Seismic Velocity Inversion and Uncertainty Quantification Using Conditional Normalizing Flows

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Motivation

- Seismic velocity inversion plays a vital role in seismic exploration.
- It is an ill-posed problem where different solutions may fit the data equally well.
Bayesian Inversion

- From the perspective of Bayesian inversion, the multiple solutions can be considered as posterior samples of the solution distribution.
- Given the observed data $d$, seismic velocity model $m$ can be expressed as the posterior distribution:

$$p(m \mid d) = \frac{p(d \mid m)p(m)}{p(d)} \propto p(d \mid m)p(m)$$

- $m$: True velocity model
- $p(d)$: Probability of seismic data $d$ being observed
- $p(m)$: Probability of seismic velocity distribution $m$
- $p(d \mid m)$: Conditional probability of observing $d$ given seismic model $m$
- $p(m \mid d)$: Conditional probability of model $m$ when $d$ is observed
Normalizing flows

• A special kind of invertible neural network
• By connecting several invertible network units in series, it could easily sample the posterior distribution via memory-efficient training
• A good choice to address the seismic inversion problem and evaluate the reliability of the inverted velocity models.
The proposed method

- HINT structure\textsuperscript{[1]} is adopted for building a conditional normalizing flow (CNF) named \textbf{i-VelInvNet}

- Instead of seismic data, RTM images are used as network input

- \textbf{Two advantages:}
  - Same dimension as the velocity model, reduce network input dimension.
  - Effectively reflect the subsurface geological features, easier for CNF to establish a mapping to the true velocity model

The proposed method

- **i-VelInvNet architecture**

Forward pass

\[
\begin{align*}
[m^1, m^2]^T &= Q_m m, \quad [d^1, d^2]^T = Q_d RTM (d) \\
Z_m &= G_m \left( \left[ m^1, m^2 \right]^T \right) \odot s + t, \quad [s, t] = G \left( \left[ d^1, d^2 \right]^T \right) \\
Z_d &= G_d \left( \left[ d^1, d^2 \right]^T \right)
\end{align*}
\]

Backward pass

\[
\begin{align*}
[d^1, d^2]^T &= G_d^{-1} (z_d) \\
[m^1, m^2]^T &= G_m^{-1} \left( (z_m - t) / s \right), \quad [s, t] = G \left( \left[ d^1, d^2 \right]^T \right) \\
m &= Q_m^{-1} \left[ m^1, m^2 \right]^T, \quad RTM (d) = Q_d^{-1} \left[ d^1, d^2 \right]^T
\end{align*}
\]

The proposed method

- **i-VelInvNet loss function**

\[
L(\theta) = \frac{1}{N} \sum_{i=1}^{N} \left[ \frac{1}{2} \left\| T(RTM, \mathbf{m}) \right\|_2^2 - \log |\det \nabla T(RTM, \mathbf{m})| \right]
\]

\( \theta \): Network parameters, \( T \): Forward pass

- **Inference process**

- ✓ for a given unseen observation data, compute \( z_d \)
- ✓ Randomly sampling \( z_m \sim N(0, I_m) \) and compute the posterior sample

\[
p_T(\mathbf{m} | \mathbf{d}) = \mathcal{T}^{-1} \left[ \begin{bmatrix} z_d \\ z_m \end{bmatrix} \right] = \mathcal{T}^{-1} \left[ \begin{bmatrix} \mathcal{T}_d(RTM(\mathbf{d})) \\ N(0, I_m) \end{bmatrix} \right]
\]

- **Uncertainty analysis of the inverted results**
Step 1: Initialization
Step 2: Compute RTM
Step 3: Train i-VellInvNet
Step 4: Posterior sampling based on the trained i-VellInvNet
Step 5: Uncertainty analysis of the inverted results
Step 6: Repeat step 2 ~ 5 for better posterior samples (optional)
Step 7: Gaussian smooth for a good initial model and perform FWI for the final inversion results
EXPT.1: Check invertibility of i-VellInvNet

- **Training dataset setup**
  - **2000 models:** 2D sections of the 3D overthrust model
  - **Model size:** $64 \times 200$
  - **Grid spacing:** 25 m
  - **Observation setup:** 21 sources, 201 receivers, 6Hz Ricker wavelet

- **How to check Invertibility?**
  - **Forward** $[z_d, z_m]^T = T(\text{RTM}, m)$,
  - **Backward** $[\text{RTM}', m']^T = T^{-1}(z_d, z_m)$
  - **Check** $m = m'$, RTM=RTM'
EXPT.1: Check invertibility of i-VellInvNet

True velocity model

RTM image
EXPT.1: Check invertibility of i-VellInvNet

Predicted velocity model

Predicted RTM image
EXPT.1: Check invertibility of i-VelInvNet

Difference of the velocity model

Difference of the RTM images
EXPT.2: Inversion results of i-VellInvNet

- Train : valid : test = 1200 : 400 : 400
- Training steps = 8000
EXPT.2: Inversion results of i-VellInvNet

- Randomly select a velocity from the test dataset
- Compute the observation data and RTM image as i-VellInvNet input
- Fix $z_d$ and feed 100 random $z_m$ for 100 posterior samples of the inverted velocity model via the backward pass of the trained i-VellInvNet.

![True velocity model](image1.png)

![RTM image](image2.png)
EXPT.2: Inversion results of i-VellInvNet

- 4 of the 100 posterior samples of the i-VellInvNet inversion results
EXPT.2: Inversion results of i-VellInvNet

- Uncertainty analysis of the posterior samples

![Posterior mean](image1)

![Standard deviation](image2)
EXPT.2: Inversion results of i-VelInvNet

- **Quantitative comparison**

<table>
<thead>
<tr>
<th>Metrics</th>
<th>Posterior samples</th>
<th>Posterior mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>MAE↓</td>
<td>0.0238</td>
<td>0.0195</td>
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<tr>
<td>MSE↓</td>
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<td>PSNR↑</td>
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<td>MSSIM↑</td>
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<td>0.9188</td>
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</tbody>
</table>

Velocity value at x=2.5km
EXPT.3: Full waveform Inversion

- FWI using the migration velocity as initial model
EXPT.3: Full waveform Inversion

- FWI using one of the i-VelInvNet posterior samples as initial model
EXPT.3: Full waveform Inversion

- FWI using the **i-VellInvNet posterior mean** as initial model

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**True velocity**

**Initial model**

**FWI result**

**Obj. value vs Iterations**
Conclusions

• CNF could provide a direct access to the conditional mean estimate and uncertainty assessment of the inverted results.

• When applied in seismic velocity inversion problem, arbitrary number of inverted velocity models and its uncertainty quantification can be obtained via a trained i-VelInvNet.

• The inverted results, either the posterior samples or posterior mean, can be used as a good initial model in the subsequent FWI for a more accurate result.

• Improvement of the proposed i-VelInvNet can be made in terms of computation efficiency and inversion accuracy.
• Open-sourced Codes:
  https://github.com/slimgroup/JUDI.jl
  https://github.com/slimgroup/InvertibleNetworks.jl

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• Please contact Dr. Yuxiao Ren (ryxchina@gmail.com) for any questions and suggestions!