SLIM Seismic Laboratory for Imaging and Modeling

Resovling Scaling Ambiguities with the measure in a blind deconvolution problem with feedback

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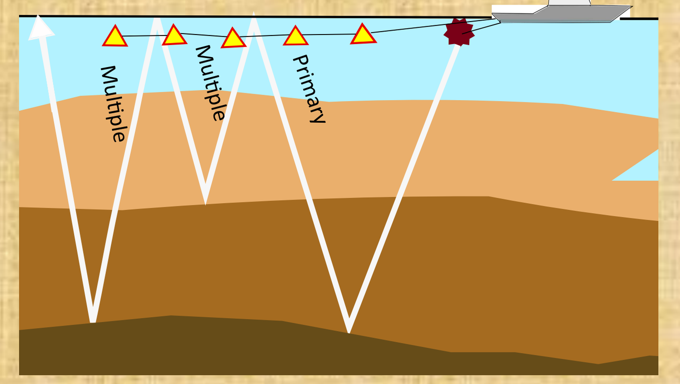
Motivation

In exploration geophysics, reflected seismology refers to methods that use principles of scattering to estimate the properties of the Earth's subsurface from reflected waves.



- Waves are generated by a man-made source towed by a ship.
- Data are collected at the surface and after some preprocessing it contains reflection events in the form of convolutions of the source wavelet Q and the subsurface Green's function G, i.e., Q*G, Q*G*G, etc.
- The sea surface is a perfect reflector (reflection coefficient is -1).
- The primary waves are sent back into the subsurface acting as a secondary source and generate echoes, called multiples.

The Surface- Related Multiple Elimination (SRME) [Verschuur, et al. 1992] formula gives the relationship between the data P, source Q , and Green's function:

$$\hat{P} = \hat{G}(\hat{Q} - \hat{P}) \qquad (1)$$

Challenges: to recover G and Q jointly from P under minimal assumptions.

Model

We consider the 1D version of (1)

$$\hat{y} = \hat{w} \otimes (\hat{x} + \hat{y}) \quad \otimes\text{: pointwise product}$$

with a time domain representation:

$$y = \underbrace{w * x}_{\text{primary}} - \underbrace{w * x * x}_{\text{first order multiple}} + \underbrace{w * x * x * x}_{\text{second order multiple}} \cdots$$
$$= w * x - y * x \qquad (2)$$

Assumptions:
- $x, \hat{x} \in L^1(R)$
- $w, \hat{w} \in L^1(R)$
- x is sparse in time.

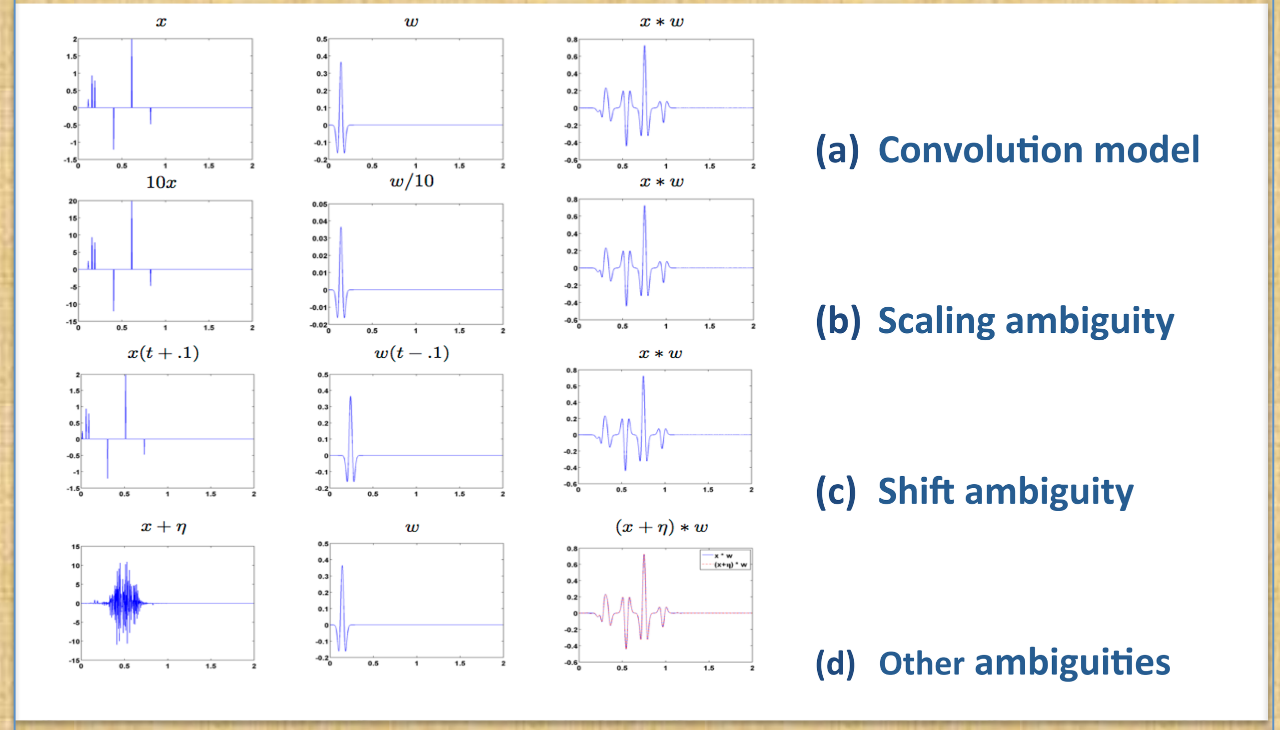
Goal: recover x and w jointly from y .

Previous method 1

By truncating the data record, the data of early times contains no multiples, therefore satisfies the simple convolution model,

$$y = w * x$$

However, this simpler model is ill-posed—has scaling and shift ambiguities, and regularization CANNOT avoid these ambiguities.



According to [Benichoux, Vincent and Gribonval 2013], the global minimizer of

$$\min_{x,w} \frac{\lambda}{2}\|y - x * w\|_2^2 + \|x\|_1 + \beta\|w\|$$

is trivial: $x \sim \delta$, for any *translation invariant seminorm* $\|\cdot\|$.

Previous method 2 (EPSI)

Estimation of Primaries by Sparse Inversion (EPSI, Groenestijn and Verschuur, 2009; Lin and Herrmann 2013):

- Alternatively update x and w
- Use L1 norm regularization when updating x

$$x_k = \arg\min_z \|y - w_k * z + y * z\|_2^2 + \lambda\|z\|_1$$

- Assume short duration of w

$$w, w = Ch, C = [I, 0]^T$$

- Solve a least-squares problem to update w

$$\widetilde{w}_k = C \arg\min_g \|y - (Cg) * x_k + y * x_k\|_2^2$$

- The success of the algorithm relies on good initial guesses as well as a manual rescaling of the wavelet

$$w_k = \alpha\widetilde{w}_k / \|\widetilde{w}_k\|_2$$

Theoretical analysis

Theorem 1 *Suppose $x, w, \hat{x}, \hat{w} \in L^1(\mathbb{R})$, and $\|x\|_1 < 1$ then the right-hand side of the following quantity exists and is integrable*

$$y := \sum_{i=1}^{\infty} (-1)^{i-1} w * x^{*i}. \qquad (3)$$

Moreover, there exist a sequence $\alpha_k \in (1, \infty)$ and a sequence of functions $x_k \in L^1(\mathbb{R})$ such that

- $\alpha_k \to 1^+$,
- $(\alpha_k w, x_k(\alpha_k))$ *is consistent with* y *(in the sense that* $(\alpha_k w, x_k(\alpha_k), y)$ *satisfies* (3),

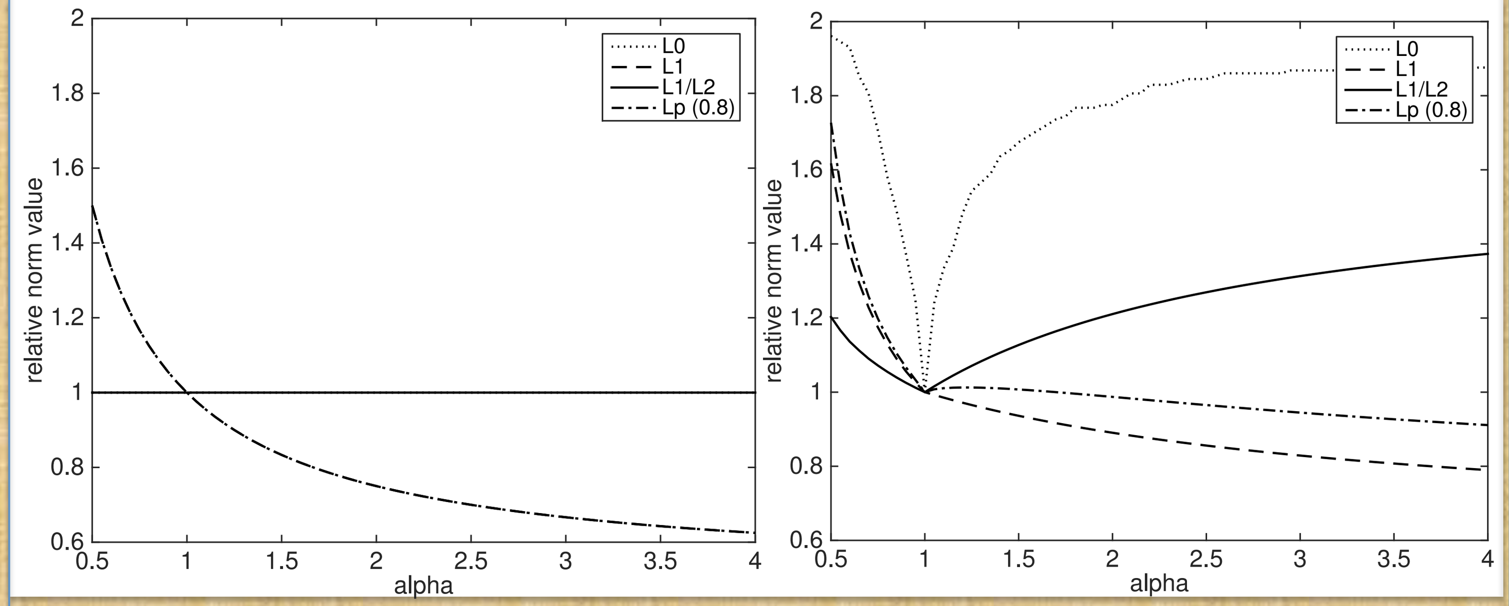
and $\|x_k\|_1 < \|x\|_1$ *for each* k*. In other words, the true solution* (w, x) *is not a local minimum to the optimization problem*

$$\min_{\widetilde{w},\widetilde{x}} \|\widetilde{x}\|_1 \text{ subject to } y = \sum_{i=1}^{\infty} (-1)^{i-1} \widetilde{w} * \widetilde{x}^{*i}.$$

Theorem 2 *Assume supp*$(x) \cap$ *supp*$(x * x) = \emptyset$*. Then there is* no *sequence* $\alpha_k \to 1$ *and* x_k*, such that*

- $(\alpha_k w, x_k(\alpha_k))$ *is consistent with* y*, and*
- $\|x_k\|_1 / \|x_k\|_2 < \|x\|_1 / \|x\|_2$.

Consequently, the scaling is locally correct when minimizing the ℓ_1/ℓ_2 *penalty.*



Values of different measures of signals $x(\alpha)$ as function of α, where $x(\alpha)$ is consistent with the data when convolved with a scaled kernel αw. For the simple convolution (3) in panel (a), all norms here are ineffective at indicating the unscaled kernel (where α=1). For the feedback-type convolution (2) shown in panel (b), the scale sensitive ℓ_1 and ℓ_p norms remain ineffective, but the scale invariant ℓ_0 and ℓ_1/ℓ_2 norms identify the unscaled kernel with clear minima.

Algorithm (method of multipliers with a lifting)

Original problem (non-convex, non differentiable)

$$\min_{x,w} \log(\|x\|_1 / \|x\|_2)$$

$$\text{subject to} \quad \|\hat{y} - \mathrm{diag}(\hat{w}\hat{x}^T) - \mathrm{diag}(\hat{y}\hat{x}^T)\|_2 \le \epsilon \quad \textbf{Data constraint}$$

$$\|x\|_\infty \le 1 \quad \textbf{Box constraint}$$

Split x into positive and negative parts (non-convex): $x = x_+ - x_-$

$$\min_{x,w} \log(\sum x_+ + \sum x_-)/(\|x_+ + x_-\|_2)$$

$$\text{subject to} \quad \|\hat{y} - \mathrm{diag}(\hat{w}\hat{x}^T) + \mathrm{diag}(\hat{y}\hat{x}^T)\|_2 \le \epsilon \quad \textbf{Data constraint}$$

$$0 \le x_+, x_-, \le 1 \quad \textbf{Box constraint}$$

$$\langle x_+, x_- \rangle = 0 \quad \textbf{Non-overlapping}$$

Lifting: increase the rank of w, x from 1 to r : $w \to R_1, x_+ \to R_2, x_- \to R_3$

$$\min_{R_1,\cdots,R_4} \mathrm{Trace}(X) - \|X\|_F + \log[\mathbf{1}^T(R_2R_2^T + R_3R_3^T + 2R_3R_2^T)\mathbf{1}] - \log[\mathrm{Trace}(R_2R_2^T + R_3R_3^T))$$

$$\text{subject to } \|\hat{y} - \mathrm{diag}(\hat{R}_1\hat{R}_2^T - \hat{R}_1\hat{R}_3^T) + \hat{y} \otimes (\hat{R}_2\hat{R}_4^T - \hat{R}_3\hat{R}_4^T))\|_2 \le \epsilon \quad \textbf{Data constraint}$$

$$\mathrm{Trace}(R_2R_3^T) = 0 \quad \textbf{Non overlapping}$$

$$R_4R_1^T(CC^T - I) = 0 \quad \textbf{Short kernel (optional)}$$

$$R_4R_2^T, R_4R_3^T \ge 0 \quad \textbf{Box constraint}$$

$$R_4R_4^T = 1 \quad \textbf{Weights summing up to 1}$$

$$X = [R_1^T \quad R_2^T \quad R_3^T \quad R_4^T]^T [R_1^T \quad R_2^T \quad R_3^T \quad R_4^T].$$

Results

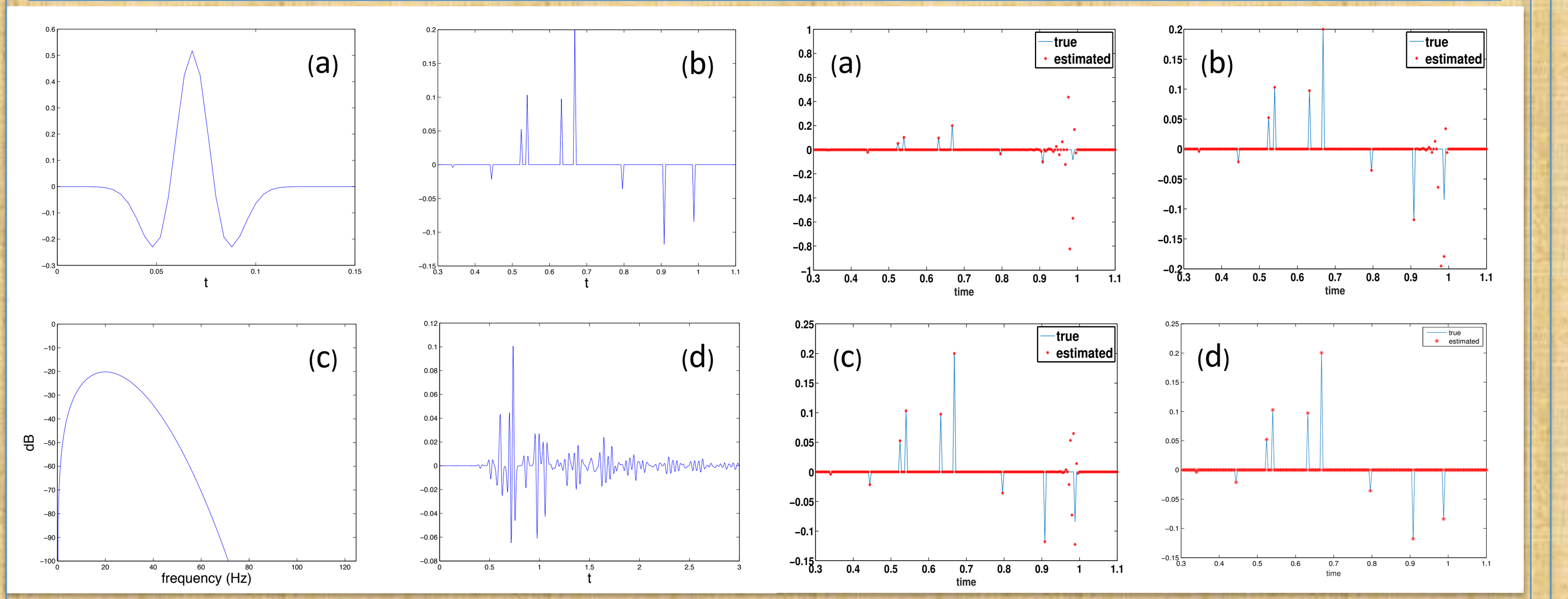
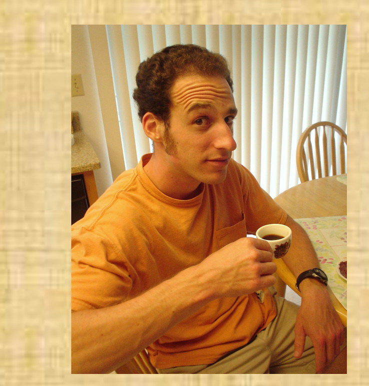


Figure 1: (a) True wavelet, (b) True signal, (c) Spectrum of the wavelet measured in dB, (d) Data record with feedback.

Figure 2: Reconstruction of x using (4) using various ranks in the lifting. (a) r=1, (b) r=2, (c) r=3, (d) r=4.



† John "Ernie" Esser
(May 19, 1980 – March 8, 2015)

This work is a reflection of Ernie's extraordinary contributions to this challenging problem. Unfortunately, Ernie was not able to see the final results of his original work. We miss him dearly, and will continue to work on this exciting approach.

Acknowledgement

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