Beating Nyquist by randomized sampling

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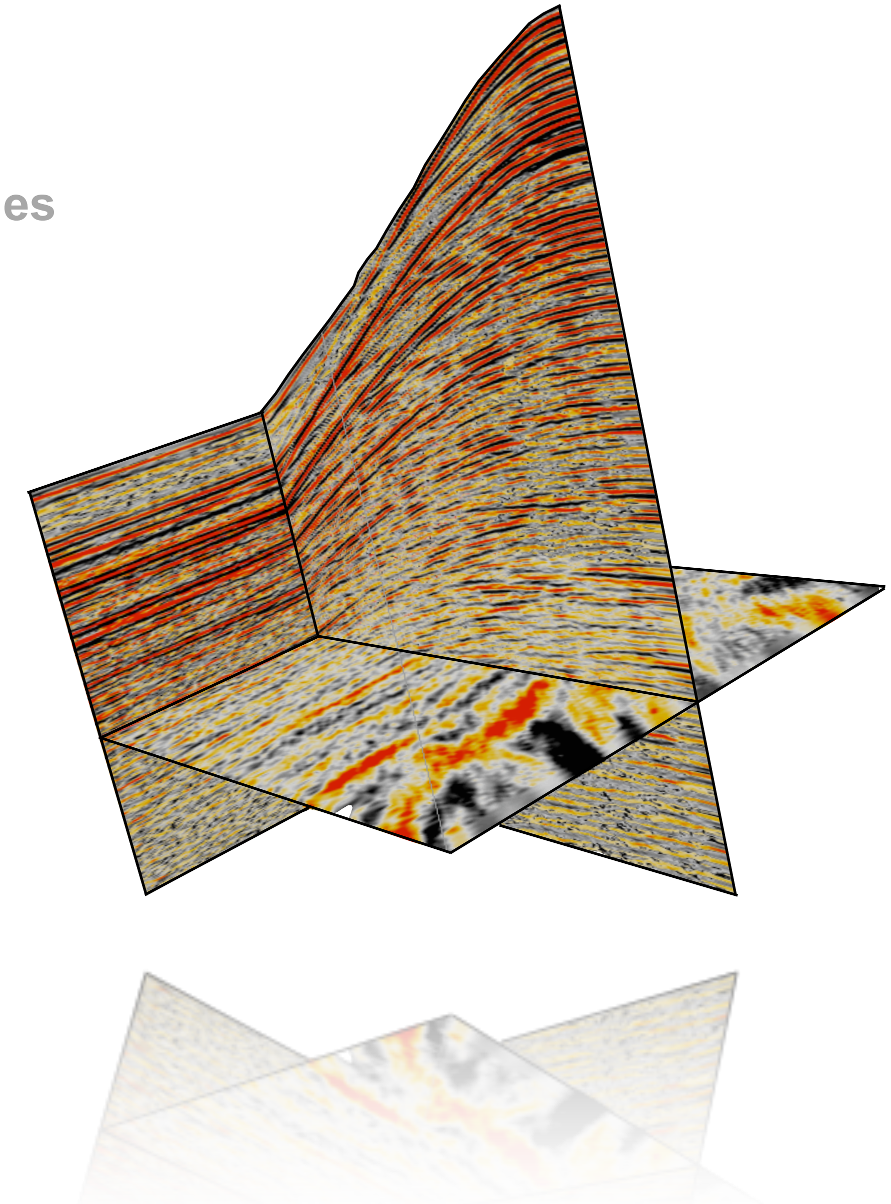
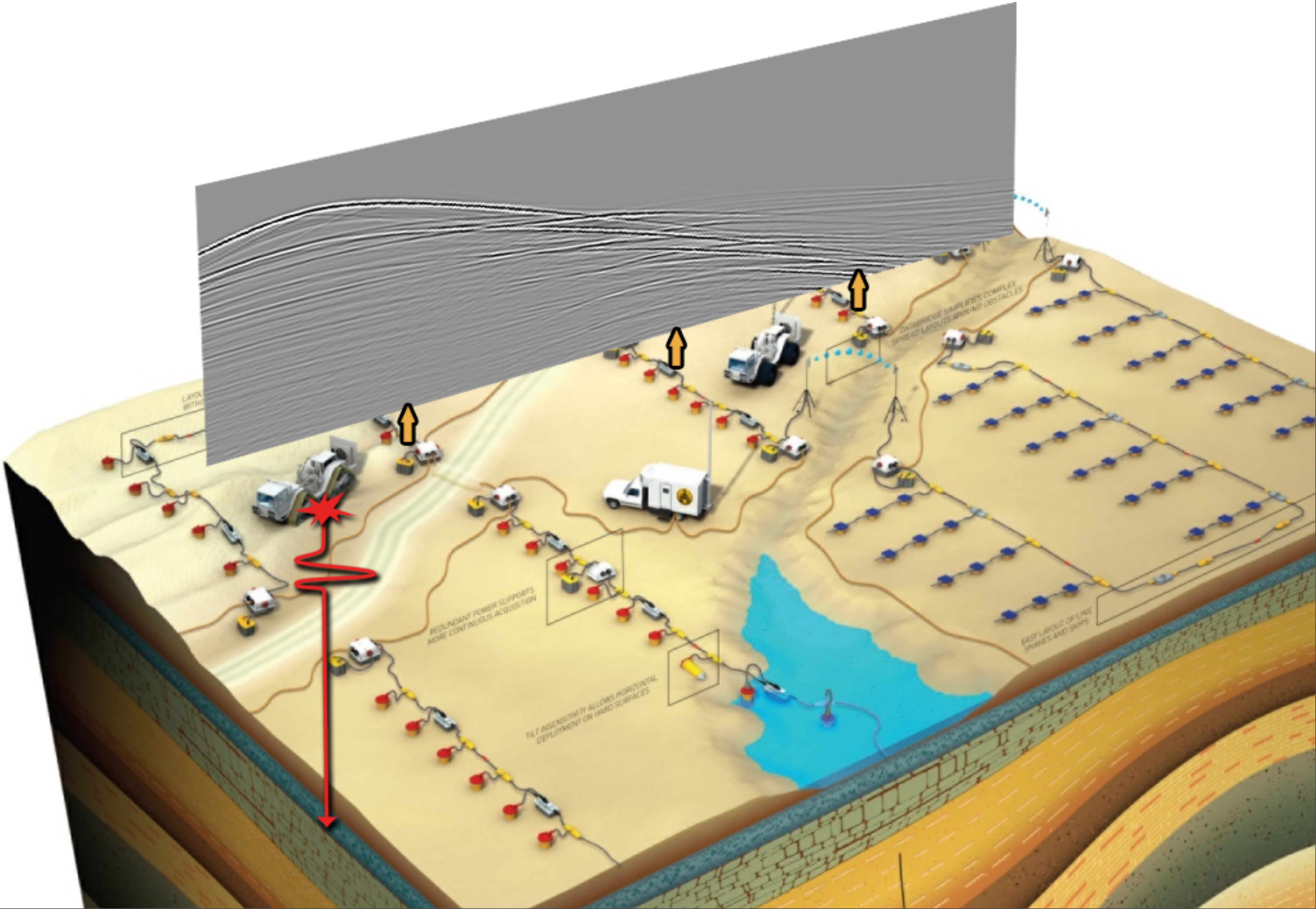


Vienna July 24, 2009

image courtesy ION (www.iongeo.com)

Seismic acquisition

Individual shots



Individual shots

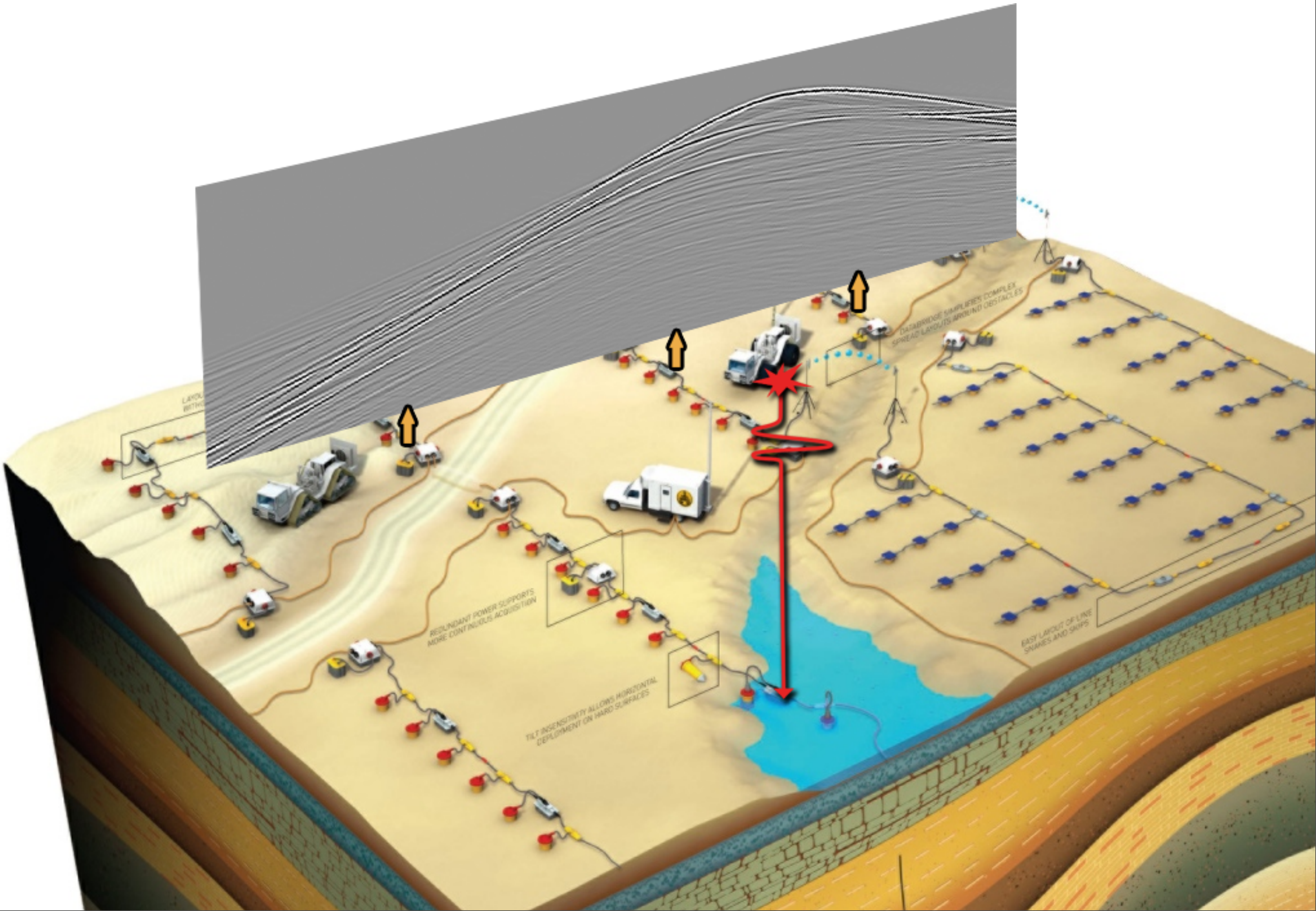
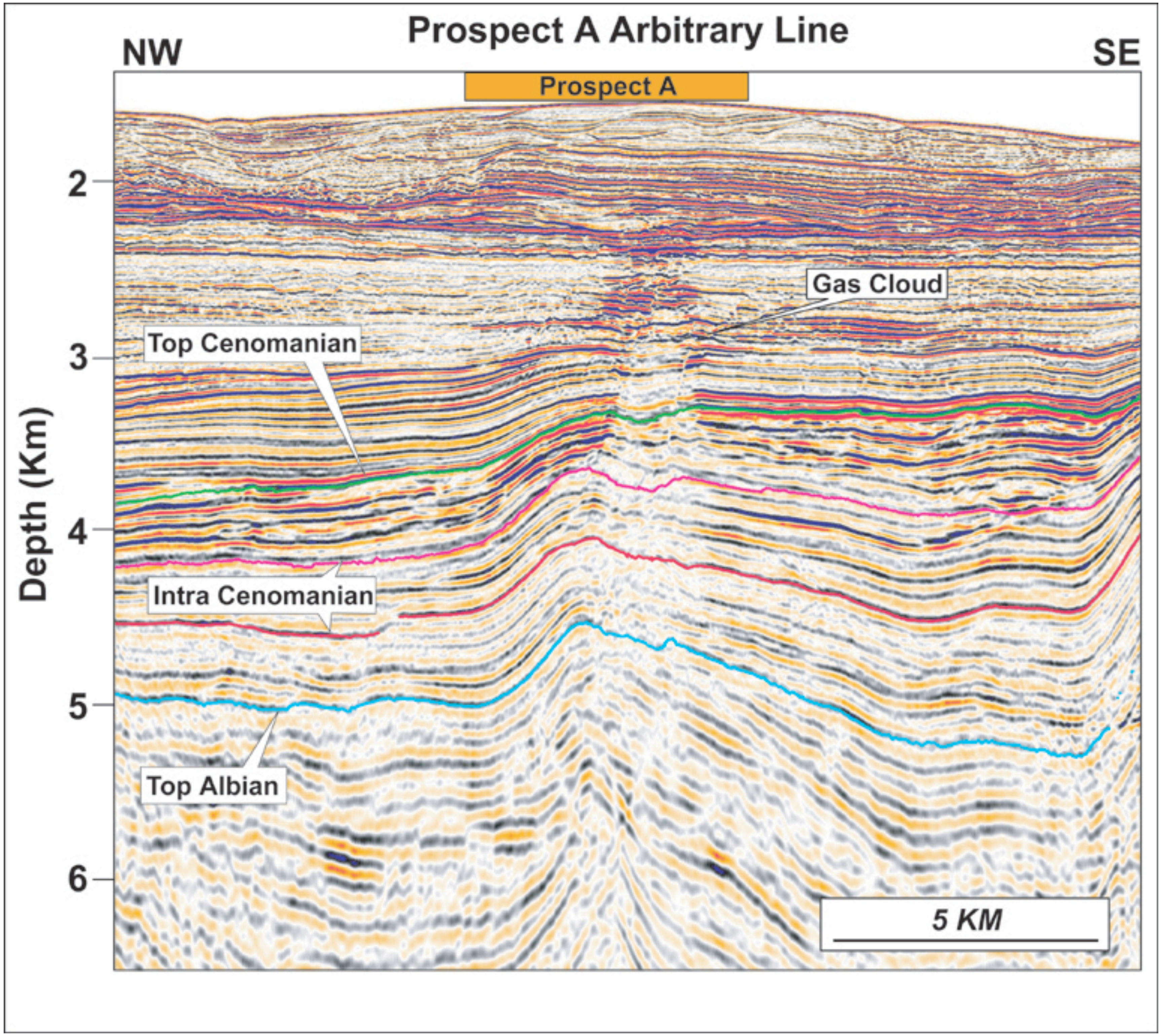


image courtesy vancoenergy (www.vancoenergy.com)

After imaging



Motivation

- **Seismic data processing, modeling & inversion:**
  - firmly rooted in Nyquist's sampling paradigm for high-dimensional wavefields
  - too *pessimistic* for signals with *structure,* i.e, there exists some sparsifying transform (e.g. Fourier, curvelets)

- **Recent theoretical & hardware developments**
  - Alternative multiscale, localized & directional transform domains that compress seismic data
  - New nonlinear sampling theory that supersedes the overly pessimistic Nyquist sampling criterion
  - New autonomous data acquisition devices that allow for more flexibility during acquisition
  - New simultaneous & continuous recording

- **Today's agenda:**
  - Extensions of ***jittered*** sampling to higher-D through ***randomized*** *blue-noise* sampling
  - Connections between ***randomized*** *simultaneous* acquisition and *compressive sampling*
  - Incorporation of additional ***physics***, e.g. include surface operator

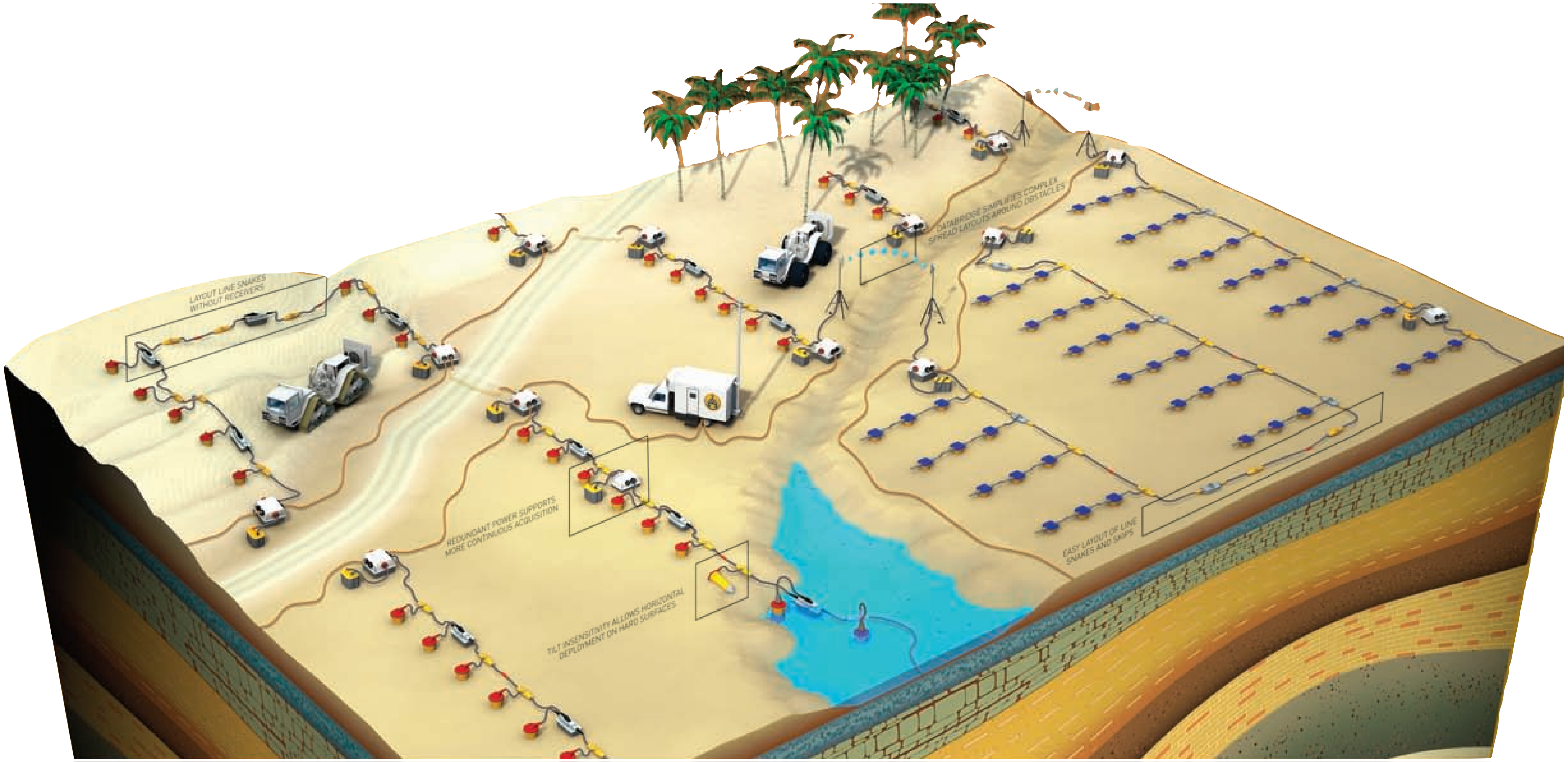
Motivation cont'd

- **Solution strategy:**
  - *leverage new* paradigm of *compressive sensing* (CS)
    - identify wavefield reconstruction from missing sources & receivers or from simultaneous acquisition as instances of CS
    - reduce acquisition, simulation, and inversion costs by ***randomization*** and deliberate ***subsampling***
  - recovery from sample **rates ≈ acquisition & computational costs** *proportional* to **transform-domain sparsity** of *data* or *model*

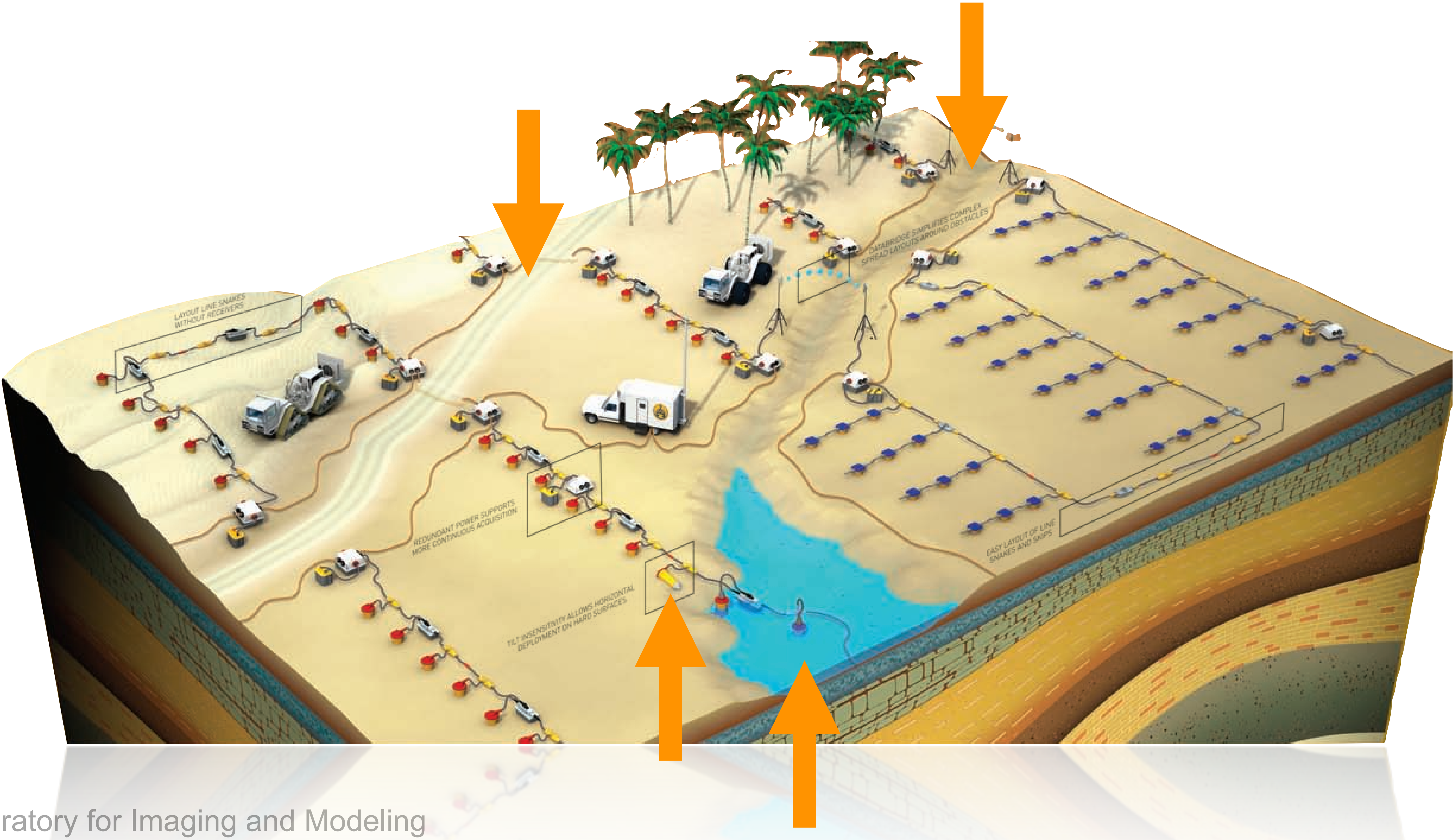
- **Remove the "curse of dimensionality" by removing constructive aliases/interferences**
  - breaking the ***periodicity of regular sampling***
  - using ***incoherent sources***

- **Turn problems into a "simple" denoising problems ....**
  - use spatial blue-noise sampling techniques from computer graphics community
  - use randomized phase encoding for simultaneous source-function design

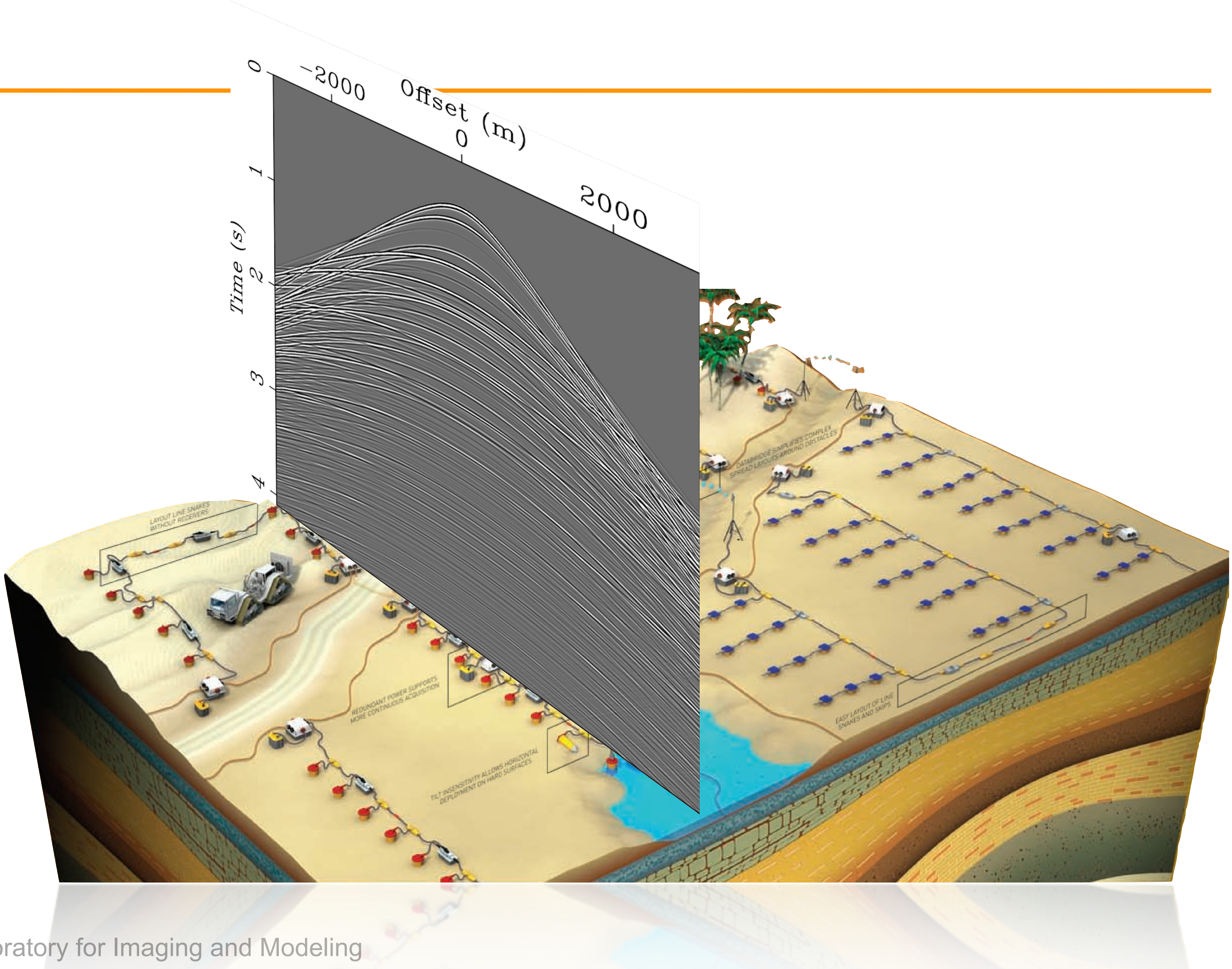
Acquisition image courtesy ION (www.iongeo.com)



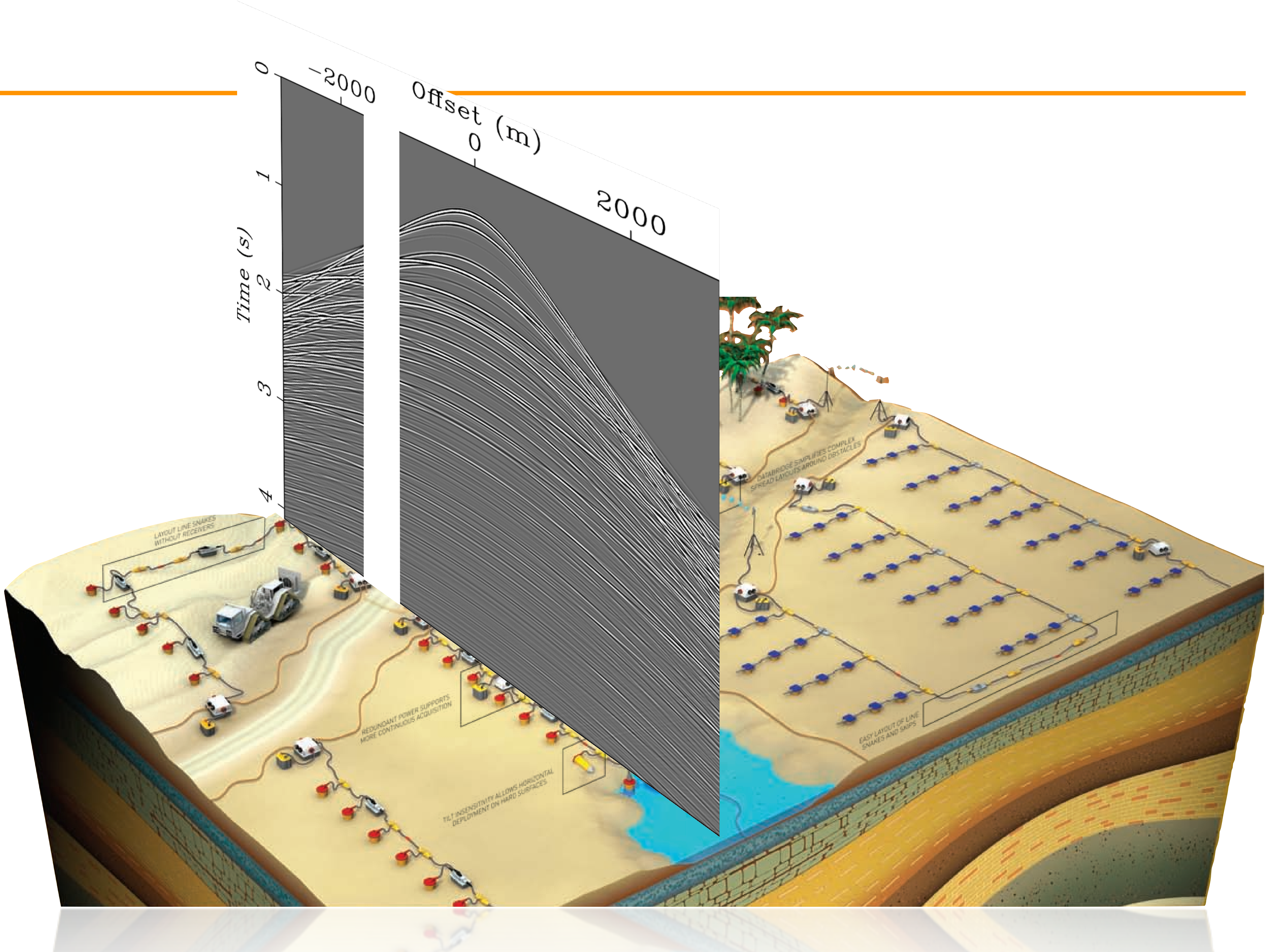
Acquisition image courtesy ION (www.iongeo.com)

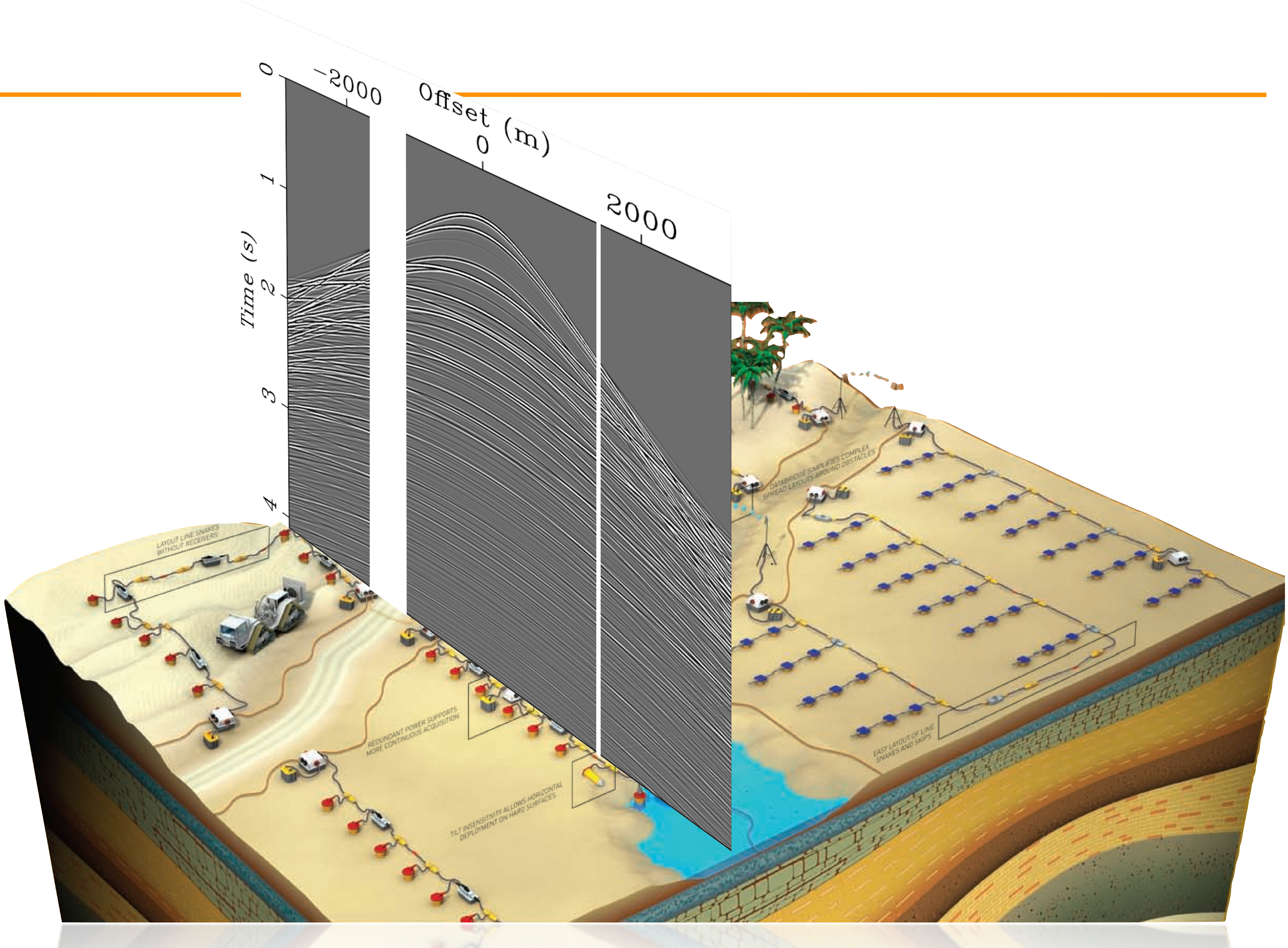


Acquisition image courtesy ION (www.iongeo.com)



Acquisition image courtesy ION (www.iongeo.com)





Acquisition image courtesy ION (www.iongeo.com)

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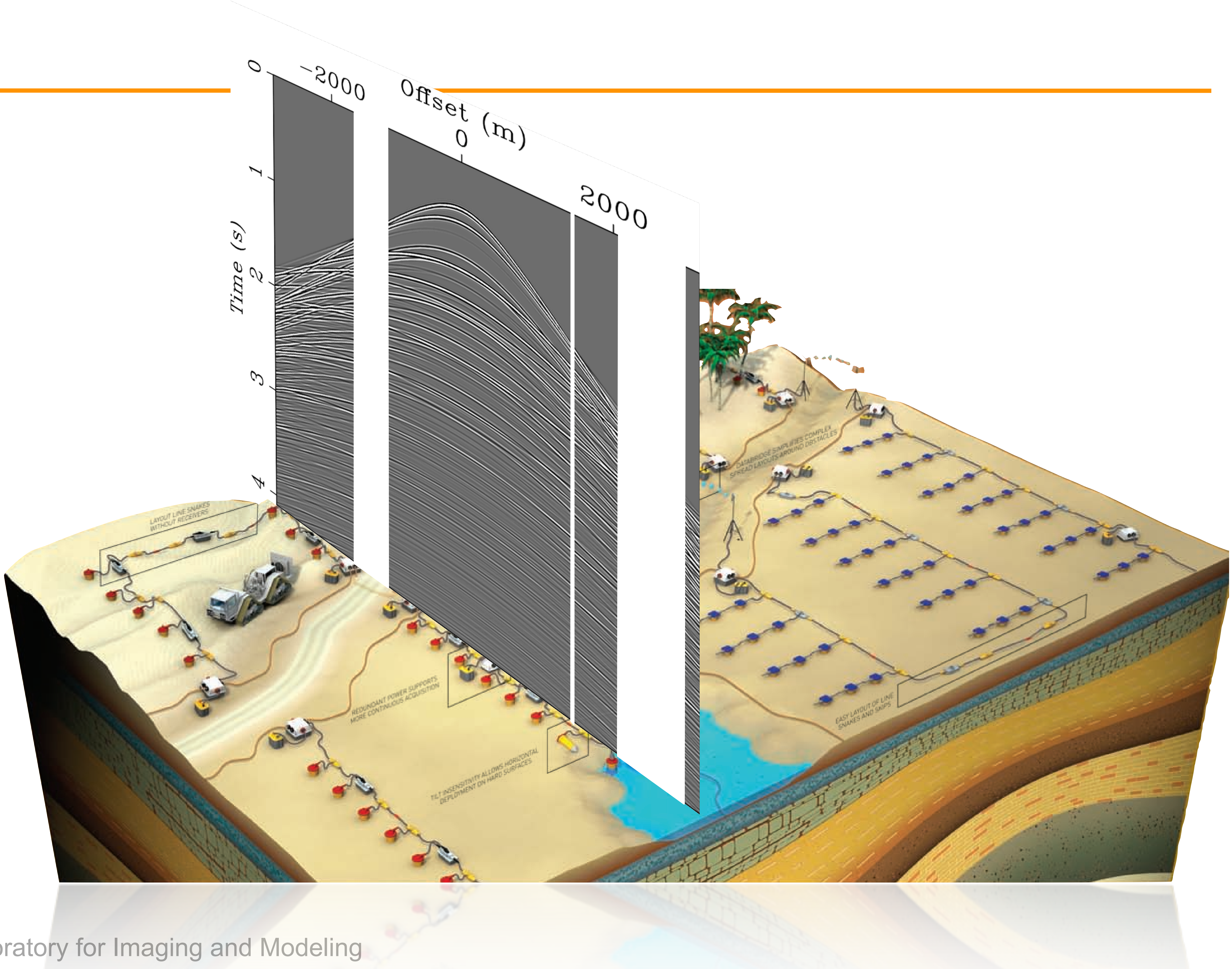
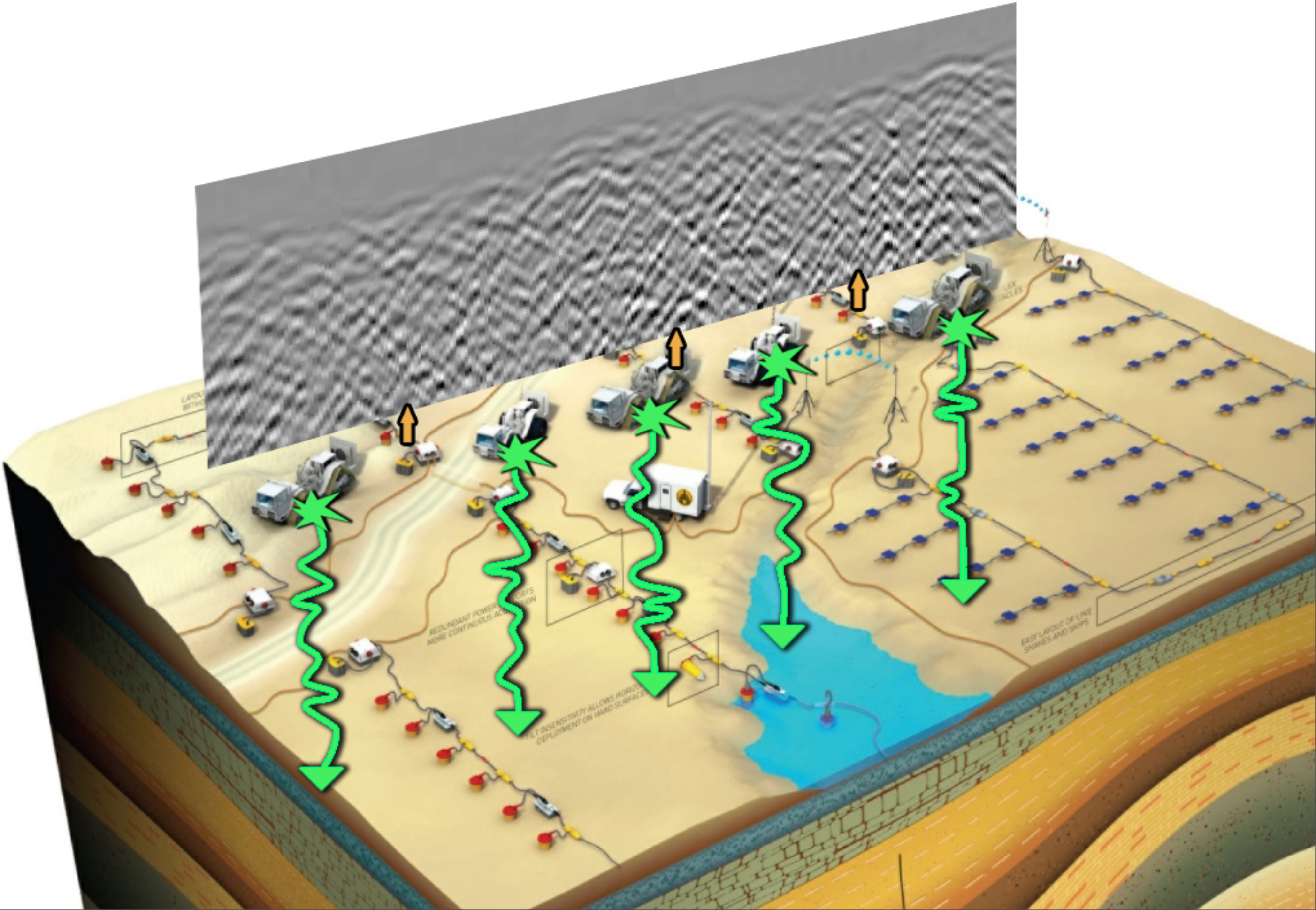



REDUNDANT POWER SUPPORTS
MORE CONTINUOUS ACQUISITION
TILT INSENSITIVITY ALLOWS HORIZ
DEPLOYMENT ON HARD SURFACE
EASY LAYOUT OF LINE
SNAKES AND SKIPS

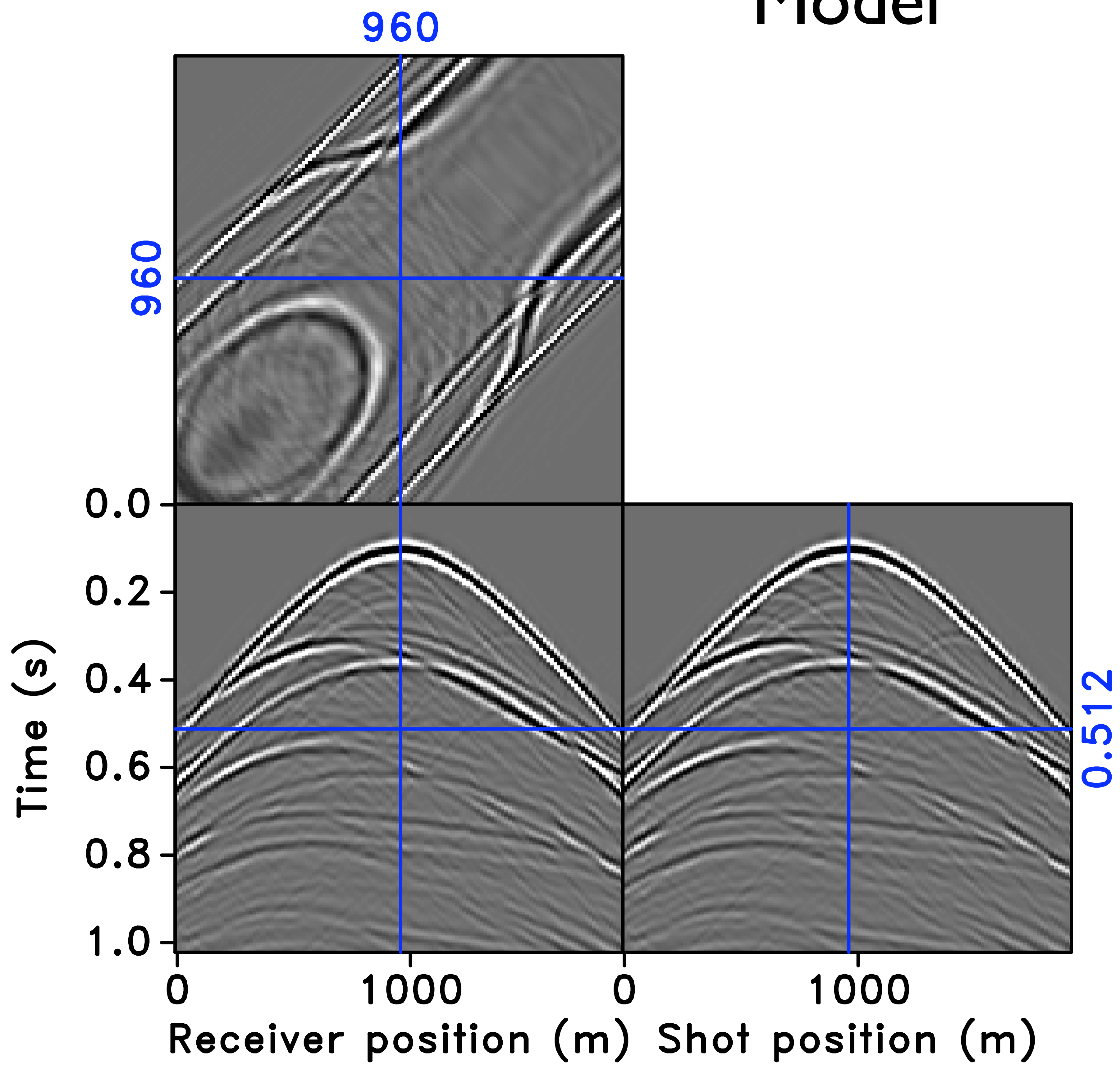
Questions

- **What is better to *periodically* sample *sequential* impulsive sources or to sample at *randomized* positions?**

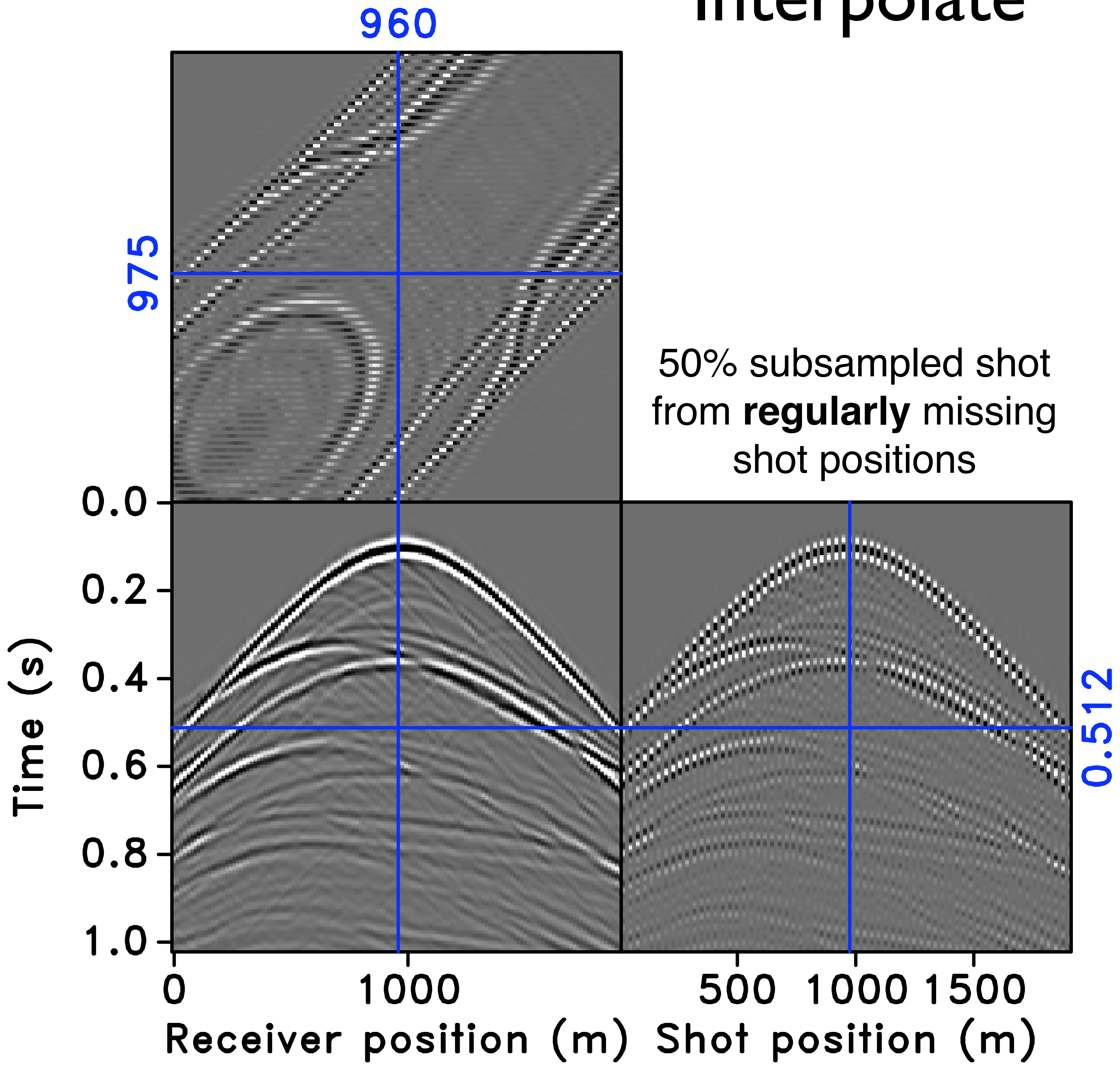
- **What is better? Having missing *single-source* or *missing randomized incoherent simultaneous* experiments?**

- Comparison between different undersampling strategies for source experiments:
  - ***Randomized*** jittered shot positions
  - ***Randomized*** simultaneous shots

Model

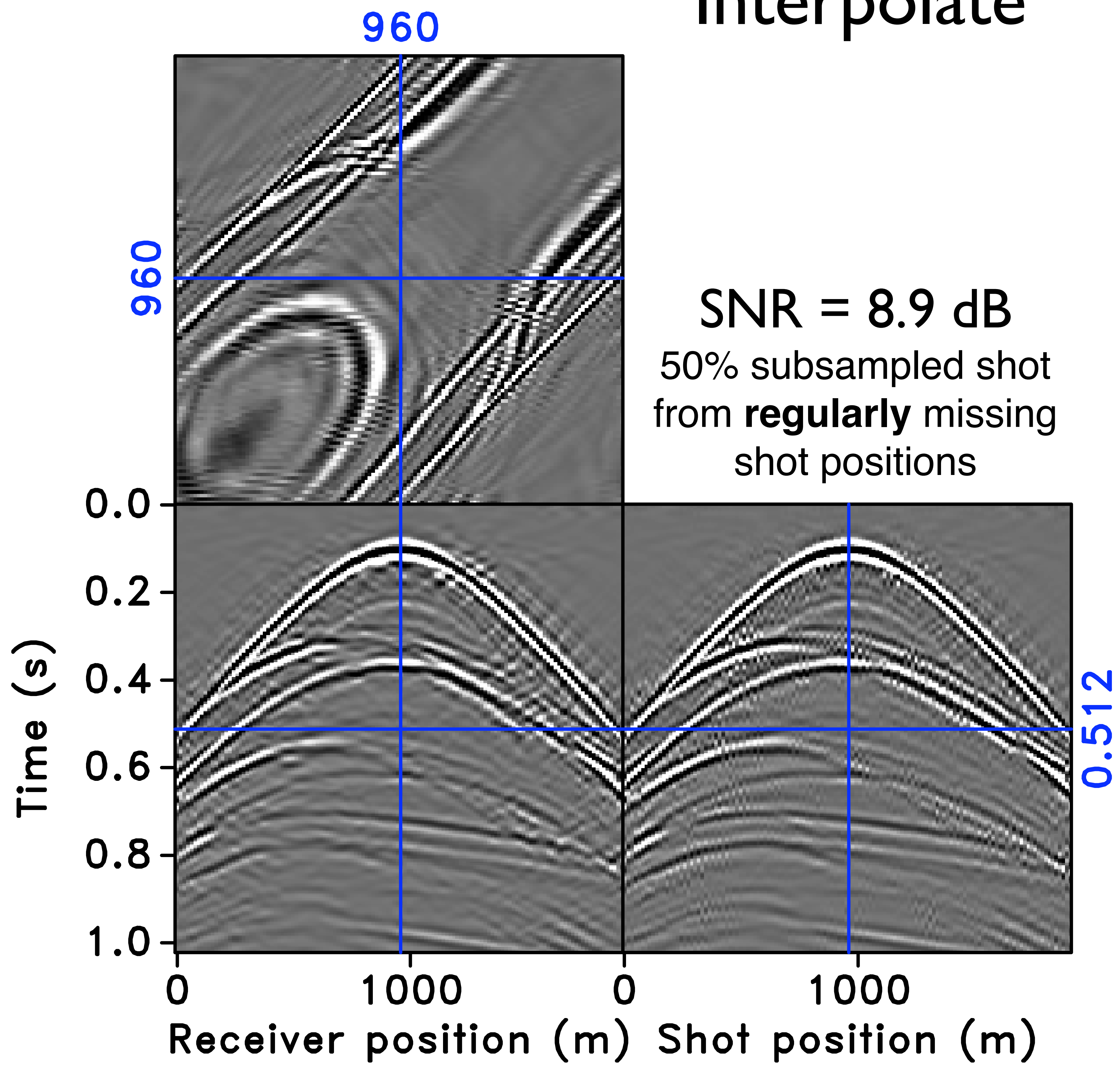

960
960
0.512
0.0
0.2
0.4
0.6
0.8
1.0
Time (s)
0
1000
0
1000
Receiver position (m)
Shot position (m)

Interpolate



50% subsampled shot from **regularly** missing shot positions

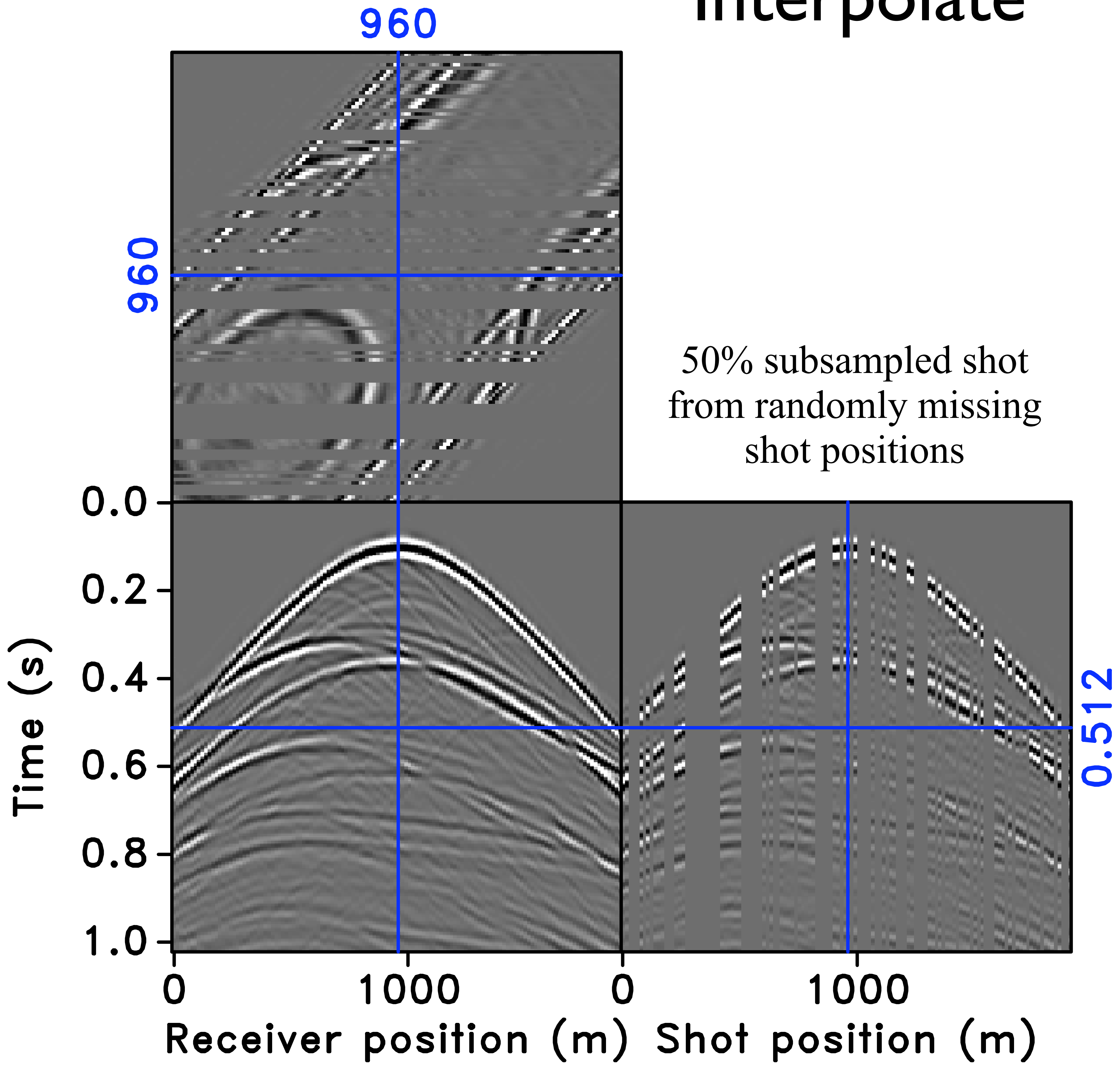
Interpolate



SNR = 8.9 dB

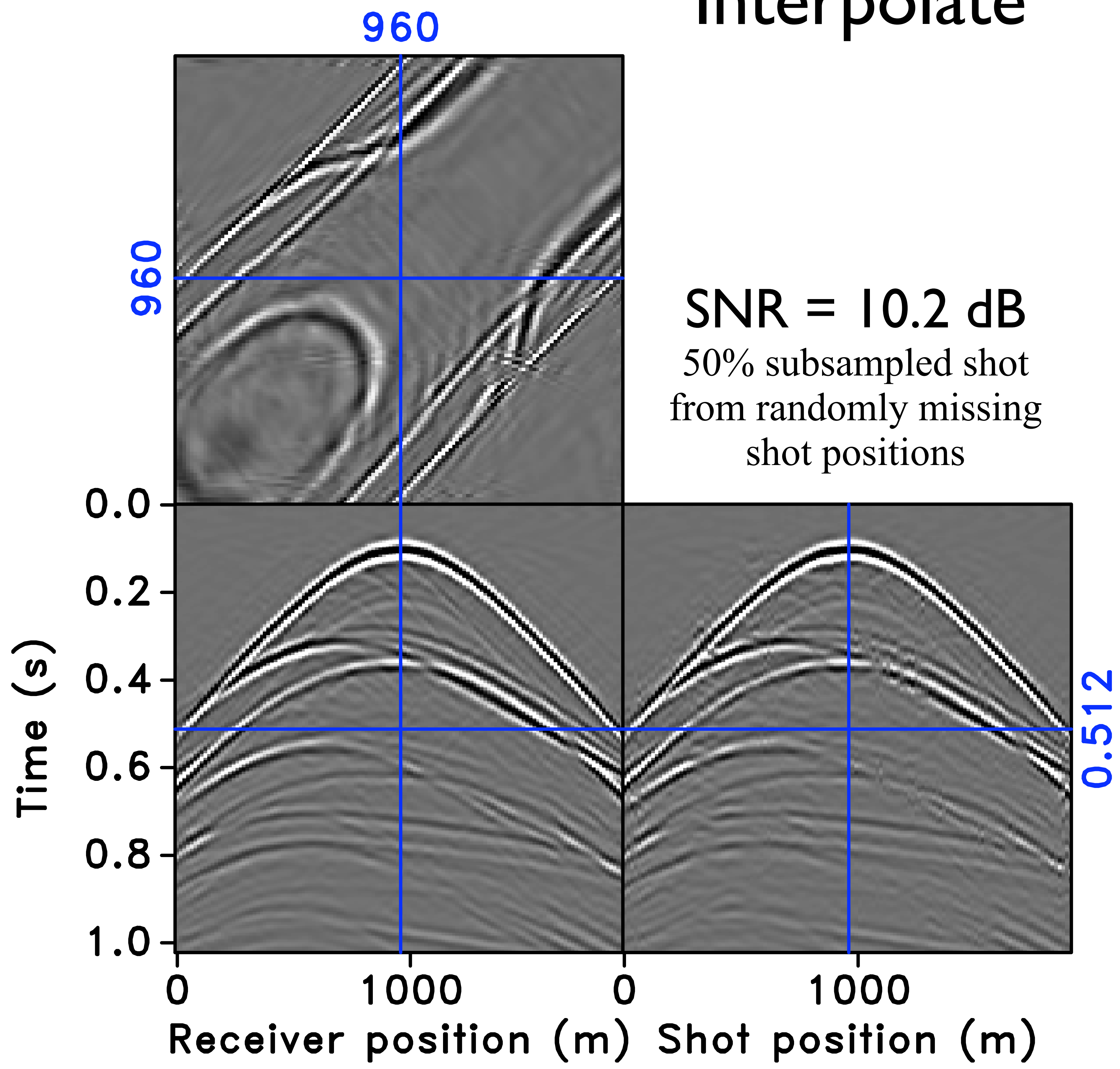
50% subsampled shot from **regularly** missing shot positions

Interpolate



50% subsampled shot
from randomly missing
shot positions

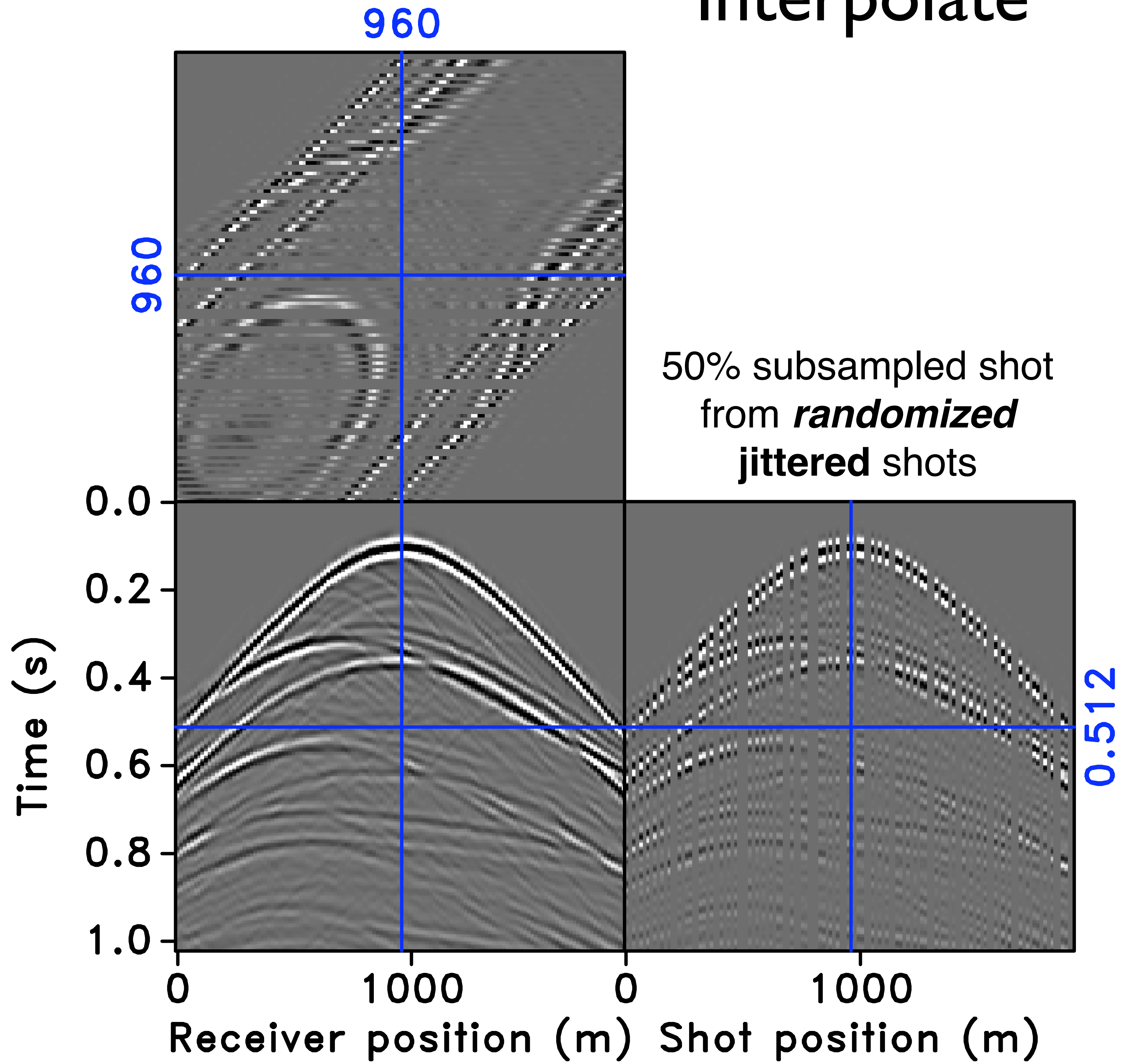
Interpolate



SNR = 10.2 dB

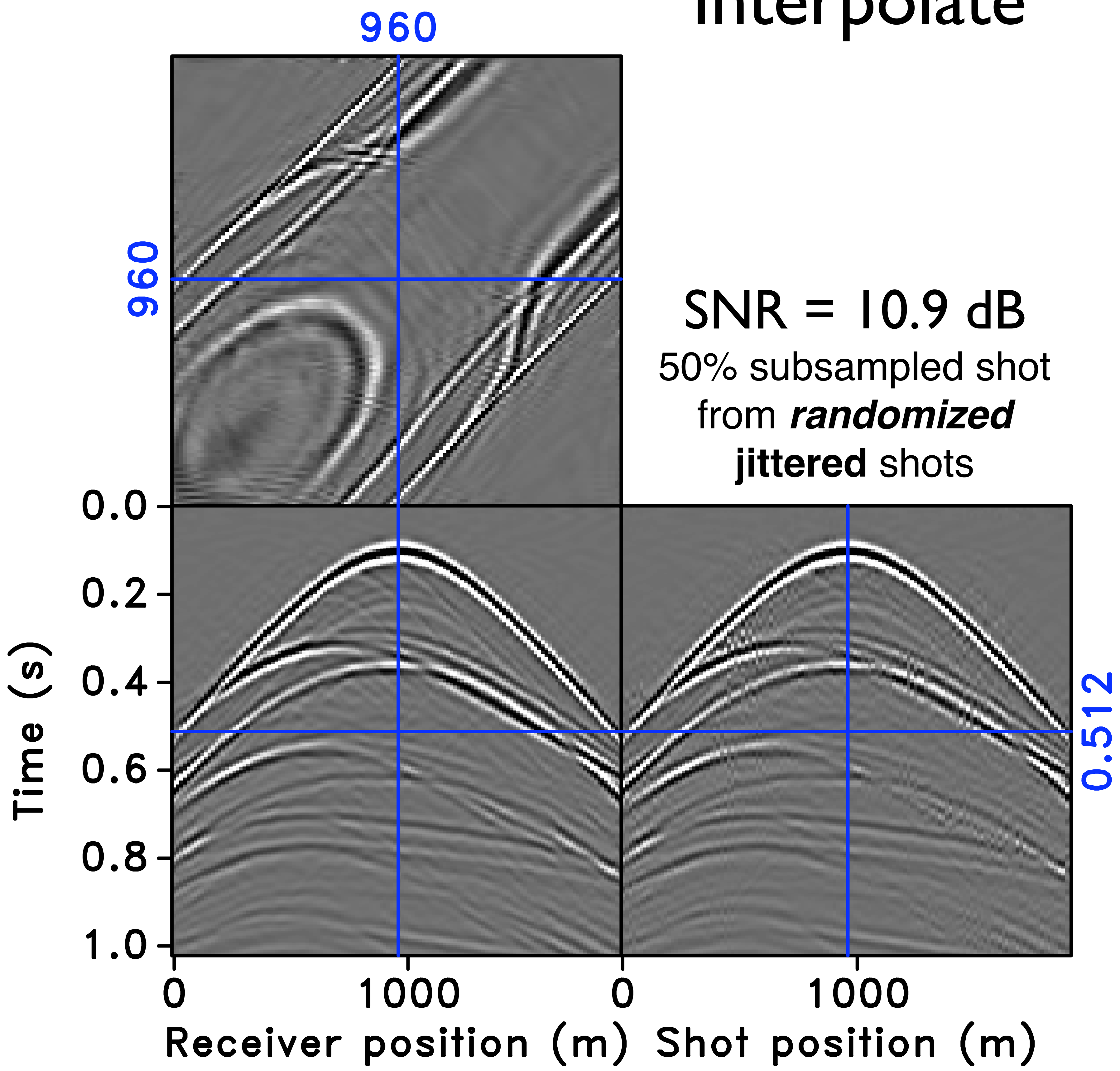
50% subsampled shot
from randomly missing
shot positions

Interpolate



50% subsampled shot
from ***randomized***
jittered shots

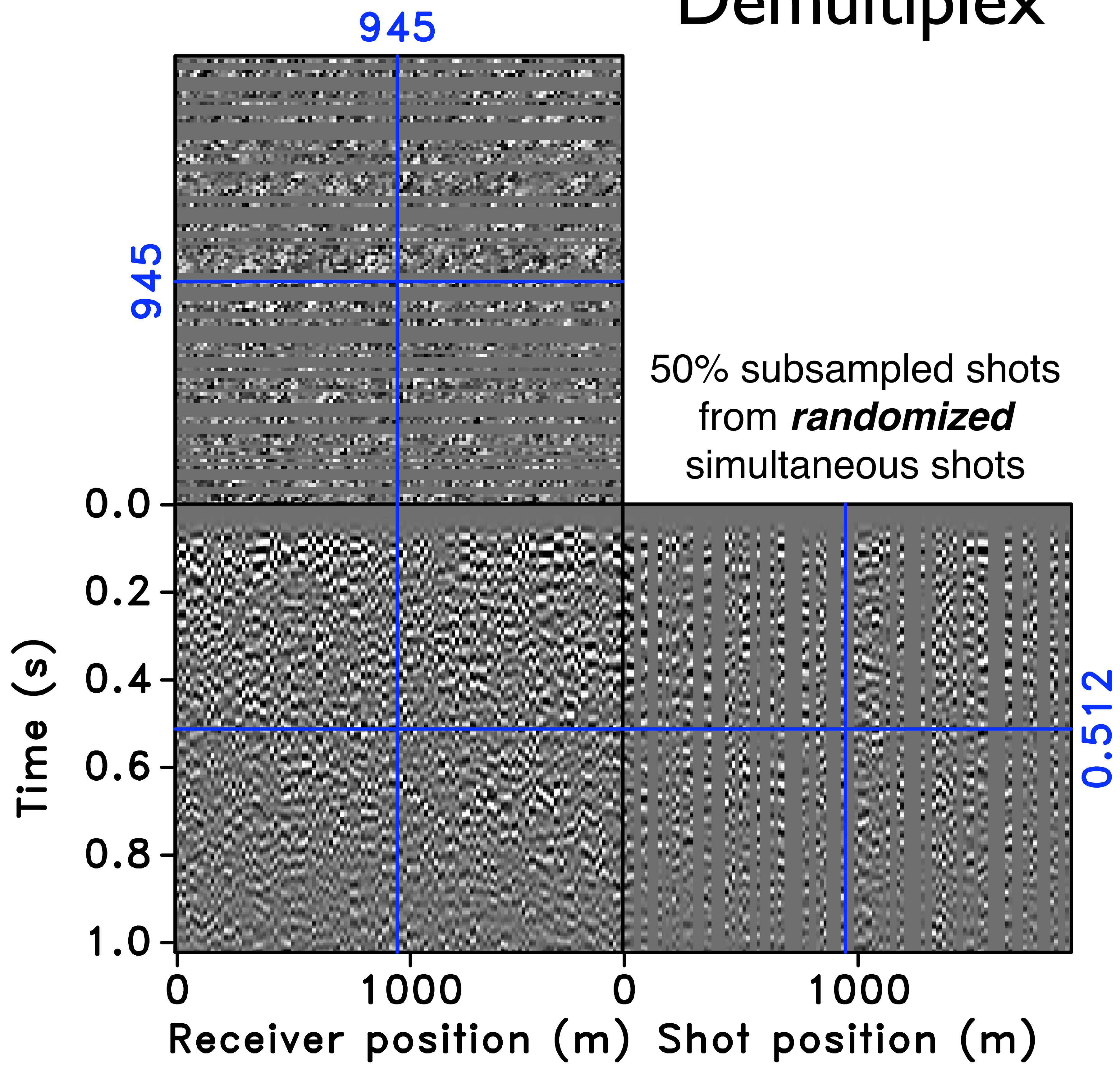
Interpolate



SNR = 10.9 dB

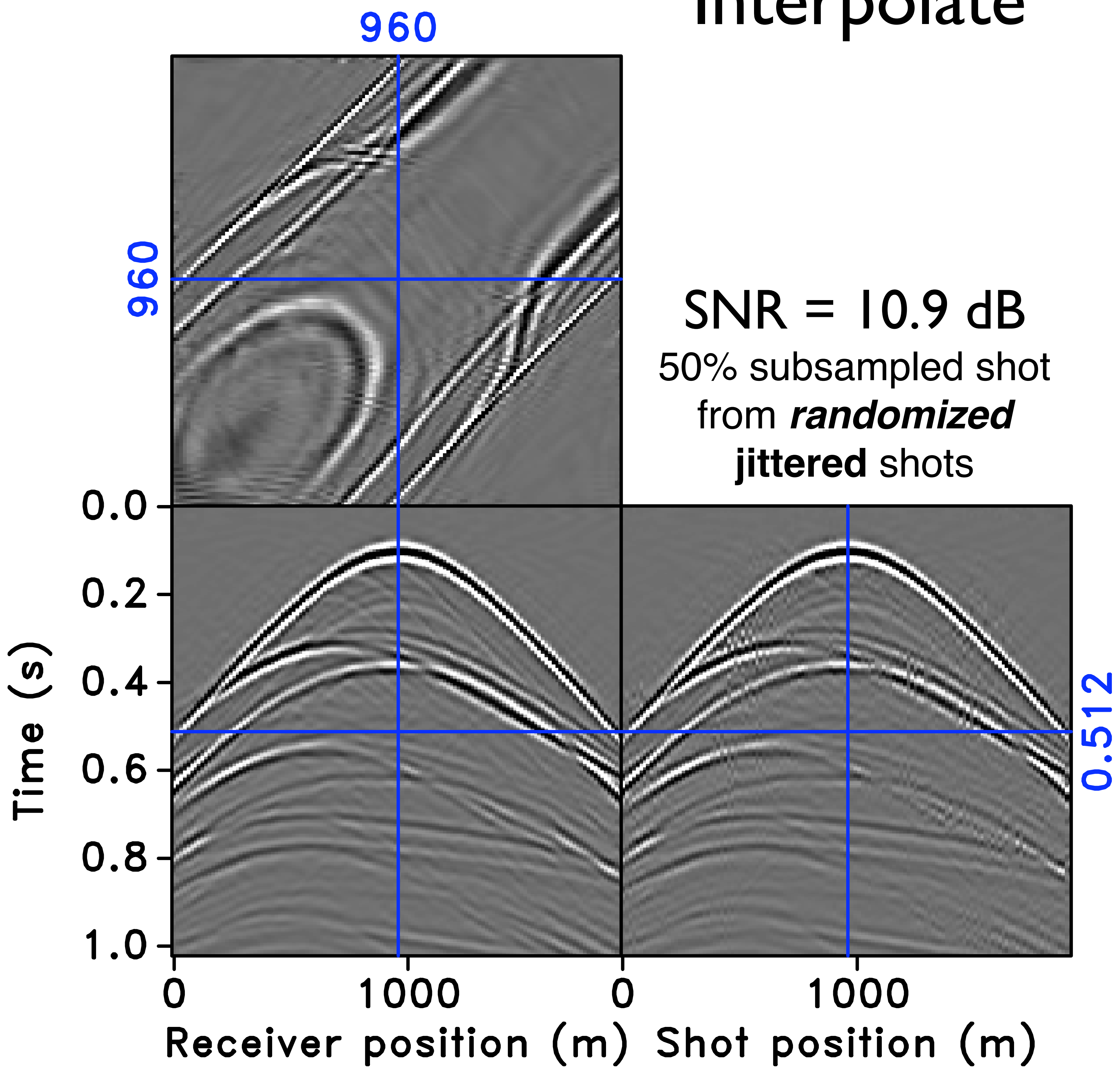
50% subsampled shot
from ***randomized***
jittered shots

Demultiplex



50% subsampled shots
from ***randomized***
simultaneous shots

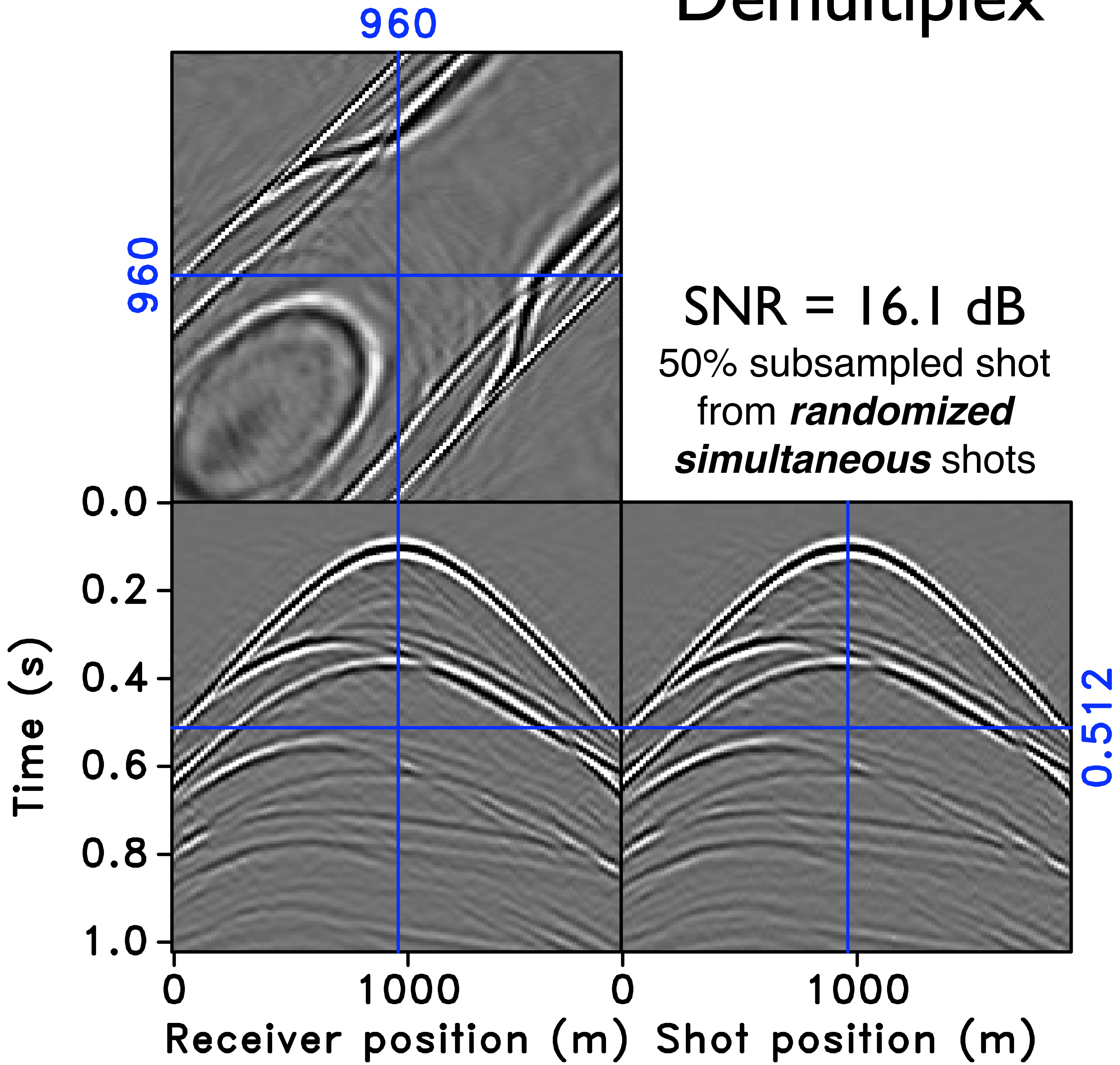
Interpolate



SNR = 10.9 dB

50% subsampled shot from ***randomized*** **jittered** shots

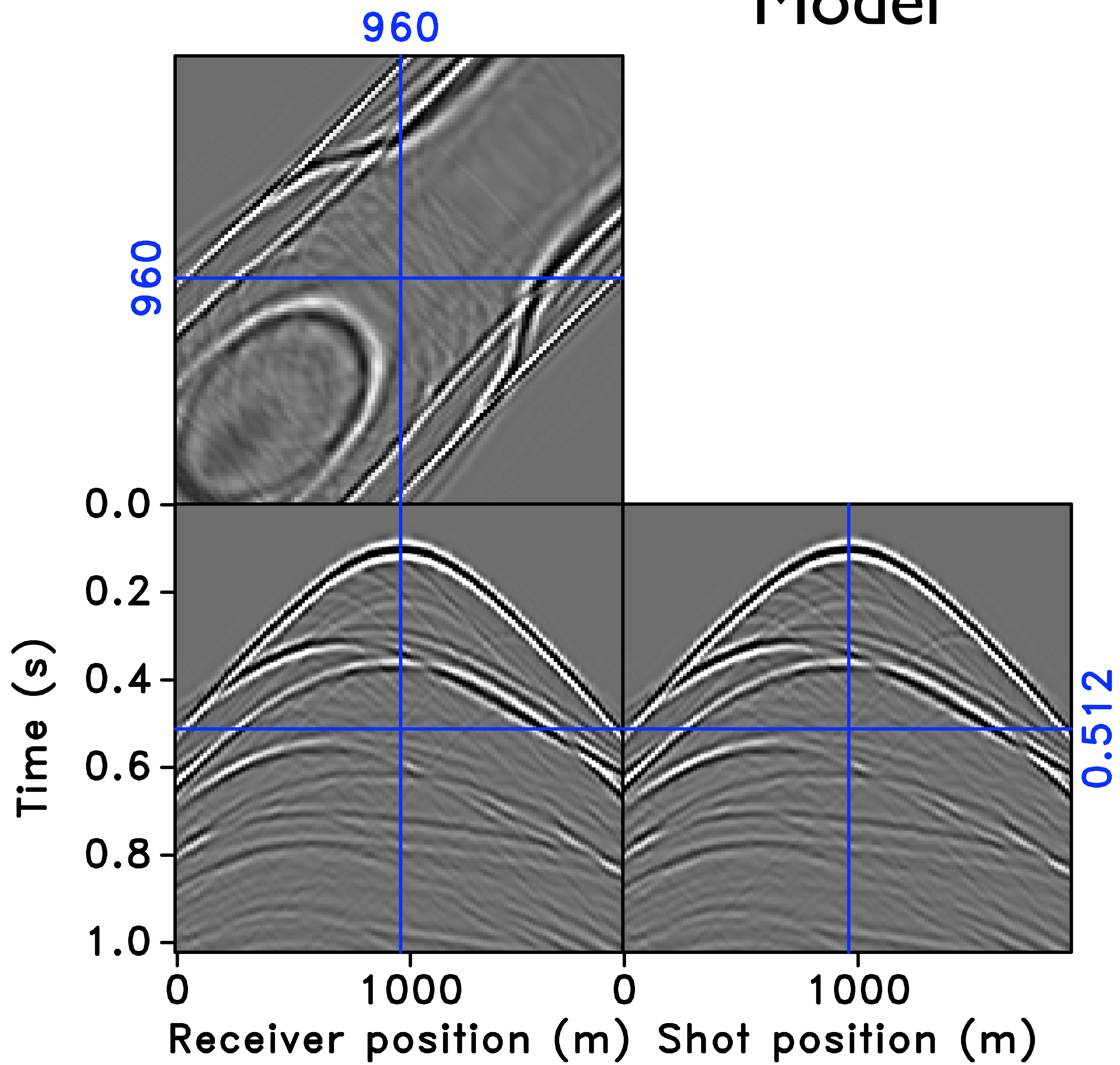
Demultiplex



SNR = 16.1 dB

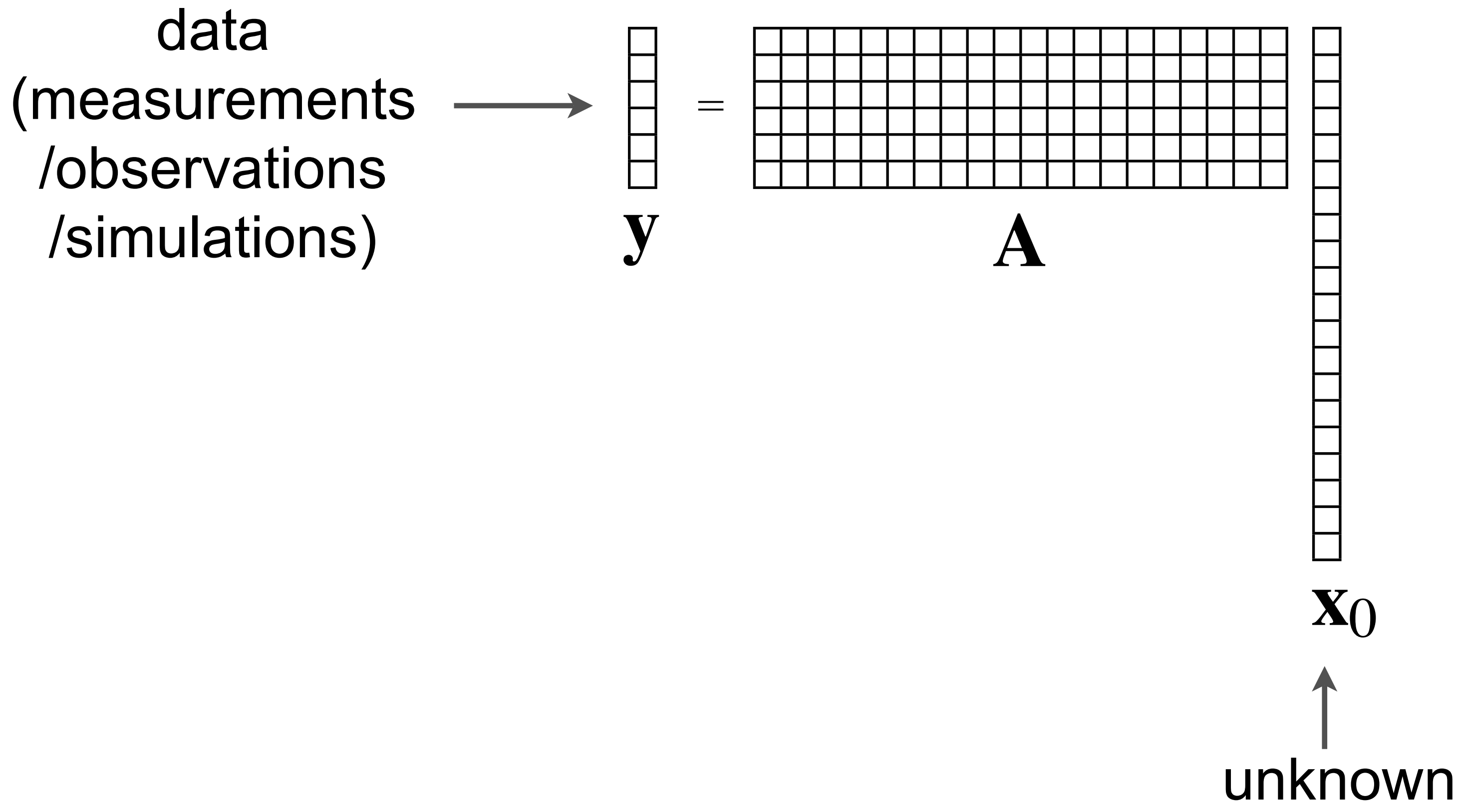
50% subsampled shot
from ***randomized***
simultaneous shots

Model


960
960
0.512
0.0
0.2
0.4
0.6
0.8
1.0
Time (s)
0
1000
0
1000
Receiver position (m)
Shot position (m)

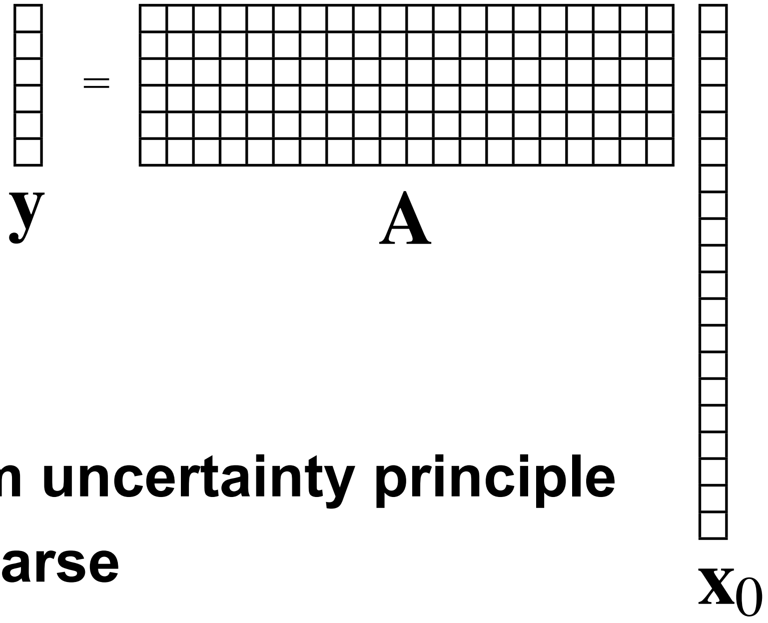
Problem statement

Consider the following (severely) underdetermined system of linear equations



Is it possible to recover $\mathbf{x}_0$ accurately from $\mathbf{y}$?

Perfect recovery



- conditions
  - **A** obeys the **uniform uncertainty principle**
  - **x**$_0$ is **sufficiently sparse**

- recovery procedure

$$\underbrace{\min_{\mathbf{x}} \|\mathbf{x}\|_{\ell_1} = \sum_n |x_i|}_{\text{"sparsity"}} \quad \text{subject to} \quad \underbrace{\mathbf{Ax} = \mathbf{y}}_{\text{perfect reconstruction}}$$

- performance
  - *k*-sparse vectors recovered from roughly on the order of *k* measurements (to within constant and *log* factors)

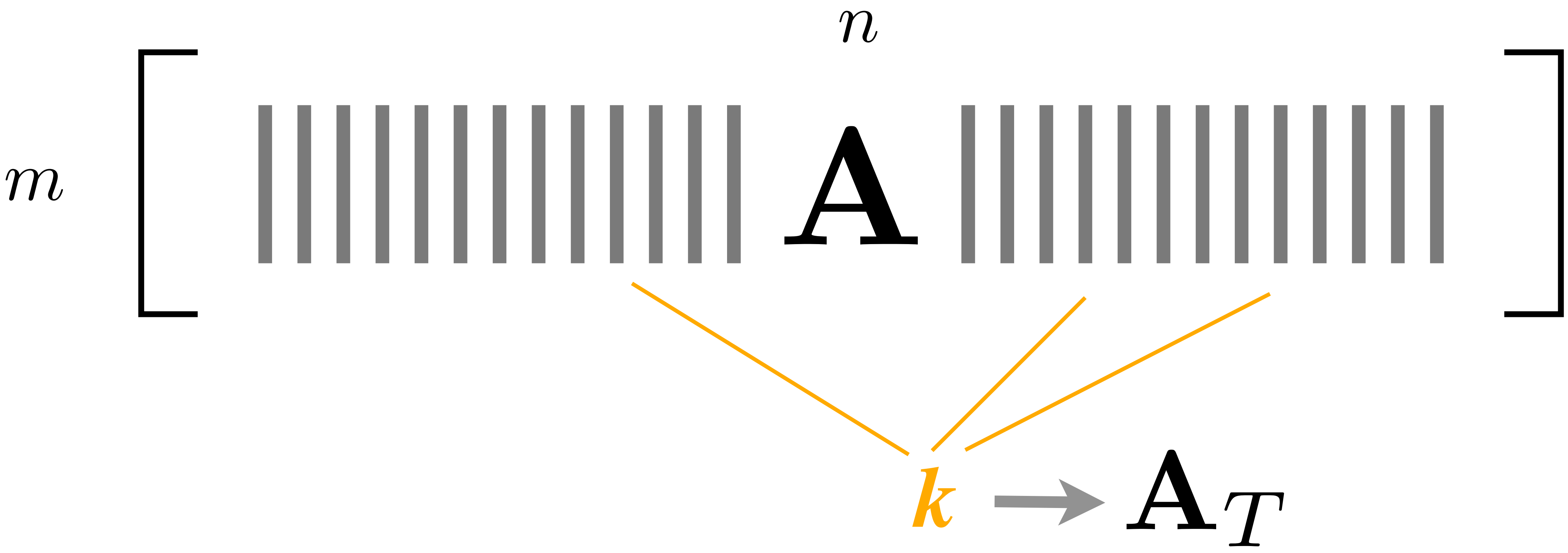
[Candès et al.'06]
[Donoho'06]

Designing CS acquisition matrix

- Restricted Isometry Property holds $m \geq C \cdot k \log(n/k)$

$$(1-\delta_k)\|\mathbf{x}_T\|_{\ell 2} \leq \|\mathbf{A}_T\mathbf{x}\|_{\ell 2} \leq (1+\delta_k)\|\mathbf{x}_T\|_{\ell 2}$$

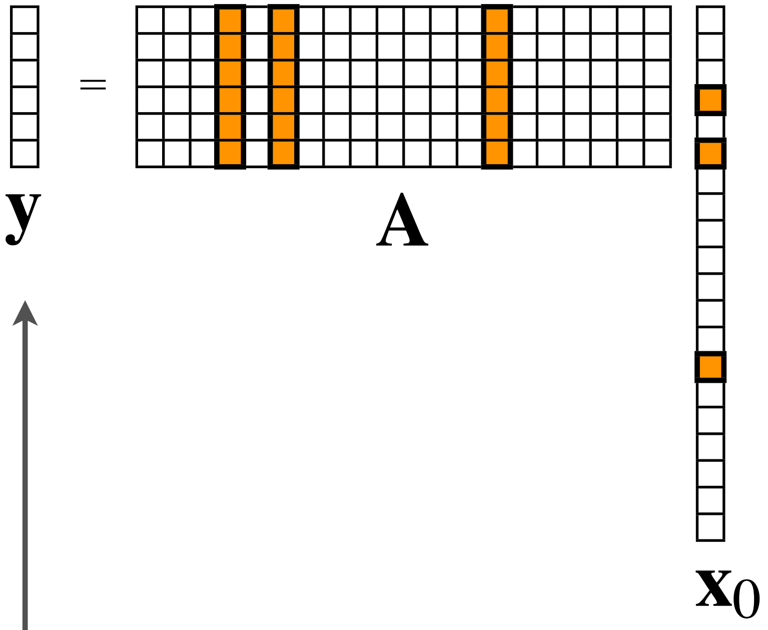
- Bounds singular values of **A**



- Subsets T of columns of **A** behave approximately as an orthonormal basis => stable
- Construction of **A** depends on ***randomization*** => spread of energy
- **A** like a matrix with iid zero-centered random Gaussian entries

Simple example

restriction operator

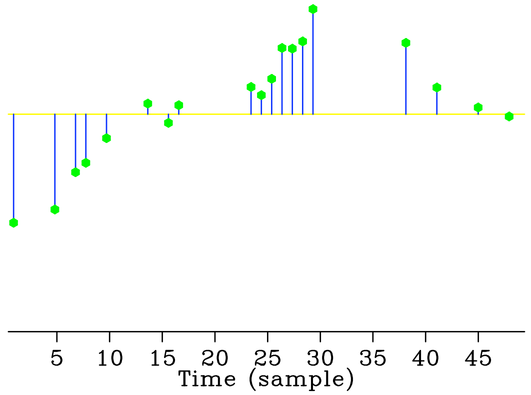


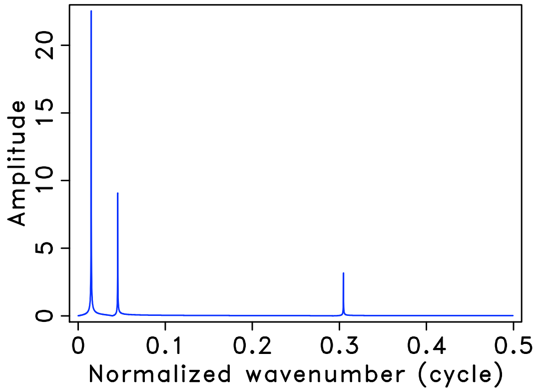
with $\mathbf{A} := \mathbf{R}\mathbf{F}^H$

Fourier transform

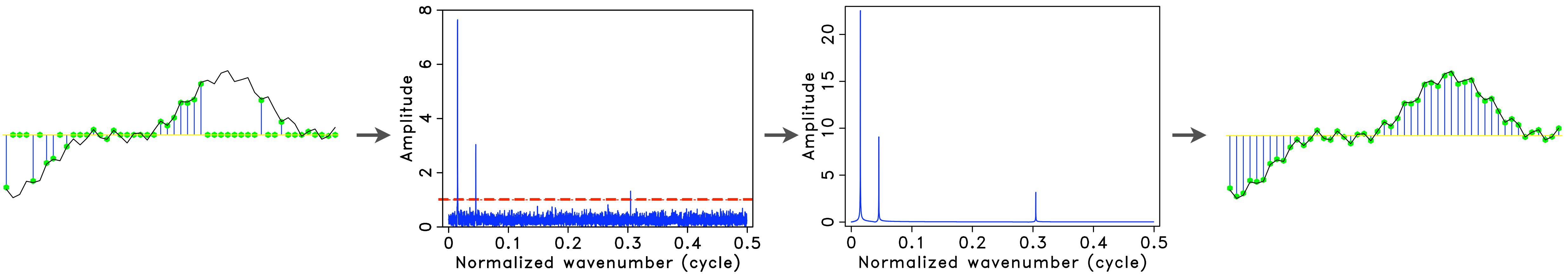
signal

Fourier coefficients (sparse)





NAIVE sparsity-promoting recovery



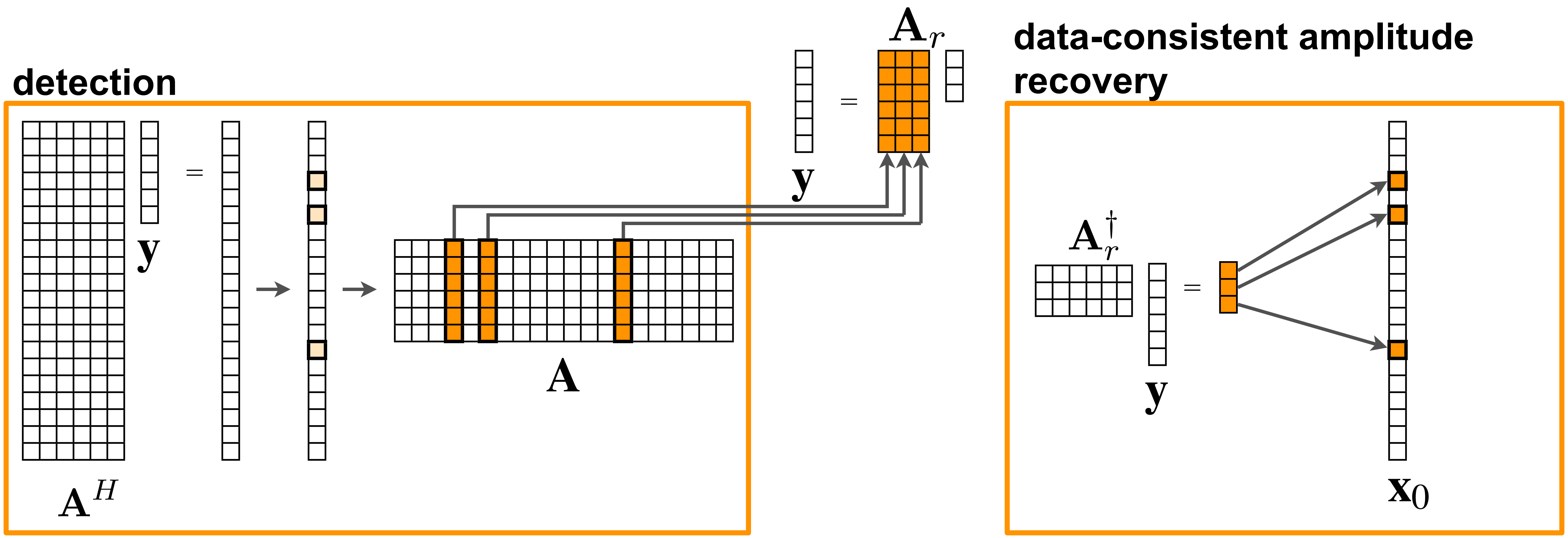
inverse Fourier transform

detection + data-consistent amplitude recovery

Fourier transform

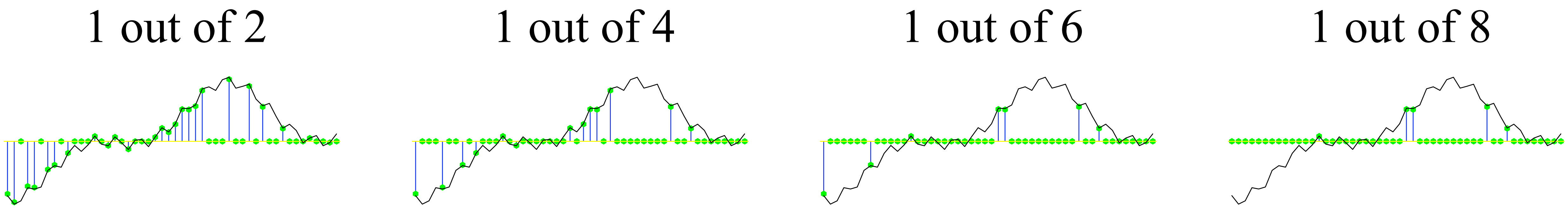
detection

data-consistent amplitude recovery



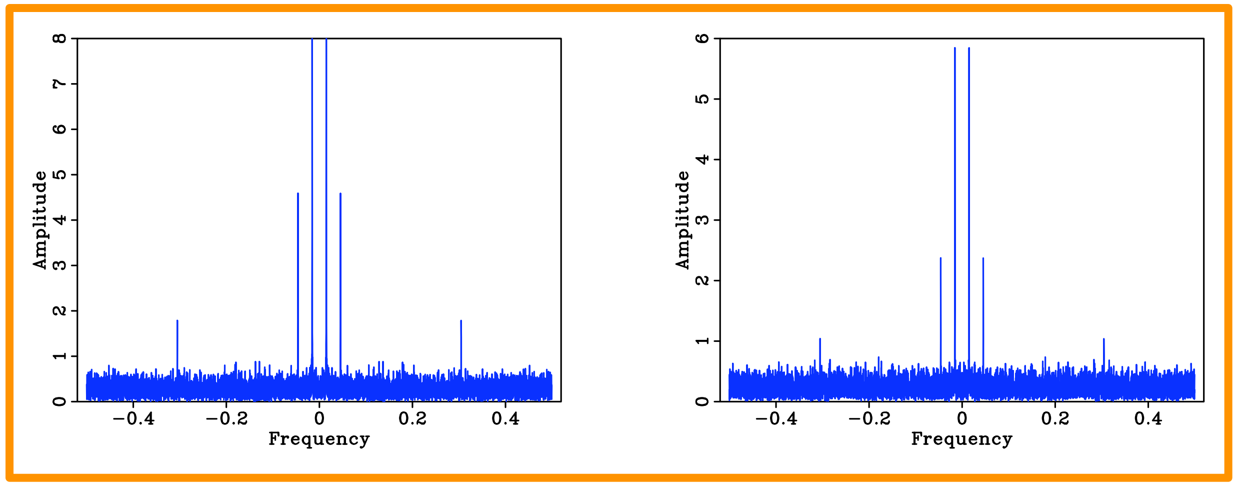
Undersampling “noise”

- “noise”
  - due to $\mathbf{A}^H\mathbf{A} \neq \mathbf{I}$
  - defined by $\mathbf{A}^H\mathbf{A}\mathbf{x}_0 - \alpha\mathbf{x}_0 = \mathbf{A}^H\mathbf{y} - \alpha\mathbf{x}_0$

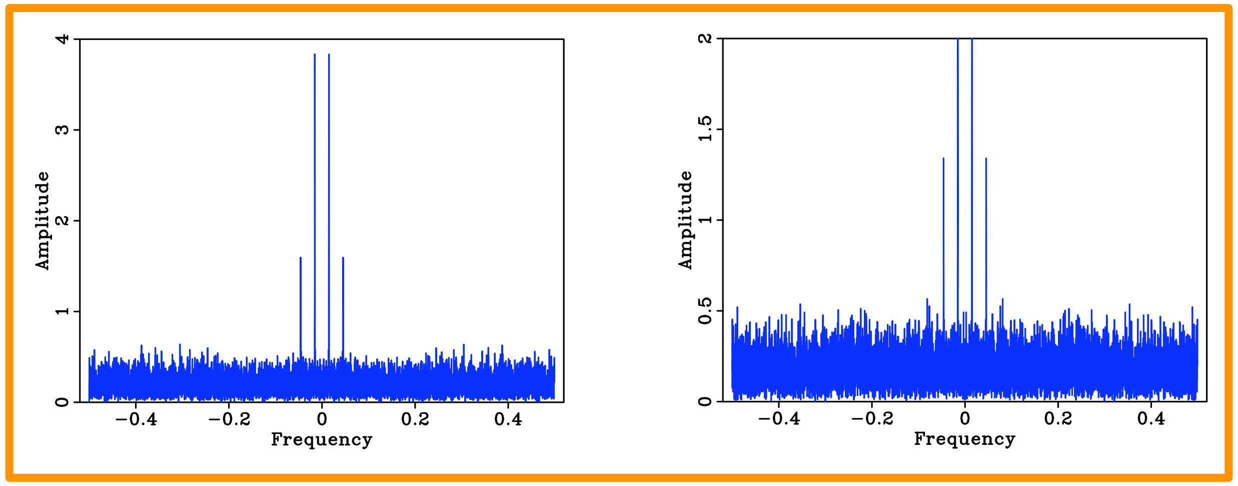


less acquired data

3 detectable Fourier modes



2 detectable Fourier modes



Extensions

- Use CS principles to select *physically* appropriate
  - ***randomized*** *restriction* matrix **R = downsampler**
  - *measurement* matrix either **I**, or ***random*** **phase encoder**, or ***randomized physics***
  - sparsifying transform **S** (e.g. curvelets)
  - driven by signal type, physics, and type of acquisition (e.g. fMRI vs seismic)
- Sparse signal representation:

$$\mathbf{y} = \mathbf{A}\mathbf{x}_0$$

with

$$\mathbf{A} = \underset{\textbf{restriction matrix}}{\mathbf{R}}\,\underset{\textbf{measurement matrix}}{\mathbf{M}}\,\underset{\textbf{sparsity matrix}}{\mathbf{S}}^{H}$$

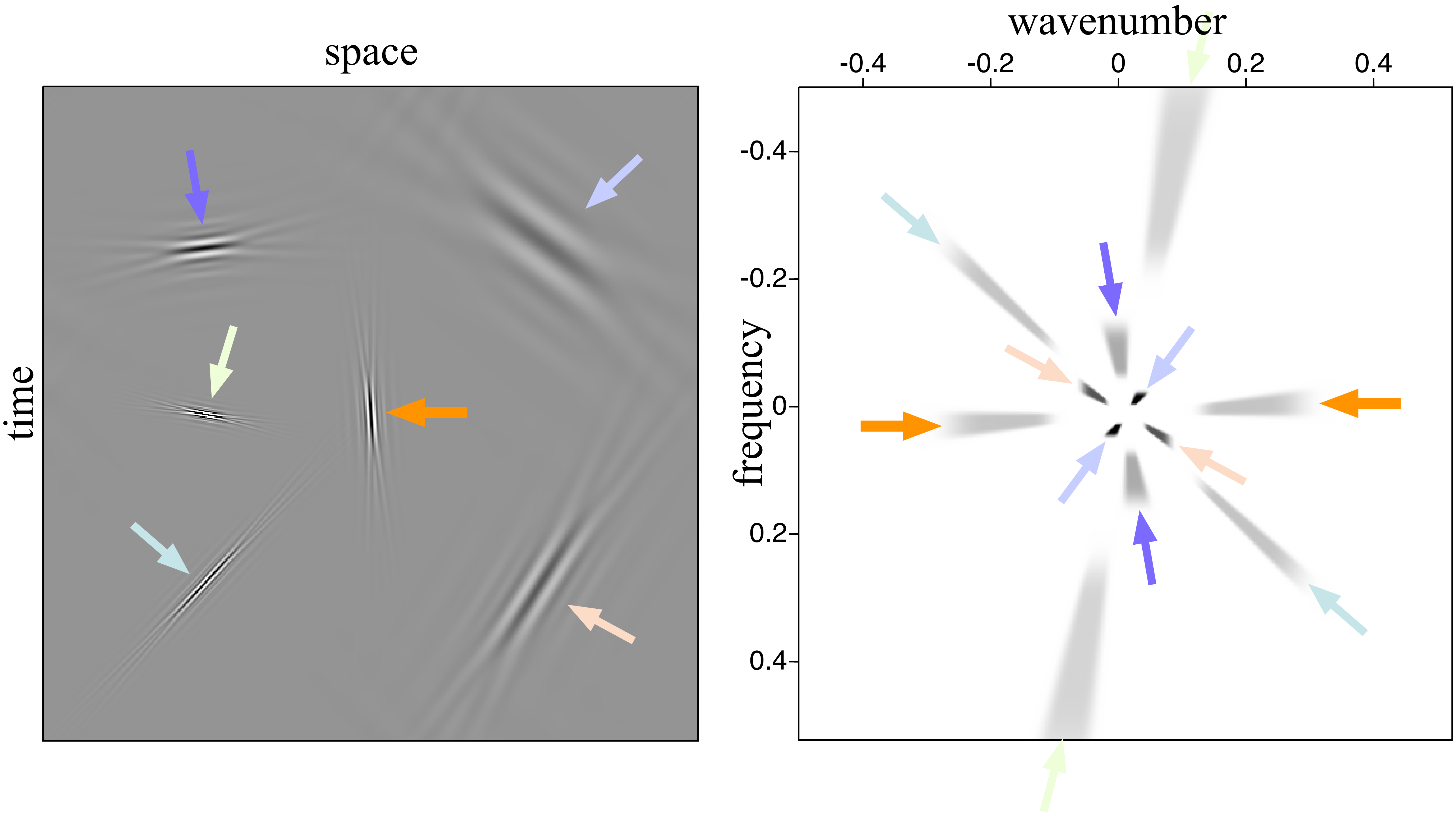
Selection turns *aliases/coherent subsampling artifacts* into harmless ***noise*** ...

Problem: CS does not yet provide practical design principles ...

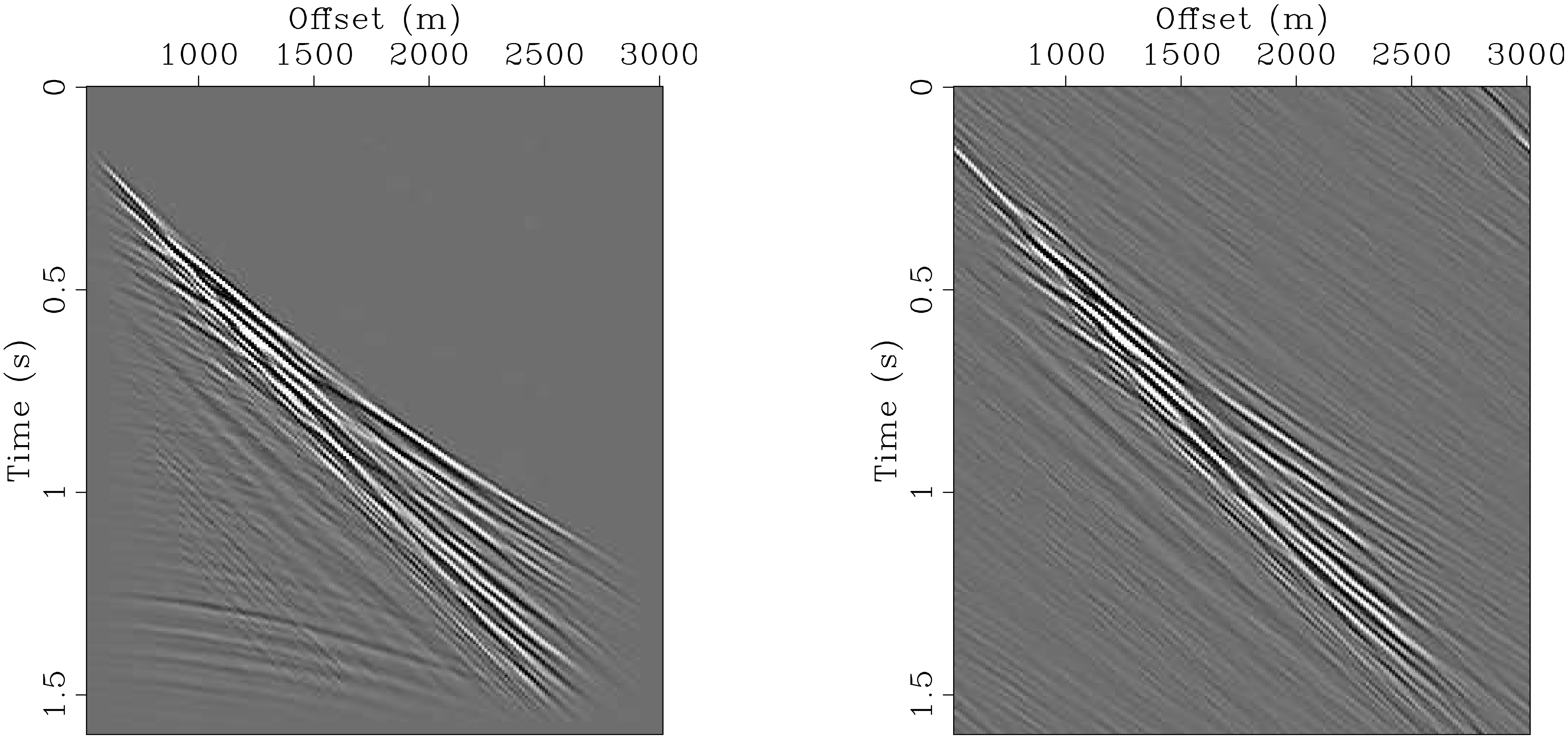
Key elements

- ☐ *sparsifying transform*
  - typically *strictly* **localized** in the Fourier space
  - rapid decay physical space to handle the complexity of seismic data
  - mutual incoherence
- ☐ *advantageous **randomized** coarse sampling*
  - generates incoherent random undersampling “noise” in sparsifying domain
  - ***spatial sampling*** that does not create large gaps
    - because of the limited spatiotemporal extent of transform elements used for the reconstruction
  - ***randomized*** subsampling of ***simultaneous-source*** experiments
    - does not create large interferences
    - leads to compression of linear systems
      - reduction # right-hand-sides
      - spectral representation operators
- ☐ *sparsity-promoting solver*
  - requires few matrix-vector multiplications

2D discrete curvelets

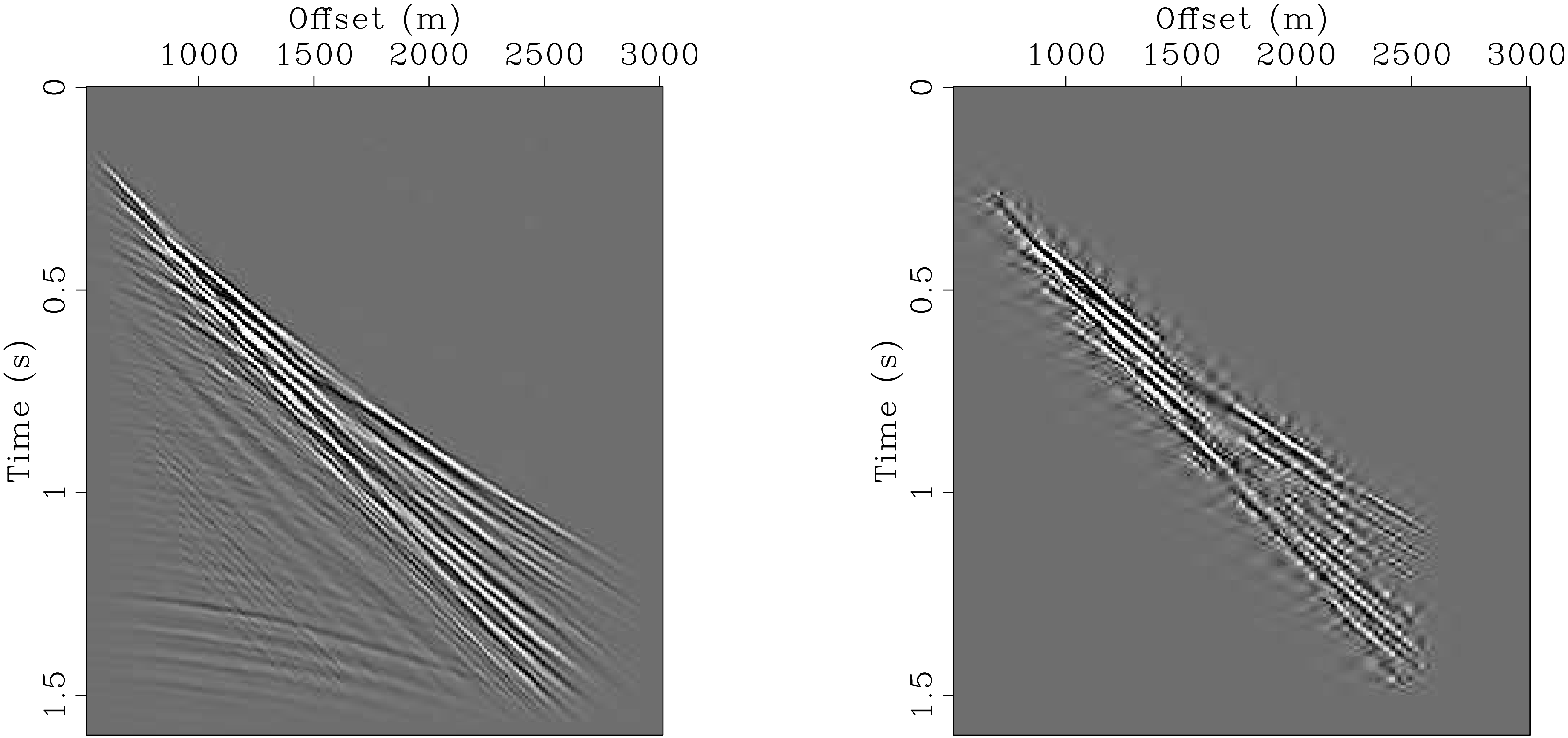


Fourier reconstruction



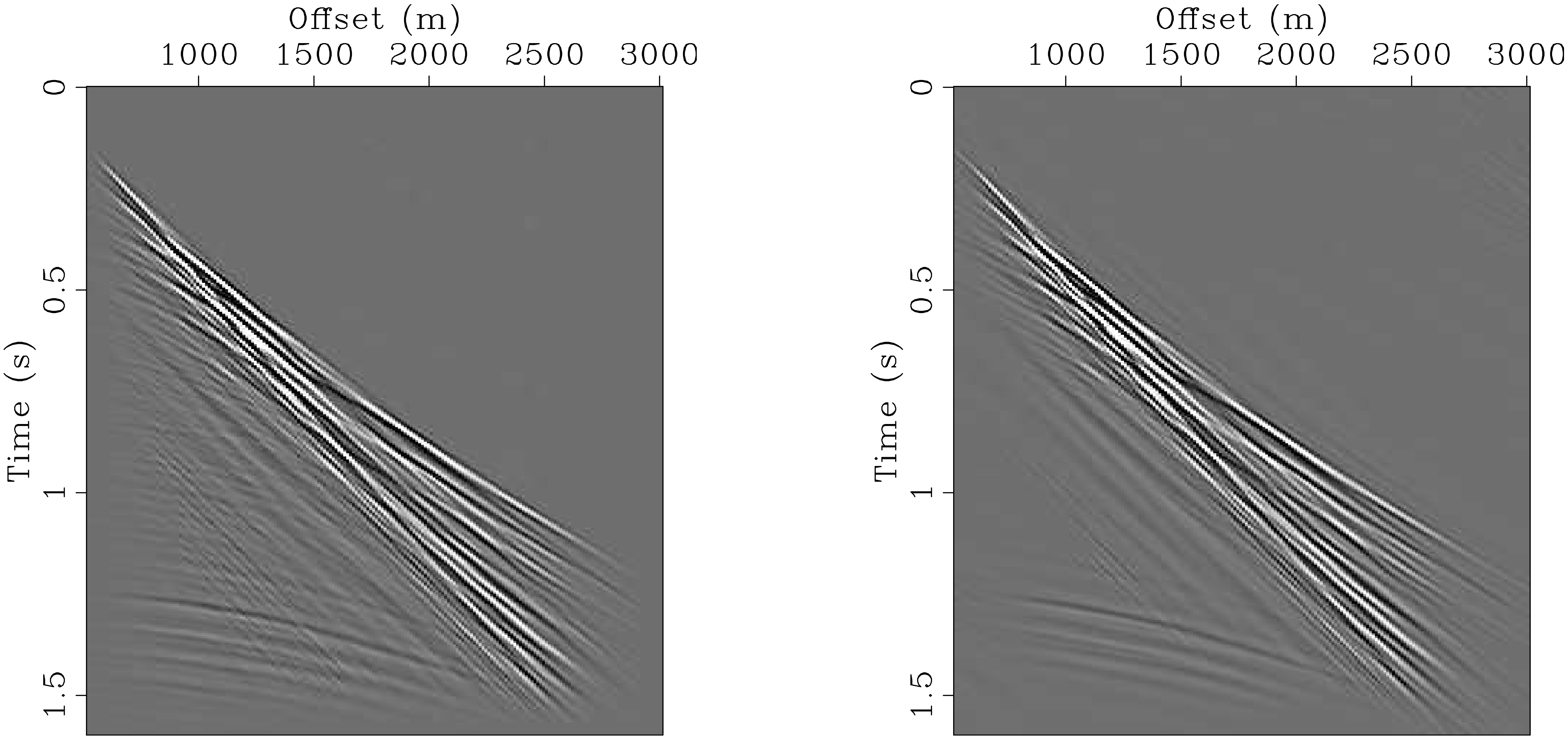
1 % of coefficients

Wavelet reconstruction



1 % of coefficients

Curvelet reconstruction



1 % of coefficients

Key elements

☑ *sparsifying transform*

- typically *strictly* **localized** in the Fourier space
- rapid decay physical space to handle the complexity of seismic data
- mutual incoherence

☐ *advantageous* ***randomized*** *coarse sampling*

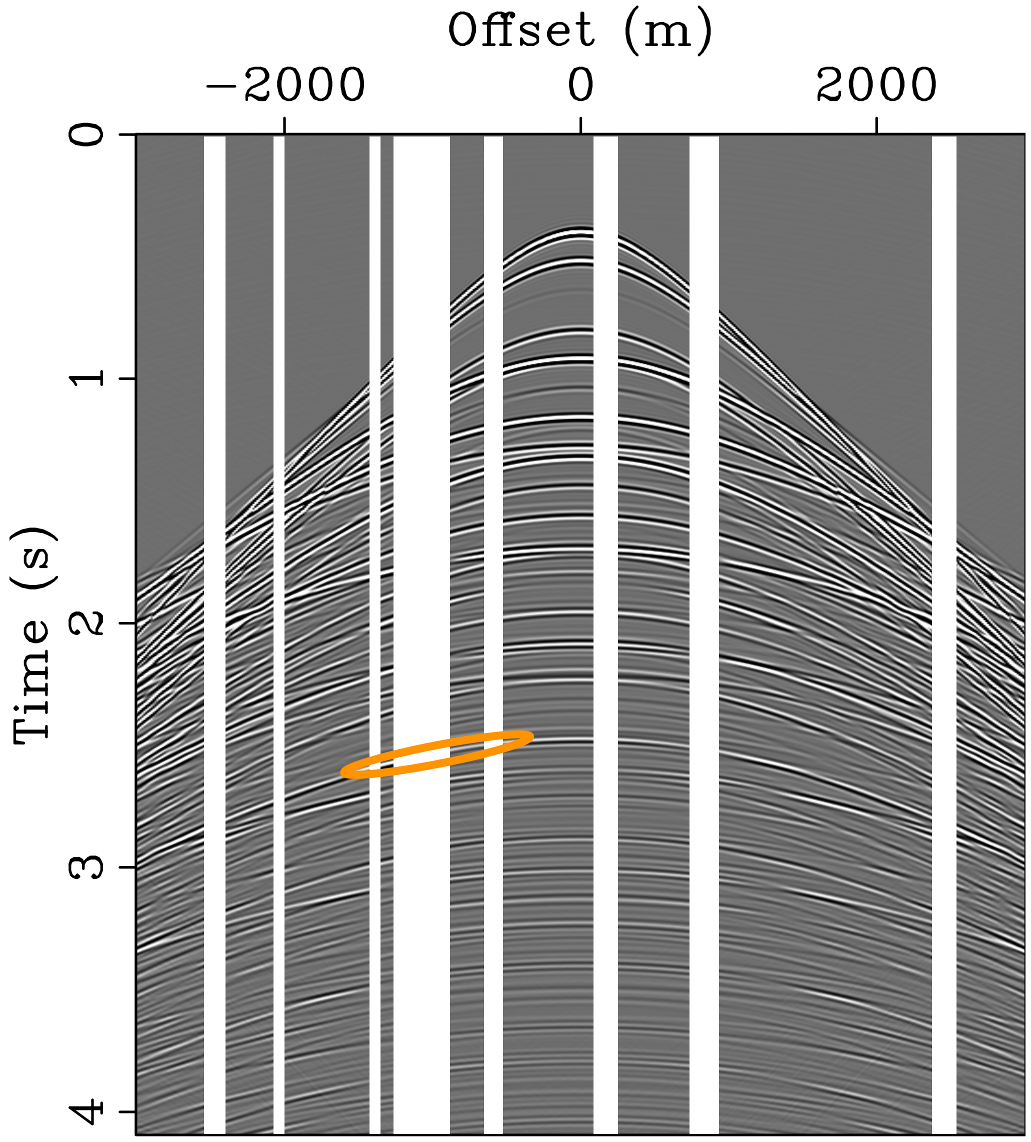
- generates incoherent random undersampling “noise” in sparsifying domain
- ***spatial sampling*** that does not create large gaps
  - because of the limited spatiotemporal extent of transform elements used for the reconstruction
- ***randomized*** subsampling of ***simultaneous-source*** experiments
  - does not create large interferences
  - leads to compression of linear systems
    - reduction # right-hand-sides
    - spectral representation operators

☐ *sparsity-promoting solver*

- requires few matrix-vector multiplications

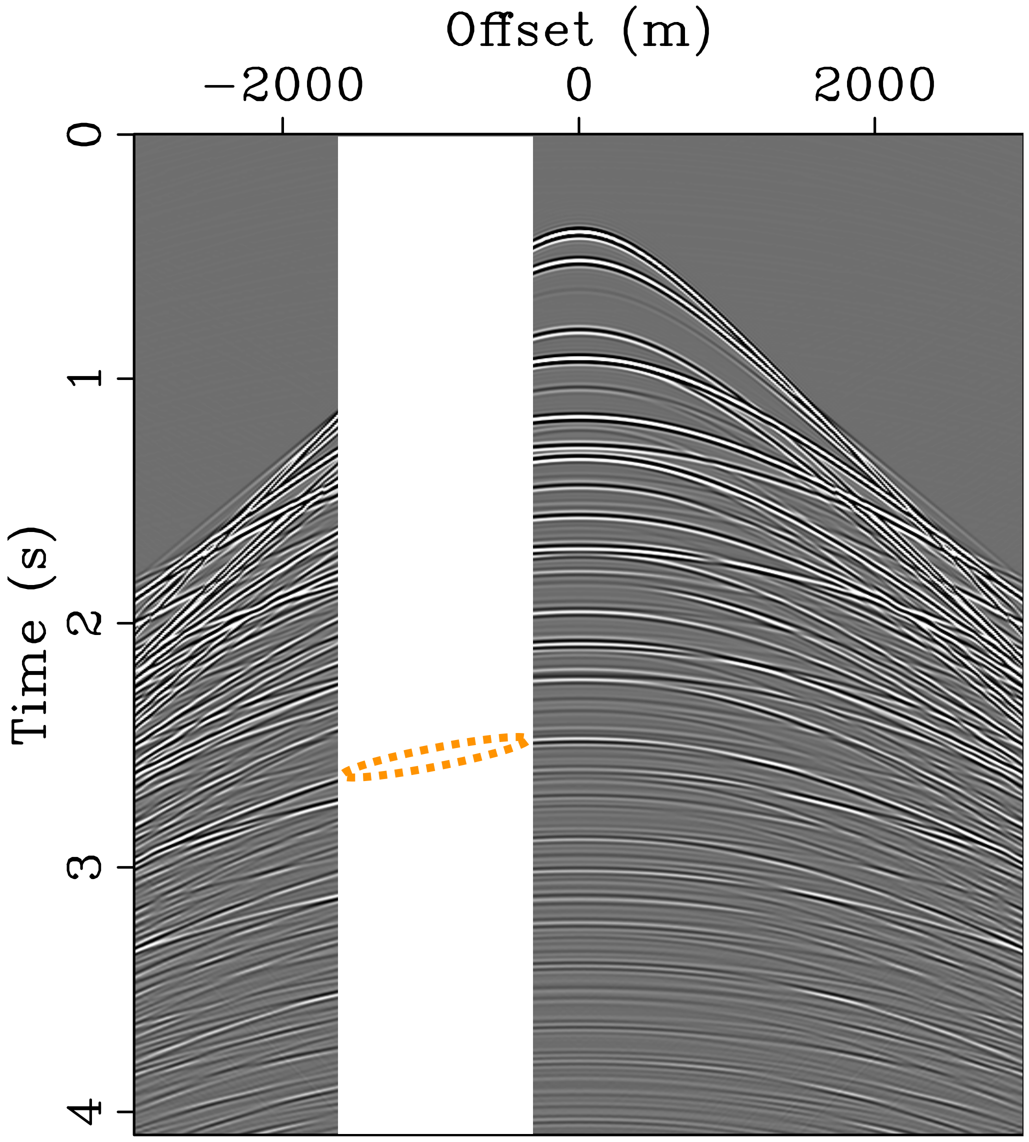
Localized transform elements & gap size

$$\tilde{\mathbf{x}} = \arg\min_{\mathbf{x}} ||\mathbf{x}||_1 \quad \text{s.t.} \quad \mathbf{y} = \mathbf{A}\mathbf{x}$$



Data

✓



Data

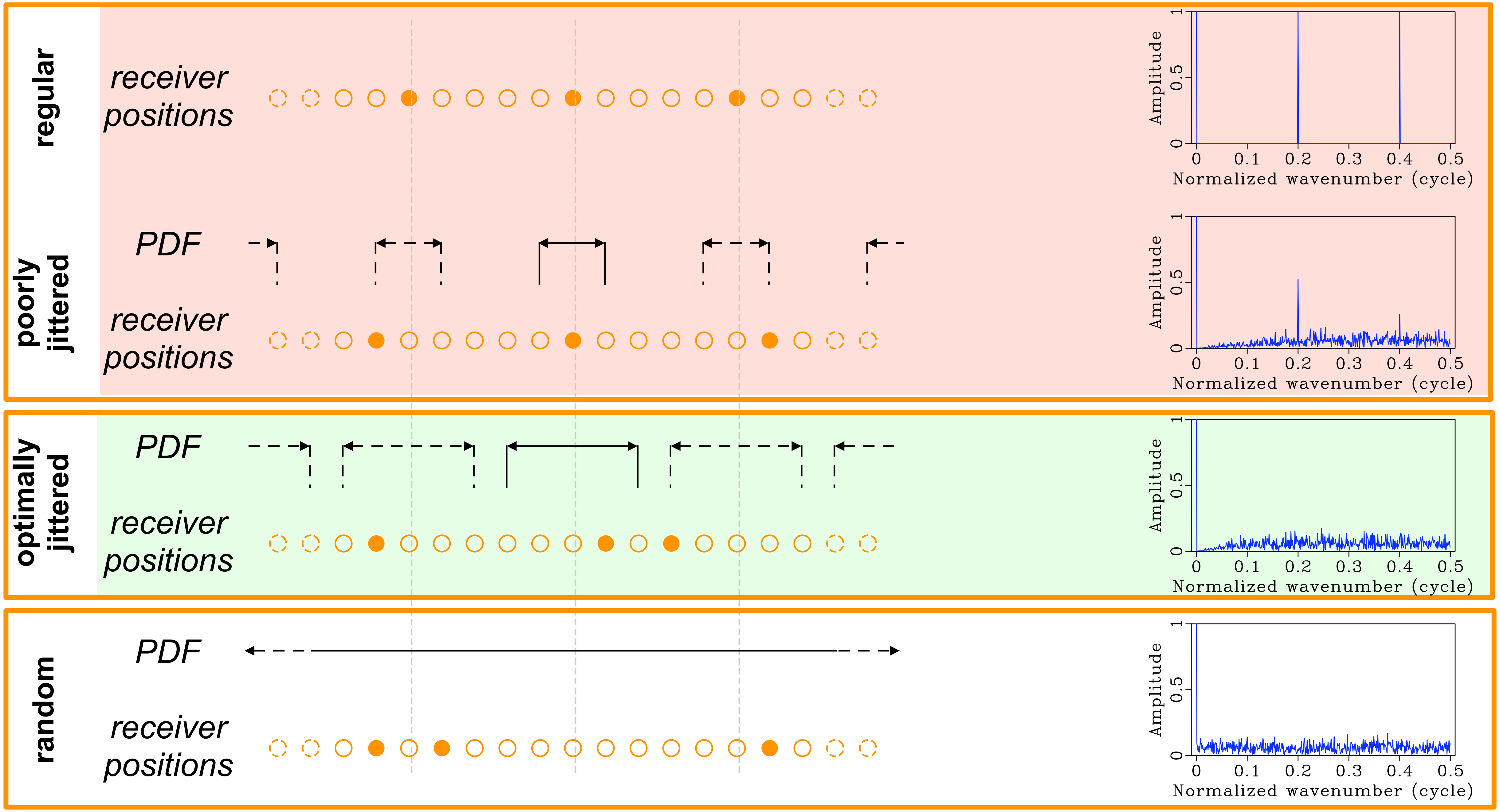
✗

Discrete *randomized* jittered undersampling

Type

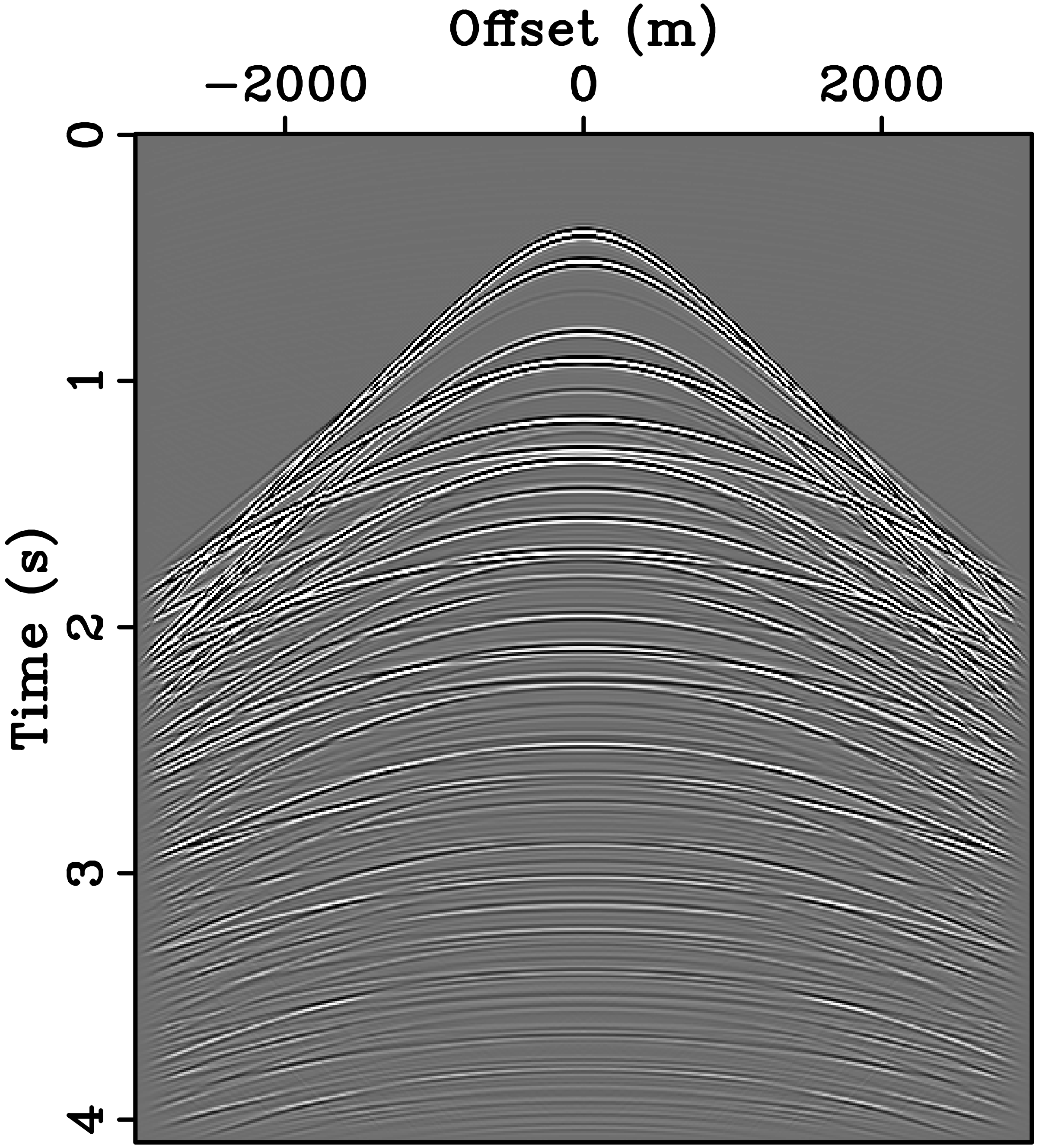
Sampling scheme

Typical spatial convolution kernel



[Hennenfent and FJH ‘08]

Model


Offset (m)
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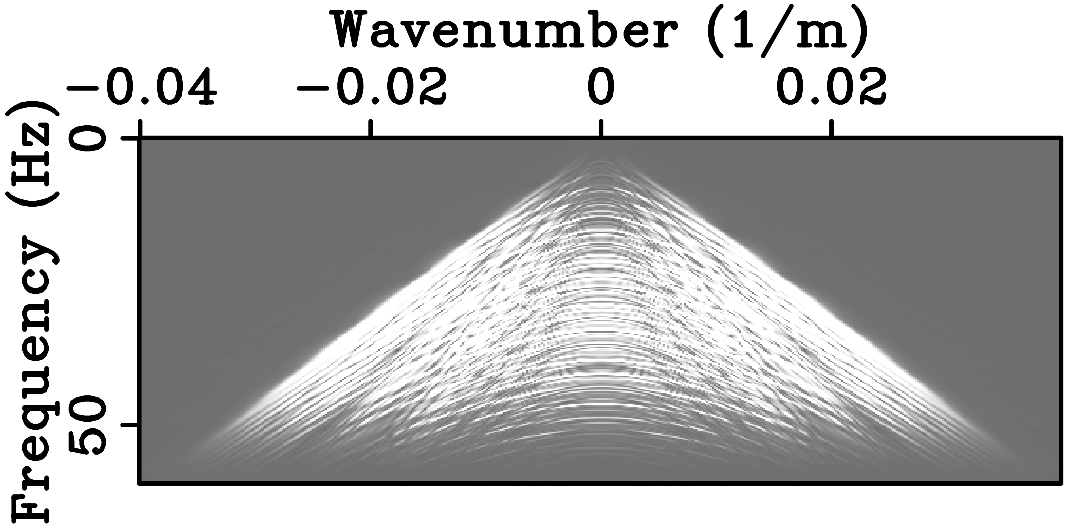
0
2000
Time (s)
0
1
2
3
4


Wavenumber (1/m)
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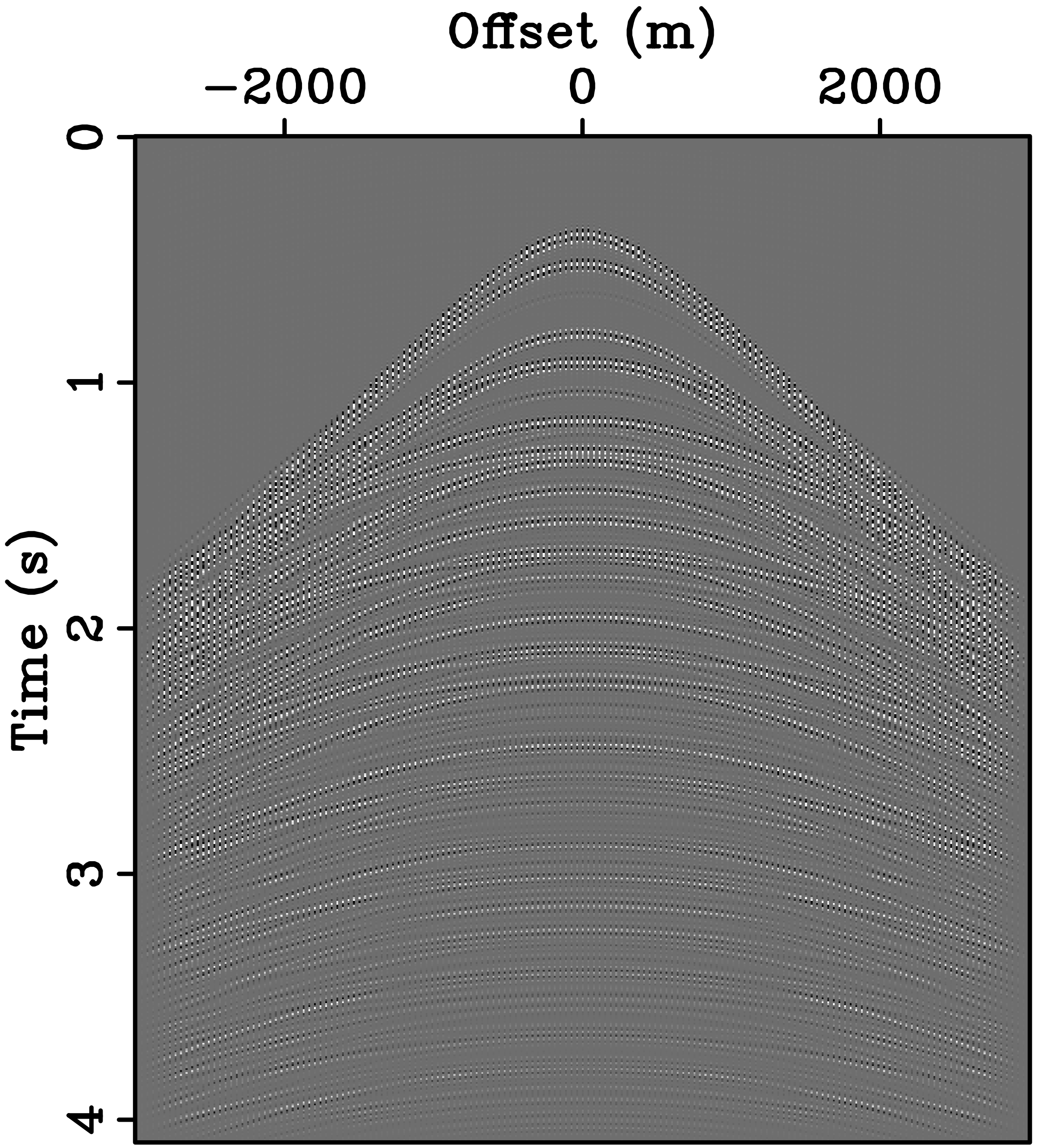
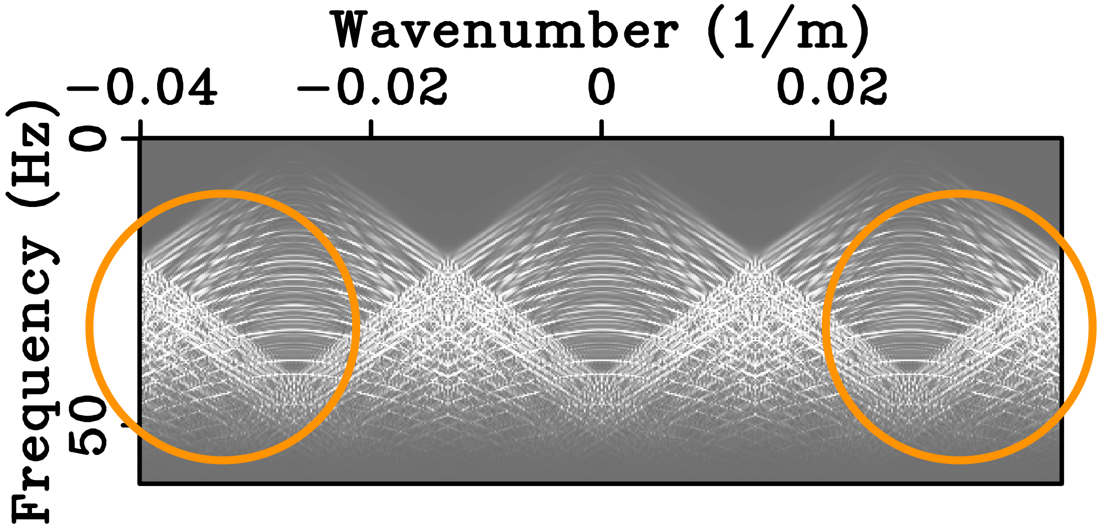
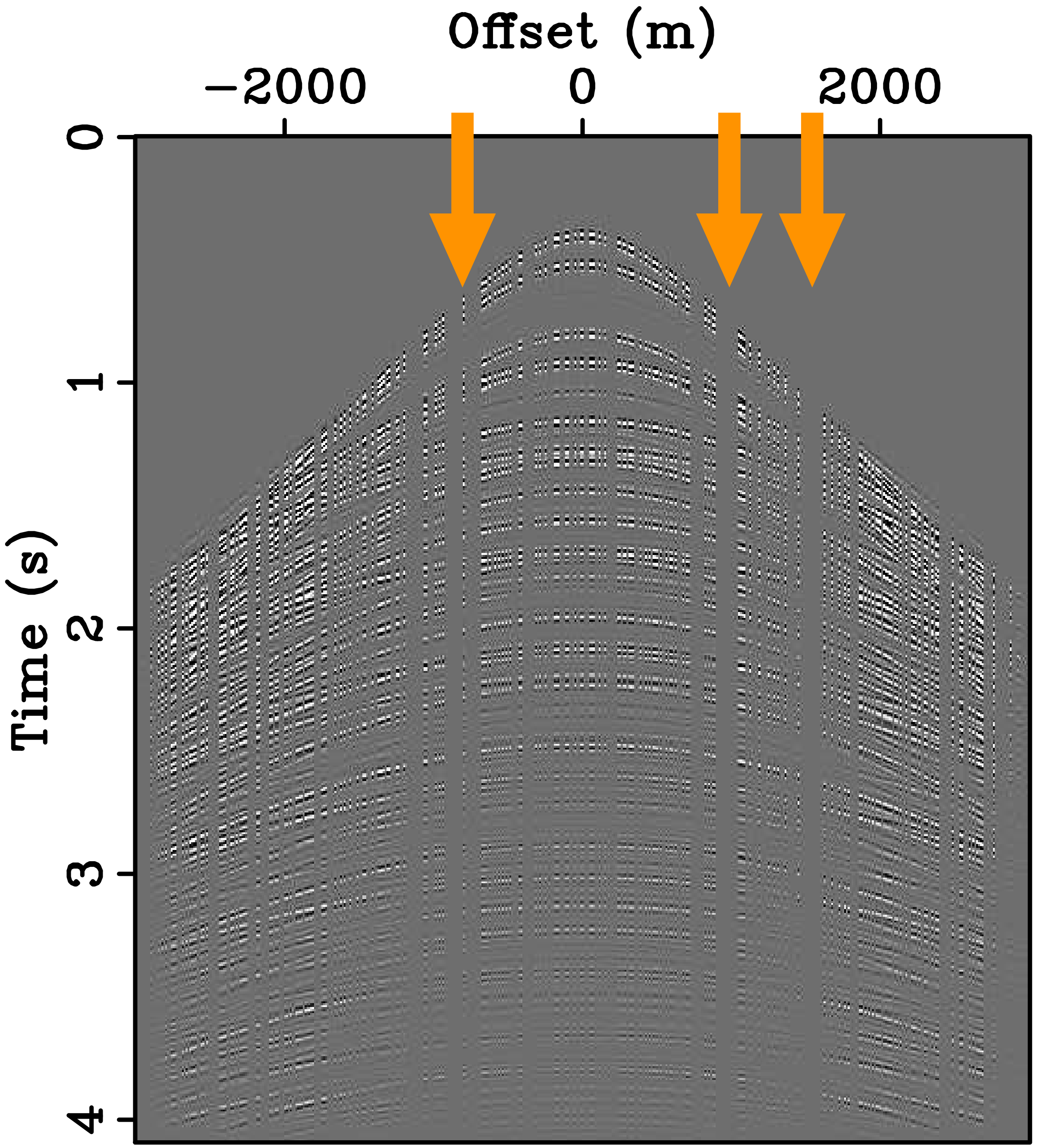
−0.02
0
0.02
Frequency (Hz)
0
50

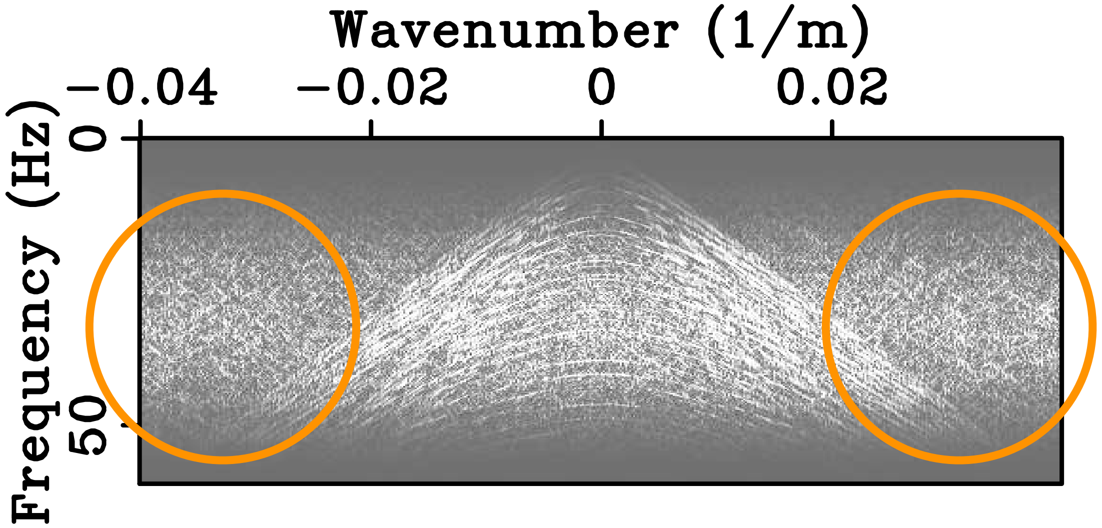
Regular 3-fold undersampling



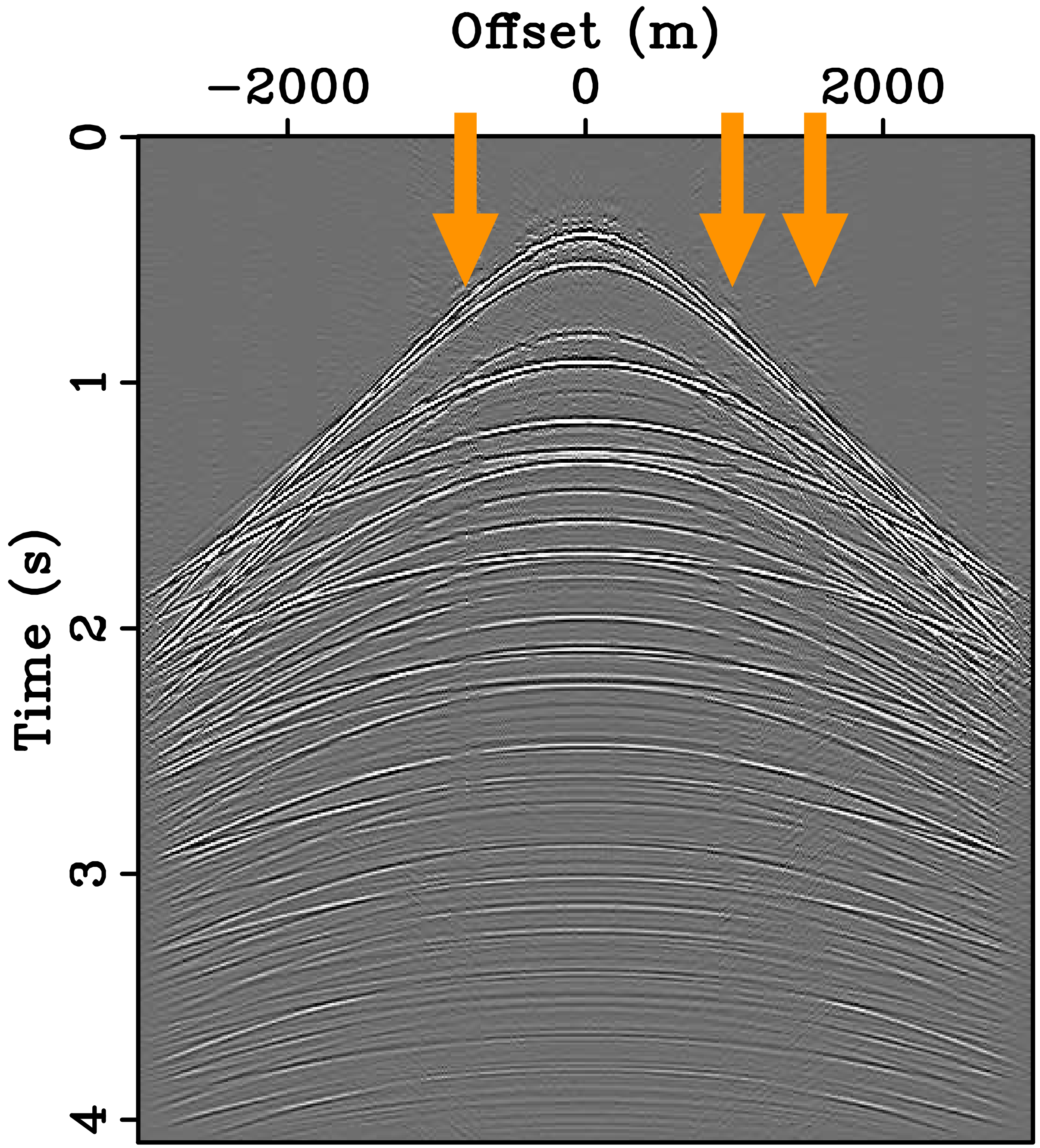


Random 3-fold undersampling

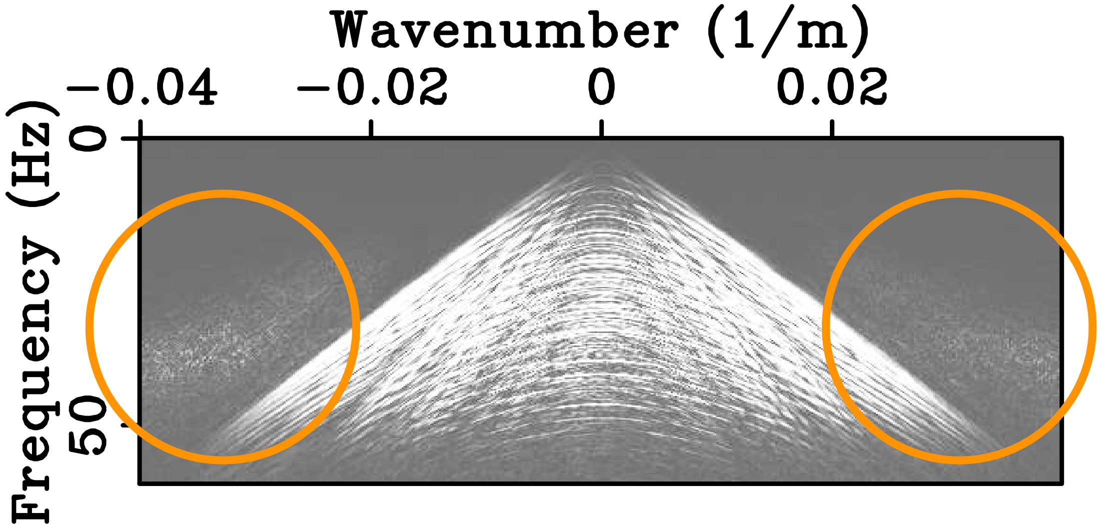




CRSI from random 3-fold undersampling

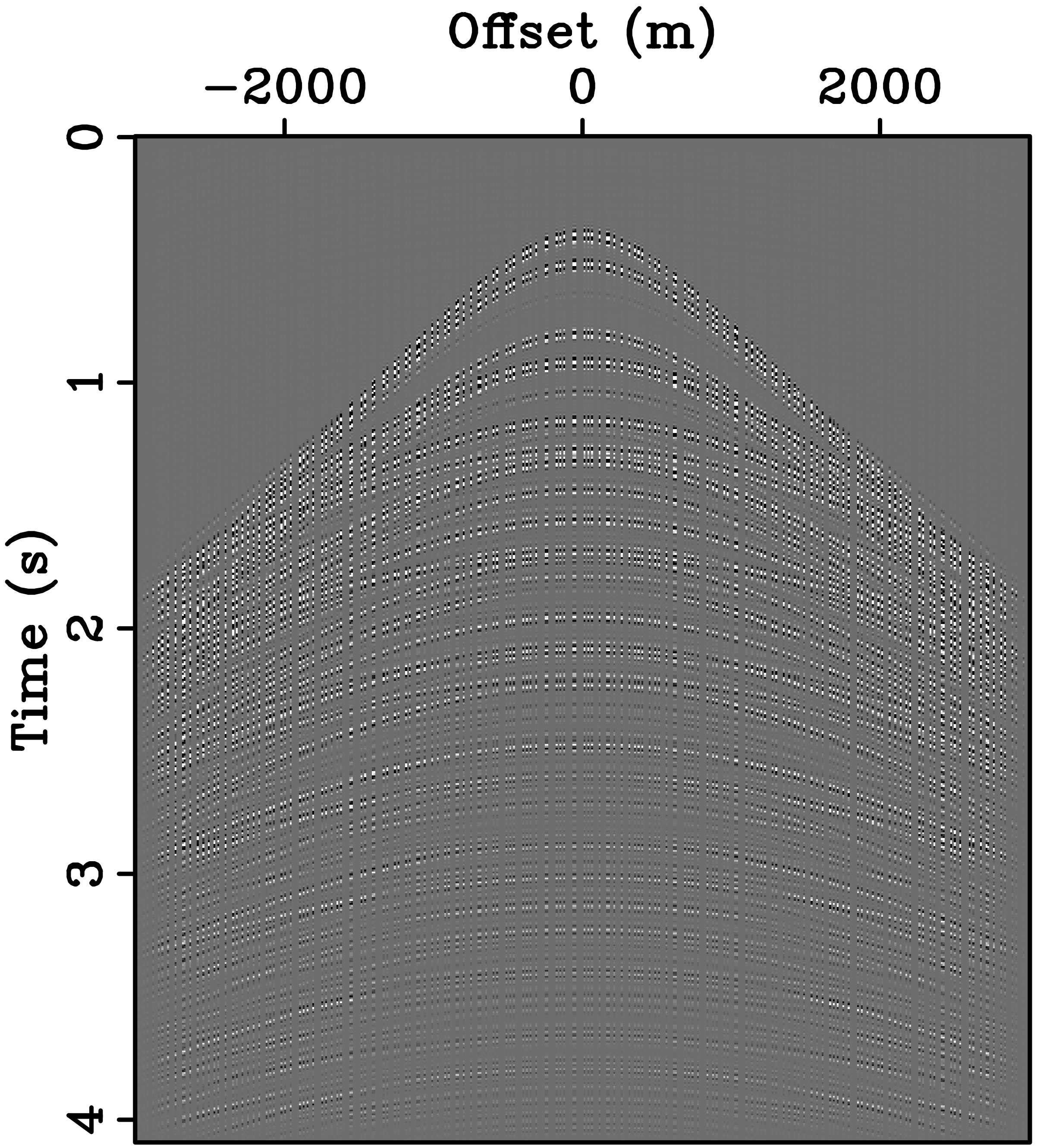


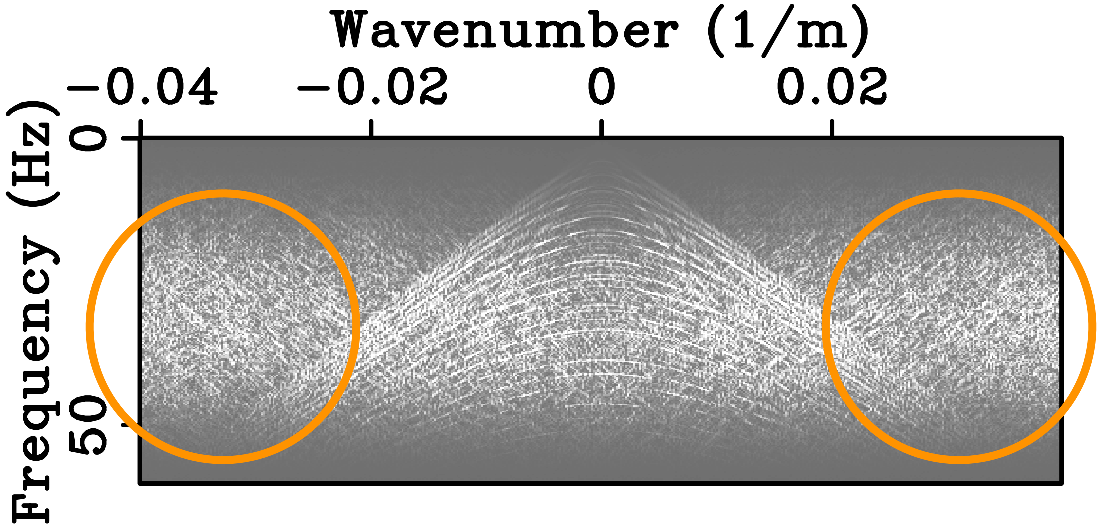
SNR = 9.72 dB



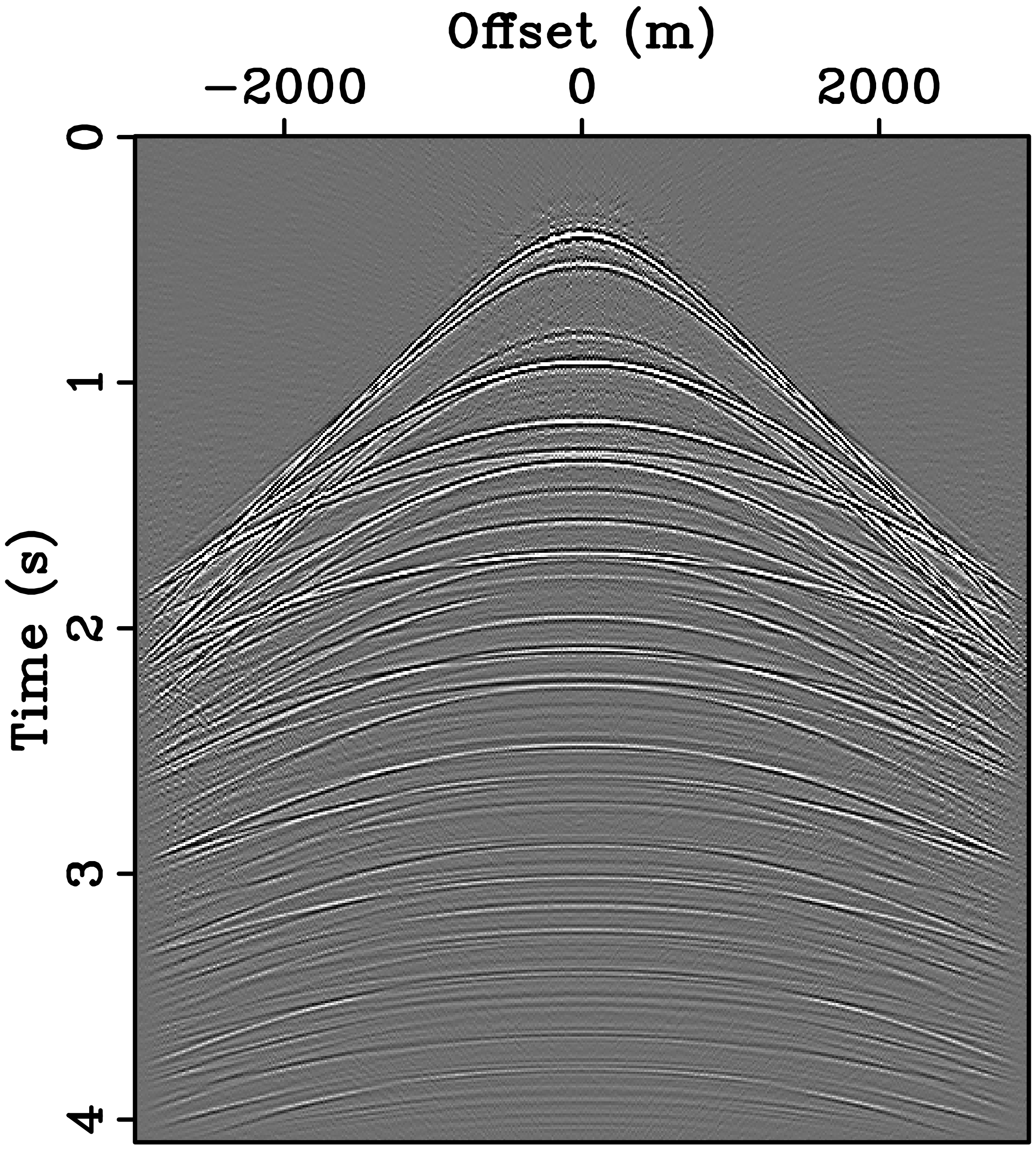
$$\mathrm{SNR} = 20 \times \log_{10}\left(\frac{\|\mathrm{model}\|_2}{\|\mathrm{reconstruction\ error}\|_2}\right)$$

Optimally-jittered 3-fold undersampling

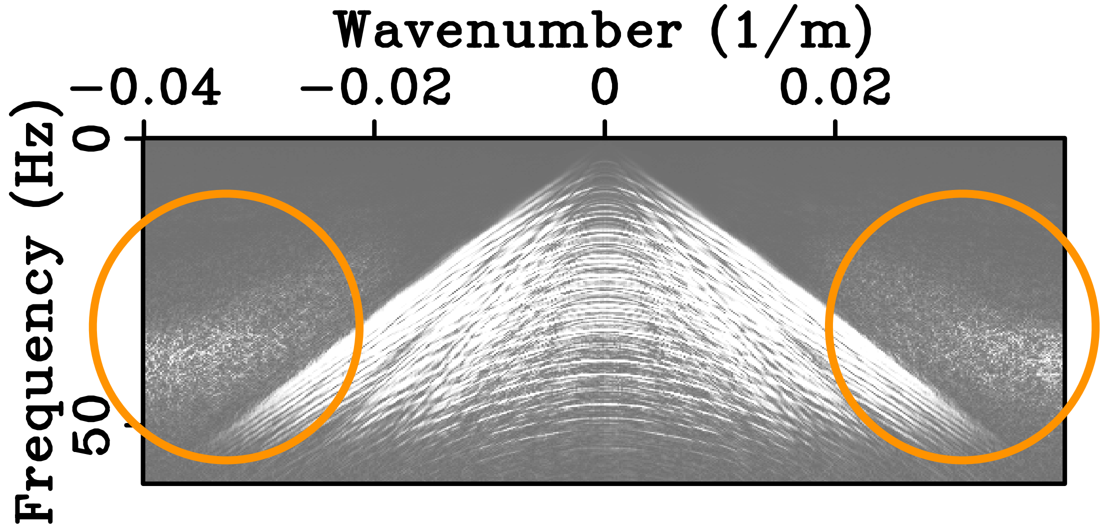




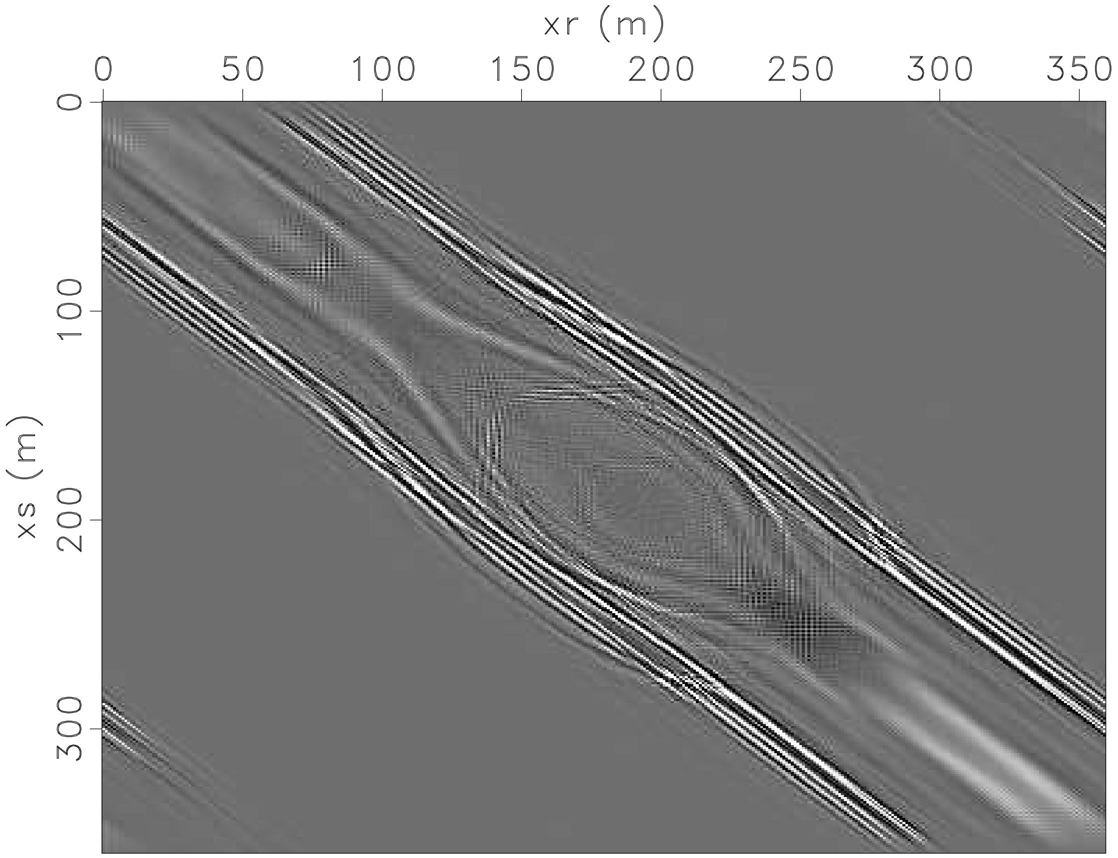
CRSI from opt.-jittered 3-fold undersampling



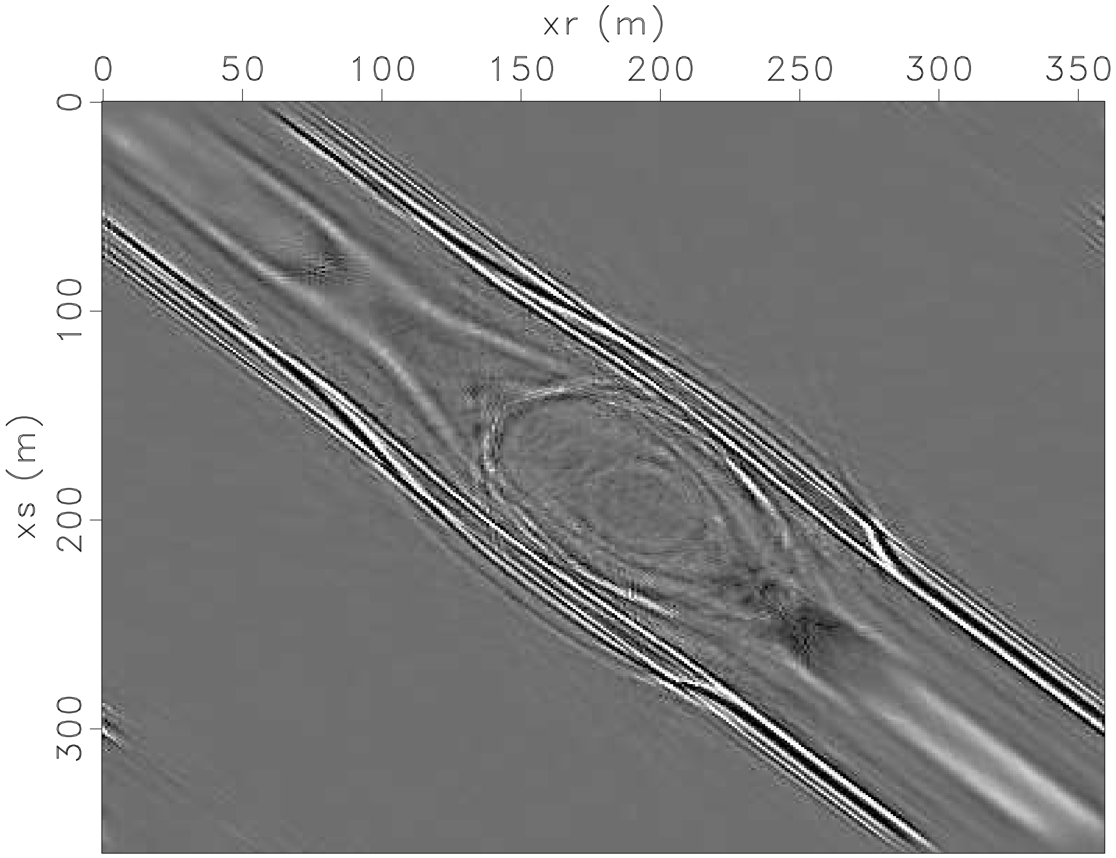
SNR = 10.42 dB



Regular vs uniform randomized 2D sampling



CRSI reconstruction from regular 2-D sampling (25% of data taken)
SNR: 4.161 dB

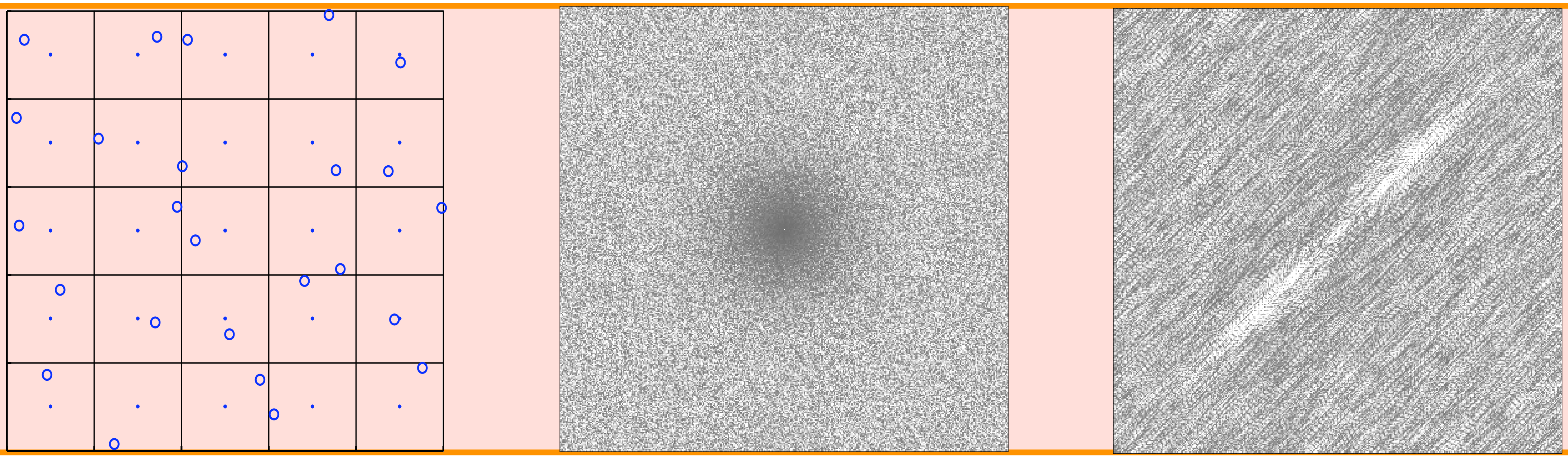


CRSI reconstruction from randomized 2-D sampling (25% of data taken)
SNR: 9.979 dB

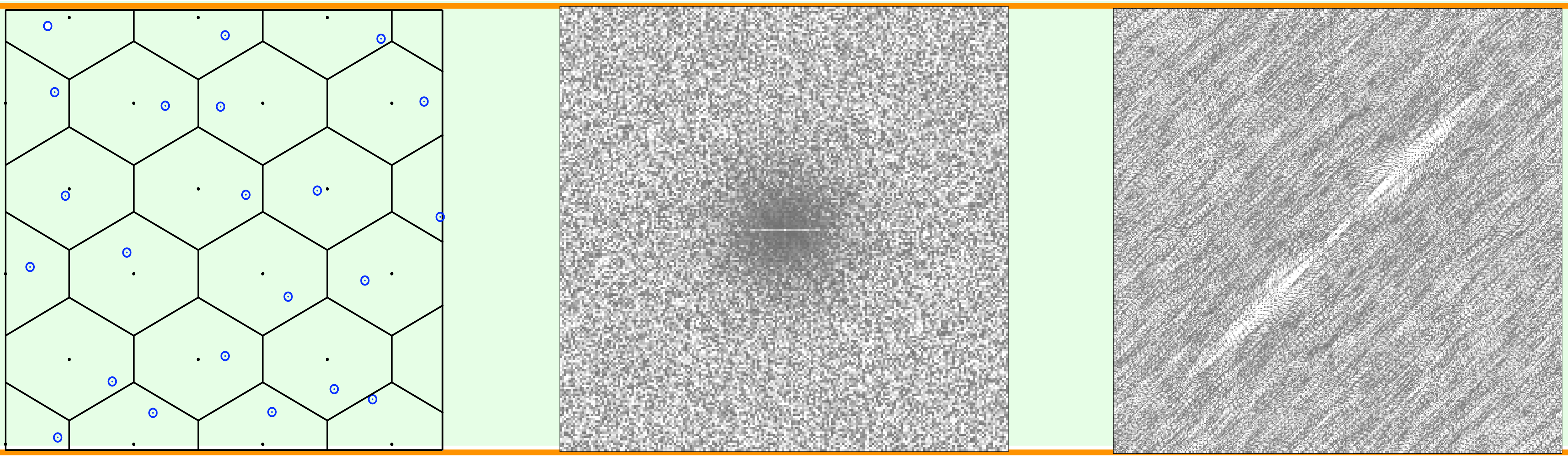
2-D discrete random *jittered* sampling

Type **Sampling scheme** **Mask's spectra** **Samples' spectra**

Square

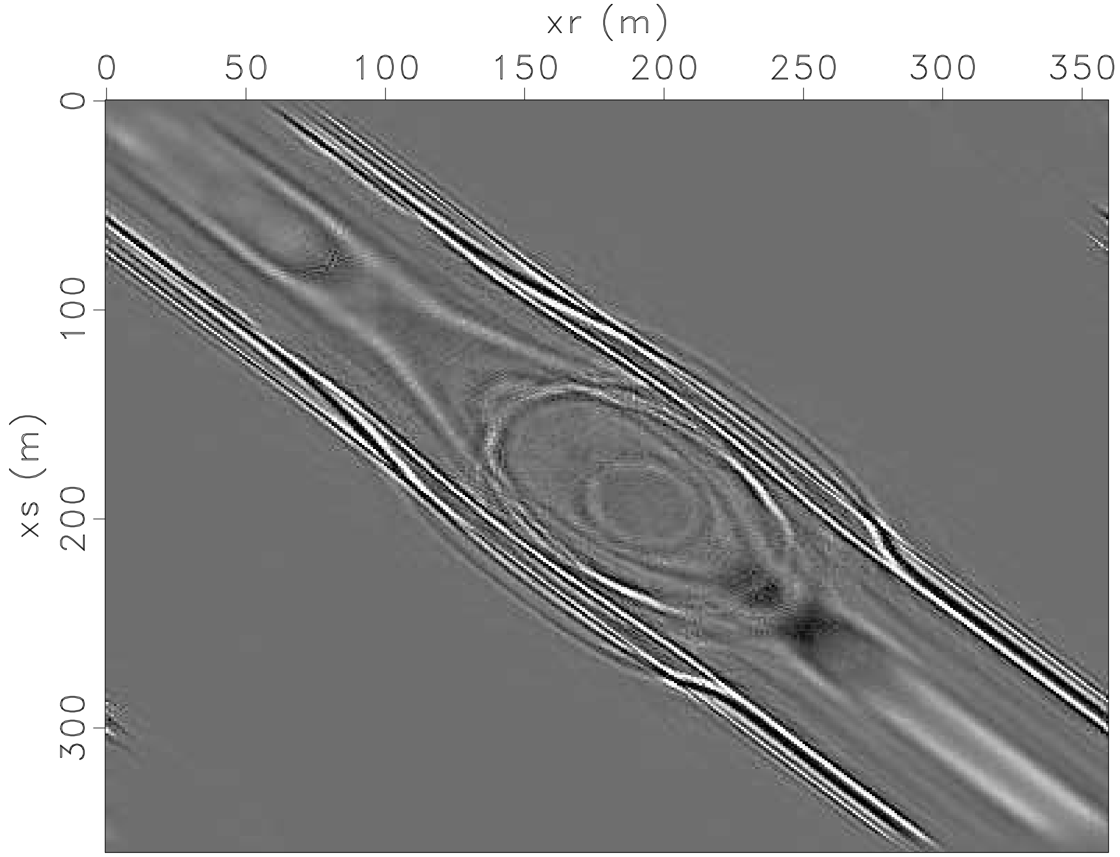


Hexagon

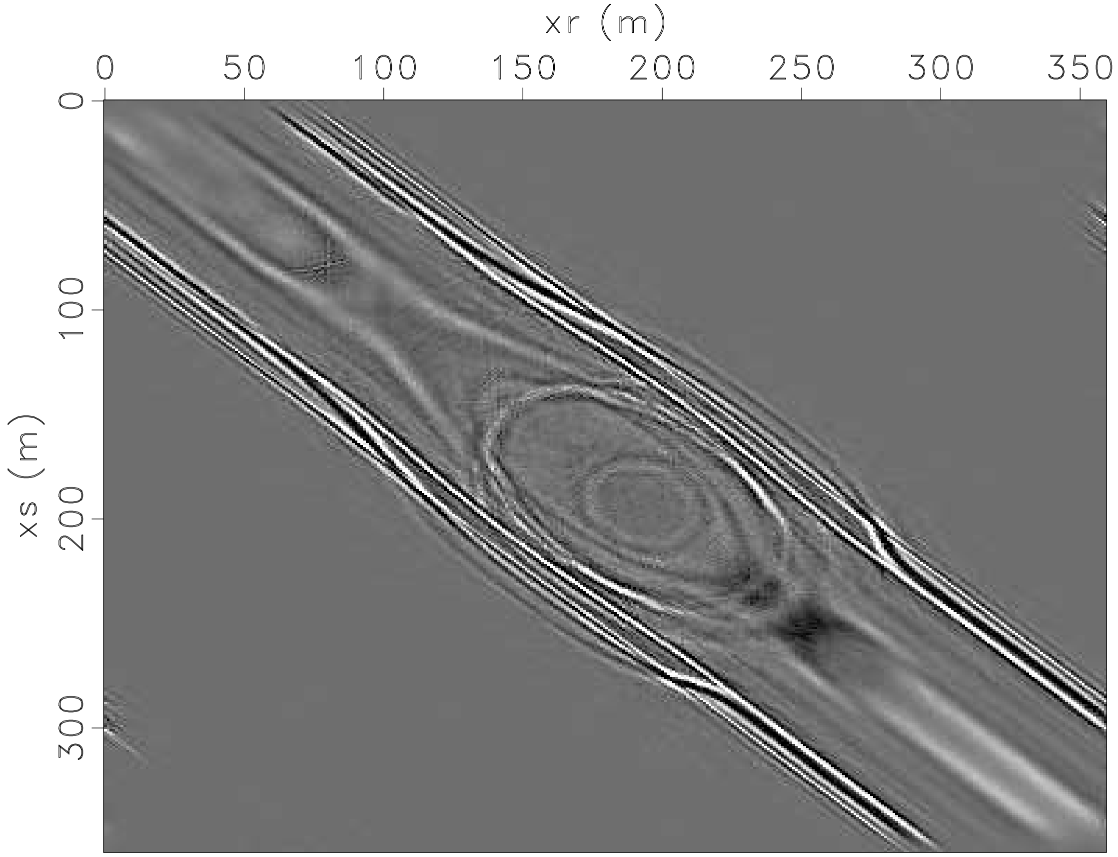


2-D discrete random jittered subsampling

- Cartesian & hexagonal jittered reconstructions almost the same.

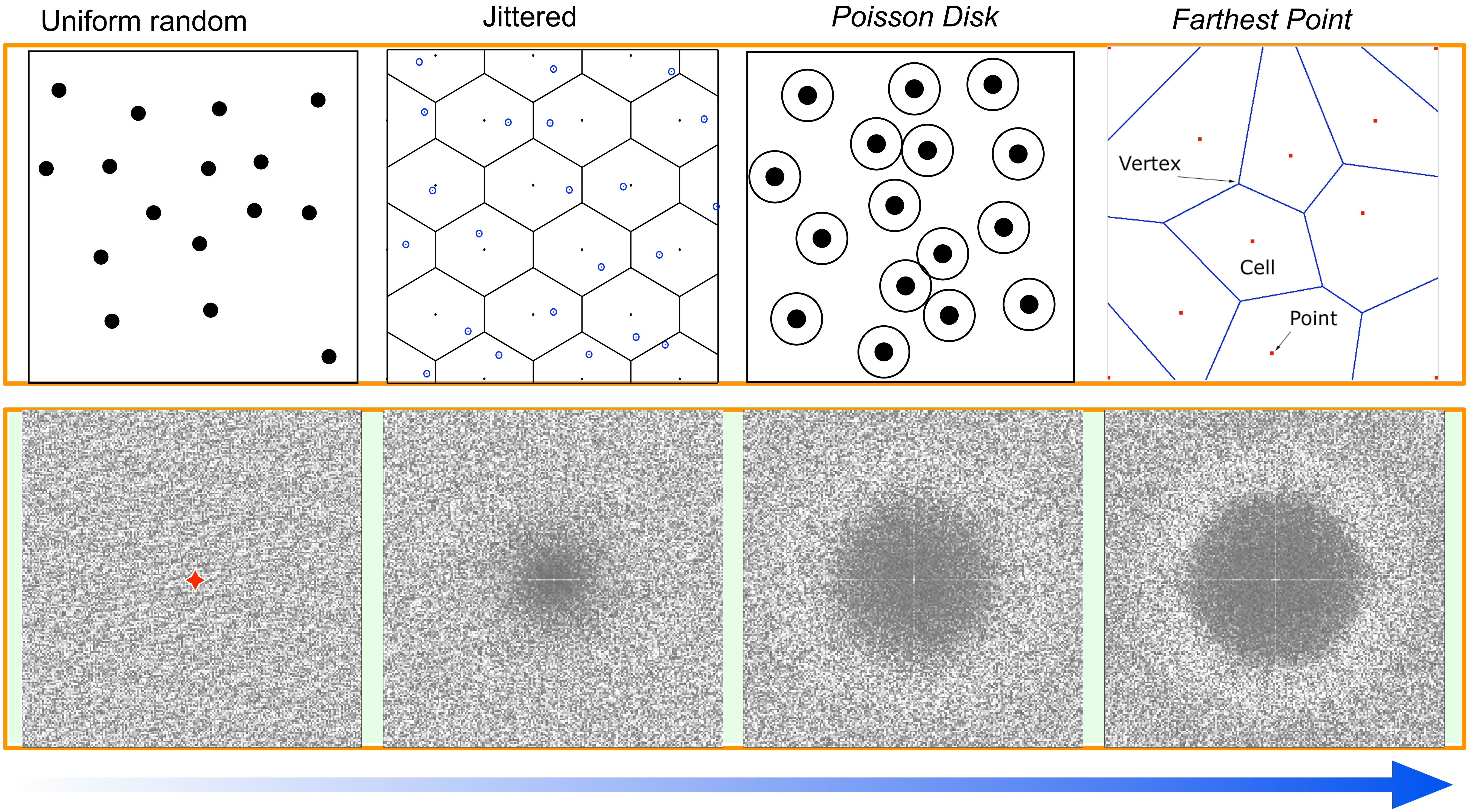


CRSI Recovery (Cartesian)
SNR = 10.820



CRSI Recovery (Hexagonal)
SNR = 10.904

Blue-noise spectra from 2D sampling methods



Uniform random

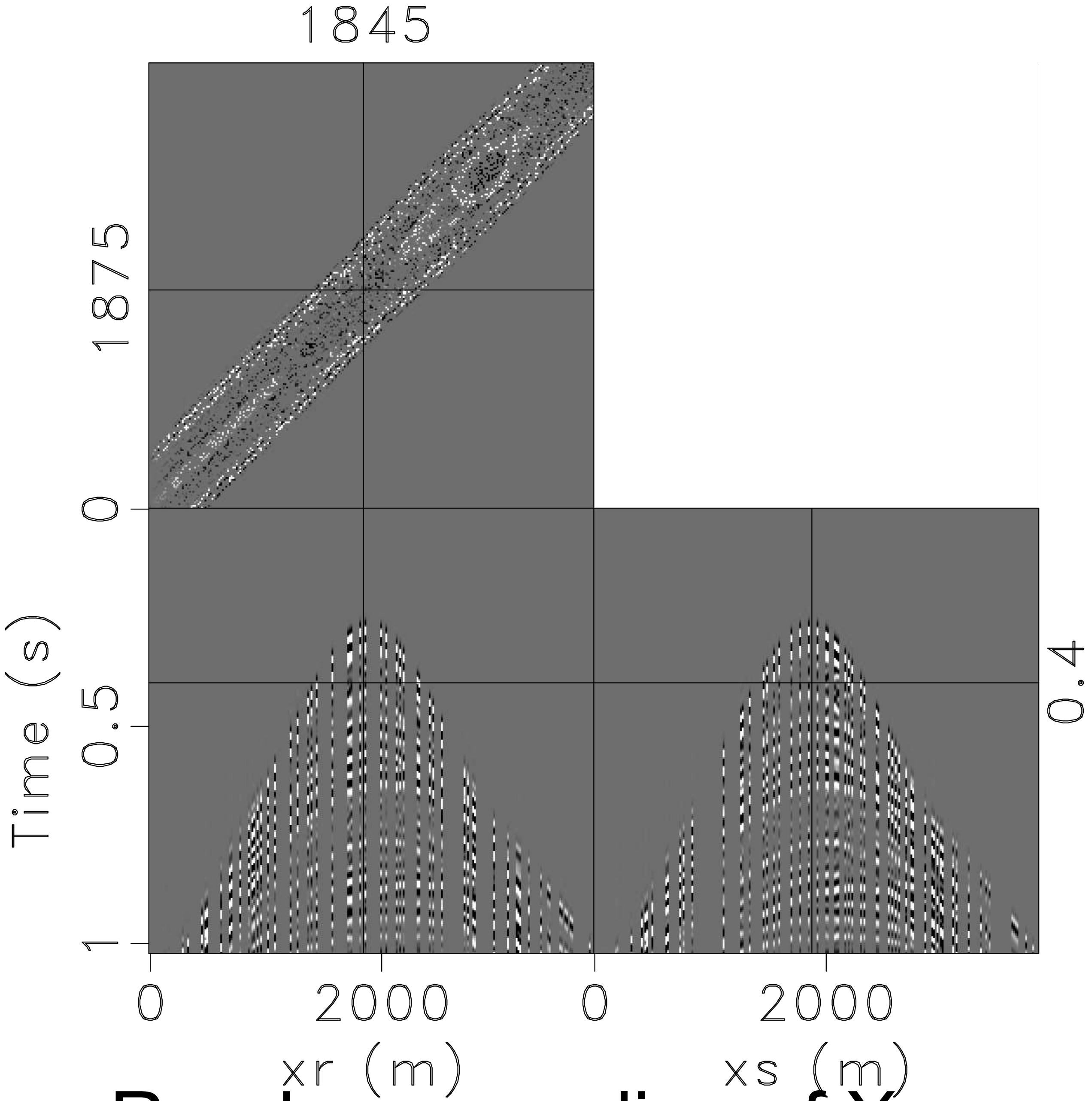
Jittered

Poisson Disk

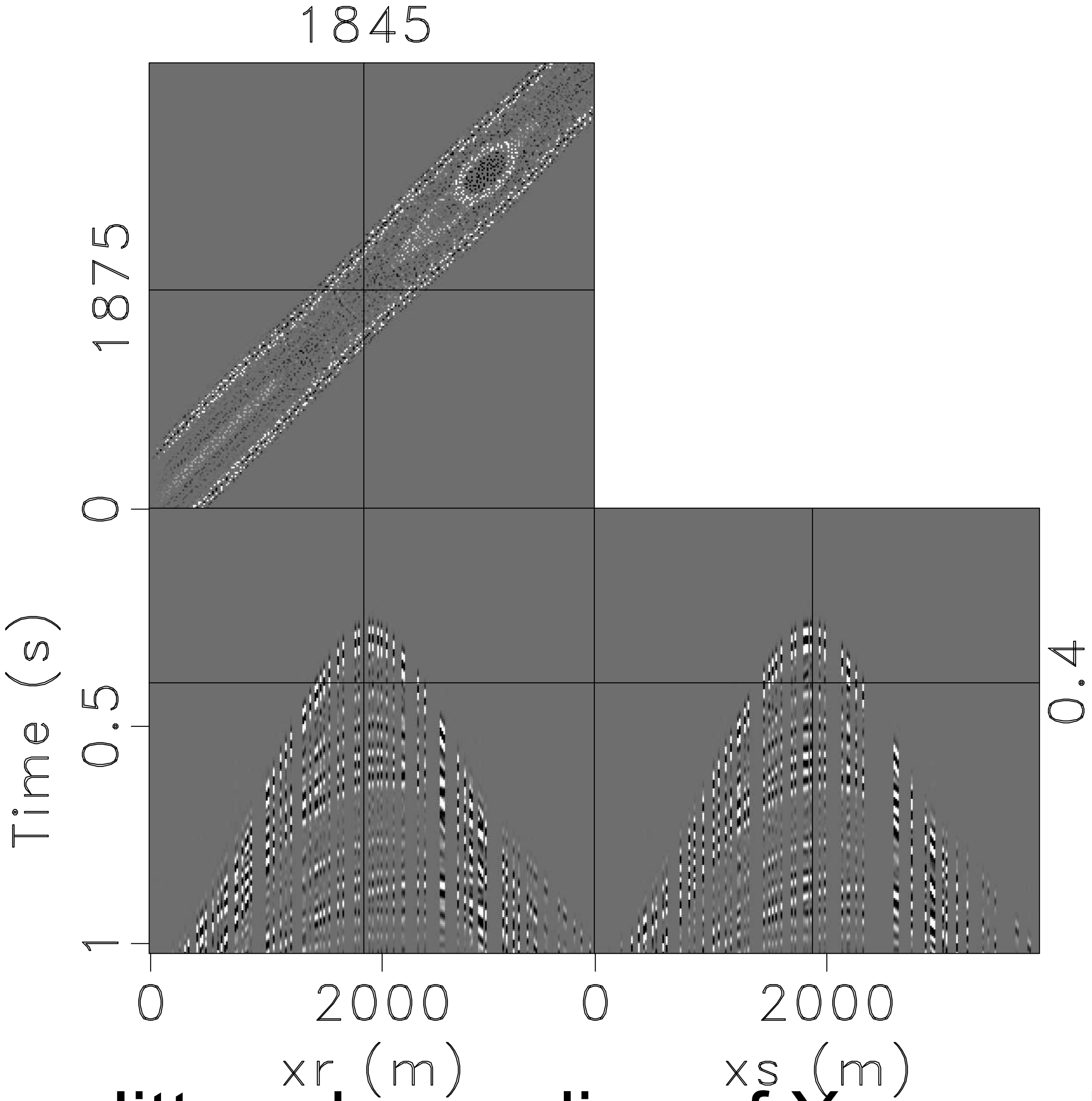
Farthest Point

Spectra become increasingly *“blue”*

Randomized 2D uniform vs jittered

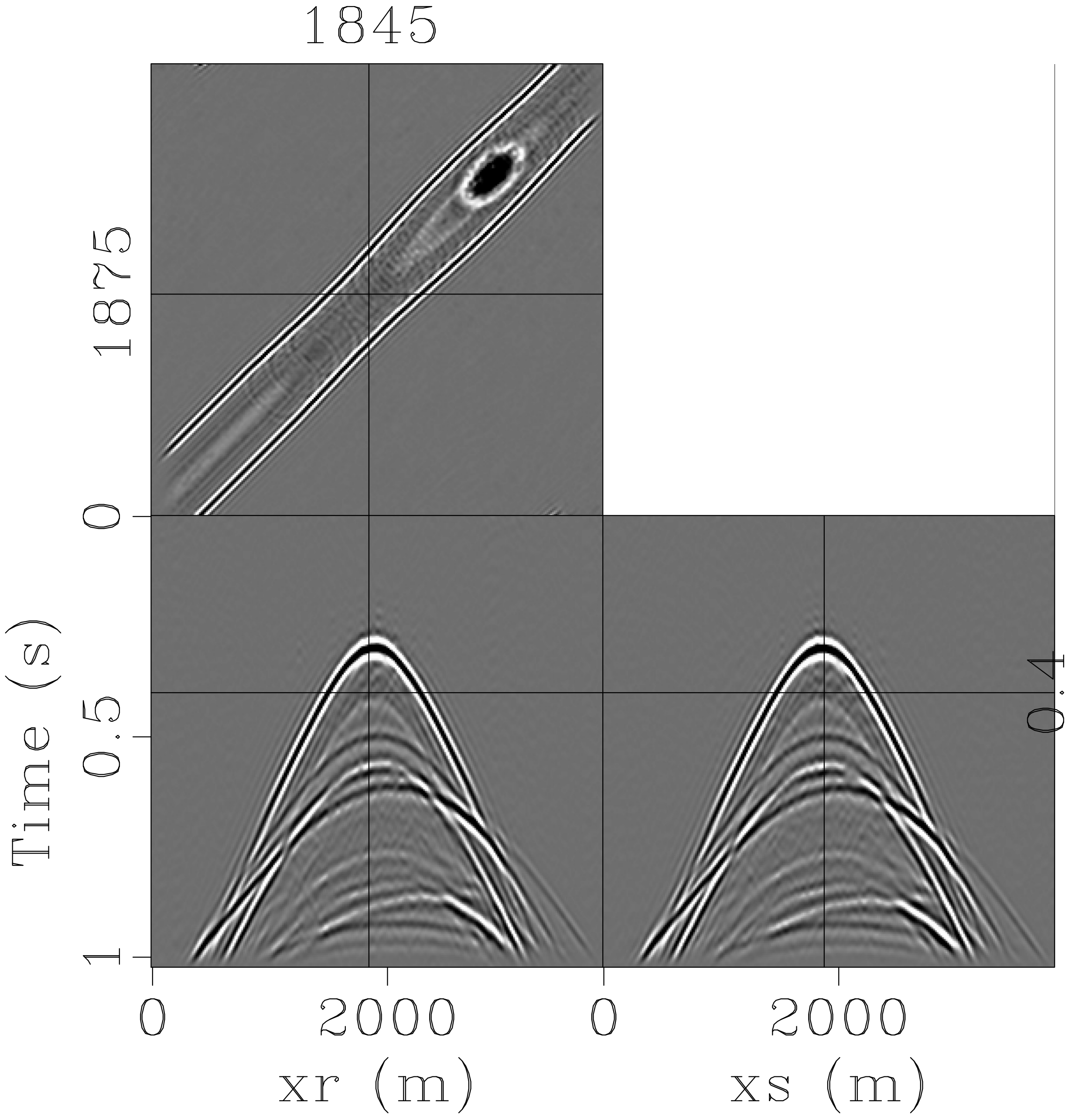


Random sampling of X-spread data (25% total data sampled)

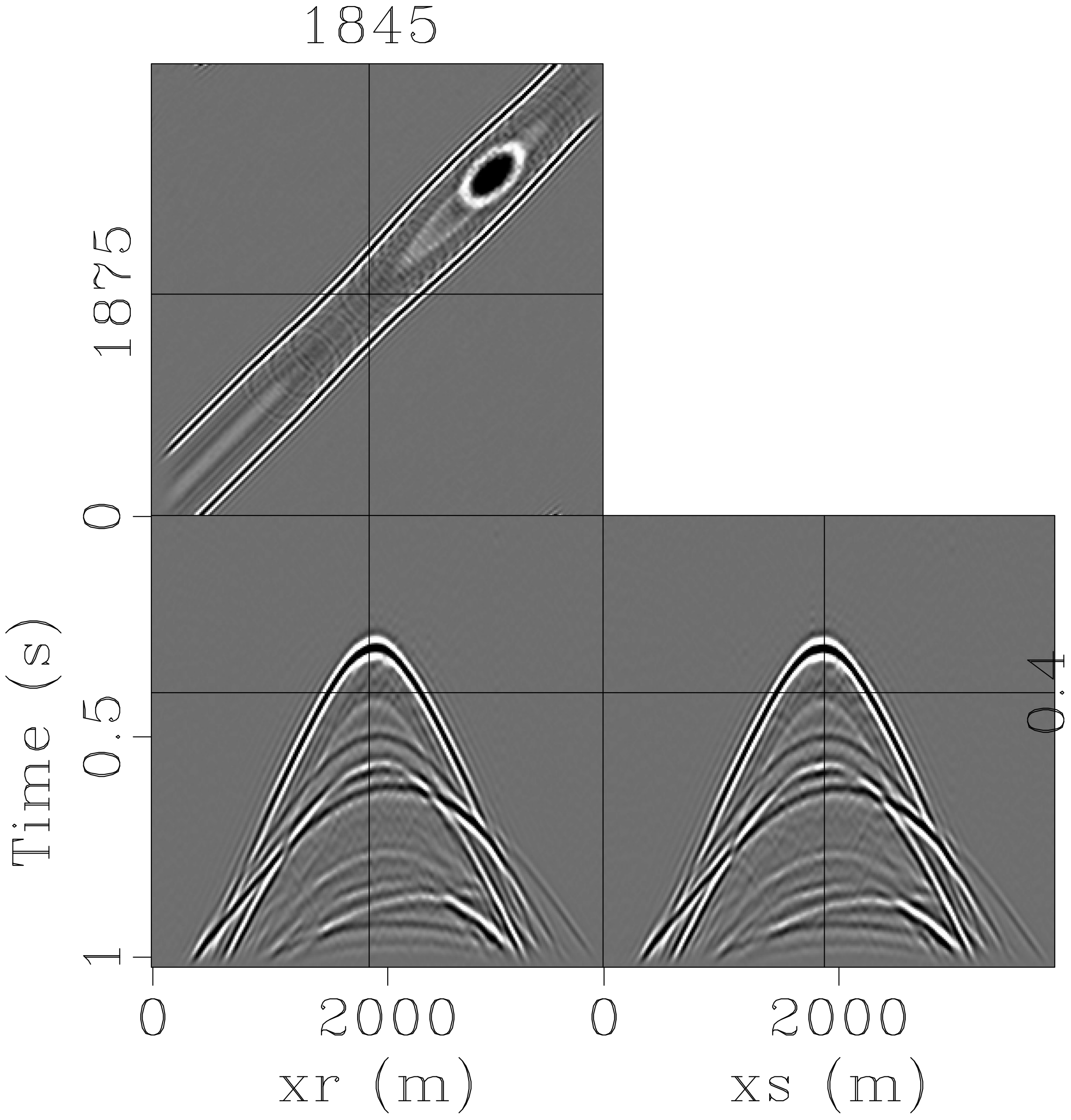


Jittered sampling of X-spread data (25% sampled)

Randomized 2D uniform vs jittered - reconstruction

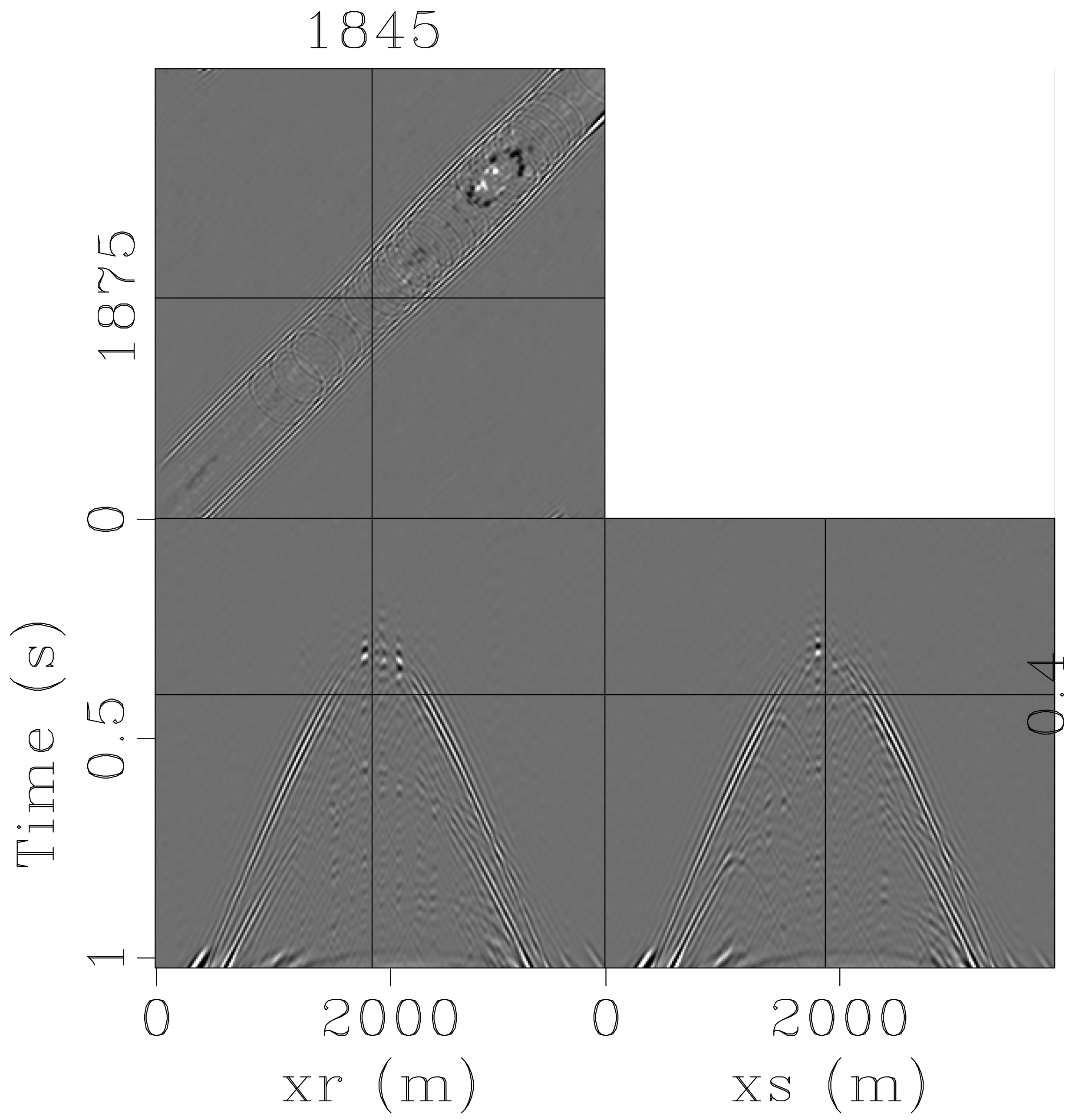


CRSI reconstruction from uniform samples
SNR=8.134

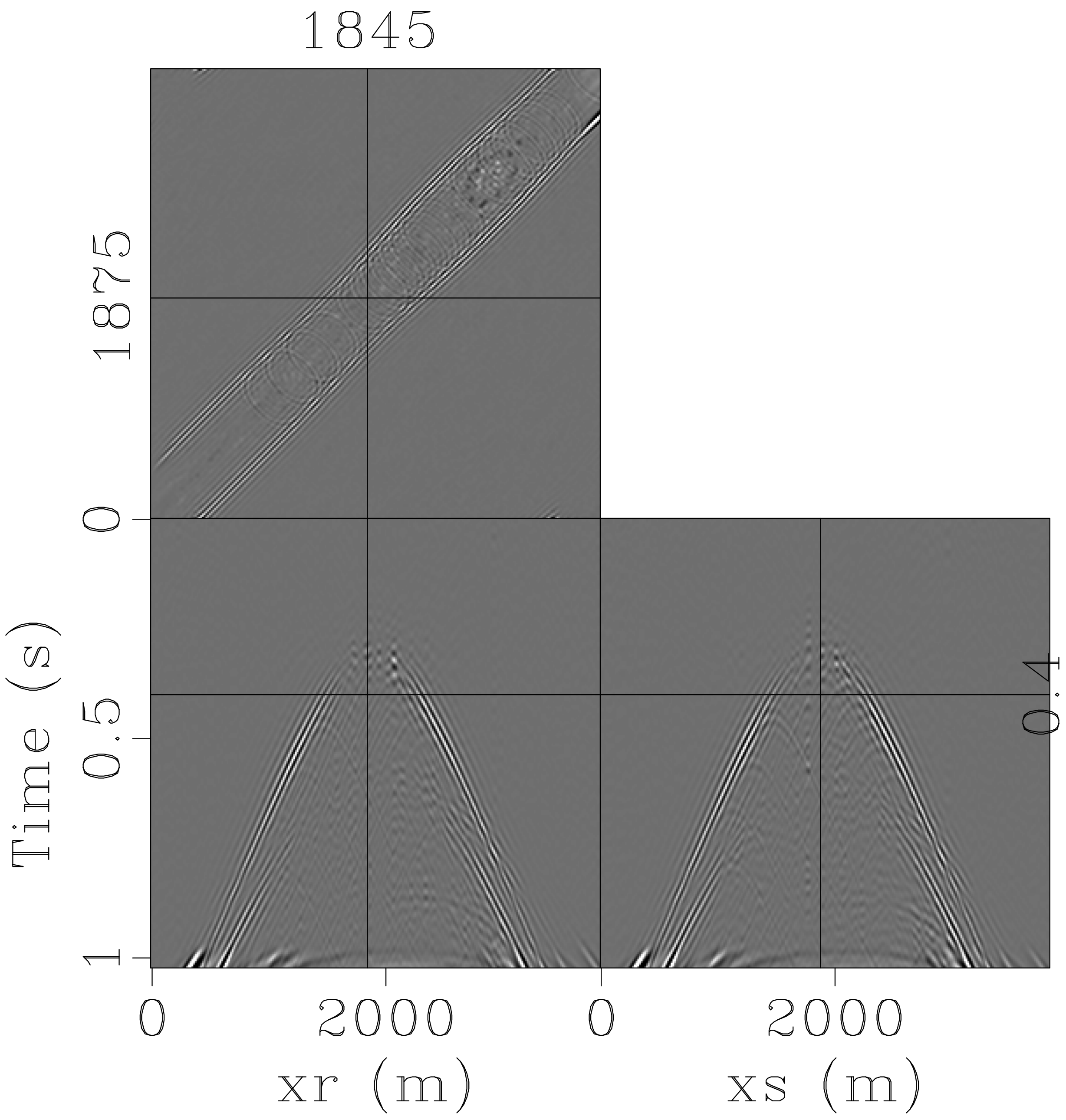


CRSI recon., 2D jittered (hexagonal) samples
SNR=8.434

Randomized 2D uniform vs jittered - residues

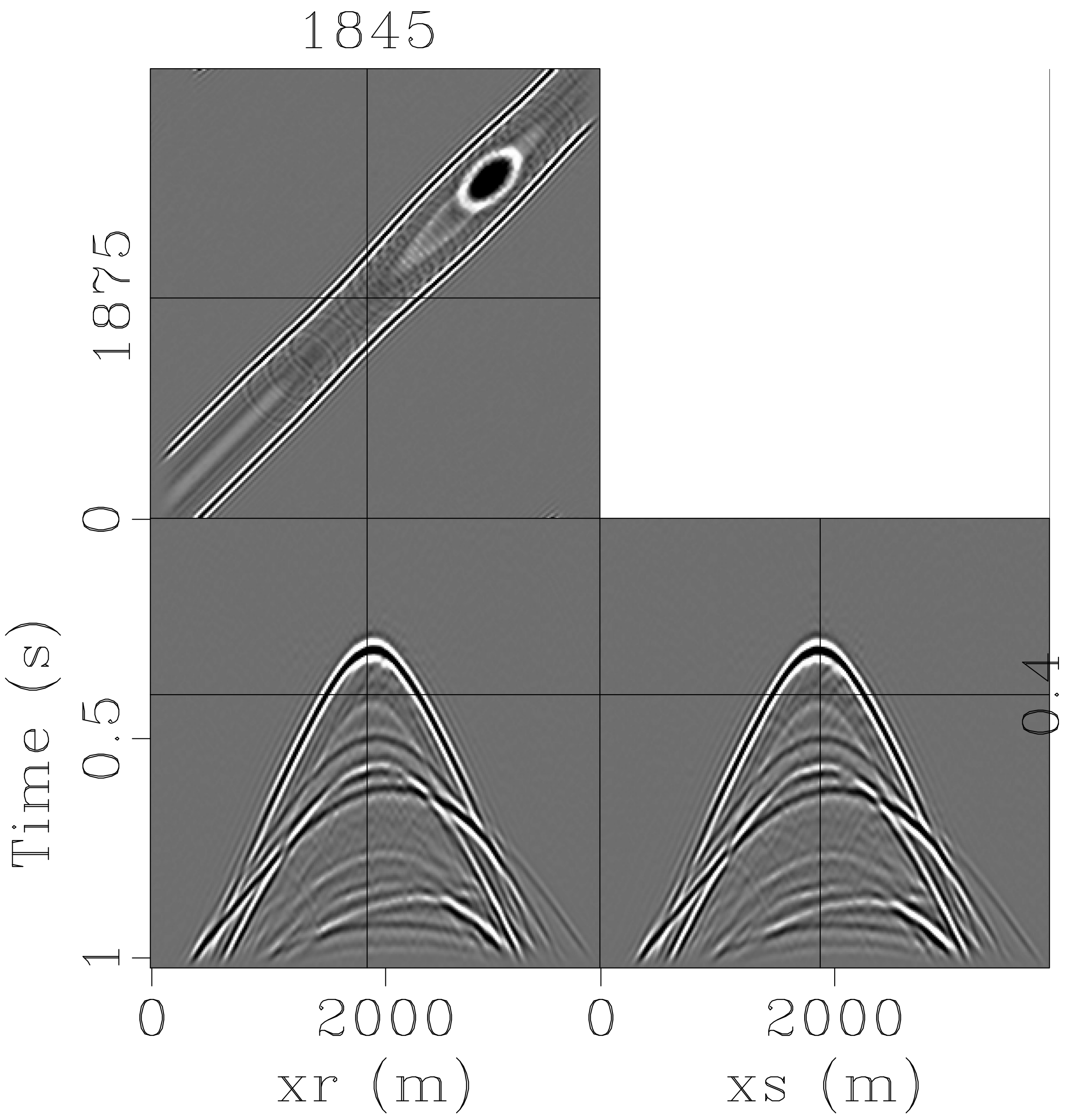


CRSI recon. residual from random samples

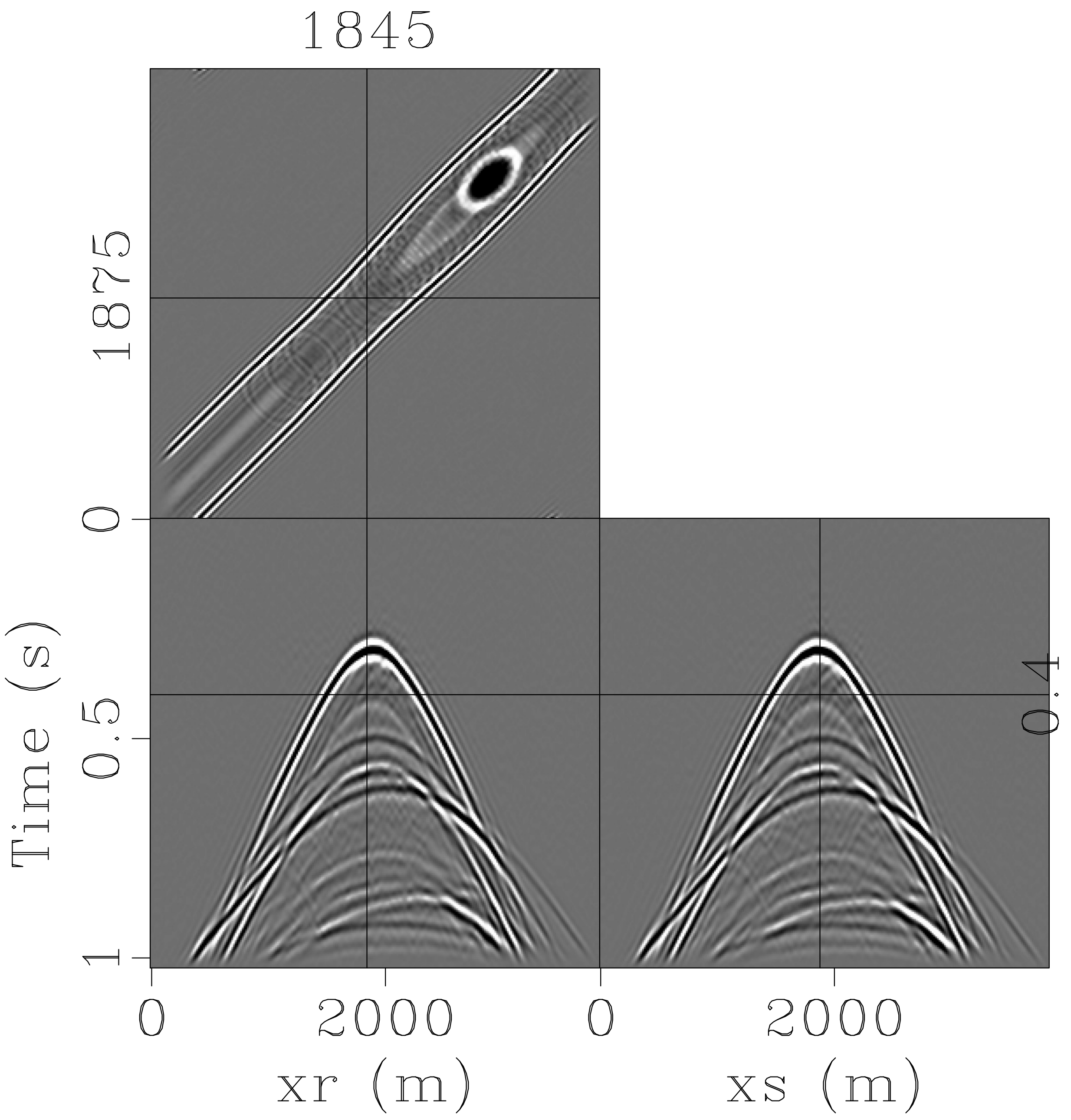


CRSI recon. residual from jittered (hexagonal) samples

Farthest point vs Poisson disk - reconstruction

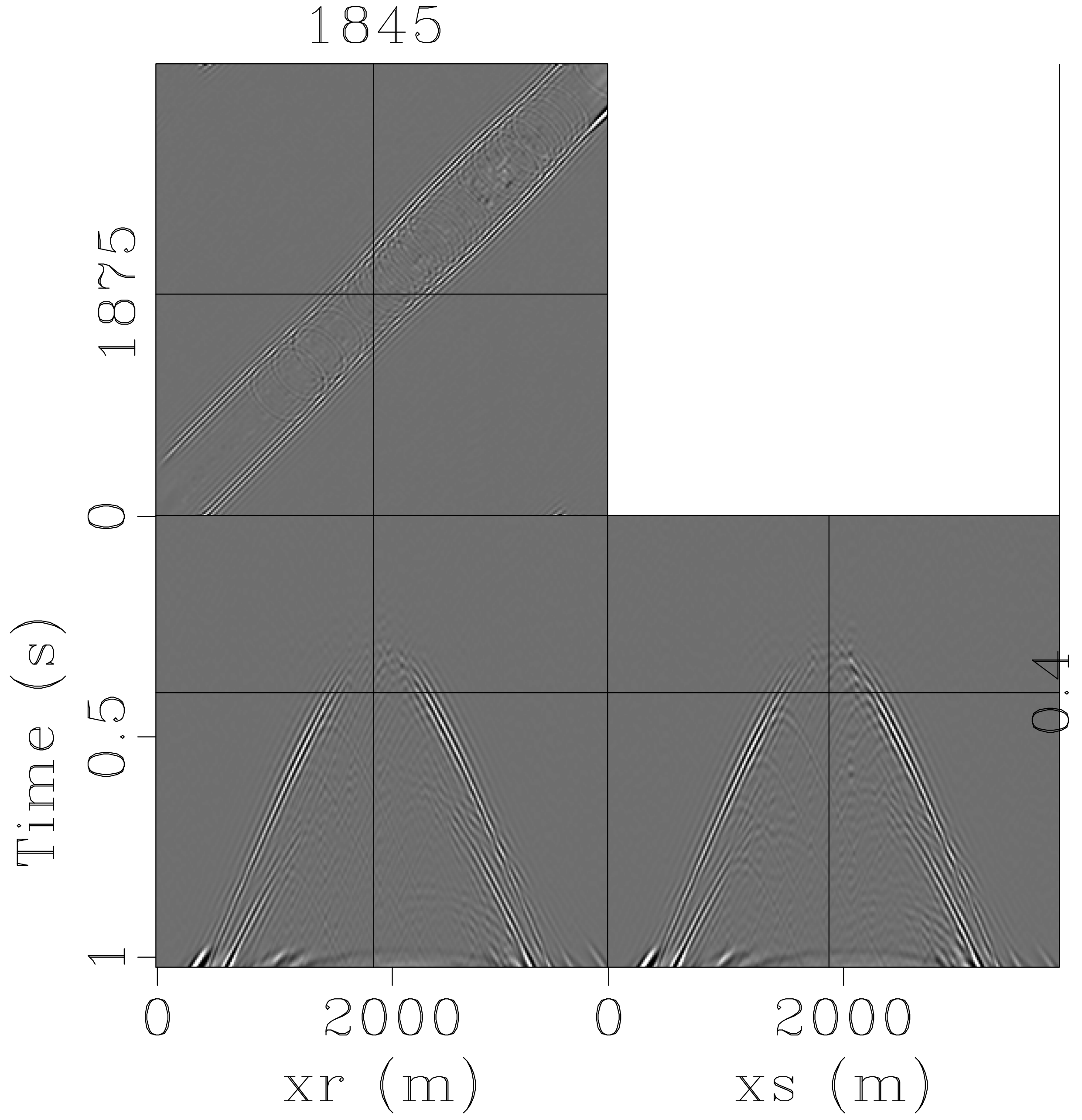


CRSI reconstruction from Farthest Point samples, SNR=8.496



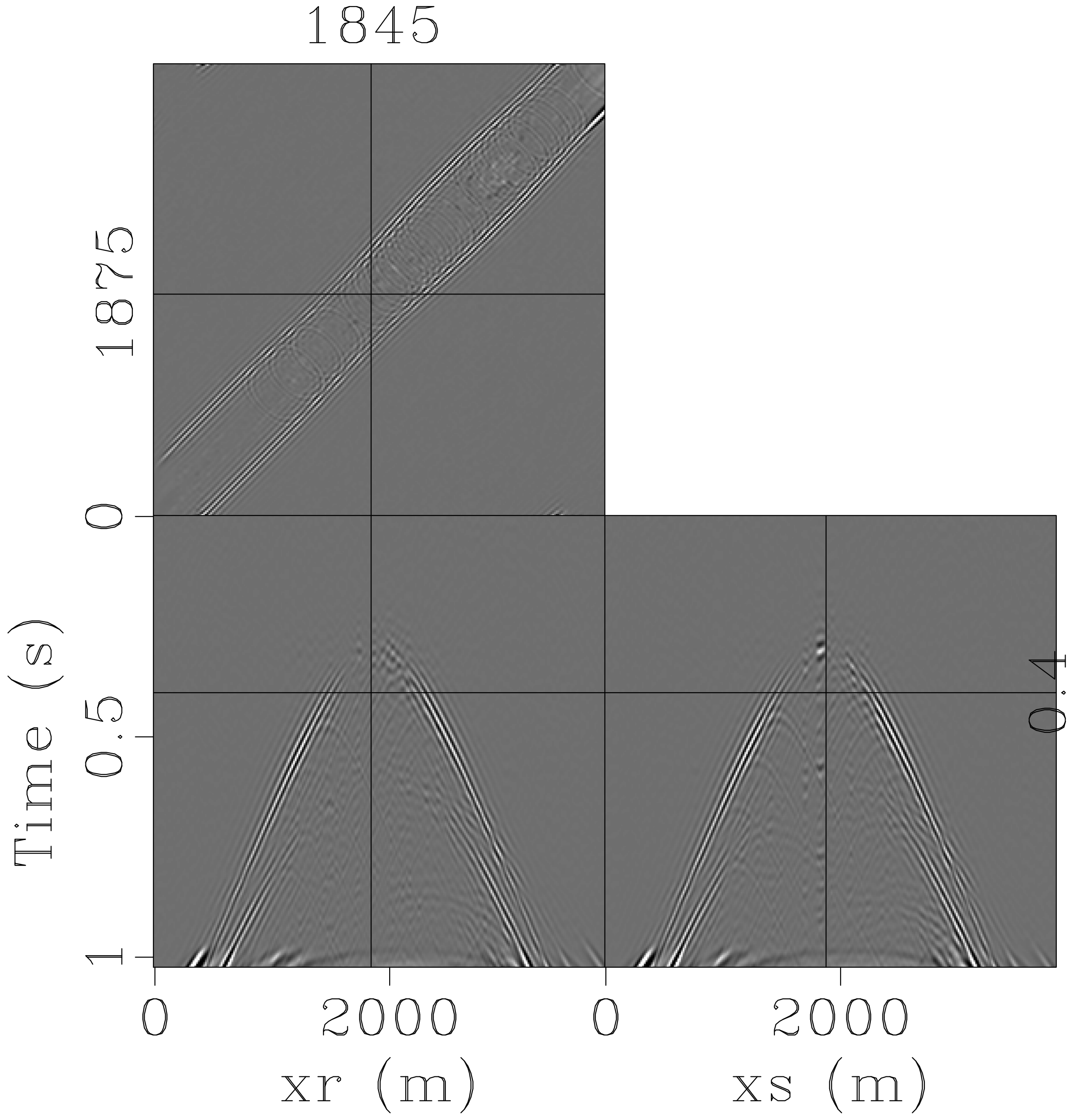
CRSI recon. from Poisson Disk samples SNR=8.483

Farthest point vs Poisson disk - residual



CRSI recon. residual from

Farthest Point samples



CRSI recon. residual from

Poisson Disk samples

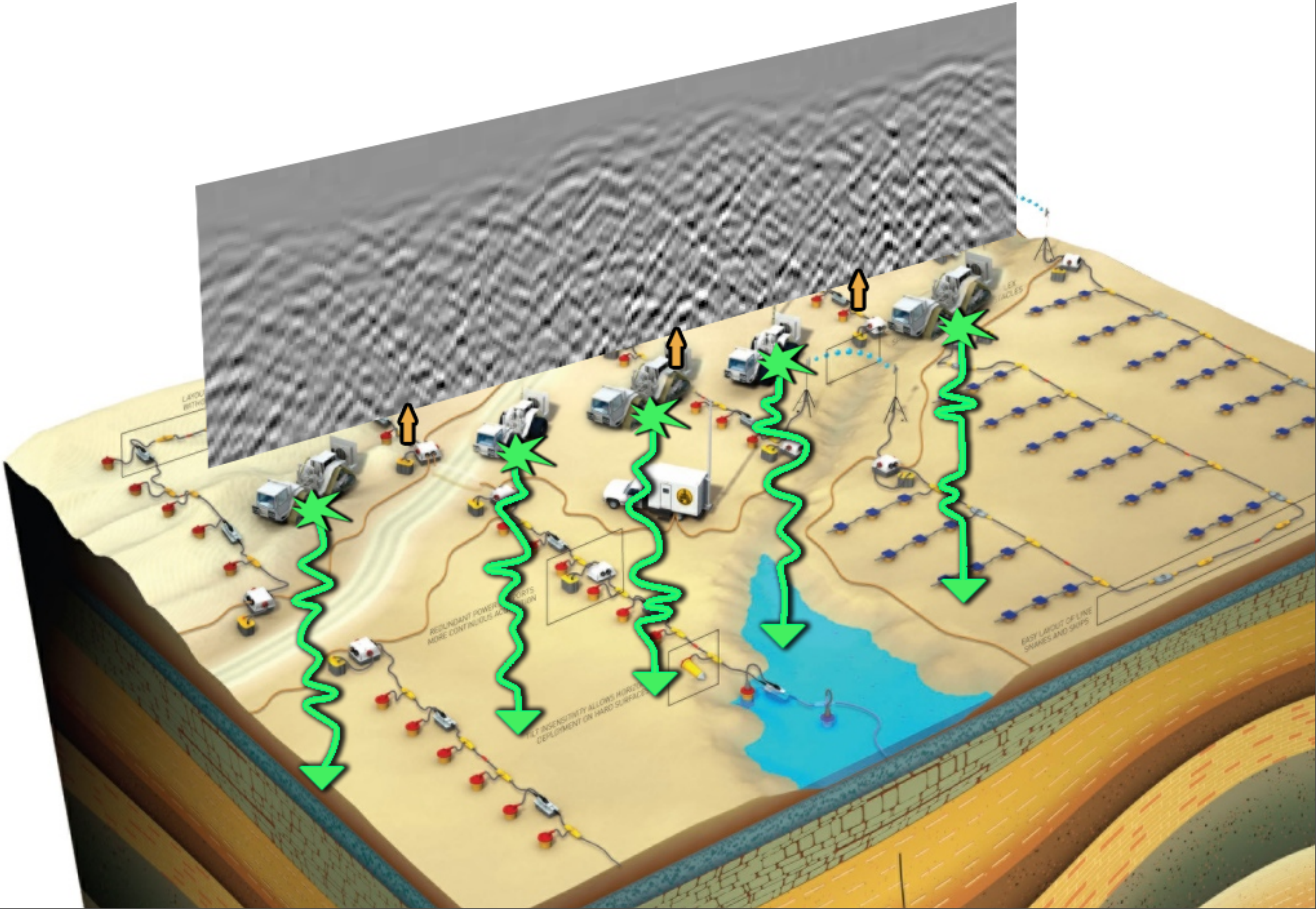
Observation & extensions

- Findings from 1D jittered sampling extend to higher dimensions
  - randomized is better than regular subsampling
  - Cartesian versus hexagonal sampling are equivalent for optimal jittered sampling
  - Furthest point and Poisson sampling lead to similar results

- Gap-size control
  - jittered sampling gives explicit control max distance between adjacent samples
  - farthest point and Poisson disk also have bounds but not explicit

- Future extensions
  - variable density sampling
  - ungridded
  - exploring symmetry (e.g. reciprocity)

- Open math. questions
  - extension of CS results to frames
  - practical design principles

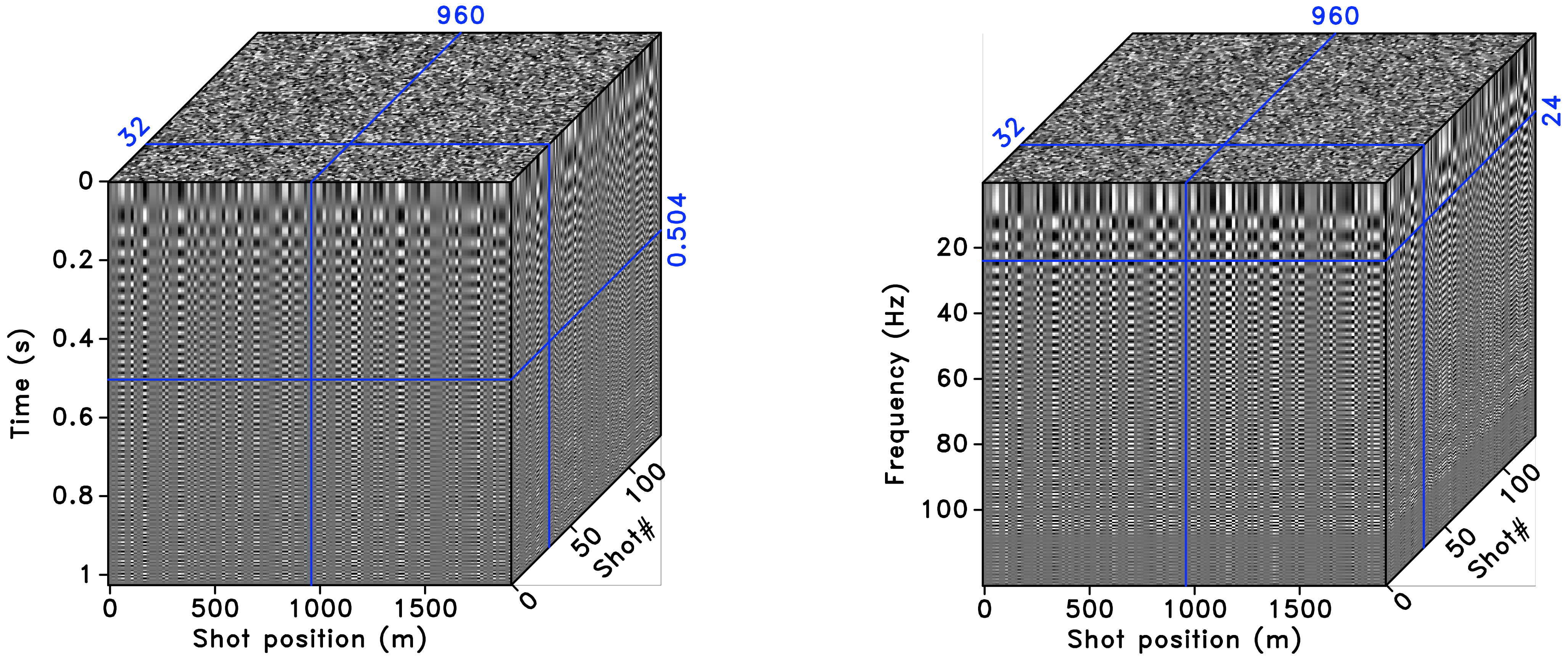

REDUNDANT POWER
MORE CONTINUOUS AC
TILT INSENSITIVITY ALLOWS HORIZ
DEPLOYMENT ON HARD SURFACE
EASY LAYOUT OF LINE
SNAKES AND SKIPS

Relation to existing work

- **Simultaneous & continuous acquisition:**
  - *A new look at marine simultaneous sources* by C. Beasley, ‘08
  - *Simultaneous Sourcing without Compromise* by R. Neelamani & C.E. Krohn, ’08.
  - *Changing the mindset in seismic data acquisition* by A. Berkout, ’08
  - *Independent simultaneous sweeping - A method to increase the productivity of land seismic crews* by D. Howe, M. Foster, T. Allen, B. Taylor, and I. Jack, ‘08

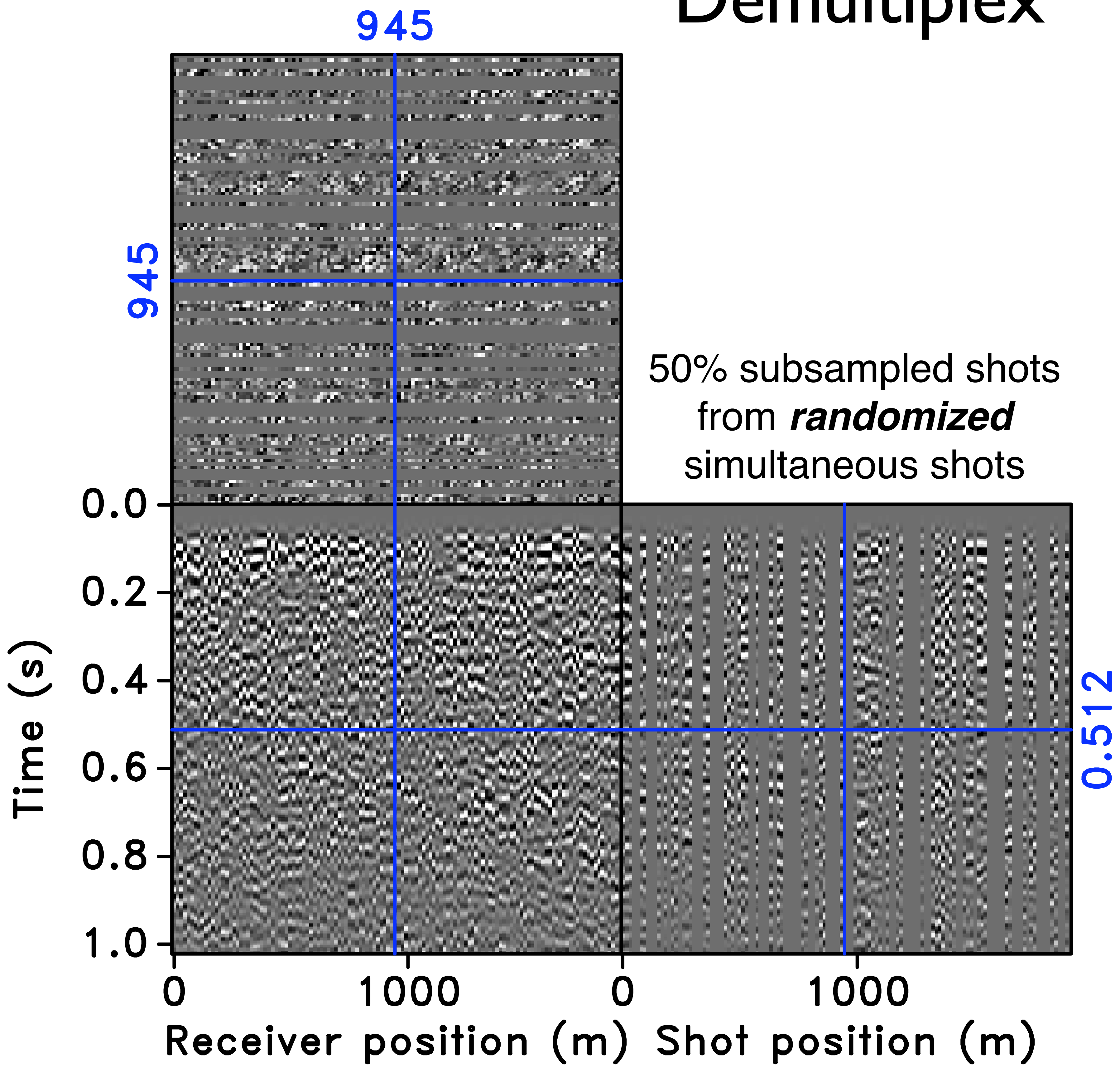
Recovery from simultaneous data

$$\mathbf{RM} = \overbrace{\left[\mathbf{R}^{\Sigma}\otimes\mathbf{I}\otimes\mathbf{I}\right]}^{\text{sub sampler}} \overbrace{\left(\mathbf{F}_{\mathrm{s}}^{*}\mathrm{diag}\left(e^{\hat{i}\boldsymbol{\theta}}\right)\mathbf{F}_{\mathrm{s}}\otimes\mathbf{I}\otimes\mathbf{I}\right)}^{\text{random phase encoder}}$$



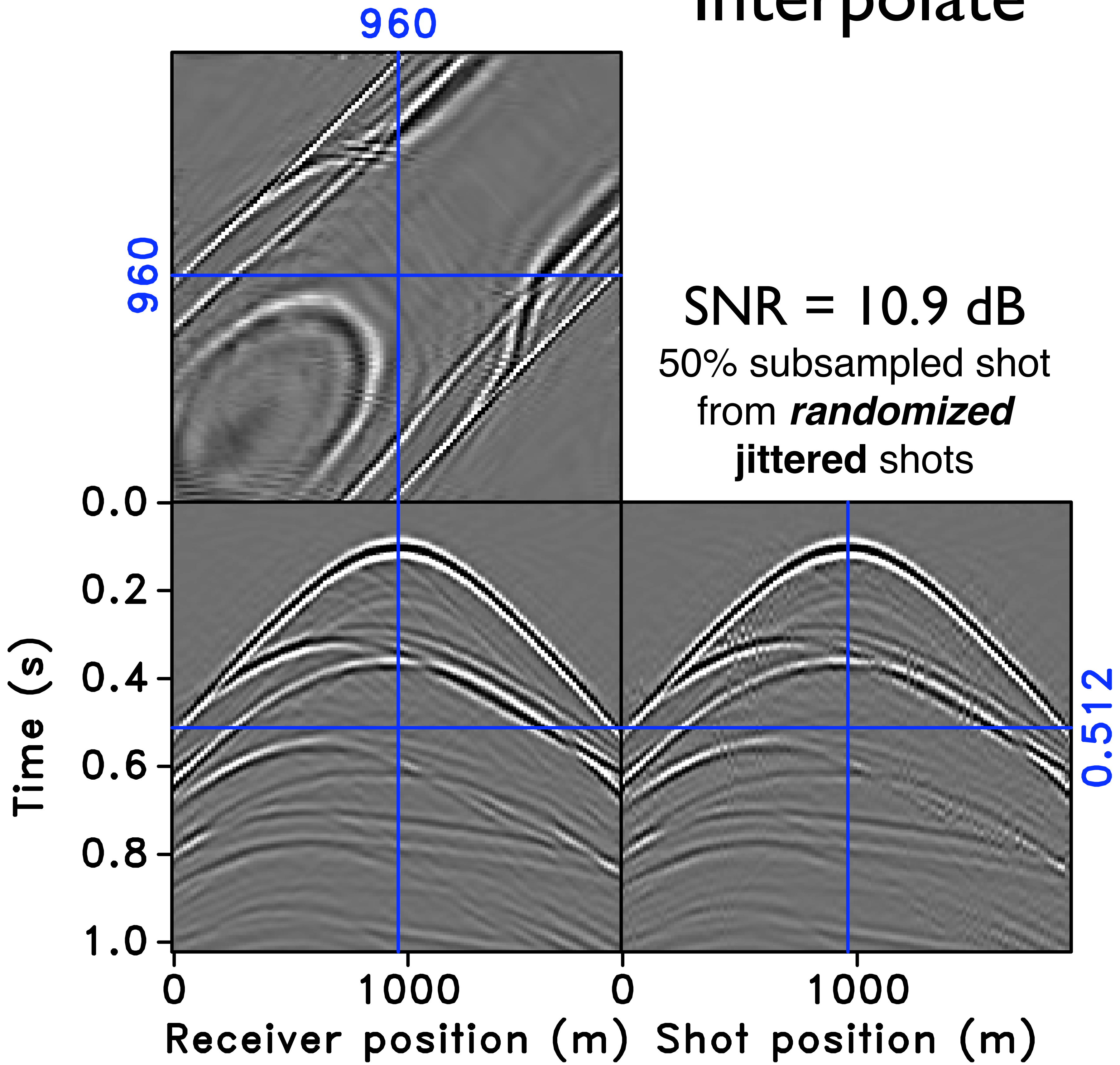
- Linearly ramping seismic sweep, 5 to 110 Hz
- Simultaneous source at all positions, each randomly phase encoded

Demultiplex



50% subsampled shots
from ***randomized***
simultaneous shots

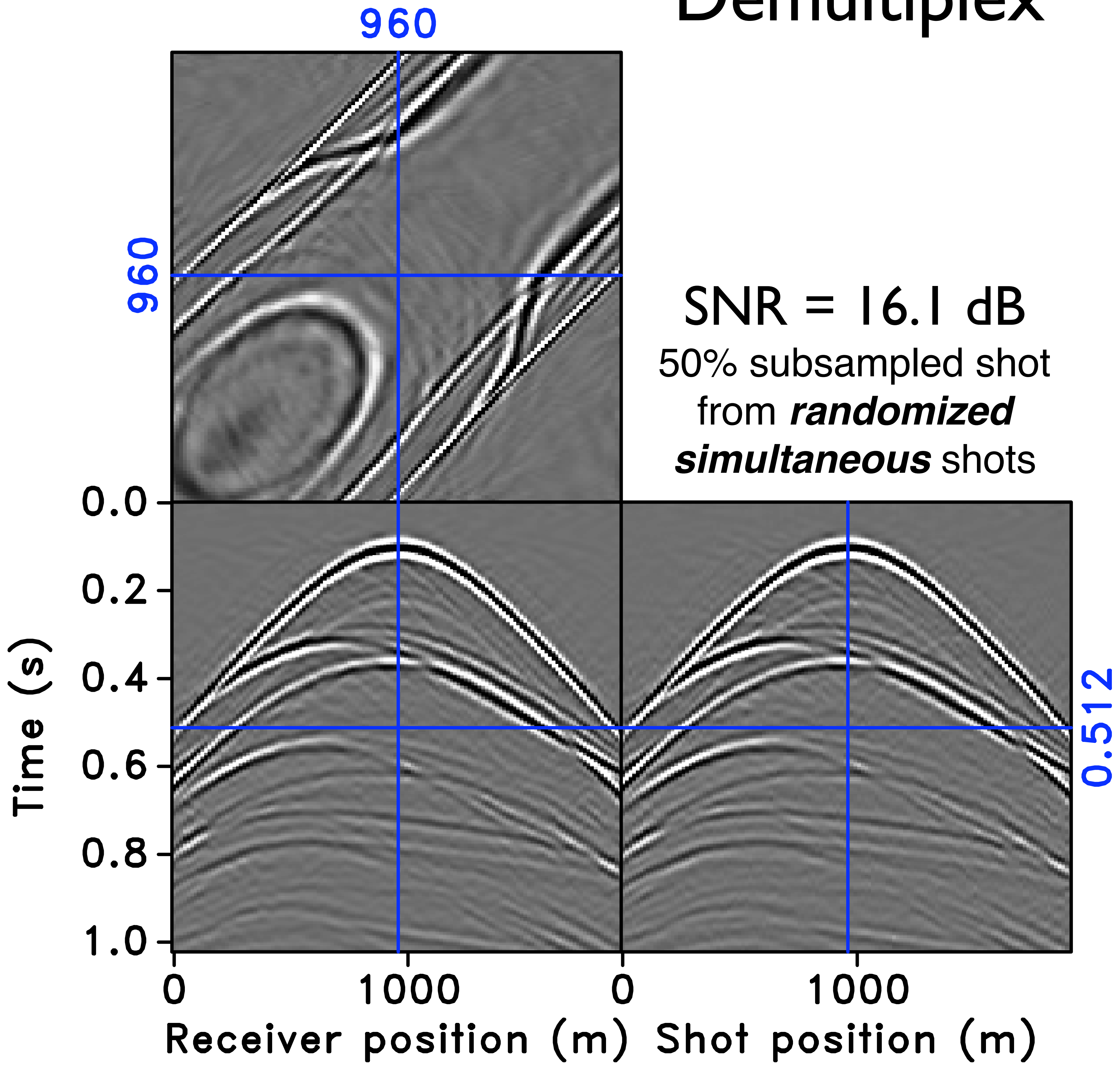
Interpolate



SNR = 10.9 dB

50% subsampled shot
from ***randomized***
jittered shots

Demultiplex



SNR = 16.1 dB

50% subsampled shot
from ***randomized***
simultaneous shots

Key elements

☑ *sparsifying transform*

- typically *strictly* **localized** in the Fourier space
- rapid decay physical space to handle the complexity of seismic data
- mutual incoherence

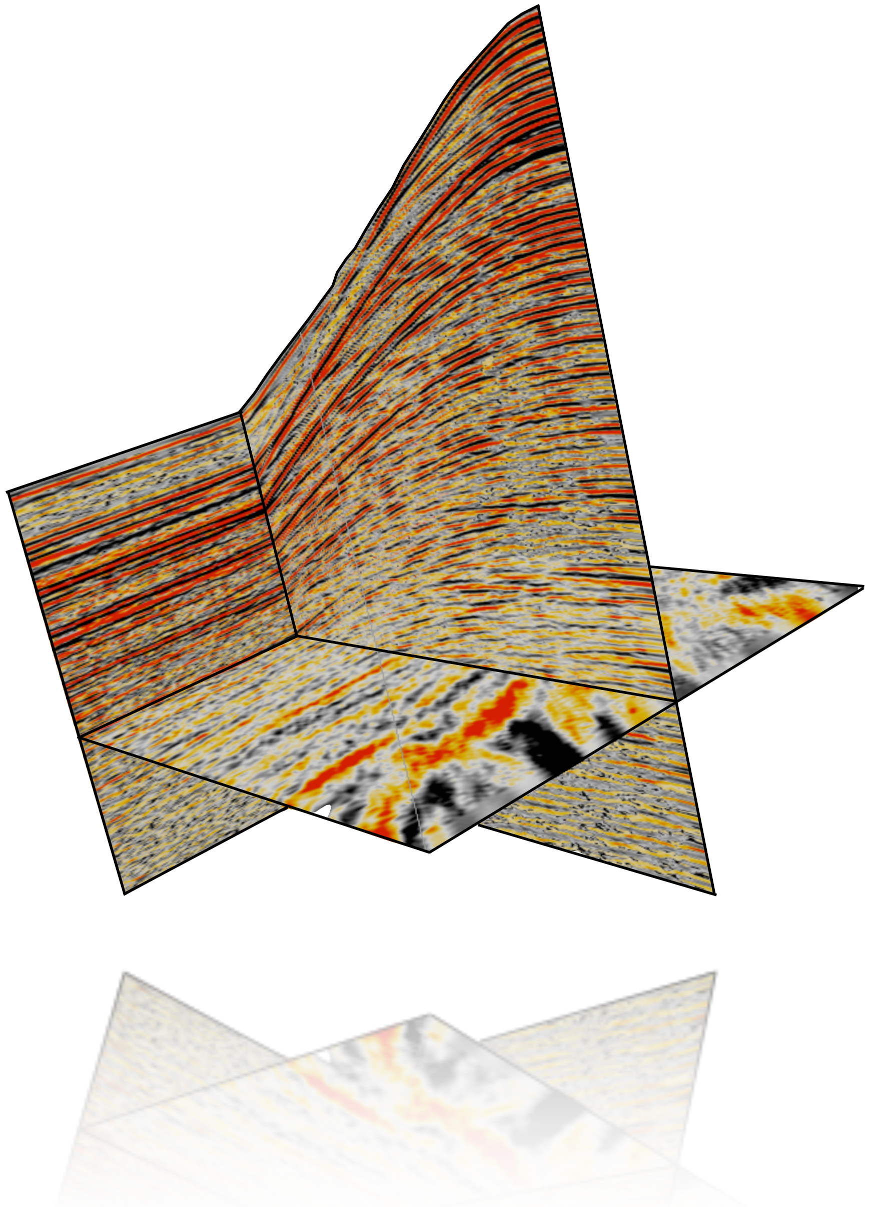
☑ *advantageous* ***randomized*** *coarse sampling*

- generates incoherent random undersampling “noise” in sparsifying domain
- ***spatial sampling*** that does not create large gaps
  - because of the limited spatiotemporal extent of transform elements used for the reconstruction
- ***randomized*** subsampling of ***simultaneous-source*** experiments
  - does not create large interferences
  - leads to compression of linear systems
    - reduction # right-hand-sides
    - spectral representation operators

☐ *sparsity-promoting solver*

- requires few matrix-vector multiplications

Recent developments: recovery of surface-free data from simultaneous acquisition



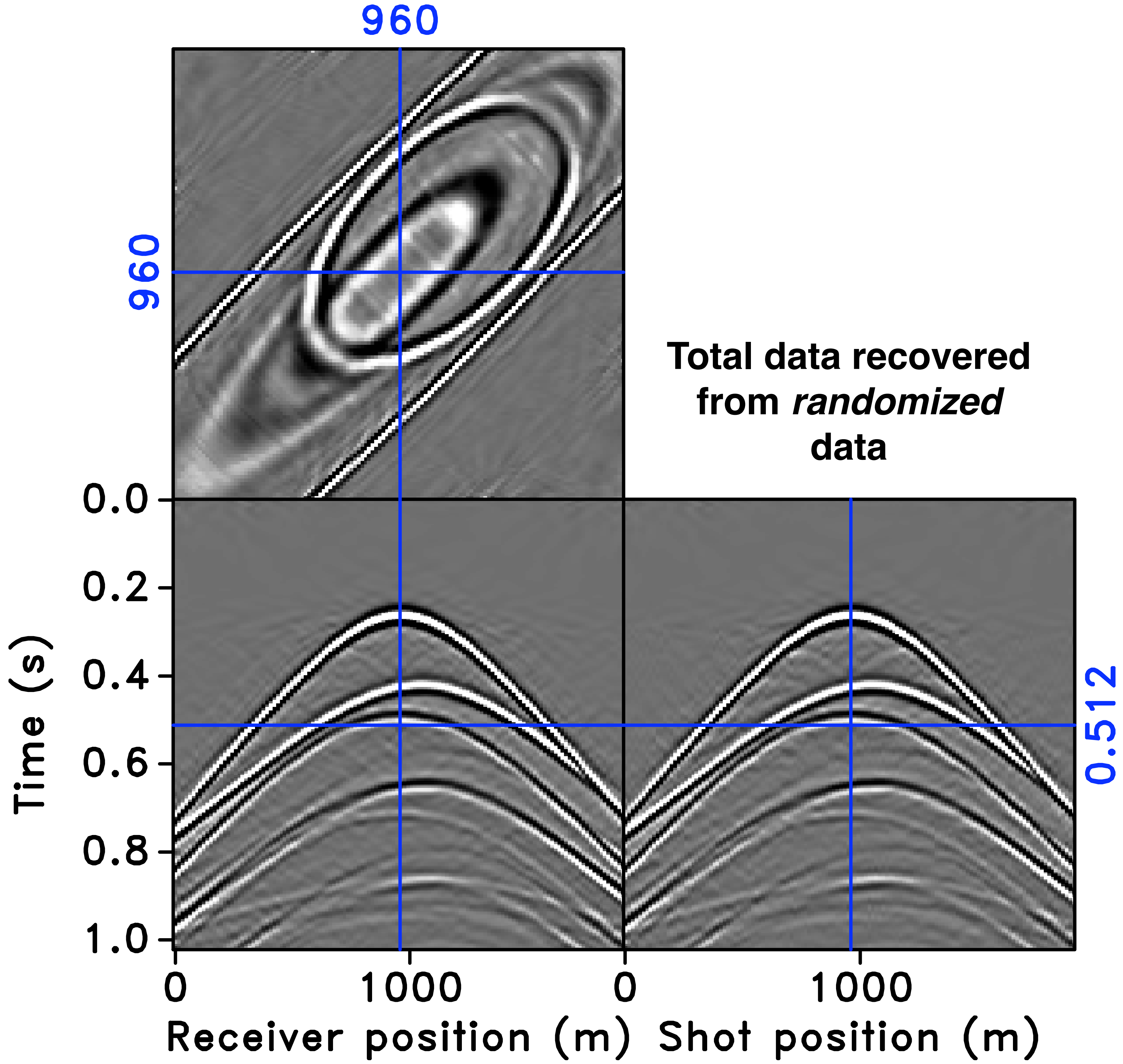
Vienna July 24, 2009

Estimation of primaries from simultaneous data

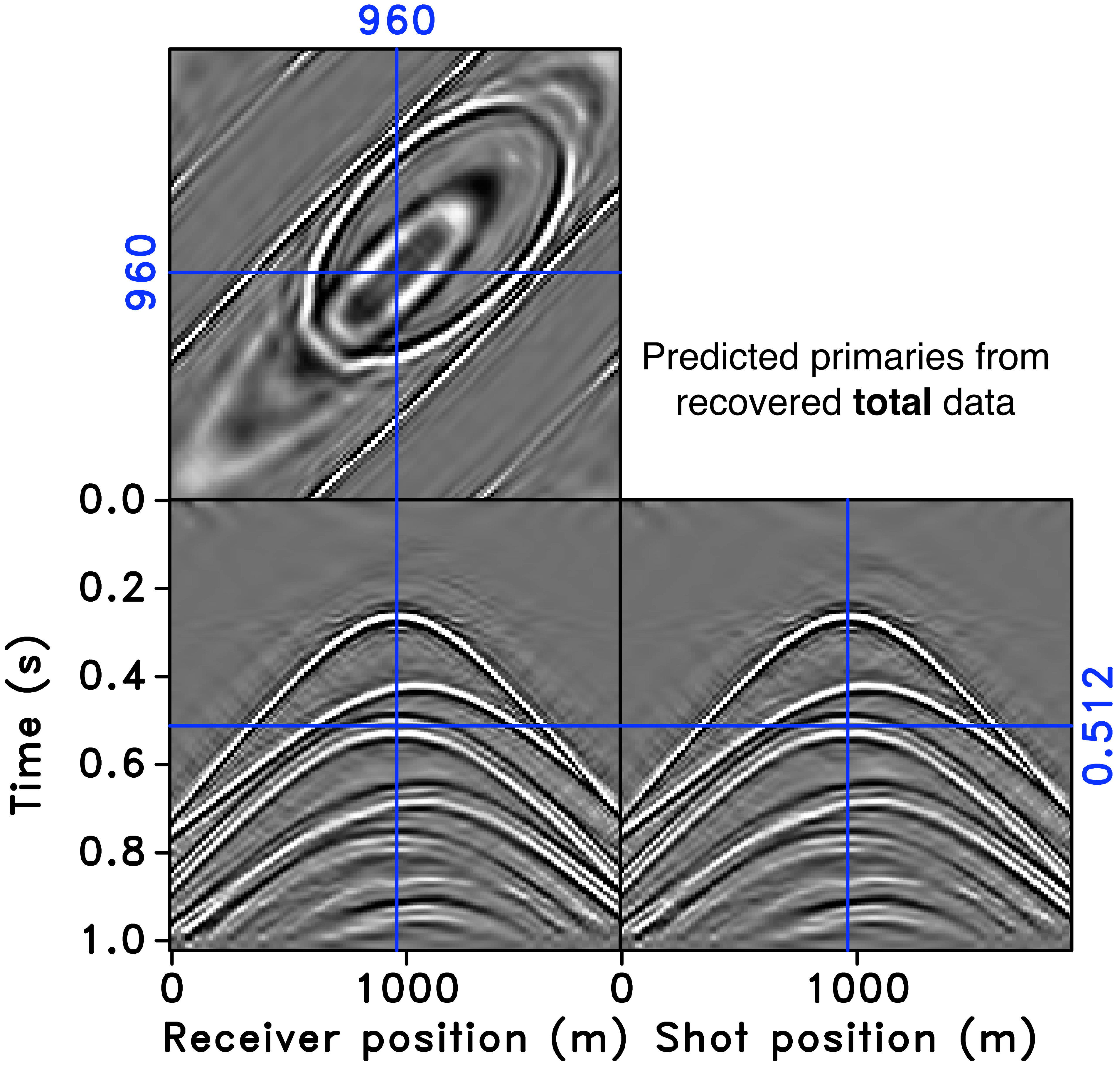
- Include multiple prediction operator
  - operator that generates all surface-related multiples
  - invert the operator as part of the sparsity-promoting recovery

$$\mathbf{RM} = \overbrace{\left[\mathbf{R}^{\Sigma} \otimes \mathbf{I} \otimes \mathbf{I}\right]}^{\text{sub sampler}} \overbrace{\left(\mathbf{F}_{\mathrm{s}}^{*} \mathrm{diag}\left(e^{\hat{i}\boldsymbol{\theta}}\right) \mathbf{F}_{\mathrm{s}} \otimes \mathbf{I} \otimes \mathbf{I}\right)}^{\text{random phase encoder}} \overbrace{\mathbf{G}}^{\text{multiple prediction}}$$

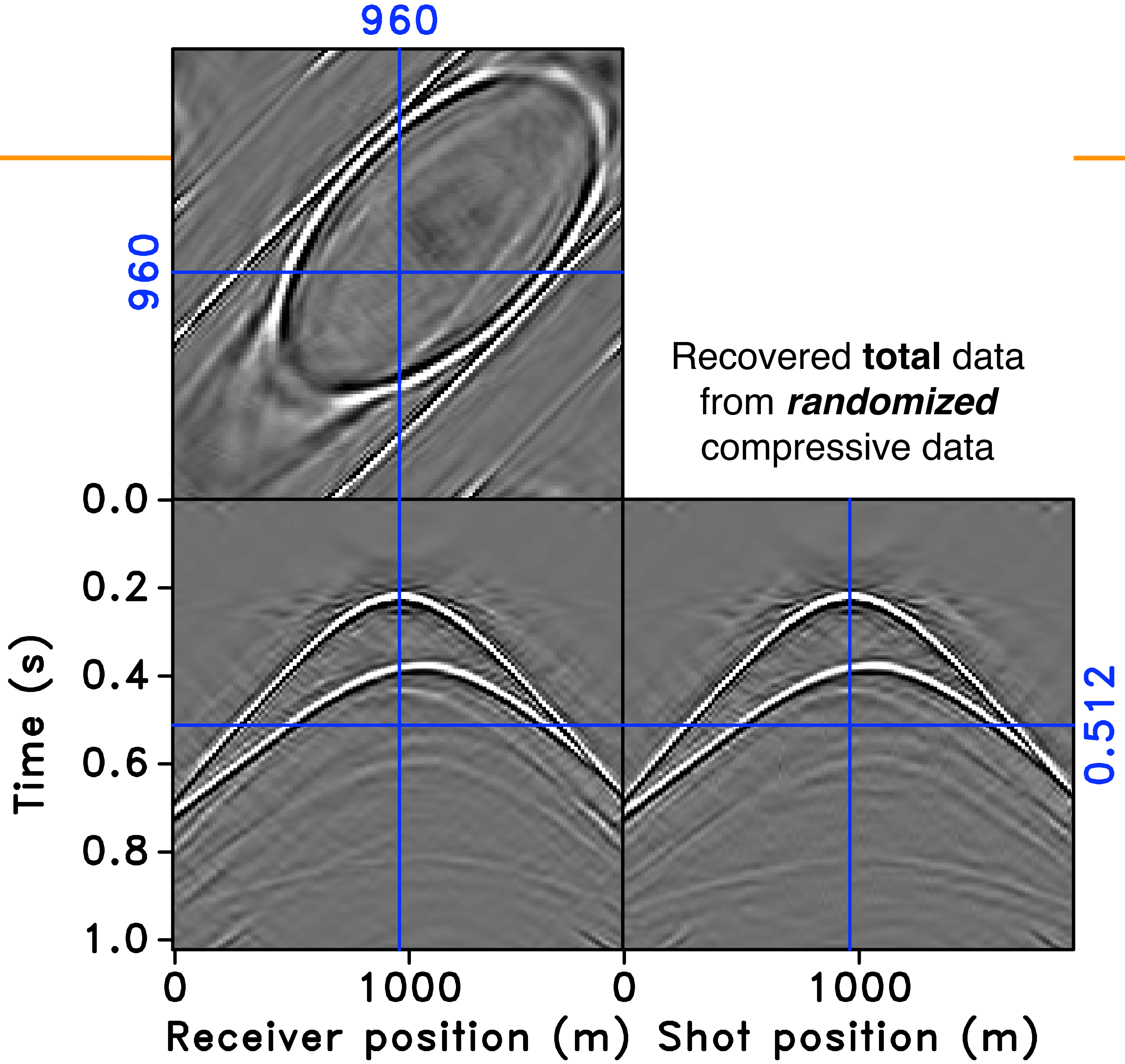
- **Primary prediction through wavefield inversion:**
  - *Elimination of free-surface related multiples without need of the source wavelet* by L. Amundsen, ‘01
  - *Primary estimation by sparse inversion and its application to near offset reconstruction* by G. van Groenenstijn and D. Verschuur, ’09



Total data recovered from *randomized* data

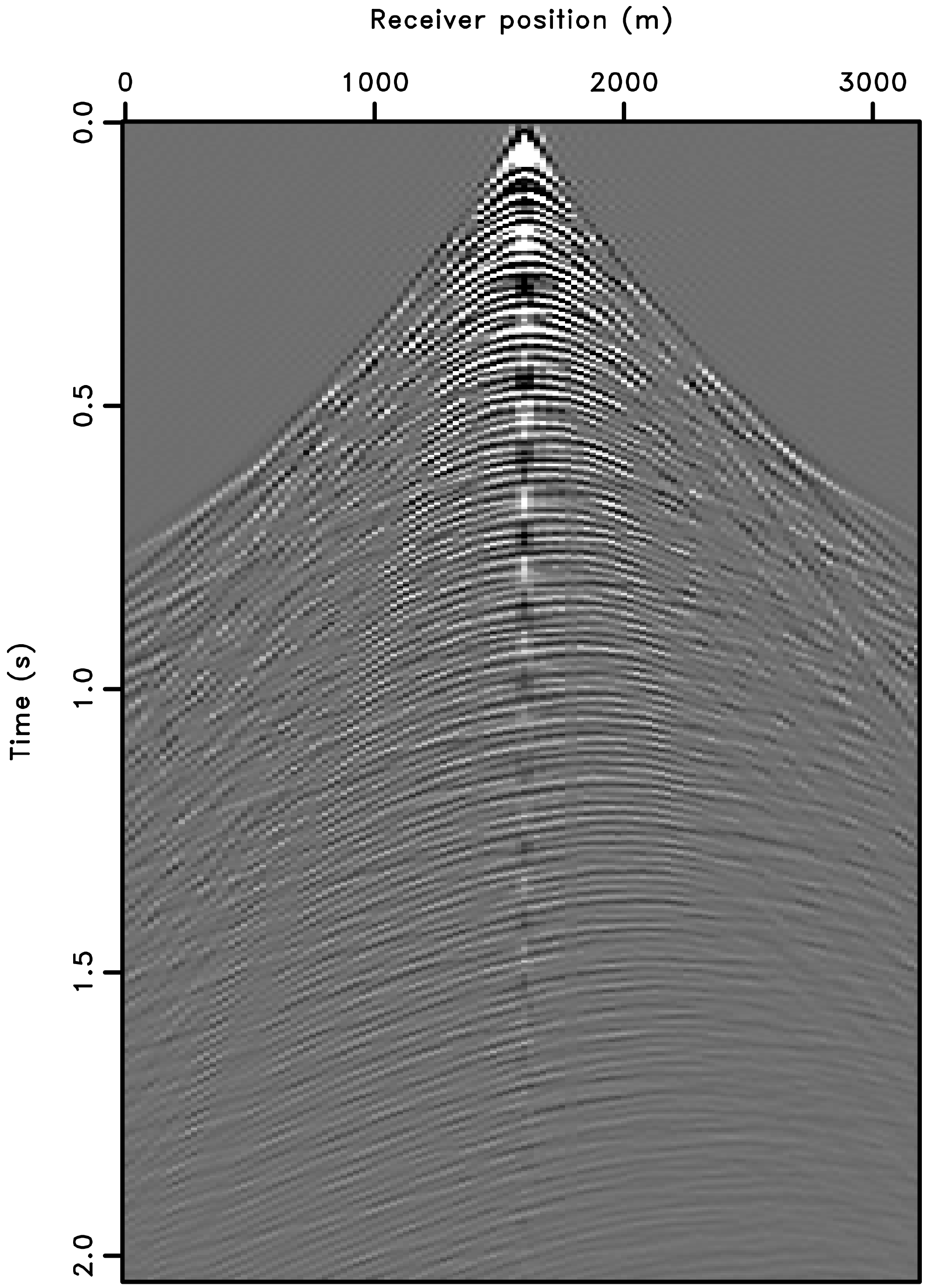


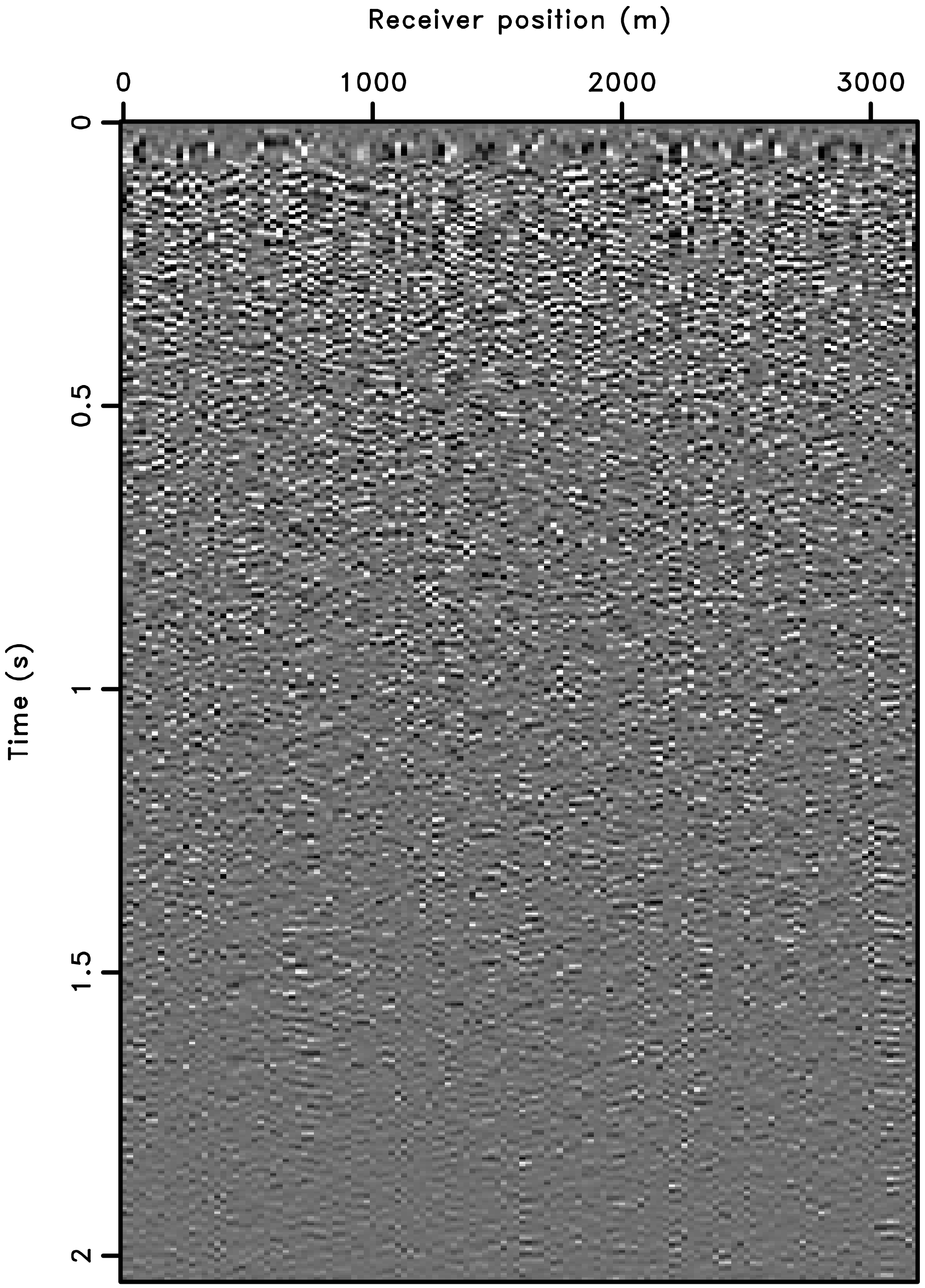
Predicted primaries from
recovered **total** data



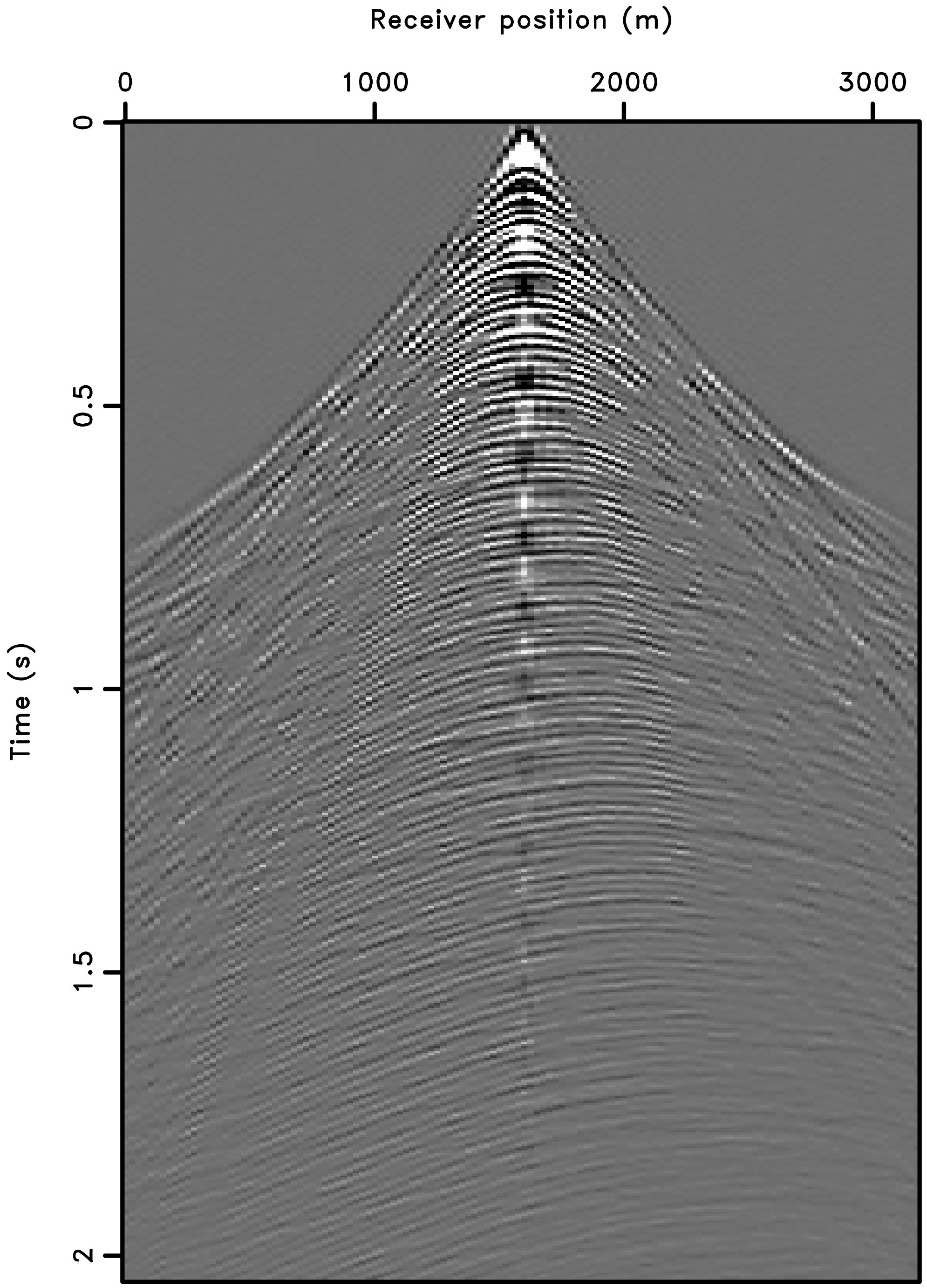
Recovered **total** data
from ***randomized***
compressive data

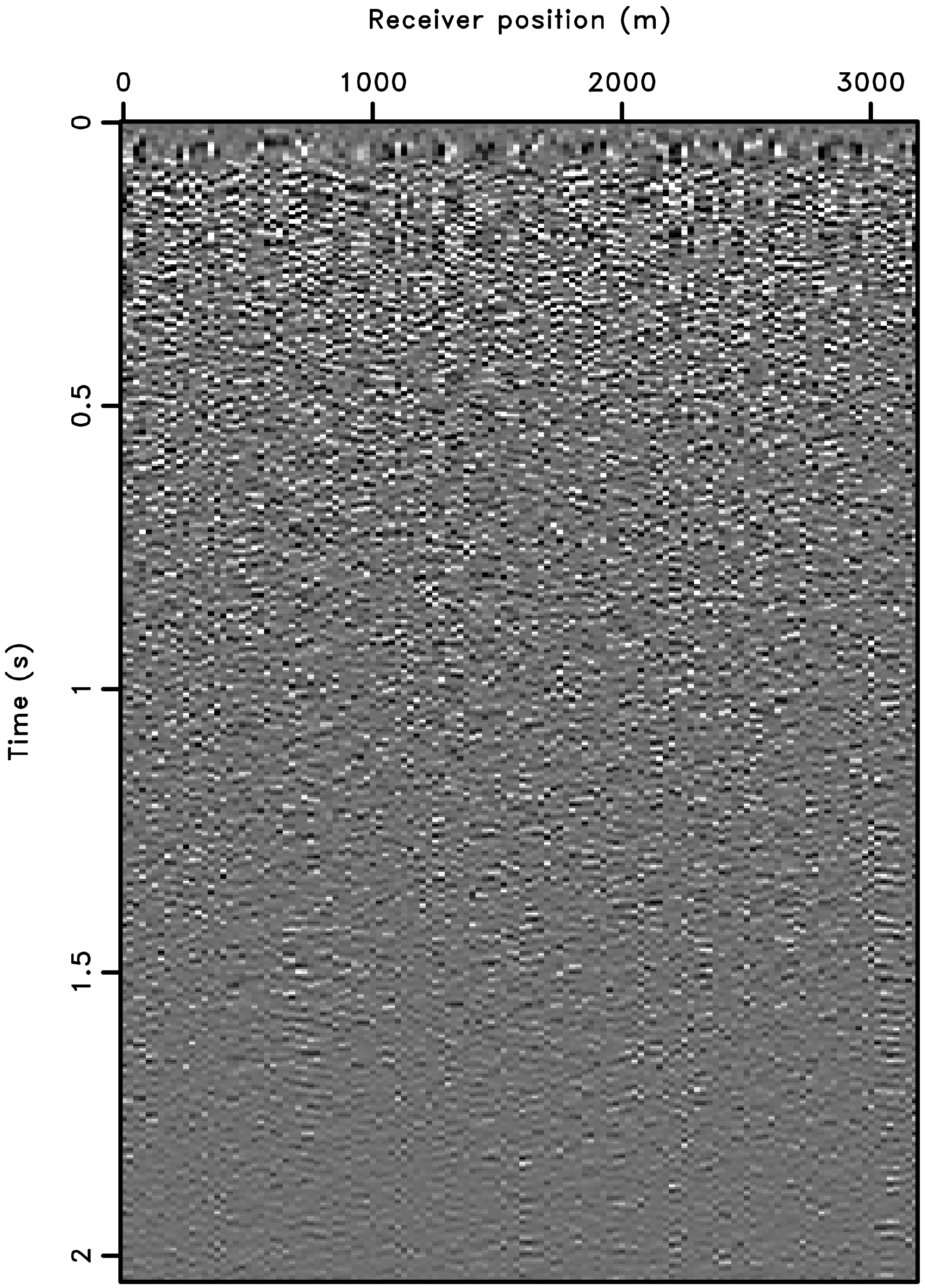
Real Marine data





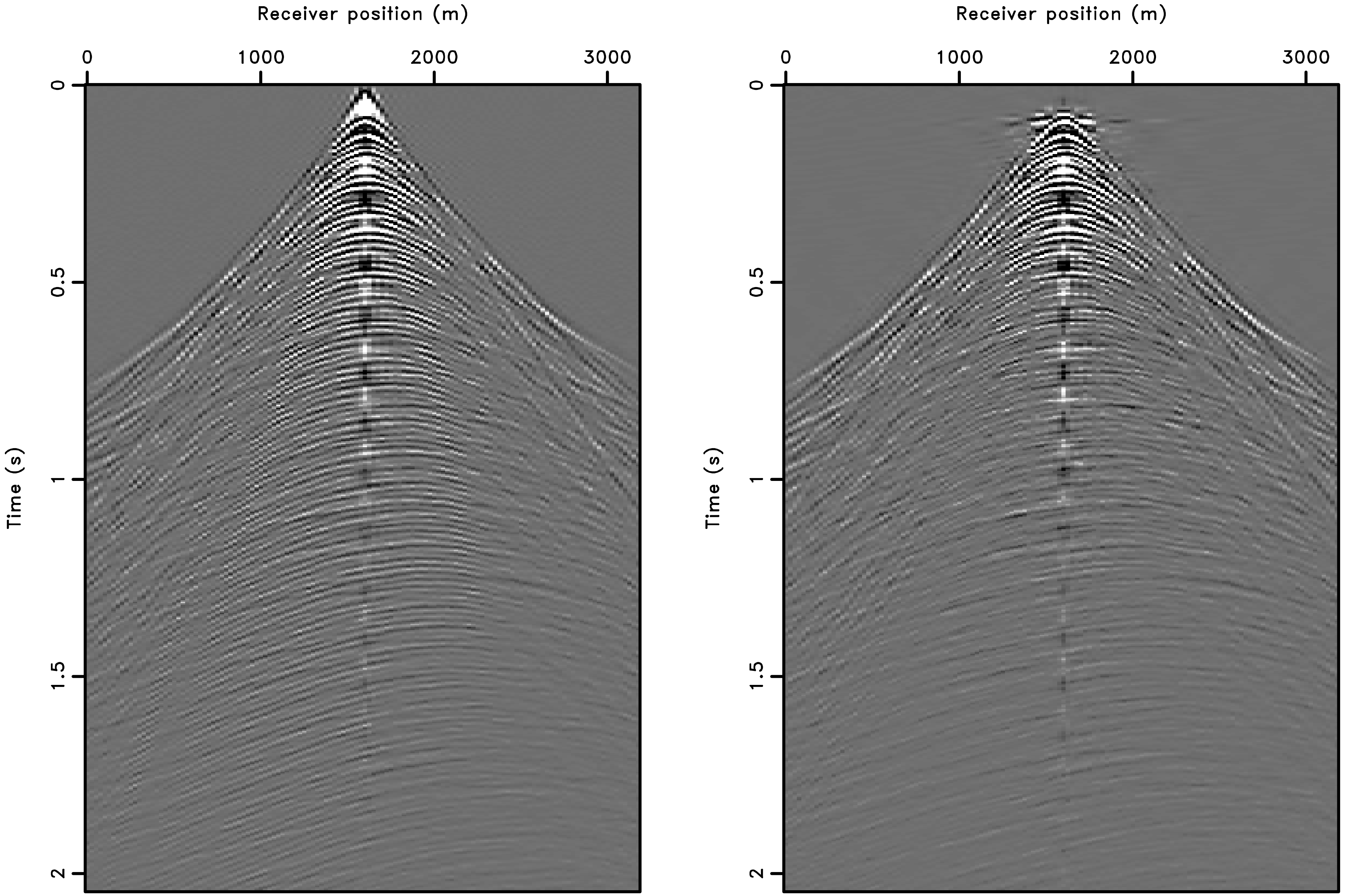
Recovery Real Marine data





85x

Primary prediction Real Marine data



Conclusions

- **Randomization** is essential for recovery from incomplete data

- Good **randomized** sampling
  - with blue-noise characteristics give good curvelet recovery
  - with simultaneous sources gives excellent curvelet recovery

- **Randomization** leads to
  - “acquisition” of *smaller* data volumes that carry the ***same*** *information* or
  - to ***improved*** *inferences* from data using the *same* resources

- **Bottom line: acquisition costs** are **no** longer determined by the ***size*** of the ***discretization*** but by ***transform-domain sparsity*** of the sampled wavefield **...**

Acknowledgments

- E. van den Berg and M. P. Friedlander for *SPGL1* (www.cs.ubc.ca/labs/scl/spgl1) & *Sparco* (www.cs.ubc.ca/labs/scl/sparco)
- Sergey Fomel and Yang Liu for Madagascar (rsf.sf.net)
- E. Candes and the Curvelab team

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and... Thank you!