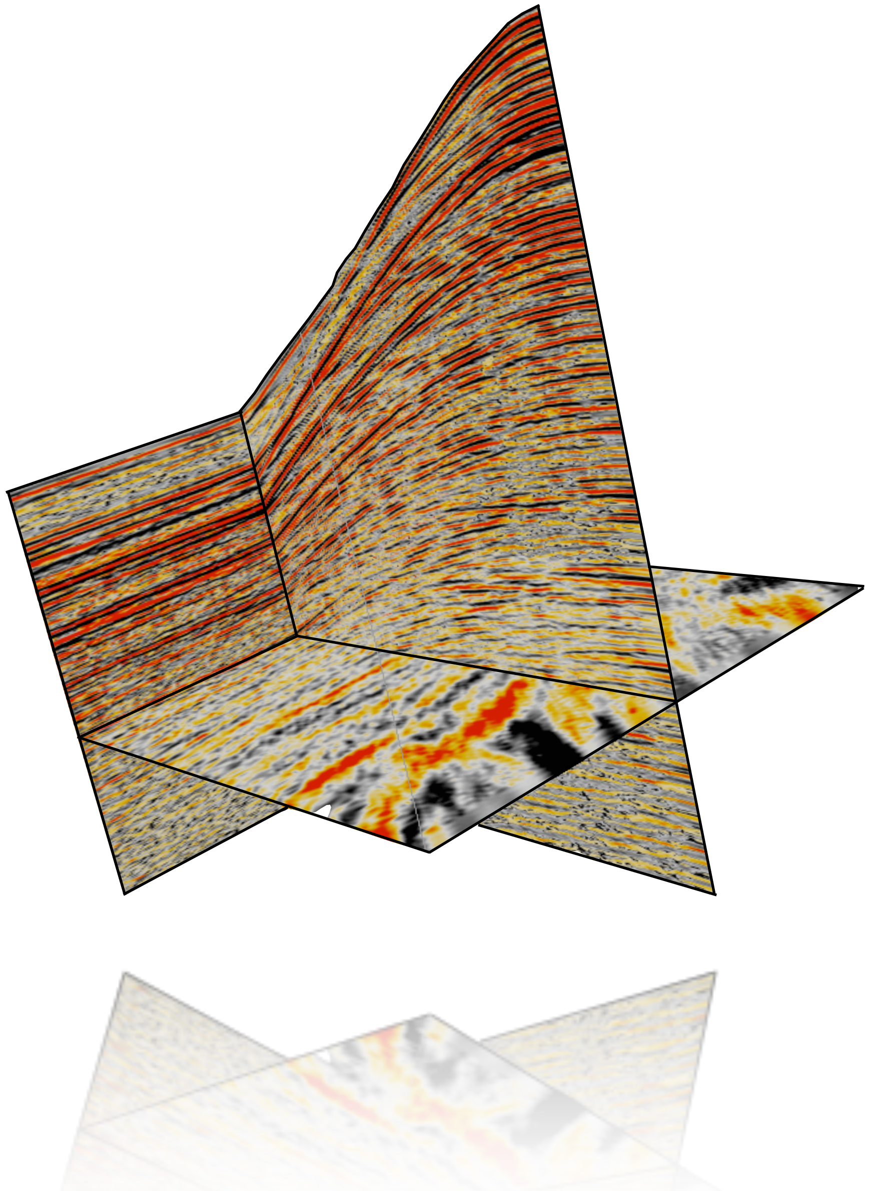
Compressive seismic imaging with simultaneous acquisition

Felix J. Herrmann*
fherrmann@eos.ubc.ca

Joint work with Yogi Erlangga, and Tim Lin

***Seismic Laboratory for Imaging & Modeling**
Department of Earth & Ocean Sciences
The University of British Columbia

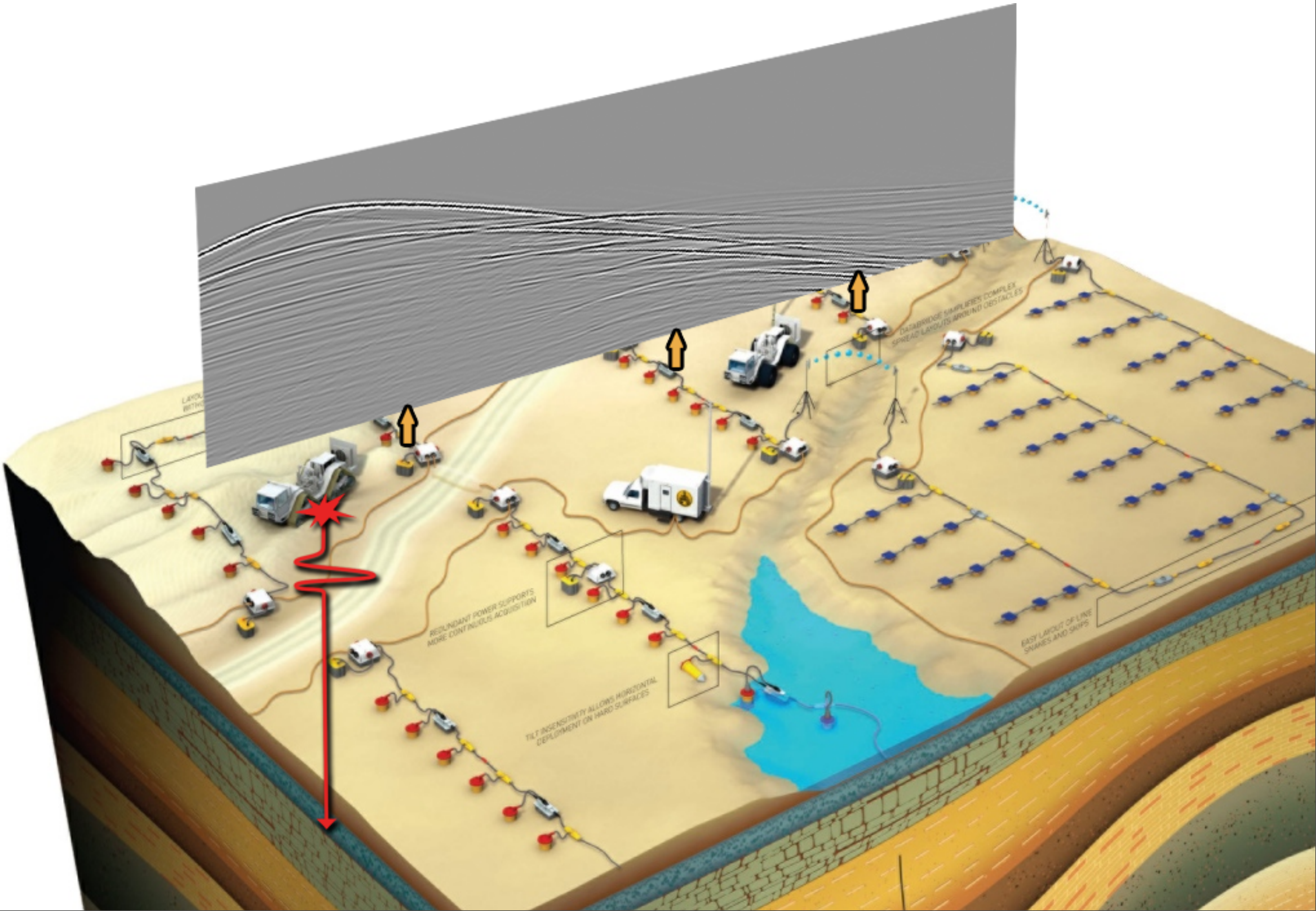


Vienna July 20, 2009

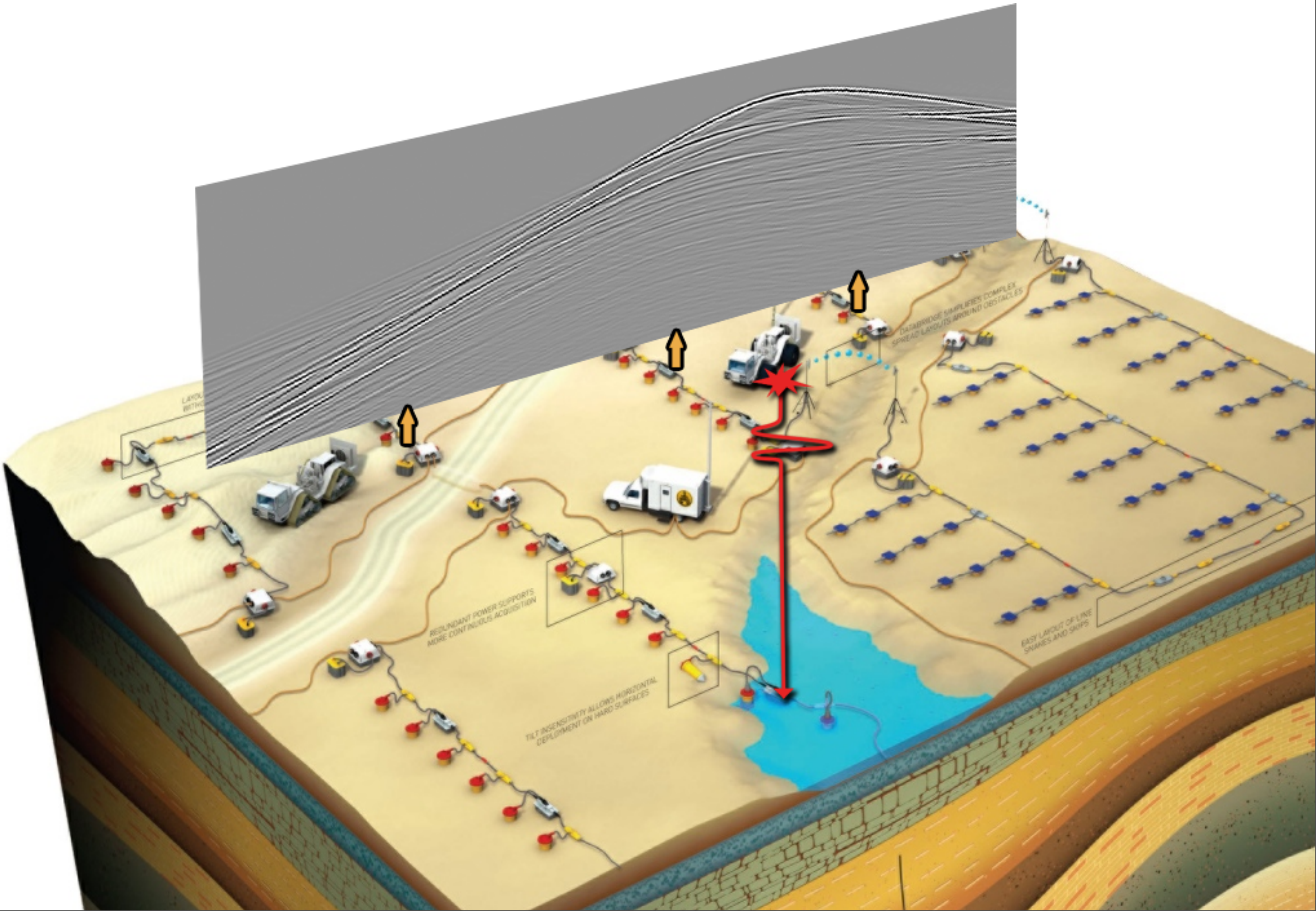
image courtesy ION (www.iongeo.com)

Seismic acquisition

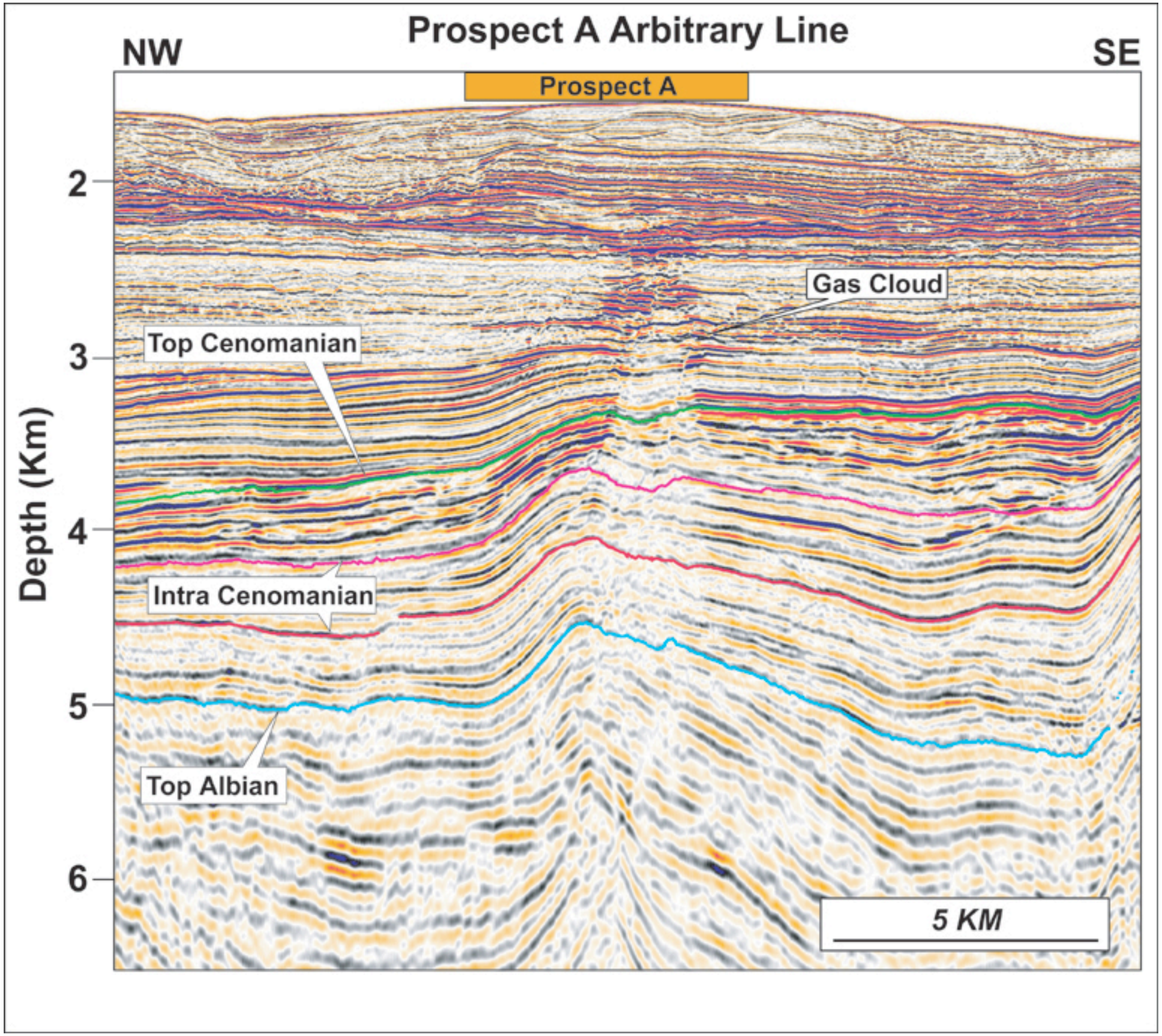
Individual shots



Individual shots



After imaging



Observations

- Seismic imaging methods are mostly based on ***linearizations***
- Seismic imaging methods are despite the *spectral* ***gap*** able to
  - locate major singularities
  - assign some sense of reflection strength
- Seismic images
  - are derived from ***multiexperiment*** data (petabytes) <=> ***redundancy***
  - permit sparse representation by multiscale & multidirection transforms that capture the "wavefront set" of the subsurface reflectors (e.g. curvelets)
- Seismic images do ***not capture the whole picture!***
- ***There is a push for full waveform inversion ...***

Seismic imaging & inversion

Multiexperiment **PDE-constrained optimization problem:**

$$\min_{\mathbf{U}\in\boldsymbol{\mathcal{U}},\,\mathbf{m}\in\boldsymbol{\mathcal{M}}} \frac{1}{2}\|\mathbf{P}-\mathbf{DU}\|_2^2 \quad \text{subject to} \quad \mathbf{H}[\mathbf{m}]\mathbf{U}=\mathbf{Q}$$

+ Free surface BC

$\mathbf{P}$ = Total multi-source and multi-frequency data volume

$\mathbf{D}$ = Detection operator

$\mathbf{U}$ = Solution of the Helmholtz equation

$\mathbf{H}$ = Discretized multi-frequency Helmholtz system

$\mathbf{Q}$ = Unknown seismic sources

$\mathbf{m}$ = Unknown model, e.g. $c^{-2}(x)$

Wavefield simulations

Based on discretization of the Helmholtz equation:

$$\mathcal{H}u = -\Delta u - \omega^2 m u = q$$

$$\begin{bmatrix} \boldsymbol{\mathcal{H}}_{\omega_1} & 0 & & \\ 0 & \boldsymbol{\mathcal{H}}_{\omega_2} & \ddots & \\ & \ddots & \ddots & 0 \\ & & 0 & \boldsymbol{\mathcal{H}}_{\omega_{n_f}} \end{bmatrix} \begin{bmatrix} \overbrace{[\mathbf{u}_1 \ \mathbf{u}_2 \ \cdots \ \mathbf{u}_{n_s}]_{\omega_1}}^{\mathbf{U}_{\omega_1}} \\ \vdots \\ \vdots \\ \underbrace{[\mathbf{u}_1 \ \mathbf{u}_2 \ \cdots \ \mathbf{u}_{n_s}]_{\omega_{n_f}}}_{\mathbf{U}_{n_f}} \end{bmatrix} = \begin{bmatrix} \overbrace{[\mathbf{q}_1 \ \mathbf{q}_2 \ \cdots \ \mathbf{q}_{n_s}]_{\omega_1}}^{\mathbf{Q}_{\omega_1}} \\ \vdots \\ \vdots \\ \underbrace{[\mathbf{b}_1 \ \mathbf{b}_2 \ \cdots \ \mathbf{b}_{n_s}]_{\omega_{n_f}}}_{\mathbf{Q}_{n_f}} \end{bmatrix}$$

$$\boldsymbol{\mathcal{H}}_{\omega_j} := \boldsymbol{\mathcal{H}}(\omega_j), \quad \omega_j = 2\pi j \Delta f, \quad j = 1, \ldots, n_f$$

Δf frequency sample interval

Adjoint state methods [Plessix '06 & many others]

For each ***separate*** source **q** solve the **unconstrained problem:**

$$\min_{\mathbf{m}\in\mathcal{M}} \frac{1}{2}\|\mathbf{p}-\mathcal{F}[\mathbf{m}]\|_2^2 \qquad \text{with} \qquad \mathcal{F}[\mathbf{m},\mathbf{q}] = \mathbf{D}\mathbf{H}^{-1}[\mathbf{m}]\mathbf{q}$$

where ***model updates*** <=> ***migrated image***

$$\delta\mathbf{m} = \Re\left(\sum_{\omega}\omega^2\sum_{s}\bar{\mathbf{u}}\odot\mathbf{v}\right) = \mathbf{K}^*[\mathbf{m},\mathbf{Q}]\delta\mathbf{d}$$

$$\text{with } \boldsymbol{\delta}\mathbf{d} = \mathrm{vec}(\mathbf{P}-\boldsymbol{\mathcal{F}}[\mathbf{m},\mathbf{Q}])$$

involve single ***implicit*** solves of Helmholtz system

$$\mathbf{H}[\mathbf{m}]\mathbf{u} = \mathbf{q} \quad \text{and} \quad \mathbf{H}^*[\mathbf{m}]\mathbf{v} = \mathbf{r}$$

with

$$\mathbf{r} = \mathbf{D}^H(\mathbf{p}-\mathcal{F}[\mathbf{m}])$$

Challenges: there are many ...

Helmholtz system is ***indefinite*** & ***ill conditioned*** => lack of convergence ***indirect*** Krylov solvers

Multiexperiment setup with ***multiple*** *right-hand-sides* is *computationally* **prohibitive** as part of ***iterative*** Newton methods

Inversion problem can be both ***over-*** and ***underdetermined*** [Symes, ‘09]

- data cannot be explained fully
- there are local minima
- many velocity models may explain data within some error

Proposed ideas to tackle *multimodality* by ***extensions & focusing*** make the situation worse by additional ***degrees of freedom***

Indirect solver

Preconditioner [Erlangga, Oosterlee, Vuik, 2006]

$$\mathcal{M} \triangleq \left(-\Delta - (1-\beta\hat{i})\omega^2 m\right)_h, \quad \beta = (0,1]$$

Deflation operator [Erlangga, Nabben, ‘08, FJH, Erlangga, ‘08]

$$\mathcal{Q} := \mathbf{I} - \mathbf{Z}\mathbf{E}^{-1}\mathbf{Y}^\top\mathcal{H}\mathcal{M}^{-1} - \mathbf{Z}\mathbf{E}^{-1}\mathbf{Y}^\top$$

with: $\mathbf{E} = \mathbf{Y}^\top\mathcal{H}\mathcal{M}^{-1}\mathbf{Z}$

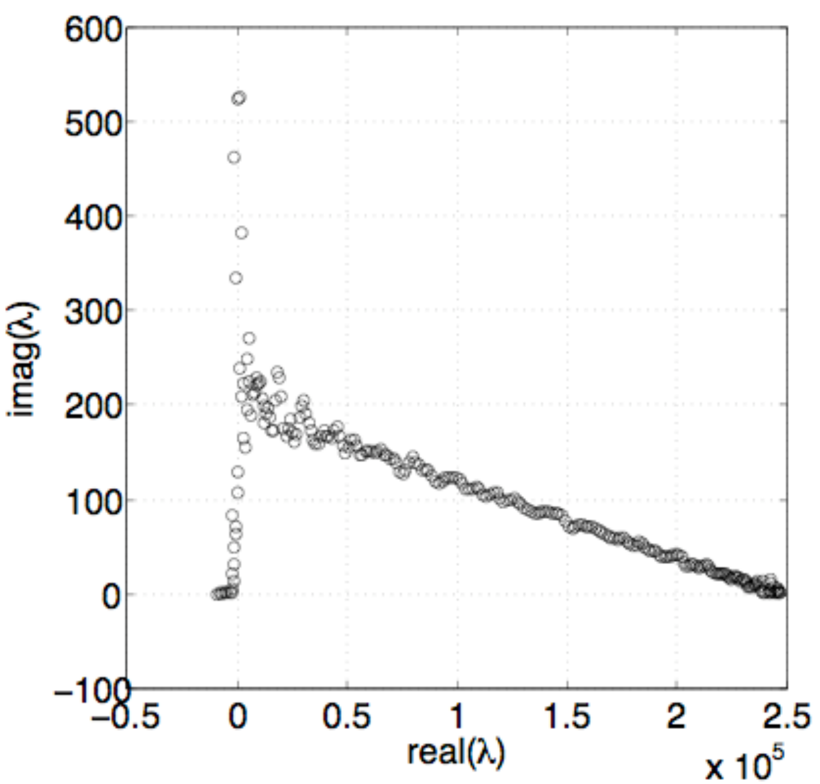
$\mathbf{Z}, \mathbf{Y}$ multigrid-type interpolation matrices

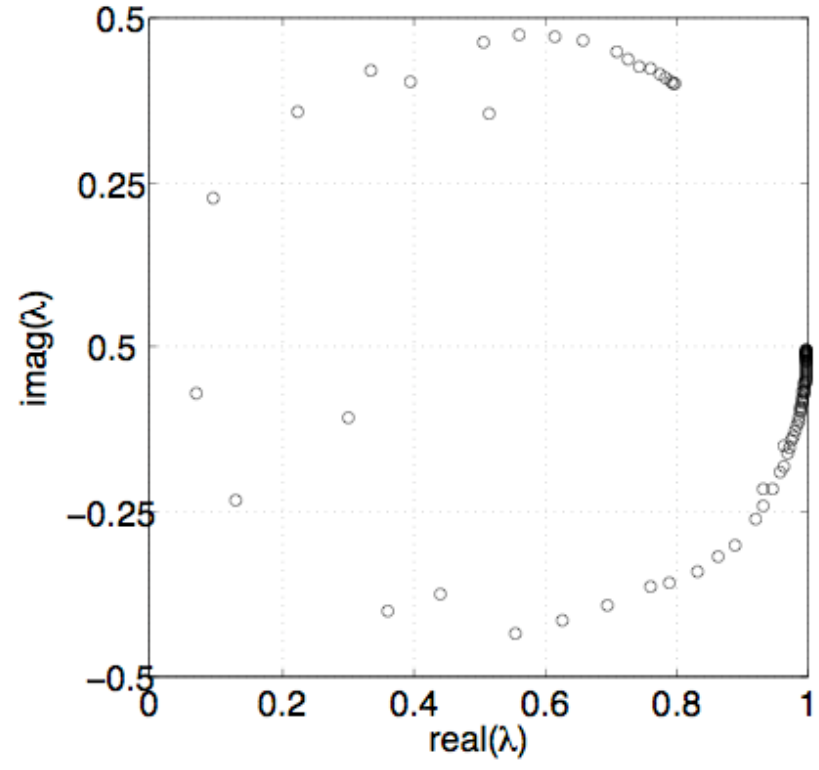
Similar computational complexity as TDFD ...

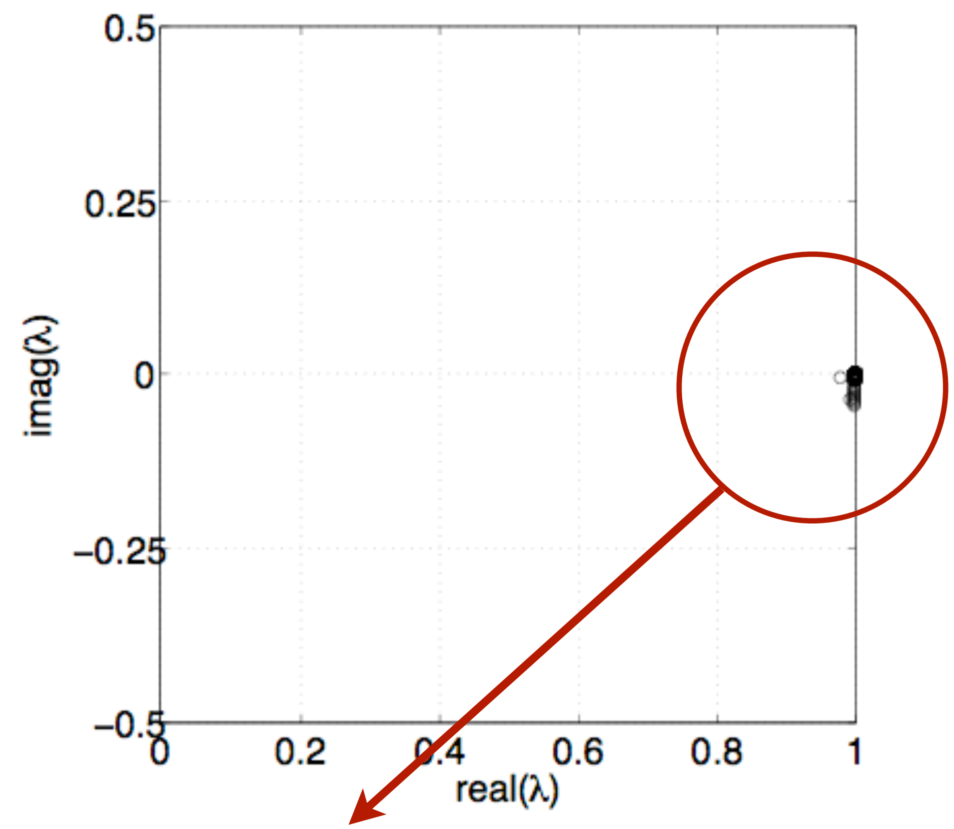
Behavior eigenvalues

1D non-constant wavenumber k, hard model $k = (50, 100)$

$\mathbf{H}$ $\mathbf{HM^{-1}}$ $\mathbf{HM^{-1}Q}$

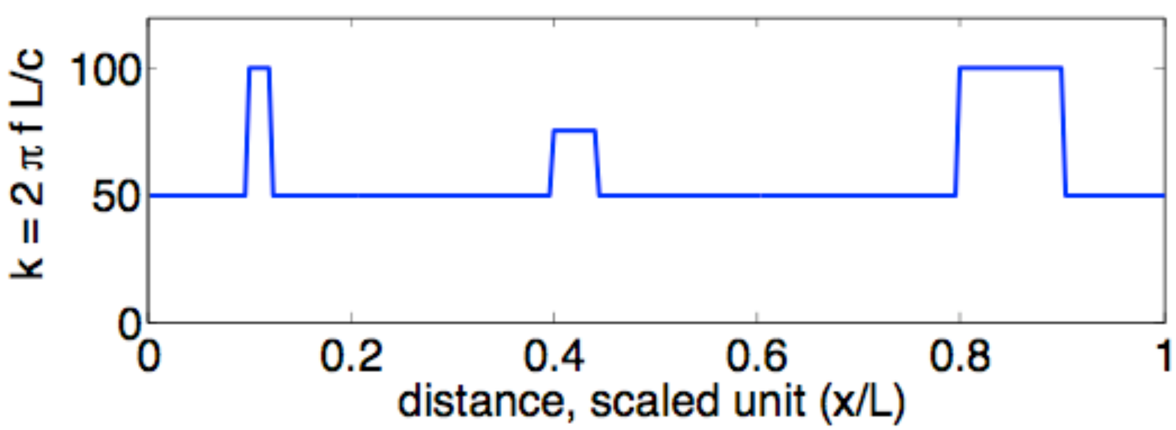






Clustering around one

For constant, smooth, or hard model, one can expect the same convergence rate



Challenges: there are many ...

✓ Helmholtz system is ***indefinite*** & ***ill conditioned*** => lack of convergence ***indirect*** Krylov solvers

Multiexperiment setup with ***multiple*** *right-hand-sides* is *computationally* **prohibitive**

Inversion problem can be both ***over-*** and ***underdetermined***

- data cannot be explained fully
- there are local minima
- many velocity models may explain data within some error

Proposed plans to tackle *multimodality* by ***extensions & focusing*** make the situation worse by additional ***degrees of freedom***

System-size reduction

- Apply CS to reduce cost of ***wavefield*** simulation with Helmholtz
  - use ***simultaneous sources*** instead of ***separated sources***
  - leverage transform-domain sparsity & randomized subsampling by **one-norm sparsity promotion**
  - reduce size Helmholtz system
    - sources (number of right-hand sides)
    - angular frequencies (number of blocks)

- Apply CS to reduce cost of computing ***image volumes*** by multi-dimensional correlations via ***explicit*** matrix-matrix multiplies
  - randomize and subsample wavefields in **model space**
  - leverage transform-domain sparsity and focusing in the ***model space*** by ***joint sparsity promotion with mixed (1,2) norms***
  - reduce costs of storage and explicit matrix-matrix multiplies
    - sources (right-hand sides), receivers, depth
    - angular frequencies (blocks)

Relation to existing work

- **Simultaneous & continuous acquisition:**
  - *Efficient Seismic Forward Modeling using Simultaneous Random Sources and Sparsity* by N. Neelamani and C. Krohn and J. Krebs and M. Deffenbaugh and J. Romberg, ‘08
- **Simultaneous simulations & migration:**
  - *Faster shot-record depth migrations using phase encoding* by Morton & Ober, ’98.
  - *Phase encoding of shot records in prestack migration* by Romero et. al., ’00.
- **Imaging:**
  - *How to choose a subset of frequencies in frequency-domain finite-difference migration* by Mulder & Plessix, ’04.
  - *Efficient waveform inversion and imaging: A strategy for selecting temporal frequencies* by Sirque and Pratt, ’04.
- **Full-waveform inversion:**
  - *3D prestack plane-wave, full-waveform inversion* by Vigh and Starr, ‘08
- **Wavefield extrapolation:**
  - *Compressed wavefield extrapolation* by T. Lin and F.J.H, ’07
  - *Compressive wave computations* by L. Demanet (SIA ’08 MS79 & Preprint)

Tools

Compressive sensing based on Johnson-Lindenstrauss embeddings

- *Compressive sensing* [Donoho, 06‘, Candes, Romberg, Tao, ‘06]

$$\mathbf{b} = \mathbf{RMx} \quad \textbf{[randomized subsampling]}$$

$$\tilde{\mathbf{x}} = \arg\min_{\mathbf{x}} \|\mathbf{x}\|_1 \quad \text{subject to} \quad \|\mathbf{RMx} - \mathbf{b}\|_2 \leq \sigma$$

$$\tilde{\mathbf{x}} \approx \mathbf{x}$$

Fast matrix computations based on Johnson-Lindenstrauss embeddings

- Improved Approximation Algorithms for Large Matrices via Random Projections by Tamás Sarlós, ’08

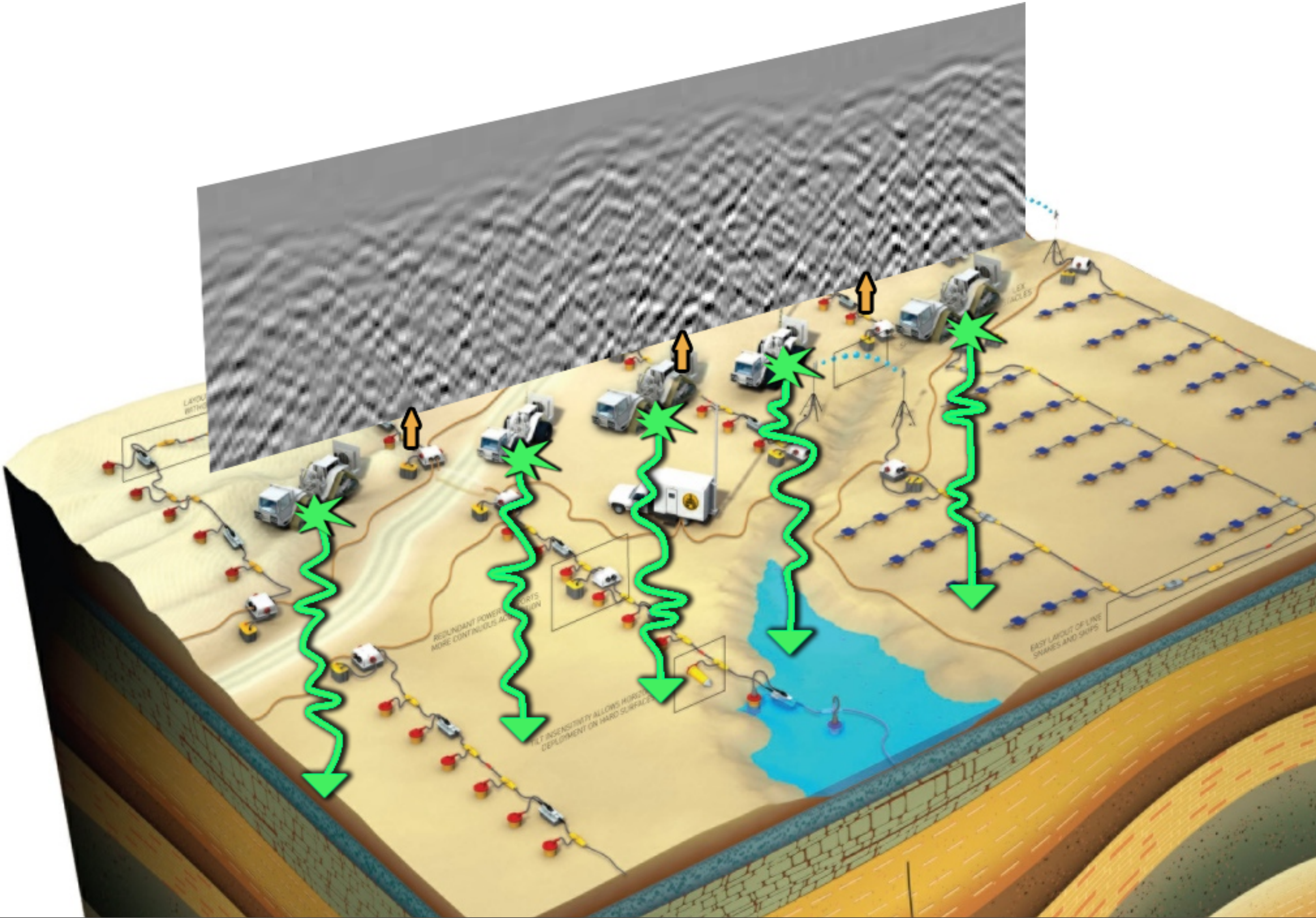
$$\mathbf{AB} \approx \mathbf{A}\left(\mathbf{RM}\right)^{*}\left(\mathbf{RM}\right)\mathbf{B}$$

Joint sparsity-promotion with mixed (1,2) norm minimization

- *Joint-sparse recovery from multiple measurements* by E. van den Berg and M. Friedlander, ‘09

$$\tilde{\mathbf{X}} = \arg\min_{\mathbf{X}} \|\mathbf{X}\|_{1,2} \quad \text{subject to} \quad \|\mathbf{AX} - \mathbf{B}\|_{2,2} \leq \sigma,$$

Simultaneous & continuous sources



System-size reduction [FJH, Lin, and Erlangga, '09]

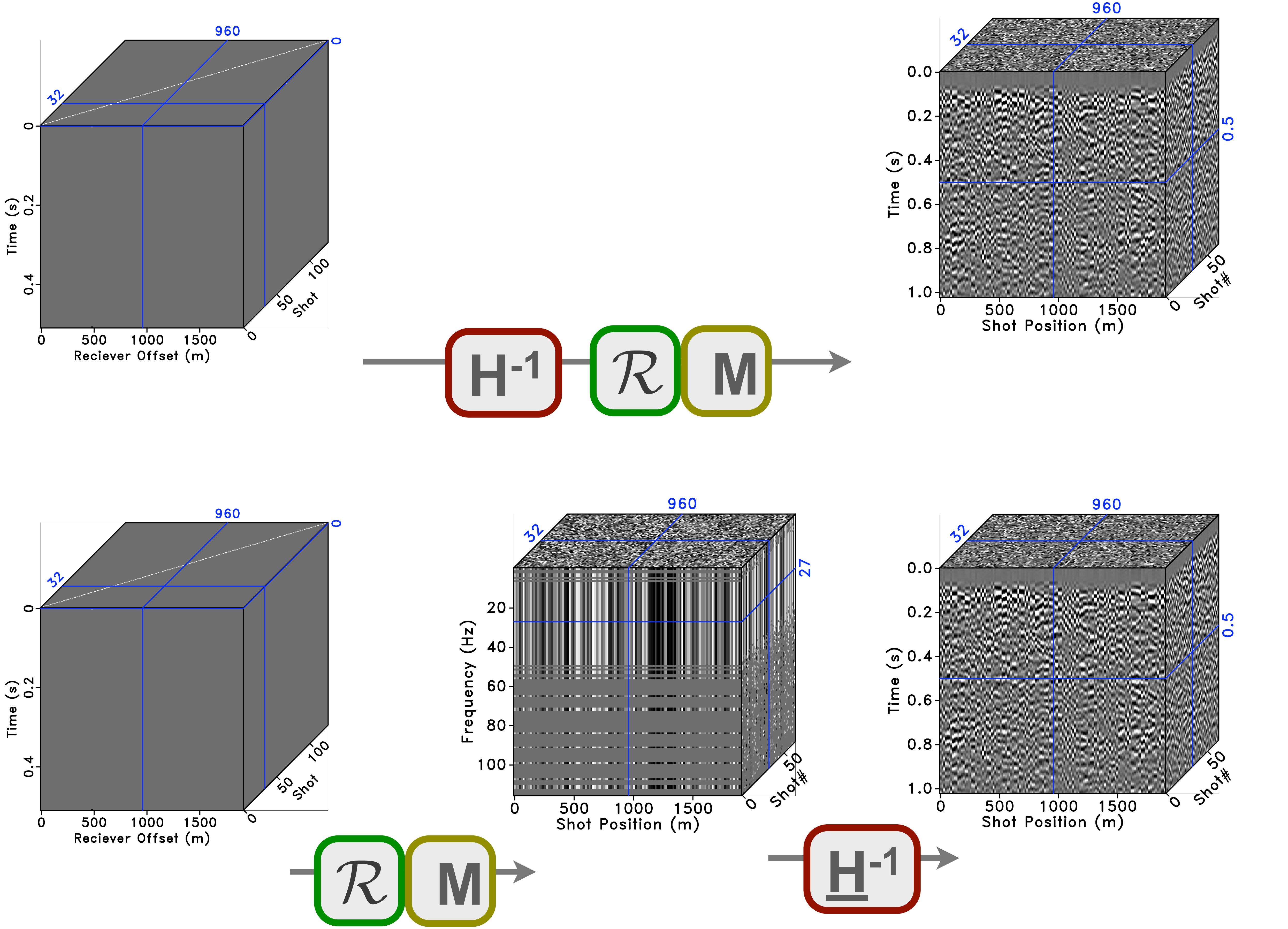
Subsample along ***source*** and ***frequency*** coordinates

Use ***fast*** transform-based sampling algorithms such as ***scrambled Fourier*** [Romberg, '08] or ***Hadamard*** ensembles [Gan et. al., '08]

$$\mathbf{RM} = \overbrace{\begin{bmatrix} \mathbf{R}_1^{\Sigma} \otimes \mathbf{I} \otimes \mathbf{R}_1^{\Omega} \\ \vdots \\ \mathbf{R}_{n_{s'}}^{\Sigma} \otimes \mathbf{I} \otimes \mathbf{R}_{n_{s'}}^{\Omega} \end{bmatrix}}^{\text{sub sampler}} \overbrace{\left(\mathbf{F}_2^* \operatorname{diag}\left(e^{\hat{i}\boldsymbol{\theta}}\right) \otimes \mathbf{I}\right) \mathbf{F}_3}^{\text{random phase encoder}},$$

$$\theta_w = \text{Uniform}([0, 2\pi])$$

- Different random restriction for each $n_s' \ll n_s$ simultaneous experiments
- Restriction reduces system size



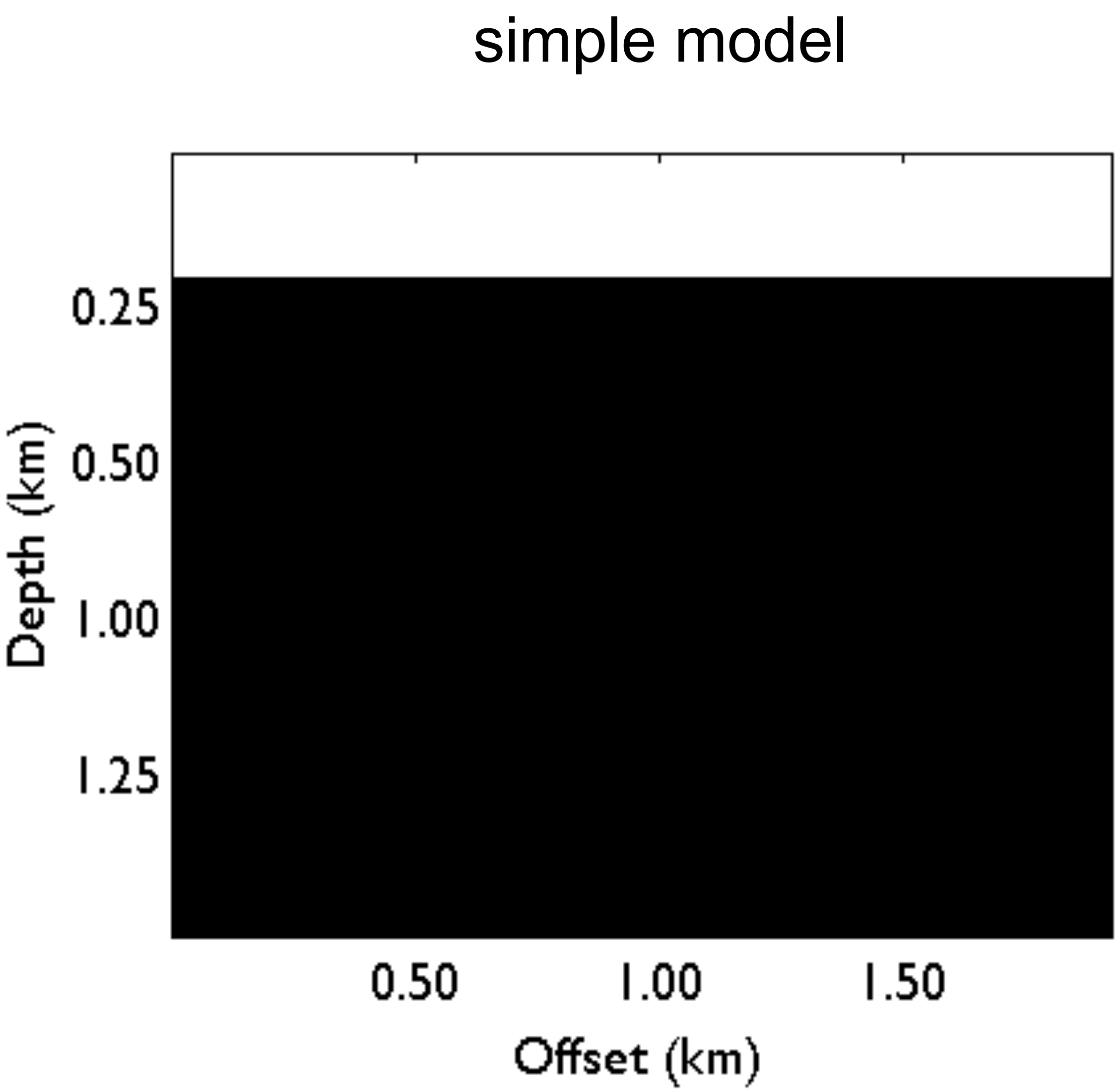
960
32
0
Time (s)
0.2
0.4
0
500
1000
1500
Reciever Offset (m)
50
100
Shot
H-1
R
M
960
32
0.0
0.2
0.4
0.6
0.8
1.0
Time (s)
0.5
0
500
1000
1500
Shot Position (m)
50
Shot#
960
32
27
20
40
60
80
100
Frequency (Hz)
0
500
1000
1500
Shot Position (m)
50
Shot#
H-1

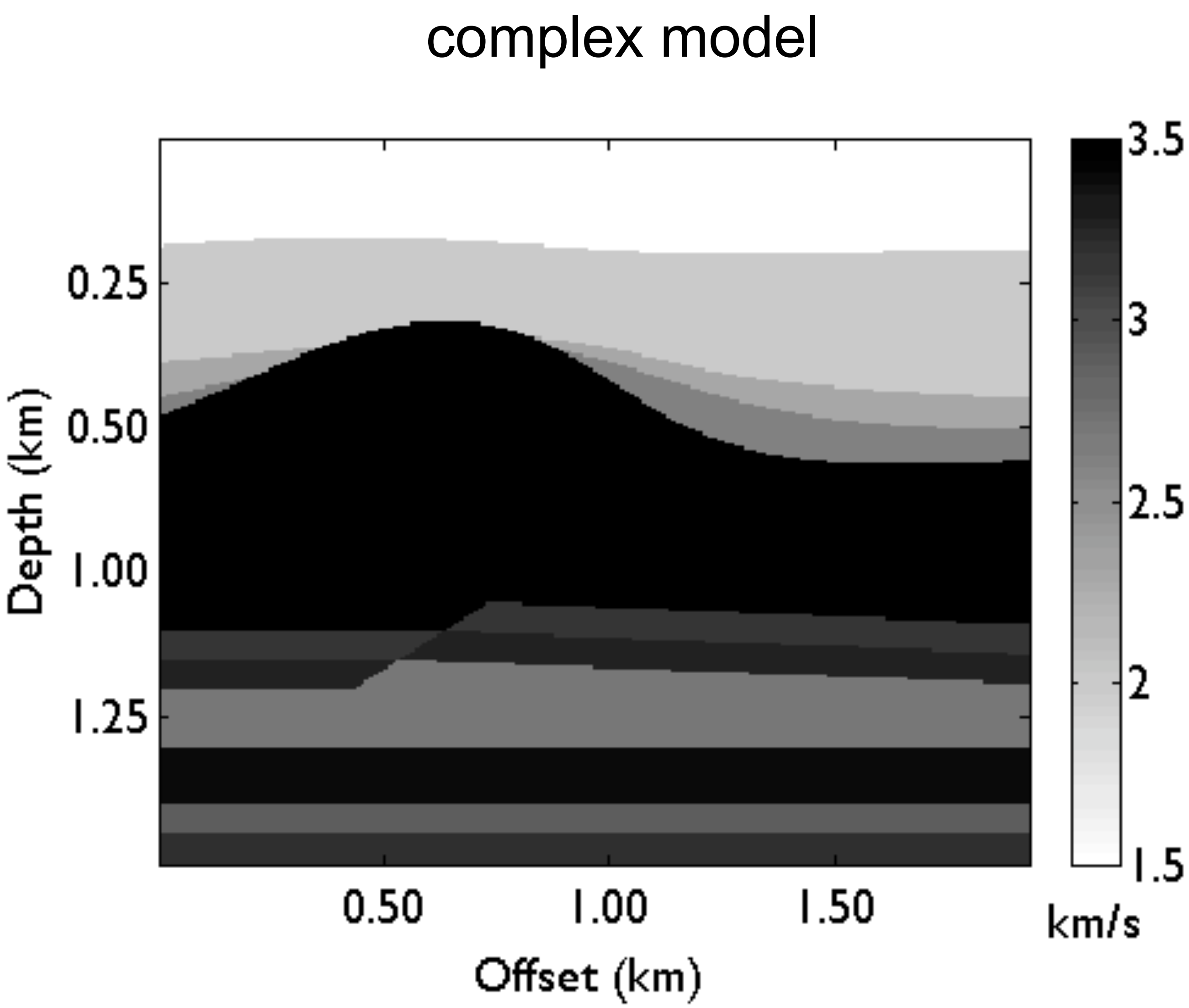
Sparsifying transform [Demanet '06]

- Use fast discrete 2-D Curvelet transform based on wrapping along shot and receiver coordinates
  - compresses highly geometrical features of monochromatic wavefields
  - incoherent with compressive-sampling matrix that acts along the source coordinate
- Use fast discrete wavelet transform along the time coordinate
  - compresses front-like features arriving along the time direction
  - reasonable incoherent with sampling of angular frequencies
- Combine both transforms through a **Kronecker** product

$$\mathbf{S} = \mathbf{C}_{2d} \otimes \mathbf{W}$$

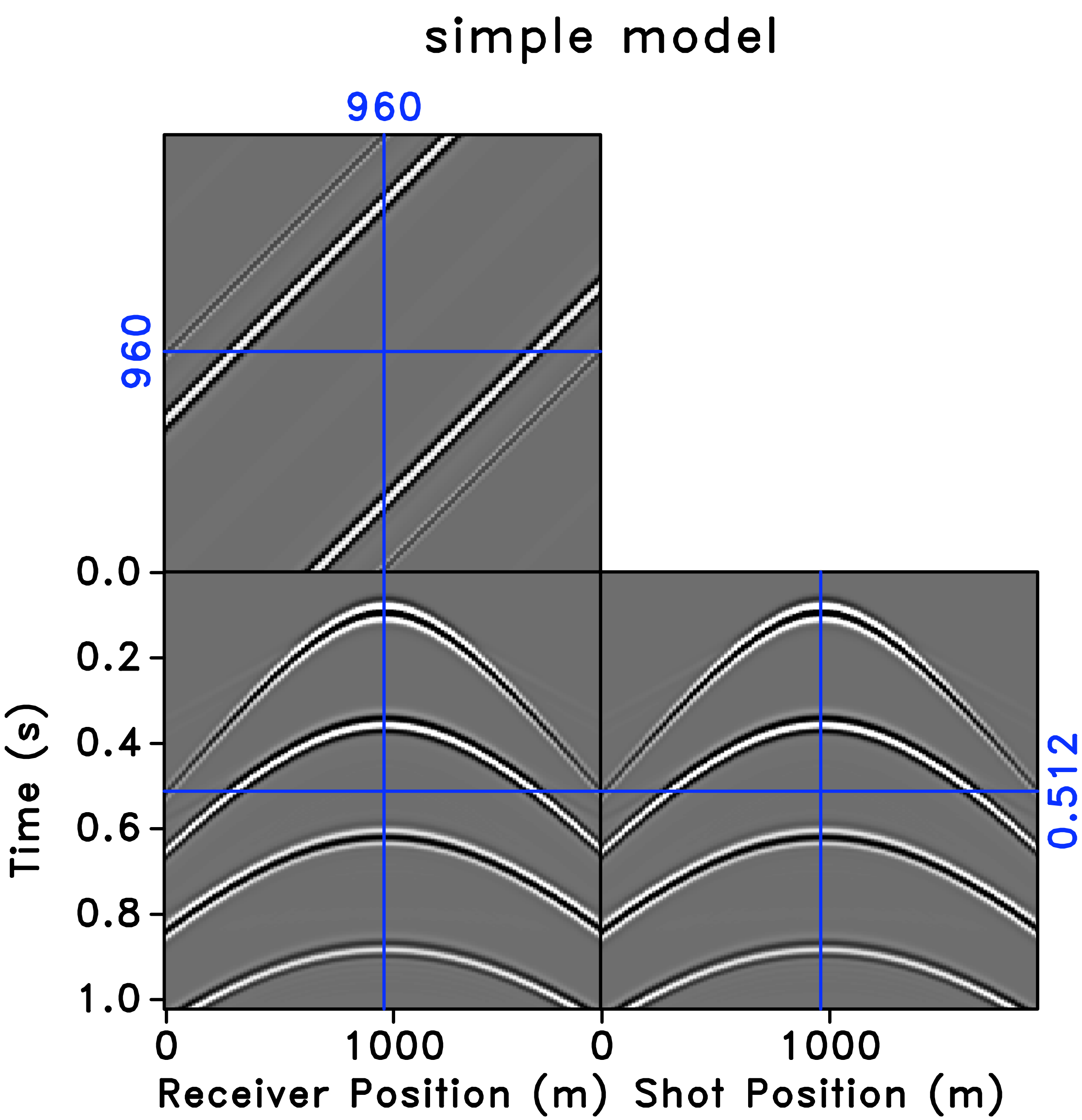
Velocity models

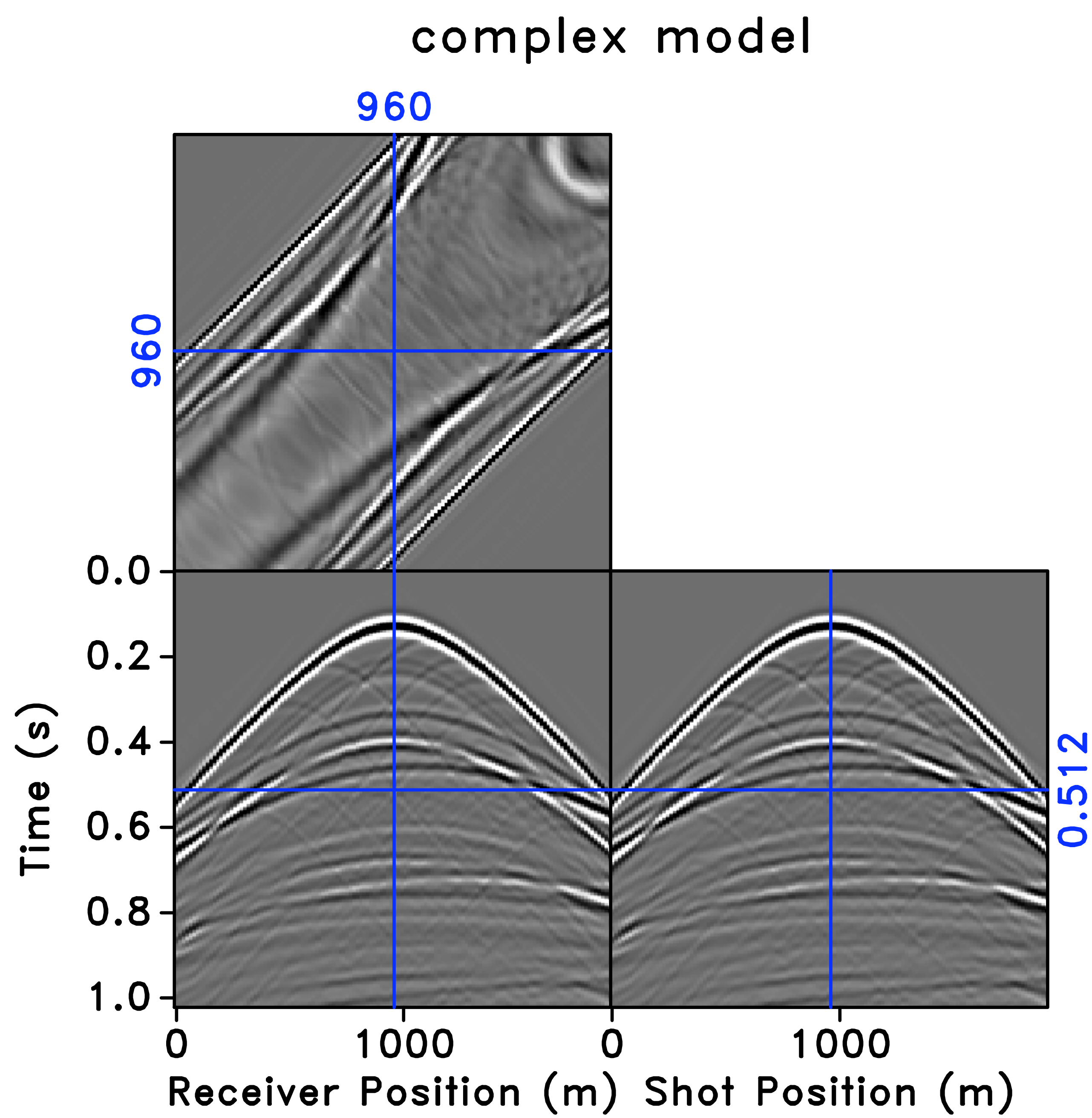




SLIM
Seismic Laboratory for Imaging and Modeling

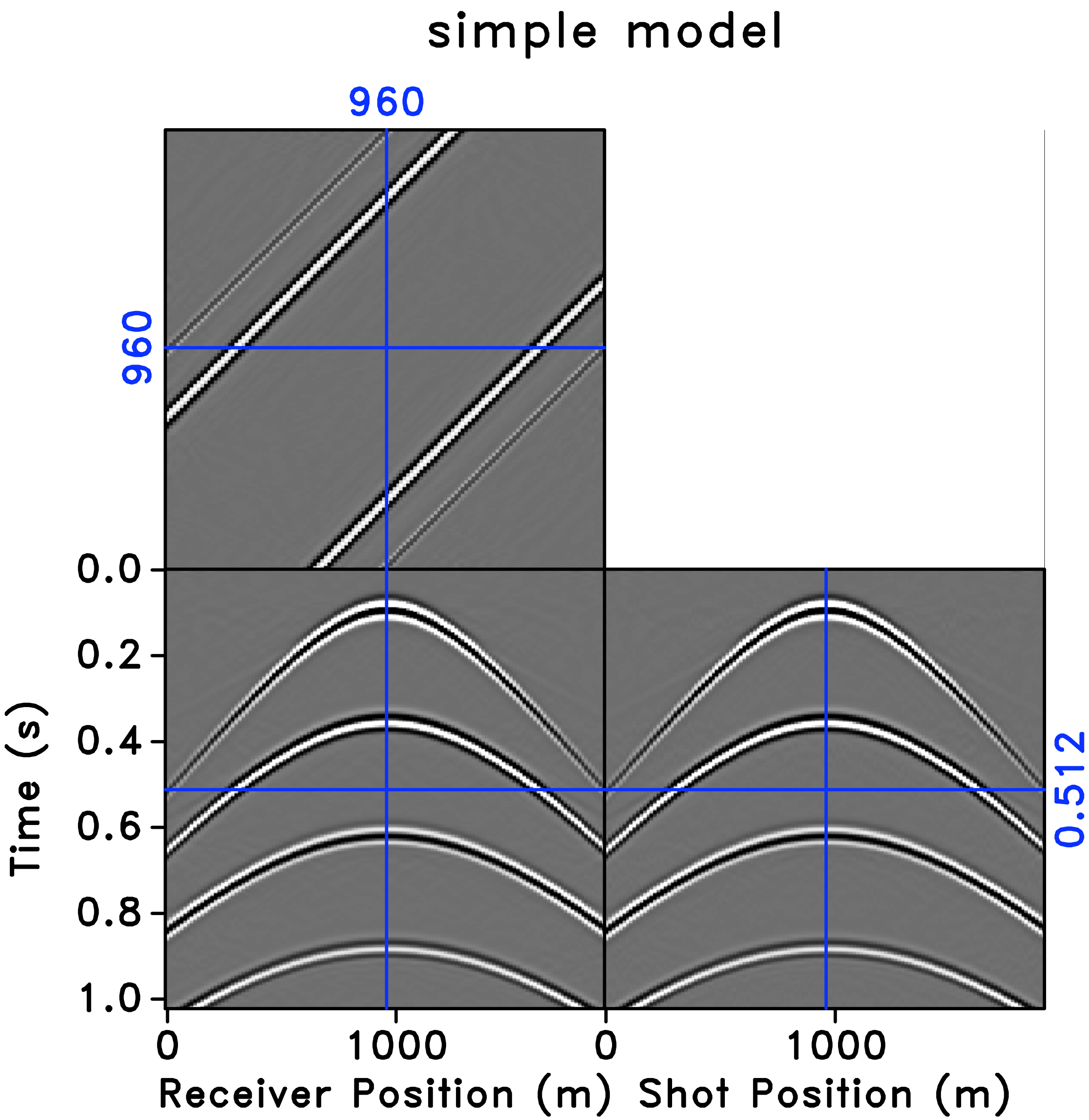
Green's functions



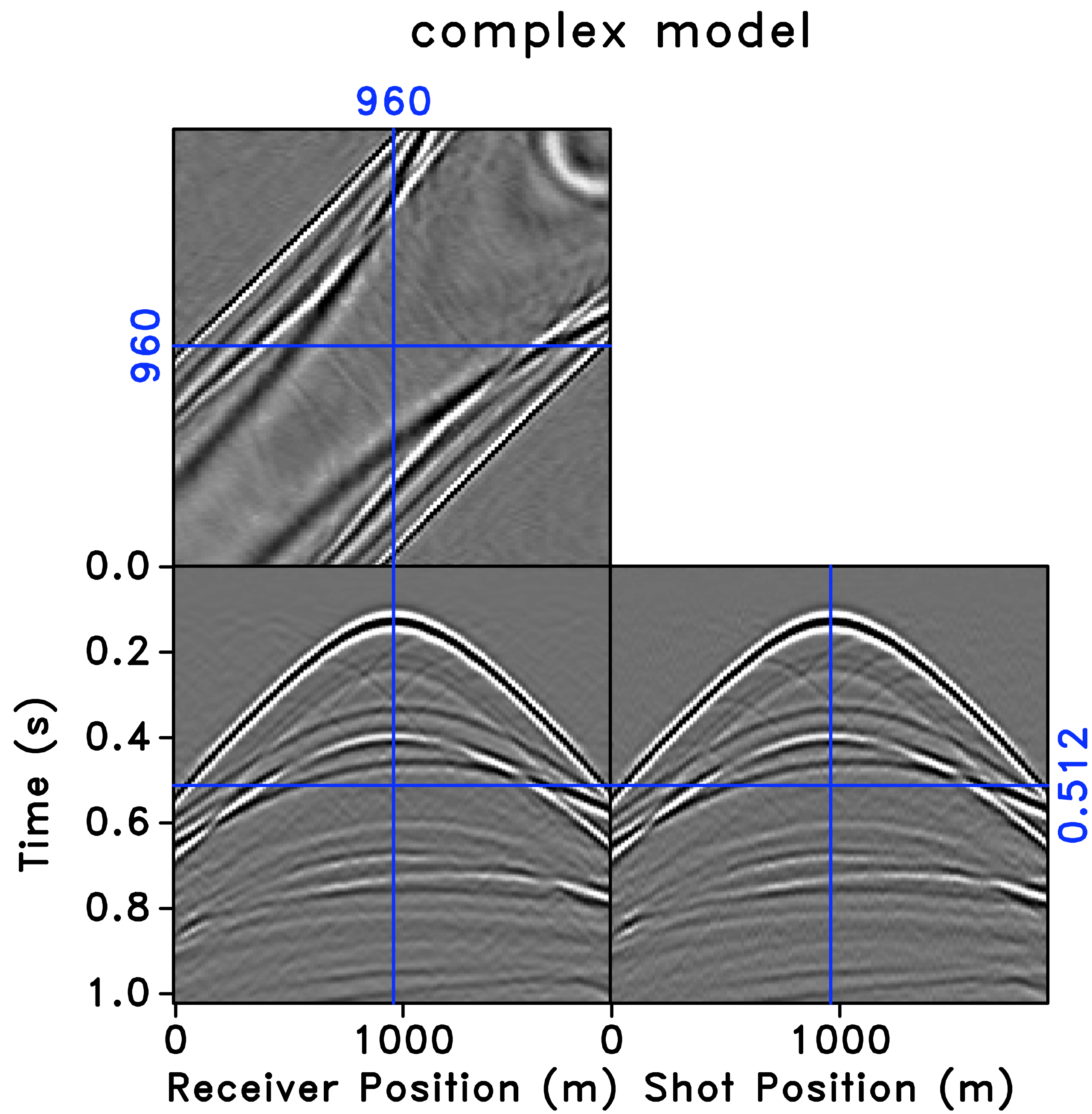


SLIM
Seismic Laboratory for Imaging and Modeling

Recovered data



28.1dB



18.2dB

300 SPGL1 iteration

SLIM
Seismic Laboratory for Imaging and Modeling

Observations & outlook

- **CS** provides a *linear sampling paradigm*
  - breaks *subsampling-related coherent interferences* by turning them into *harmless noise*
  - ***degree*** of *subsampling commensurate* with transform-domain ***sparsity***
  - *subsampling* of solutions to PDEs

- Works as long as *recovery costs* are ***smaller*** than *simulation-cost reductions*

- ***Robust*** (via sparsity promotion) instance of exploiting ***invariance*** = *"sparsity conservation"* of *multiscale transform* under certain ***solution operators***

- Bottom line: **numerical modeling costs** are **no** longer determined by the ***size*** of the ***discretization*** but by ***transform-domain compressibility*** of the **solution ...**

Challenges: there are many ...

✓ Helmholtz system is ***indefinite*** & ***ill conditioned*** => lack of convergence ***indirect*** Krylov solvers

±✓ Multiexperiment setup with ***multiple*** *right-hand-sides* is *computational* **prohibitive**

Inversion problem can be both ***over-*** and ***underdetermined***

- data cannot be explained fully
- there are local minima
- many velocity models may explain data within some error

Proposed plans to tackle *multimodality* by ***extensions & focusing*** make the situation worse by additional ***degrees of freedom***

System-size reduction

- Apply CS to reduce cost of ***wavefield*** simulation with Helmholtz
  - use ***simultaneous sources*** instead of ***separated sources***
  - leverage transform-domain sparsity & randomized subsampling by **one-norm sparsity promotion**
  - reduce size Helmholtz system
    - sources (number of right-hand sides)
    - angular frequencies (number of blocks)

- Apply CS to reduce cost of computing ***image volumes*** by multi-dimensional correlations via ***explicit*** matrix-matrix multiplies
  - randomize and subsample wavefields in **model space**
  - leverage transform-domain sparsity and focusing in the ***model space*** by ***joint sparsity promotion with mixed (1,2) norms***
  - reduce costs of storage and explicit matrix-matrix multiplies
    - sources (right-hand sides), receivers, depth
    - angular frequencies (blocks)

Tools

Compressive sensing based on Johnson-Lindenstrauss embeddings

– *Compressive sensing* [Donoho, 06‘, Candes, Romberg, Tao, ‘06]

$$\begin{aligned} \mathbf{b} &= \mathbf{RMx} \quad \textbf{[randomized subsampling]} \\ \tilde{\mathbf{x}} &= \arg\min_{\mathbf{x}} \|\mathbf{x}\|_1 \quad \text{subject to} \quad \|\mathbf{RMx}-\mathbf{b}\|_2 \leq \sigma \\ \tilde{\mathbf{x}} &\approx \mathbf{x} \end{aligned}$$

Fast matrix computations based on Johnson-Lindenstrauss embeddings

– Improved Approximation Algorithms for Large Matrices via Random Projections by Tamás Sarlós, ’08

$$\mathbf{AB} \approx \mathbf{A}\left(\mathbf{RM}\right)^{*}\left(\mathbf{RM}\right)\mathbf{B}$$

Joint sparsity-promotion with mixed (1,2) norm minimization

– *Joint-sparse recovery from multiple measurements* by E. van den Berg and M. Friedlander, ‘09

$$\tilde{\mathbf{X}} = \arg\min_{\mathbf{X}} \|\mathbf{X}\|_{1,2} \quad \text{subject to} \quad \|\mathbf{AX}-\mathbf{B}\|_{2,2} \leq \sigma,$$

Differential semblance

- Invoke *physical* principle of ***focusing*** [Claerbout & many others] <=> *mathematical* principle of ***extensions*** [Symes]

- Motivated by Symes' differential semblance principle [Symes '09]: "*Amongst all possible quadratic forms in the data, parameterized by velocity, of the form*

$$\min_{\mathbf{m}} \|(\mathsf{P}_h \overbrace{\boldsymbol{\delta}\mathbf{I}(\cdot,\ \underset{\substack{\uparrow \\ \text{redundant coordinate}}}{h}; \mathbf{m}, \delta\mathbf{d})}^{\text{image volume}})\|_2 \quad \text{with } \overbrace{\mathsf{P}_h \cdot = \mathbf{h}\cdot}^{\text{annihilator}},$$

only differential semblance is smooth jointly as function of smooth perturbations in velocity and finite energy perturbations in data [Stolk & Symes, '03]"

- Forms the basis of ***nonlinear*** migration velocity analysis on ***linearized*** data [Symes, '09].

Image volume

Compute multi-D ***cross-correlations*** on ***multiexperiment*** solutions of the forward- and reverse-time Helmholtz systems--i.e,

$$\boldsymbol{\delta}\mathbf{I}(m,h,t) = \left(\bar{\mathbf{U}} * \mathbf{V}^T\right)$$

with

$$\mathbf{U}_f = \left[\mathbf{u}_1 \cdots \mathbf{u}_{n_f}\right] \text{ and } \mathbf{V}_f = \left[\mathbf{v}_1 \cdots \mathbf{v}_{n_f}\right]$$

and

$$\left(\bar{\mathbf{U}} * \mathbf{V}^T\right) := \mathbf{T}_{(x_s,x_r,\omega)\mapsto(m,h,t)} \begin{bmatrix} \bar{\mathbf{U}}_1 & & \\ & \ddots & \\ & & \bar{\mathbf{U}}_{n_f} \end{bmatrix} \begin{bmatrix} \mathbf{V}_1^T \\ \vdots \\ \mathbf{V}_{n_f}^T \end{bmatrix}$$

where

$$m = \frac{1}{2}(x_s + x_r) \quad \text{and} \quad h = \frac{1}{2}(x_s - x_r)$$

High **dimensional** and highly ***redundant ...***

Imaging condition

Claerbout's imaging principle:

$$\begin{aligned} \delta\mathbf{m} &= \boldsymbol{\delta}\mathbf{I}(\cdot, h=0, t=0) \\ &= \mathbf{K}^{*}\boldsymbol{\delta}\mathbf{d} \end{aligned}$$

- implicit in adjoint state method
- Image volume
  - very large because of additional degree of freedom
  - expensive to store

System-size reduction by CS

For each angular frequency, subsample with CS matrix

$$\mathbf{RM} := \overbrace{\begin{bmatrix} \mathbf{R}_1^\sigma \otimes \mathbf{R}_1^\rho \otimes \mathbf{R}_1^\zeta \\ \vdots \\ \mathbf{R}_{n'_f}^\sigma \otimes \mathbf{R}_{n'_f}^\rho \otimes \mathbf{R}_{n'_f}^\zeta \end{bmatrix}}^{\text{sub sampler}} \overbrace{\left(\mathbf{F}_3^* \left(e^{\hat{i}\boldsymbol{\theta}}\right)\right) \mathbf{F}_3}^{\text{random phase encoder}} ,$$

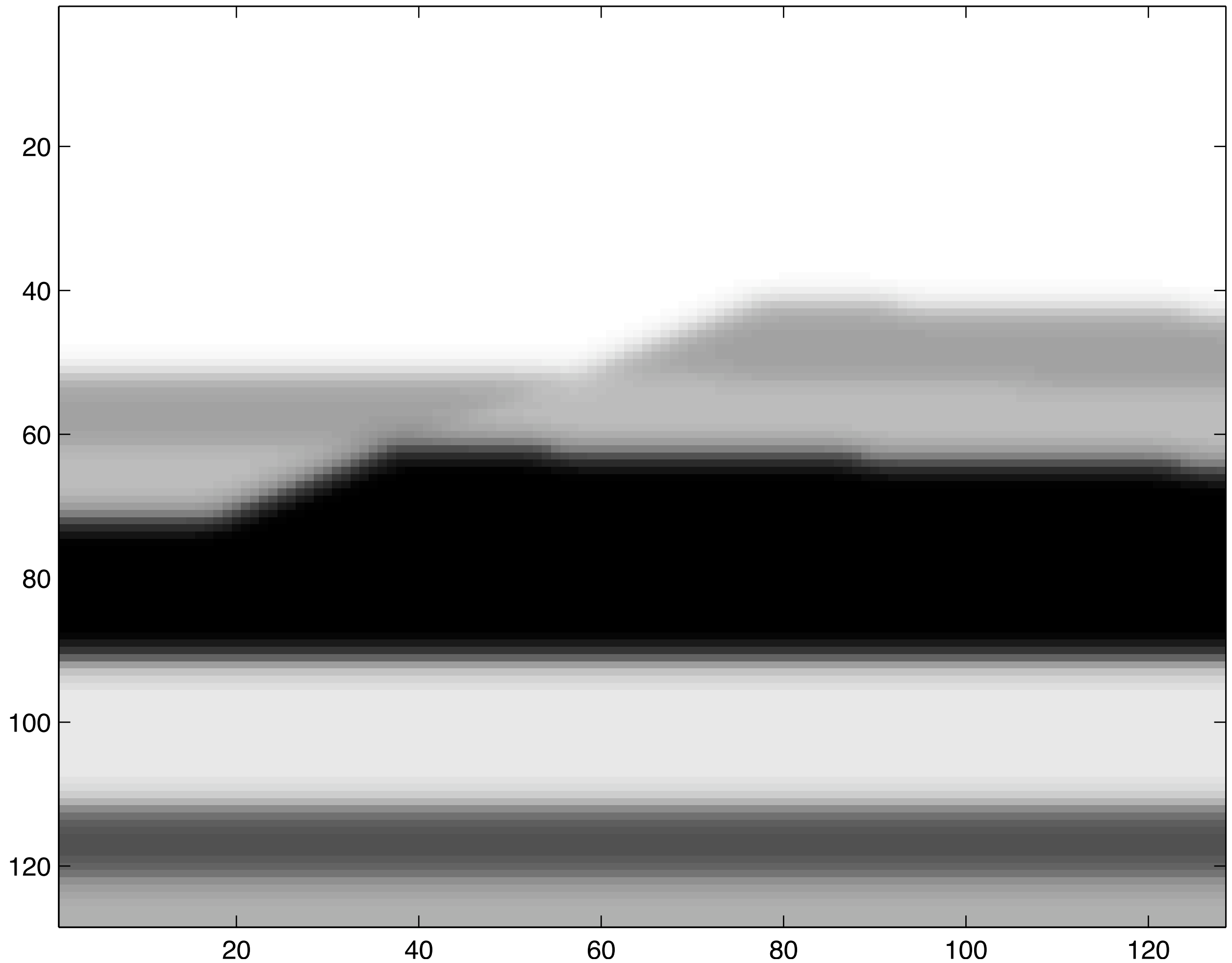
with

$$n'_f \times n'_\sigma \times n'_\rho \times n'_\zeta \ll n_f \times n_s \times n_r \times n_z$$

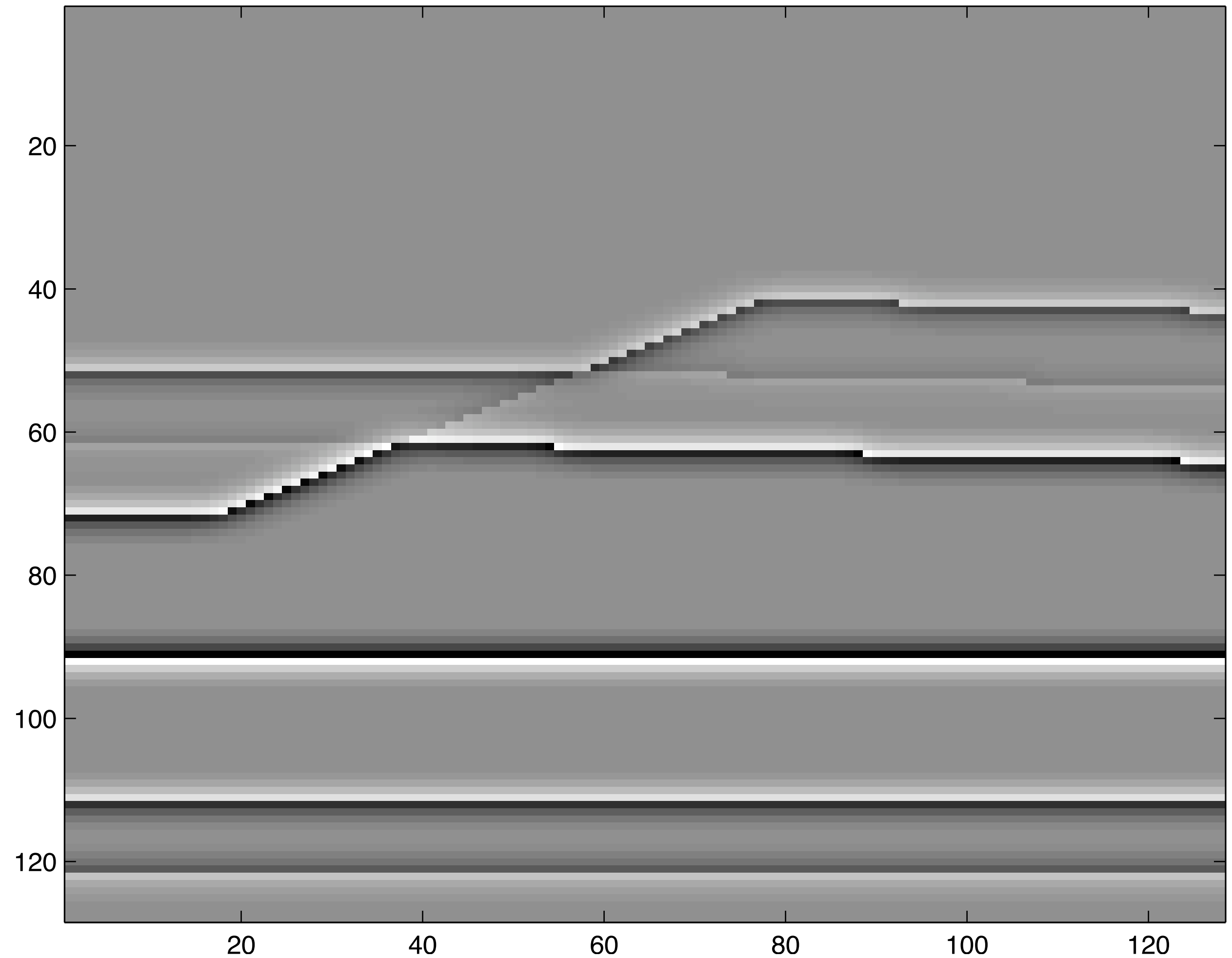
Model-space CS subsampling along subsurface source, receiver, and depth coordinates yielding an *approximate* **extended** image

$$\boldsymbol{\delta}\mathbf{I}(m,h,t) \approx \left(\bar{\mathbf{U}}(\mathbf{RM})^* * \mathbf{RMV}^T\right)$$

Example

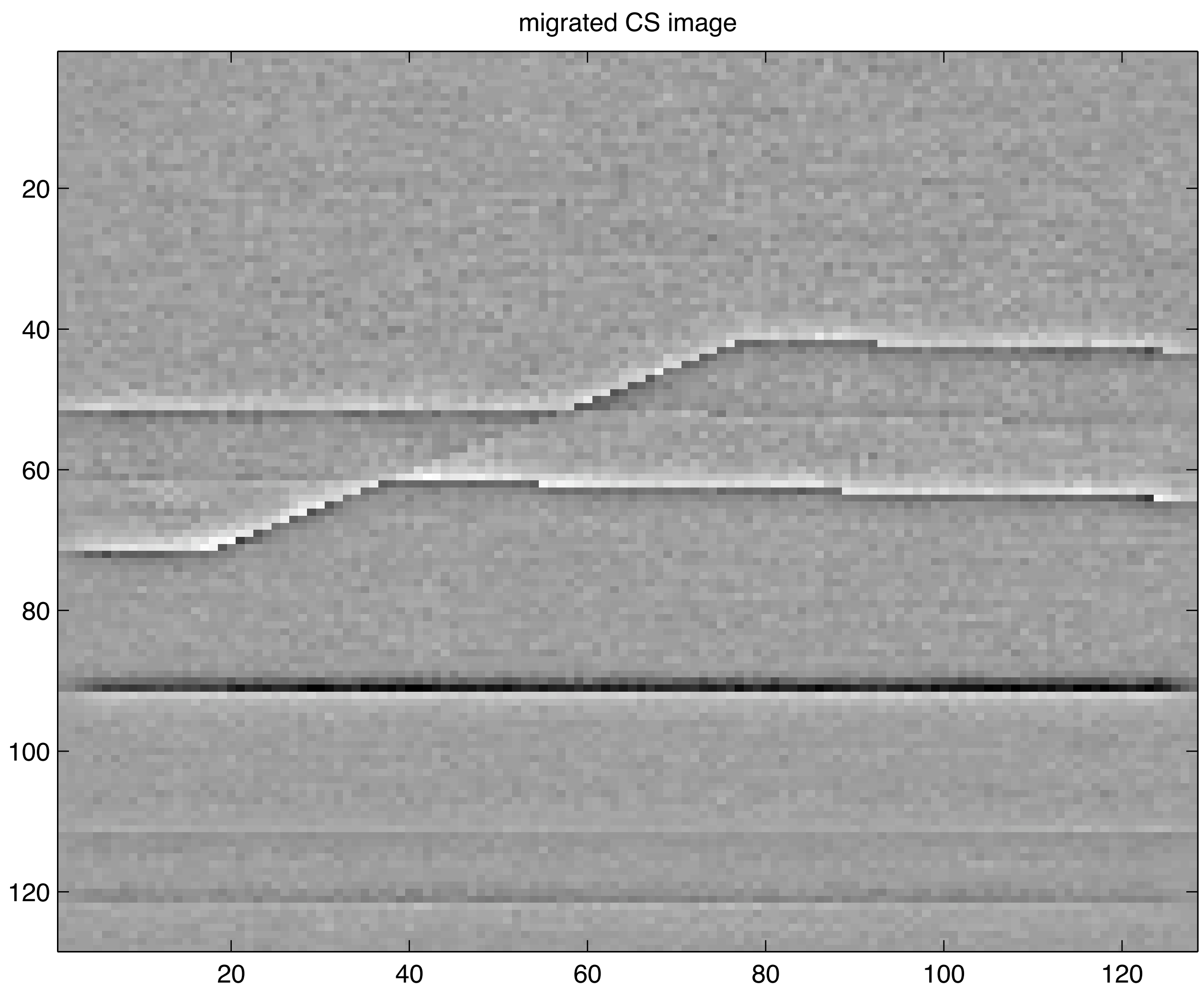


background velocity model

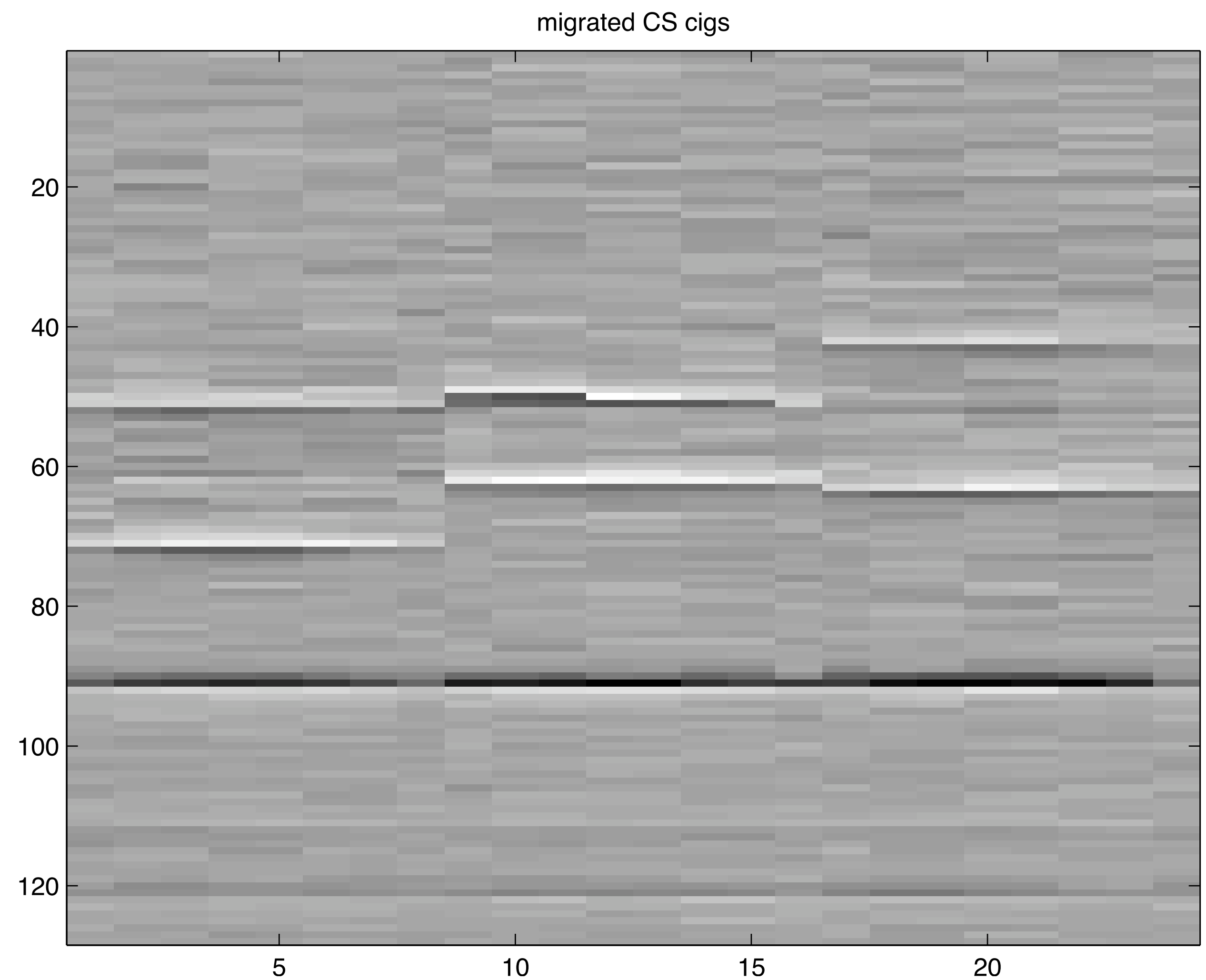


perturbation

Example: matched filter



$$\boldsymbol{\delta}\mathbf{I}(\cdot, h = 0, t = 0)$$



$$\boldsymbol{\delta}\mathbf{I}([m_1,\ m_2,\ m_3], h, t = 0)$$

Recovery from 64-fold subsampling ...

- **Noisy**
- **Not focused**

Tools

Compressive sensing based on Johnson-Lindenstrauss embeddings

– *Compressive sensing* [Donoho, 06‘, Candes, Romberg, Tao, ‘06]

$$\mathbf{b} = \mathbf{RMx} \quad \textbf{[randomized subsampling]}$$

$$\tilde{\mathbf{x}} = \arg\min_{\mathbf{x}} \|\mathbf{x}\|_1 \quad \text{subject to} \quad \|\mathbf{RMx} - \mathbf{b}\|_2 \leq \sigma$$

$$\tilde{\mathbf{x}} \approx \mathbf{x}$$

Fast matrix computations based on Johnson-Lindenstrauss embeddings

– Improved Approximation Algorithms for Large Matrices via Random Projections by Tamás Sarlós, ’08

$$\mathbf{AB} \approx \mathbf{A}\left(\mathbf{RM}\right)^{*}\left(\mathbf{RM}\right)\mathbf{B}$$

Joint sparsity-promotion with mixed (1,2) norm minimization

– *Joint-sparse recovery from multiple measurements* by E. van den Berg and M. Friedlander, ‘09

$$\tilde{\mathbf{X}} = \arg\min_{\mathbf{X}} \|\mathbf{X}\|_{1,2} \quad \text{subject to} \quad \|\mathbf{AX} - \mathbf{B}\|_{2,2} \leq \sigma,$$

Extended Born & focusing

Define ***extended*** *linearized* forward model [Symes, '09]:

$$\bar{\mathbf{K}}[\mathbf{m},\mathbf{Q}]\boldsymbol{\delta}\mathbf{I} \approx \boldsymbol{\delta}\mathbf{D}$$

- multiexperiment form *amenable* for ***joint sparsity promotion***
- introduce penalty term that penalizes ***defocusing***

Form augmented system with **focusing**:

$$\begin{array}{rcll} \bar{\mathbf{K}}\boldsymbol{\delta}\mathbf{I} & \approx & \boldsymbol{\delta}\mathbf{D} & \text{data fit} \\ \lambda^2 \mathsf{P}_h \boldsymbol{\delta}\mathbf{I} & \approx & \mathbf{0} & \text{focusing} \end{array}$$

with $\mathsf{P}_h \cdot = \mathbf{h}\cdot$ *annihilator* that increasingly *penalizes* non-zero offsets.

Solution involves multi-D "deconvolution" (adjoint of cross correlation):

$$(\mathbf{U}^* \star \boldsymbol{\delta}\mathbf{I}) \quad \approx \quad \mathbf{V}^T$$

Compressed linearized inversion

Compressively sample augmented system that includes sparsity synthesis operator--i.e,

$$\begin{aligned} \mathbf{RM}\left(\mathbf{U}^* \star \mathbf{S}^*\mathbf{X}\right) &\approx \mathbf{RMV}^T \\ \mathsf{P}_h\mathbf{X} &\approx \mathbf{0} \end{aligned}$$

$$\mathbf{AX} \approx \mathbf{B}$$

with the sparsifying transform **S** for each offset h given by the curvelet or wavelet transform

Recover focused solution by mixed (1,2)-norm minimization.

Promote sparsity amongst images though one-norm on columns

Penalize energy amongst rows => focusing

Joint-sparsity promotion [van den berg & Friedlander, '09]

Recover focused solution by mixed (1,2)-norm minimization:

$$\tilde{\mathbf{X}} = \arg\min_{\mathbf{X}} \|\mathbf{X}\|_{1,2} \quad \text{subject to} \quad \|\mathbf{A}\mathbf{X} - \mathbf{B}\|_{2,2} \leq \sigma,$$

with

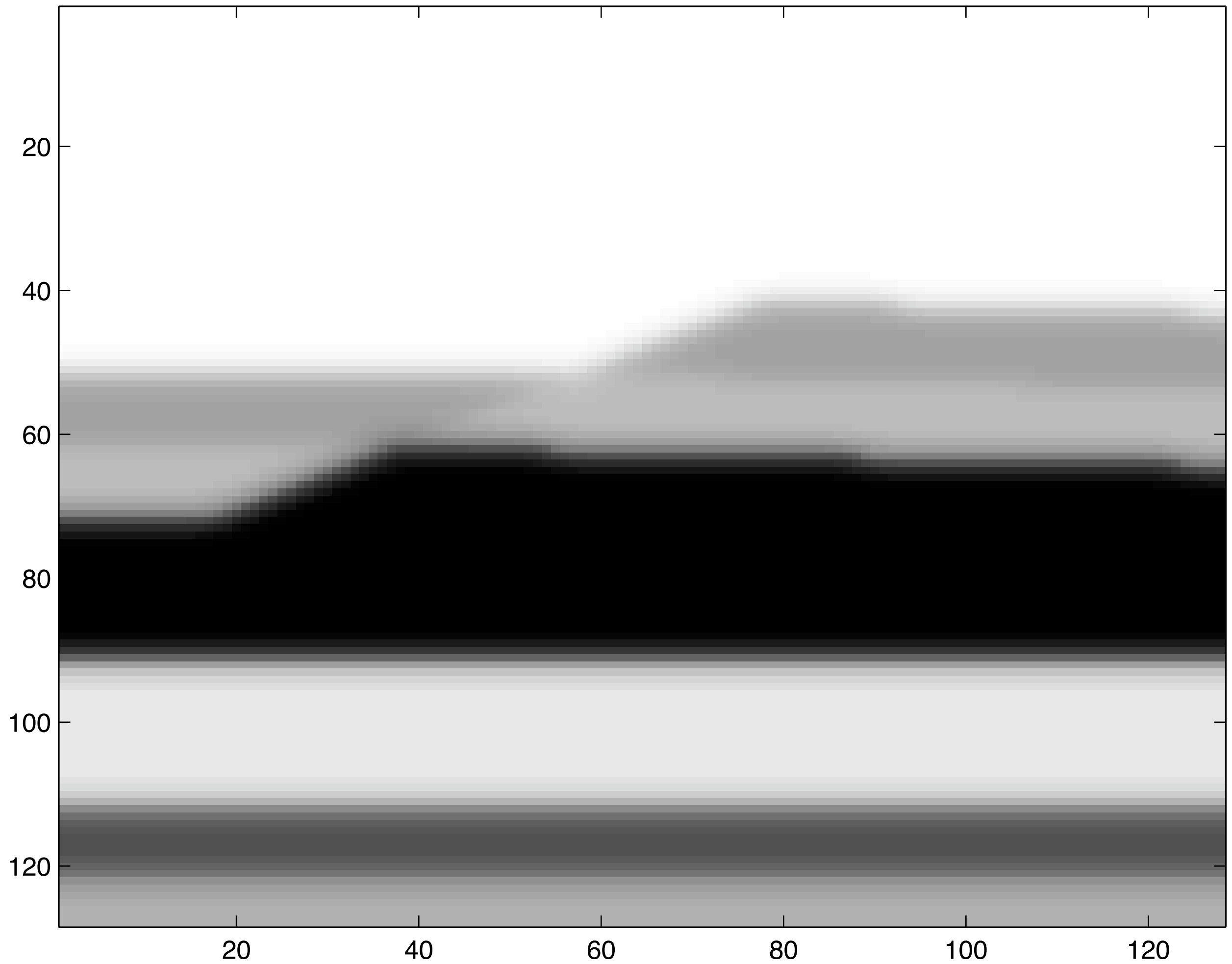
$$\|\mathbf{X}\|_{1,2} := \sum_{i \in \text{rows}(\mathbf{X})} \|\text{row}_i(\mathbf{X})^*\|_2$$

and

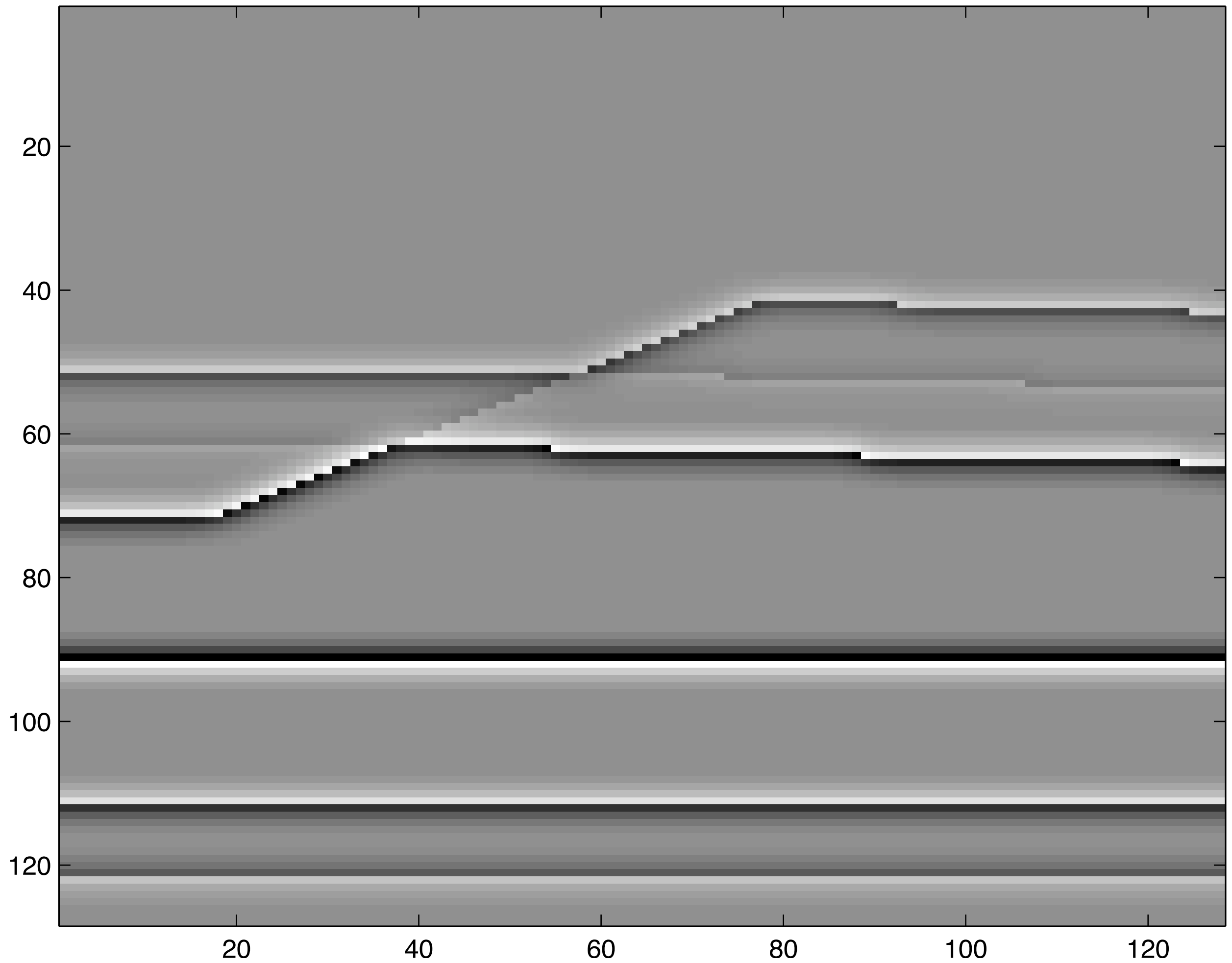
$$\|\mathbf{X}\|_{2,2} := \left(\sum_{i \in \text{rows}(\mathbf{X})} \|\text{row}_i(\mathbf{X})^*\|_2^2 \right)^{\frac{1}{2}}.$$

Solved with SPGL1.

Example

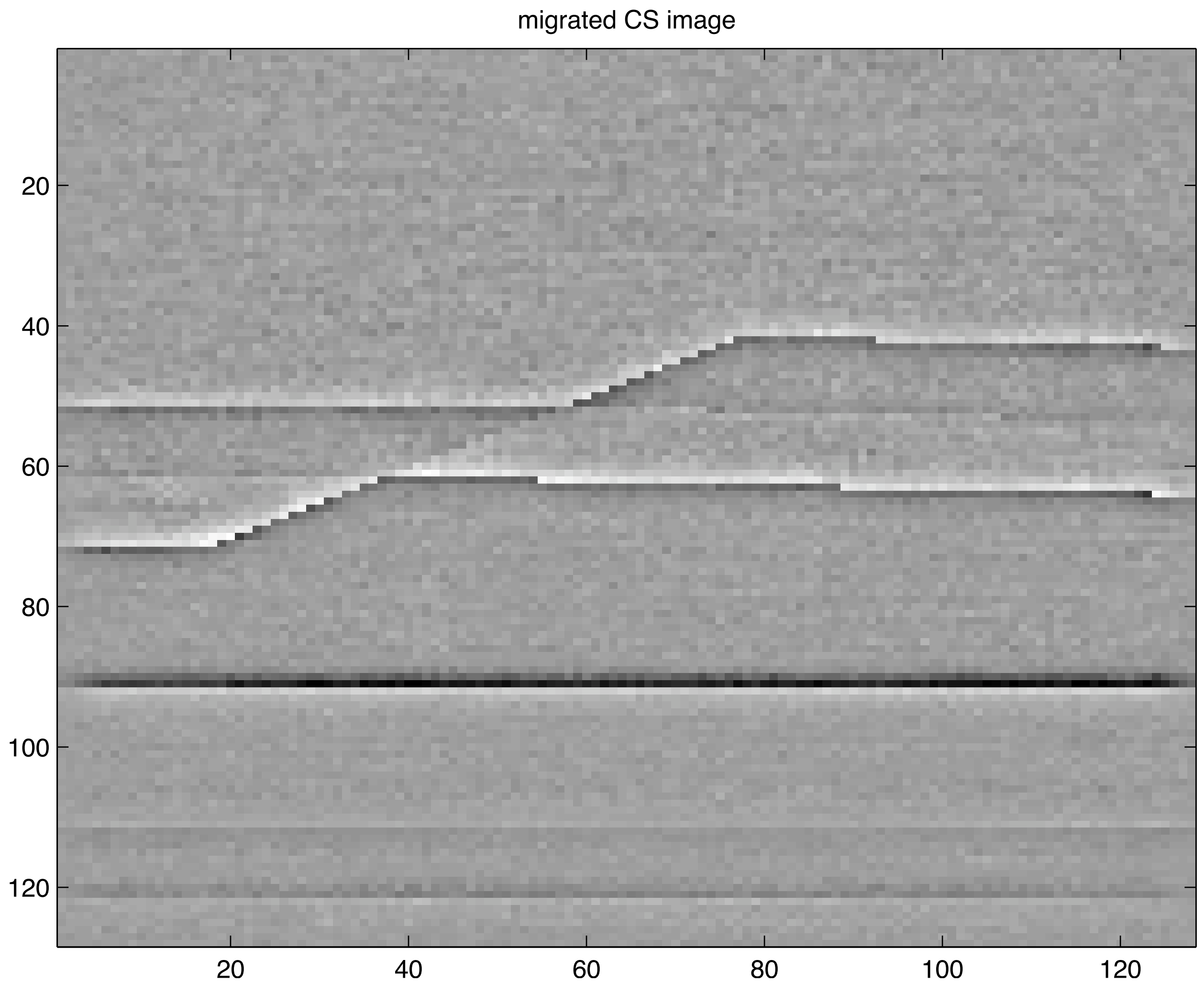


background velocity model

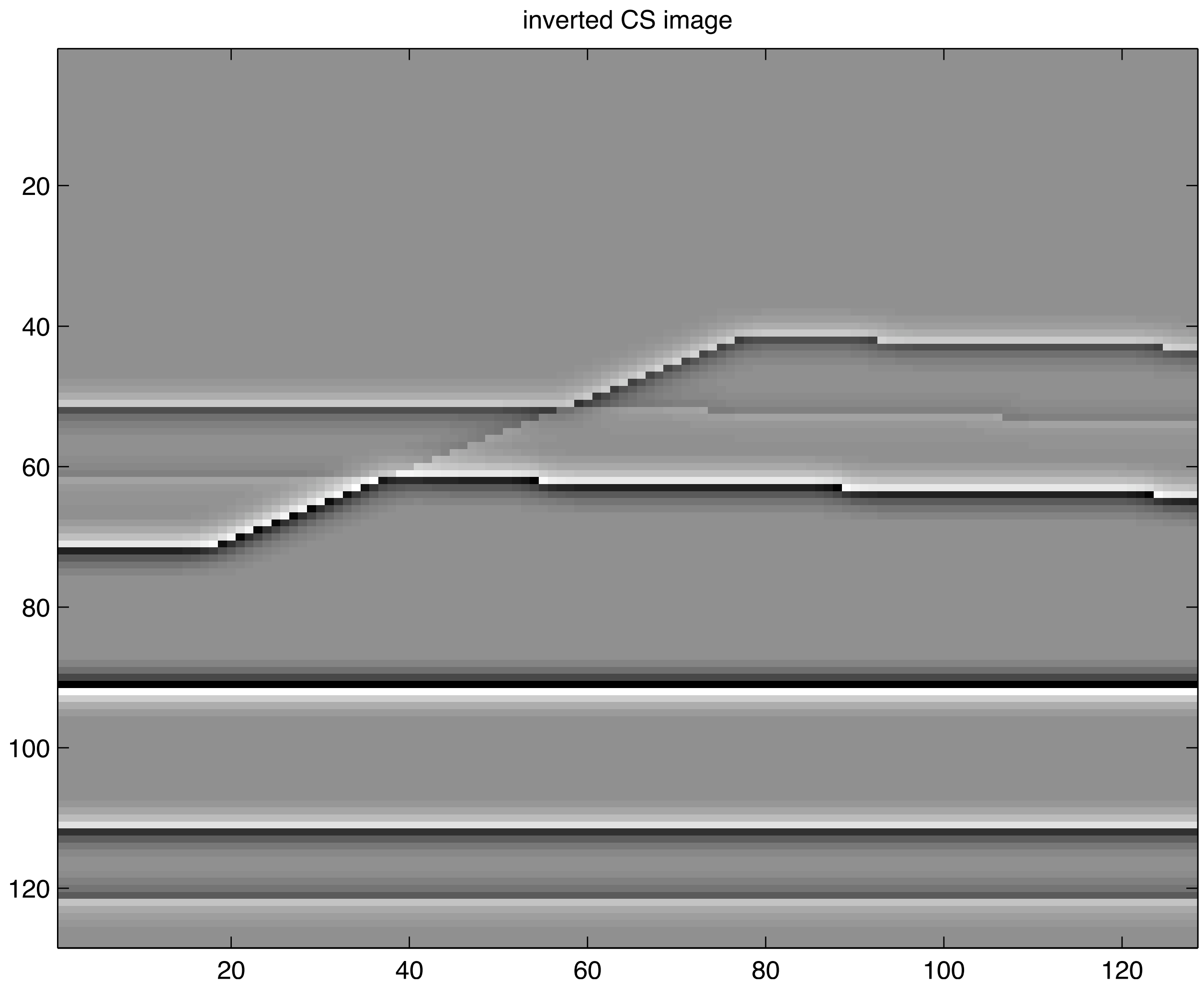


perturbation

Example



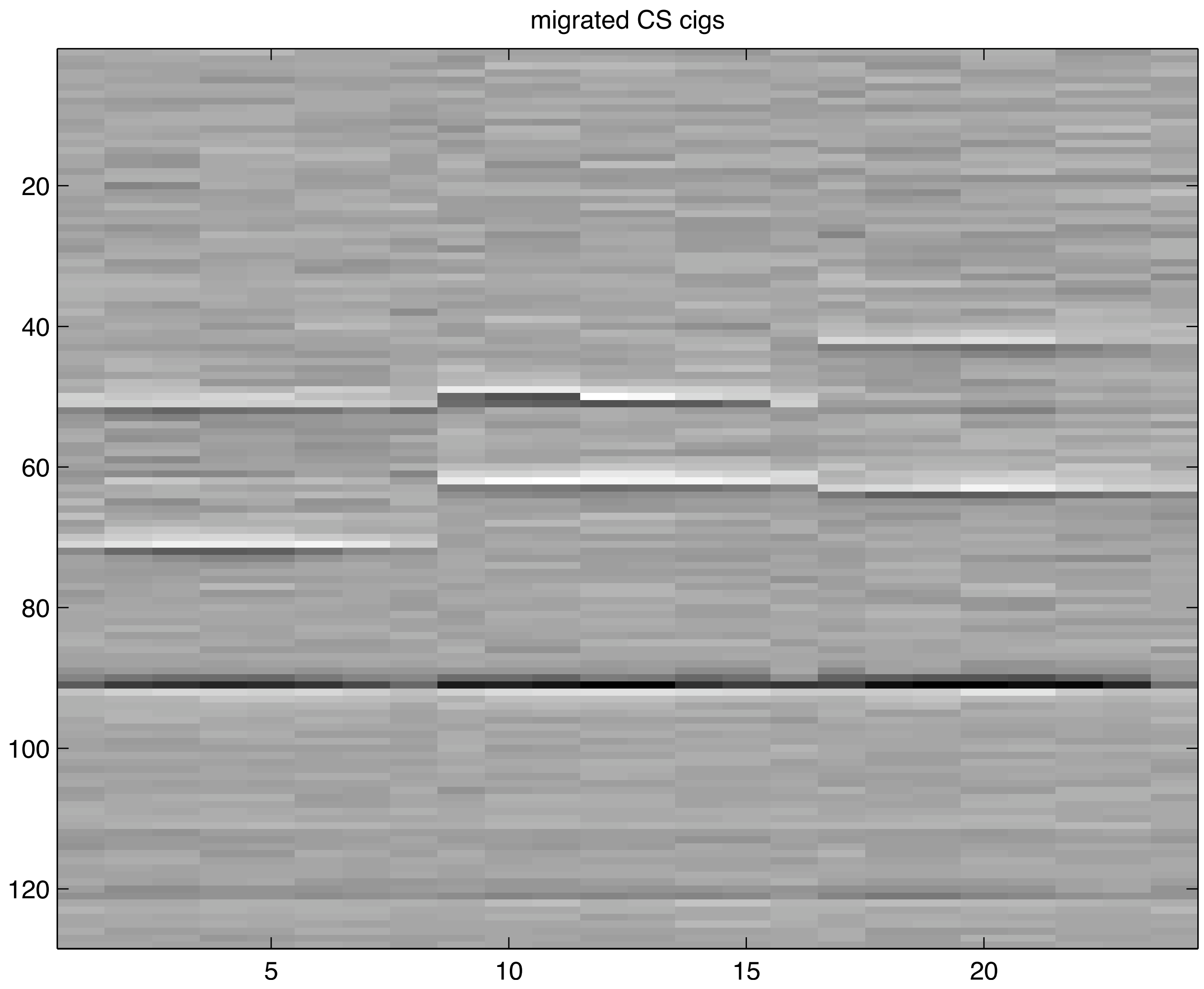
matched filter



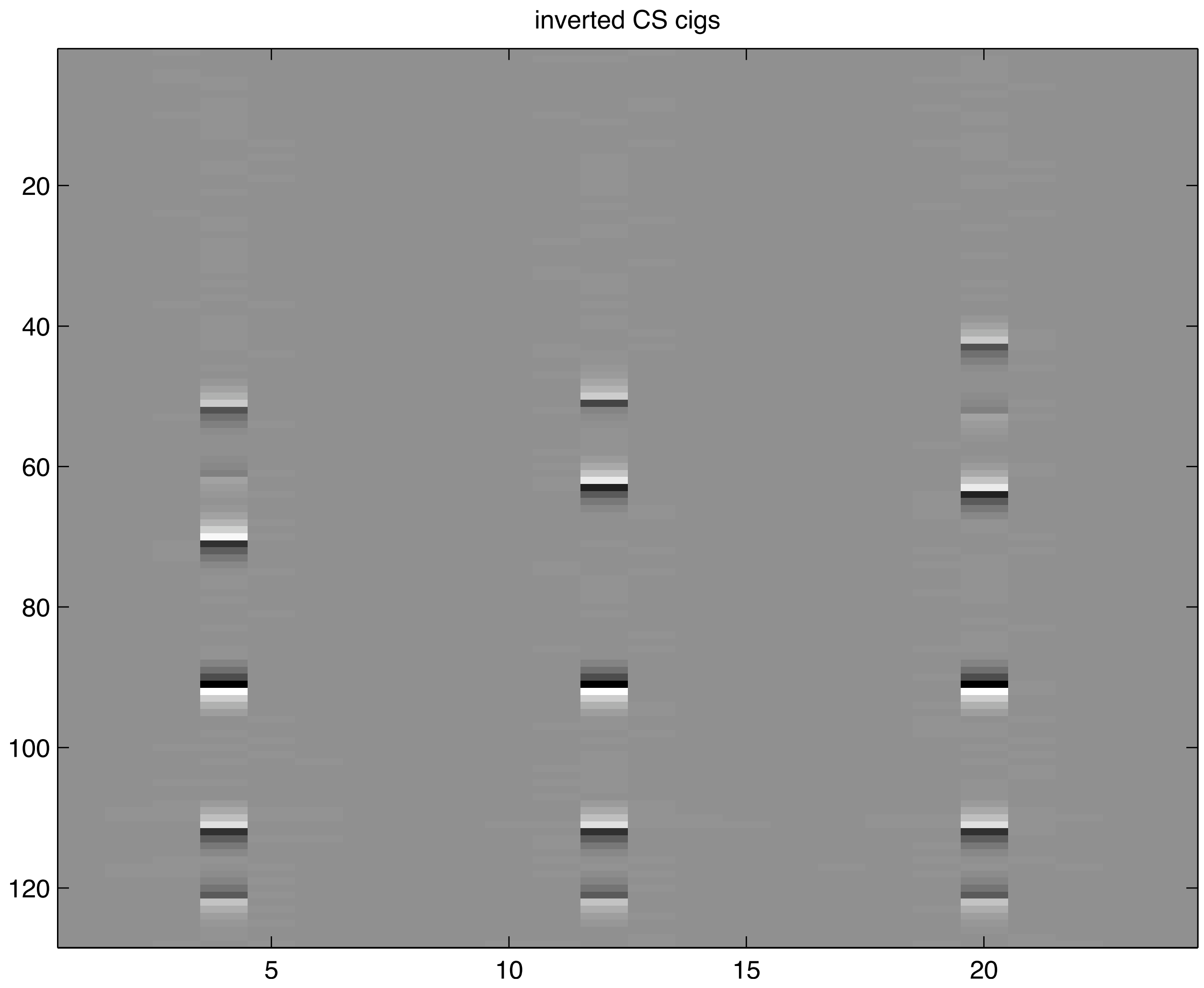
sparsity promotion

Recovery from 64-fold subsampling ...

Example



matched filter



sparsity promotion

Common-image gathers are focussed.

Observations & outlook

- CS allows for a ***compression*** of data volumes without *significant* ***loss*** of ***information*** yielding a ***reduction*** in *computational* costs

- CS has ***direct*** implications for seismic acquisition--from ***sequential*** to ***simultaneous*** acquisition

- ***Joint*** sparsity promotion allows for ***focusing***

- **Speculation:** Proposed approach may be suitable to handle Symes's proposal to add a degree of freedom yielding a ***nonlocal*** *forward* model in tandem with an inverse problem that penalizes ***nonlocality*** through ***focusing ...***

Acknowledgments

- E. van den Berg and M. P. Friedlander for *SPGL1* (www.cs.ubc.ca/labs/scl/spgl1) & *Sparco* (www.cs.ubc.ca/labs/scl/sparco)
- Sergey Fomel and Yang Liu for Madagascar (rsf.sf.net)
- E. Candes and the Curvelab team

This work was carried out as part of the SINBAD project with financial support from the collaborative research & development (CRD) grant DNOISE (334810-05) funded by the Natural Science and Engineering Research Council (NSERC) and matching contributions from BG, BP, Chevron, ExxonMobil and Shell.

slim.eos.ubc.ca

and... Thank you!