Curvelet imaging & processing: an overview

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thanks to: Gilles, Peyman and Candes, Sacchi

Earth and Ocean Sciences UBC

Research program

How **to improve seismic images?**

What **is in the image?**

Why **is it in the image?**

Curvelet imaging & processing series is devoted to the *'How'*.

Goals

Processing & imaging scheme

- **increases resolution & SNR**
- **preserves edges = freq. content**
- **works with and extends existing**
  - **noise removal approaches**
  - **imaging schemes**

Develop the right *language* to deal with SNR ≤ 0 ....

Wish list

Seek a *transform domain* that is

- ***relative insensitive* to *local phase***
- ***sparse & local* (position/dip)**
- ***optimal* for *curved* reflectors**
- ***well-behaved* under *operators* (e.g imaging)**

Aim to bring out those high frequencies with *low* SNR!

Basic idea

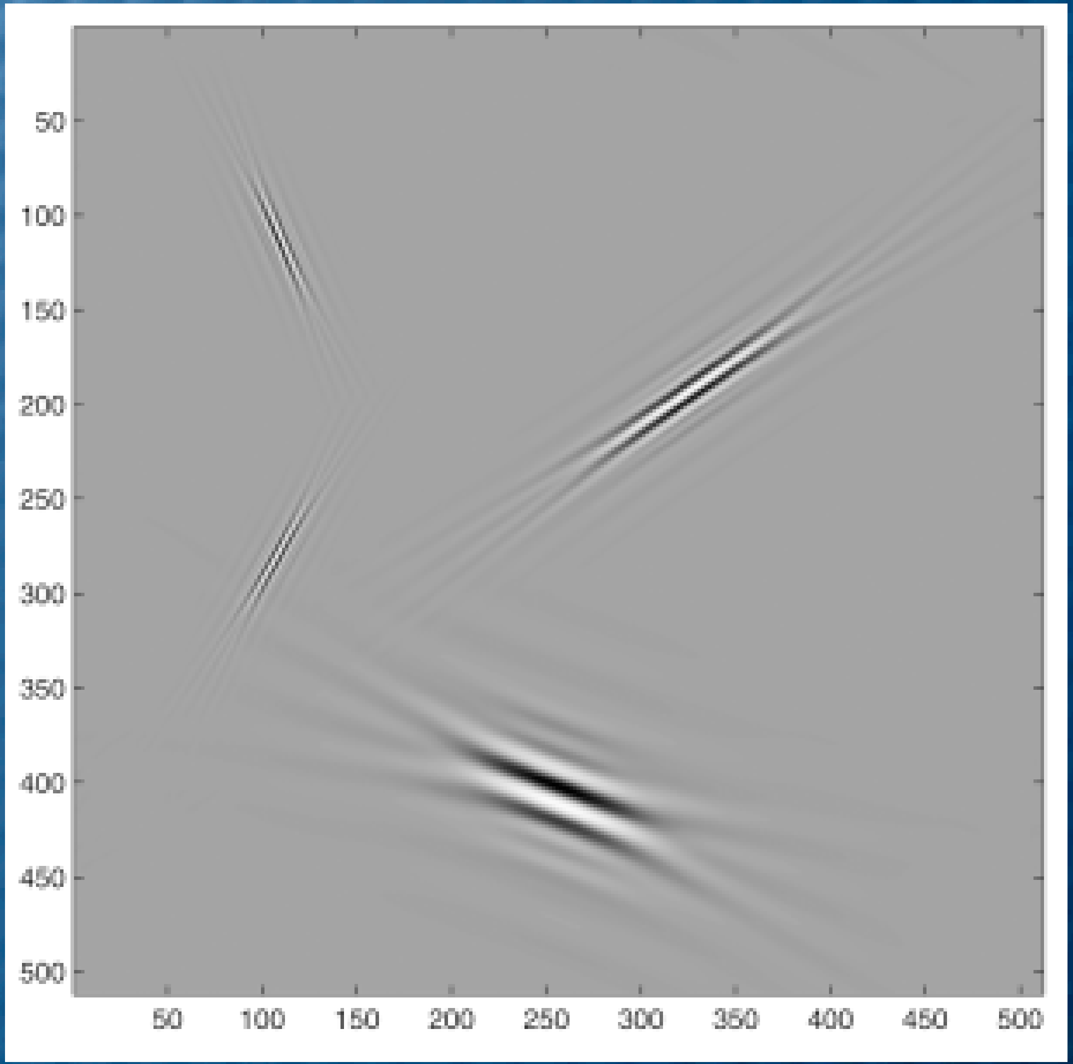
Build on the premise that you stand a much better chance of solving a problem when the model is represented optimally ...

- **local**
- **sparse**
- **multi-scale and multi-directional**

Well behaved under operators (e.g migration)!

Why curvelets

- Nonseparable
- Local in 2-D space
- Local in 2-D Fourier
- Anisotropic
- Multiscale
- Almost orthogonal
- Tight frame
- Optimal



Curvelets

Curvelets/Contourlets:

- **Anisotropic scaling law:**

$$\textbf{width} \equiv 2^{-2j} \qquad \textbf{length}^2 \equiv 2^{-j}$$

- **Directional selective:**

$$\textbf{\#orientations} = 1/\sqrt{\text{scale}}$$

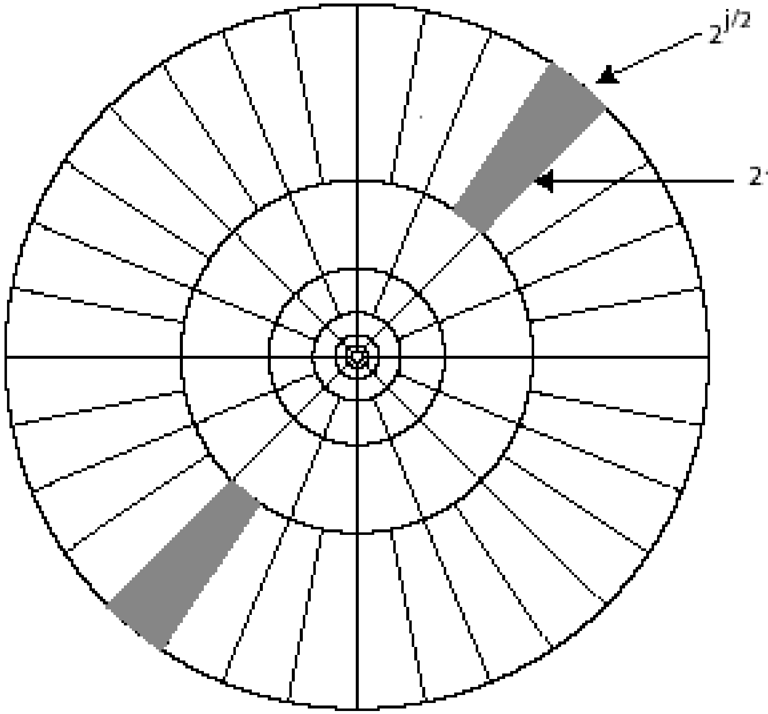
- **Close to optimal for functions with singularities on C^2-curves:**

$$\|m - \tilde{m}_m^{\text{improved wavelet}}\|_2 \qquad C \cdot m^{-2} \left(\log m\right)^3$$

Candes '02, Do '02

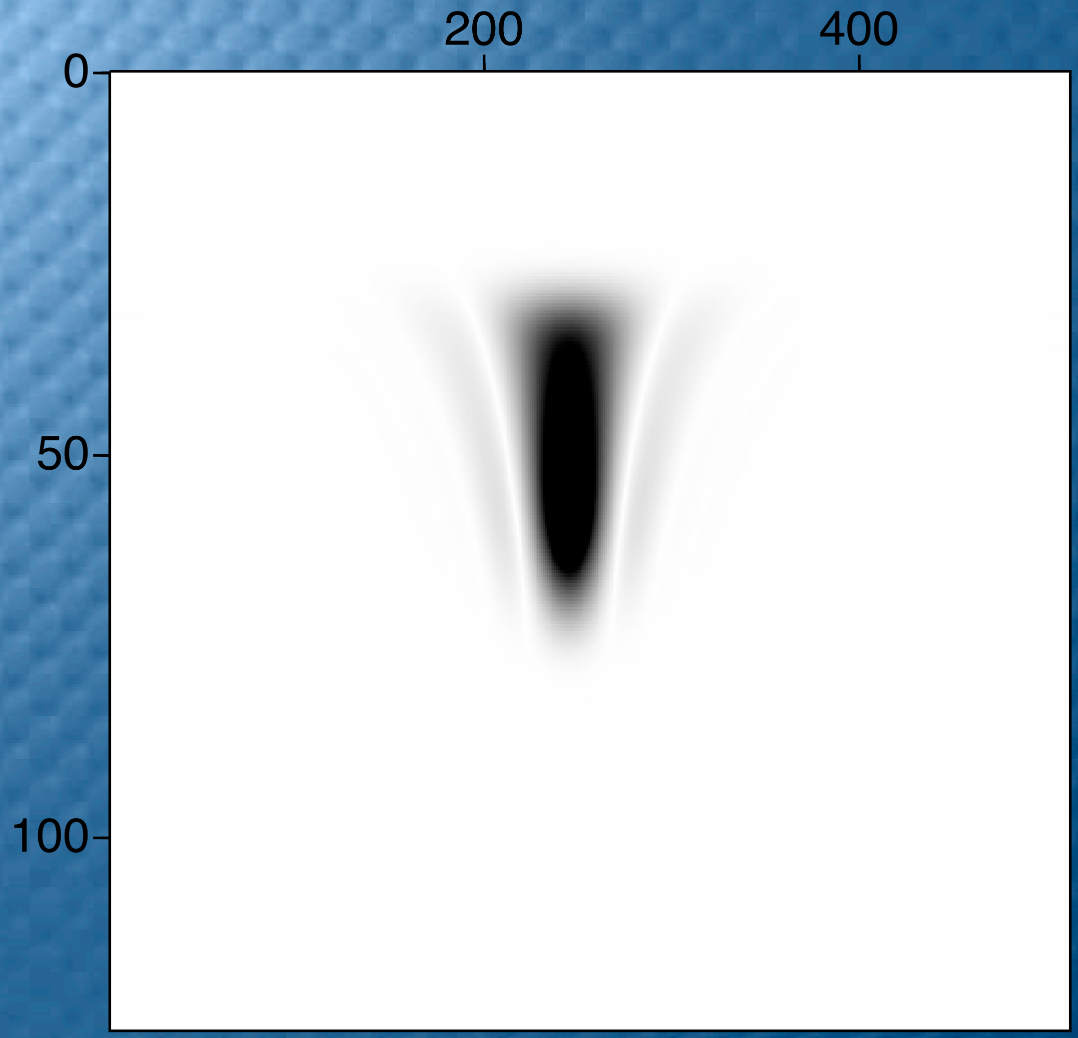
Why curvelets

$$W_j = \{\zeta, \quad 2^j \le |\zeta| \le 2^{j+1}, |\theta - \theta_J| \le \pi \cdot 2^{\lfloor j/2 \rfloor}\}$$



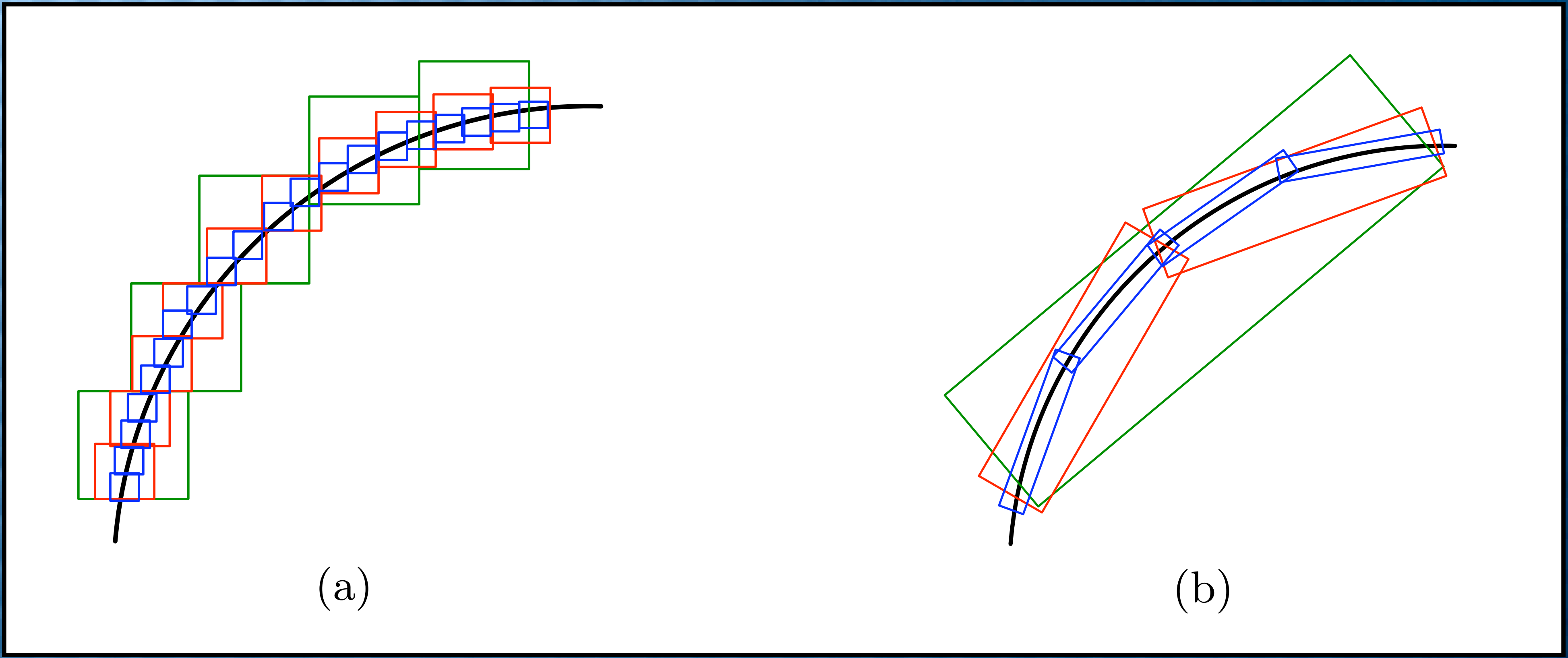
second dyadic partitioning

Why curvelets



Curvelet in FK-dom□in

Approximation rates



Do ‘01

Other domains

Most techniques are ***global:***

Non-adaptive:

- FFT
- Radon

Adaptive:

- Principle & Independent components
- SVD & KL

Approximation rates

- **Fourier/SVD/KL**

$$||f - \tilde{f}_m^F|| \propto m^{-1/2},\ m \to \infty$$

- **Wavelet**

$$||f - \tilde{f}_m^W|| \propto m^{-1},\ m \to \infty$$

- **Optimal data adaptive**

$$||f - \tilde{f}_m^A|| \propto m^{-2},\ m \to \infty$$

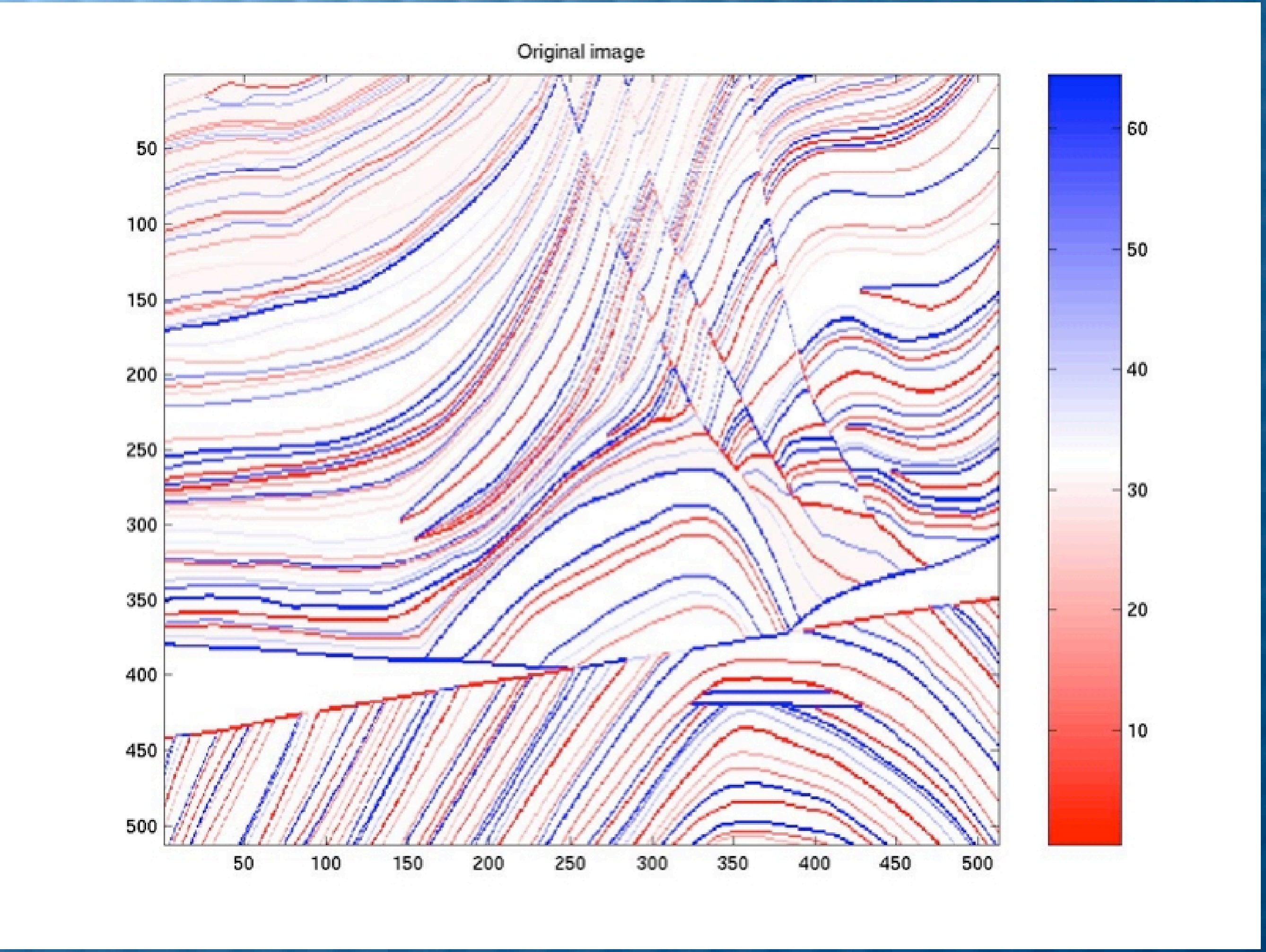
- **Close to optimal Curvelet**

$$||f - \tilde{f}_m^C|| \leq C \cdot m^{-2}(\log m)^3,\ m \to \infty$$

Gain orders of magnitude ... that's ruler of the game

Candes '02

Denoising



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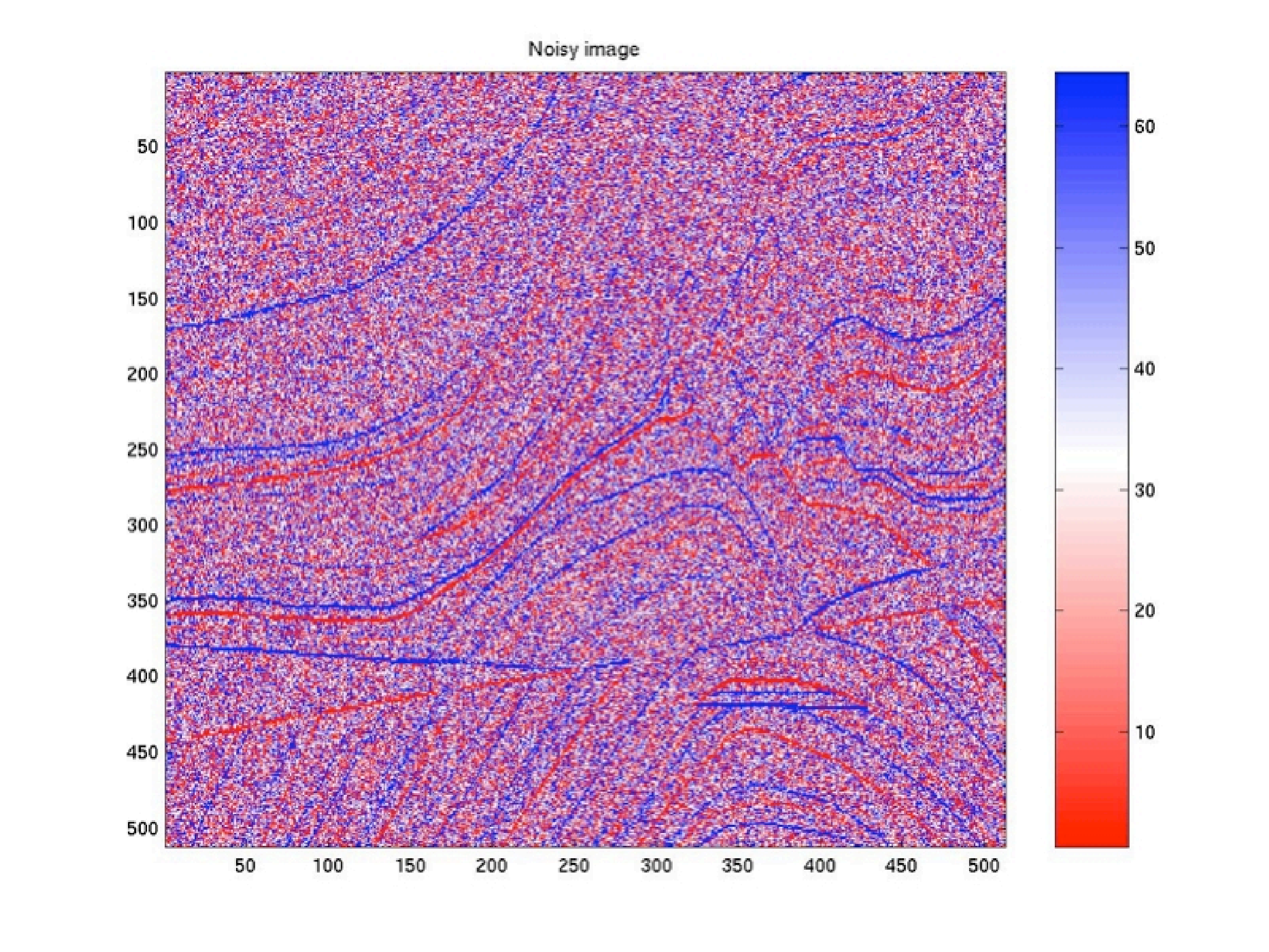
Denoising



Noisy image
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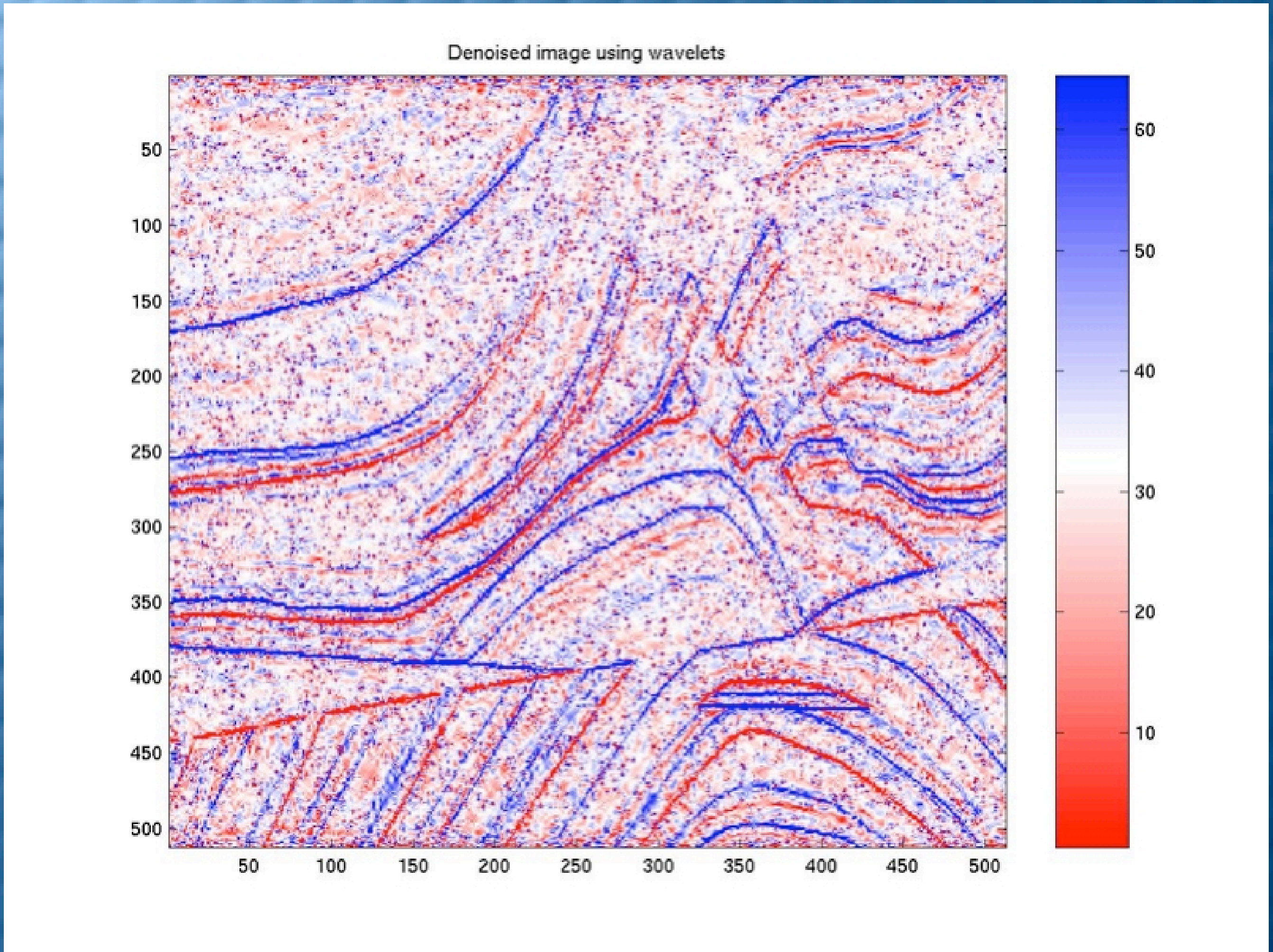
Wavelets



Denoised image using wavelets
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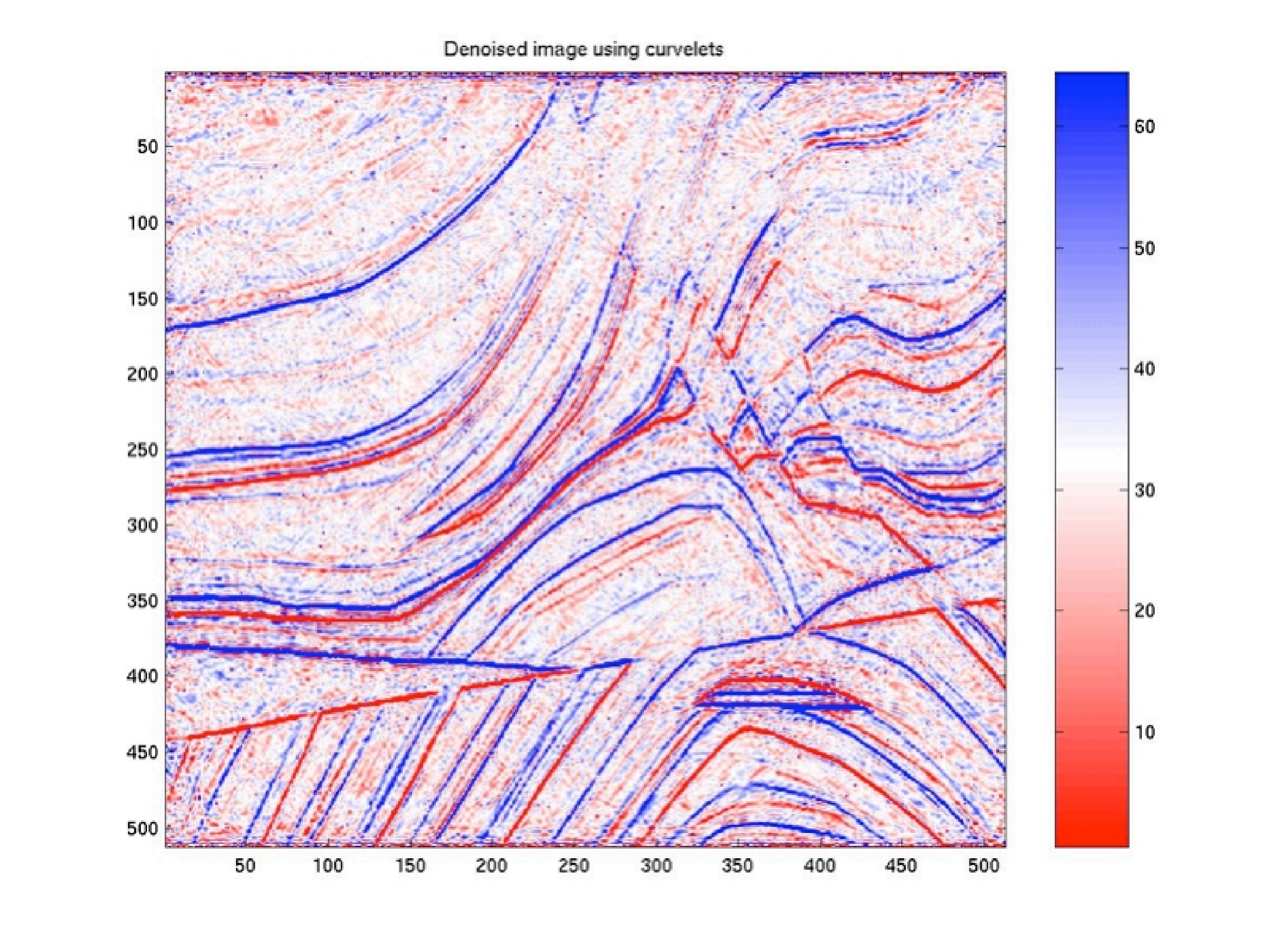
Curvelets



Denoised image using curvelets
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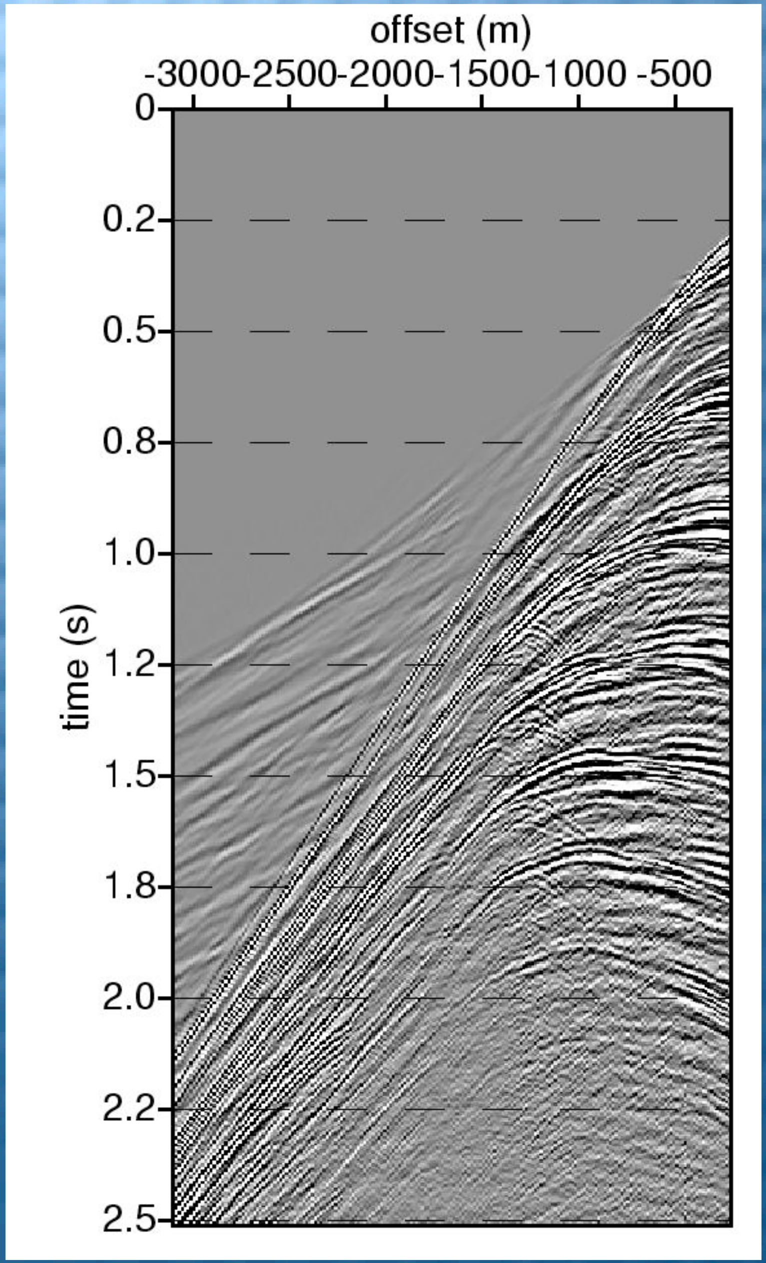
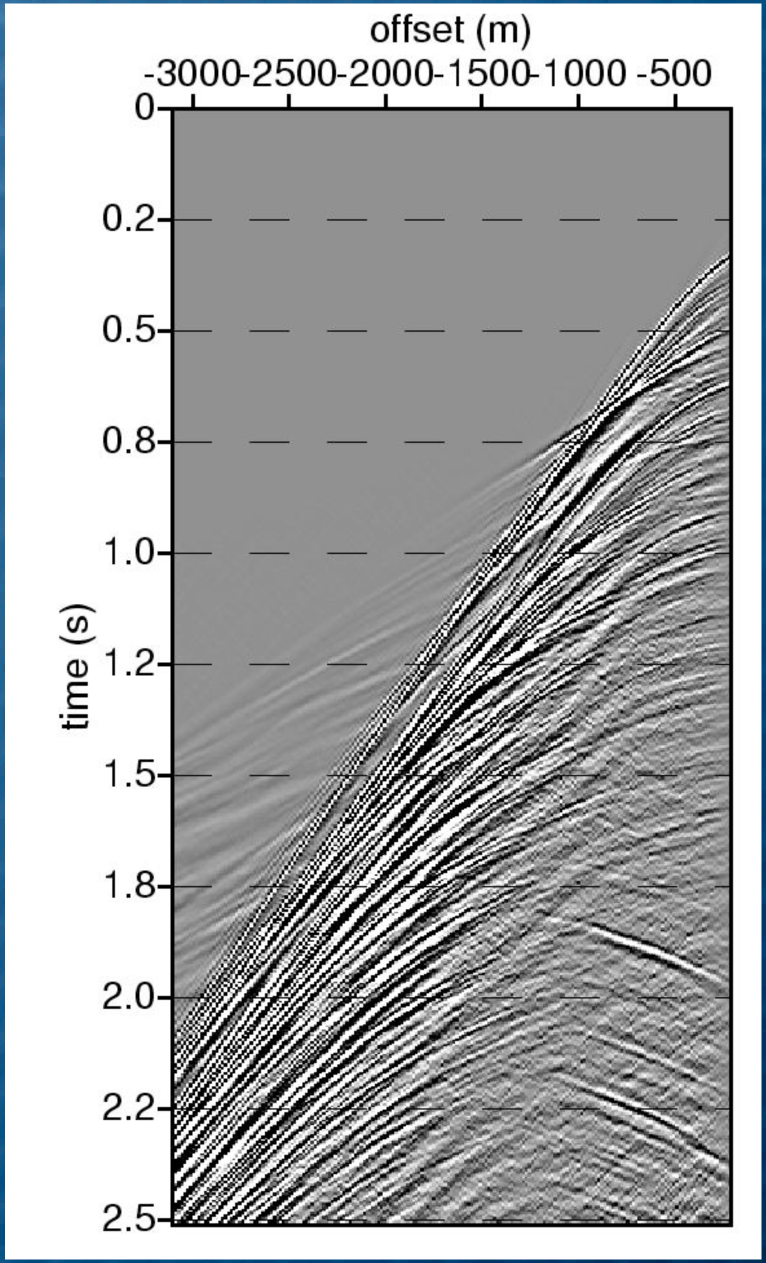
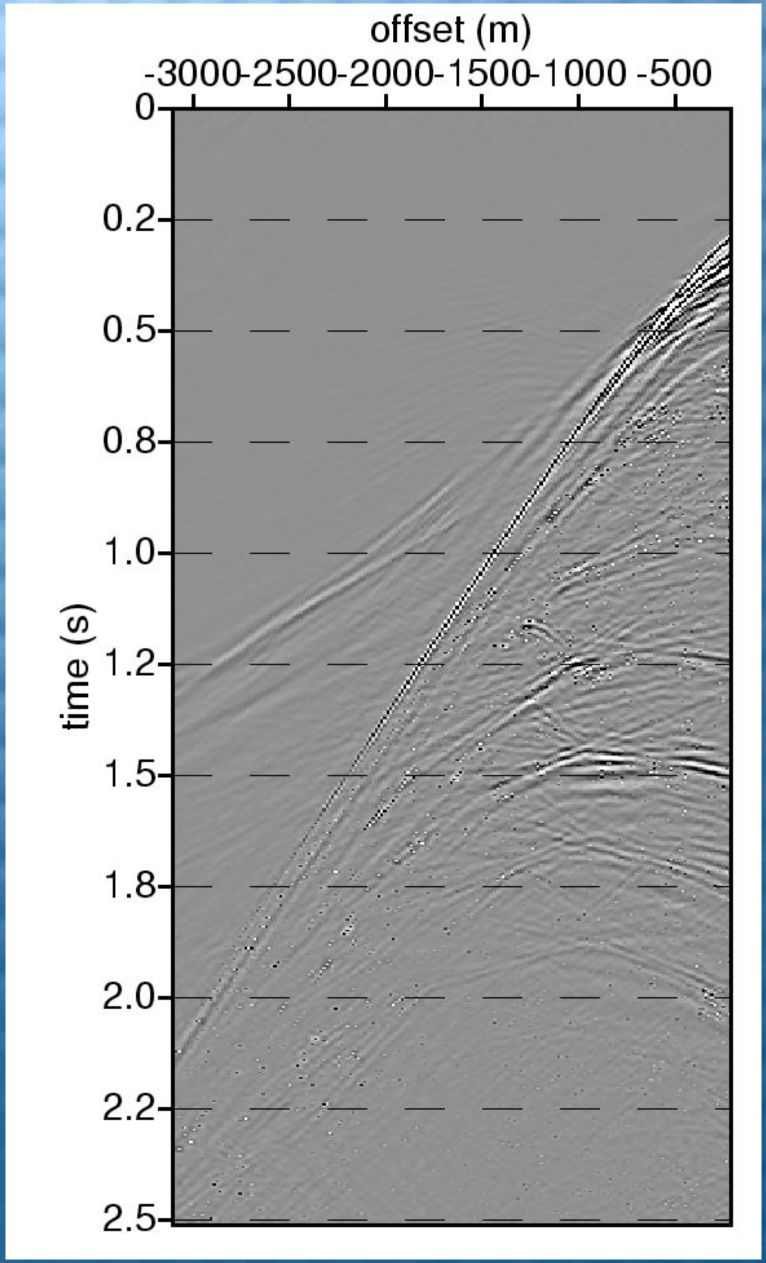
Multiple suppression with curvelets



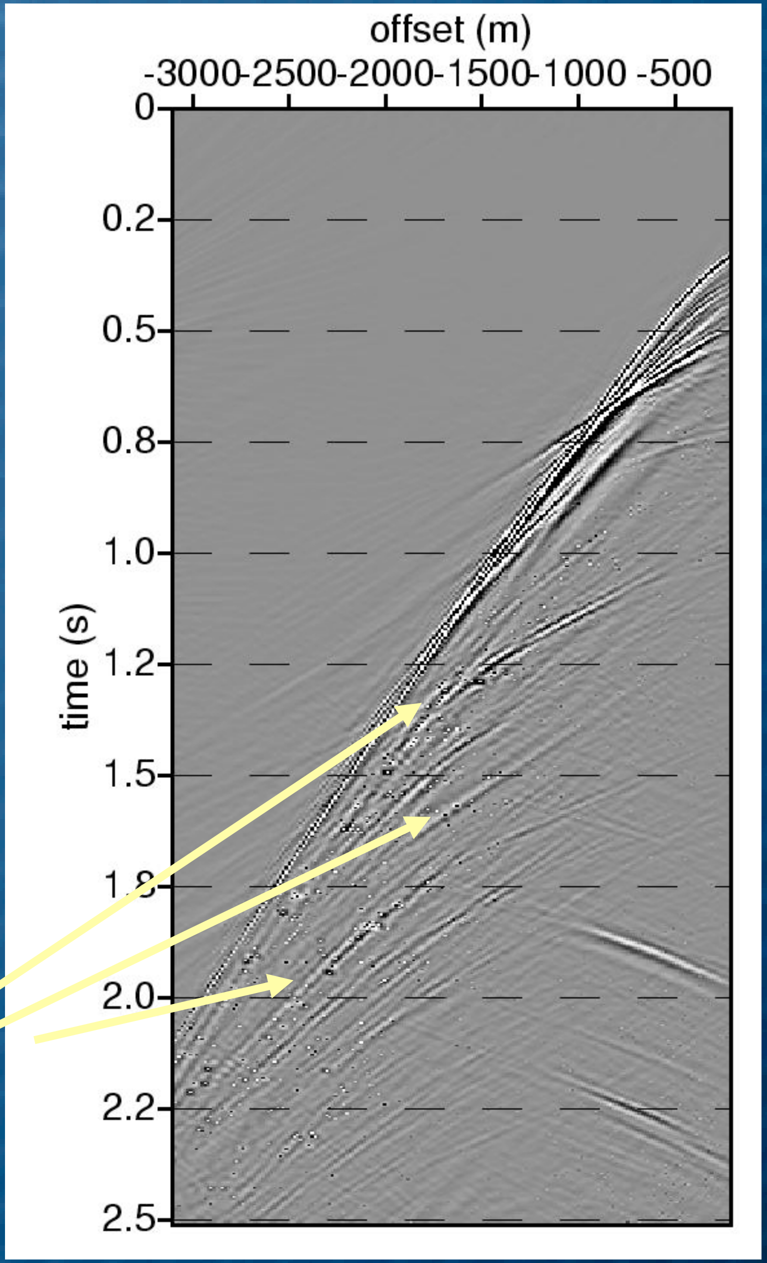
Input with multiples



Multiple suppression with curvelets



Output curvelet filtering with stronger threshold



Preserved primaries

What did we do?

Used

$$\hat{\mathbf{m}} = \mathbf{B}^{\dagger}\boldsymbol{\Theta}_{\lambda\Gamma}\left(\mathbf{B}\mathbf{d}\right) \text{ with } \Gamma = \left|\mathbf{B}\mathbf{n}_{\mathbf{p}}\right|$$

to denoise

$$\overbrace{\mathbf{d}}^{\text{noisy data}} = \underbrace{\mathbf{m}}_{\text{noise-free}} + \overbrace{\mathbf{n}}^{\text{col. noise}}$$

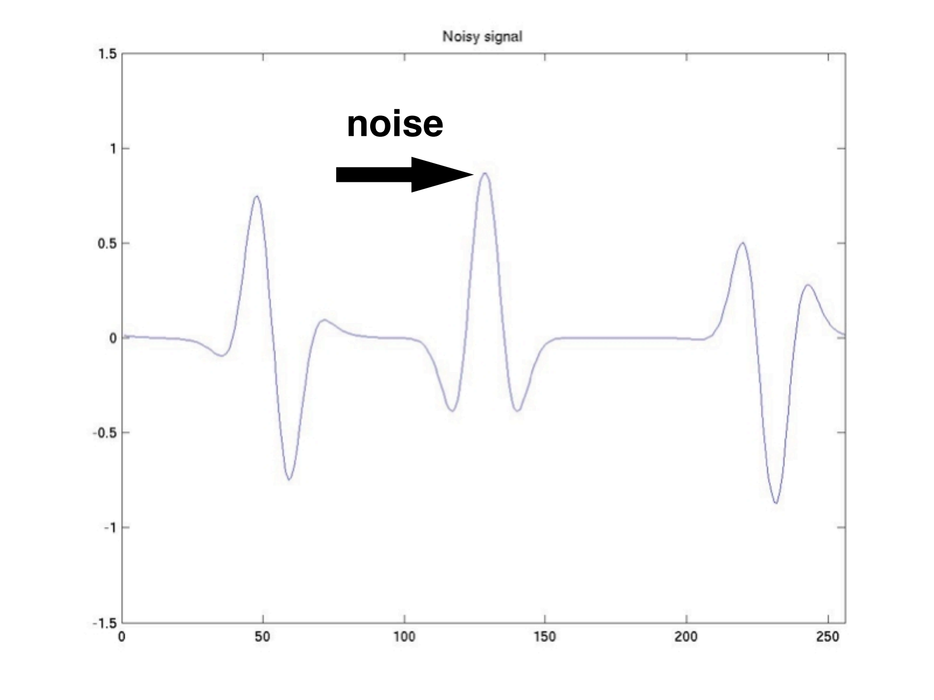
with a simple ***mute*** with λ control parameter.

Some theory

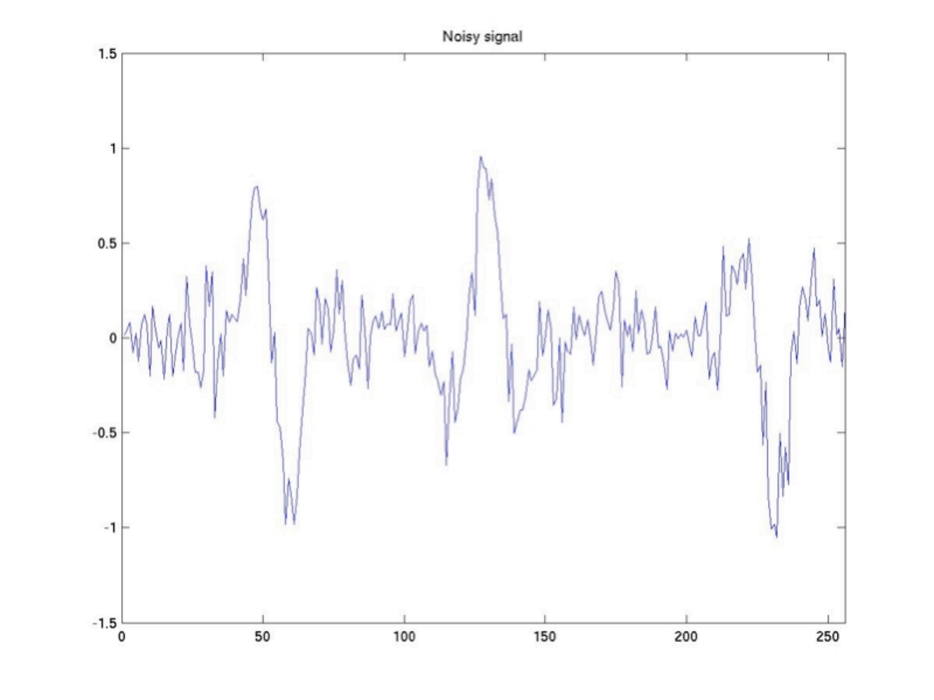
- ***Unconditional* bases:**
  - **norm *always* shrinks when shrinking coef.**
- ***Denoising*** by ***minimax* estimation:**
  - ***diagonal*** thresholding (non-linear)
- ***Extension*** to ***colored noise:***
  - ***almost diagonalizes Covariance***

Ideal *edge-preserving* tool for seismic processing.

Unconditional bases

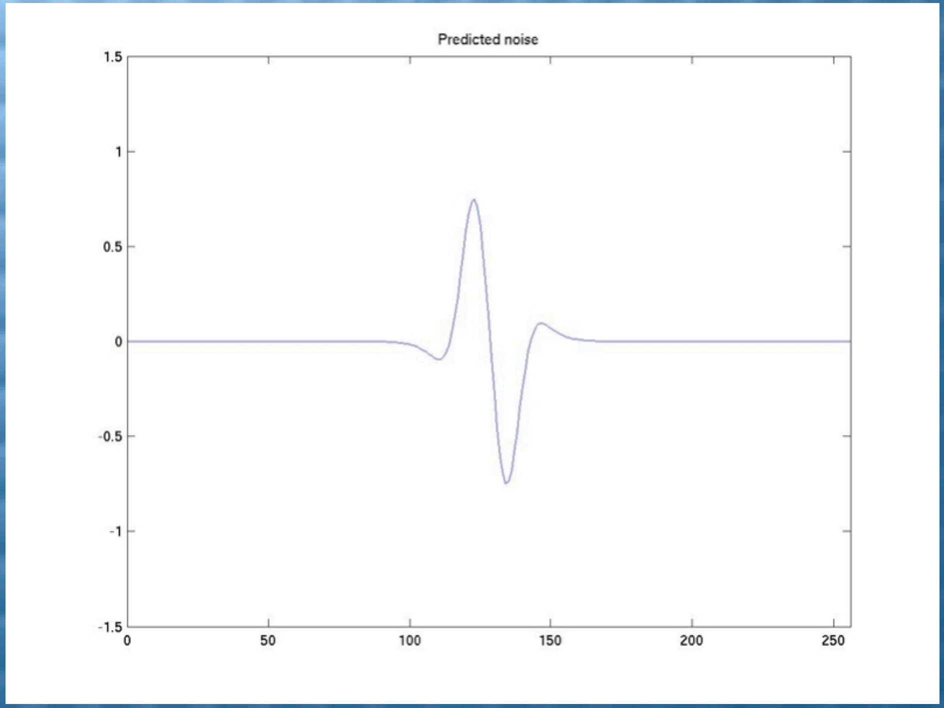


signal + coherent noise

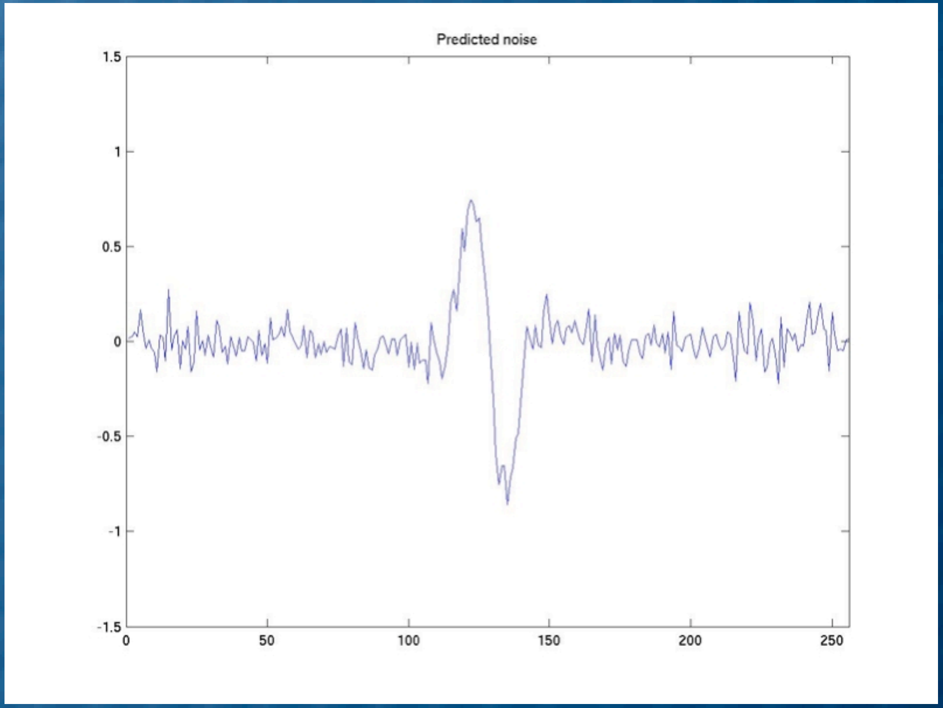


signal + coherent & incoherent noise

Unconditional bases

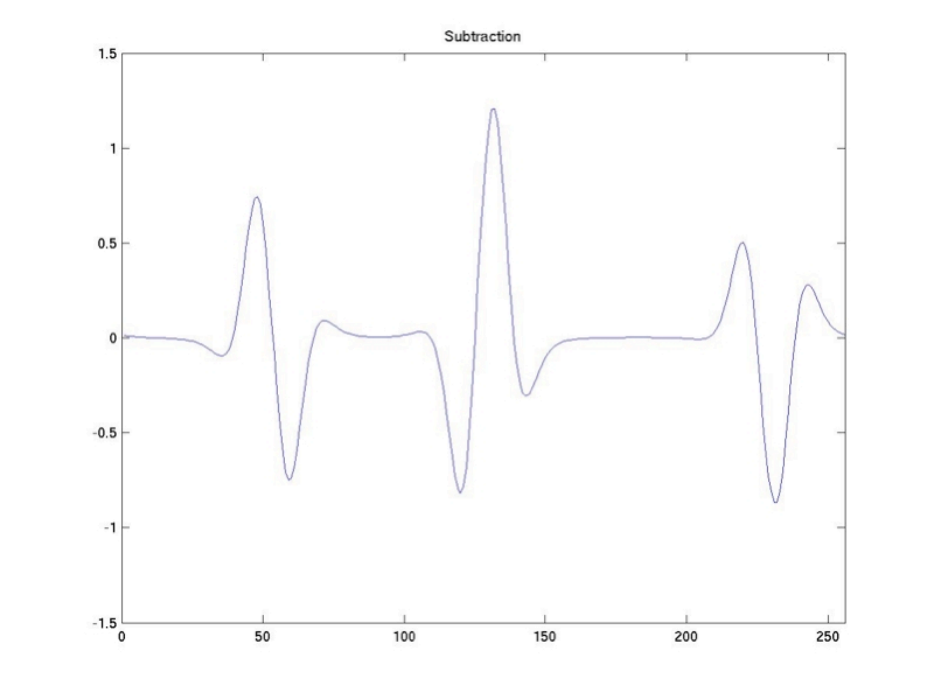


‘wrongly’ predicted noise

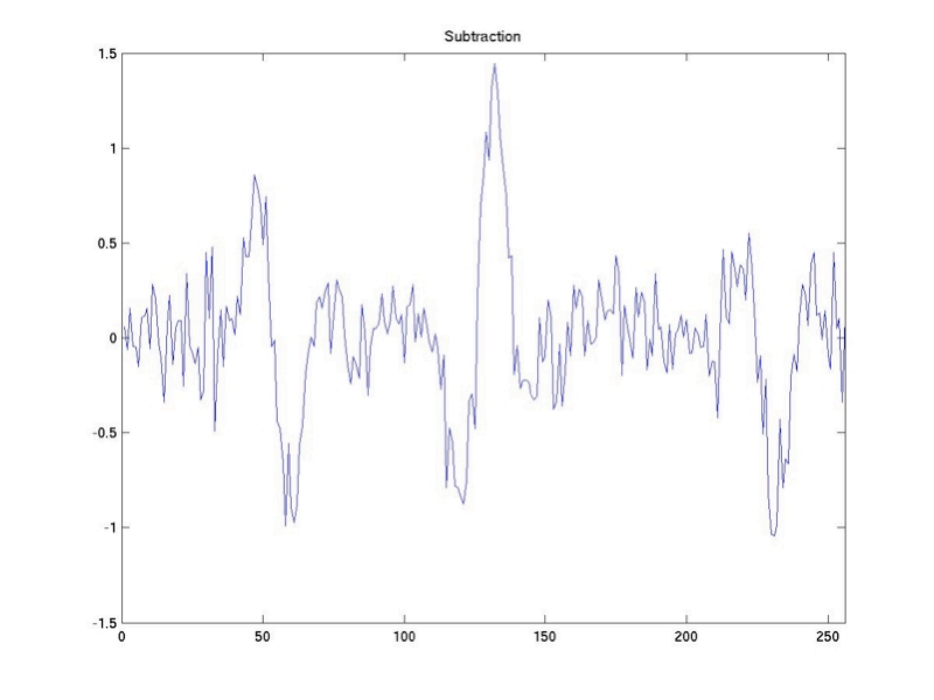


‘wrongly’ noisy predicted noise

Unconditional bases

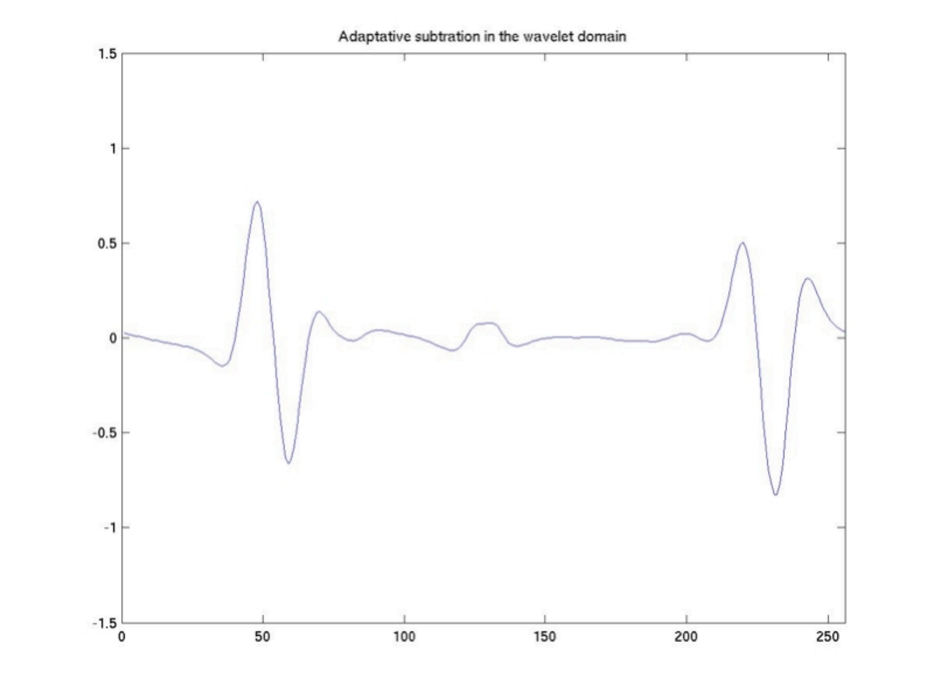


noise-free subtraction

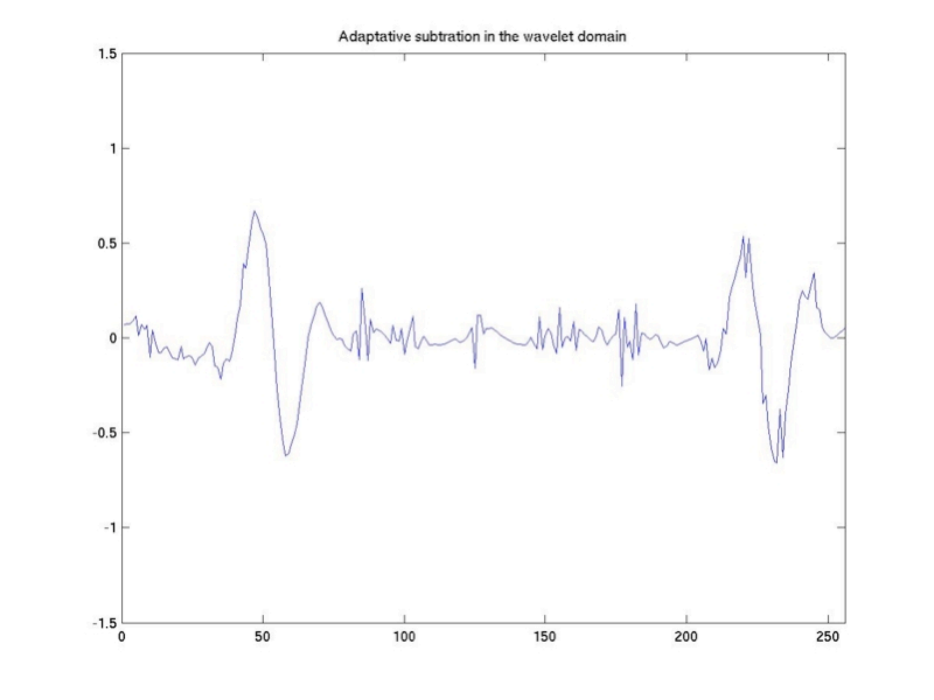


noisy subtraction

Unconditional bases



noise-free adaptive subtraction



noisy adaptive subtraction

Unconditional bases

Works because wavelets are

- ***local* in space**
  - ***accounts for bulk of the phase***
- ***local* in frequency**
  - ***adapts* to amplitude spectrum**

Adds robustness to the equation ...

Minimax estimation

$$\hat{\mathrm{m}} = \mathbf{B}^{\dagger}\Theta_t\left(\mathbf{B}\mathbf{d}\right)$$

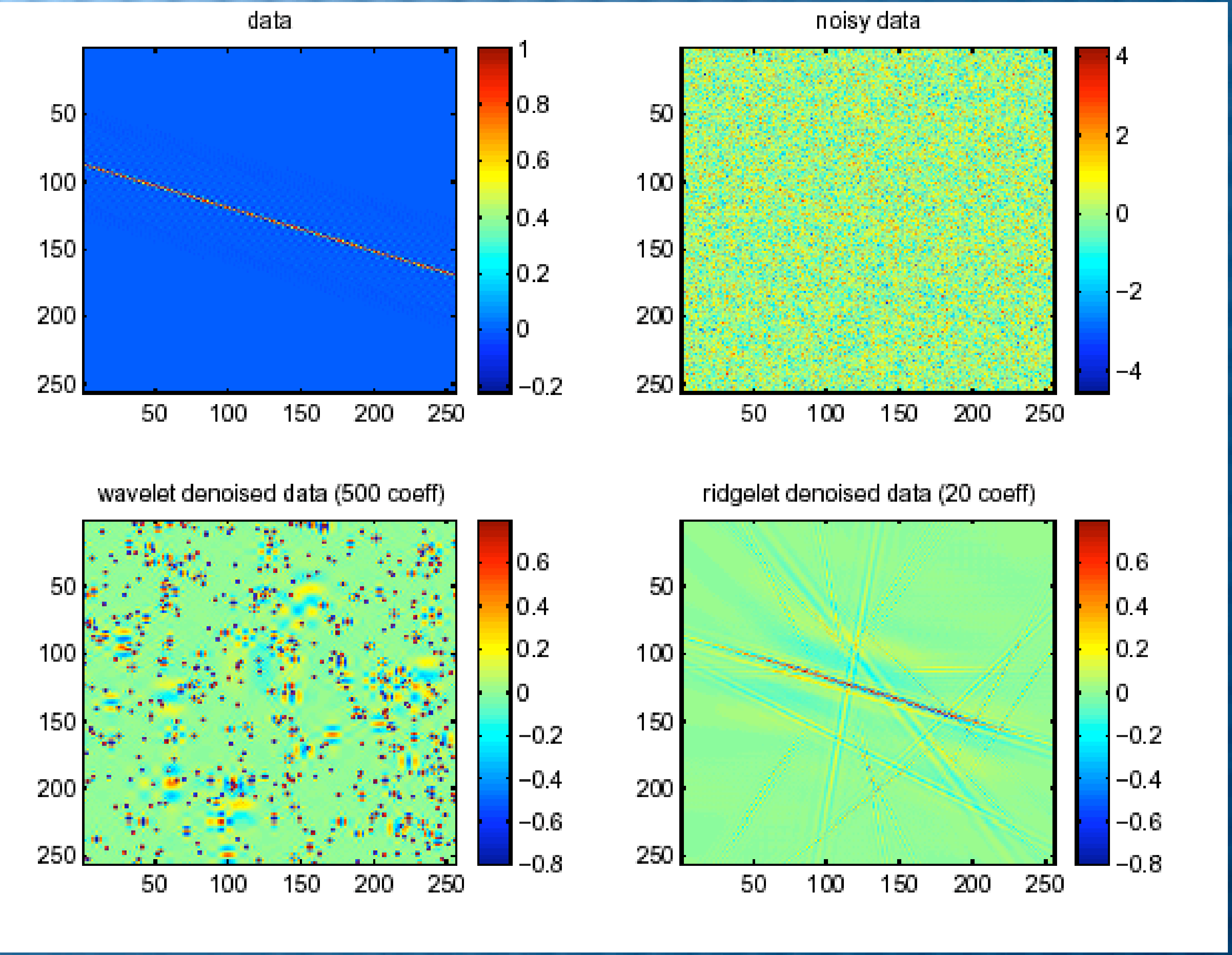
- approximates **minimax**, minimizes max. risk without *prior* **info**
- **Bayes for 'least favorable' *prior***
- **preserves edges**
- **optimal/unconditional** basis functions

Mallat '97
Donoho '92

Wavelets

- Represent **piece-wise smooth** functions at **“no”** additional cost.
- Do **not** have to know where the **singularities** are.
- **Only** good for **point-scatterers** or **horizon/vertically-aligned reflectors.**

Directional wavelets



courtesy WaveleLab + Beamlab '02

Non-linear estimation

Reformulate

$$\hat{\mathbf{m}} \; : \; \min_{\mathbf{m}} \frac{1}{2}\|\mathbf{d} - \mathbf{m}\|_2^2$$

into

$$\hat{\tilde{\mathbf{m}}} \; : \; \min_{\tilde{\mathbf{m}}} \frac{1}{2}\|\tilde{\mathbf{d}} - \tilde{\mathbf{m}}\|_2^2 + \lambda^2\|\tilde{\mathbf{m}}\|_p$$

with

$$\tilde{\mathbf{m}} \triangleq \mathbf{B}\mathbf{m} \text{ and } \tilde{\mathbf{d}} \triangleq \mathbf{B}\mathbf{d}$$

Non-linear estimation

Hard thresholding for p=0:

$$\Theta_\lambda^h\left(\tilde{\mathbf{d}}\right) \triangleq \begin{cases} \tilde{\mathbf{d}} & \text{if} \quad |\tilde{\mathbf{d}}| > \lambda \\ 0 & \text{if} \quad |\tilde{\mathbf{d}}| \leq \lambda \end{cases}$$

Soft thresholding for p=1:

$$\Theta_\lambda^s\left(\tilde{\mathbf{d}}\right) \triangleq \begin{cases} \text{sign}(\tilde{\mathbf{d}})(|\tilde{\mathbf{d}}| - \lambda)_+ & \text{if} \quad |\tilde{\mathbf{d}}| > \lambda \\ 0 & \text{if} \quad |\tilde{\mathbf{d}}| \leq \lambda \end{cases}$$

Adaptive subtraction

Adaptive subtraction by matched filter:

$$\overbrace{\hat{\mathbf{n}}}^{\text{denoised}} : \min_{\mathbf{\Phi}} = \| \underbrace{\mathbf{d}}_{\text{noisy data}} - \overbrace{\mathbf{\Phi}}^{\text{matched filter}} \overset{t}{*} \underbrace{\mathbf{m}}_{\text{pred. noise}} \|_p$$

- **residue is the denoised data**
- **risk of over fitting**

May loose primary reflection events ...

Guitton& Verschuur'04

Non-linear adaptive subtraction

Extend to *colored noise:*

$$\overbrace{\mathbf{d}}^{\text{noisy data}} = \underbrace{\mathbf{m}}_{\text{noise-free}} + \overbrace{\mathbf{n}}^{\text{col. noise}}$$

Solve

$$\hat{\mathbf{m}} : \min_{\mathbf{m}} \frac{1}{2} \|\mathbf{C}_n^{-1/2} \left(\mathbf{d} - \mathbf{m}\right)\|_2^2$$

with

$$\mathbf{C}_n \triangleq \mathbf{E}\{\mathbf{n}\mathbf{n}^T\}$$

Non-linear adaptive subtraction

Recast in *Curvelet* domain:

$$\hat{\tilde{\mathbf{m}}} \ : \ \min_{\tilde{\mathbf{m}}} \frac{1}{2}\|\mathbf{C}_{\tilde{n}}^{-1/2}\left(\tilde{\mathbf{d}} - \tilde{\mathbf{m}}\right)\|_2^2 + \lambda^2\|\tilde{\mathbf{m}}\|_p$$

Use ***unconditional-basis*** **property:**

$$\mathbf{C}_{\tilde{n}} \triangleq \mathbf{E}\{\tilde{\mathbf{n}}\tilde{\mathbf{n}}^T\} \approx \mathrm{diag}\left(\mathrm{diag}\left(\mathbf{C}_{\tilde{n}}\right)\right) \triangleq \Gamma^2$$

‘Challenge’ to find the Γ’s

Non-linear adaptive subtraction

Solve

$$\hat{\tilde{\mathbf{m}}} \; : \; \min_{\tilde{\mathbf{m}}} \frac{1}{2} \|\mathbf{\Gamma}^{-1}\left(\tilde{\mathbf{d}} - \tilde{\mathbf{m}}\right)\|_2^2 + \lambda^2 \|\tilde{\mathbf{m}}\|_p$$

can be written as

$$\hat{\mathbf{m}} = \mathbf{B}^{\dagger}\mathbf{\Gamma}\Theta_{\lambda}\left(\mathbf{\Gamma}^{-1}\mathbf{B}\mathbf{d}\right) = \mathbf{B}^{\dagger}\Theta_{\boldsymbol{\lambda}\mathbf{\Gamma}}\left(\tilde{\mathbf{d}}\right).$$

No matched filter required!

Noise prediction

Model or predict noise and set

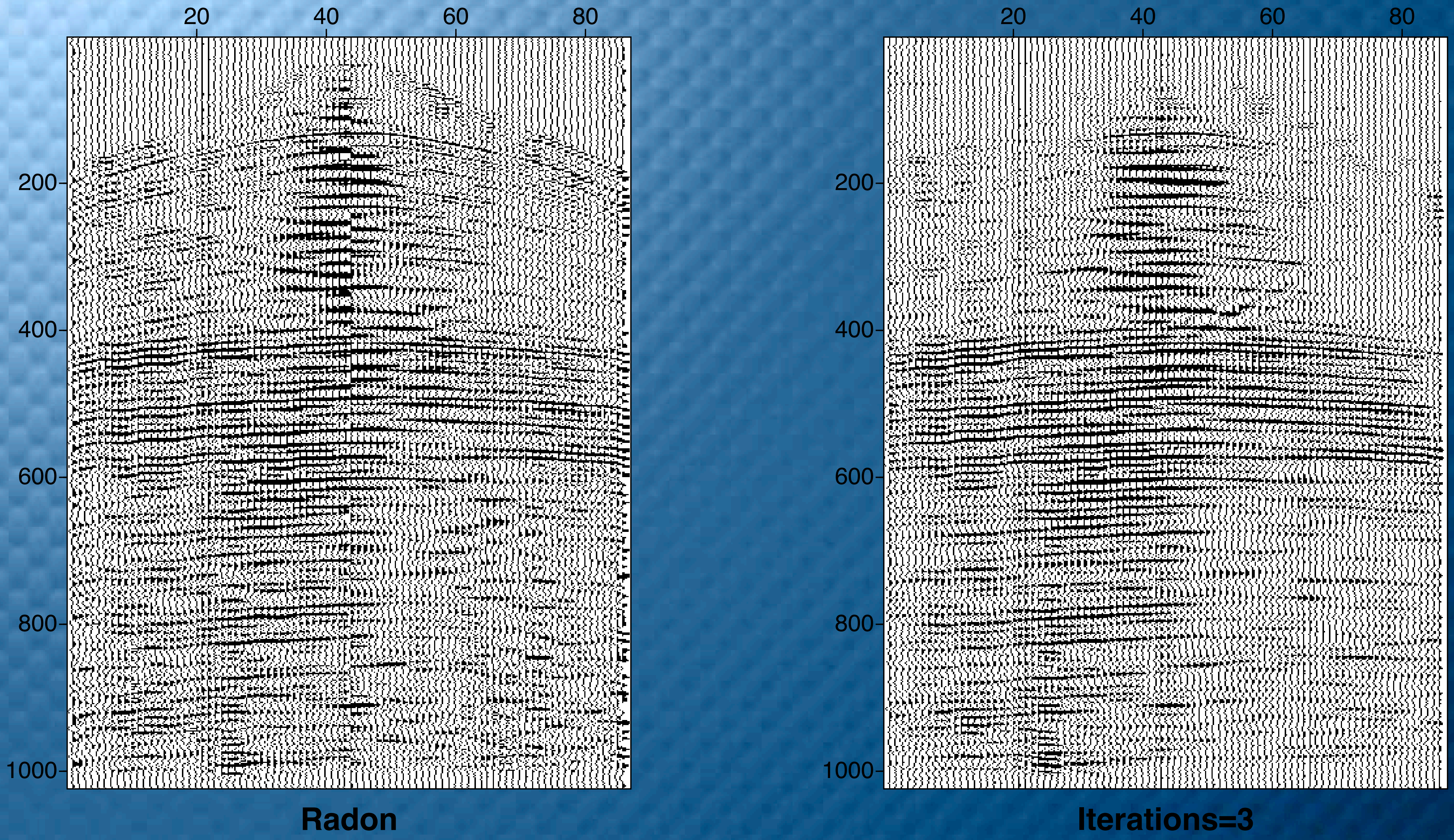
$$\Gamma = |\mathbf{B}\mathbf{n}_p|$$ **with n_p predicted noise**

- **ground roll, multiples, 4-D vintages**
- **conventional techniques (e.g. Radon)**

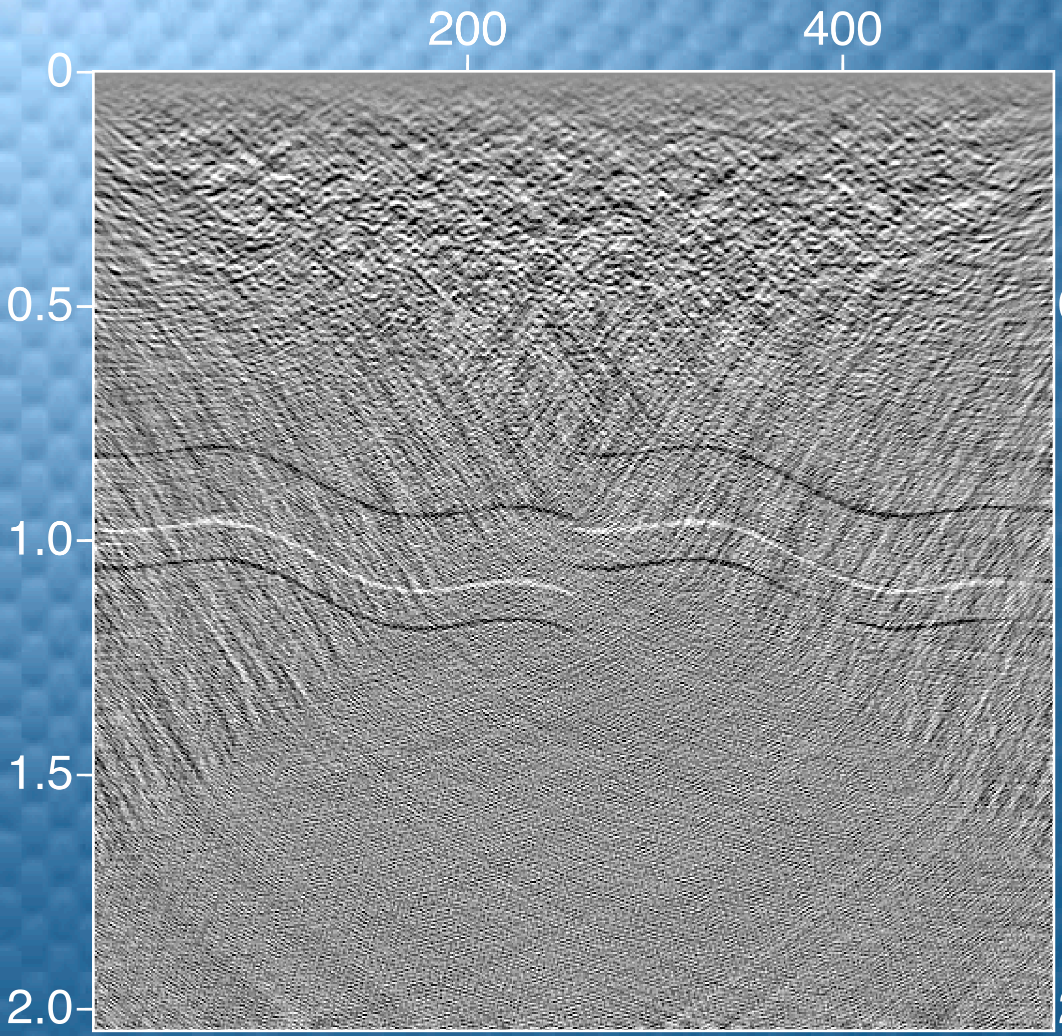
Monte-Carlo sample diag. covariance

- **migration**

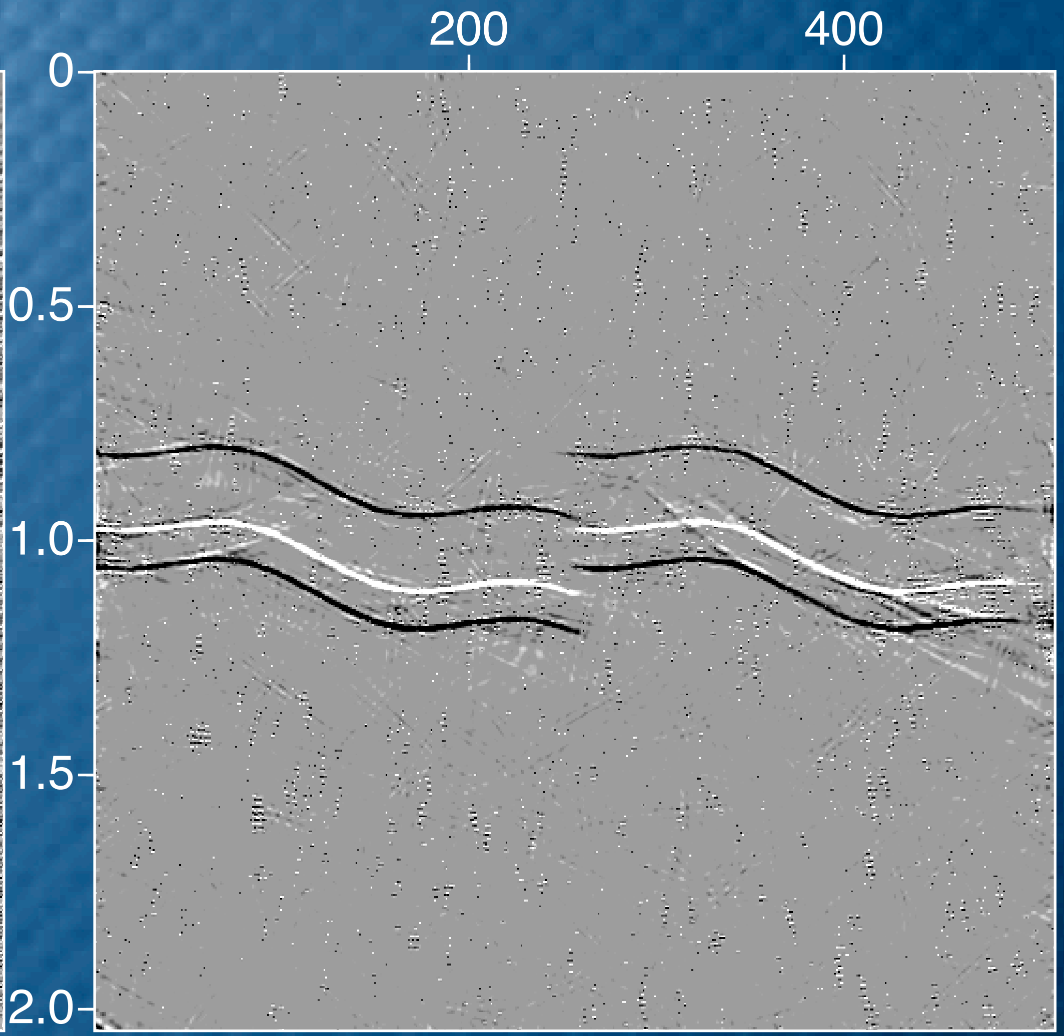
Ground-roll removal with curvelets



Imaging with Curvelets

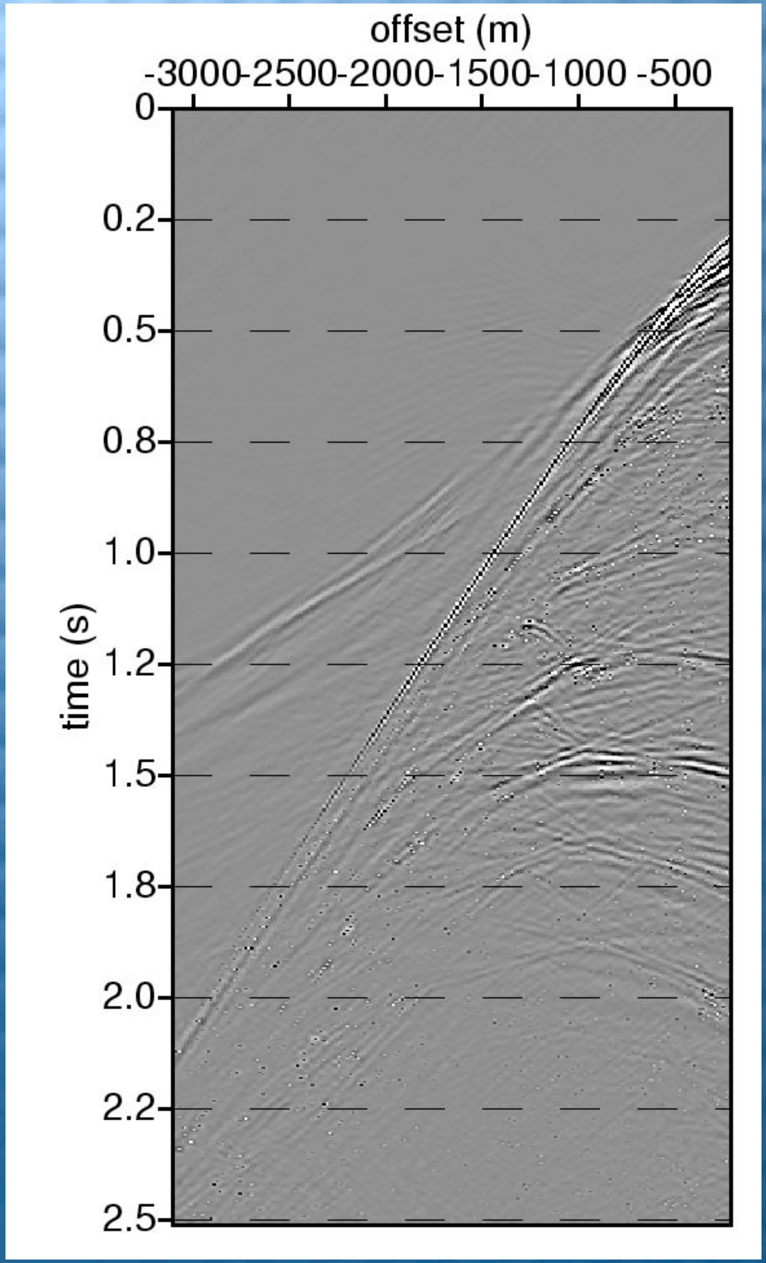


Least-squares migrated Image

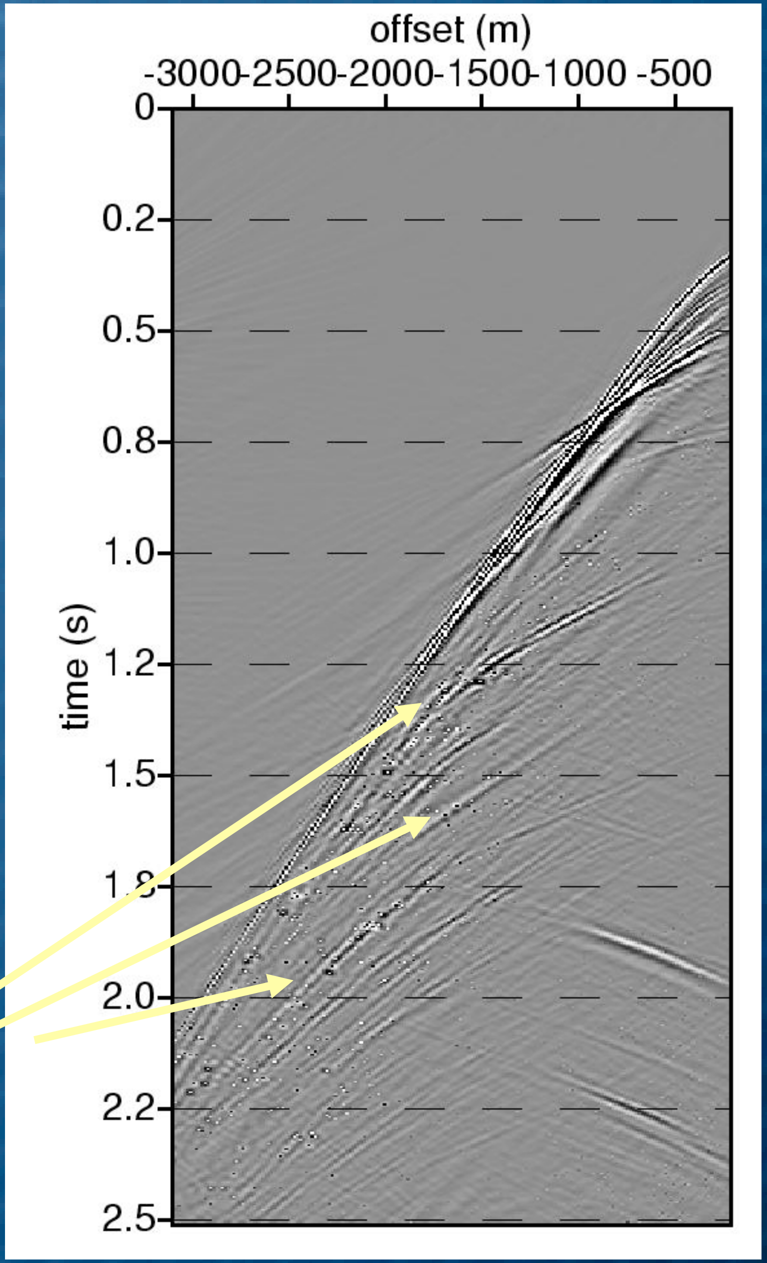


Constrained Optimization

Multiple suppression with curvelets



Output curvelet filtering with stronger threshold



Preserved primaries

Global optimization

After thresholding

- **remove artifacts & ‘miss fires’**
- **normal operator (inversion)**

Impose additional *penalty functional*

- ***prior* information**
- ***sparseness & continuity***

Defining the right norm is crucial ...

Global optimization

Formulate constrained optimization:

$$\hat{\mathbf{m}}: \quad \min_{m} J(\mathbf{m}) \quad \text{s.t.} \quad |\tilde{\mathbf{m}} - \hat{\tilde{\mathbf{m}}}_0|_\mu \leq \mathbf{e}_\mu, \quad \forall \mu$$

with

$$\hat{\mathbf{m}}_0 = \mathrm{B}^\dagger \Theta_{\boldsymbol{\lambda\Gamma}} \left(\tilde{\mathbf{d}} \right)$$

and with e_μ **threshold and noise-dependent** ***tolerance*** **on curvelet coeff.**

Global optimization

Set tolerances

$$\mathbf{e}_\mu = \begin{cases} \boldsymbol{\Gamma}_\mu & \text{if} \quad |\hat{\tilde{\mathbf{m}}}_0|_\mu \quad \geq \quad |\lambda\boldsymbol{\Gamma}|_\mu \\ \boldsymbol{\lambda\Gamma}_\mu & \text{if} \quad |\hat{\tilde{\mathbf{m}}}_0|_\mu \quad < \quad |\lambda\boldsymbol{\Gamma}|_\mu \end{cases}$$

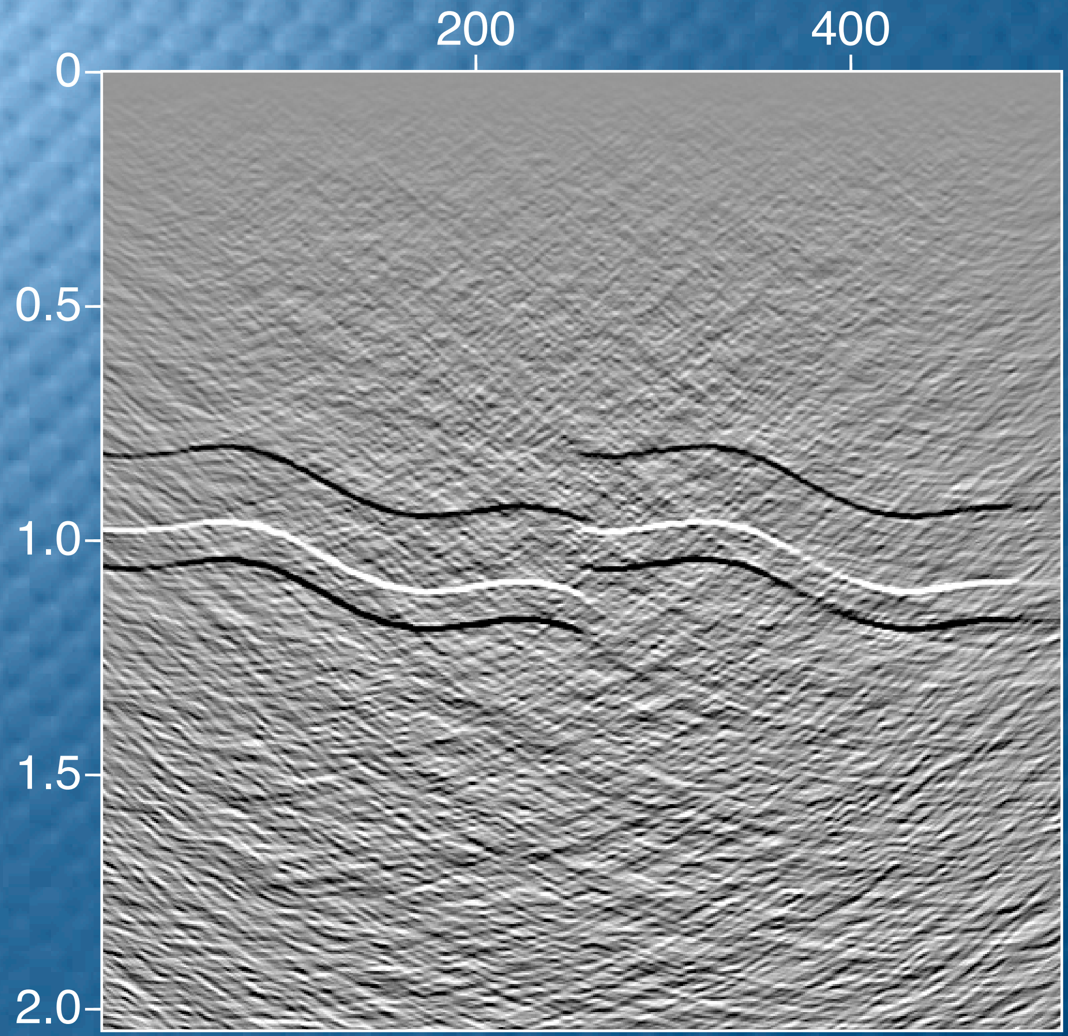
with $\sqcap$ **defining the confidence interval,**
e.g $\sqcap = 3$ **corresponds to 95 %.**

Global optimization

Constrained optimization problem:

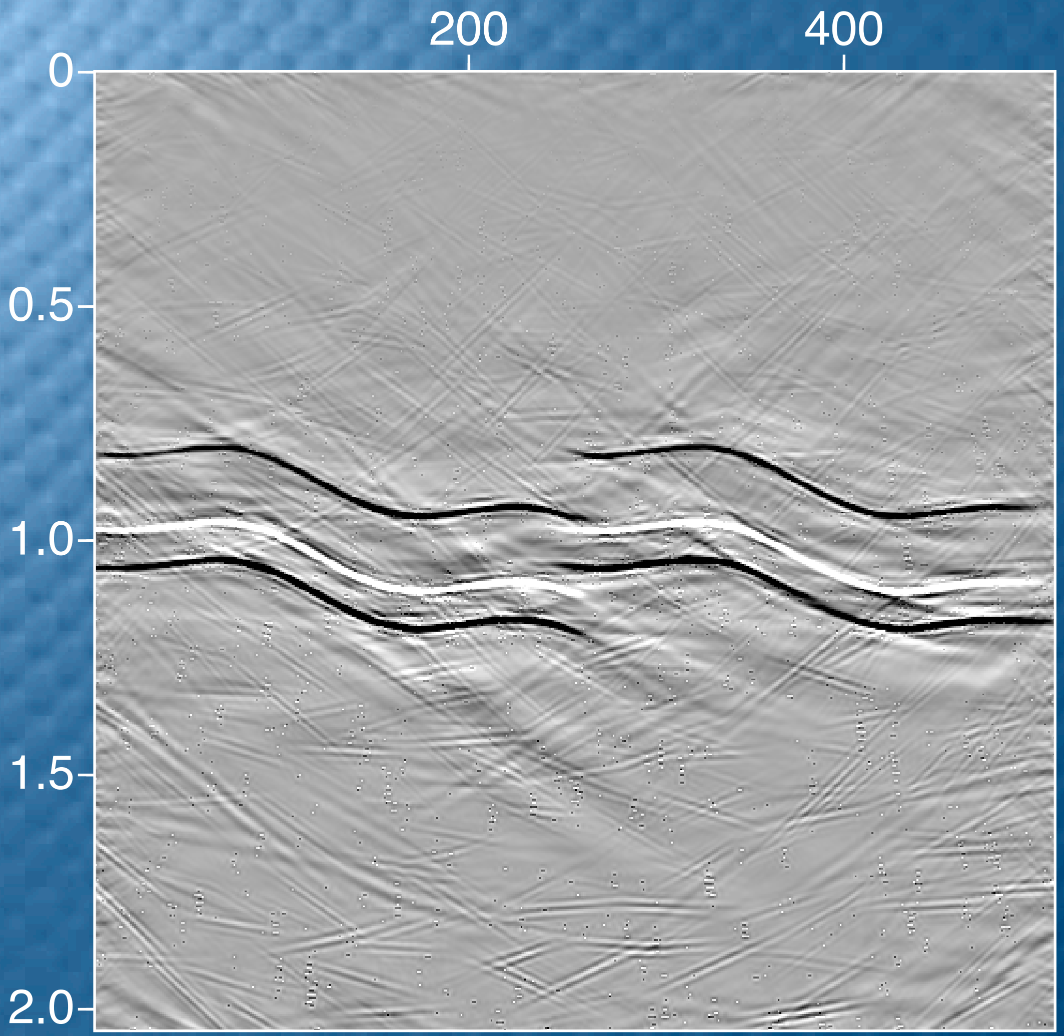
- **Uses augmented Lagrangian (Nocedal and Wright 2001)**
- **L1-penalty function**
- **Initial Lagrangian multipliers given by gradient of** $\hat{m}_0$
- **Uses Steepest Decent and line search**

Examples



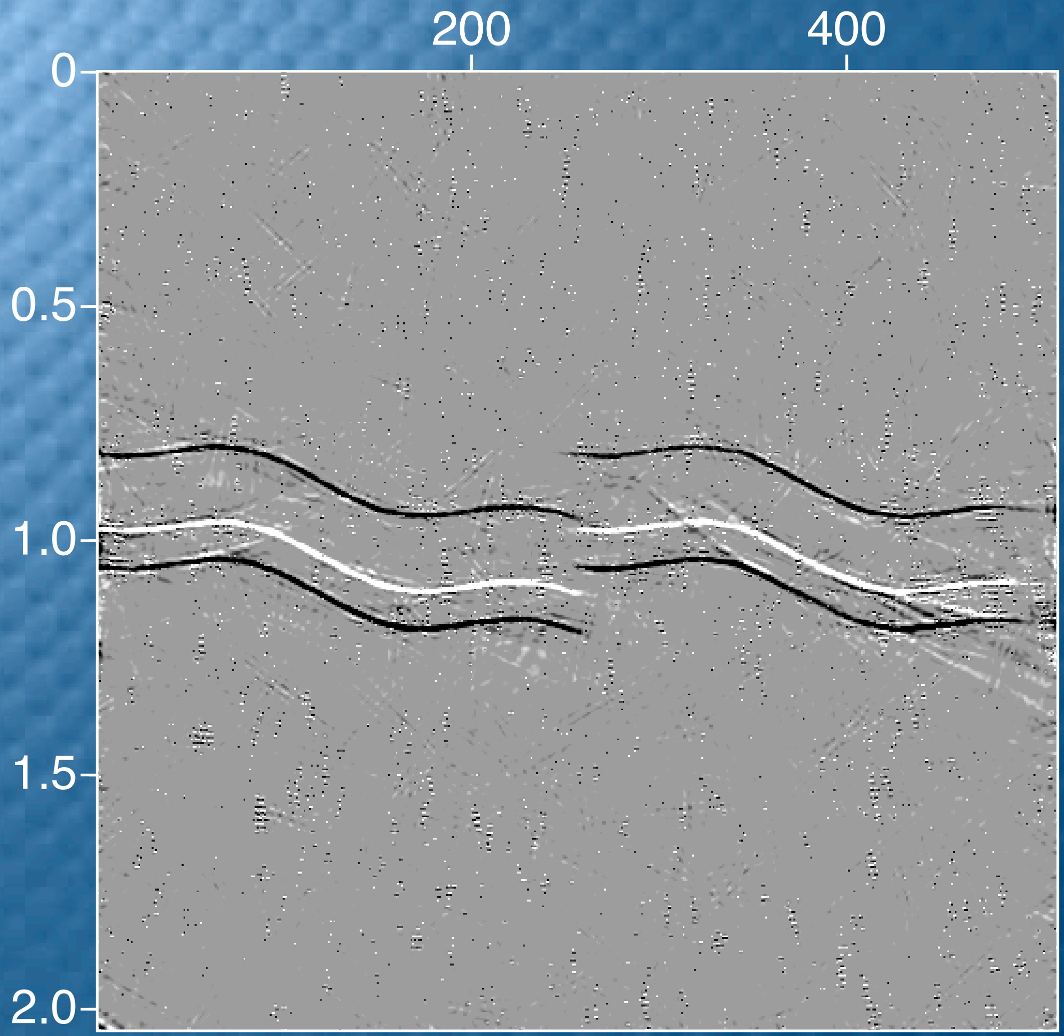
Noisy Image

Examples



Denoised after Thresholding

Examples



Constrained Optimization

Applications

Presented a framework that

- **removes (coherent) noise *via* adaptive subtraction:**
  - Ground-roll & Multiple removal (Thursday)
  - Compute 4-D difference Cubes (Wednesday)
- **improves imaging & inversion:**
  - sparseness constrained imaging (next & Thursday)

- **ample opportunity to expand!**

Conclusions

- Succeeded in partly full filling our “dream”.
- Devised a robust estimation method relatively insensitive to local phase.
- Right norm is an important & open problem.
    - exploit redundancy seismic data
    - function spaces & learning functionals
- So far focussed on ‘simple denoising’ let’s include an operator ....

Acknowledgements

Candes & Donoho for making their Curvelet code available.

Partially supported by a NSERC Grant.

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CRSNG Investir dans les gens, la découverte et l'innovation