Phase transitions in exploration seismology: statistical mechanics meets information theory

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joint work with
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Complexity in the oil industry meeting 2007,

Natal, Aug 9

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Research interests

- Develop techniques to obtain higher quality images from (incomplete) data **<=> seismic** imaging of transitions
- Characterization of reflectors **<=>** estimation of singularity orders of imaged reflectors
- Understand physical processes that generate singular transitions in the earth **<=>** Percolation phenomena
- Singularity-preserved upscaling

Today's topics

Phase diagrams in the recovery of seismic data from incomplete measurements

- old ideas in geophysics reincarnated in the new field of "compressive sampling"
- describes regions that favor recovery

Phase diagrams in the description of seismic reflectors

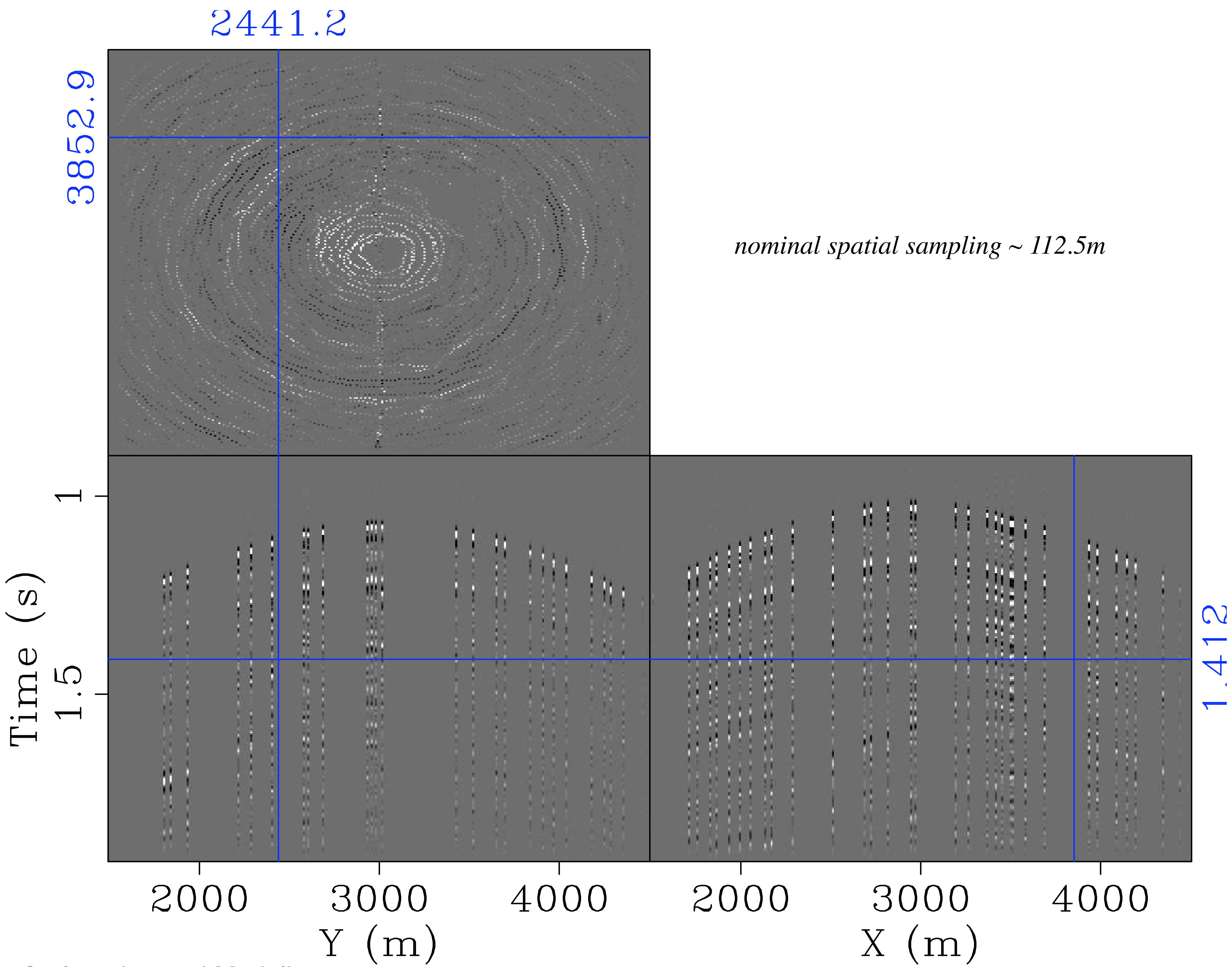
- mixture models with critical points <=> reflectors
- a first step towards singularity-preserved upscaling

Phase-transition behavior in compressive sampling

joint work with Gilles Hennenfent

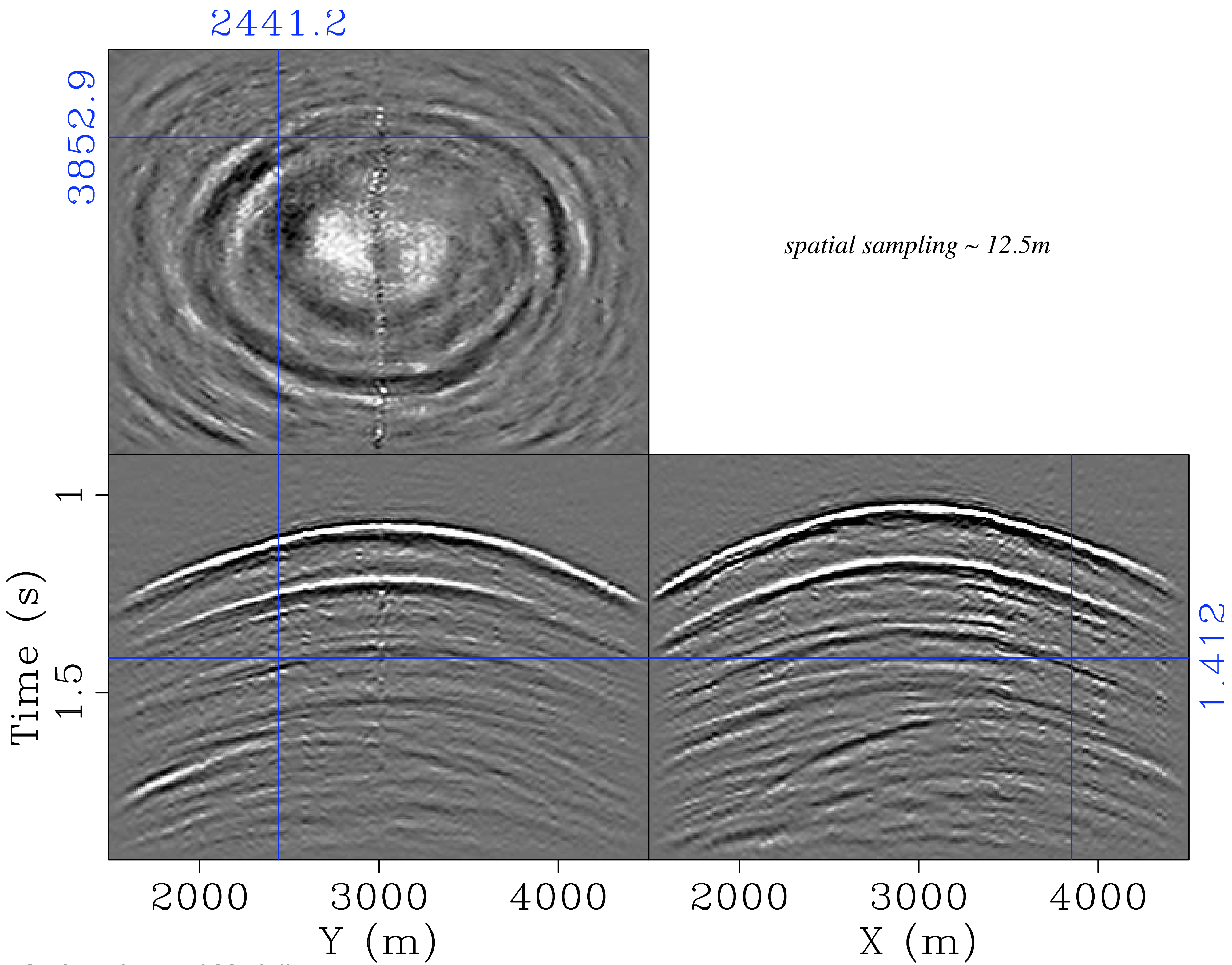
"Non-parametric seismic data recovery with curvelet frames" in revision for GJI

Data



nominal spatial sampling ~ 112.5m

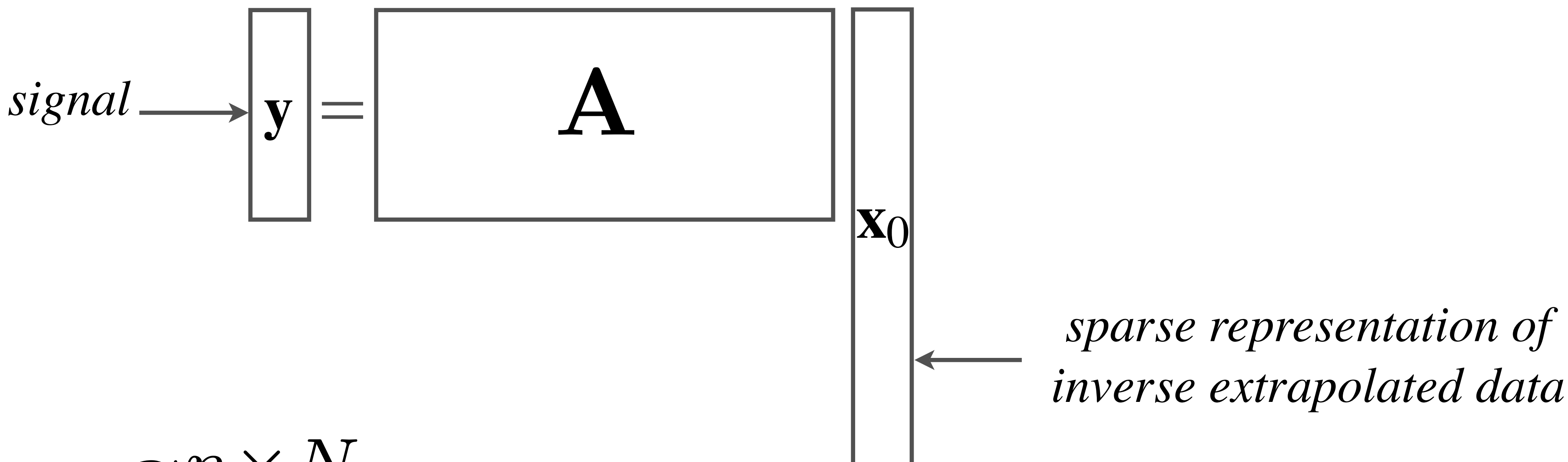
CRSI



spatial sampling ~ 12.5m

Fourier example

Consider n-random time samples from a signal with k-sparse Fourier spectrum, i.e.



with $\mathbf{A} \in \mathbb{C}^{n \times N}$ the time restricted inverse Fourier transform.

The signal

$$\mathbf{f}_0 = \mathbf{F}^H \mathbf{x}_0$$

with the k-non-zero spectrum can ***exactly*** be recovered.

Fourier example cont'd

Solve

sparsity enhancement ↓ *data consistency* ↓

$$\mathbf{P_1}: \begin{cases} \tilde{\mathbf{x}} = \arg\min_{\mathbf{x}\in\mathbb{R}^N} \|\mathbf{x}\|_1 = \sum_{i=1}^N |x_i| \quad \text{s.t.} \quad \mathbf{y} = \mathbf{A}\mathbf{x} \\ \tilde{\mathbf{f}} = \mathbf{F}^H\tilde{\mathbf{x}}. \end{cases}$$

When a traveler reaches a fork in the road, the l1 -norm tells him to take either one way or the other, but the l2 -norm instructs him to head off into the bushes. [Claerbout and Muir, 1973]

Recovery for Gaussian matrices when [Donoho and Tanner '06]

$$n = k \times 2\log(N/k)$$

For arbitrary measurement sparsity bases [Candes, Romberg & Tao '06]

$$n = \mu^2 k \times \log N$$

↑ *mutual coherence*

for certain conditions on the matrix and sampling ....

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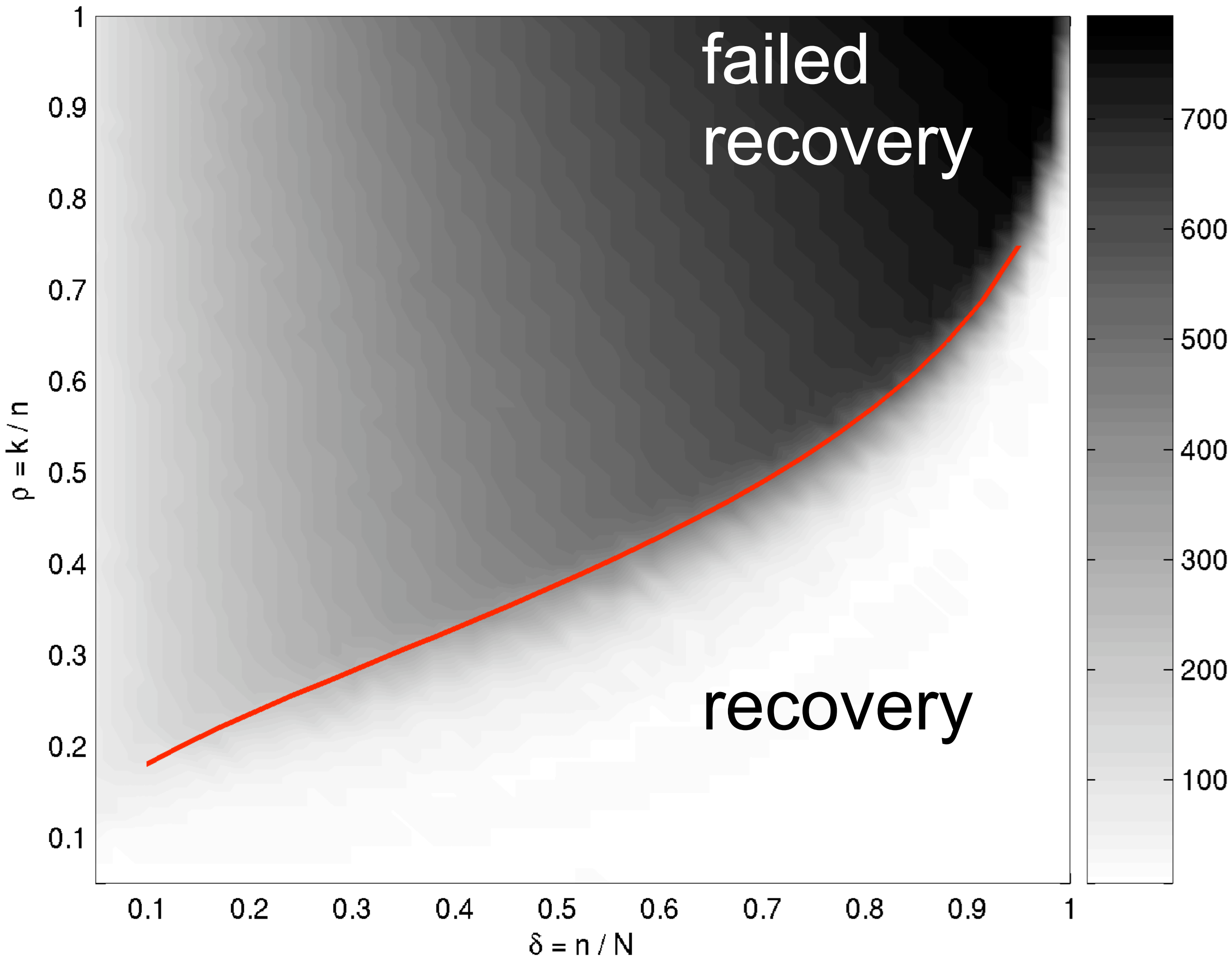
Phase diagrams

l1 solver [from Donoho et al ‘06]

undersampled
rich signal

fully sampled
rich signal



undersampled
sparse signal

fully sampled
sparse signal

In the white region

$$\hat{\mathbf{x}} = \arg\min_{\mathbf{x}} \|\mathbf{x}\|_1 \quad \text{subject to} \quad \mathbf{A}\mathbf{x} = \mathbf{y}$$

recovers exactly.

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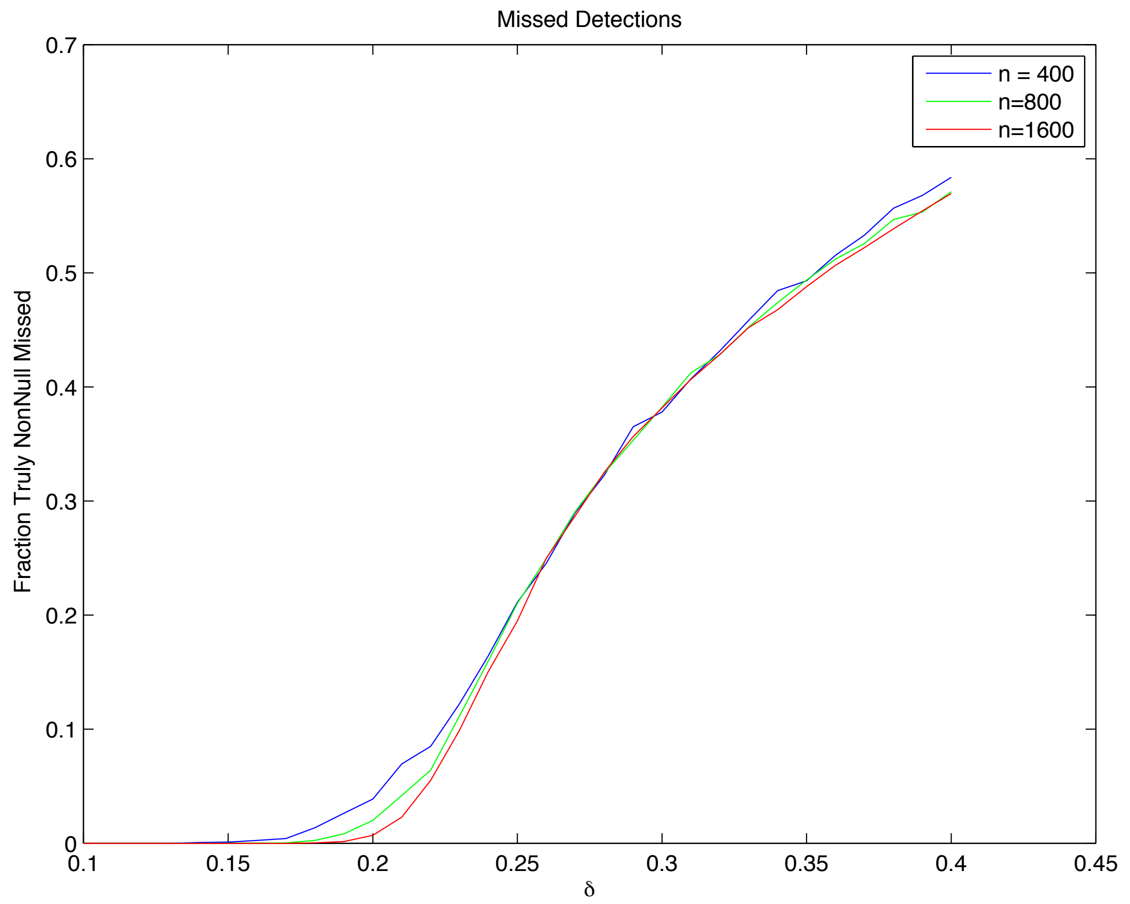
Phase transition

Has a second-order phase transition at a oversampling of 5



- transition becomes sharper for $n \longrightarrow \infty$
- conceptual but unexplored 'link' with percolation theory
- k-neighborhoodness of polytopes undergoes a phase transition

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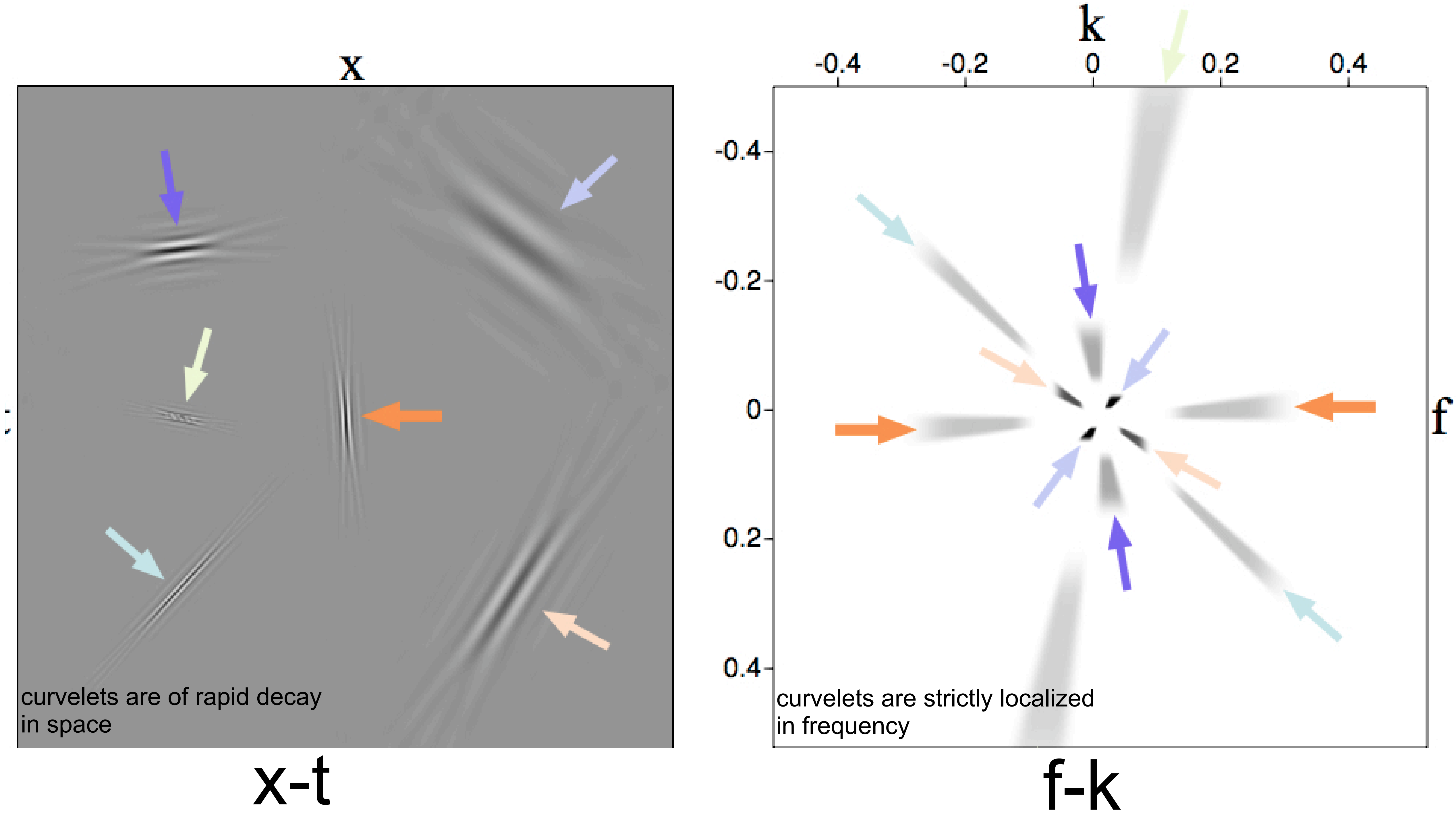
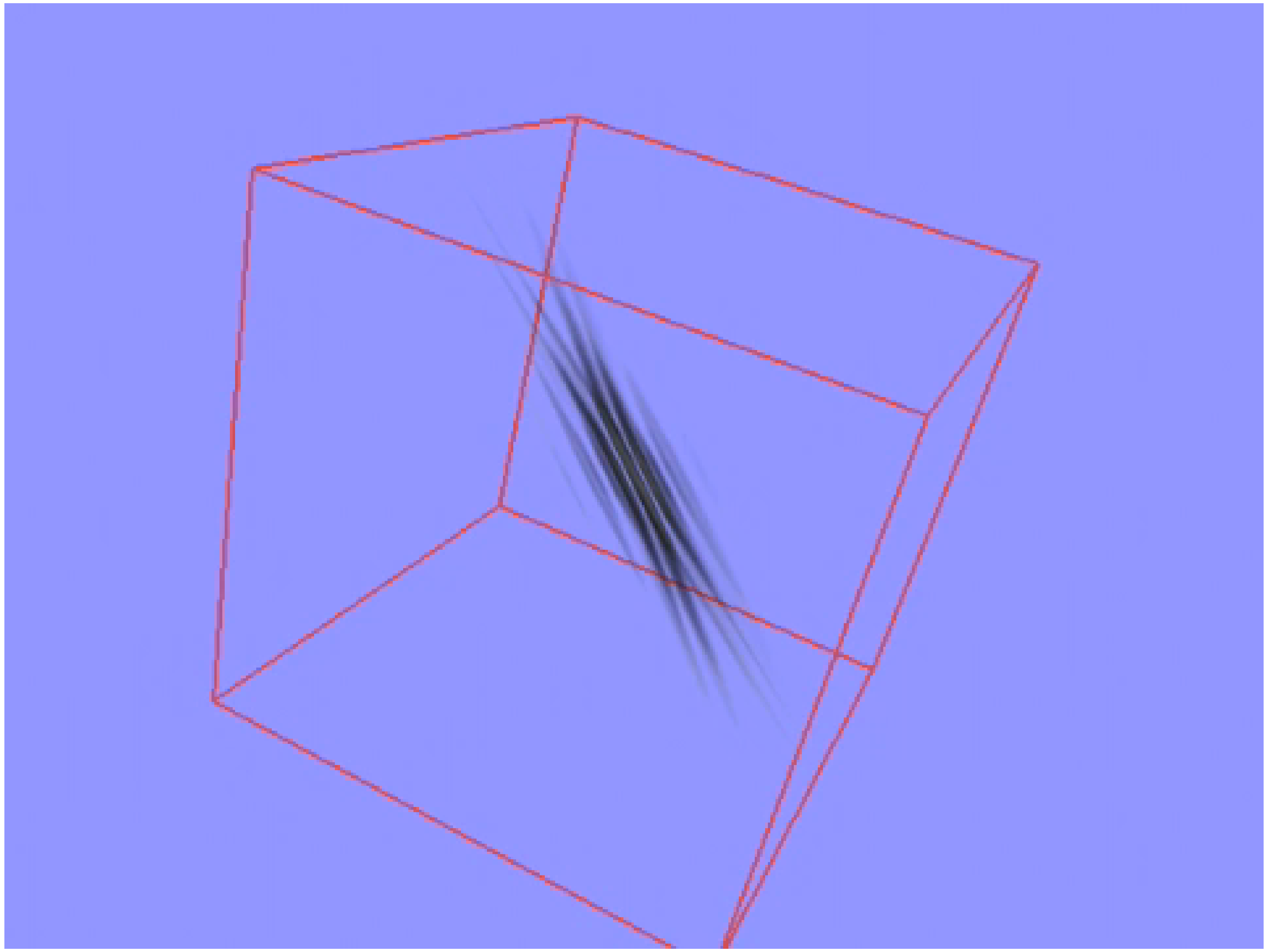
2-D curvelets



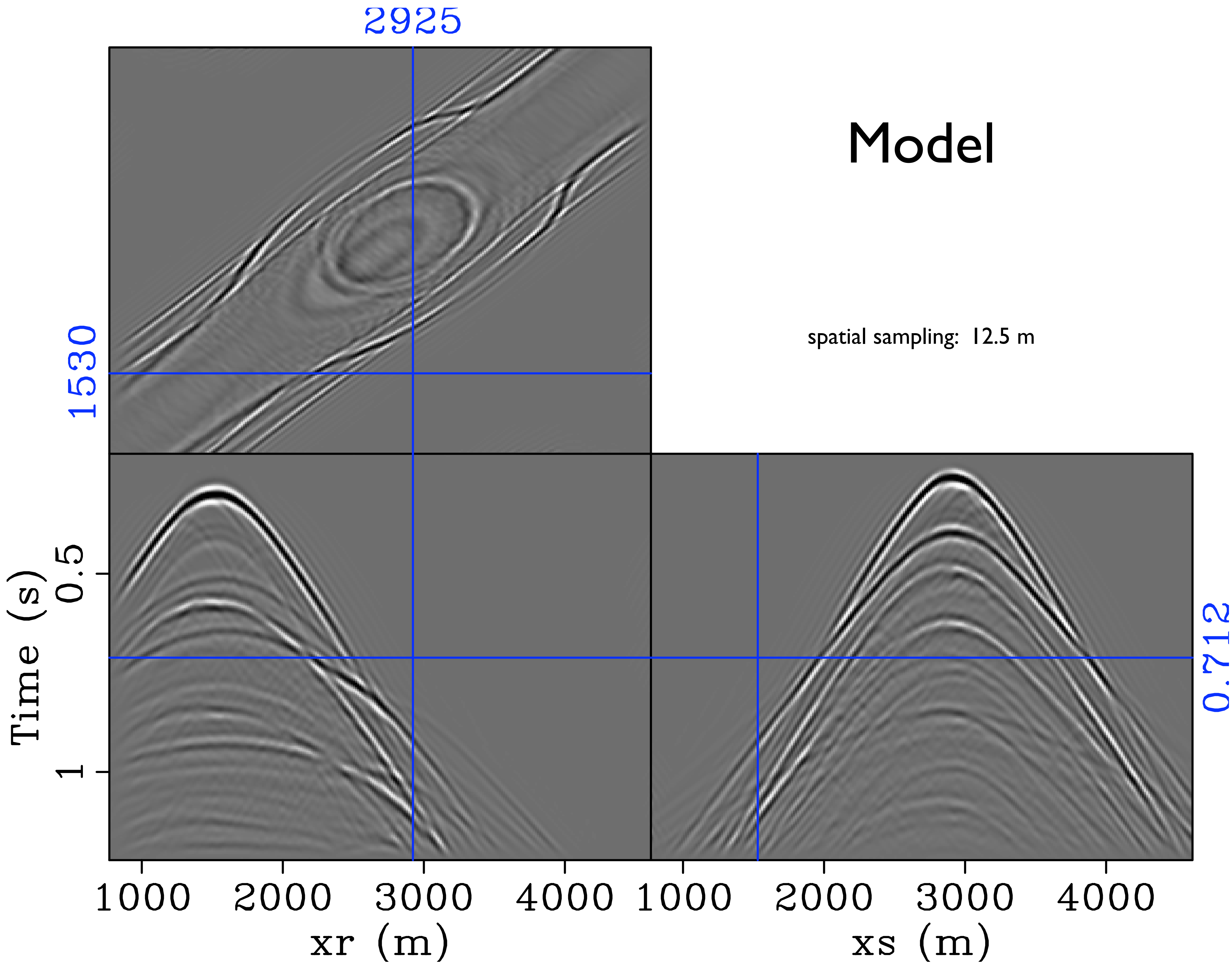
Oscillatory in one direction and smooth in the others!

3-D curvelets



Curvelets live in wedges in the 3 D Fourier plane...

Model

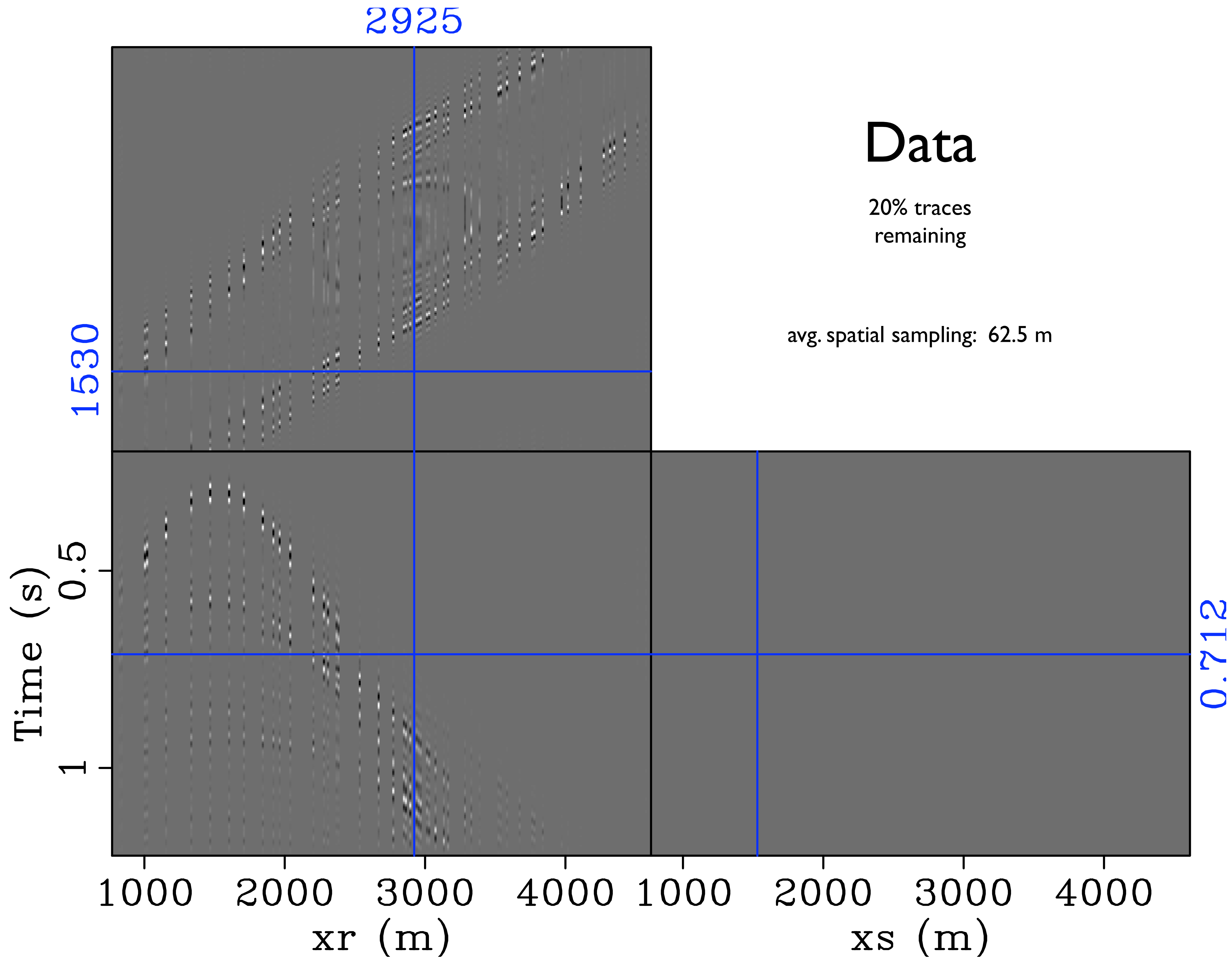


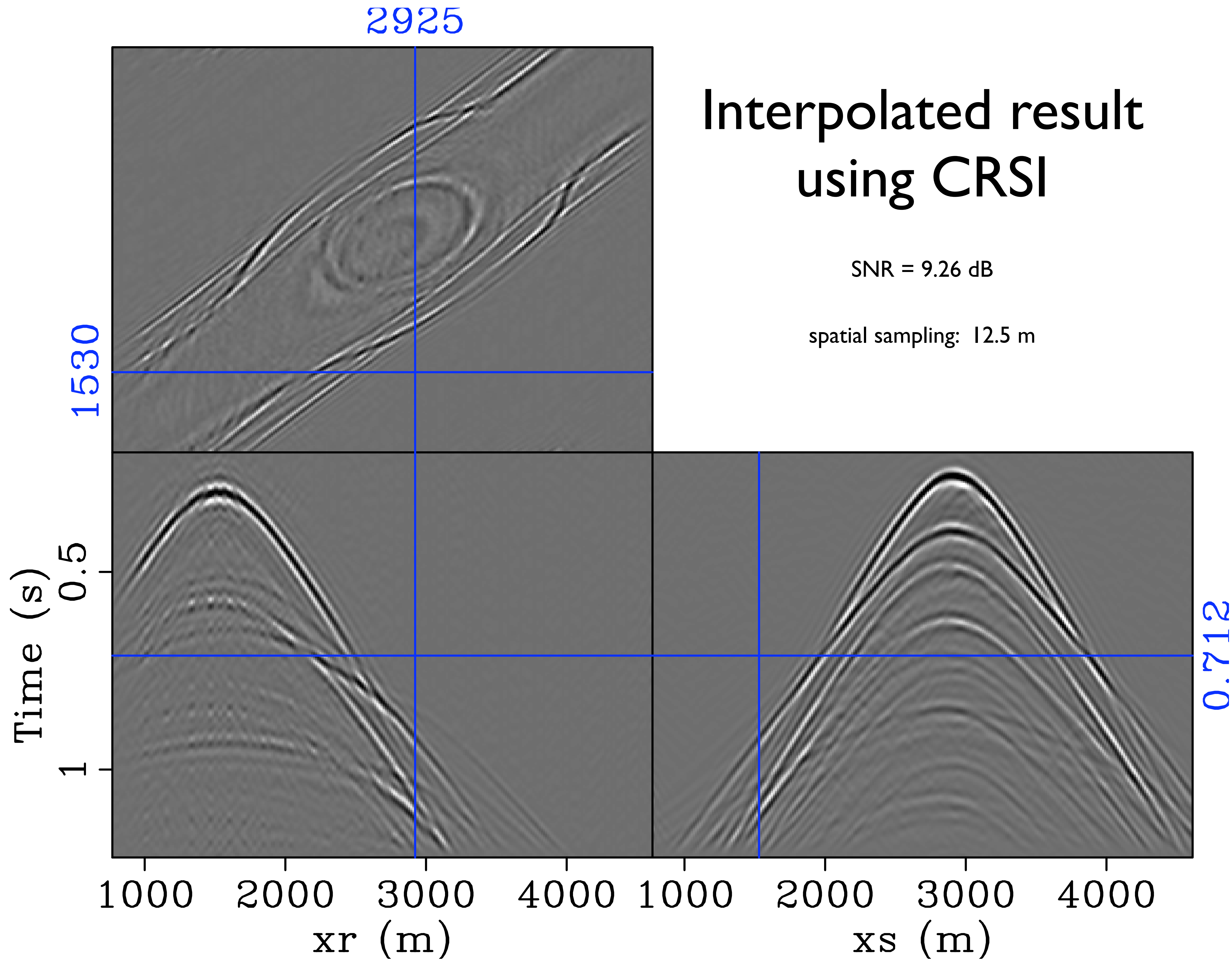
spatial sampling: 12.5 m

Data

20% traces
remaining

avg. spatial sampling: 62.5 m

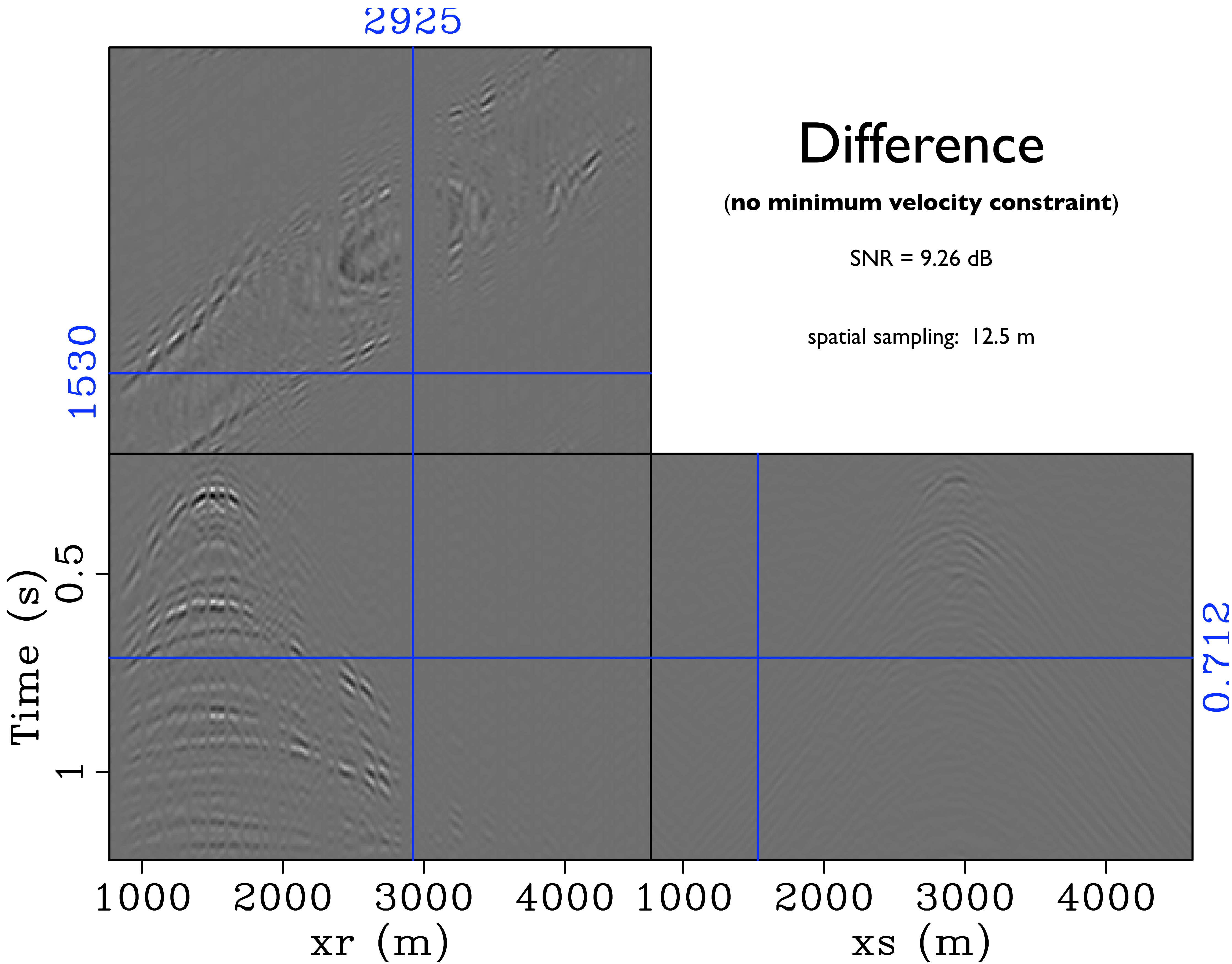




Interpolated result using CRSI

SNR = 9.26 dB

spatial sampling: 12.5 m



Difference

(no minimum velocity constraint)

SNR = 9.26 dB

spatial sampling: 12.5 m

New paradigm

Traditional data collection & compression paradigm

- ‘over emphasis’ on data collection
- extract essential features
- throw away the rest ....

New paradigm compression during sampling

- project onto measurements that breaks aliases
- recover with sparsity promotion

Exploration seismology

- ‘random’ sampling of seismic wavefields [Hennenfent & F.J.H ‘06]
- compressive wavefield extrapolation where eigenfunctions of the Helmholtz operator are used as the measurement basis [Lin & F.J.H ‘07]

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Characterizing singularities

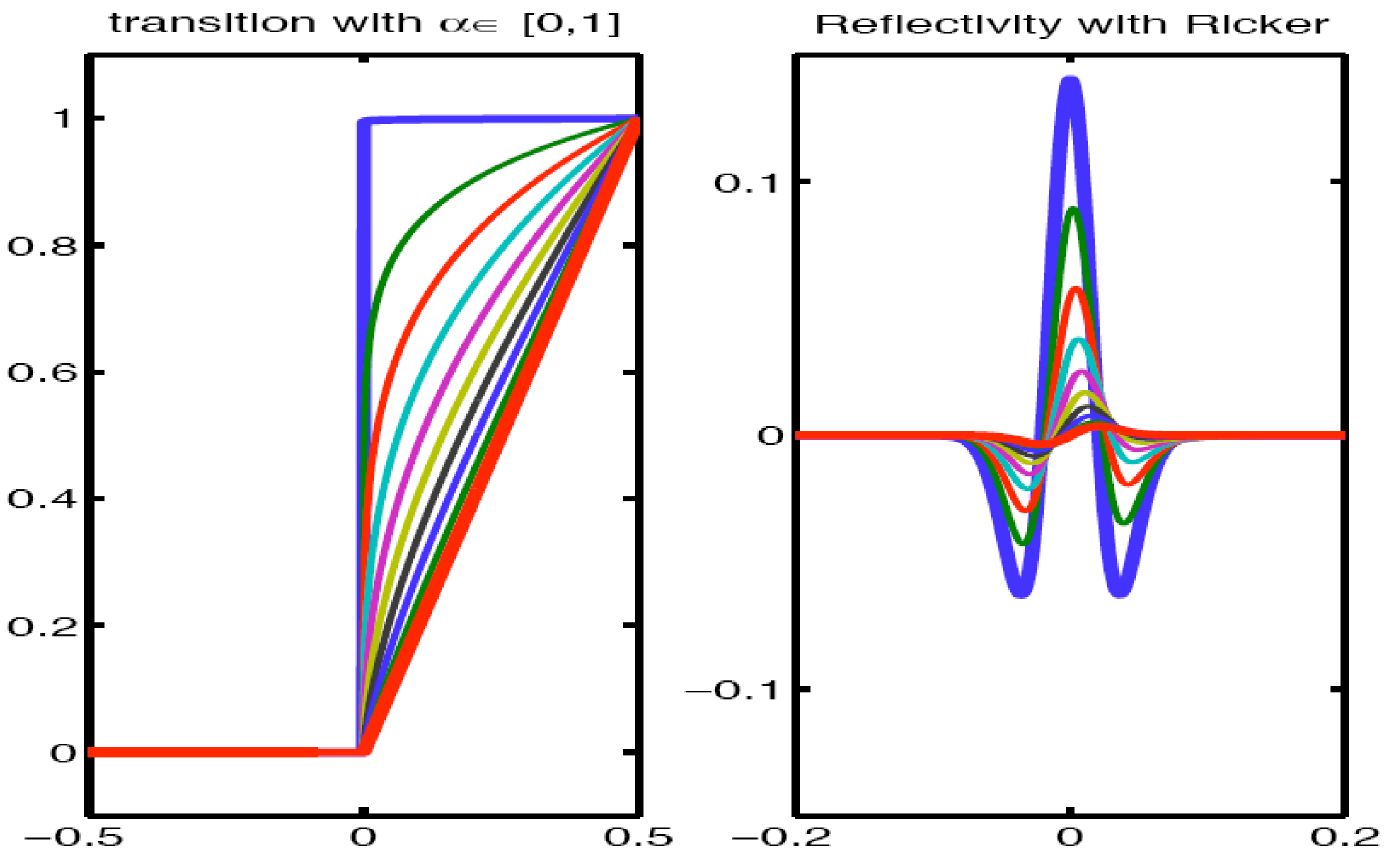
joint work Mohammad Maysami

"Seismic reflector characterization by a multiscale detection-estimation method" '07

Problem

- Delineate the stratigraphy from seismic images.
- Parameterize seismic transitions
  - beyond simplistic reflector models
  - consistent with observed intermittent behavior of sedimentary records
- Estimate the parameters from seismic images:
  - location
  - singularity order
  - instantaneous phase

Singularity characterization
through waveforms
[F.J.H '98, '00, '03, '07]

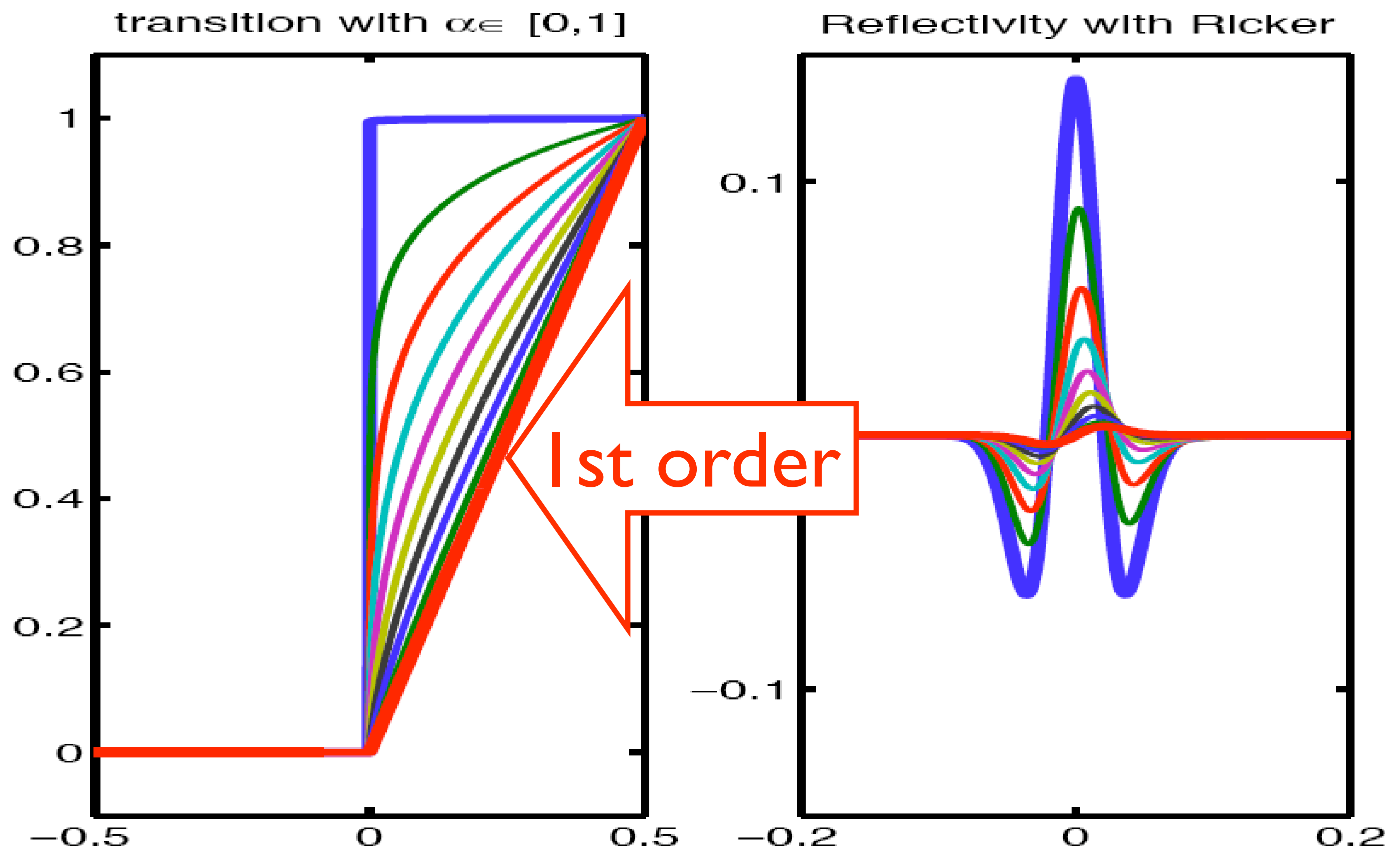


- **generalization of zero- & first-order discontinuities**
- **measures wigglyness / # oscilations / sharpness**

Singularity characterization

through waveforms

[F.J.H '98, '00, '03, '07]



- **generalization of zero- & first-order discontinuities**
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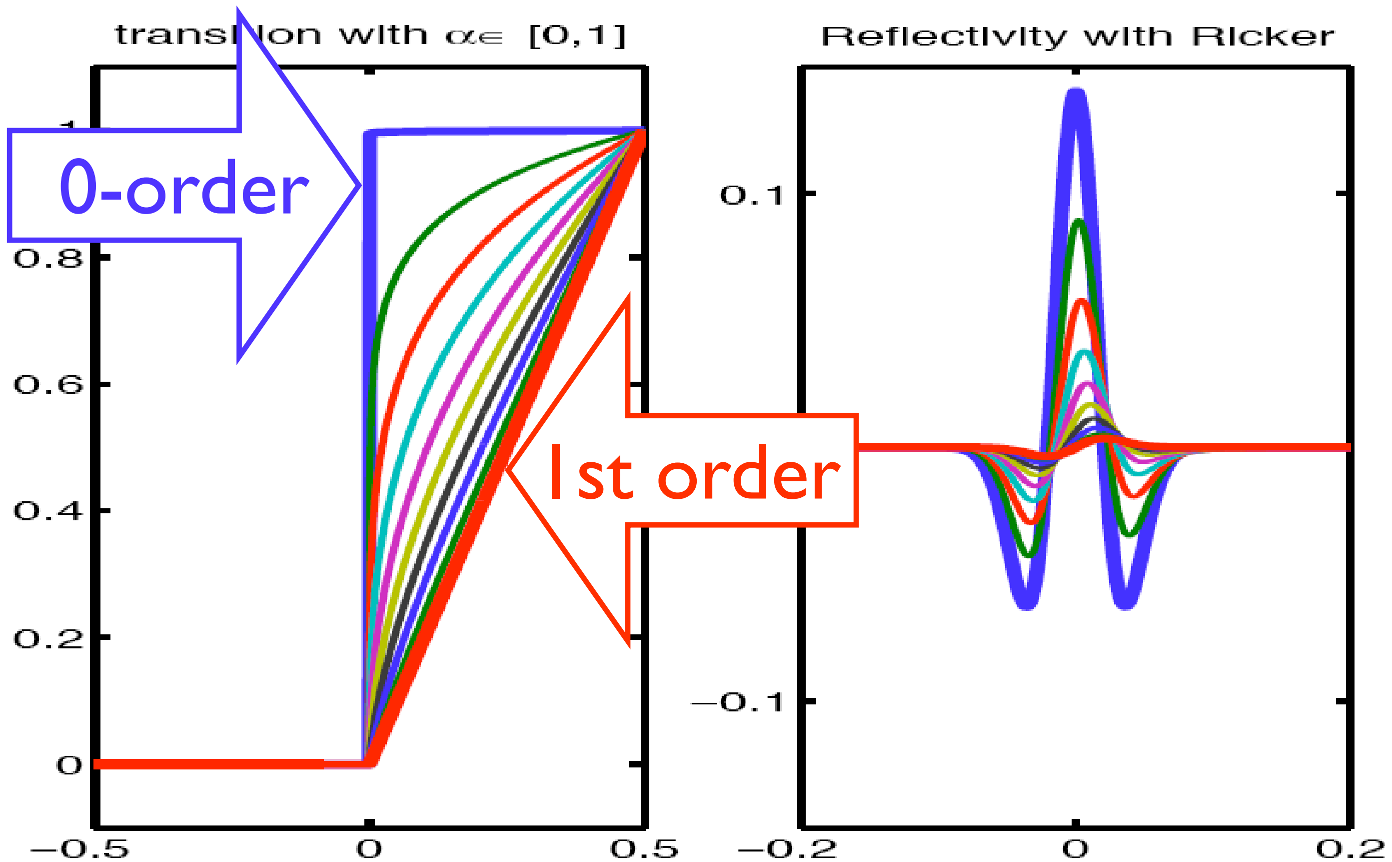
Singularity characterization
through waveforms
[F.J.H '98, '00, '03, '07]

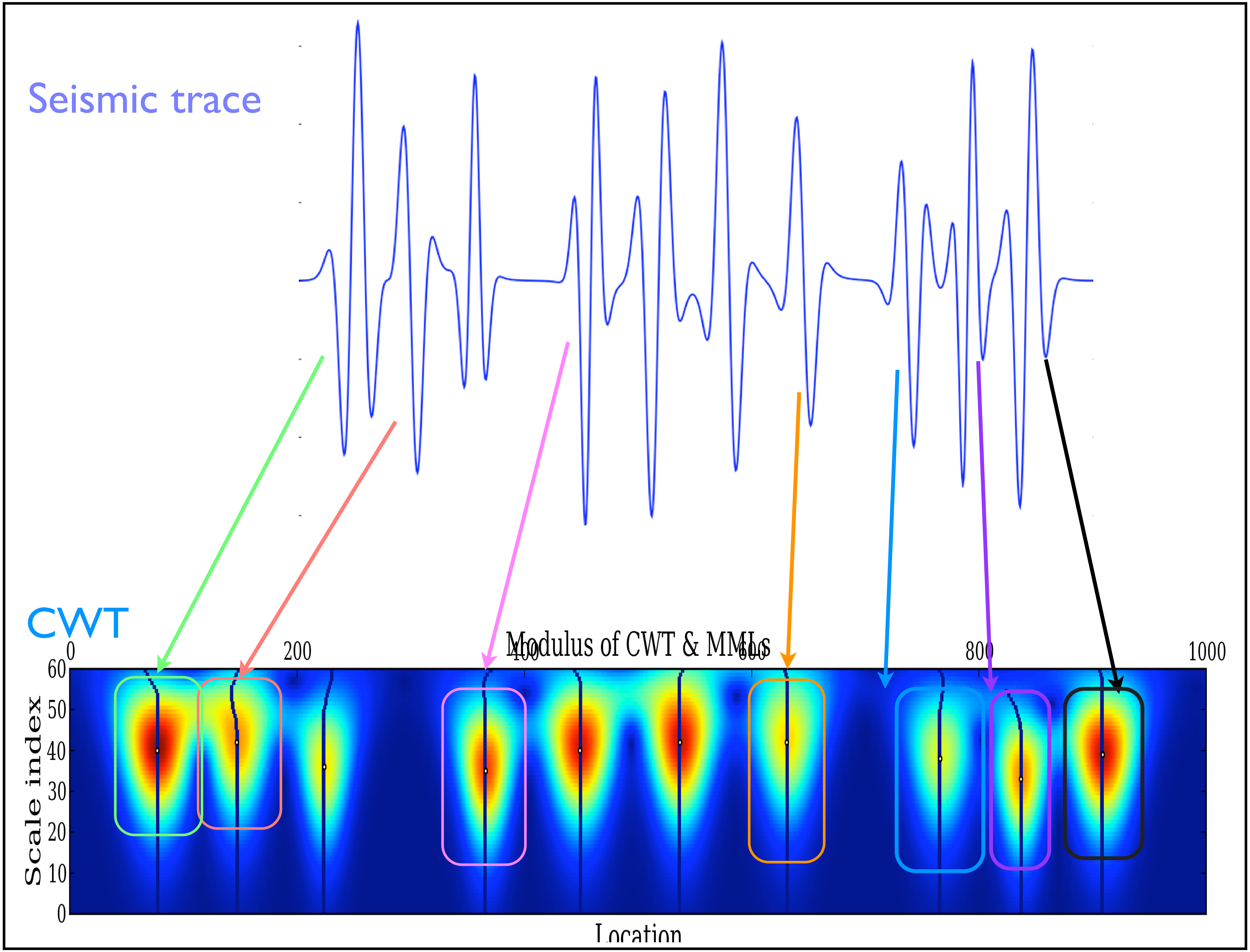


- **generalization of zero- & first-order discontinuities**
- **measures wigglyness / # oscillations / sharpness**

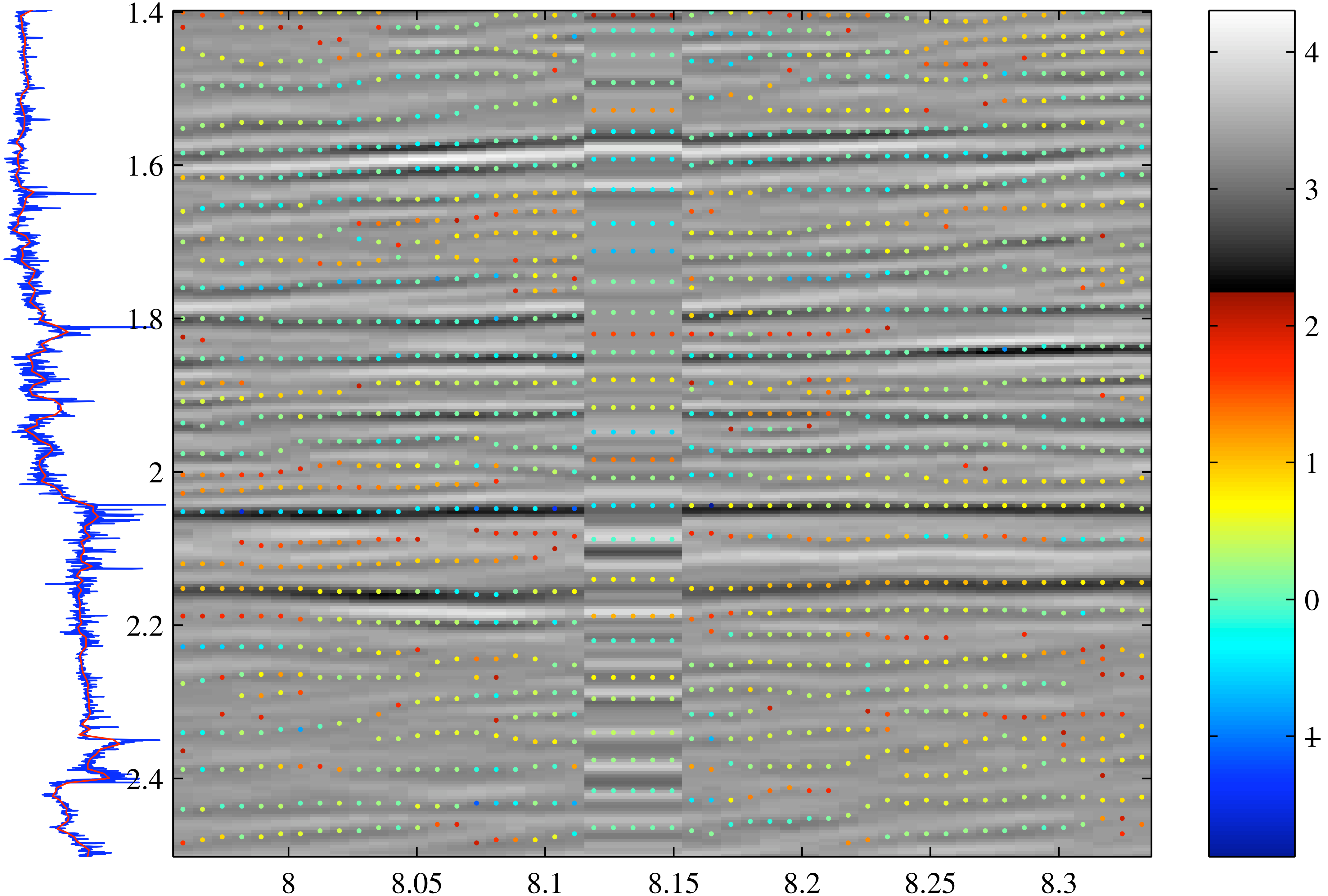
Approach

[Wakin et al ‘05-’-07, M&H ‘07]

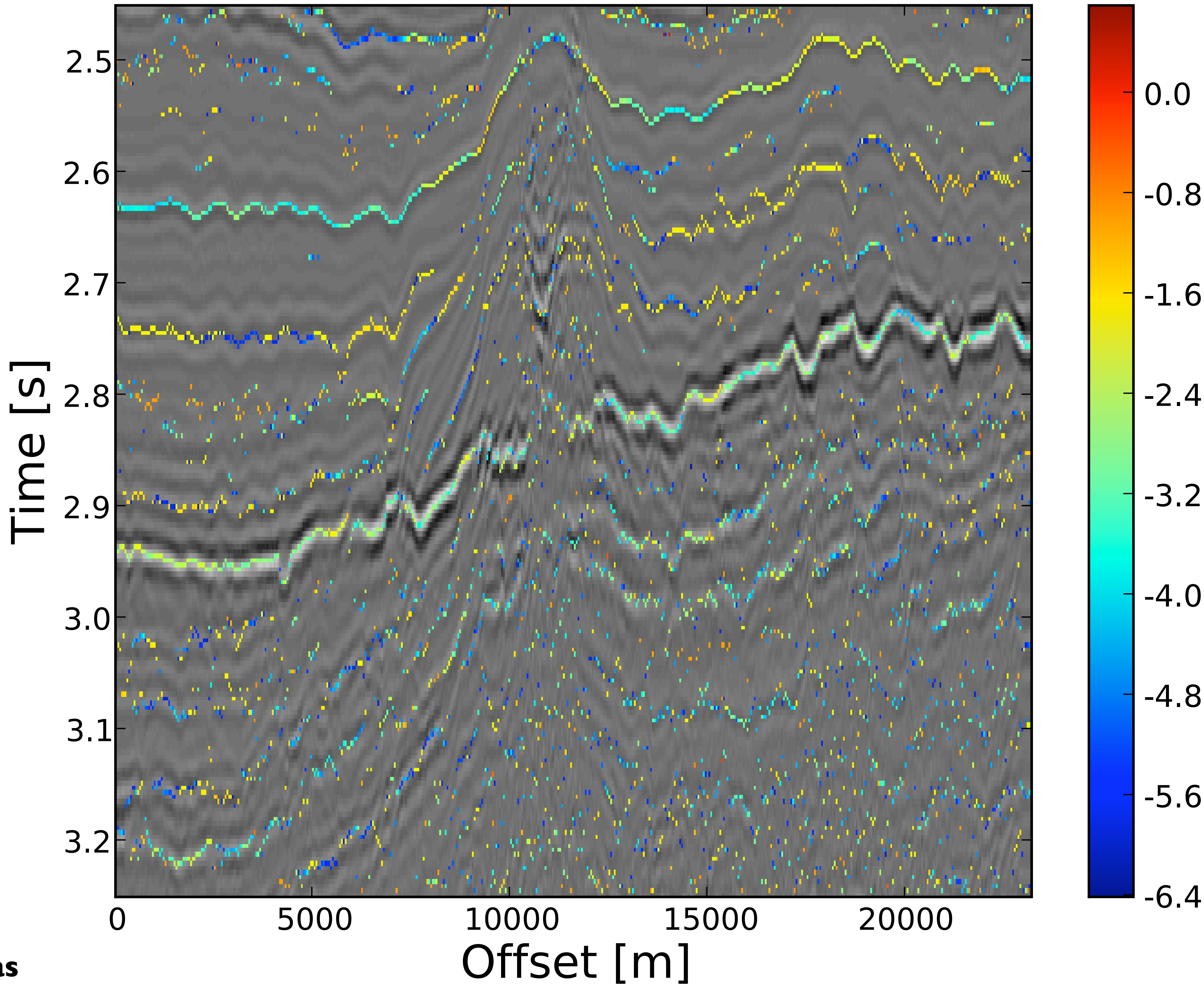
- Use a detection-estimation technique
  - multiscale detection => segmentation
  - multiscale Newton technique to estimate the parameterization
- Overlay the image with the parameterization


Seismic trace
CWT
Modulus of CWT & MMLs
0
200
400
600
800
1000
60
50
40
30
20
10
0
Scale index
Location

Singularity map



Estimated alpha



Courtesy
CGG Veritas

Observations

- Stratigraphy is detected
- Parameterization provides information on the lithology
    - evidence of changes in exponents along clinoforms
- Method suffers from curvature in the imaged reflectors
- Extension to higher dimensions necessary
- Model that explains different types of transitions
- A step beyond the zero-& first-order discontinuities

Modeling seismic singularities

Joint work with Yves Bernabe (MIT)

“Seismic singularities at upper-mantle phase transitions: a site percolation model” GJI ‘04

Problem

Earth subsurface is highly heterogeneous

- sedimentary crust, upper-mantle transition zone & core-mantle boundary

Smooth relation volume fractions and the transport properties.

Homogenization/equivalent medium (EM) theory *smoothes* the singularities during *upscaling*

- relatively *easy* for *volumetric* properties (density)
- *notoriously difficult* for *transport* properties (velocity)

Q: How to preserve singularities in effective properties?

Our approach

Include *connectivity* in models for the *effective* properties of bi-compositional mixtures **<=> SWITCH**

Start with *binary* mixtures, e.g.

- sand-shale
- gas-hydrate, Opal-Opal CT
- upper-mantle mineralogy

Studied two cases:

- elastic properties upper mantle (H & B '04)
- fluid-flow properties synthetic rock (B & H '04)

Mixture laws for binary mixtures

Elastic case:

- Controlled by *connectivity* of stiffest phase

Fluid-flow case:

- Controlled by *connectivity* of high-conductivity phase

Note: Stiff phase = low porosity, low conductivity phase

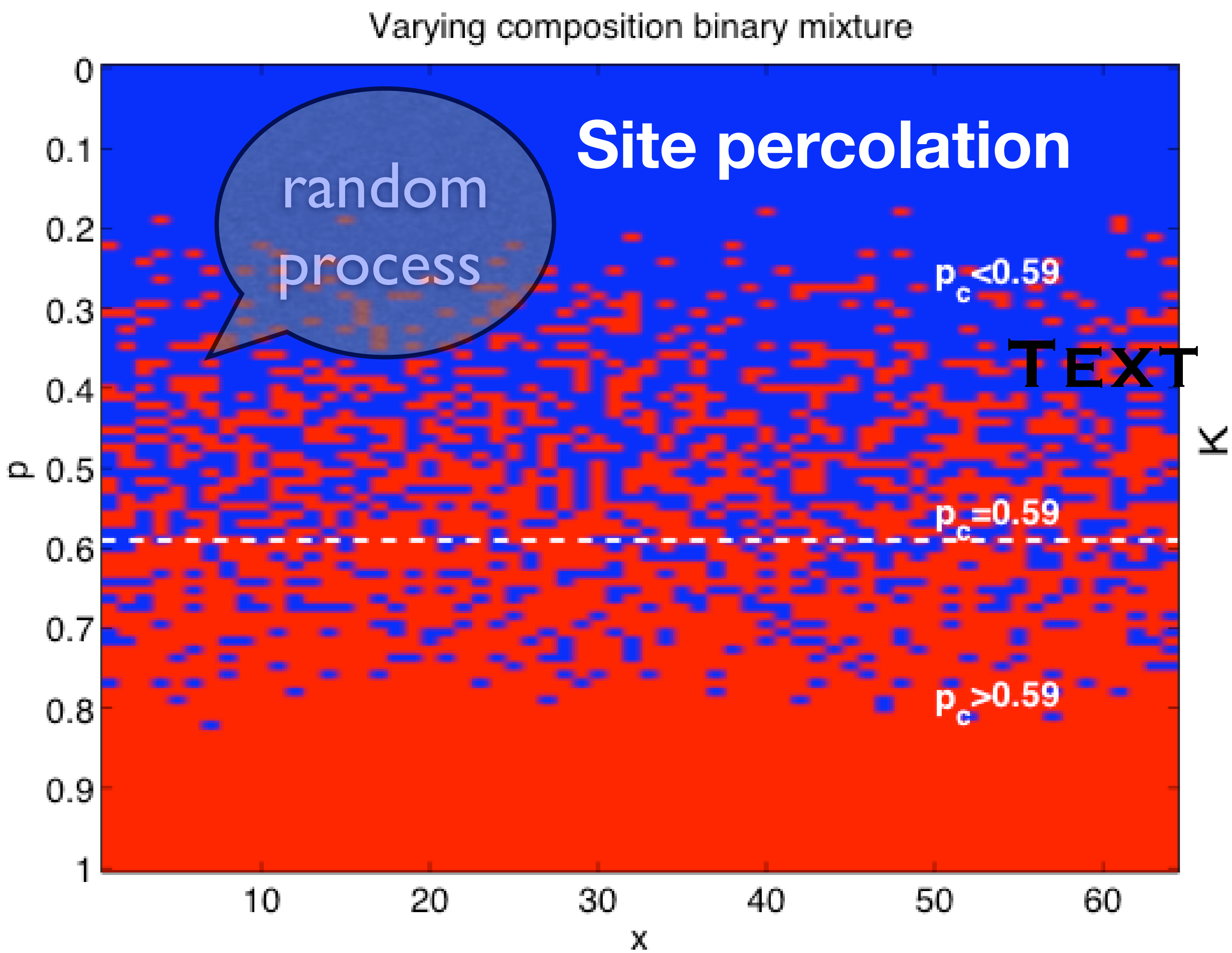
No obvious link elastic-fluid flow properties ...

Singularity modeling

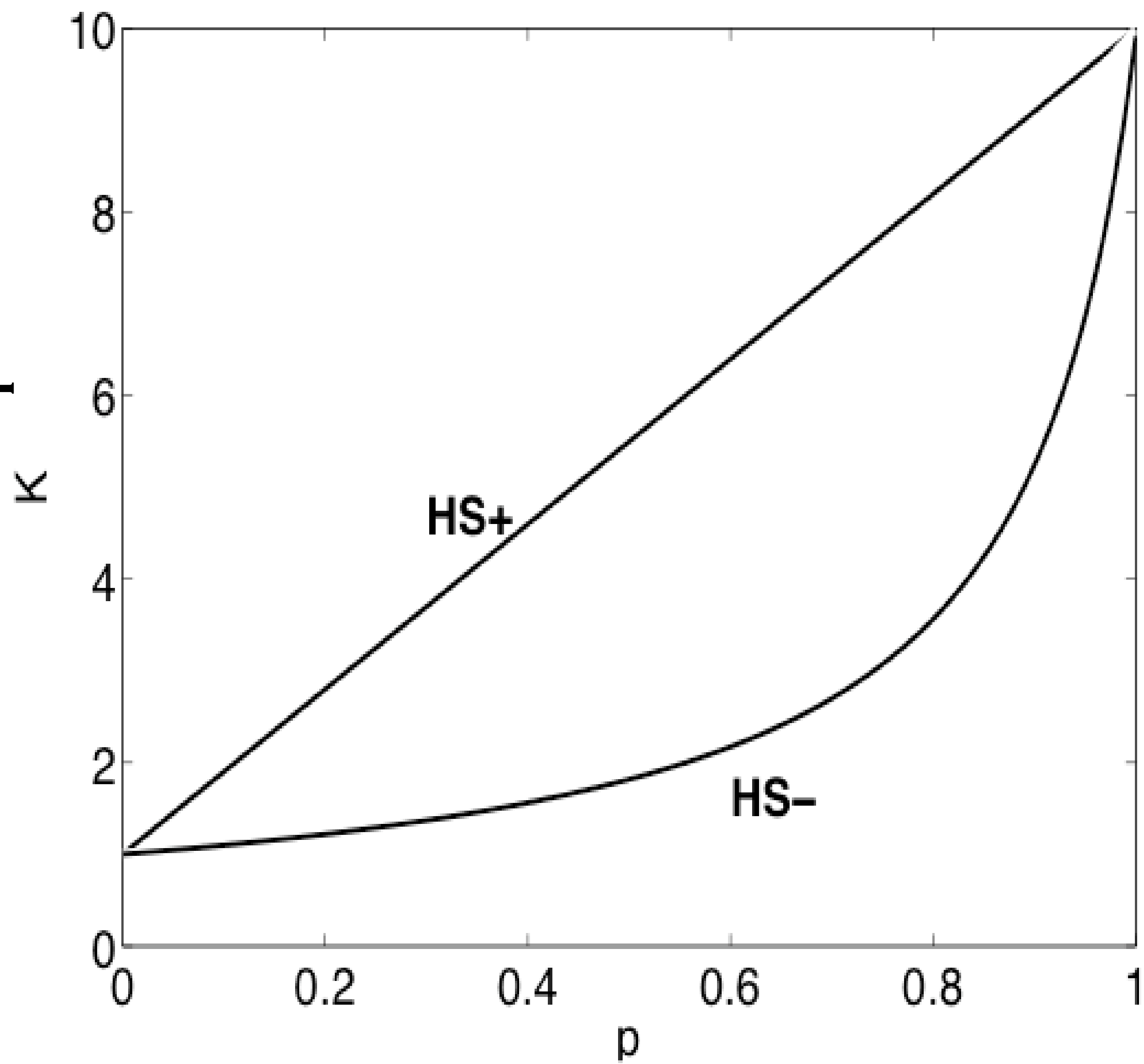
binary mixtures

LP olivine

HP β-spinel



elastic properties

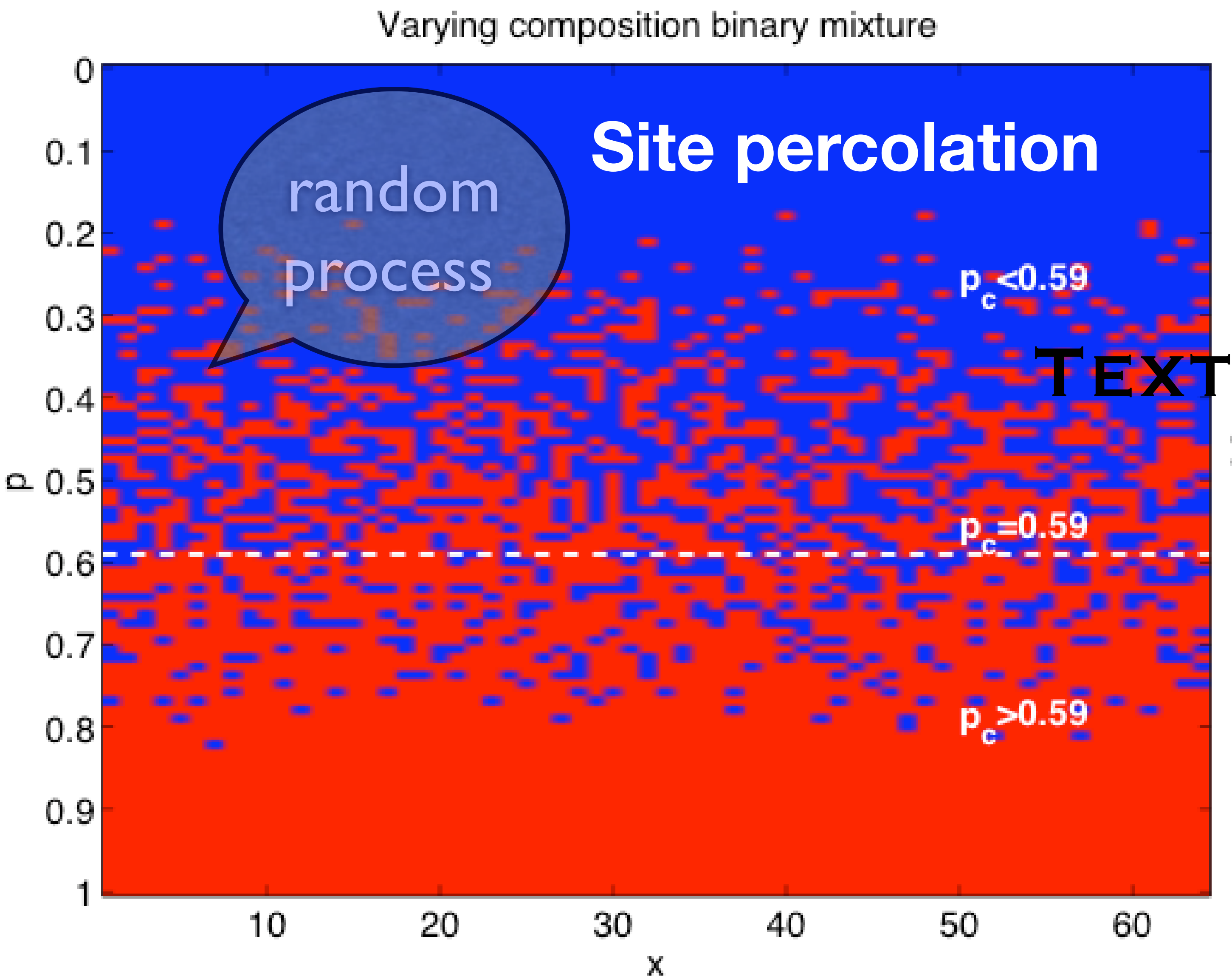


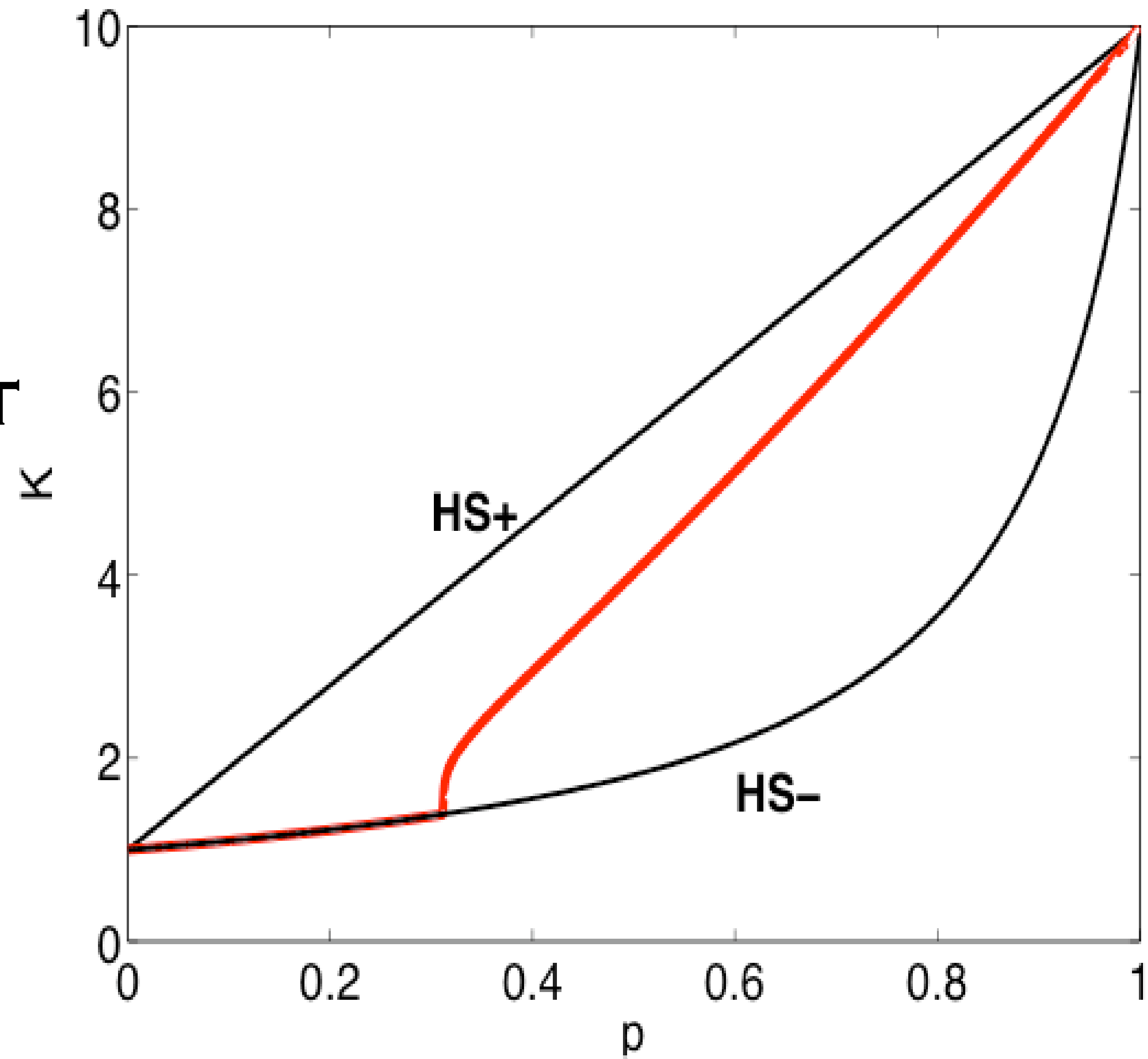
volume fraction

Singularity modeling

binary mixtures

LP olivine





HP β-spinel

volume fraction

Singularity modeling

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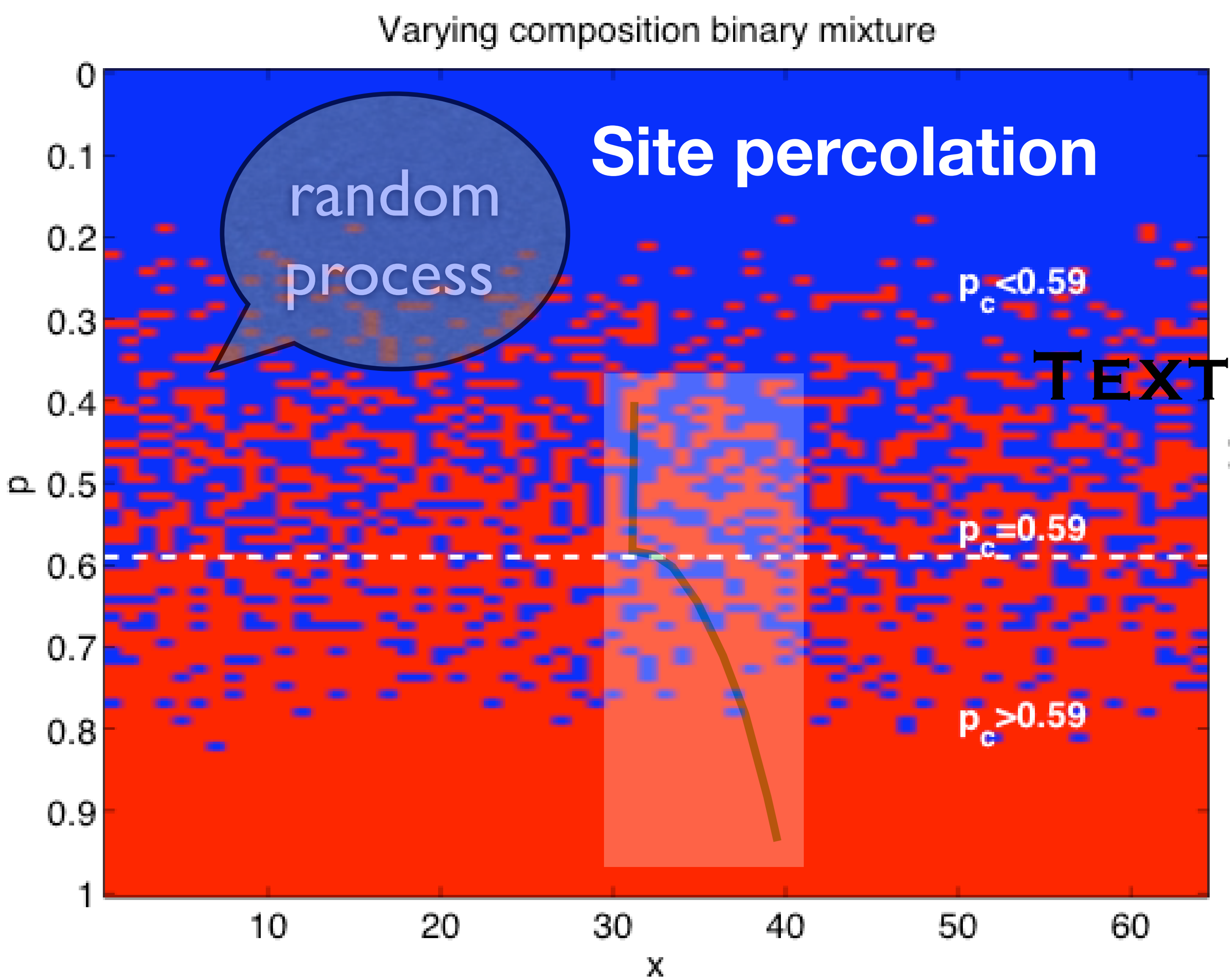
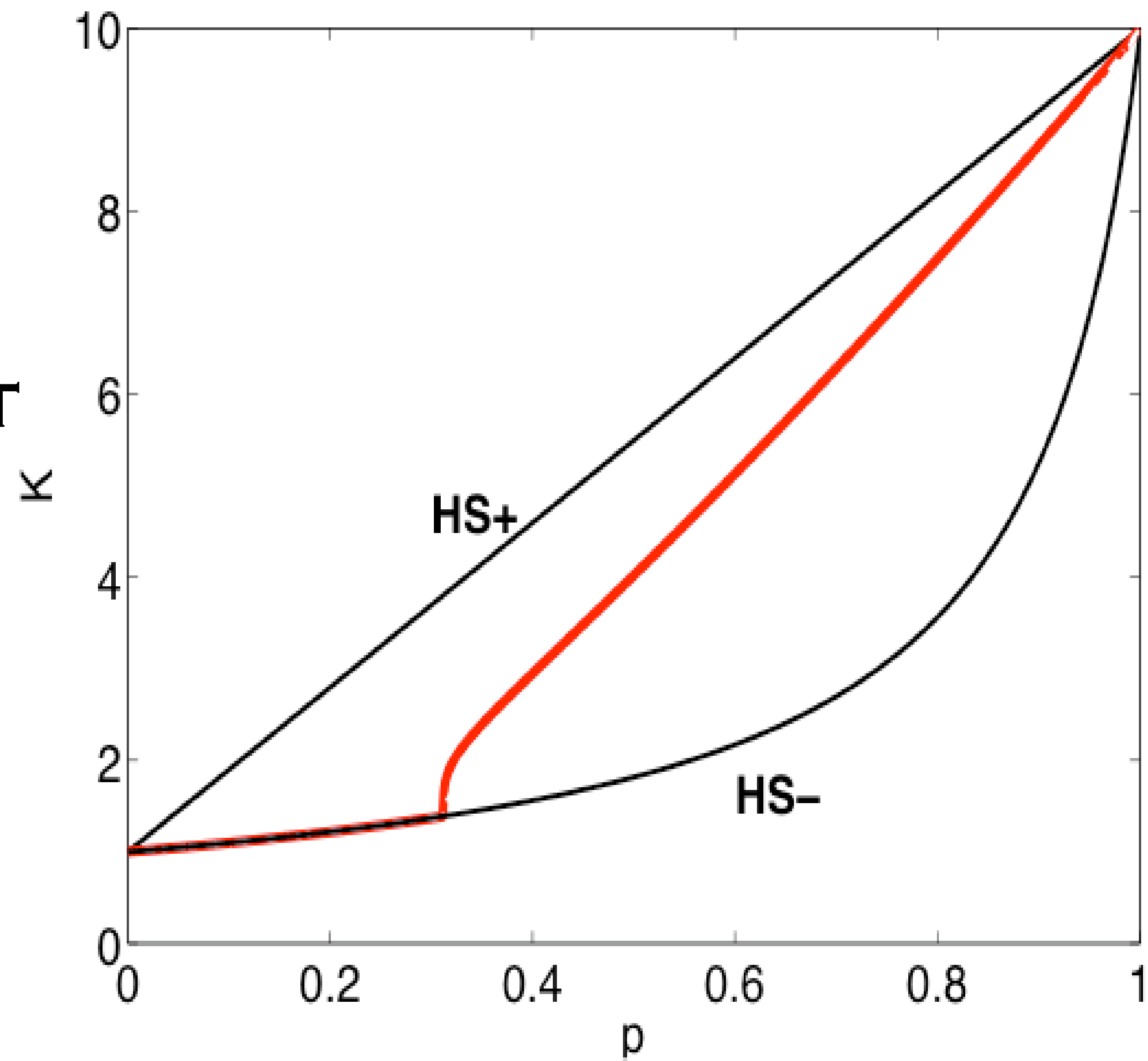
elastic properties





HP β-spinel

volume fraction

Mixing model

Homogeneous mixing (e.g., solid solution) of **two** *phases* (**LP** weak and **HP** strong) can only produce ***gradually*** varying *elastic* properties.

If **Heterogeneous** (e.g. *emerging random macroscopic* inclusions) mixing, then a **singularity** in the elastic properties ***must*** arise at the depth where the **strong**, **HP** phase becomes **connected** (observed in binary alloys).

Site-percolation model

Assume volume fractions p and $q = 1 - p$, are **linear** functions of depth z.

At a **critical** depth z_c, which corresponds to the percolation threshold $p_c = p(z_c)$, an "infinite", **connected HP** *cluster* is formed.

For $z \geq z_c$

- not all HP inclusions belong to the *infinite* cluster.
- isolated HP inclusions can still be found, embedded in the remaining LP material and forming with it a ***mixture*** (**M**).

Site-percolation model

Above z_c we have a ***weak*** **LP** *matrix* containing *randomly* distributed, non-percolating, **strong** HP inclusions.

Below z_c, a ***strong*** HP *skeleton* is intertwined with the **weaker**, ***mixed*** material **M**.

Site-percolation model

Volume fraction p^* of **HP** material that belongs to the ***infinite*** cluster

- is zero for $p < p_c$ (i.e., above z_c)
- has a **power-law** dependence on $(p - p_c)$ for

  $p \geq p_c$.

Hence, p^* is given by:

$$p^* = p \left(\frac{p - p_c}{1 - p_c} \right)^{\beta}$$

Site-percolation model

Mixed **M** is given by $q^* = (1 - p^*)$.

For **M**, we need the *volume fractions* of its **LP** and **HP** parts,

$$q_M = (1 - p)/\{(1 - p) + (p - p^*)\}$$

$$p_M = (1 - q_M),$$

yielding

$$p_M = 1 - \frac{q}{1 - p\left(\frac{p - p_c}{1 - p_c}\right)^{\beta}}$$

Site-percolation model

Binary mixture:

- **Strong** when its **strong** component is **connected.**
- **Weak** *otherwise.*

Assume *locally* **isotropic**

- Bin. mixtures bounded by Hashin-Shtrikman (**HS**).
- *upper* **HS** bound when *strong* component connects, the *lower* one applies *otherwise*.

Bulk modulus K of the co-existence region *above* z_c is given by the *lower* **HS** bound:

$$K = K_{LP}\left(1 + \frac{p(K_{HP} - K_{LP})}{q(K_{HP} - K_{LP})a_{LP} + K_{LP}}\right)$$

Site-percolation model

Below z_c we must switch to the *higher* **HS** bound:

$$K = K_{HP}\left(1 + \frac{q^*(K_M - K_{HP})}{p^*(K_M - K_{HP})a_{HP} + K_{HP}}\right)$$

Since the HP inclusions in **M** are *isolated*, K_M is calculated using the *lower* **HS** bound:

$$K_M = K_{LP}\left(1 + \frac{p_M(K_{HP} - K_{LP})}{q_M(K_{HP} - K_{LP})a_{LP} + K_{LP}}\right)$$

Site-percolation model

Major consequence of this model is that it predicts:

- a β-**order**, cusp-like ***singularity*** in the ***elastic*** moduli as the ***critical*** *depth* z_c is approached from **below** (instead of a first-/ zero-order discontinuity).
- **Singularities that** persist for vanishing *contrasts*.
- Density that does **not** behave **singularly**.

Site-percolation model

Elastic ***contrasts*** between **LP** and **HP** are **small**:

- Nearly *coincident* **HS** bounds.
- *Excessively* **small** contrasts.

Discard ***isotropy*** assumption:

- *Horizontally*-oriented *oblate ellipsoidal* inclusions which *coalesce* below z_c into long, vertical *dendrites*, leaving *prolate* **M** inclusions between them.
- ***Transversely*** *isotropic* structure.

Site-percolation model

Near **normal** incidence, V_{P} and V_{S} approach limiting values as the ***aspect*** ratio is goes to *zero*.

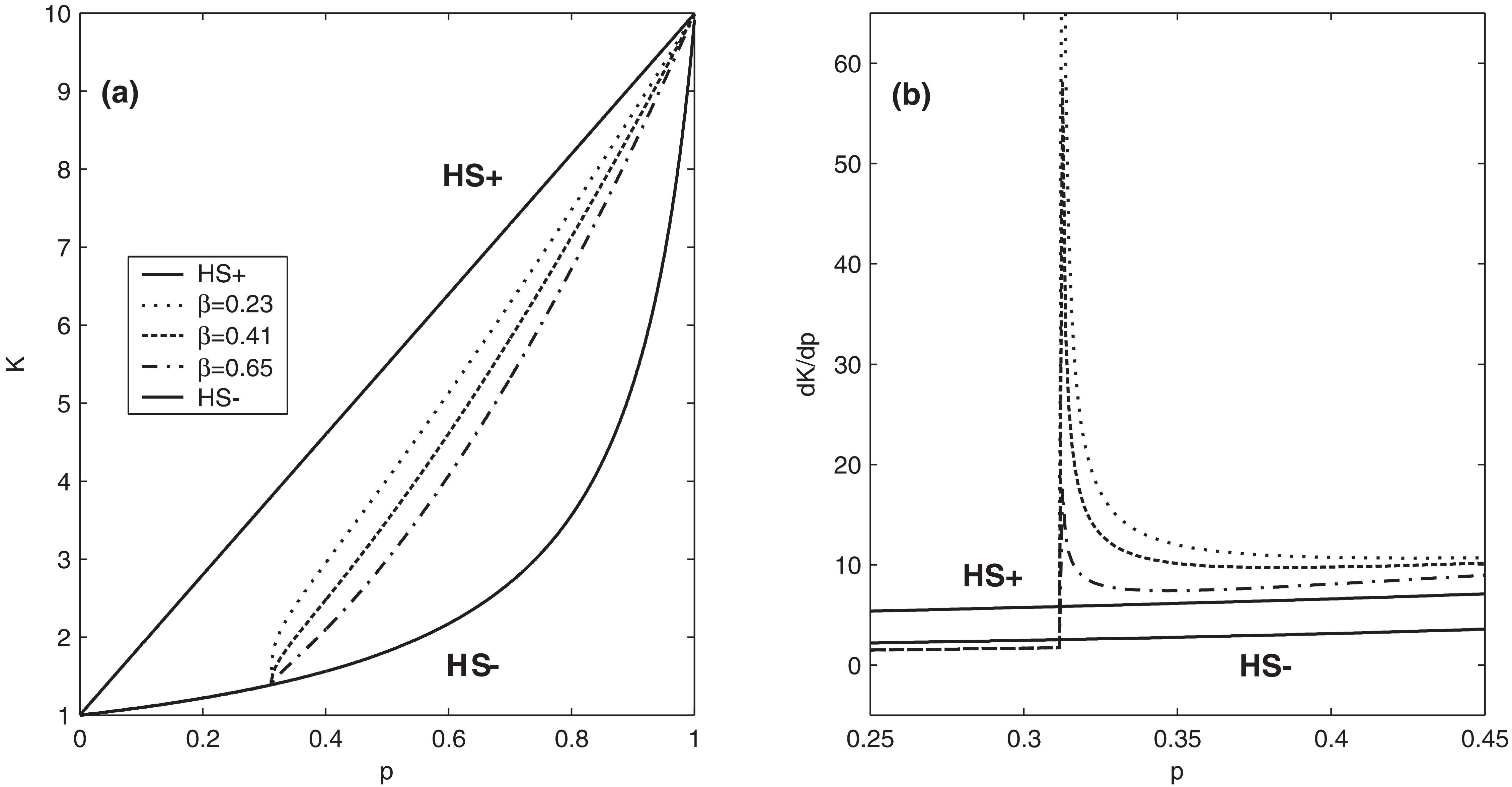
Same as replacing ***lower*** and ***higher*** **HS** bounds by Reuss and Voigt averages:

$$K = \left(\frac{q}{K_{\text{LP}}} + \frac{p}{K_{\text{HP}}} \right)^{-1} \qquad (\text{for } z < z_{\text{c}})$$

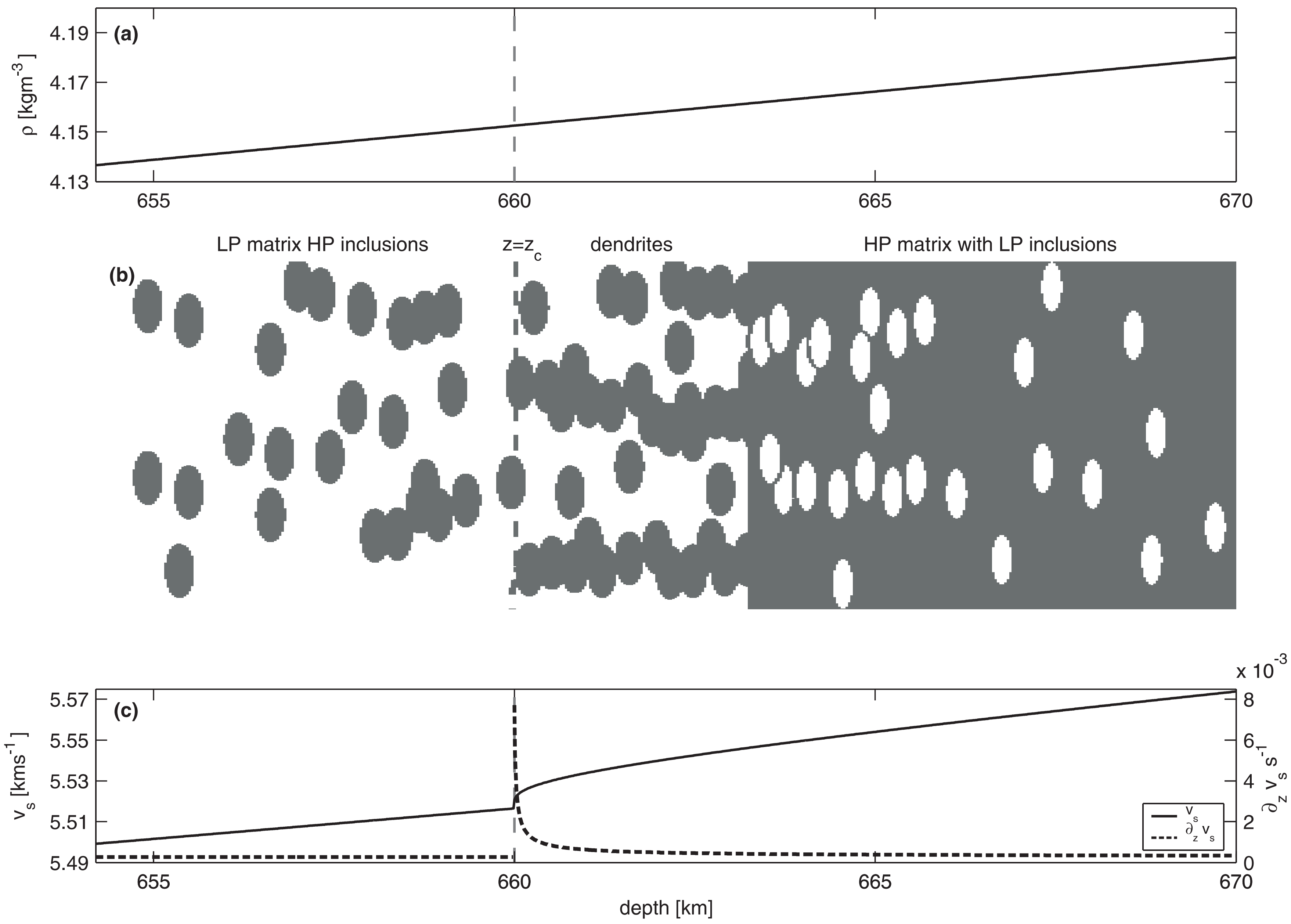
$$K = q * K_{\text{M}} + p * K_{\text{HP}} \qquad (\text{for } z \geq z_{\text{c}})$$

$$K_{\text{M}} = \left(\frac{q_{\text{M}}}{K_{\text{LP}}} + \frac{p_{\text{M}}}{K_{\text{HP}}} \right)^{-1}$$

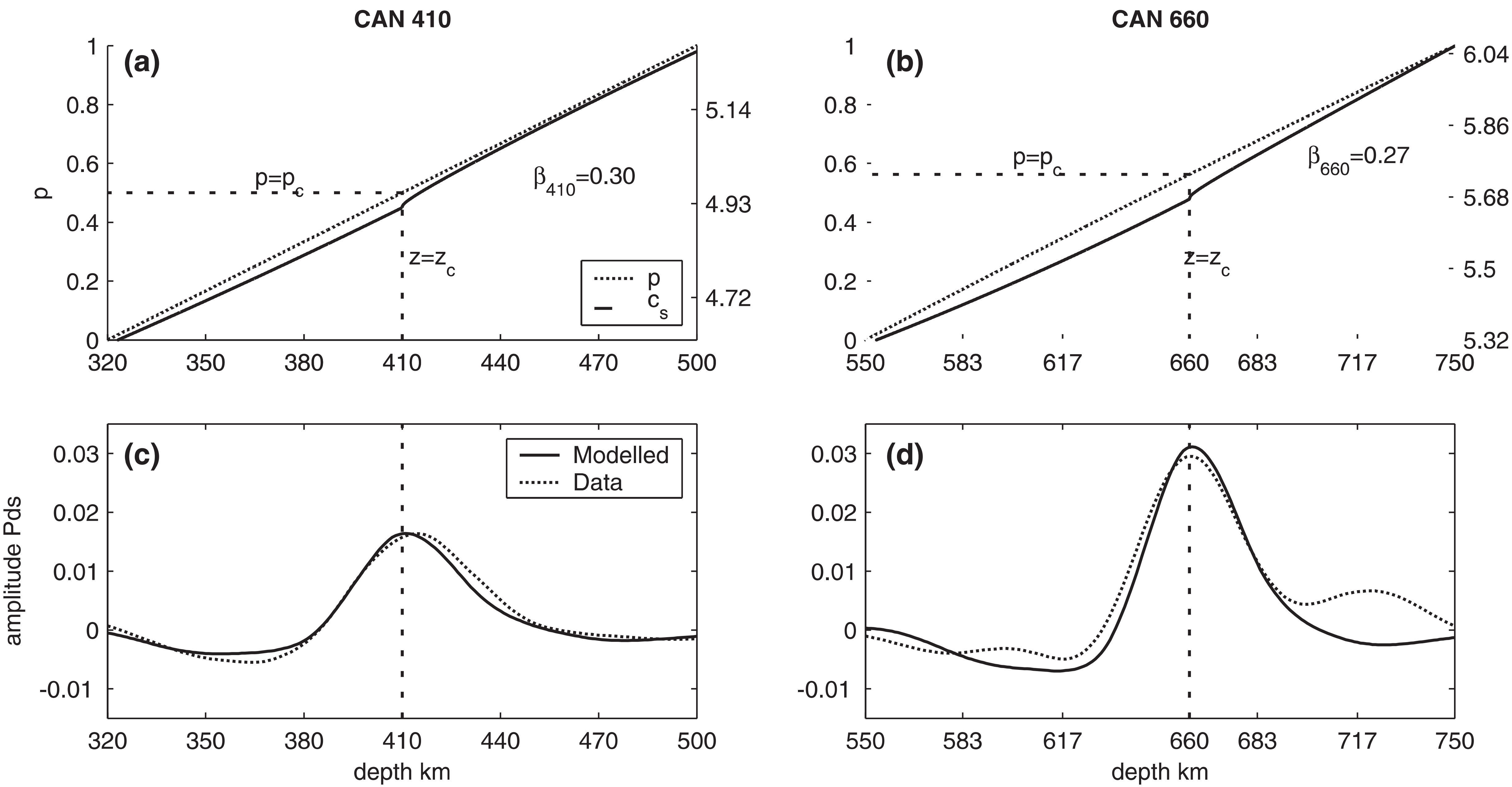
Singularity model



Singularity model upper-mantle transitions



Modeled data vs seismic



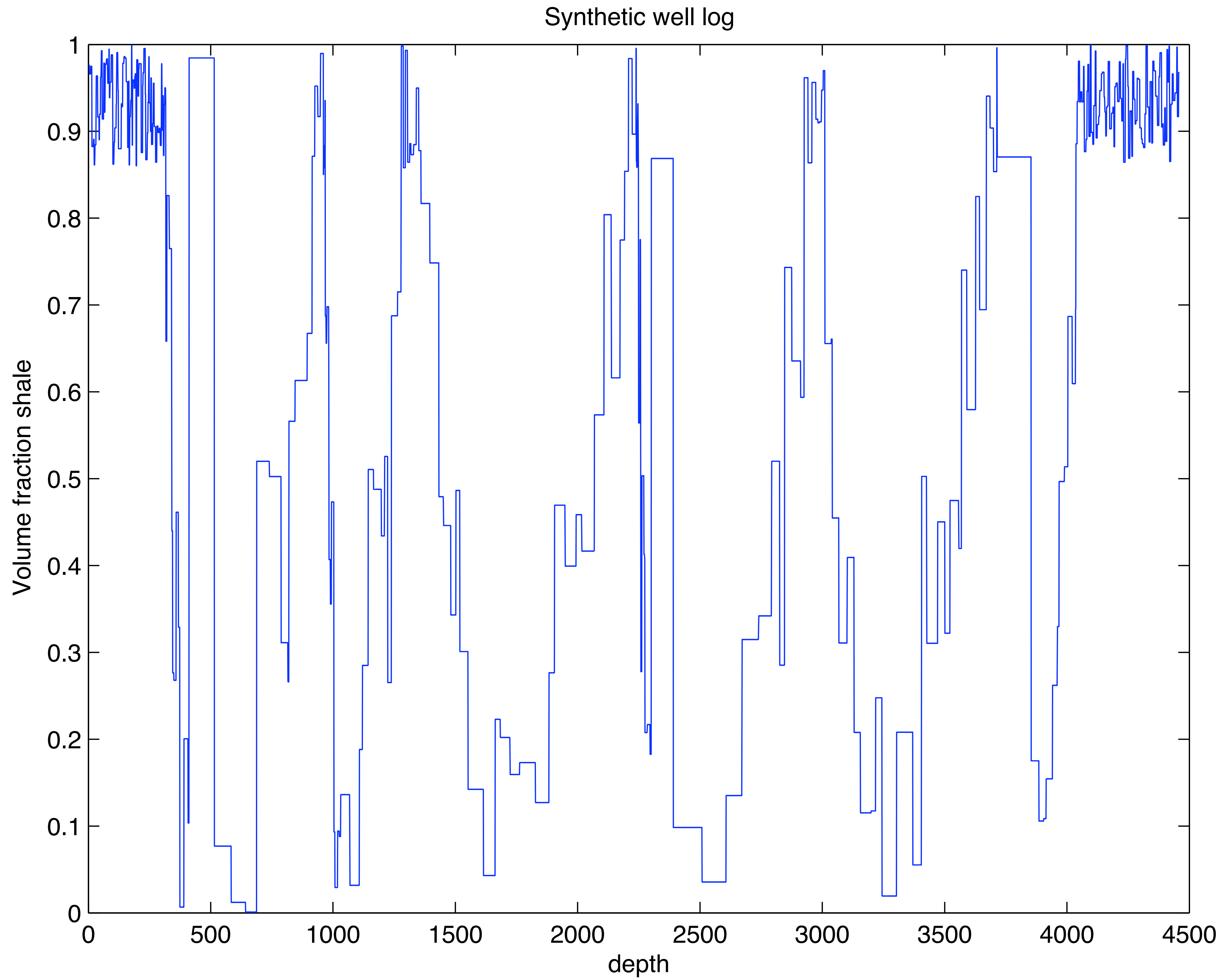
Singularity-preserved upscaling

Joint work with Yves Bernabe (MIT)

The problem

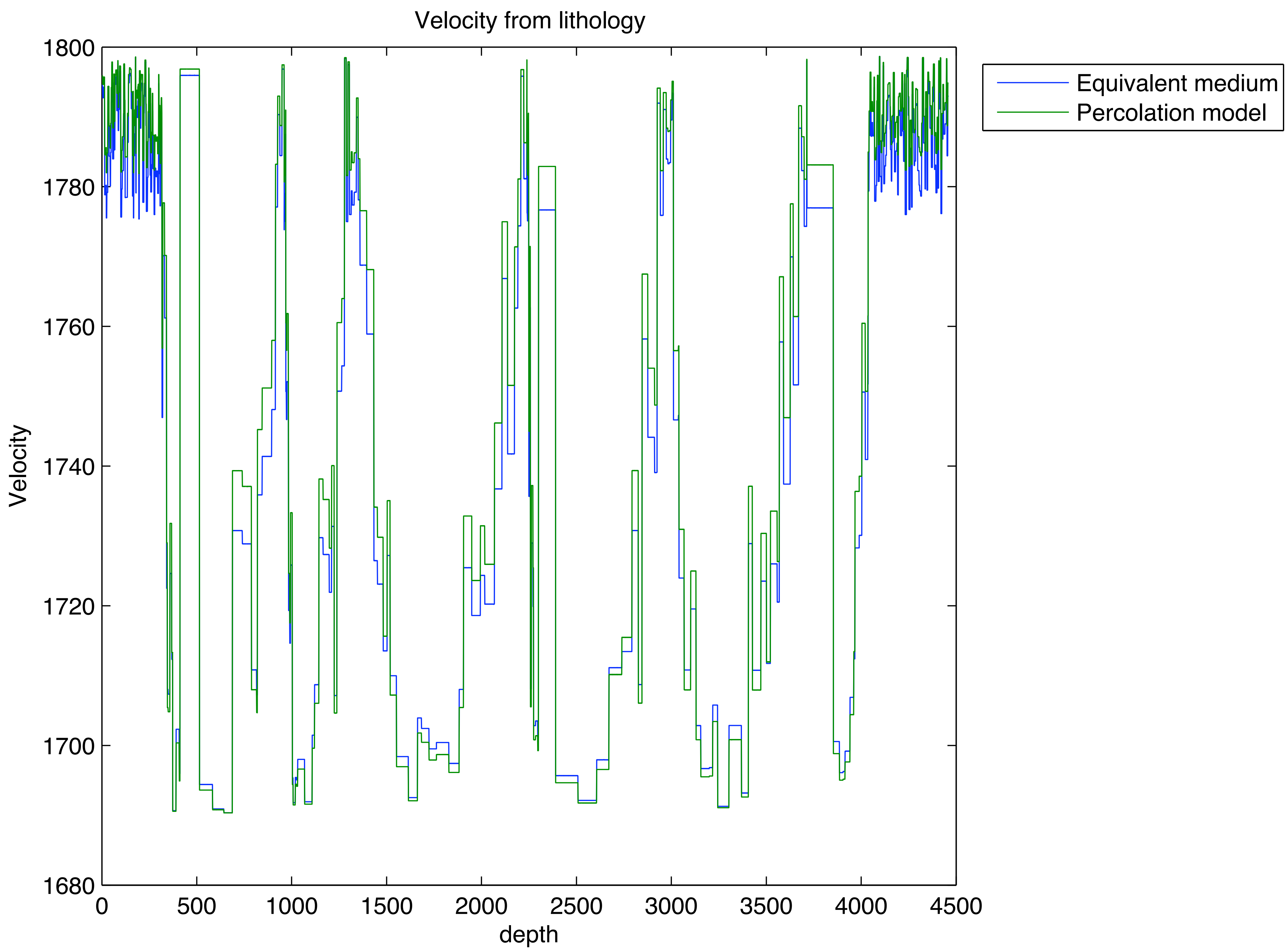
- Equivalent medium based upscaling washes out the singularities
- Reflection seismology lives by virtue of singularities in the elastic moduli (transport properties)
- Propose a singularity preserving upscaling method:
  - upscales the lithology rather than the velocities
- Singularities can be due to sharp changes in composition or due to the switch ...

Volume fraction synthetic well

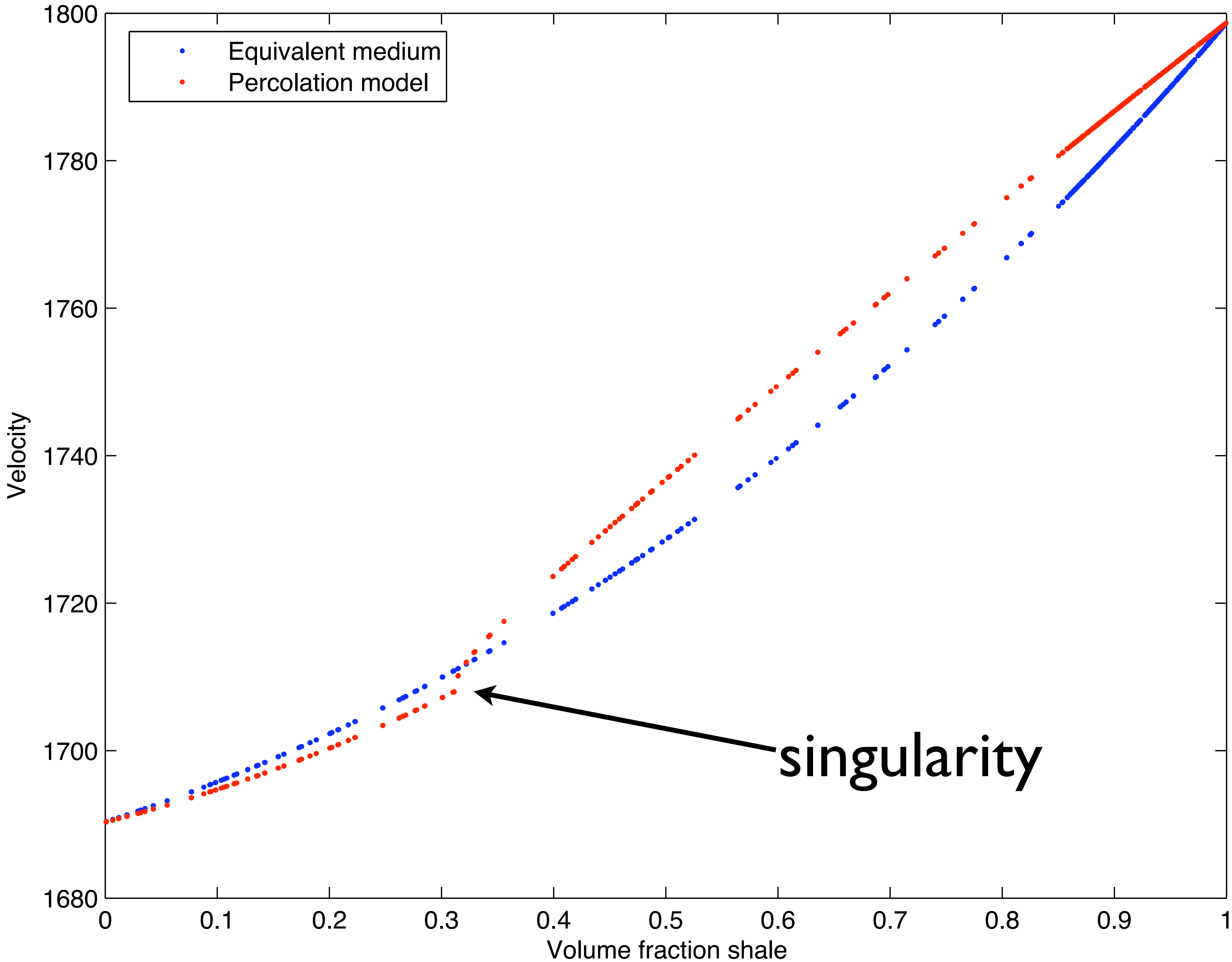


Courtesy Chevron

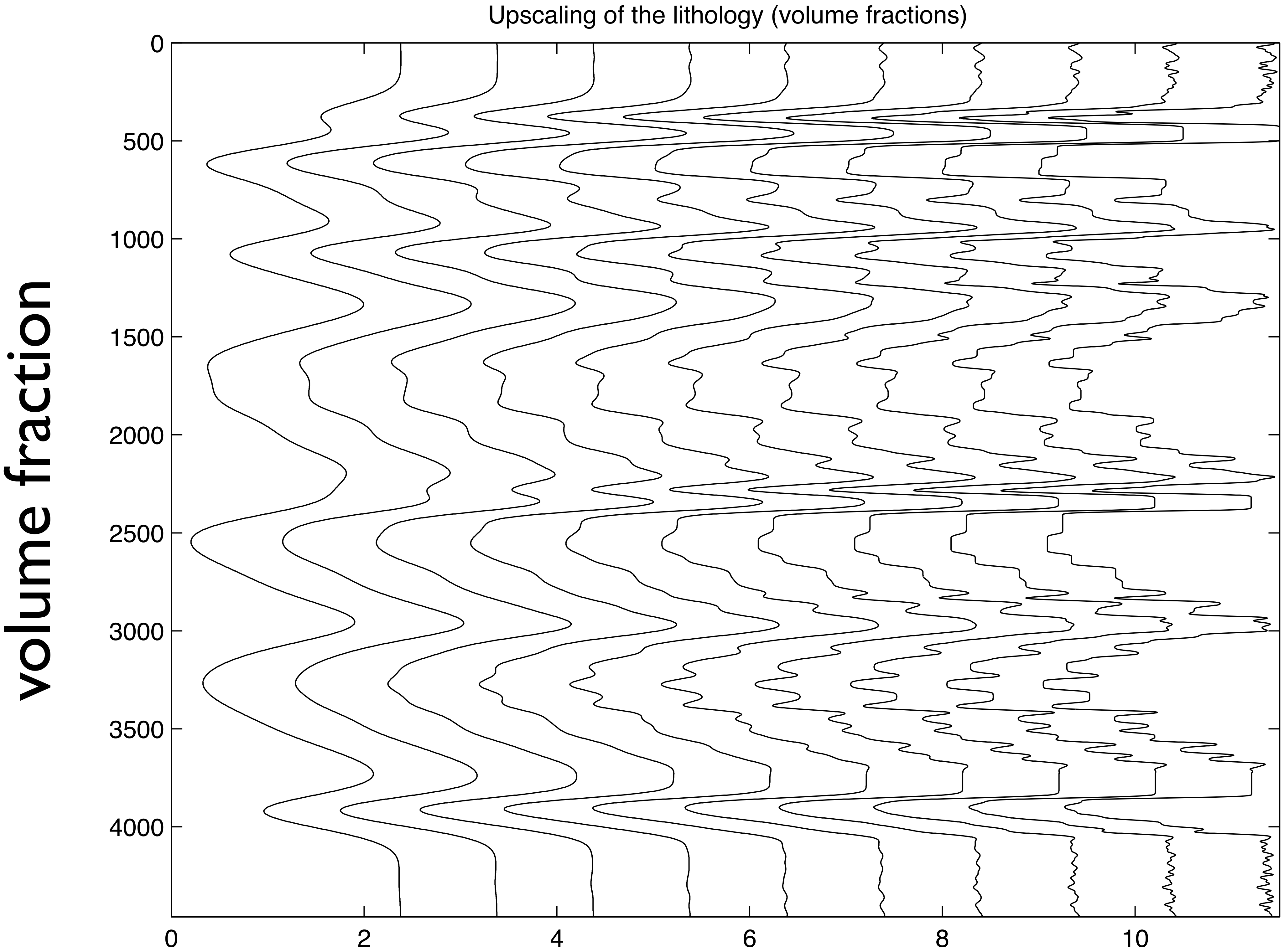
Switch vs no switch



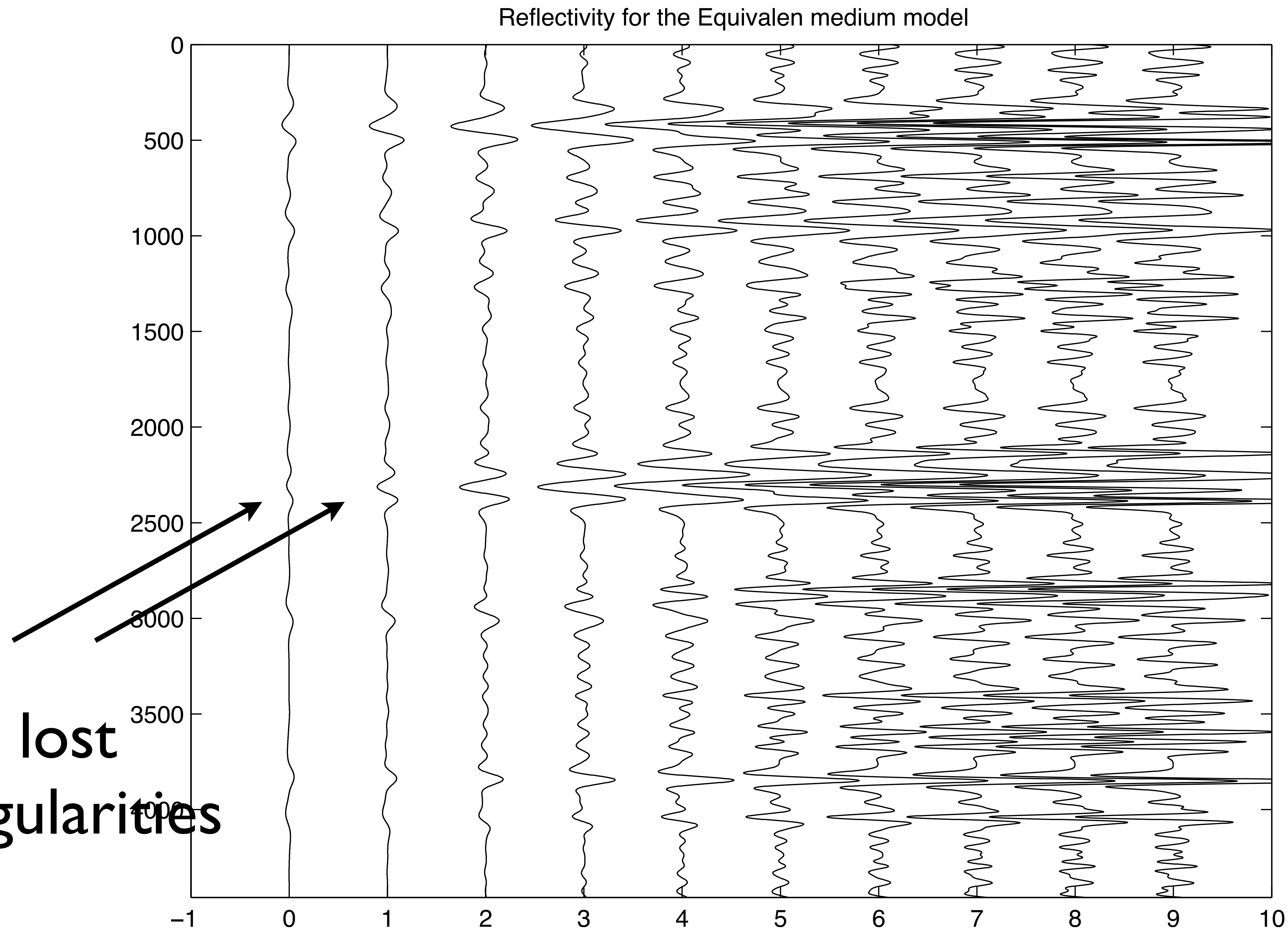
Switch vs no switch



Lithological upscaling

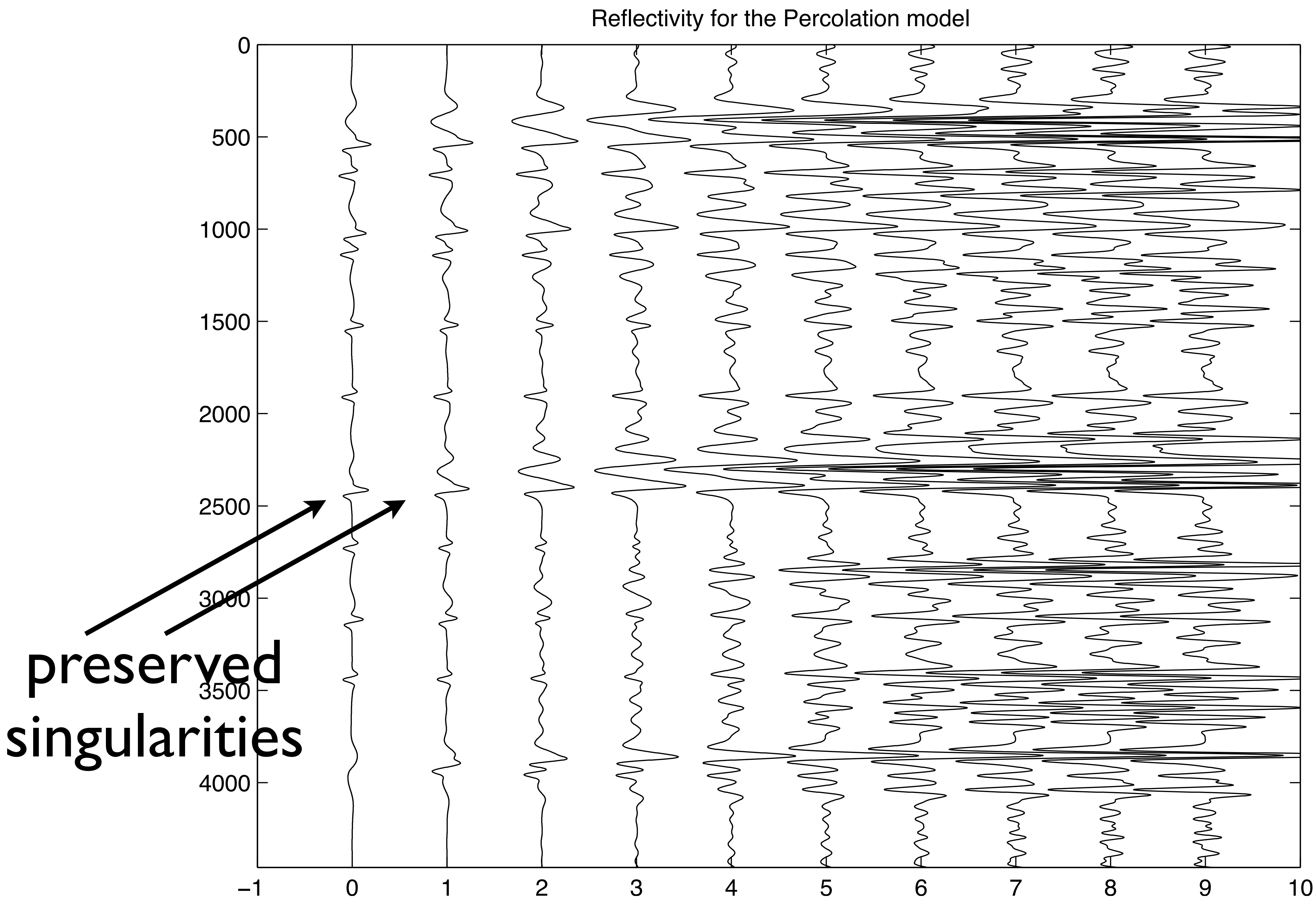


EM-upscaled reflectivity



lost singularities

Perco.-upscaled reflectivity



Relation to fluid flow & open problems

joint work with Yves Bernabe (MIT)

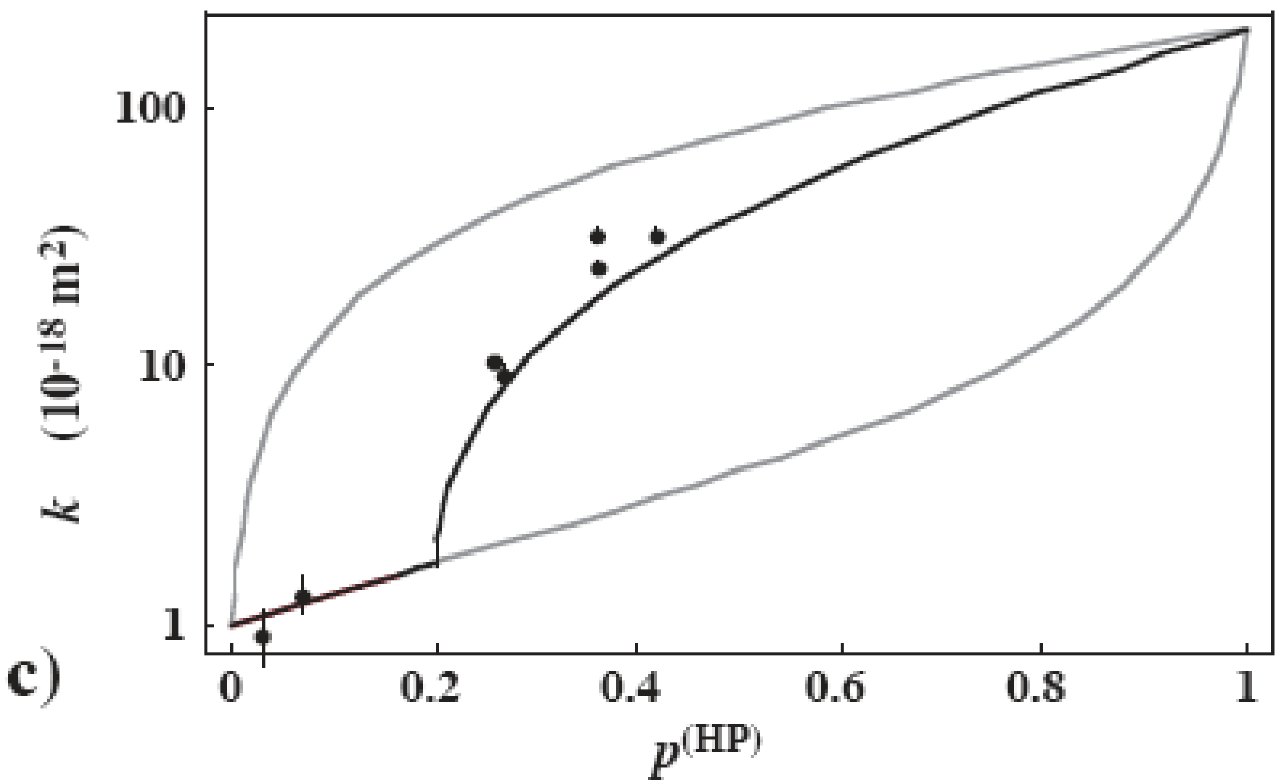
Sedimentary crust

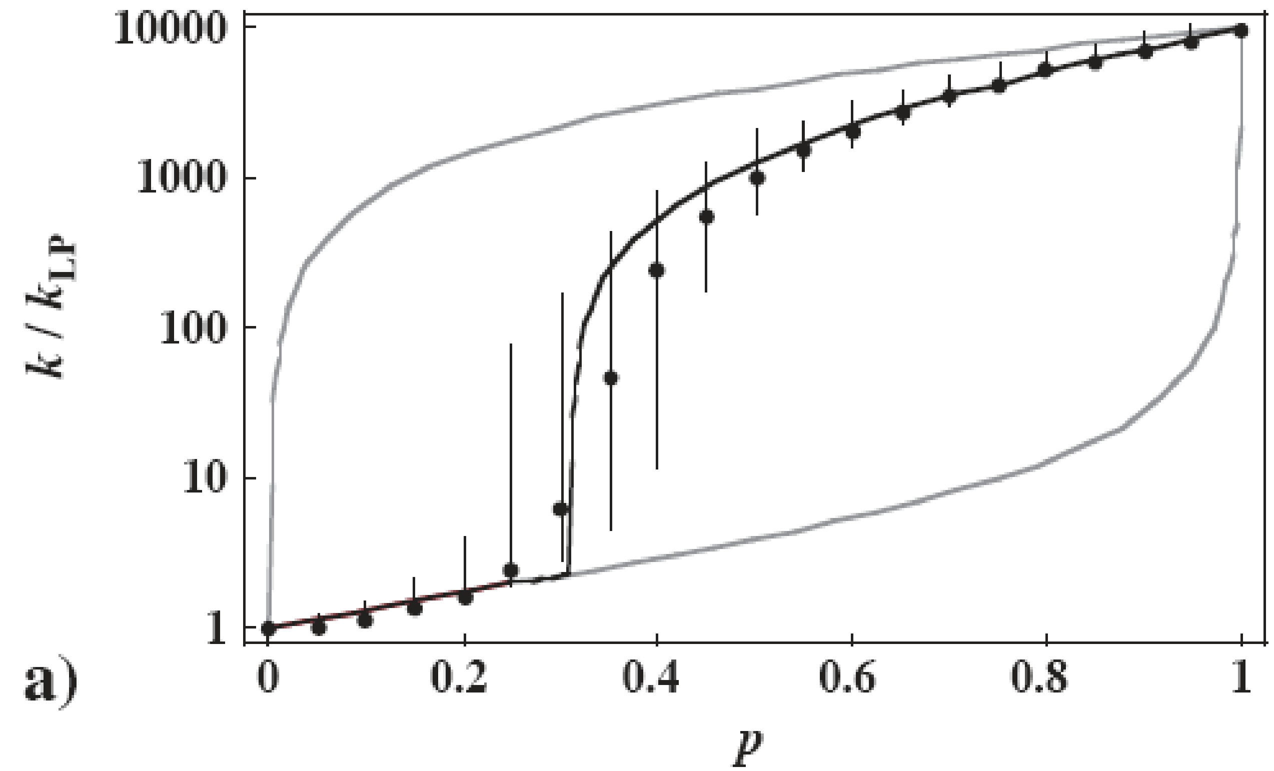
What does this model buy us? Some insight in

- the ***complexity*** of transitions
- the creation of a ***singularity*** for smooth varying ***composition***, e.g. when ***clay lenses connect ...***
- the ***morphology*** at transitions
- linking ***elastic*** and ***fluid*** properties remains a ***challenge*** ....

Fluid flow

Connectivity **of the** ***high conductive*** **phase**





measured

modeled

- **difficult to model**
- **difficult to measure**

[B&H '04]

NUMERICAL SIMULATIONS

Grid up to 50 X 50 X 50

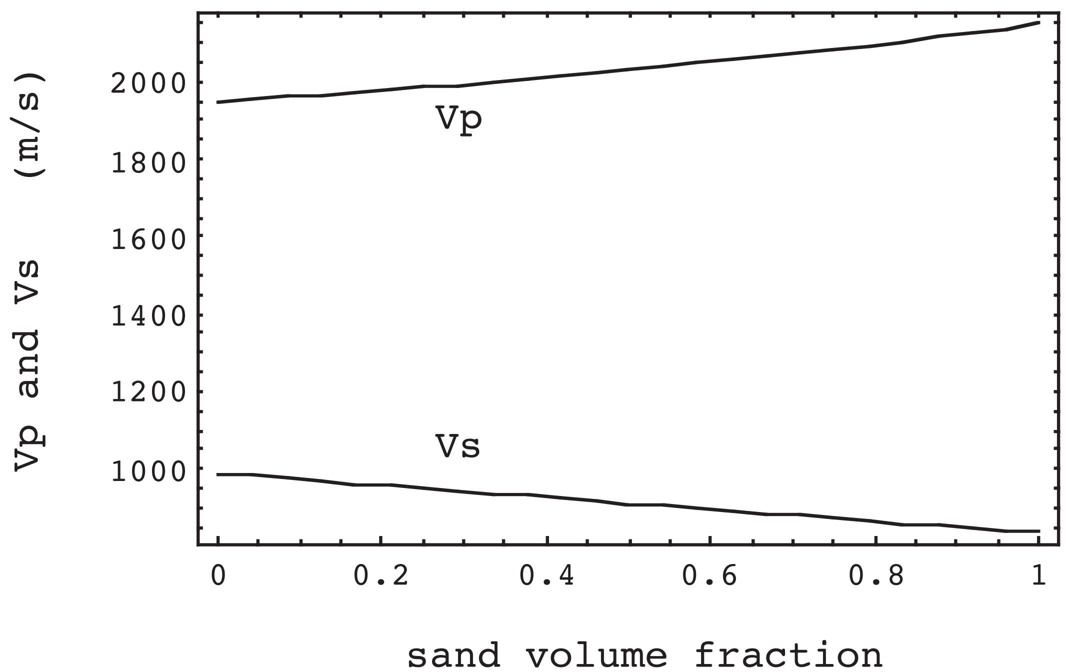
k_{HP}/k_{LP} from ~1 to 10^6

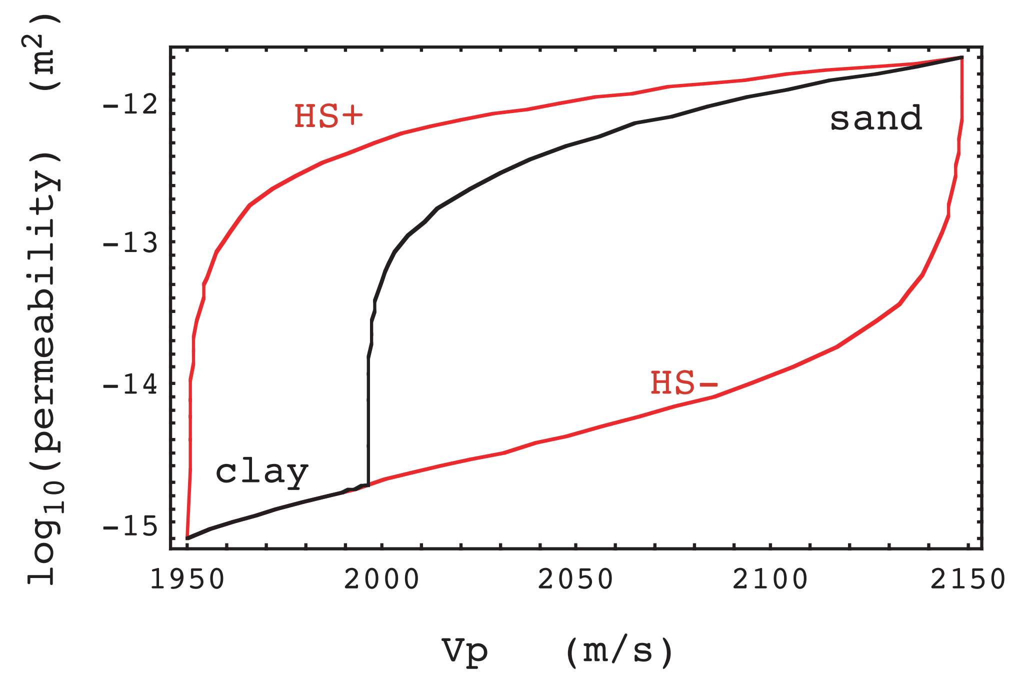
Steady-state flow equation:

$$\frac{\partial}{\partial x}\left(\frac{k}{\mu}\frac{\partial P_{\mathrm{p}}}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{k}{\mu}\frac{\partial P_{\mathrm{p}}}{\partial y}\right)+\frac{\partial}{\partial z}\left(\frac{k}{\mu}\frac{\partial P_{\mathrm{p}}}{\partial z}\right)=0$$

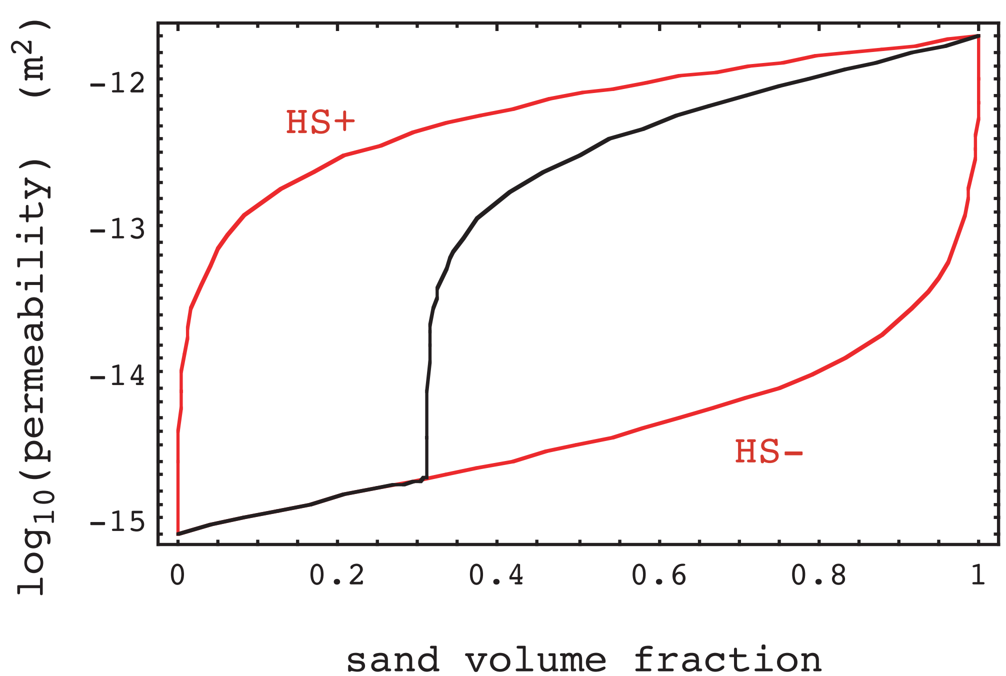
Sand-clay model

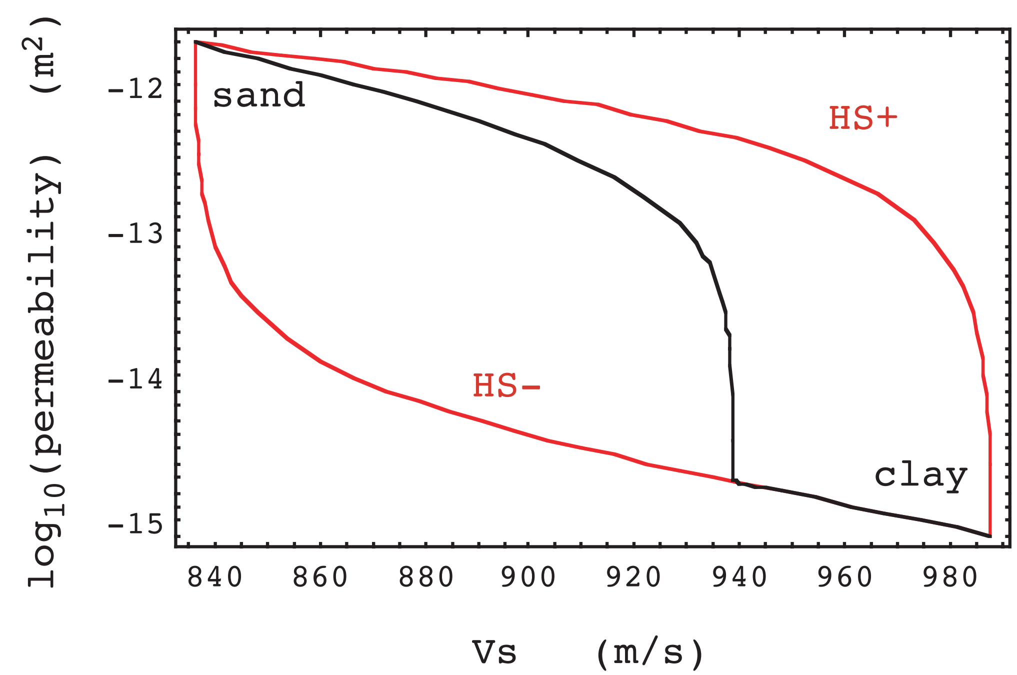
according HB model [HB '04]





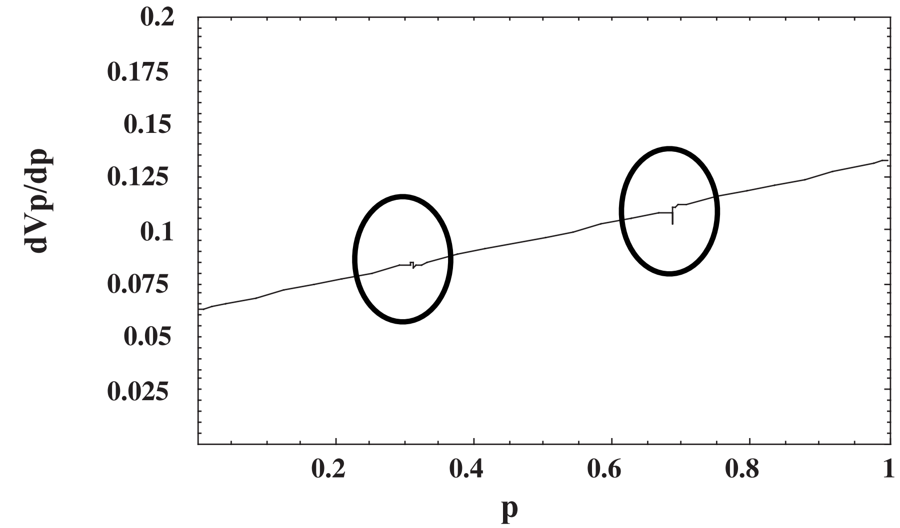
velocities <=> permeability

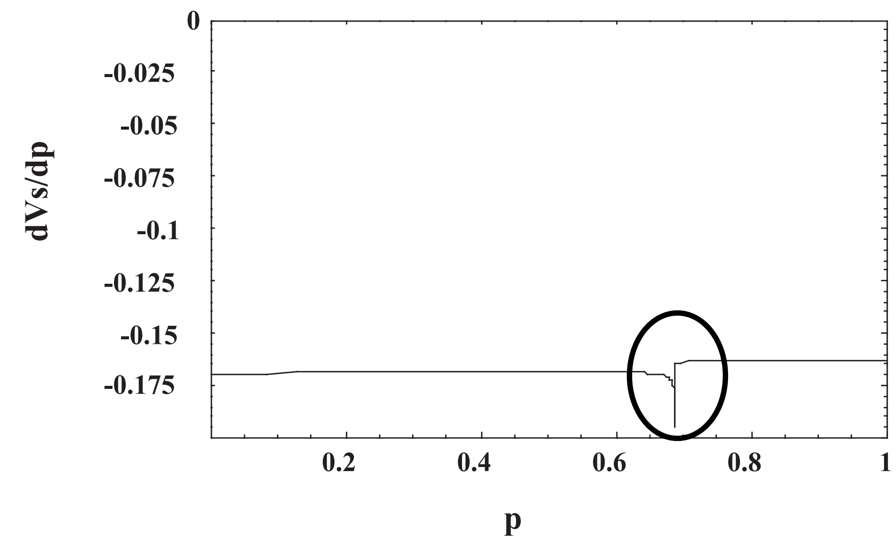


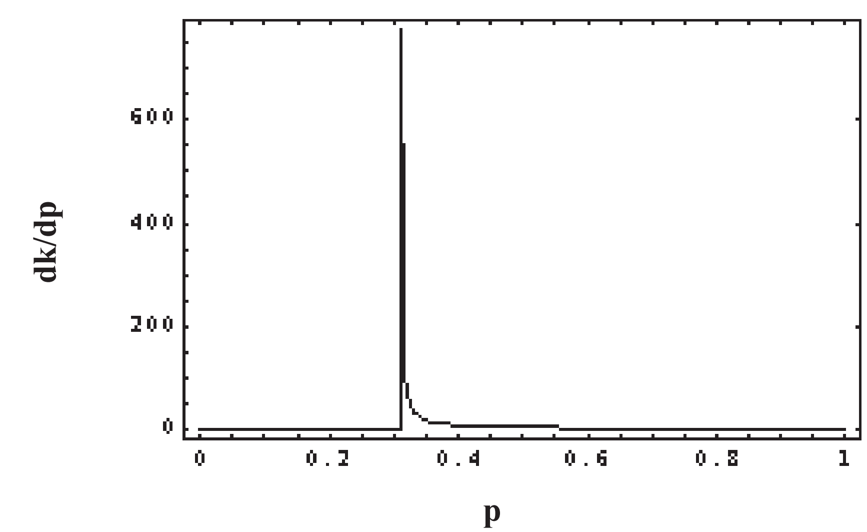


Elastic versus Fluid

- ***singularities*** in ***elastic*** properties are ***small***
- ***singularities*** in fluid properties are ***large***
- ***waves*** are way more ***sensitive*** to singularities then ***diffusion*** driven ***fluid*** flow
- models for fluid and elastic transport are not integrated
- incorporate in Biot?







Conclusions

- Multiscale compressible signal representations that exploit higher-dim. geometry are indispensable for acquiring accurate information on the imaged waveforms.
- Imaged waveforms carry information on the fine structure of the reflectors.
- Percolation model provides an interesting perspective
  - on linking the micro-connectivity to singularities detected by waves
  - on providing an upscaling that preserves features = singularities that matter ....

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