Curvelet-Regularized Seismic Deconvolution

Vishal Kumar and Felix Herrmann

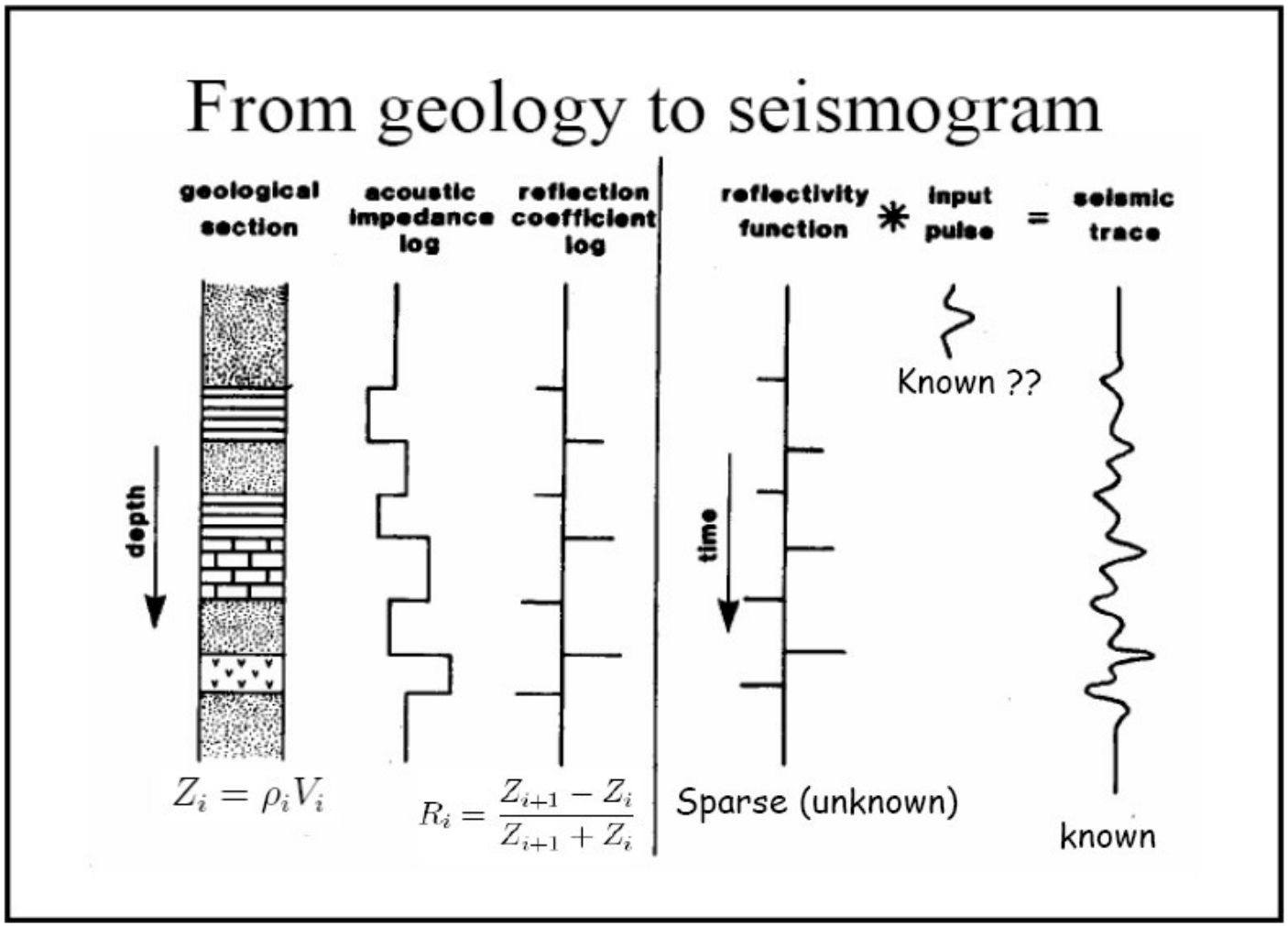
Earth and Ocean Sciences
University of British Columbia
Vancouver, BC, Canada

May 12, 2008

SLIM
Seismic Laboratory for Imaging and Modeling

UBC

The Premise

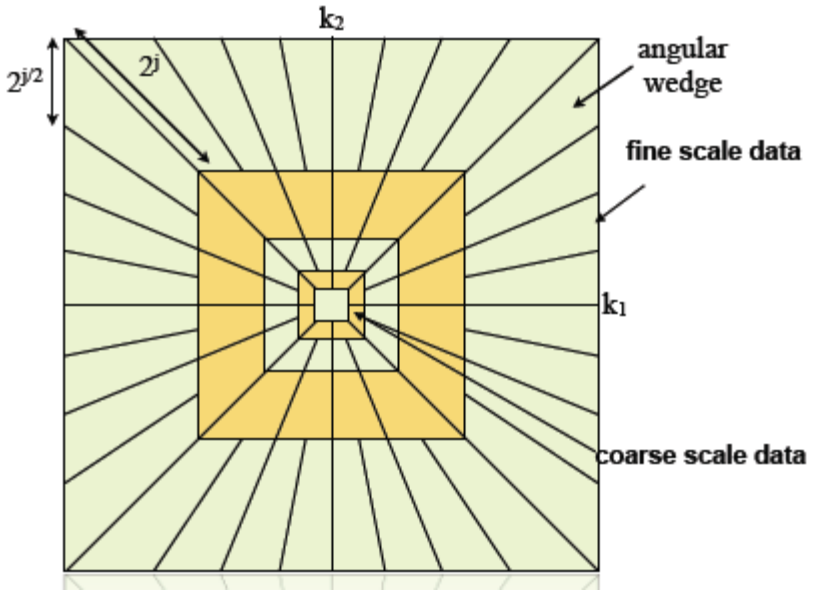

From geology to seismogram
geological section
acoustic impedance log
reflection coefficient log
reflectivity function
input pulse
seismic trace
depth
time
Known ??
$Z_i = \rho_i V_i$
$R_i = \frac{Z_{i+1} - Z_i}{Z_{i+1} + Z_i}$
Sparse (unknown)
known

Motivation

- Spiky assumption is too limited to model seismic reflectivity. [Herrmann, 2005]
- There is an inherent continuity along the reflectors. [Hennenfent et al., 2005]

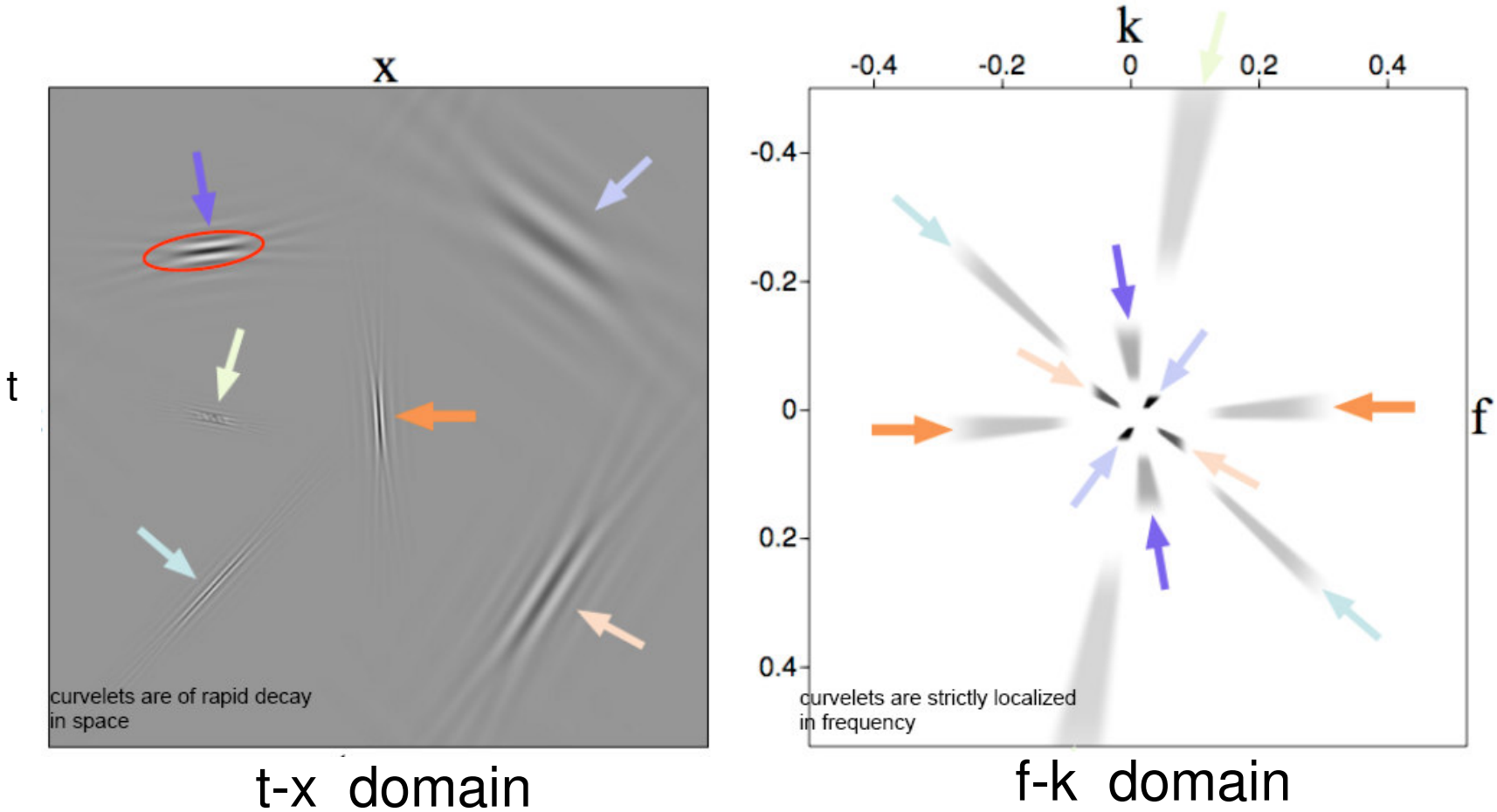
Curvelet transform

- **multiscale:** tiling of the FK domain into dyadic coronae
- **multi-directional:** coronae sub-partition into angular wedges
- **anisotropic:** width ≈ length^2
- **pseudo-localized in spatial domain:** rapid decay in space
- **localized in frequency domain**



[Candes et al.,2006]

2-D Curvelets



t-x domain

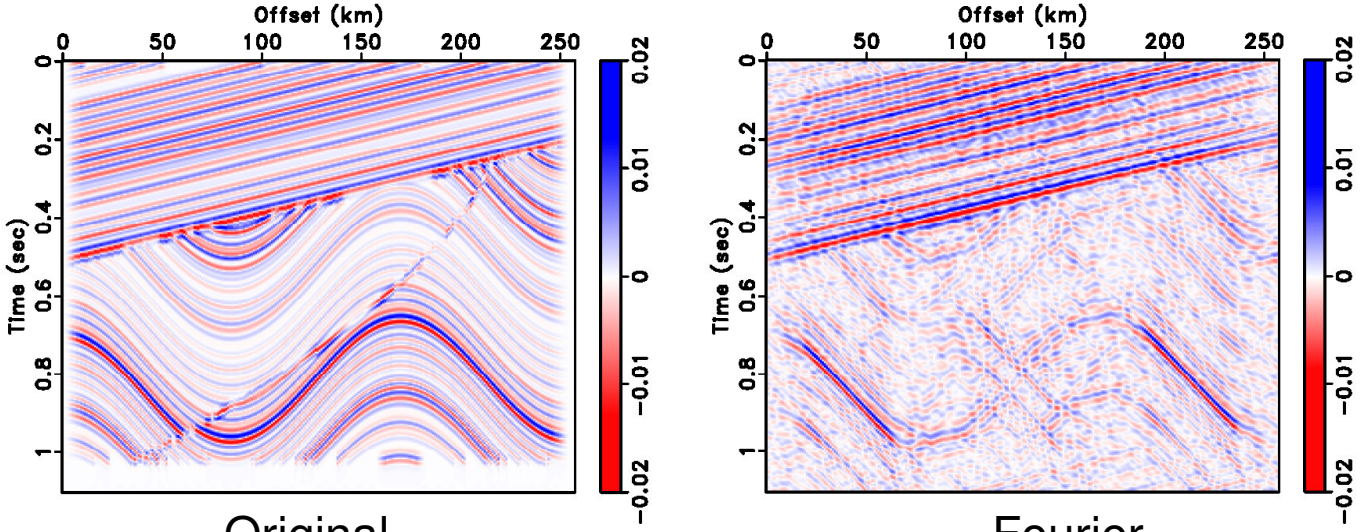
f-k domain

[Adapted from Herrmann and Hennenfent, 2008]

Why curvelets ?

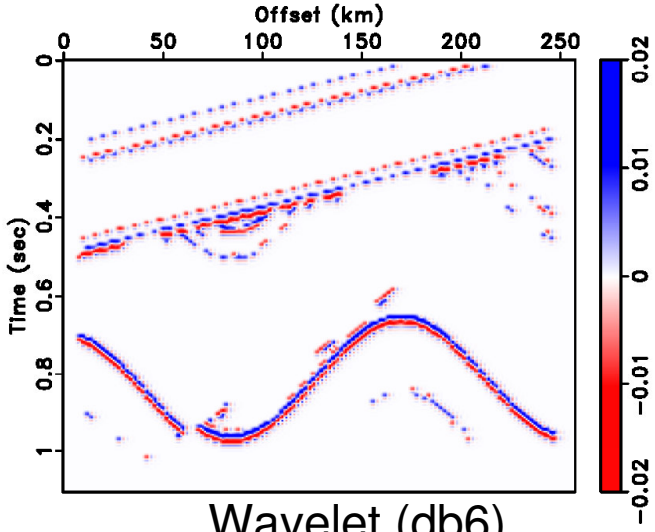
Transform	Underlying assumption
FK	plane waves
linear/parabolic Radon transform	linear/parabolic events
wavelet transform	point-like events (1D singularities)
curvelet transform	**curve-like events (2D singularities)**

Partial Reconstruction of model with 1% Amplitude largest coefficients

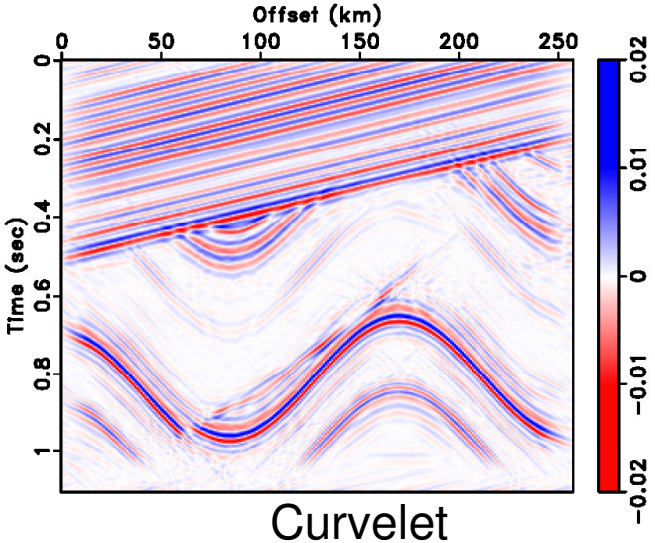


Original

Fourier



Wavelet (db6)



Curvelet

Forward problem

$$y = Am + n$$

$y \rightarrow$ Data

$A \rightarrow$ Convolution Operator

$m \rightarrow$ Reflectivity

$n \rightarrow$ Noise

Deconvolution
as an optimization problem

$$\min_{x} \|x\|_1 \quad \text{s.t} \quad \|y - AC^T x\|_2 \leq \varepsilon$$

$$\hat{m} = C^T x$$

$$\varepsilon^2 = \sigma^2 \left[N + 2\sqrt{2N} \right]$$

$C^T \rightarrow$ Curvelet synthesis operator,

$\sigma \rightarrow$ Standard deviation of noise,

$N \rightarrow$ No. of data points

Data & Model

- Sigmoid model is half differentiated in the frequency domain to obtain non-spiky reflectivity.

- Reflectivity model is convolved with Ricker wavelet (central frequency = 25 Hz) and random noise (σ=.002) is added.

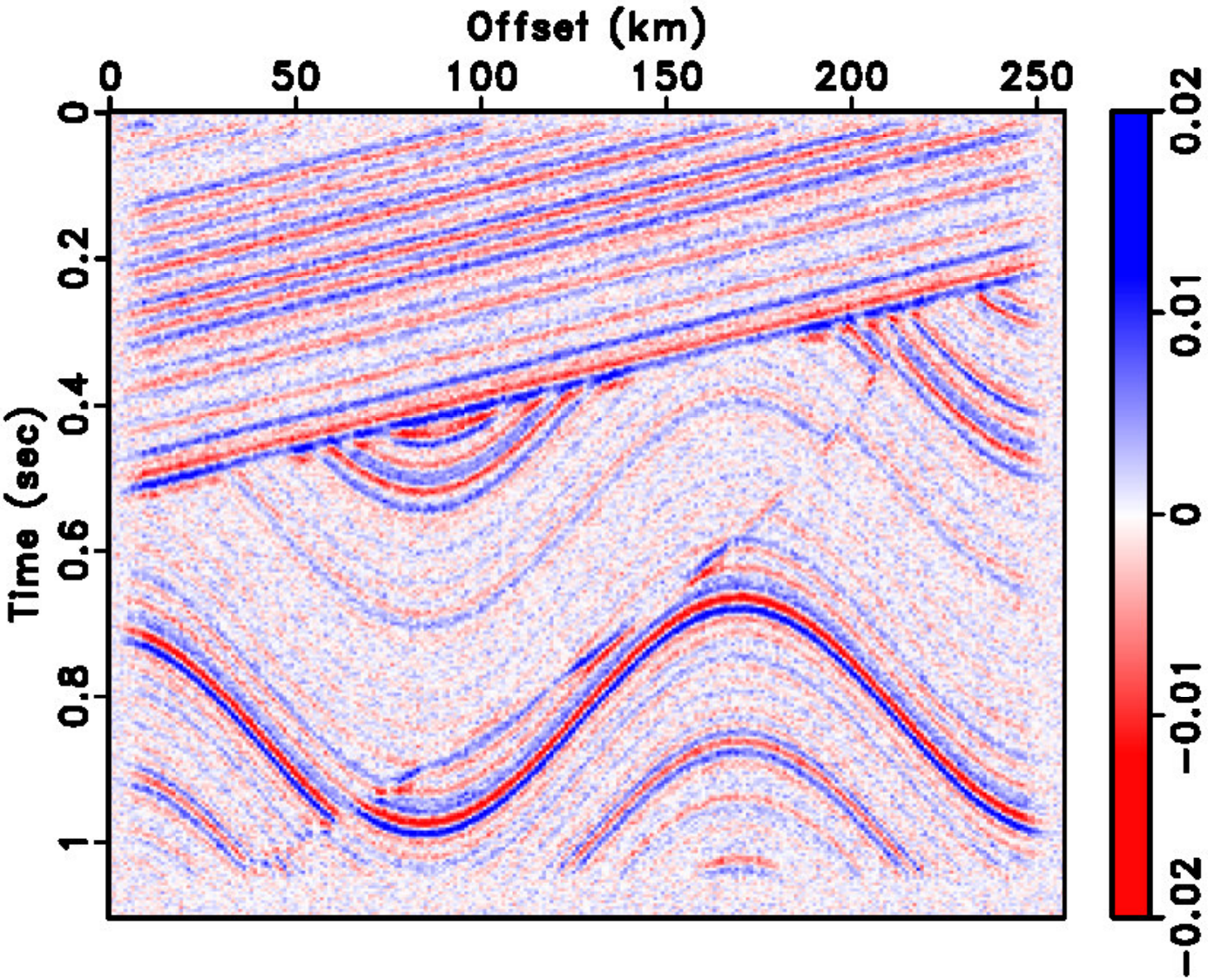
Approach

- Inversion operators were formed in SPARCO. [Van den Berg et. al]
- Solution is found by SPGl_1 algorithm. [Van den Berg and Friedlander]
- Results are compared with those of Spiky Deconvolution defined as:

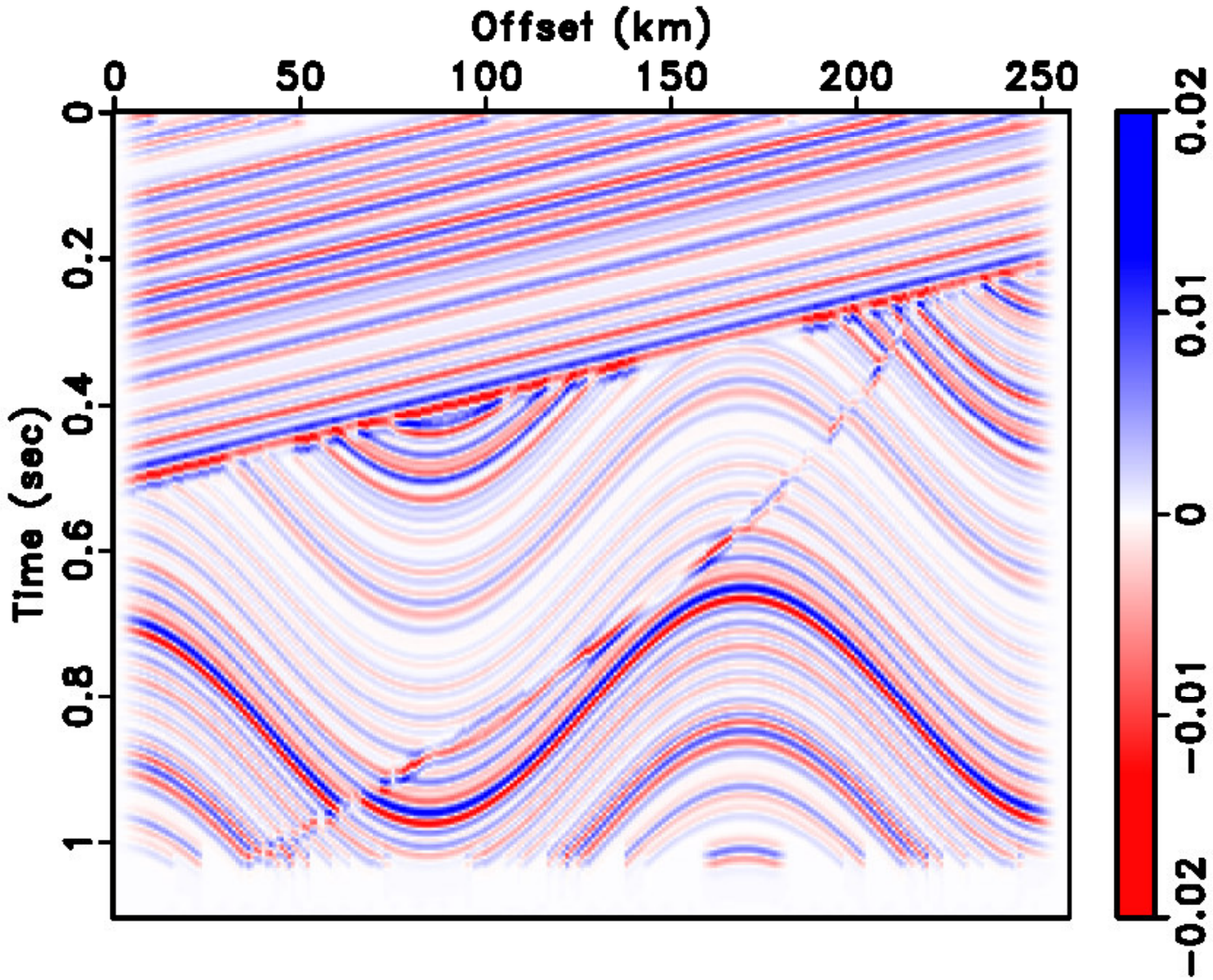
$$\min_{m} \|m\|_1 \quad \text{s.t} \quad \|y - Am\|_2 \leq \varepsilon$$

[Taylor et al., 1979; Oldenburg et al., 1981]

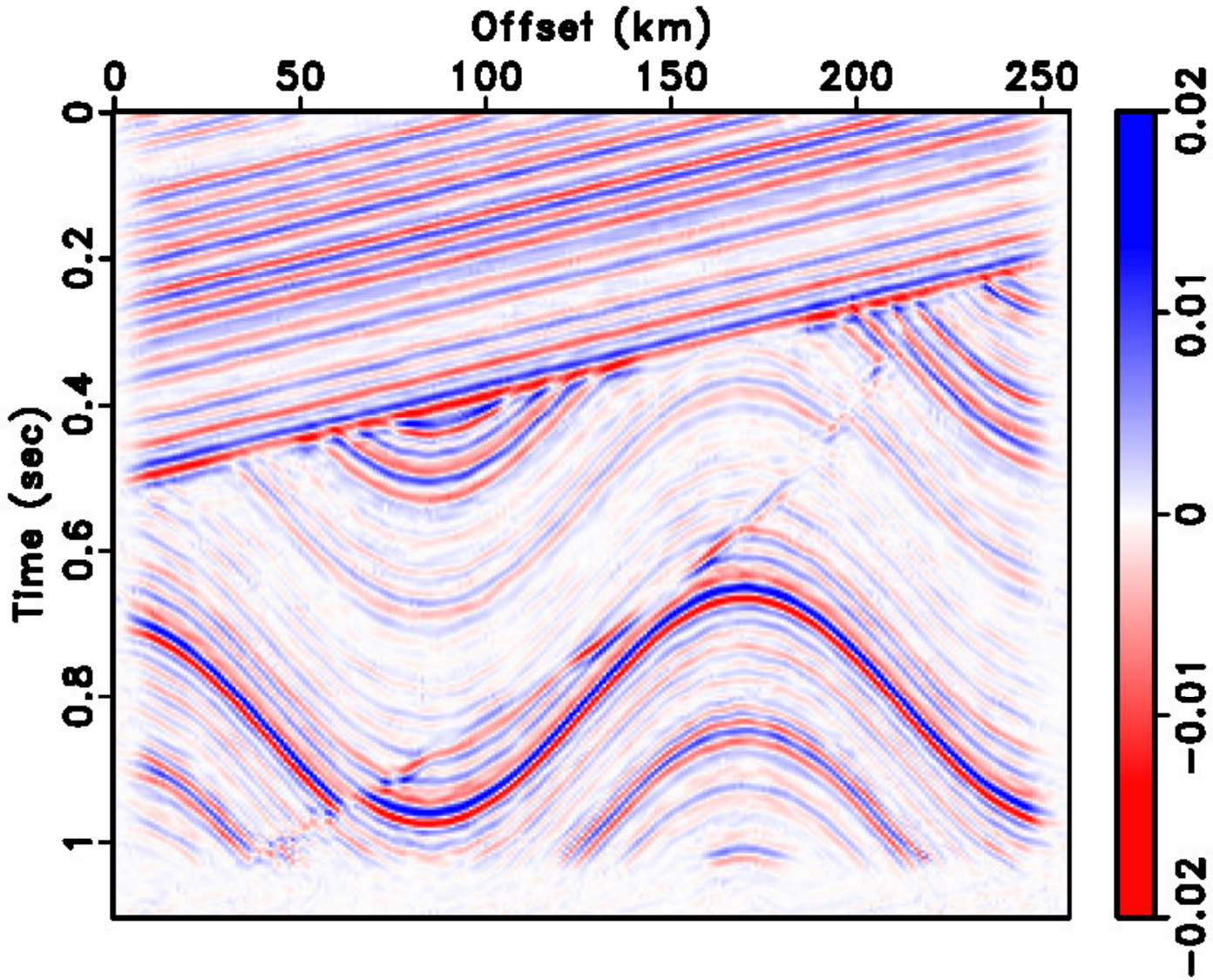
Data


Offset (km)
0
50
100
150
200
250
Time (sec)
0
0.2
0.4
0.6
0.8
1
0.02
0.01
0
−0.01
−0.02

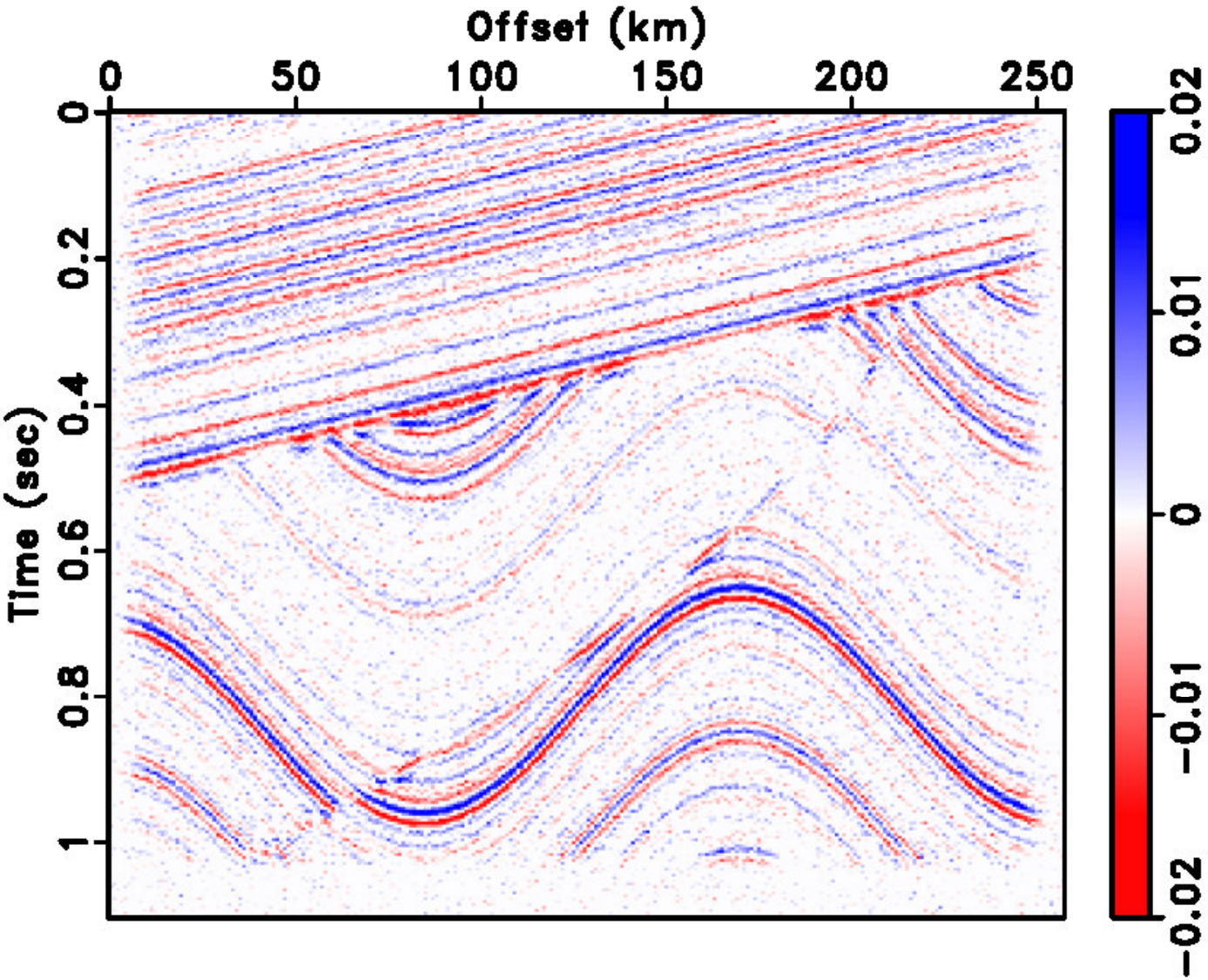
Original Reflectivity



Curvelet Inversion

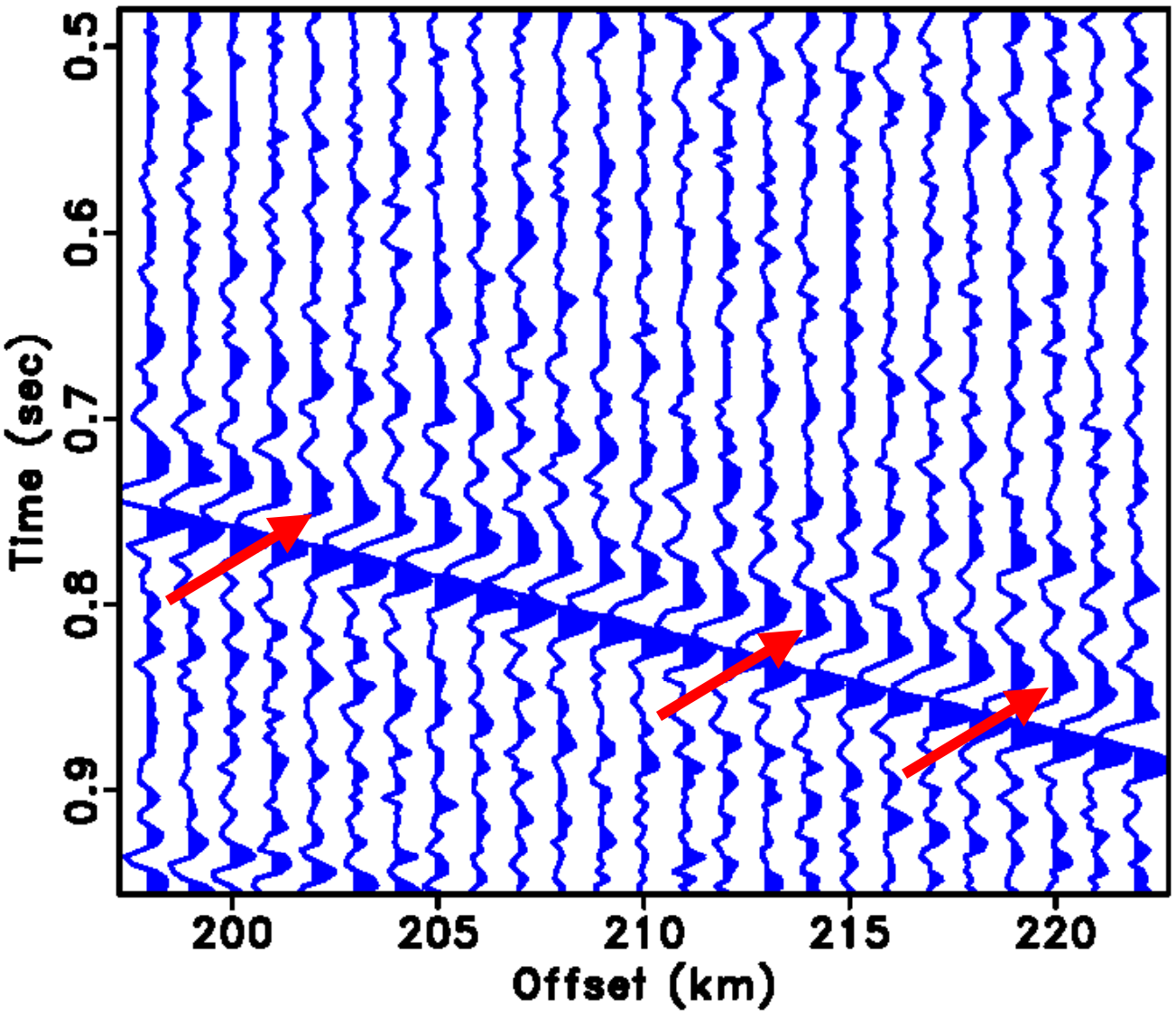


Spiky Inversion

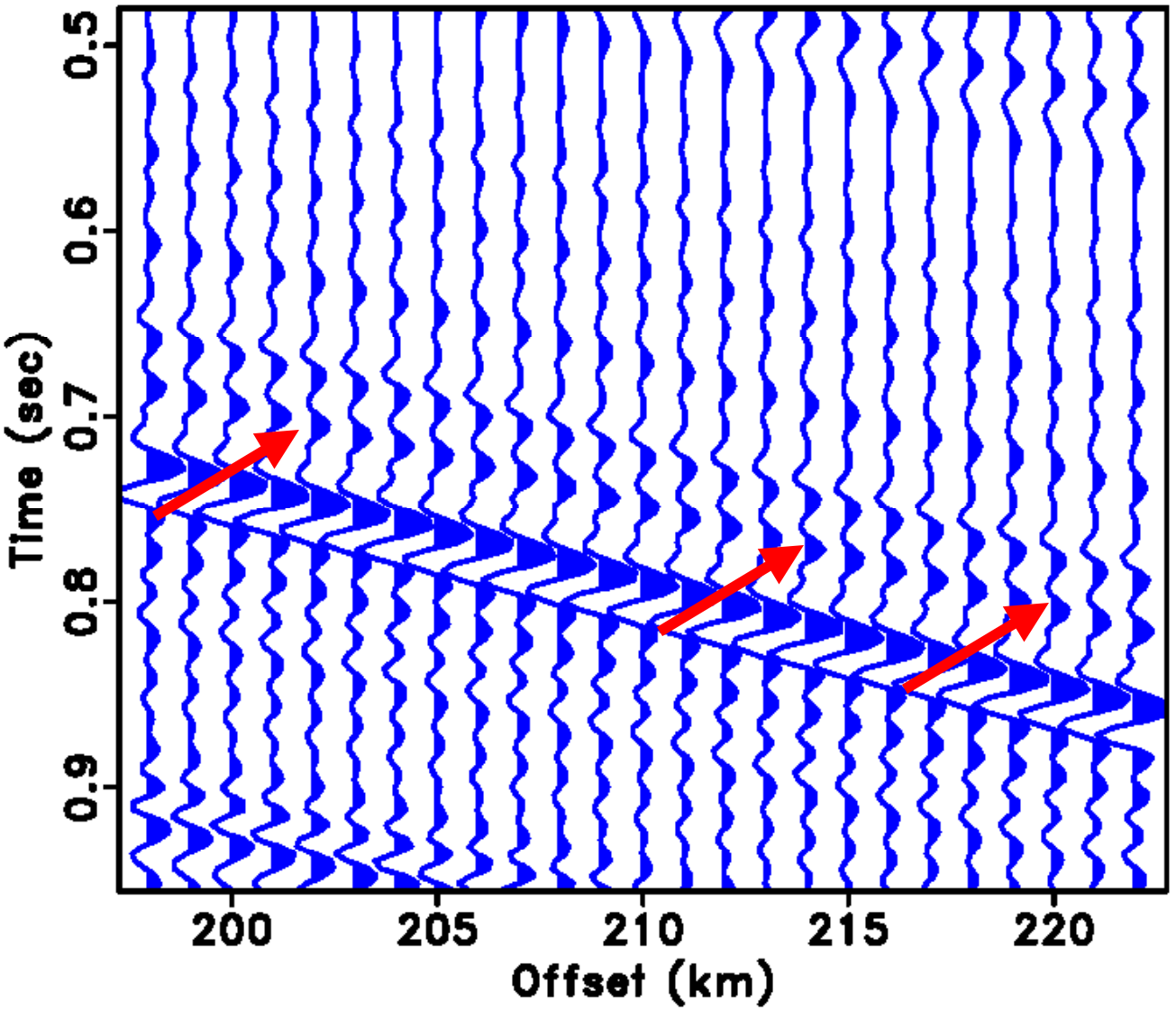


Now Zoom-in wiggle plots!

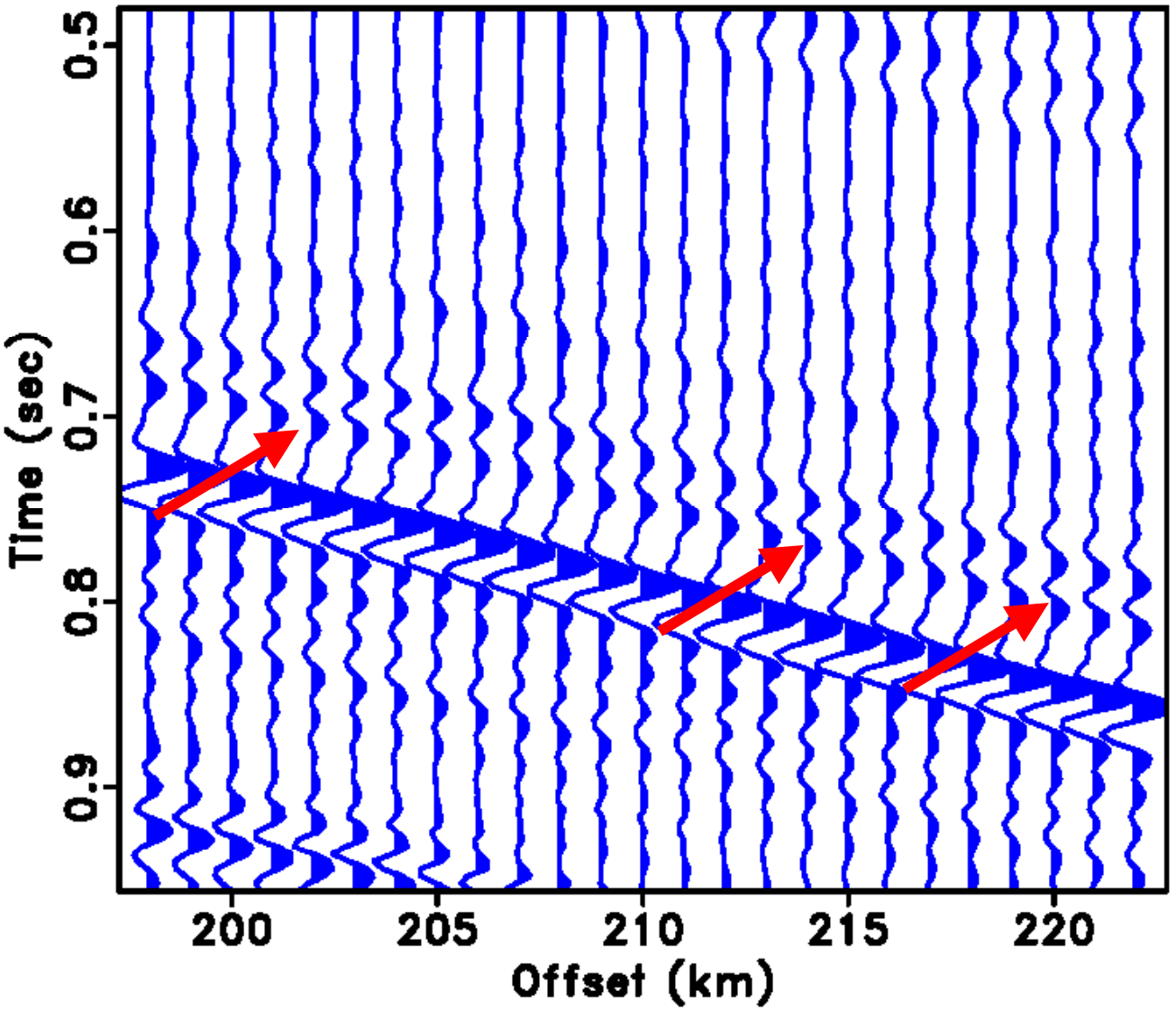
Data



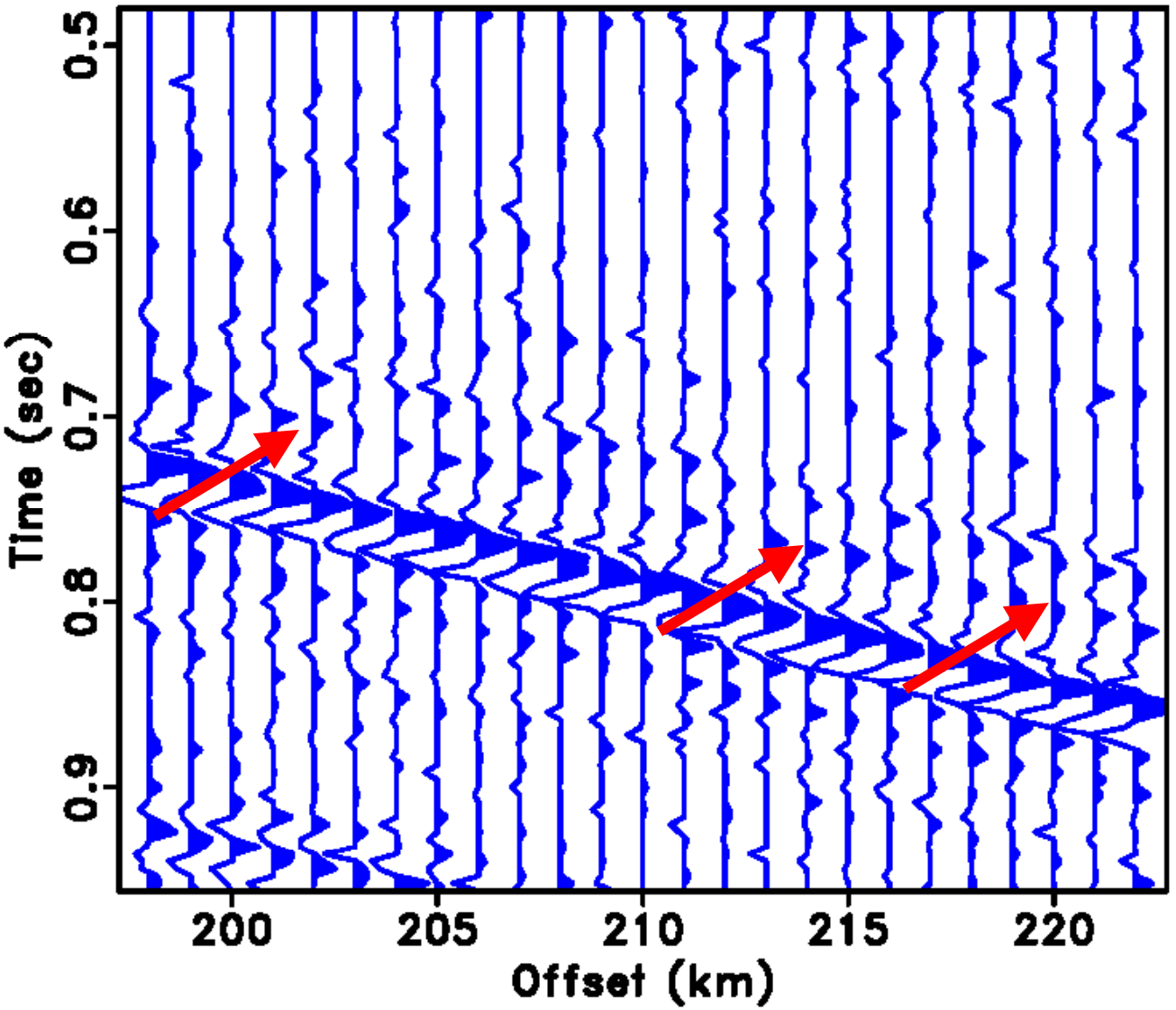
Original Reflectivity



Curvelet Inversion



Spiky Inversion

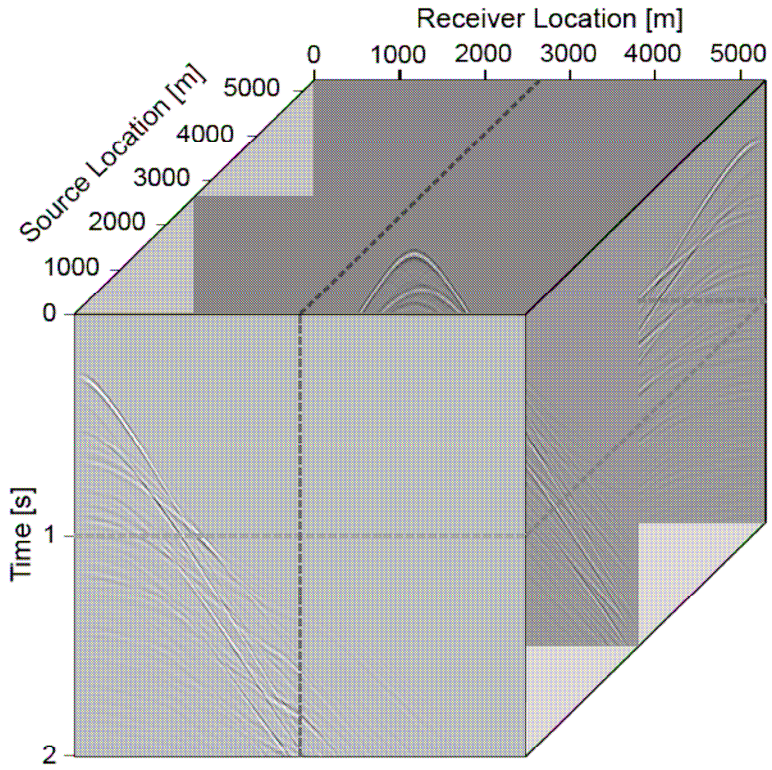


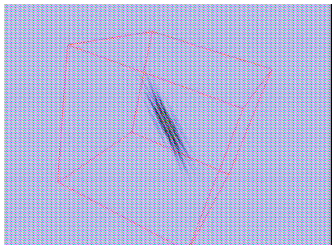
Conclusions

- The algorithm can be applied to estimate non-spiky reflectivity.
- The approach shows promising results by providing better resolution and noise attenuation compared to “Spiky Deconvolution”.
- Unlike trace by trace, our approach exploits coherency between the neighborhood traces.

Future work

- Pre-stack 2-D data can be sorted in shot-time-receiver coordinate to output as a 3-D data cube
- 3-D curvelets can then be used for deconvolution
- 3-D curvelets exploit additional coherency among the different shot gathers.





References

Candes, E., L. Demanet, D. Donoho, and L. Ying, 2006, Fast discrete curvelet transforms: Multiscale Modeling and Simulation, 5, 861–899.

Candes, E., J. Romberg, and T. Tao, 2005, Stable signal recovery from incomplete and inaccurate measurements: Comm. Pure Appl. Math., 59, 1207–1223.

Hennenfent, G., F. J. Herrmann, and R. Neelamani, 2005, Sparseness-constrained seismic deconvolution with Curvelets: Presented at the CSEG National Convention.

Hennenfent, G., E. van den Berg, M. P. Friedlander, and F. J. Herrmann, 2008, New insights into one-norm solvers from the pareto curve: Geophysics.

Herrmann, F. J., 2005, Seismic deconvolution by atomic decomposition: a parametric approach with sparseness constraints: Integr. Computer-Aided Eng., 12, 69–91.

Herrmann, F. J. and G. Hennenfent, 2008, Non-parametric seismic data recovery with curvelet frames: Geophysical Journal International, 173, 233–248.

Oldenburg, D. W., S. Levy, and K. P. Whittall, 1981, Wavelet estimation and deconvolution: Geophysics, 46, 1528–1542.

Taylor, H. L., S. C. Banks, and J. F. McCoy, 1979, Deconvolution with l1-norm: Geophysics, 44, 39–52.

van den Berg, E. and M. P. Friedlander, 2008, Probing the Pareto frontier for basis pursuit solutions: Technical Report TR-2008-01, UBC Computer Science Department. (http://www.optimization-online.org/DB_HTML/2008/01/1889.html).

Acknowledgements

S. Fomel, P. Sava, and the other developers of Madagascar (rsf.sourceforge.net).

E. van den berg and M. P. Friedlander for SPG*l*1 (www.cs.ubc.ca/labs/scl/spgl1).

E. van den Berg, M. P. Friedlander, G. Hennenfent, F. J. Herrmann, R. Saab, and O. Yilmaz for Sparco (www.cs.ubc.ca/labs/scl/sparco/).

Divestco for granting the CSEG Foundation Scholarship.

This work was carried out as part of the SINBAD project with financial support, secured through ITF, from the following organizations: BG, BP, Chevron, ExxonMobil, and Shell. SINBAD is part of the collaborative research & development (CRD) grant number 334810-05 funded by the Natural Science and Engineering Research Council (NSERC).

Thank you!
(Merci Beaucoup)