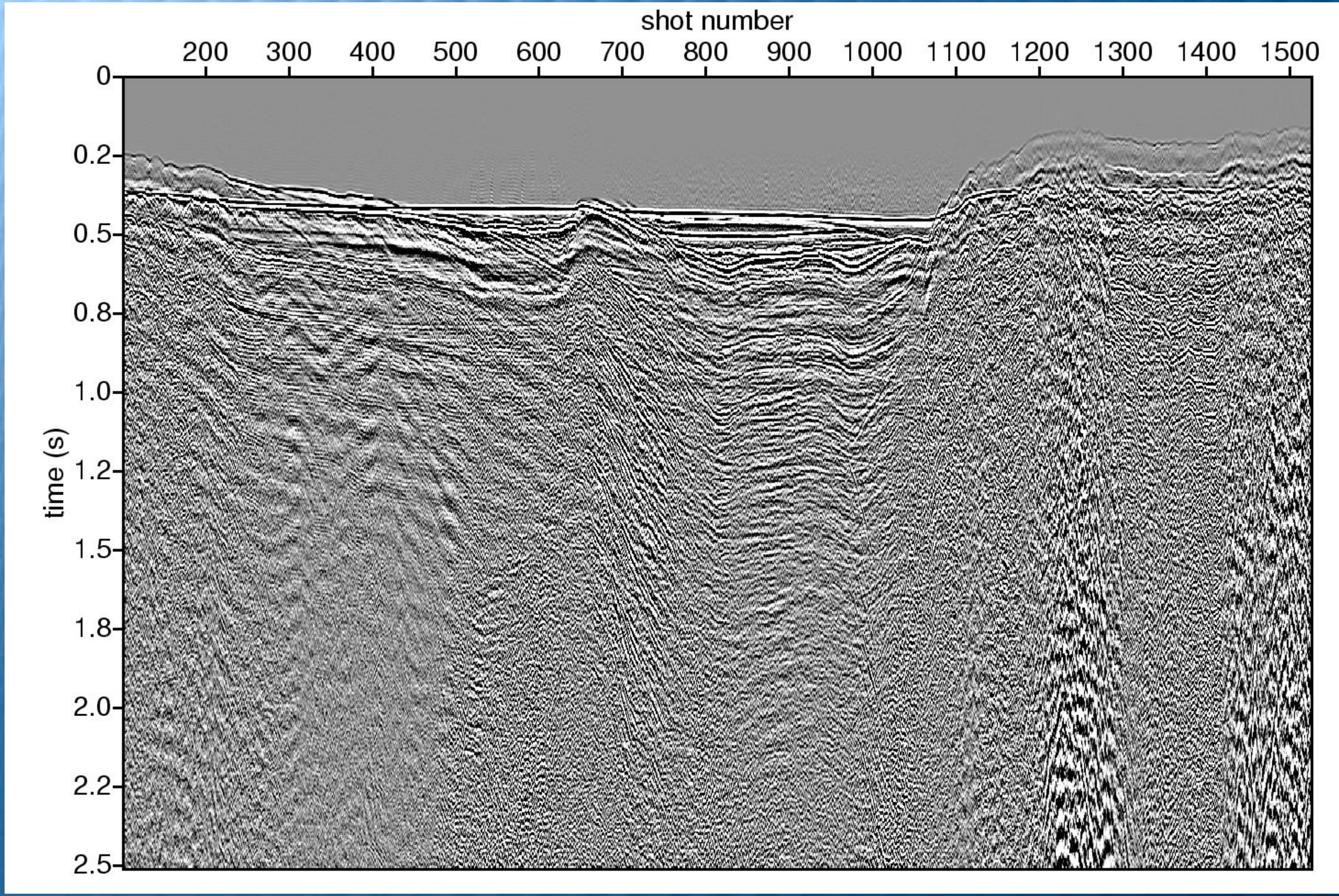


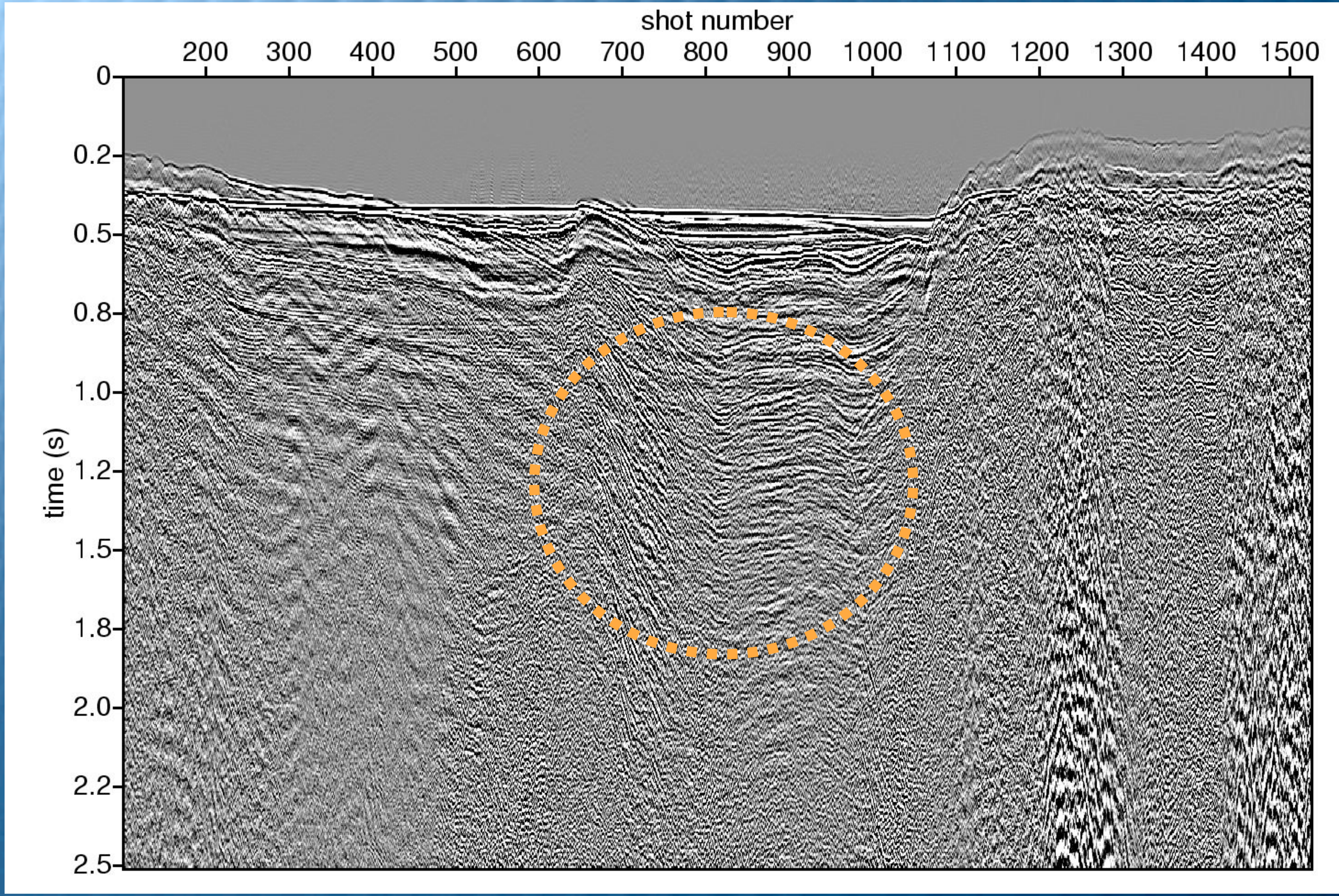
Offshore Scotland example

Common offset (500 m) after 2D SRME



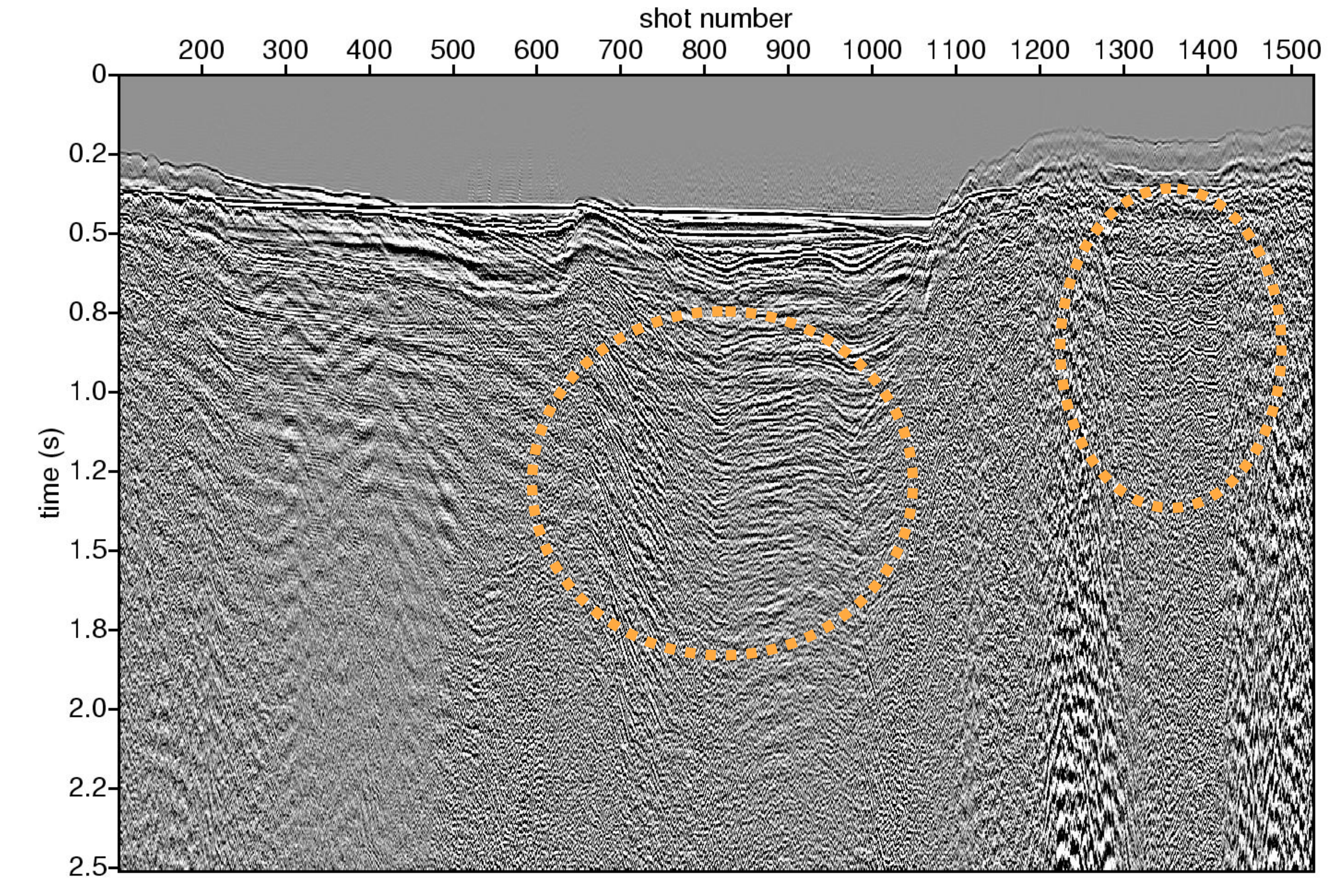
Offshore Scotland example

Common offset (500 m) after 2D SRME

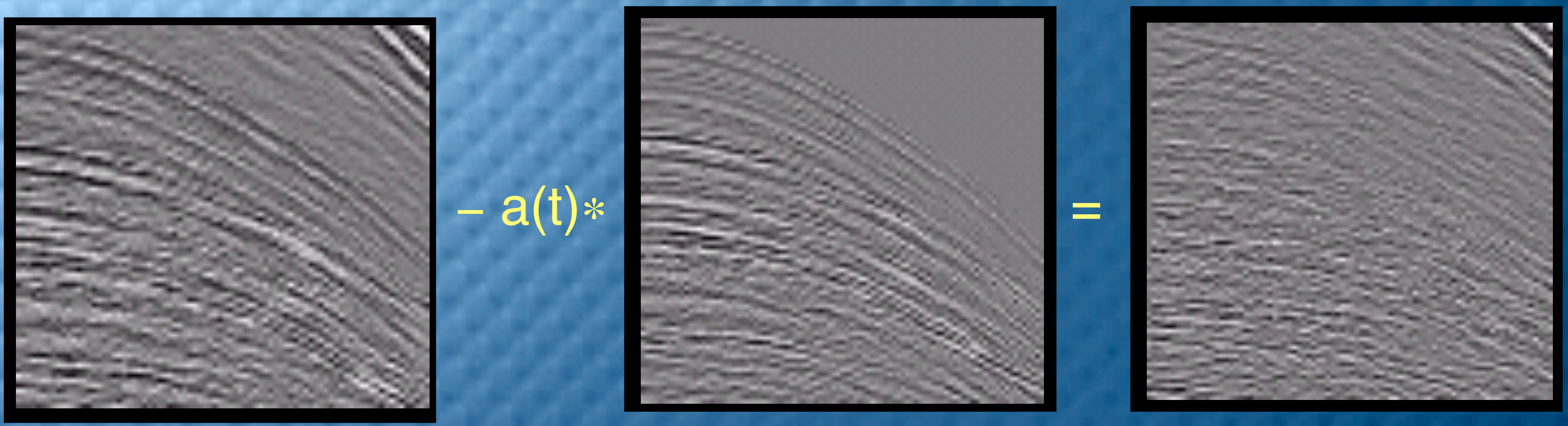


Offshore Scotland example

Common offset (500 m) after 2D SRME



Least-squares subtraction



Input data with multiples **Predicted multiples** **output data**

Adaptive subtraction based on minimum energy in the output

Adaptive subtraction

Matched filter:

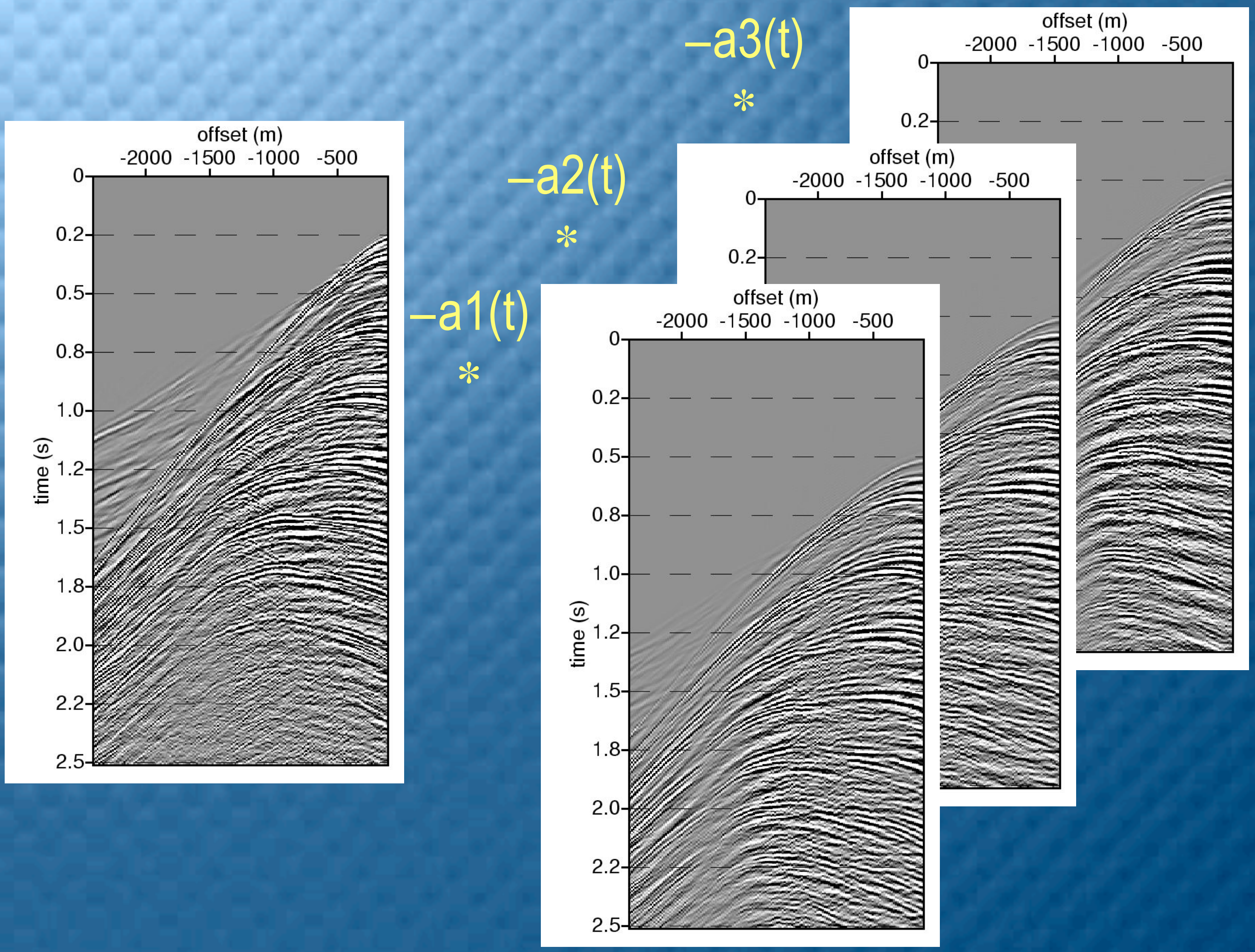
$$\overbrace{\hat{\mathbf{n}}}^{\text{denoised}} : \min_{\boldsymbol{\Phi}} = \| \underbrace{\mathbf{d}}_{\text{noisy data}} - \overbrace{\boldsymbol{\Phi}}^{\text{matched filter}} \overset{t}{*} \underbrace{\mathbf{m}}_{\text{pred. noise}} \|_p$$

- ★ **p=1 enhances sparseness**
- ★ **residue is the denoised data**
- ● **risk of over fitting**

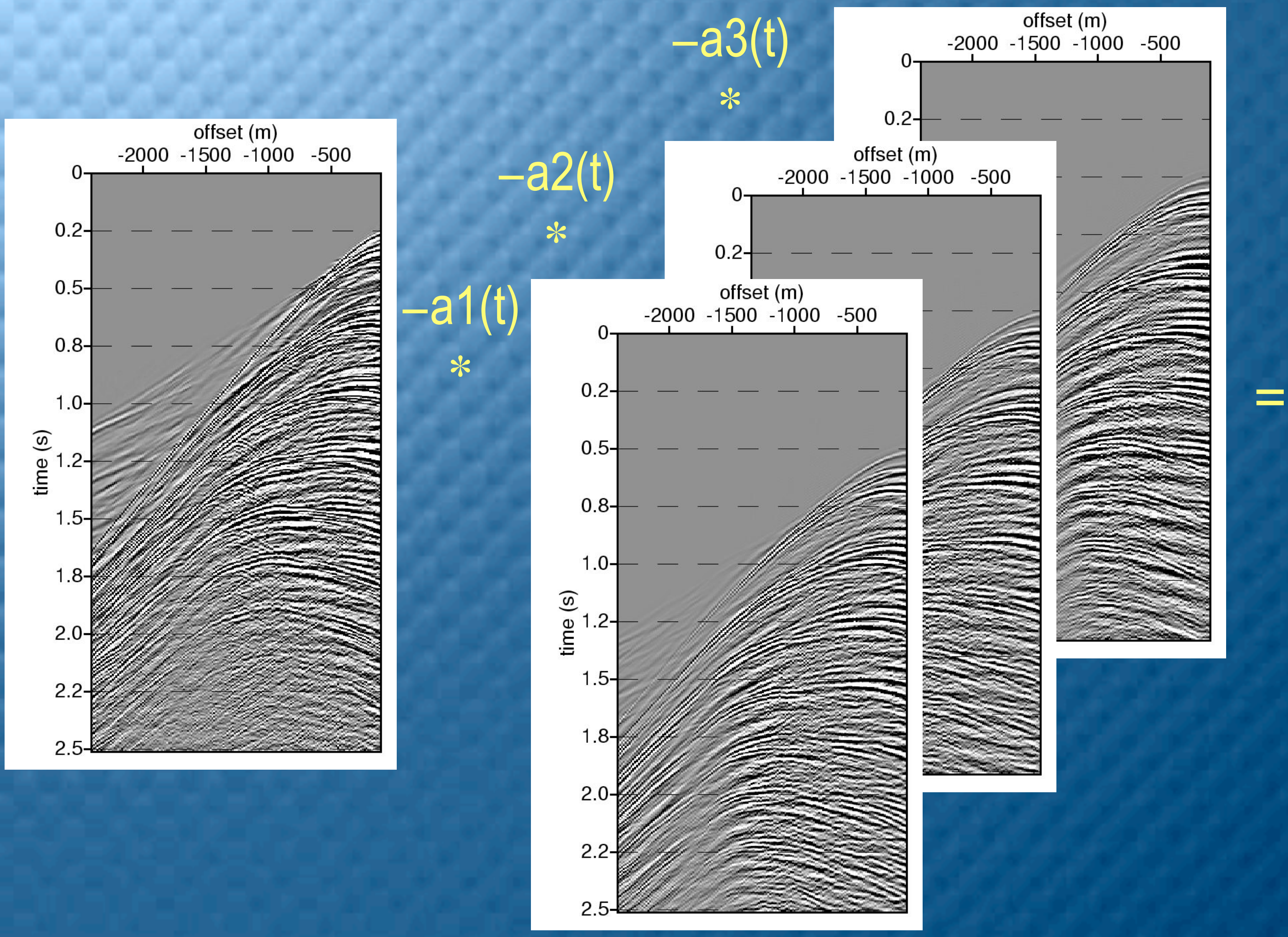
May loose primary reflection events ...

Guitton&
Verschuur'04

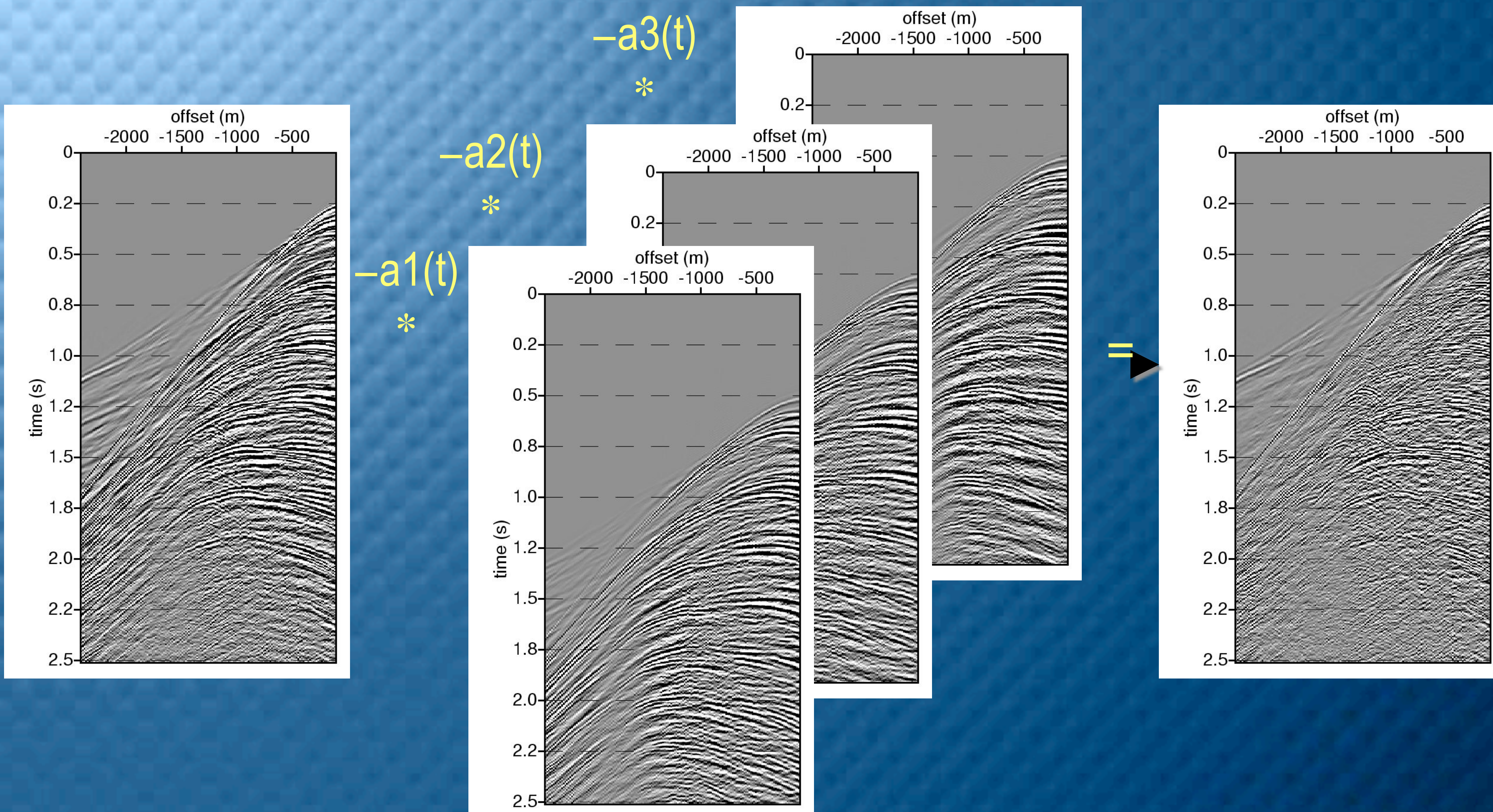
Multi-gather subtraction



Multi-gather subtraction



Multi-gather subtraction



Least-squares subtraction

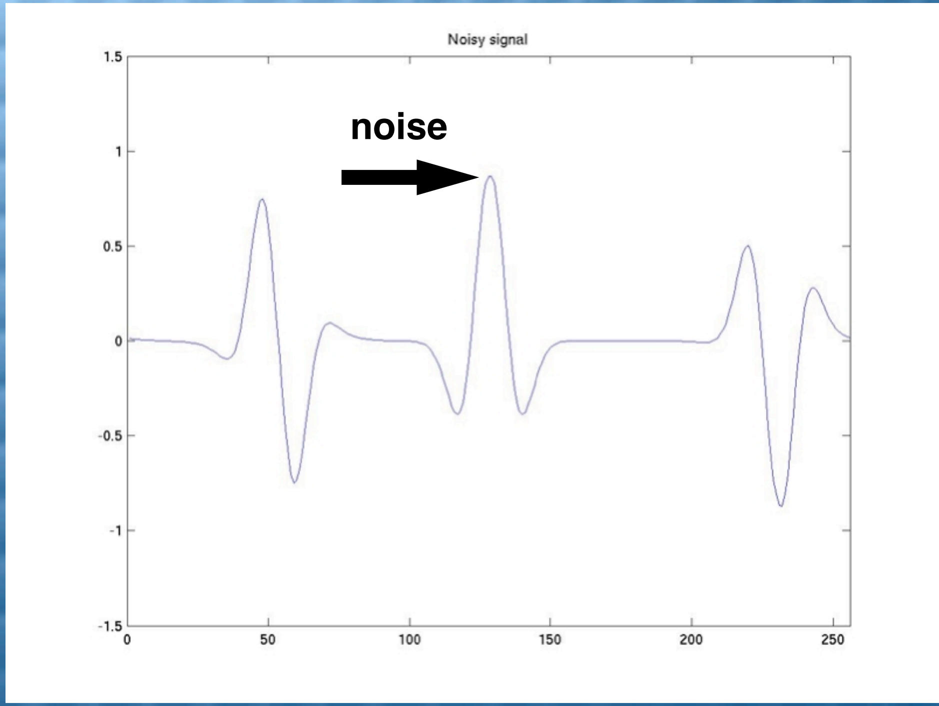
- Can be effective in many situations
- Relatively fast and easy to apply
- But …. *minimum energy* assumption is ***not*** always valid
- Danger of distorting the primaries due to overfitting
- Explore other subtraction domains that

Other domains

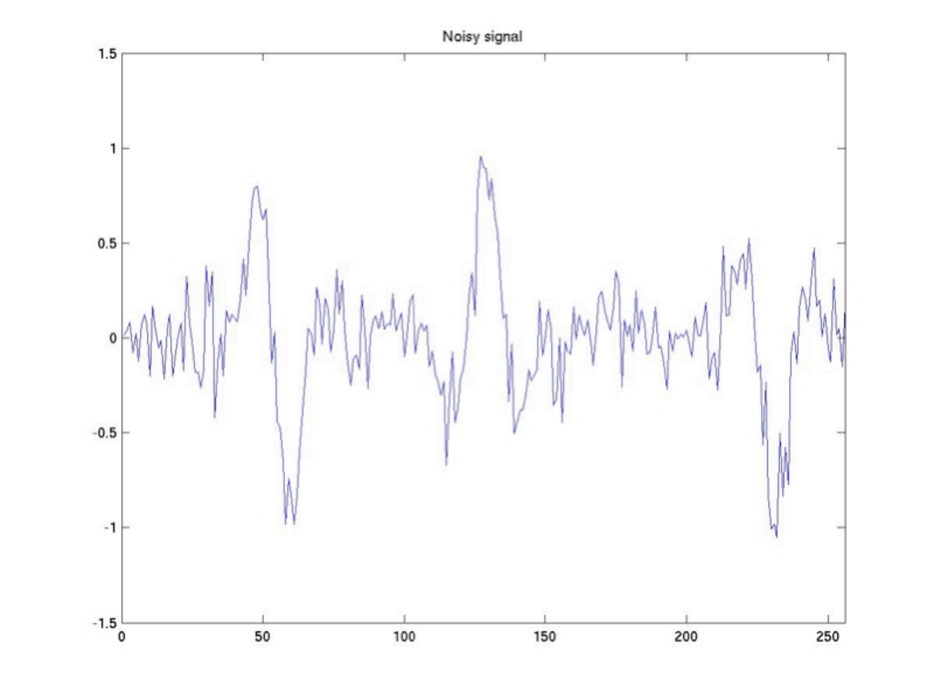
- sparse & local
- relatively insensitive to
  - phase rotations
  - misalignments
- almost diagonalize Covariance operator of multiples & primaries

We know that wavelets are unconditional bases ....

Unconditional bases

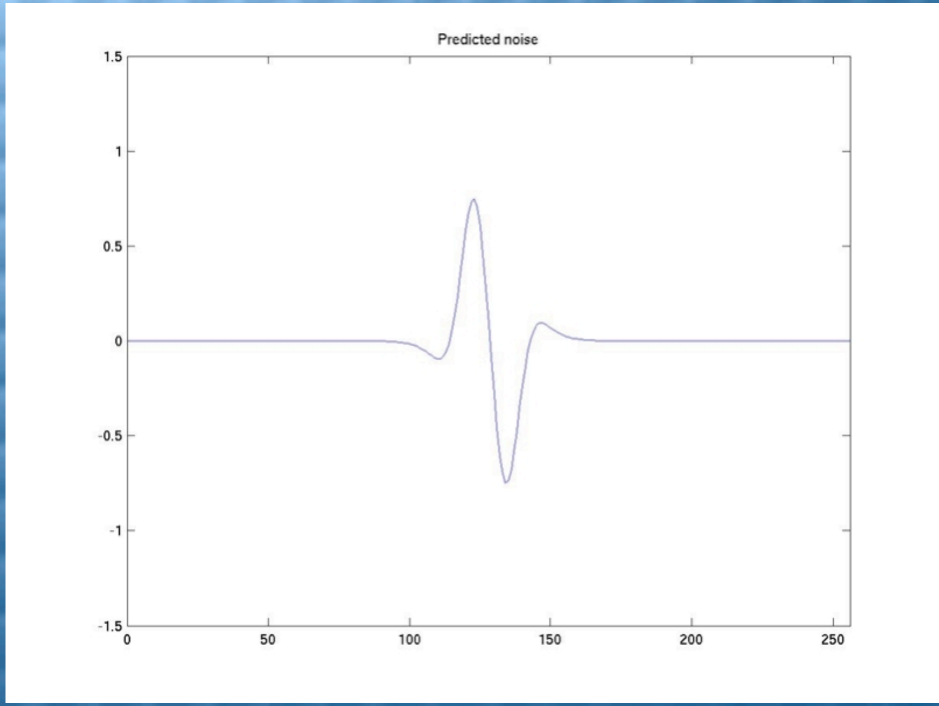


signal + coherent noise

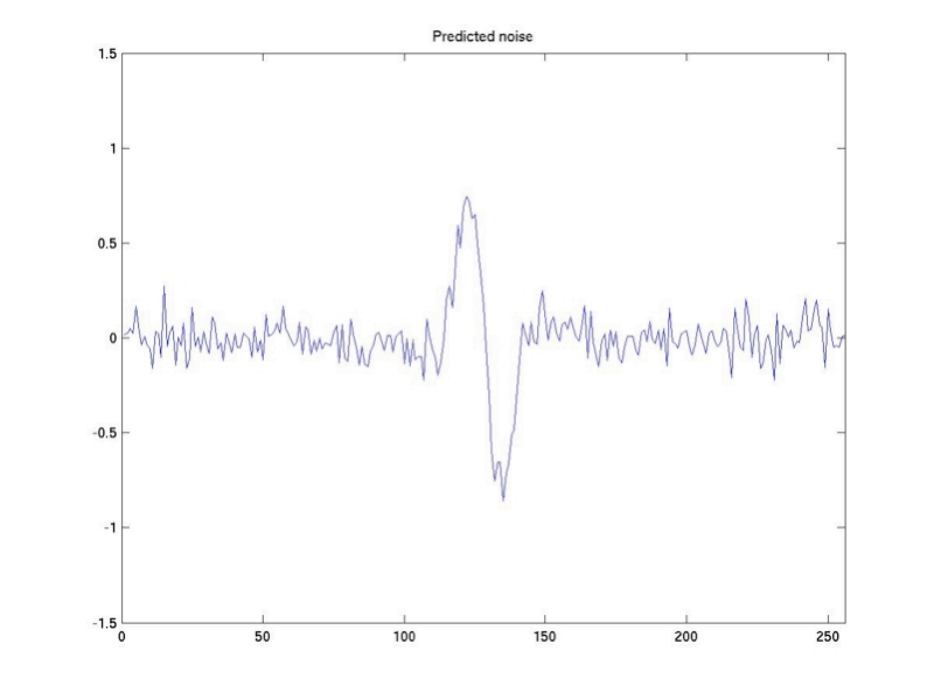


signal + coherent & incoherent noise

Unconditional bases

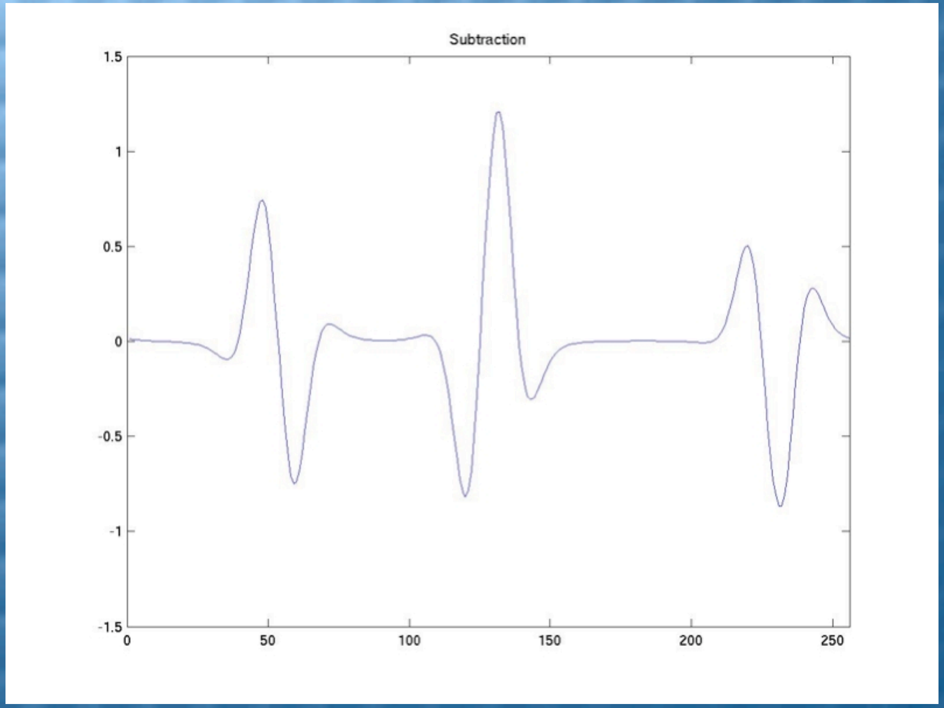


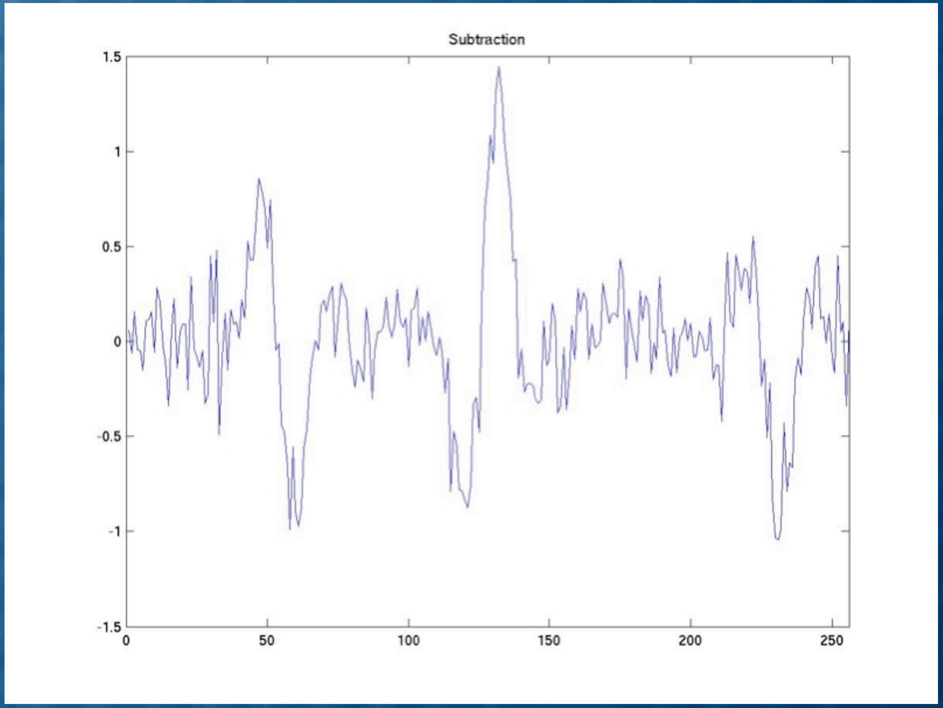
‘wrongly’ predicted noise



‘wrongly’ noisy predicted noise

Unconditional bases

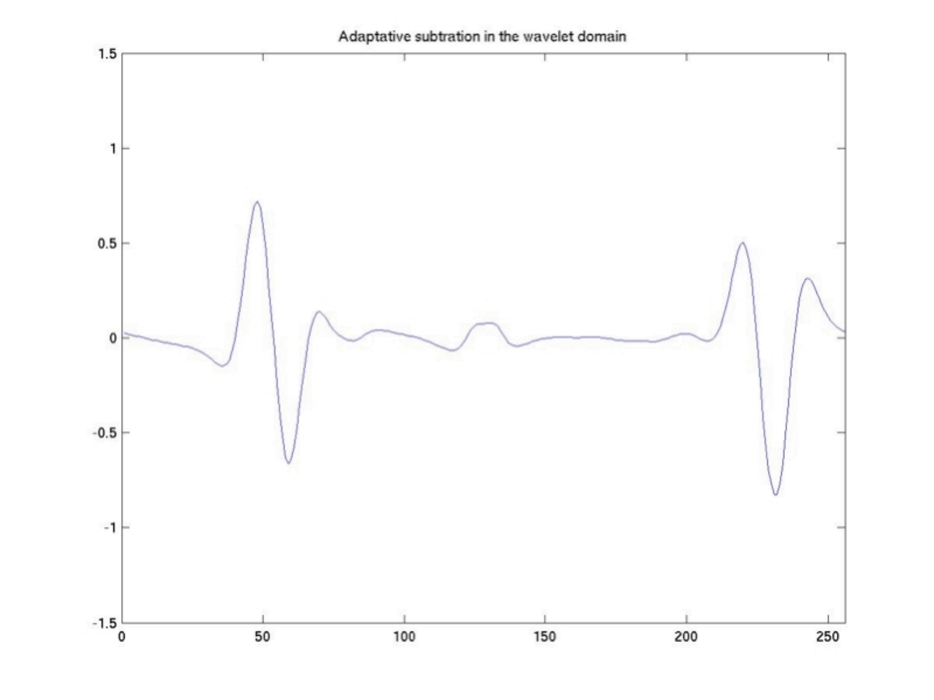


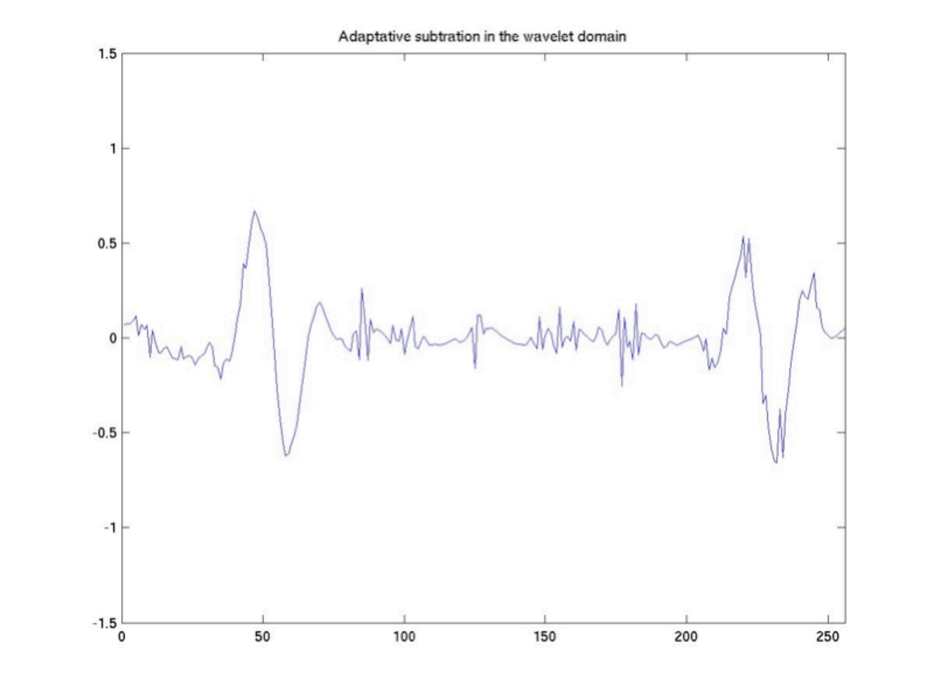


noise-free subtraction

noisy subtraction

Unconditional bases





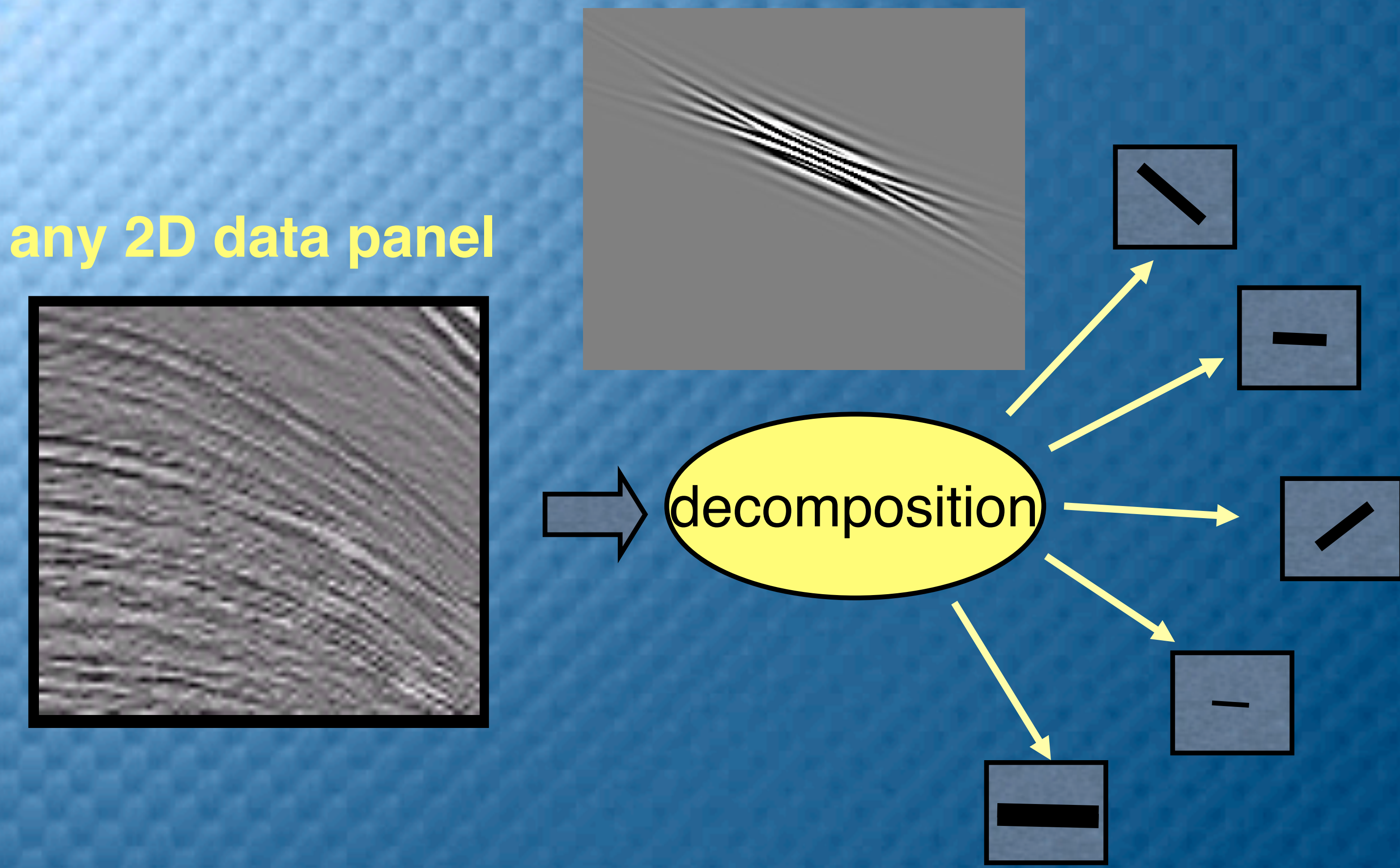
noise-free adaptive subtraction

noisy adaptive subtraction

Wavelets

- **Represent piece-wise smooth functions at “no” additional cost.**
- Do **not** have to know ***where*** the **singularities** are.
- Are **unconditional basis.**
- **Only** good for **point-scatterers** or **horizon/vertically-aligned reflectors.**
- **Lack directional selectivity.**
- **Do *NOT* work well with waves.**

Curvelet domain



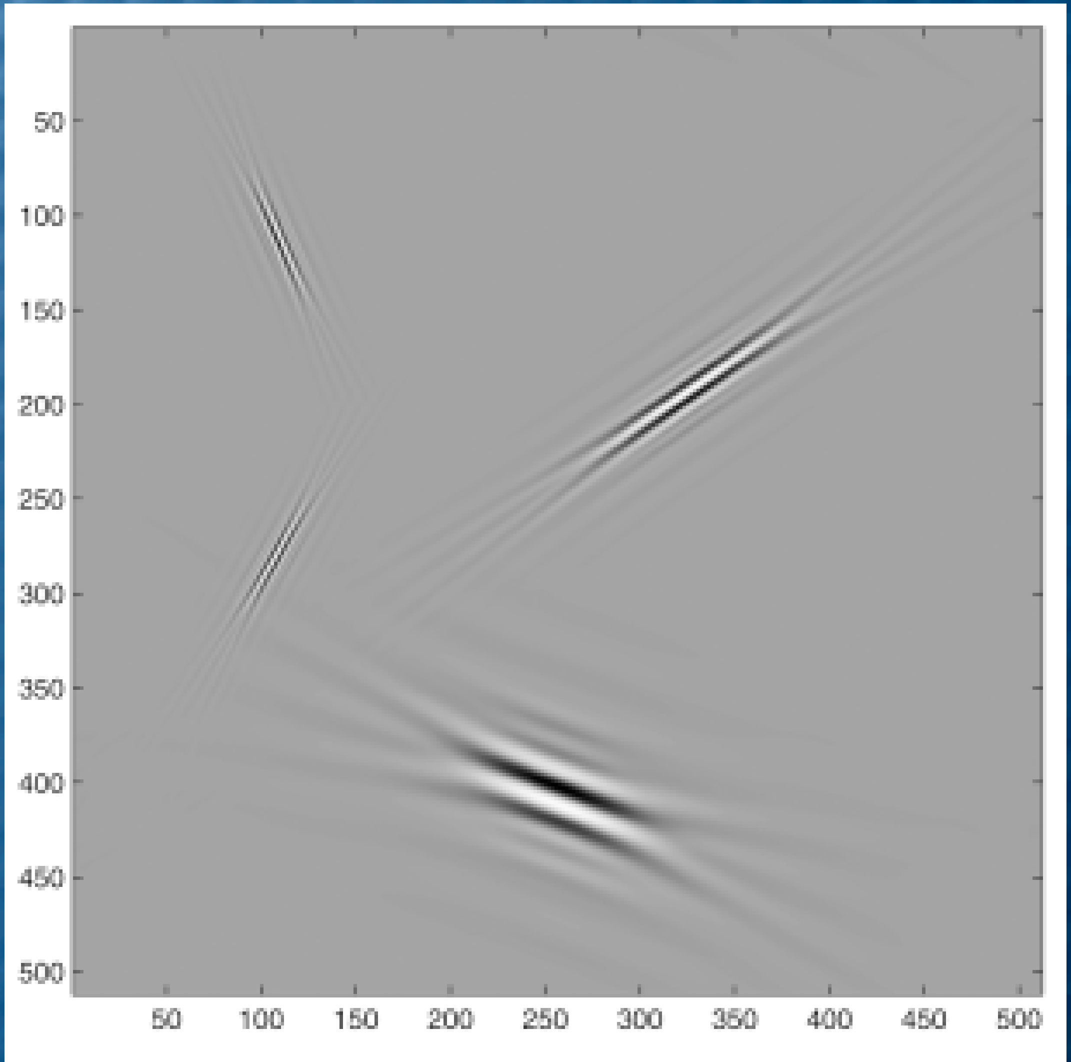
- **Almost orthogonal decomposition into multiscale basis functions with local frequency and local dip properties**
- **Natural basis for wave equations**
- **Consist of plane wavelets invariant under convolution**

Curvelet domain

- Curvelet transform developed by Candes and Donoho (2002)
- Geophysical applications:
  - Multiple & ground-roll suppression
  - New imaging algorithms
  - 4D data matching
- Curvelets sense *local* **dip** and *local* **frequency content** and can ***discriminate*** on these properties

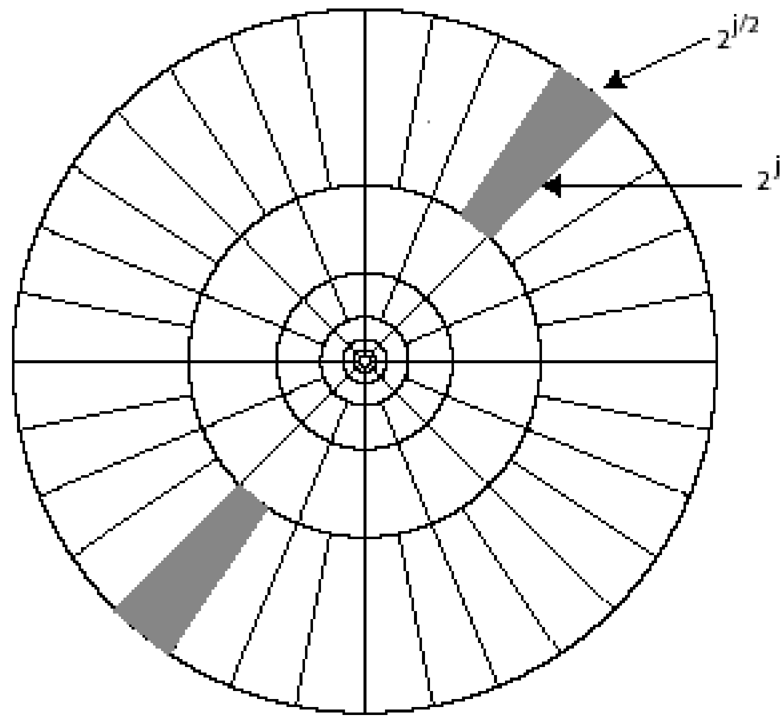
Why curvelets

- Nonseparable
- Local in 2-D space
- Local in 2-D Fourier
- Anisotropic
- Multiscale
- Almost orthogonal
- Tight frame



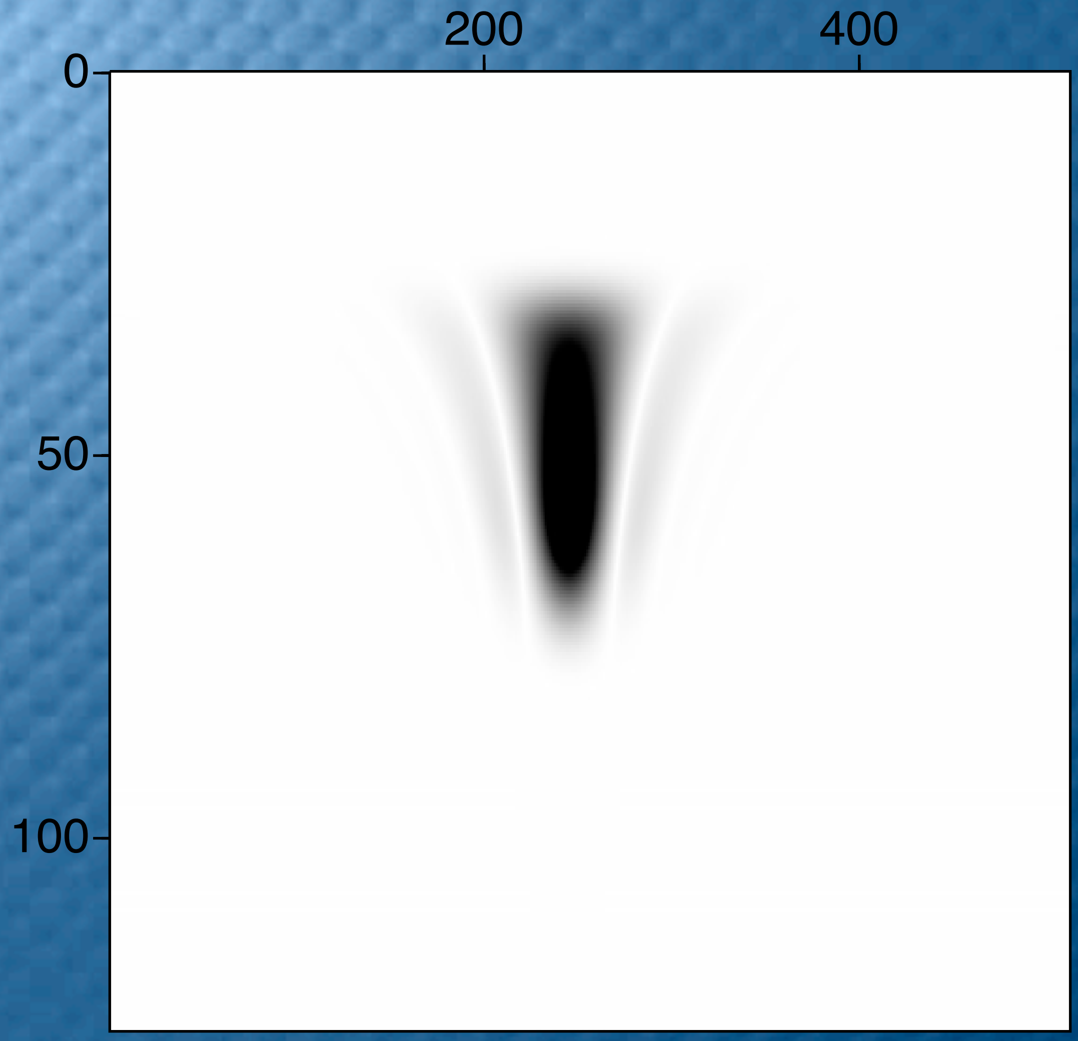
Why curvelets

$$W_j = \{\zeta, \quad 2^j \le |\zeta| \le 2^{j+1}, |\theta - \theta_J| \le \pi \cdot 2^{\lfloor j/2 \rfloor}\}$$



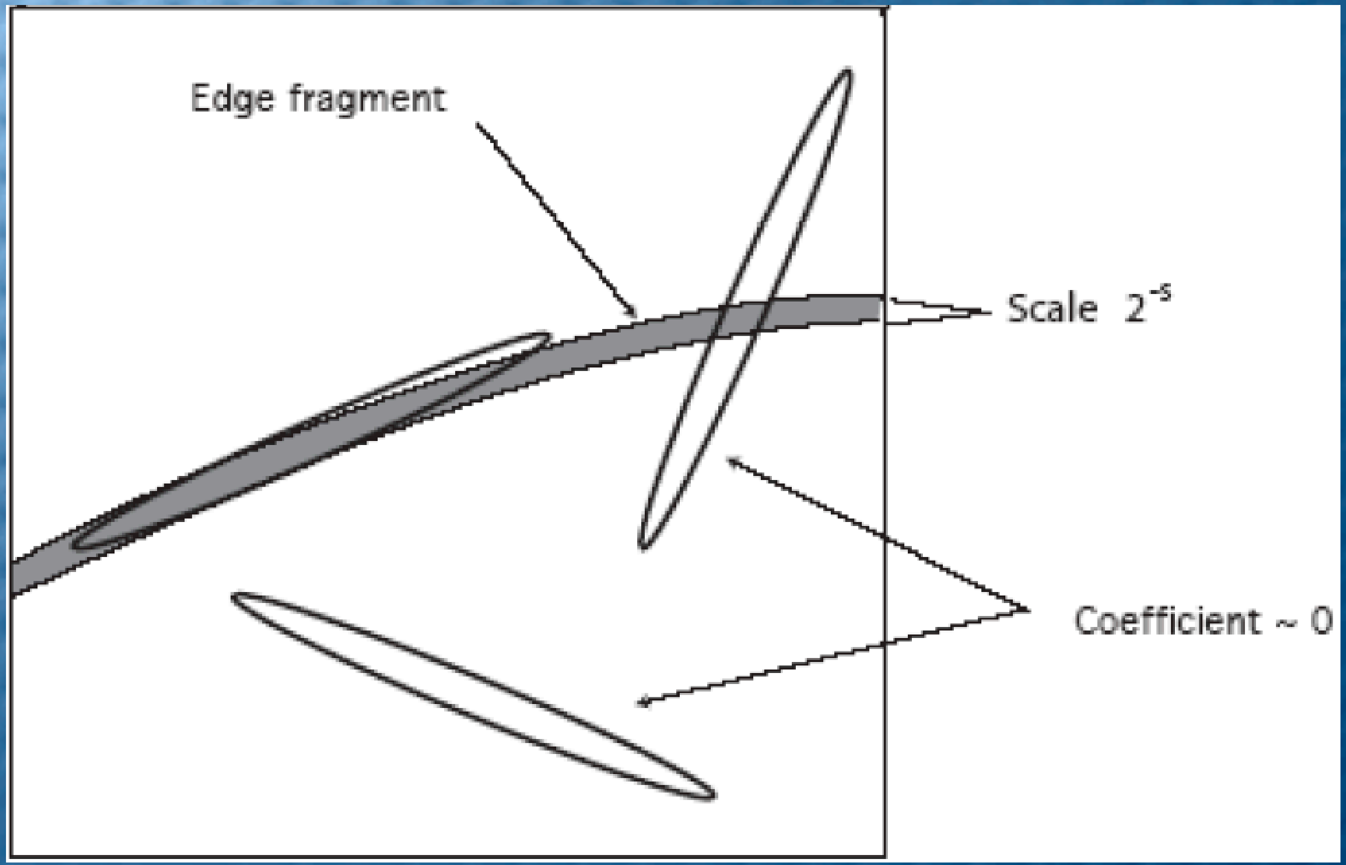
second dyadic partitioning

Why curvelets



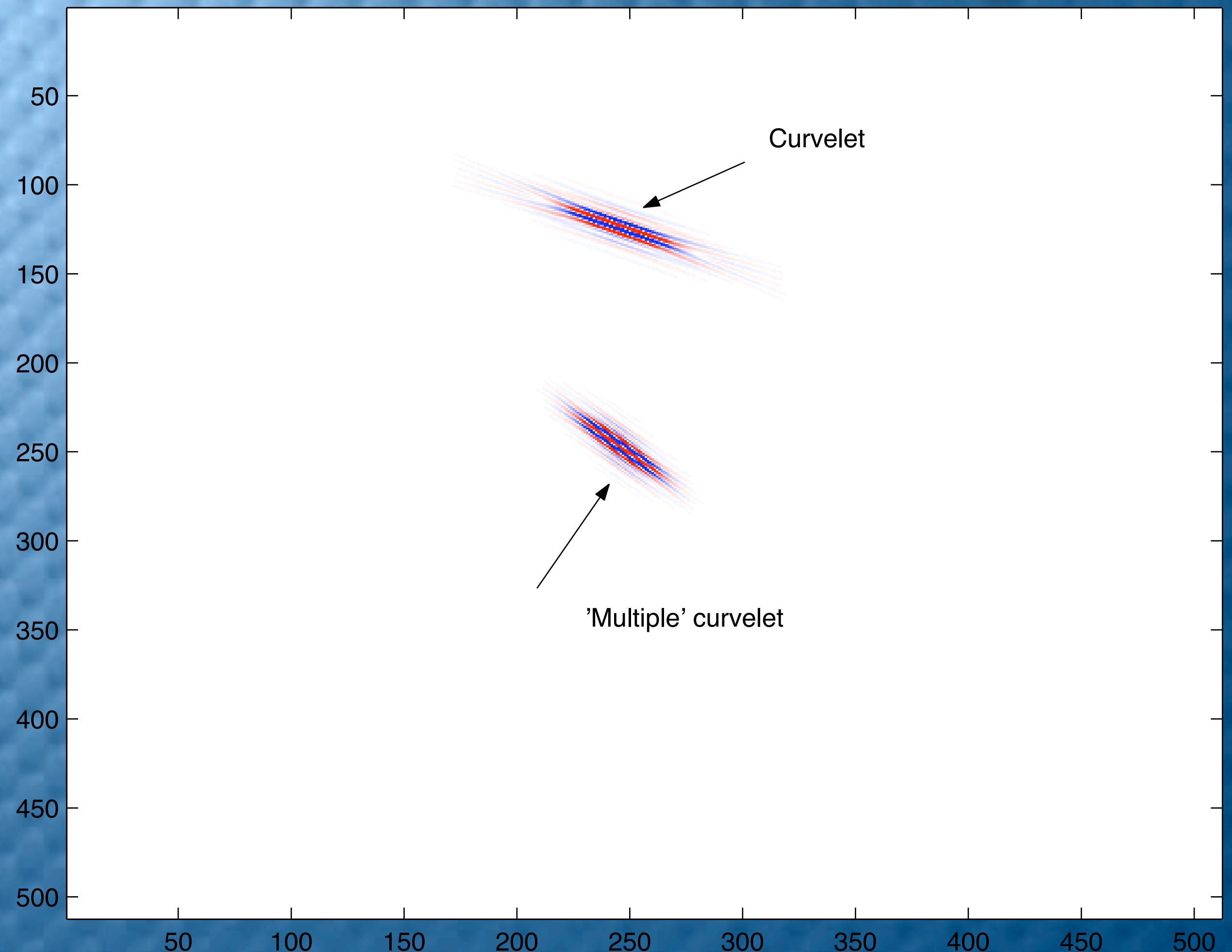
Curvelet in FK-domain

Dot products



Candes '02

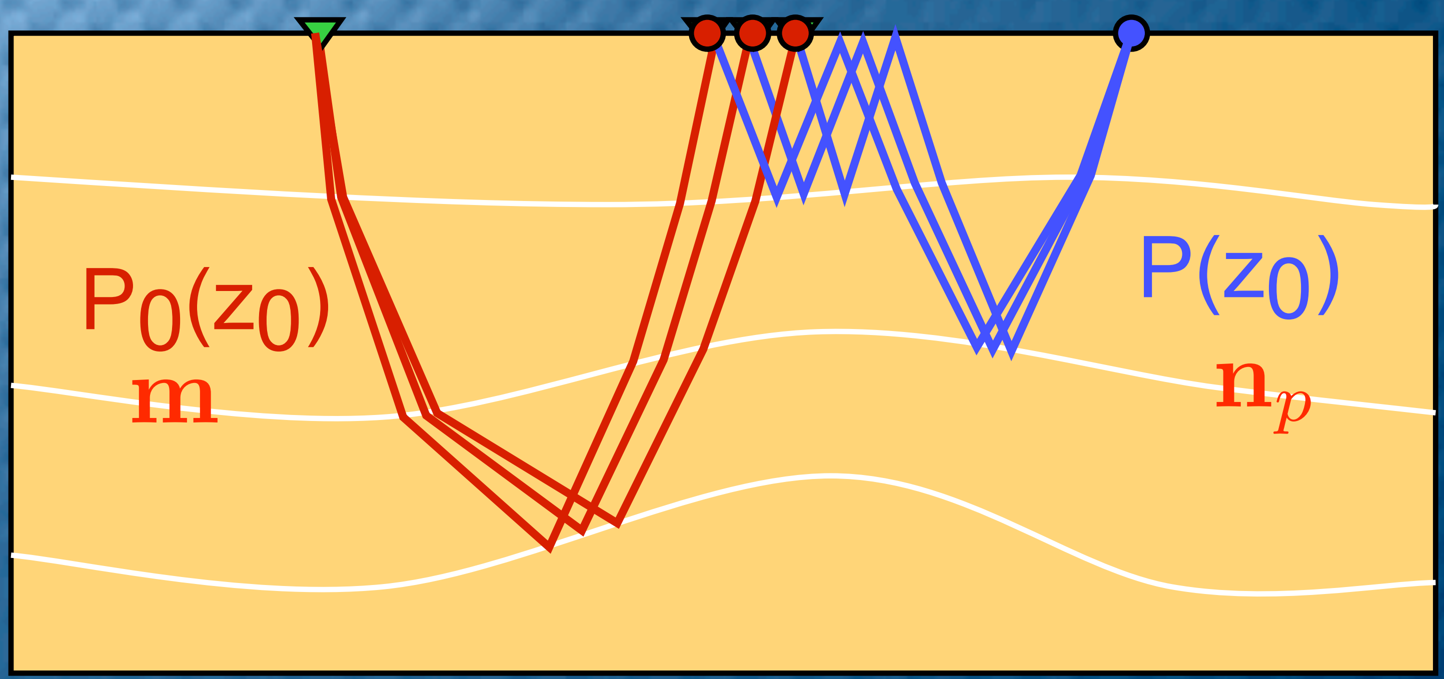
Curvelet 'Multiples'



- **Almost diagonalize Green's functions (Candes & Demanet '04)**
- **Natural basis for wave equations**
- **Invariant under convolution, i.e. 'multiple multiple' = curvelet-like**

Surface multiple elimination

Multiple prediction: data convolution along surface



What do we do?

Use as alternative to matched adapt. sub.

$$\hat{\mathbf{m}} = \mathbf{B}^{\dagger}\boldsymbol{\Theta}_{\lambda\Gamma}\left(\mathbf{Bd}\right) \text{ with } \Gamma = |\mathbf{Bn}_{\mathbf{p}}|$$

to denoise

$$\overbrace{\mathbf{d}}^{\text{noisy data}} = \underbrace{\mathbf{m}}_{\text{noise-free}} + \overbrace{\mathbf{n}}^{\text{col. noise}}$$

with a simple ***mute*** with λ control parameter

$$\lambda = 3 \leftrightarrow 90\% \text{ confidence interval}$$

Non-linear adaptive subtraction

Extend to *colored noise:*

$$\overbrace{\mathbf{d}}^{\text{noisy data}} = \underbrace{\mathbf{m}}_{\text{noise-free}} + \overbrace{\mathbf{n}}^{\text{col. noise}}$$

Solve

$$\hat{\mathbf{m}} : \min_{\mathbf{m}} \frac{1}{2} \|\mathbf{C}_n^{-1/2} (\mathbf{d} - \mathbf{m}) \|_2^2$$

with

$$\mathbf{C}_n \triangleq \mathbf{E}\{\mathbf{n}\mathbf{n}^T\}$$

Non-linear adaptive subtraction

Recast in *Curvelet* domain:

$$\hat{\tilde{\mathbf{m}}} \ : \ \min_{\tilde{\mathbf{m}}} \frac{1}{2} \|\mathbf{C}_{\tilde{n}}^{-1/2} \left(\tilde{\mathbf{d}} - \tilde{\mathbf{m}} \right) \|_2^2 + \lambda^2 \|\tilde{\mathbf{m}}\|_p$$

Use ***unconditional-basis* property:**

$$\mathbf{C}_{\tilde{n}} \triangleq \mathbf{E}\{\tilde{\mathbf{n}}\tilde{\mathbf{n}}^T\} \approx \mathrm{diag}\left(\mathrm{diag}\left(\mathbf{C}_{\tilde{n}}\right)\right) \triangleq \Gamma^2$$

‘Challenge’ to find the Γ’s

Non-linear adaptive subtraction

Solve

$$\hat{\tilde{\mathbf{m}}} \;:\; \min_{\tilde{\mathbf{m}}} \frac{1}{2}\|\boldsymbol{\Gamma}^{-1}\left(\tilde{\mathbf{d}} - \tilde{\mathbf{m}}\right)\|_2^2 + \lambda^2\|\tilde{\mathbf{m}}\|_p$$

can be written as

$$\hat{\mathbf{m}} = \mathbf{B}^{\dagger}\boldsymbol{\Gamma}\Theta_{\lambda}\left(\boldsymbol{\Gamma}^{-1}\mathbf{B}\mathbf{d}\right) = \mathbf{B}^{\dagger}\Theta_{\boldsymbol{\lambda}\boldsymbol{\Gamma}}\left(\tilde{\mathbf{d}}\right).$$

No matched filter required!

Non-linear estimation

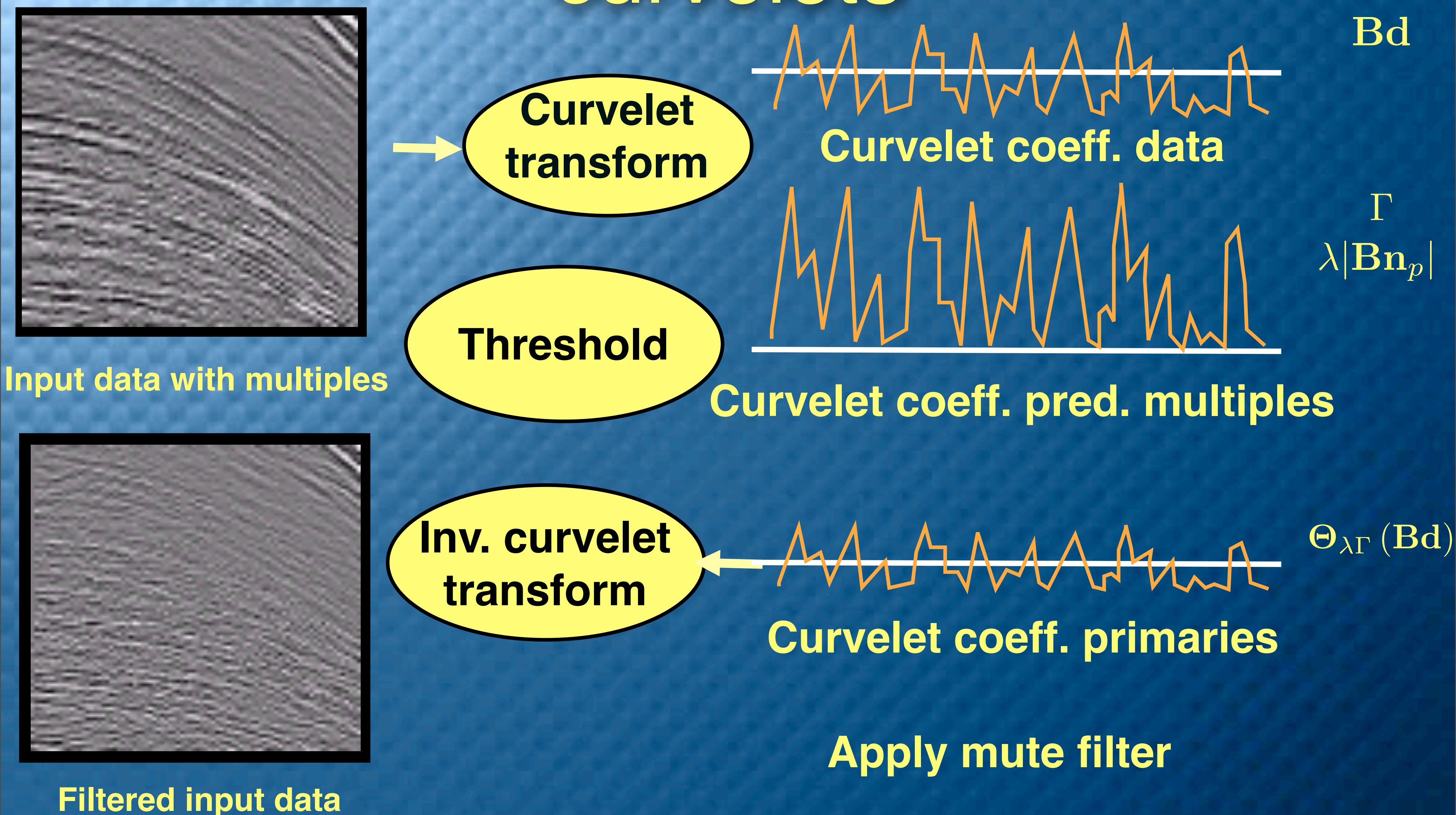
Hard thresholding for p=0:

$$\Theta_\lambda^h\left(\tilde{\mathbf{d}}\right) \triangleq \begin{cases} \tilde{\mathbf{d}} & \text{if} \quad |\tilde{\mathbf{d}}| > \lambda \\ 0 & \text{if} \quad |\tilde{\mathbf{d}}| \leq \lambda \end{cases}$$

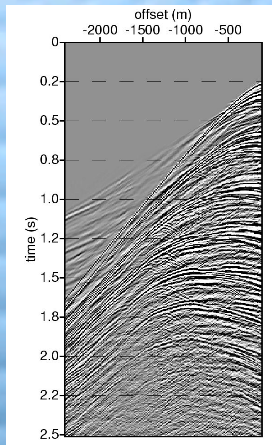
Soft thresholding for p=1:

$$\Theta_\lambda^s\left(\tilde{\mathbf{d}}\right) \triangleq \begin{cases} \operatorname{sign}(\tilde{\mathbf{d}})(|\tilde{\mathbf{d}}| - \lambda)_+ & \text{if} \quad |\tilde{\mathbf{d}}| > \lambda \\ 0 & \text{if} \quad |\tilde{\mathbf{d}}| \leq \lambda \end{cases}$$

Multiple suppression with curvelets



Multiple suppression with curvelets

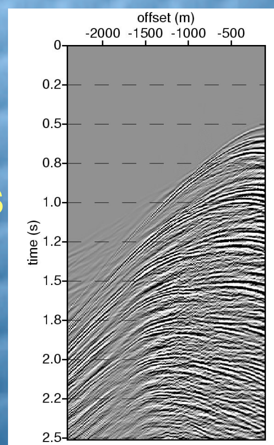


Input data with multiples

Curvelet transform

$\mathbf{Bd}$

Curvelet coeff. data



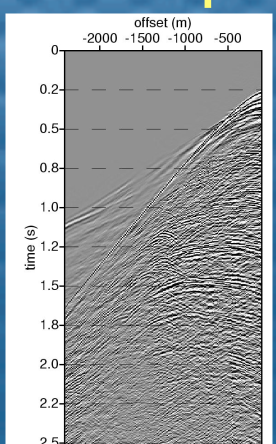
predicted multiples

Threshold

Γ

$\lambda|\mathbf{Bn}_p|$

Curvelet coeff. pred. multiples



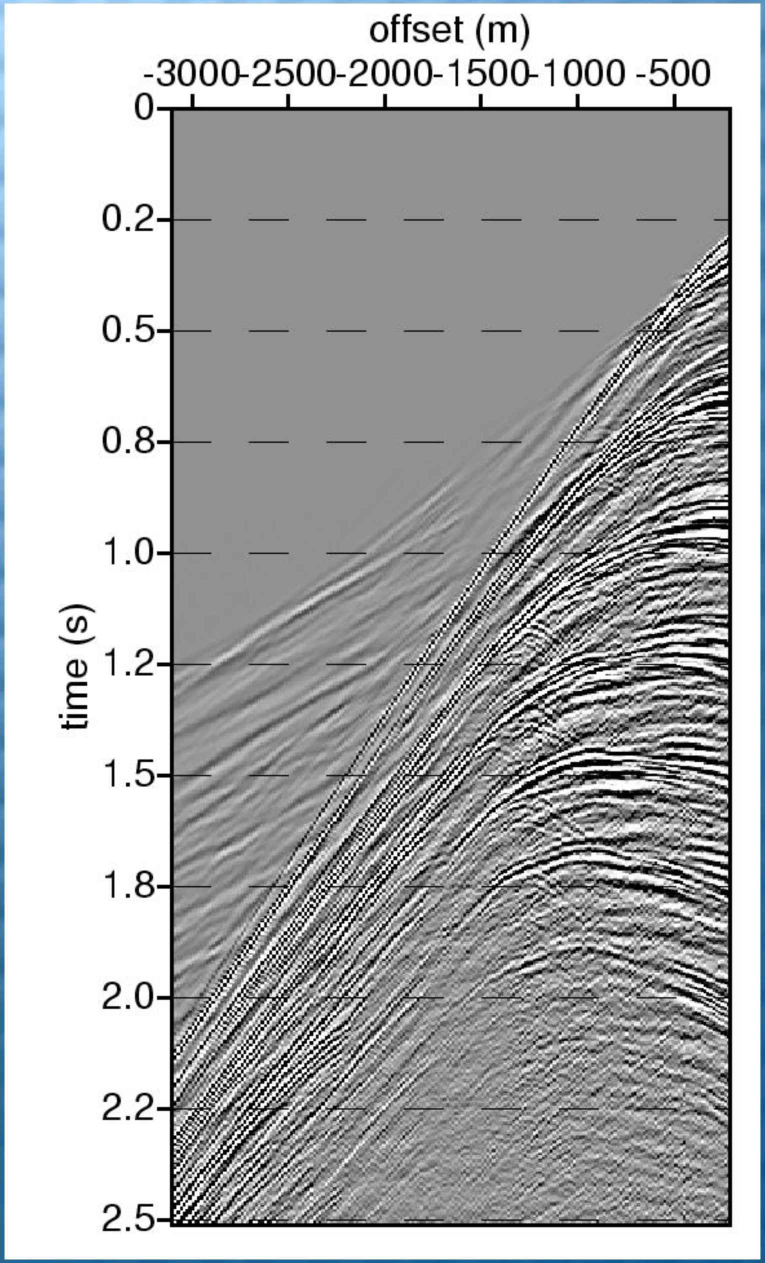
Filtered input data

Inv. curvelet transform

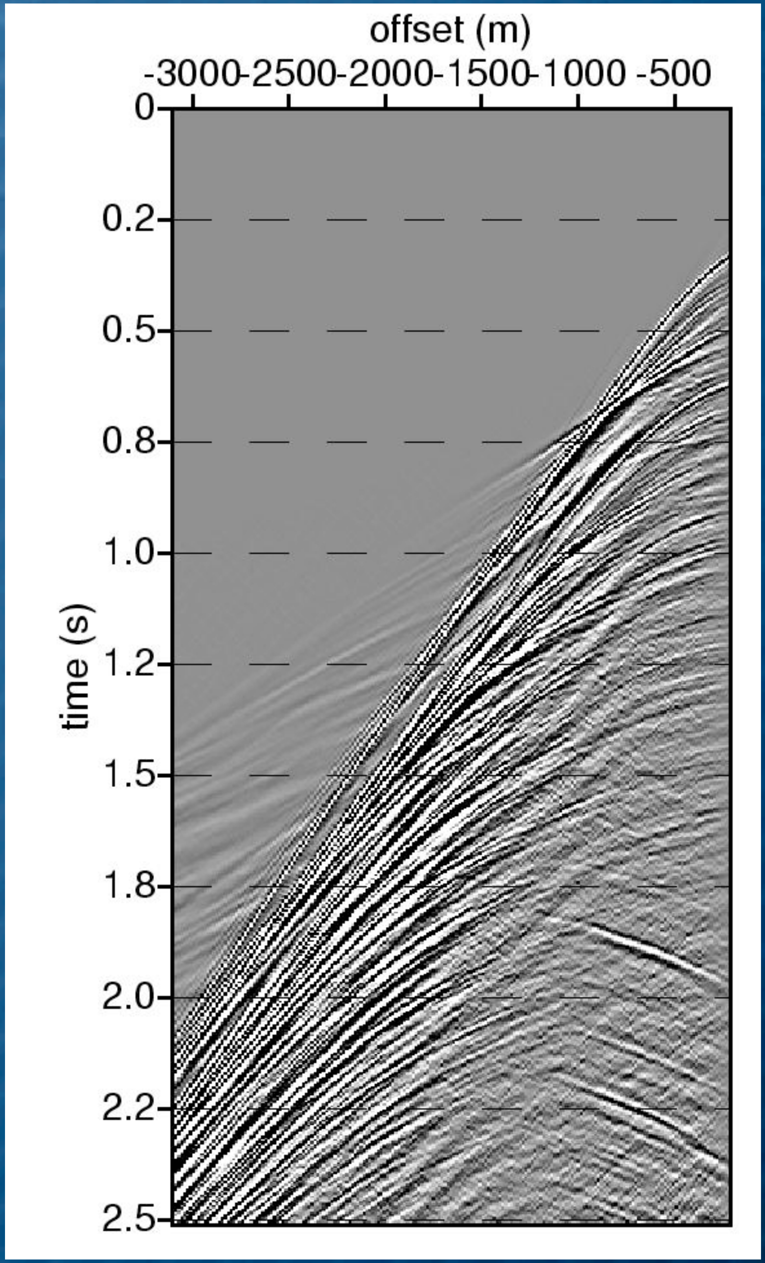
$\Theta_{\lambda\Gamma}(\mathbf{Bd})$

Curvelet coeff. primaries

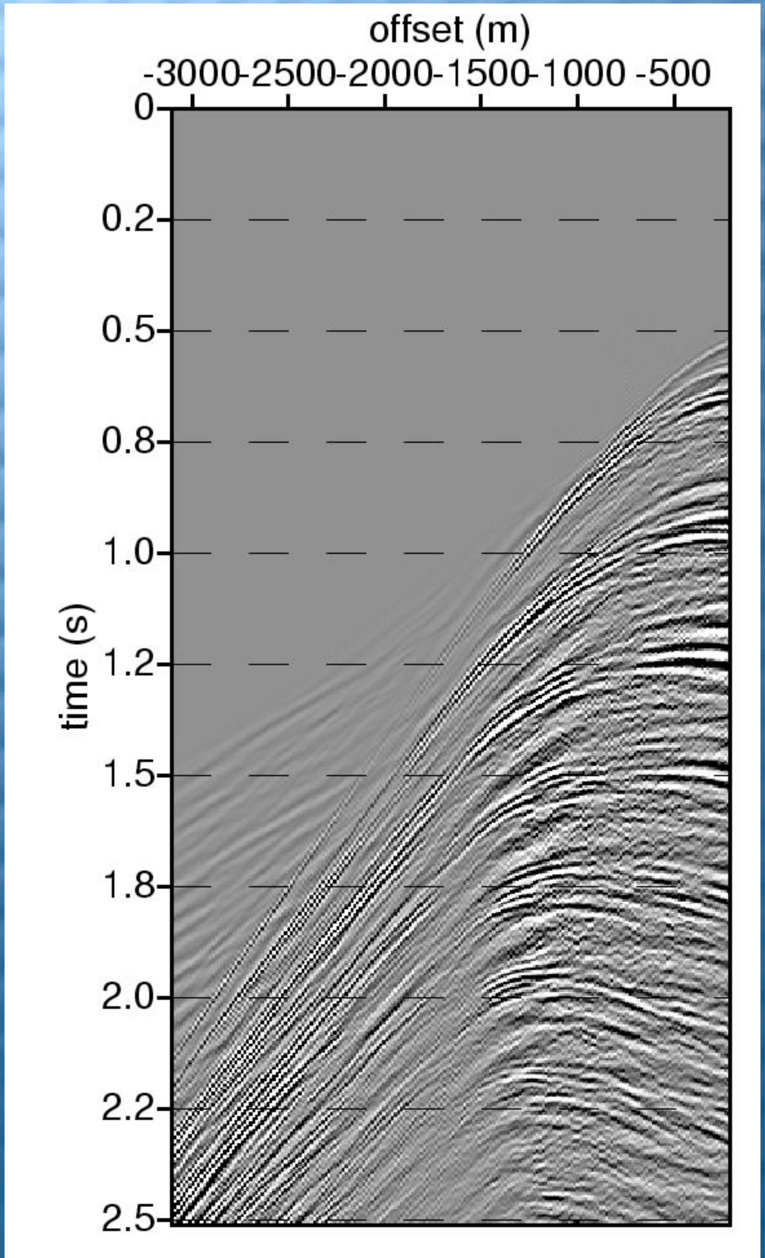
Subtraction with L2 norm



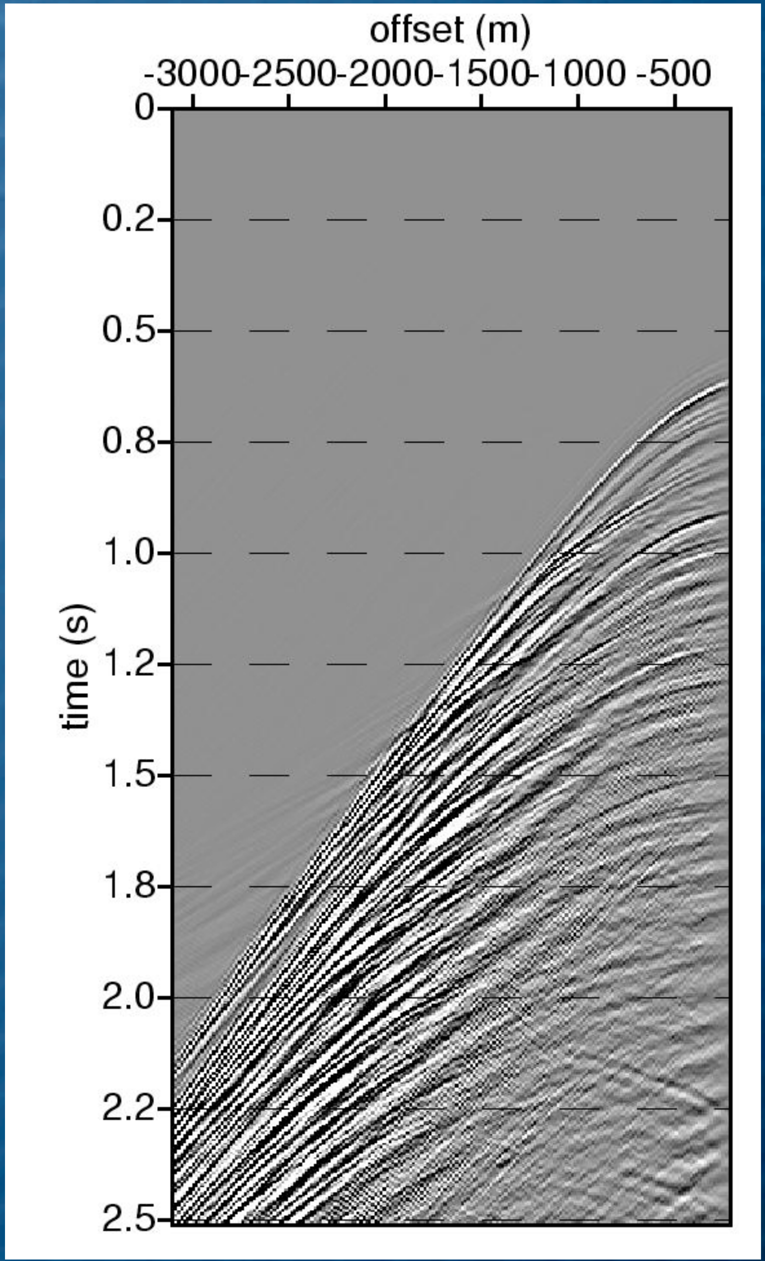
Input
with
multiples



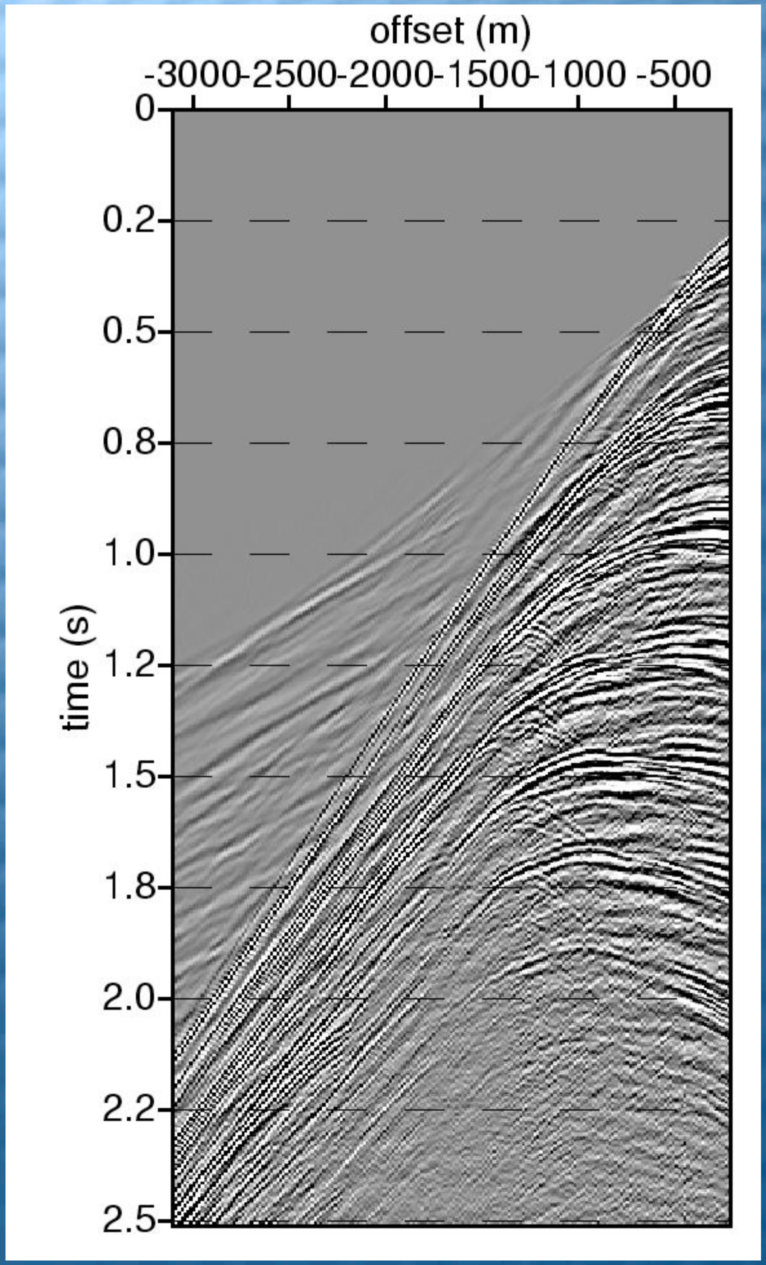
Subtraction with L2 norm



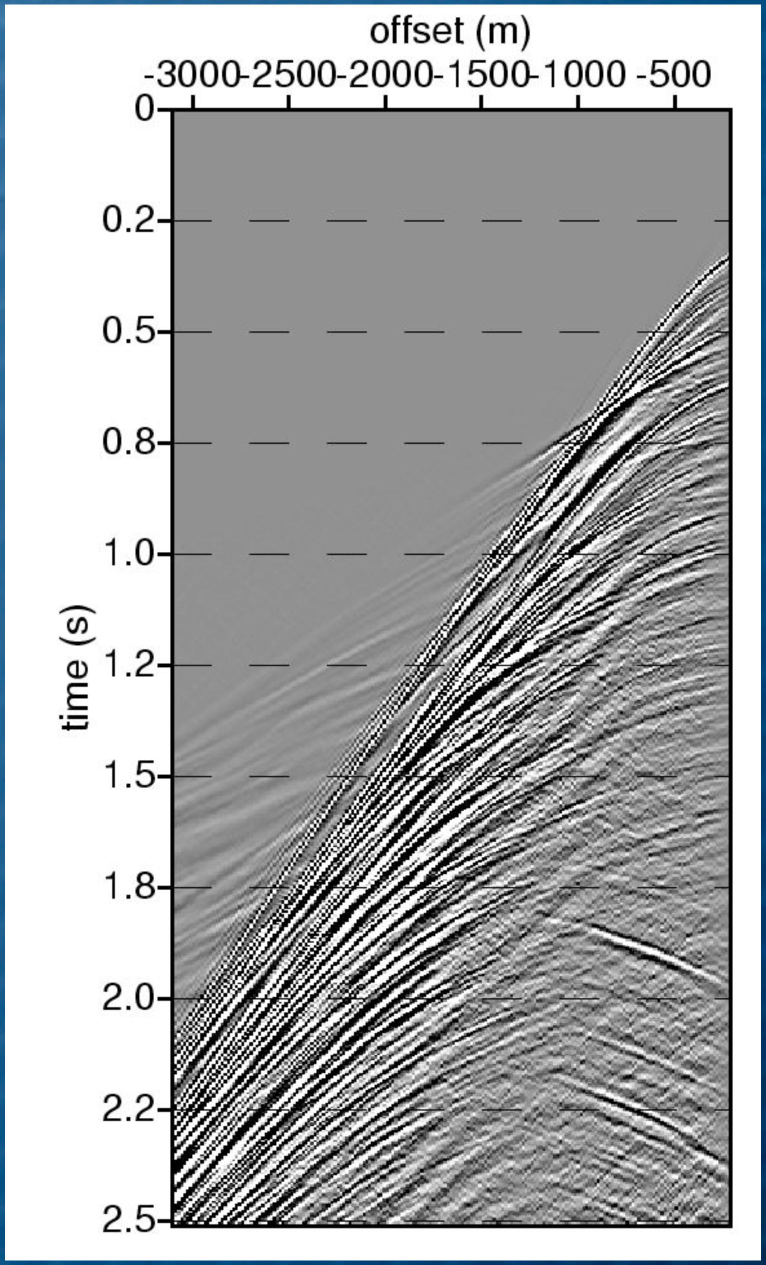
predicted multiples



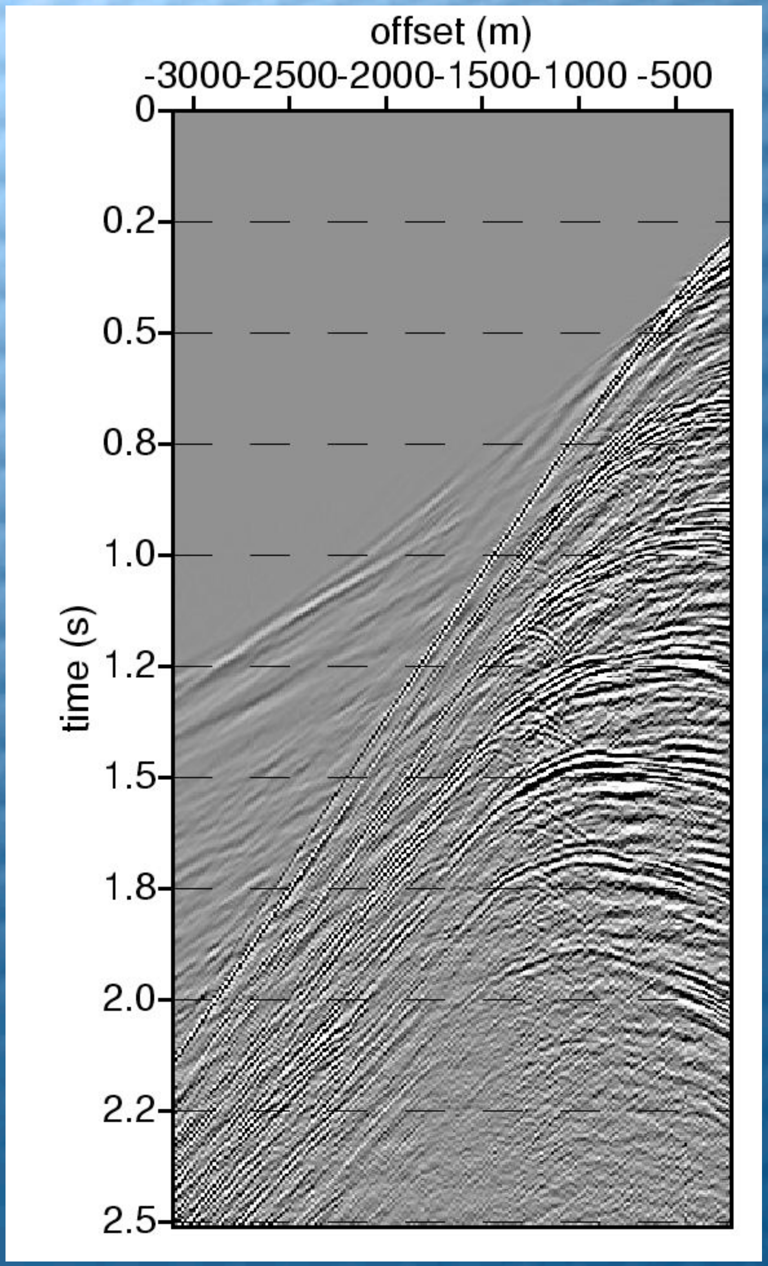
Subtraction with L2 norm



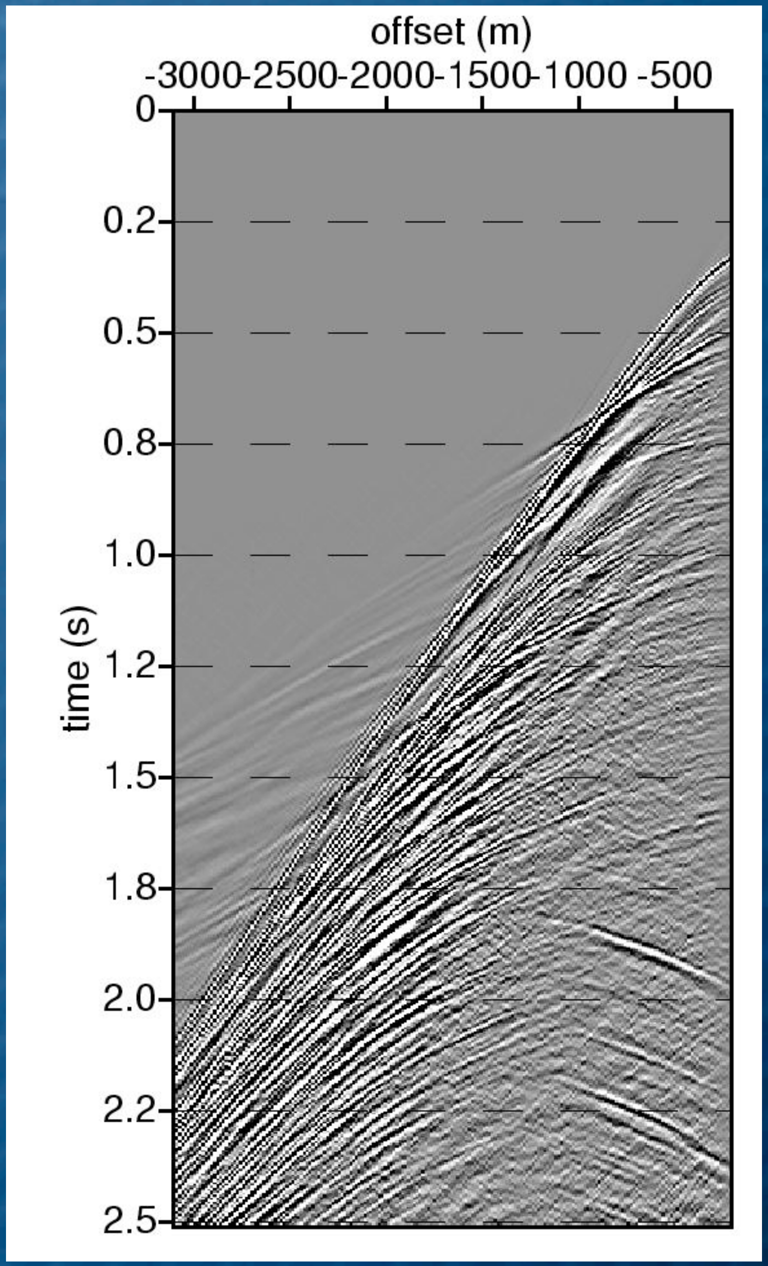
Input
with
multiples



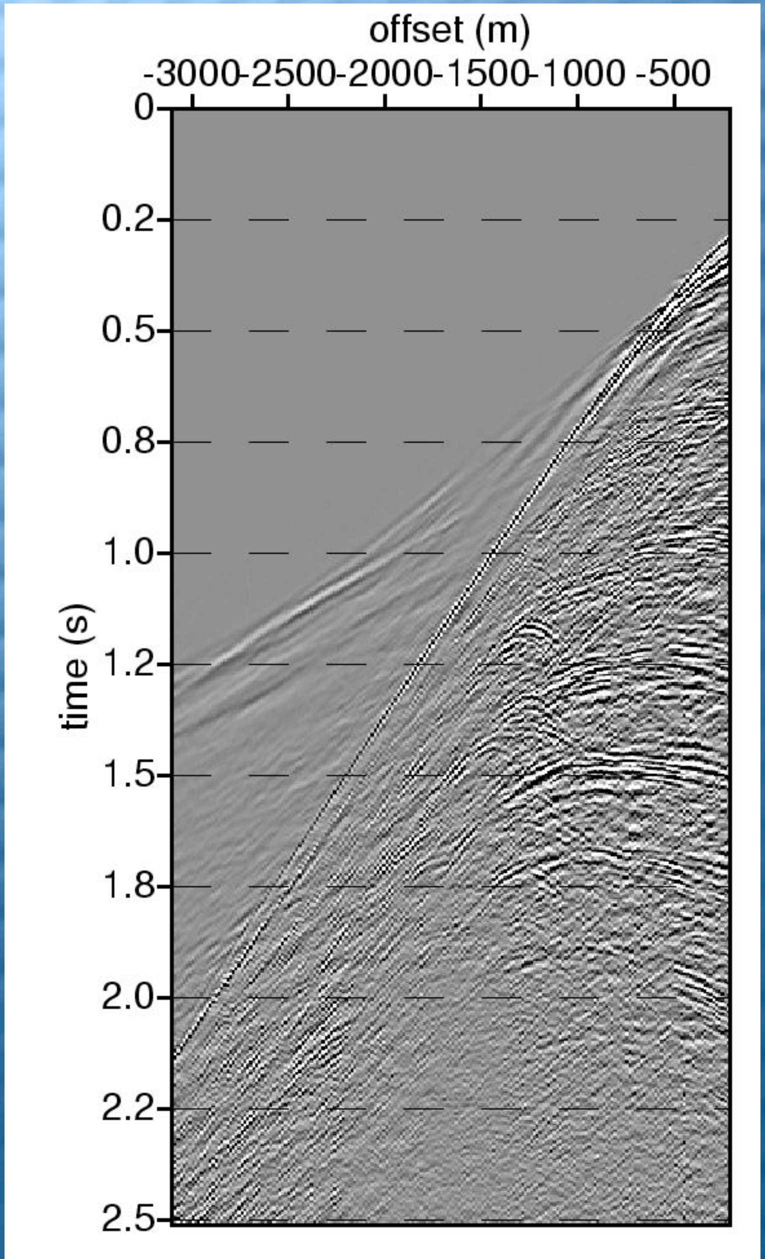
Subtraction with L2 norm



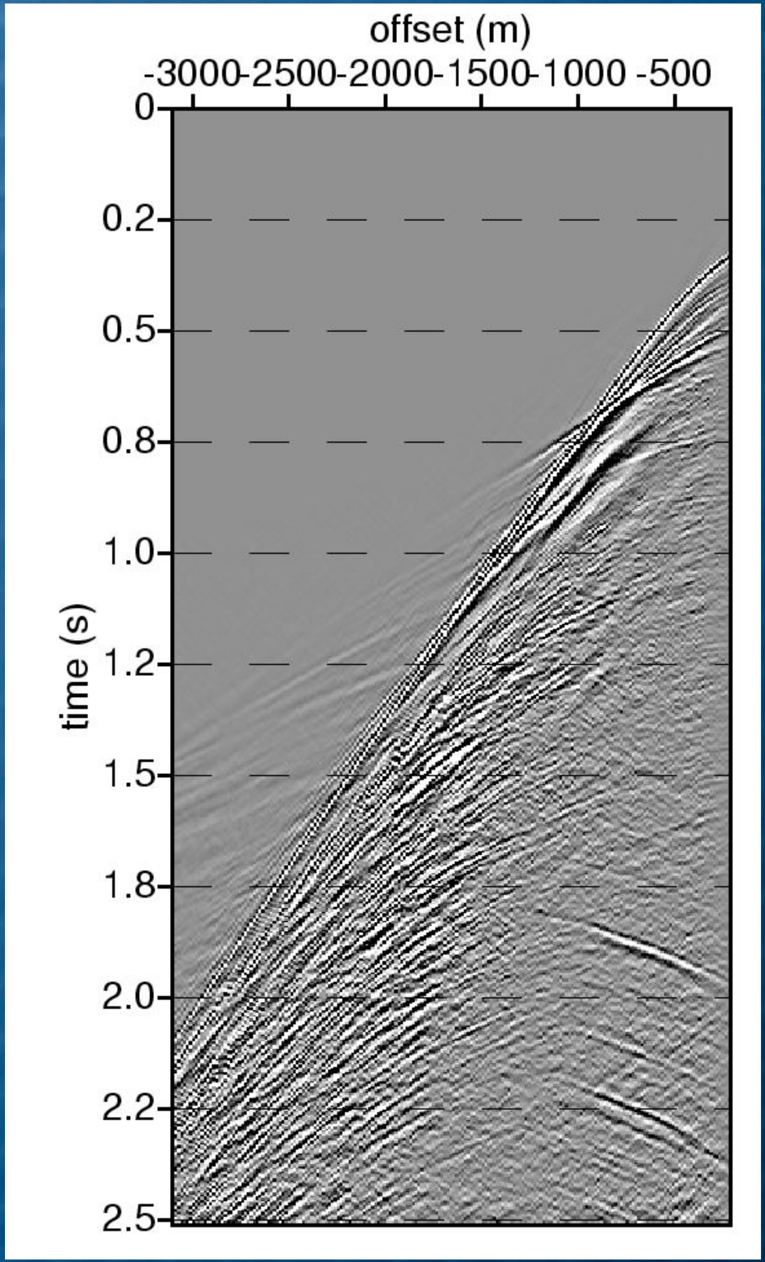
Output
SRME
L2
subtraction



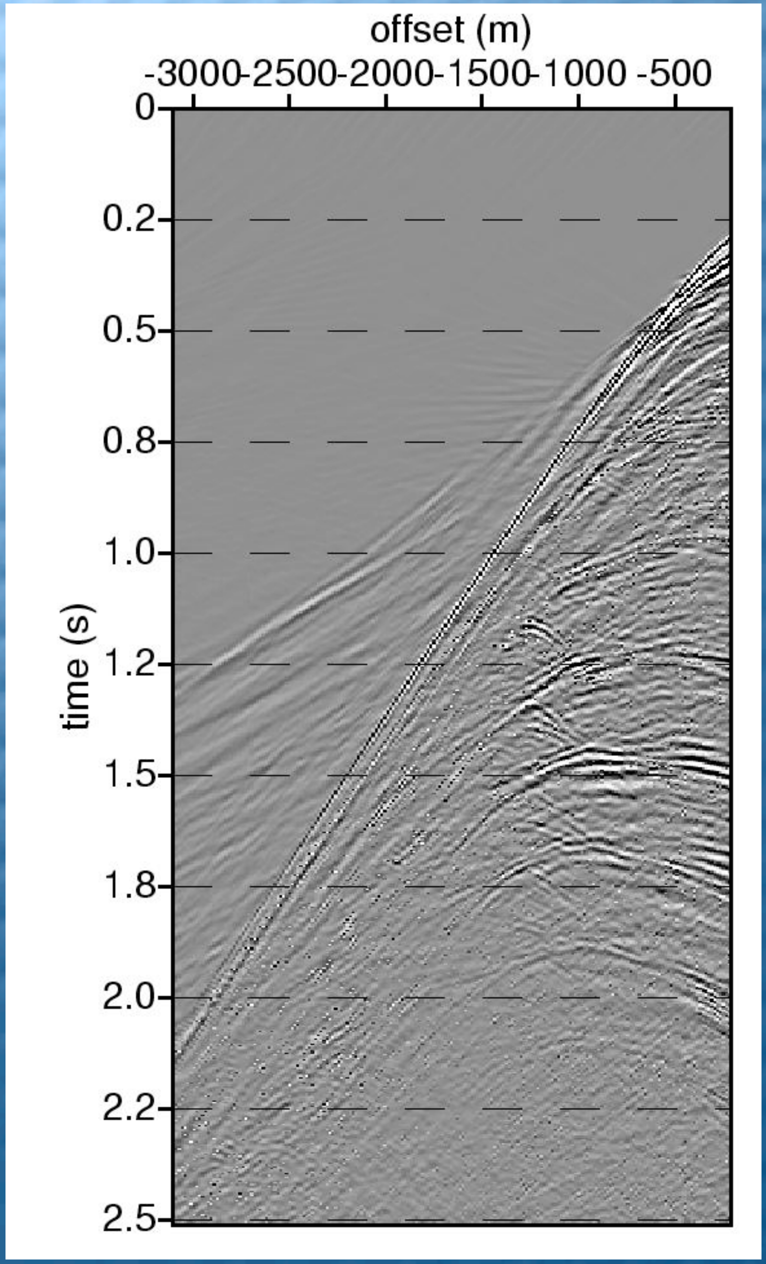
Subtraction with L2 norm



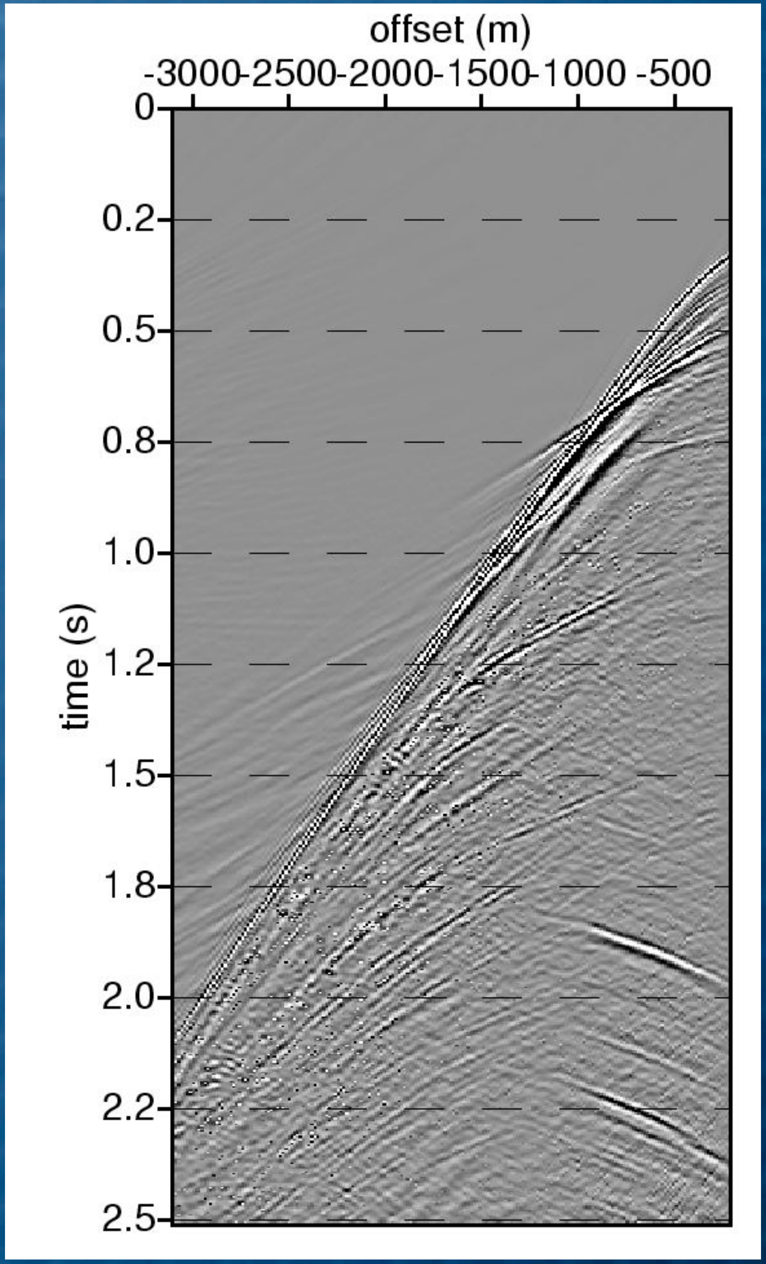
Output
SRME
multi-L2
subtraction



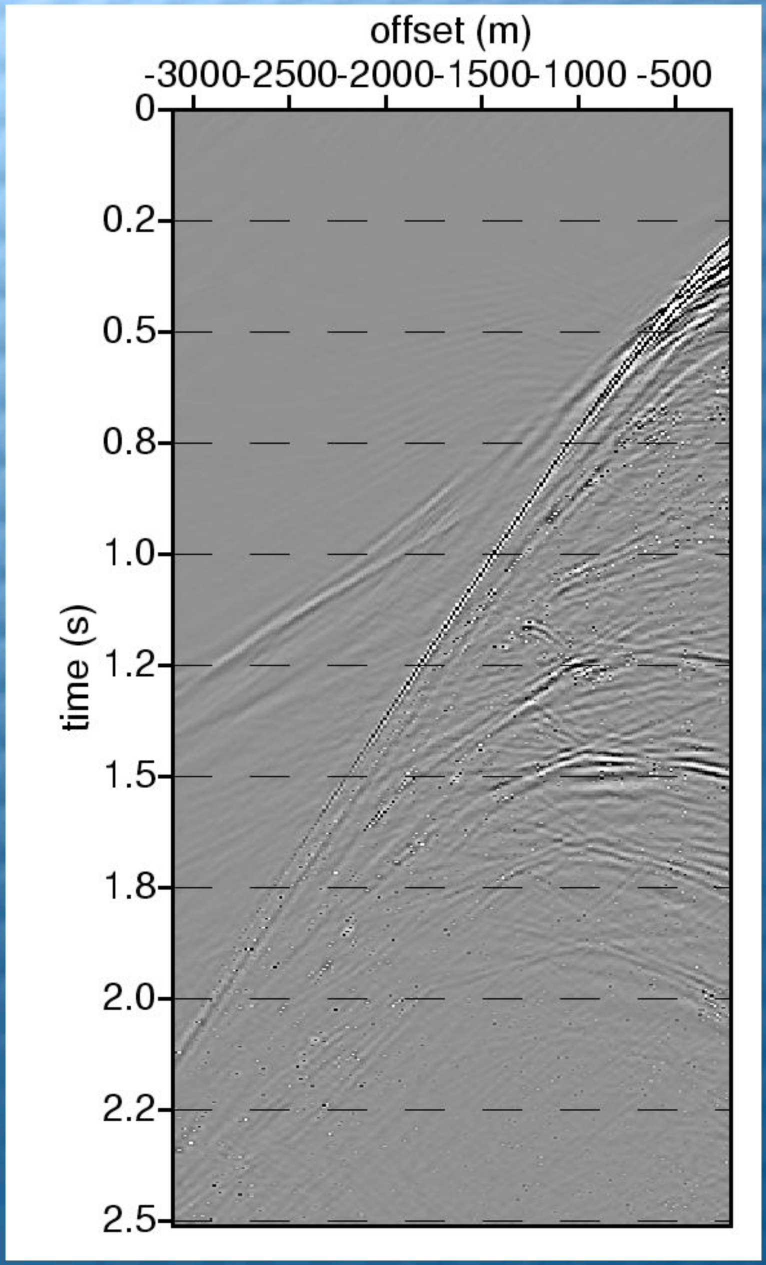
Multiple suppression with curvelets



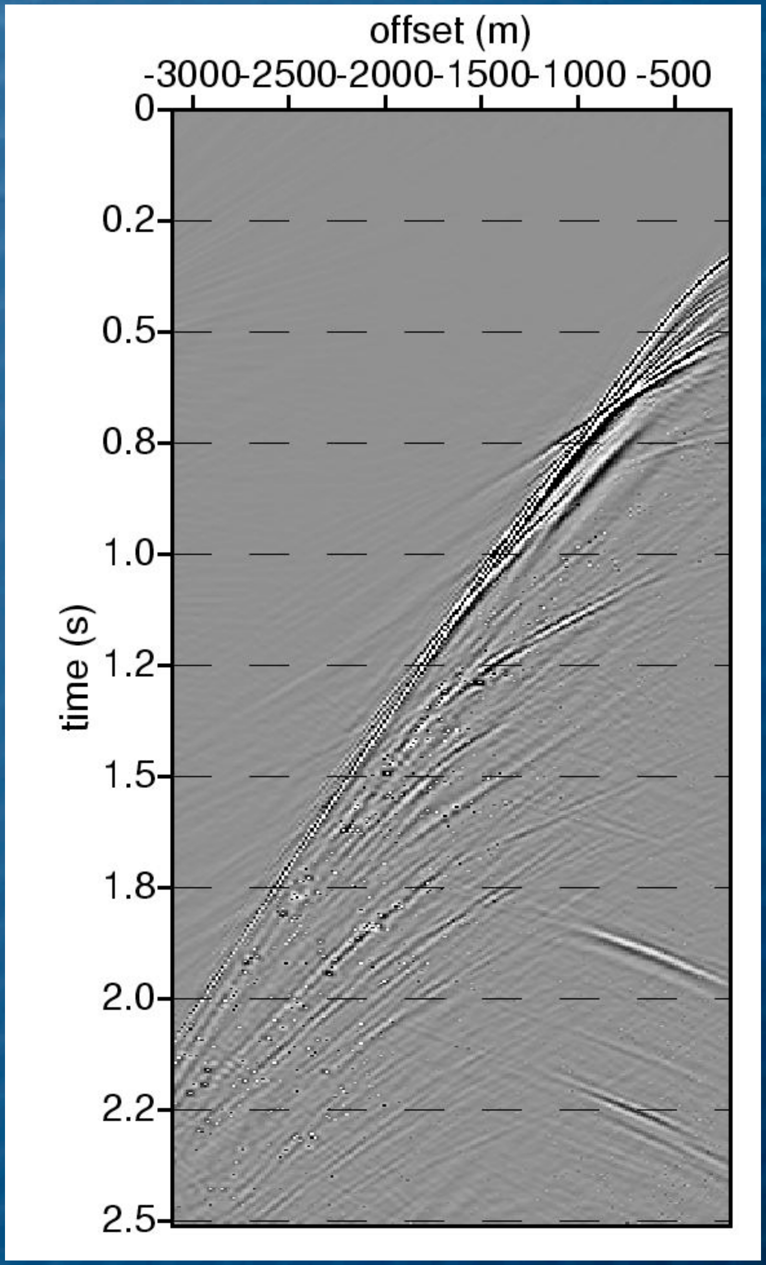
Output curvelet filtering



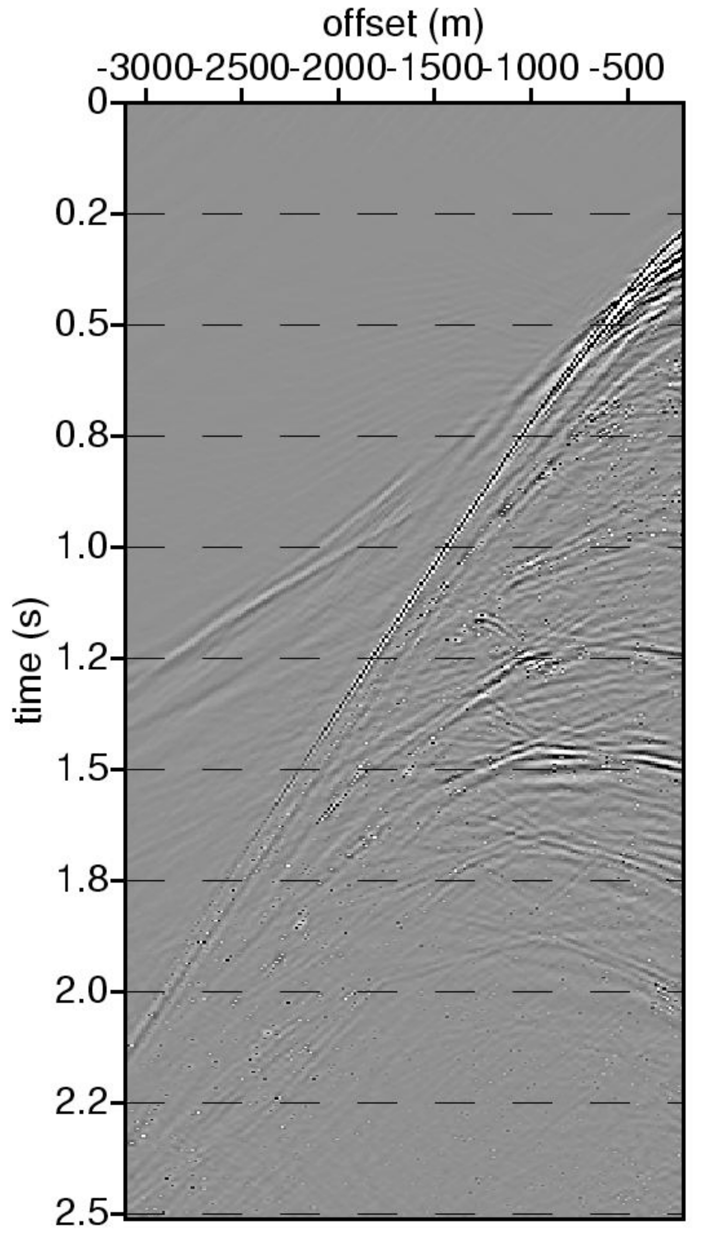
Multiple suppression with curvelets



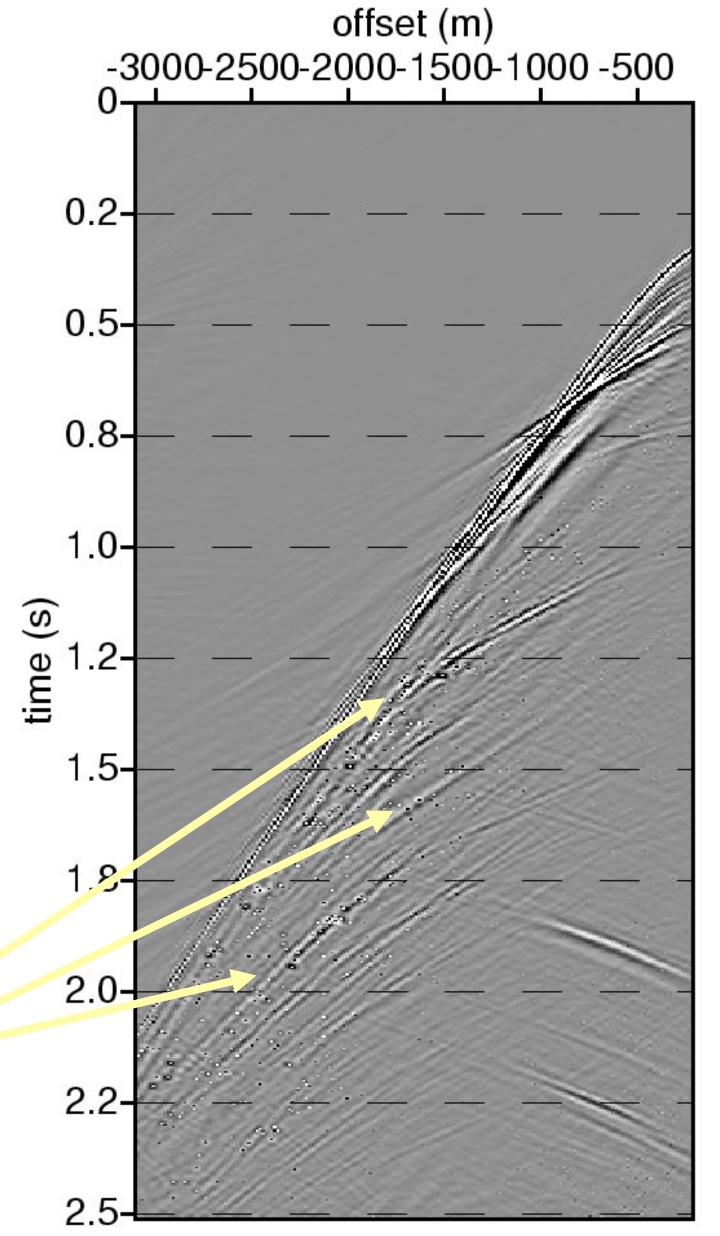
Output curvelet filtering with stronger threshold



Multiple suppression with curvelets

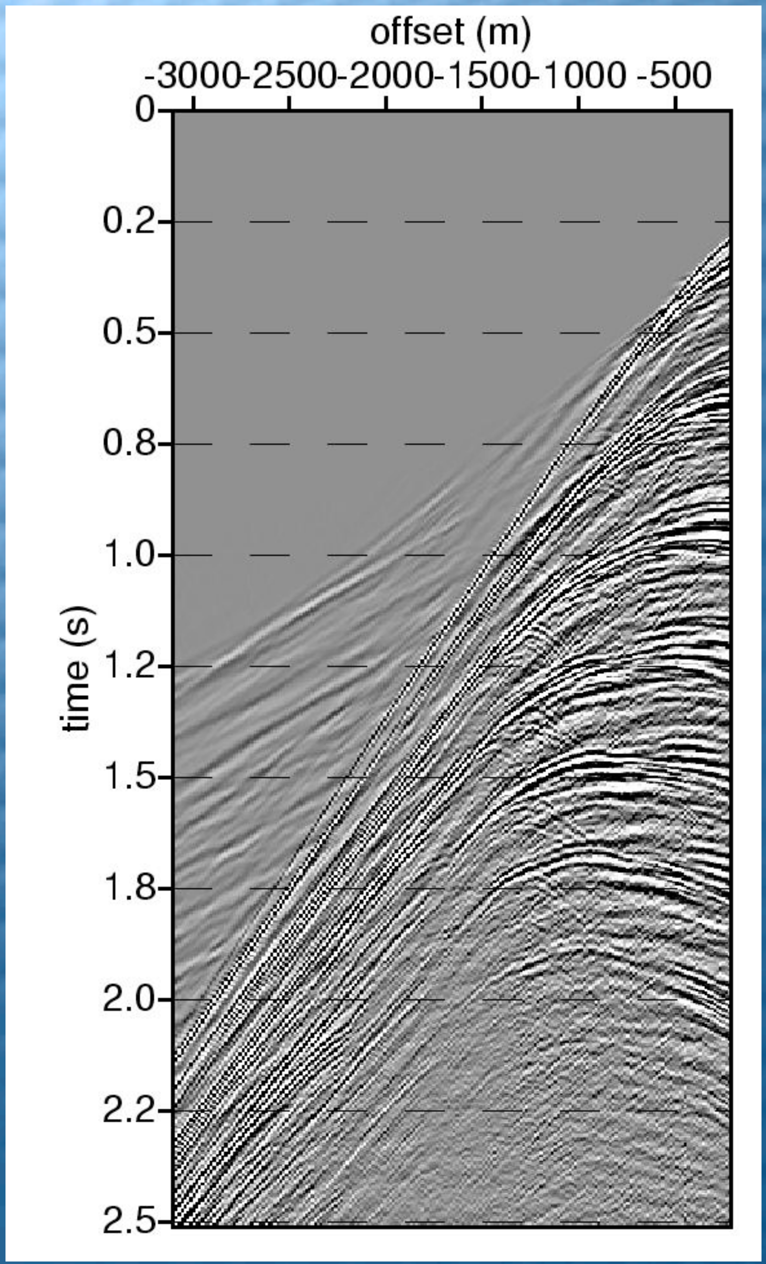


Output curvelet filtering with stronger threshold

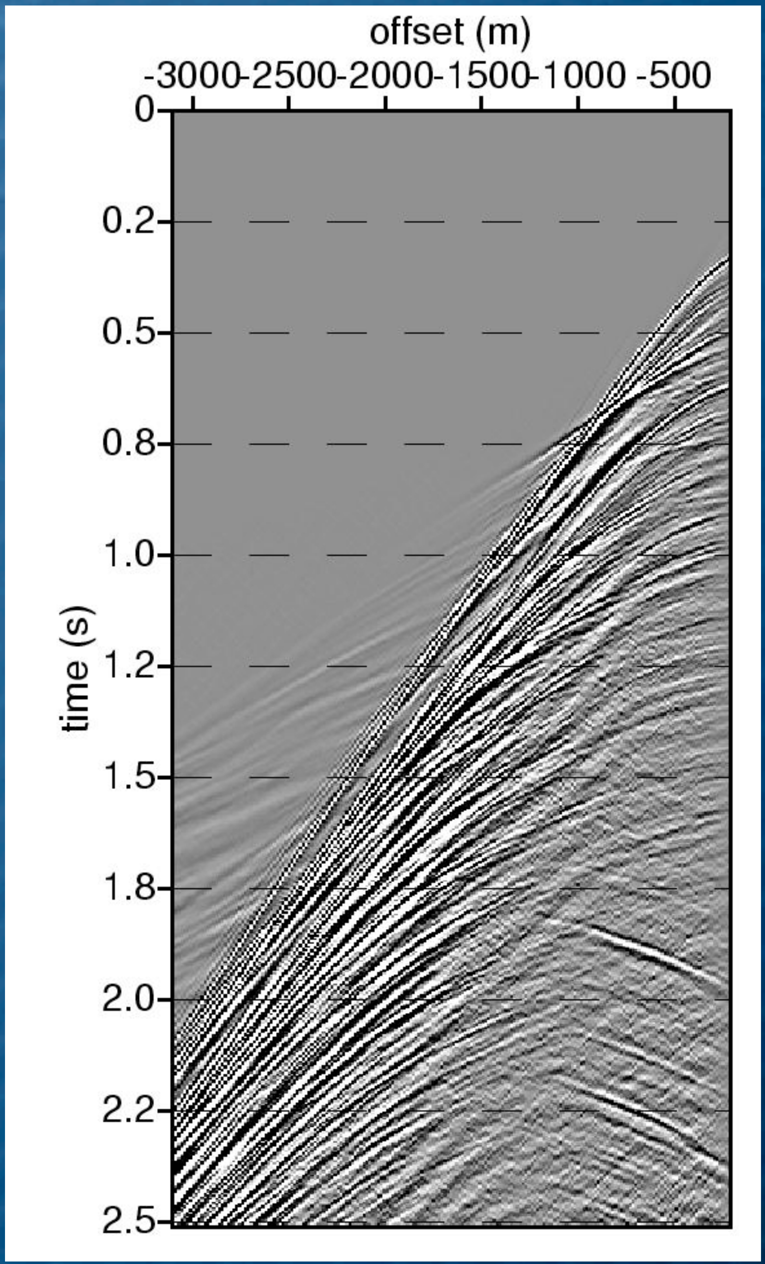


Preserved primaries

Subtraction with L2 norm



Input
with
multiples



Noise prediction

Used 3D SRME to predict noise

Can use other noise predictions (e.g. Radon)

We do **NOT** ***subtract*** rather ***mute***

- put to zero or preserve coherent features
- less sensitive to errors

Impose additional (sparseness) norms

Preserve the edges & primaries!

Global optimization

Formulate constrained optimization:

$$\hat{\mathbf{m}} : \quad \min_{m} J(\mathbf{m}) \quad \text{s.t.} \quad |\tilde{\mathbf{m}} - \hat{\tilde{\mathbf{m}}}_0|_\mu \leq \mathbf{e}_\mu, \quad \forall \mu$$

with

$$\hat{\mathbf{m}}_0 = \mathbf{B}^\dagger \Theta_{\boldsymbol{\lambda\Gamma}} \left(\tilde{\mathbf{d}} \right)$$

and with e_μ **threshold and noise-dependent *tolerance* on curvelet coeff.**

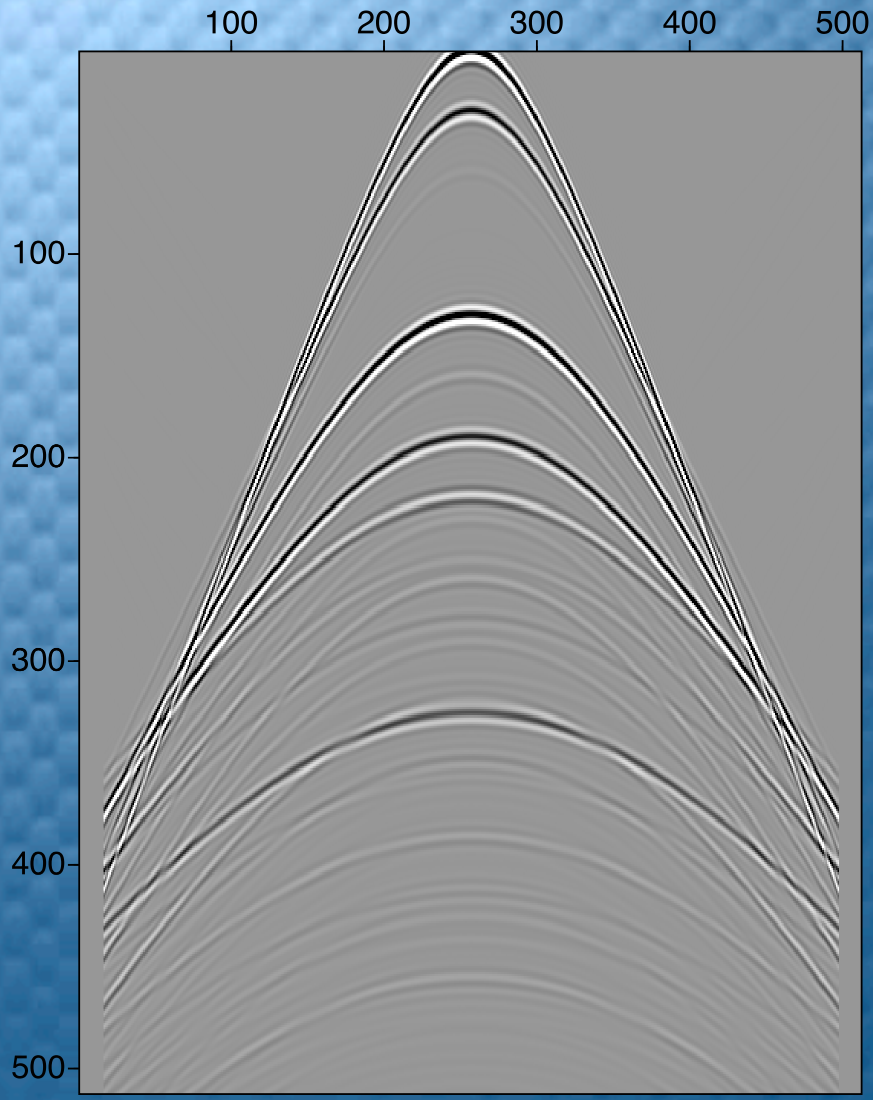
Global optimization

Set tolerances

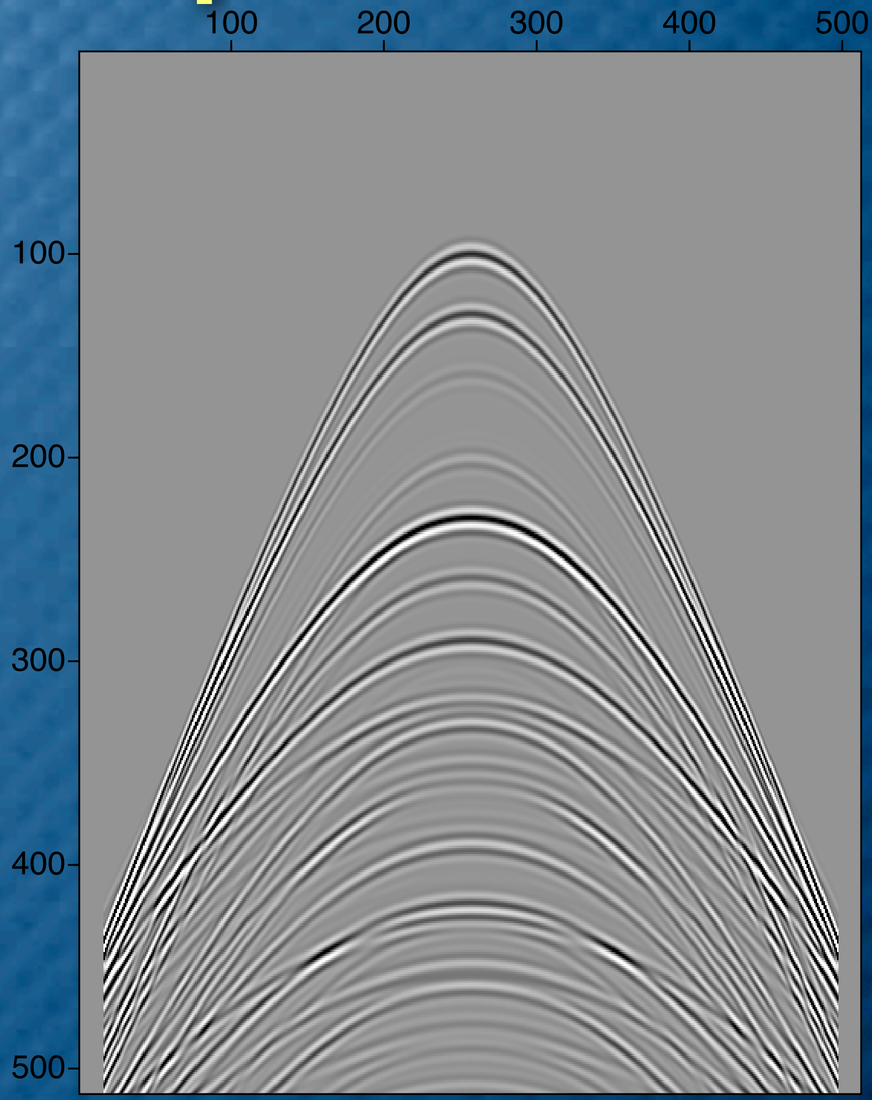
$$\mathbf{e}_\mu = \begin{cases} \mathbf{\Gamma}_\mu & \text{if} \quad |\hat{\tilde{\mathbf{m}}}_0|_\mu \quad \geq \quad |\boldsymbol{\lambda}\mathbf{\Gamma}|_\mu \\ \boldsymbol{\lambda}\mathbf{\Gamma}_\mu & \text{if} \quad |\hat{\tilde{\mathbf{m}}}_0|_\mu \quad < \quad |\boldsymbol{\lambda}\mathbf{\Gamma}|_\mu \end{cases}$$

with λ defining the confidence interval, e.g $\lambda = 3$ corresponds to 95 %.

1-D example

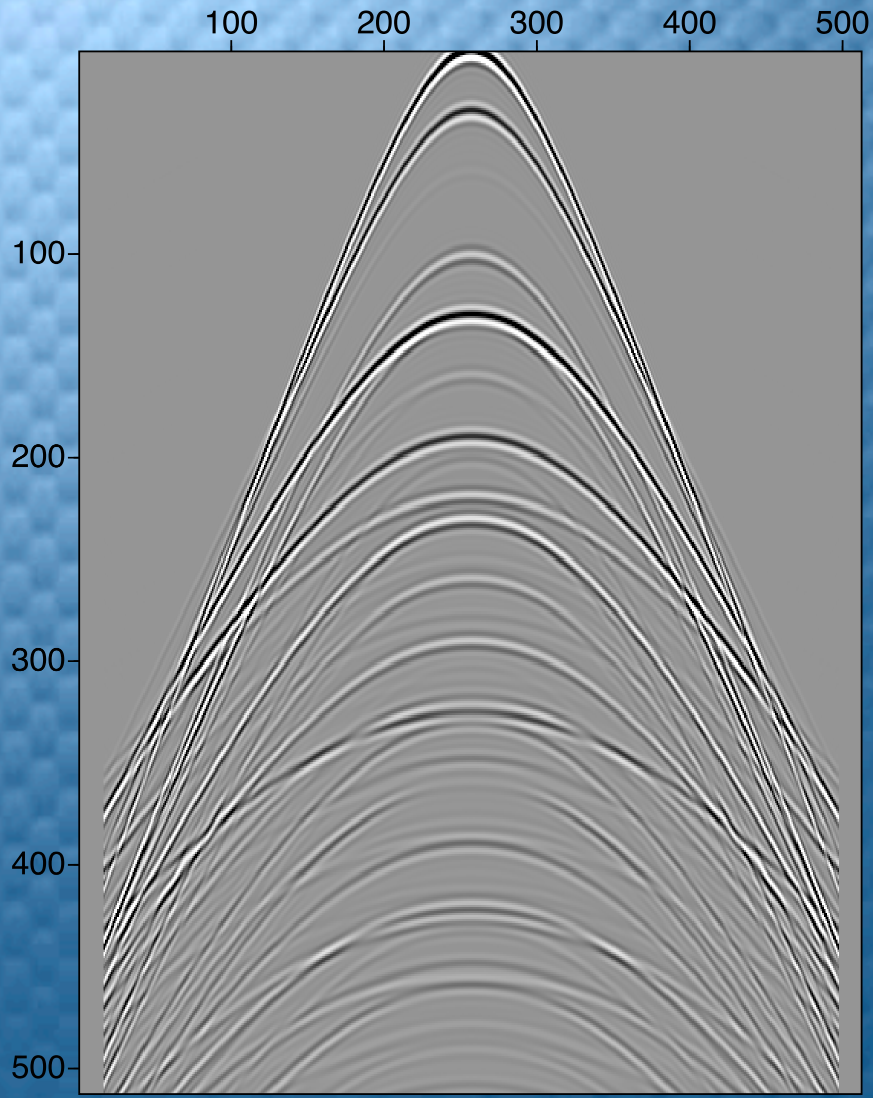


Primaries

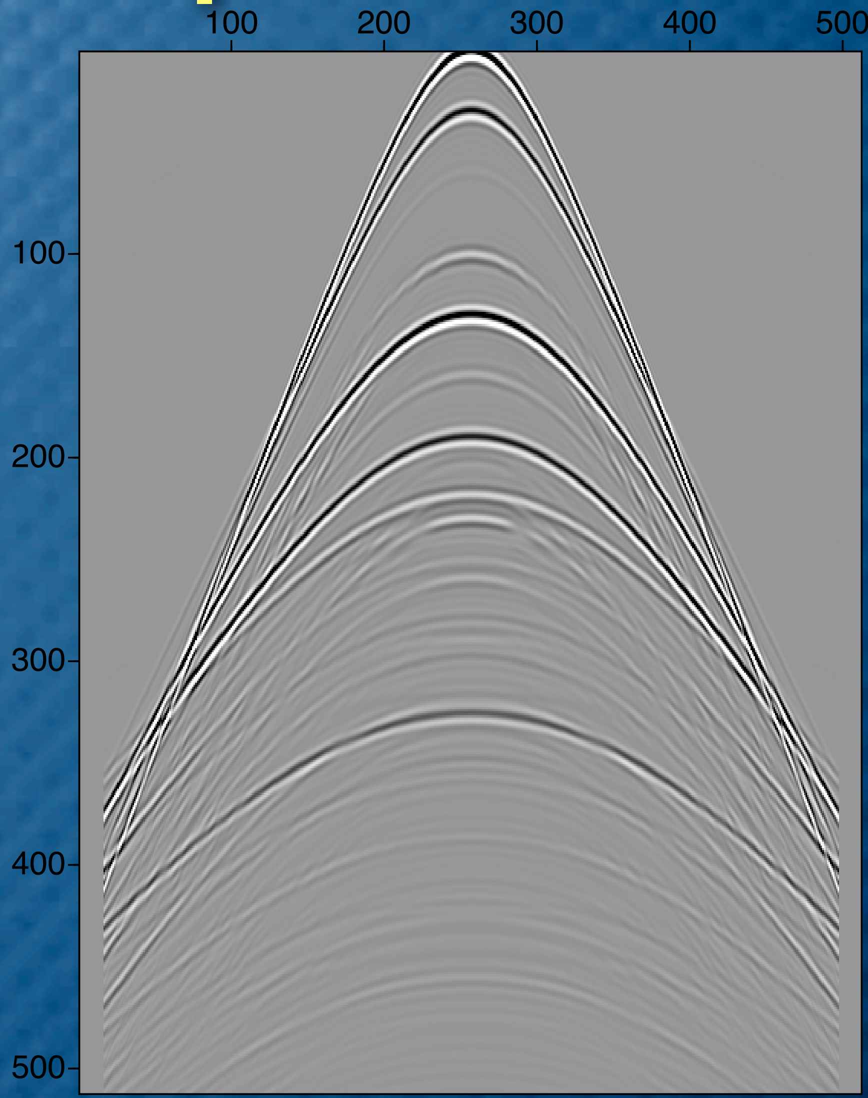


Multiples

1-D example

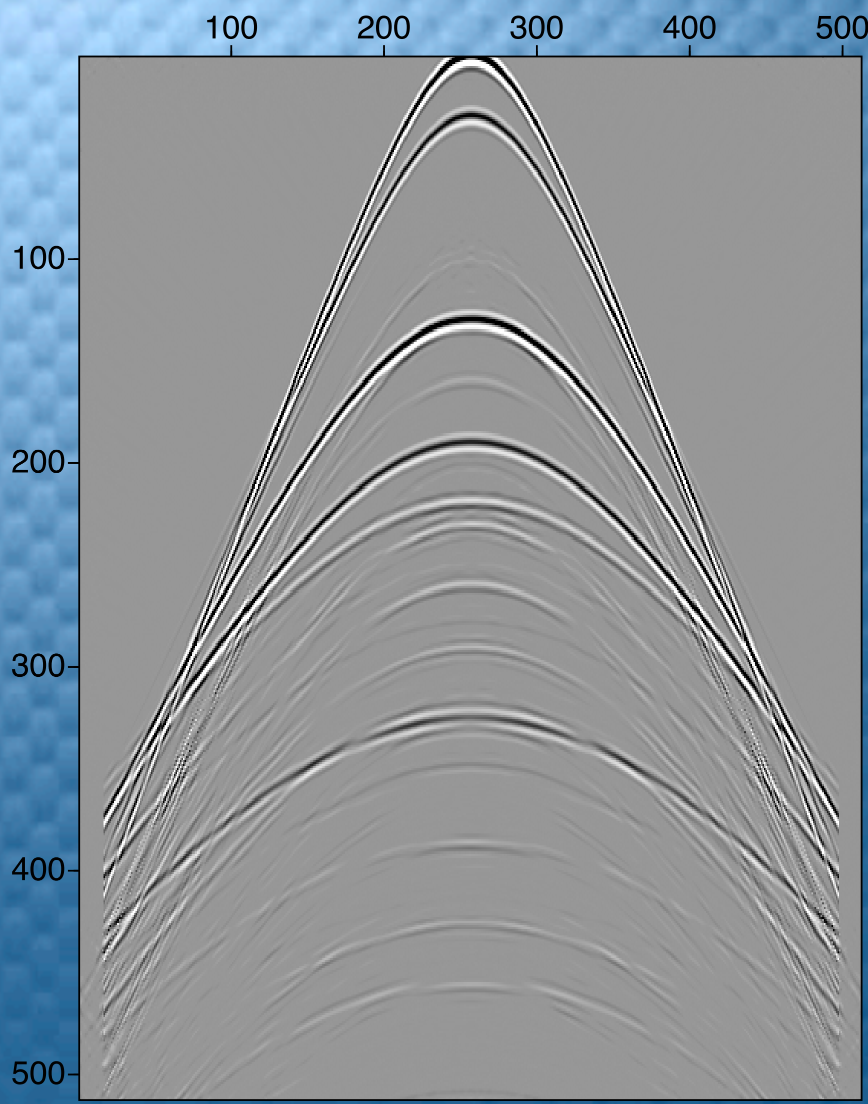


Data with multiples



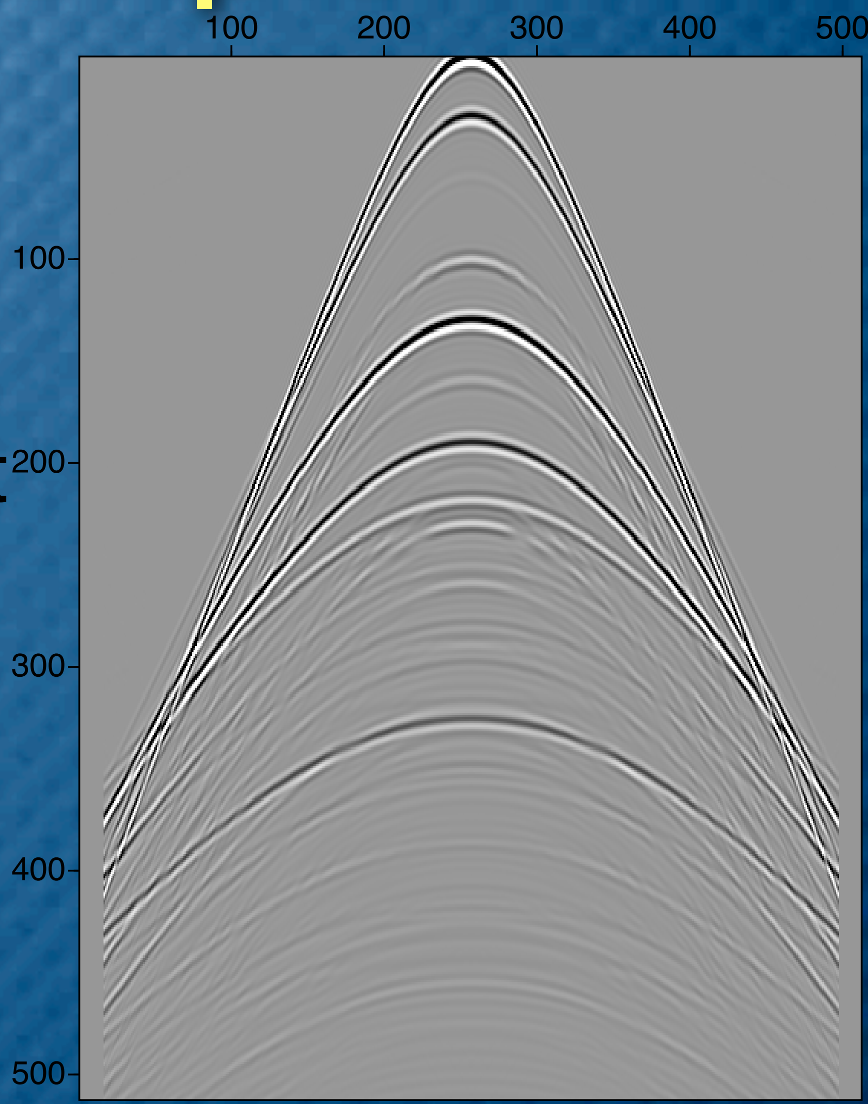
Least-squares adaptive subtraction

1-D example



Curvelet adaptive subtraction with optimization

Text



Least-squares adaptive subtraction

Conclusions

- For 3D SRME the acquisition geometry determines the prediction quality and possibilities
- New domains extend the adaptive subtraction toolbox and hence the quality of the end result
- Non-linear Curvelet thresholding adds robustness
- Finding a norm that enhances the wave-front set is an open problem
- Estimating the seismic wavelet should be feasible

Acknowledgements

Frank Kempe (Cray) for conducting the 3D SRME tests on the Cray-X1 and George Stephenson (Cray) for his support

Candes & Donoho for making their Curvelet code available.

Partially supported by a NSERC Grant.

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